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SUMMARY

Empirical equations were obtained for electron backscattering coefficients in the energy range 0.6 to 1.8 MeV. These equations give normal-incidence backscattering coefficients and approximate angular-incidence backscattering coefficients as functions of thickness, atomic number, and energy. Materials involved were metals with atomic numbers ranging from 13 to 79. Additional data are presented to allow extrapolation of the formulas to include incident energies between 5.0 keV and 3.0 MeV.

By using the empirical equations, an upper limit was calculated for the fraction of electrons backscattered from the collector to the emitter of a beta radioisotope cell. For the particular proposed design considered, the upper limit indicated that electron backscattering from the collector would cause, with the proper material, only moderate efficiency losses in the cell.

Author

INTRODUCTION

The beta radioisotope cell, which has been proposed as a lightweight electric powerplant (refs. 1 and 2), has a beta source deposited on a metal foil enclosed inside a thin metal collector. Backscattered electrons from the metal surfaces (that is, primary electrons turned around by multiple scatterings), as well as secondary electrons, will detract from the performance of the cell. An analysis of the reduction of efficiency requires a knowledge of the backscattering coefficient, the ratio of backscattered to incident electrons. A theoretical expression for the backscattering coefficient does not exist. Moreover, prior to reference 3, published data on backscattering was scanty in the energy range of a disintegrating beta emitter. In reference 3, backscattering coefficients were determined by the retarding potential method as functions of target thickness and atomic number as well as primary electron energy and angle of incidence. A variety of metals from aluminum to gold were measured in the energy range 0.6 to 1.8 MeV. Thicknesses ranged from 12.7 to 494 milligrams per square centimeter.

It is the purpose of this report to present general empirical relations for the backscattering coefficient. First, an equation for the normal-incidence backscattering coefficient as a function of thickness is fitted to the data. This formula holds for materials with atomic numbers between 13 and 79. Subsequently, relations for the backscattering coefficient as a function of the angle

of incidence of the primary electrons are found. Within their range of validity, then, the empirical equations give the backscattering coefficient for arbitrary angular incidence and arbitrary thickness.

The energy and atomic number dependent parameters of the empirical equations are presented either as formulas or in graphical form. Extrapolation of the equations to include incident electron energies from 5.0 keV to 3.0 MeV is made possible by using additional data from the literature (refs. 4 and 5). In an application to the beta radioisotope cell, an upper limit on the fraction of electrons backscattered from collector to emitter of a spherical cell of specific design is calculated using the empirical equations.

ANALYSIS AND RESULTS

The practical range R of an electron is the maximum distance an electron of given energy can penetrate a given material (neglecting straggling). It plays an important role in backscattering. For materials of thickness t less than $R/2$, many electrons penetrate the material with sufficient energy to repenetrate the same thickness. For materials of thickness equal to $R/2$, none of the electrons that pass through the material have sufficient energy to repenetrate the thickness. Thus, if straggling is neglected, materials of thickness greater than half range stop more electrons, but do not backscatter more electrons than materials of thickness equal to half range. Therefore, the backscattering coefficient η has a constant maximum value for materials of thickness equal to or greater than the half range thickness. The backscattering coefficient for these thicknesses ($t \geq R/2$) is defined as the maximum backscattering coefficient η_{max}.

The practical range of an electron in aluminum is given by an empirical equation from Katz and Penfold (ref. 6) for incident energies between 0 and 2.5 MeV,

$$R = 412\, E^{1.265-0.0954 \ln E} \tag{1}$$

where R is the range in milligrams per square centimeter and E is the kinetic energy in MeV. (A list of symbols is given in appendix A.) With the assumption that the practical range is inversely proportional to the number of electrons in the target material (ref. 7), the practical range in a material of atomic number Z and atomic weight A is given by a generalization of equation (1):

$$R = \frac{A}{Z}\frac{13}{27}\, 412\, E^{1.265-0.0954 \ln E} \tag{2}$$

It was shown in reference 3 that, for a given material, the dependence of the measured backscattering coefficient on target thickness t in milligrams per square centimeter can be described simply in terms of the dimensionless variables η/η_{max} and $2t/R$. At normal incidence, the relative backscattering η/η_{max} is a function only of the relative thickness $2t/R$ and does not depend

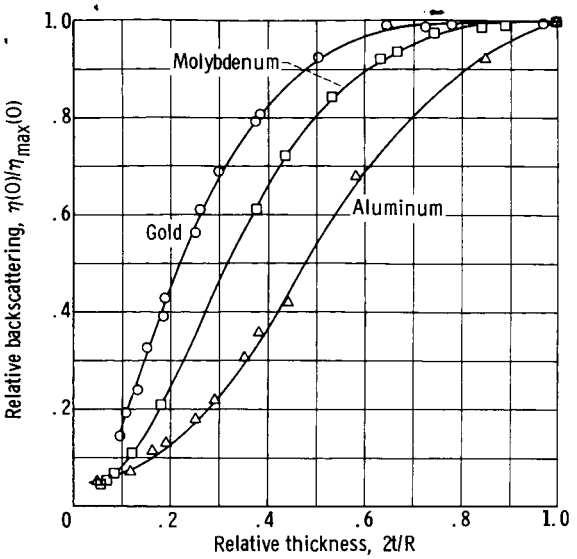

Figure 1. - Relative backscattering at normal incidence as function of relative thickness. (From ref. 3.)

explicitly on primary electron energy from 0.6 to 1.8 MeV. In the following section, this fact is used as the starting point for developing the empirical equations.

Relative Backscattering at Normal Incidence

Curves for the experimentally determined relative backscattering at normal incidence against relative thickness are replotted from reference 3 for aluminum, molybdenum, and gold in figure 1. The corresponding curves for other materials reported in reference 3 are similar.

With the proper constants, the following equation is found to give a good fit to the entire range of these normal incidence data

$$\frac{\eta(0)}{\eta_{max}(0)} = 1 - e^{-\alpha(2t/R)^n} + a \tag{3}$$

The constants, which depend on atomic number (the primary electron energy enters the equation through $\eta_{max}(0)$), are determined by plotting

$$\ln\left\{-\ln\left[1 - \frac{\eta(0)}{\eta_{max}(0)} + a\right]\right\}$$

against $\ln(2t/R)$ for the experimental data. A straight line results whose slope is n and whose intercept is $\ln \alpha$. The parameter a (a small correction) is chosen to bring the values corresponding to the high and low relative thicknesses onto the line. The constants determined in this way for the seven metals appear in table I.

From examining the values in table I, it was found that the following two equations accurately represent α and n and permit interpolation for metals of other atomic numbers

$$n = 2.32 - 8.40\times10^{-3} Z \tag{4}$$

$$\alpha = 0.760 Z^{0.55} \tag{5}$$

The parameter a can be determined only approximately from the present data.

TABLE I. - PARAMETERS OF EQUATION (3) FOR RELATIVE BACKSCATTERING AT NORMAL INCIDENCE

Material	Atomic number, Z	Parameters of equation (3)		
		α	n	a
Aluminum	13	3.13	2.21	0.05
Iron	26	4.81	2.11	.015
Nickel	28	4.90	2.08	.015
Molybdenum	42	6.05	1.94	.005
Silver	47	6.82	1.92	.005
Tantalum	73	8.41	1.70	~0
Gold	79	8.58	1.65	~0

It has a small effect on the equation for most of the range of relative thickness, however, and its value can be interpolated for other Z materials from table I.

The fit of the previous equations can be seen by comparison to the experimental data of reference 3. (Experimental error in the data is estimated to be less than +7 percent and -2 percent of the measured value (ref. 3).) Equation (3) becomes $y = 1 - e^{-x}$ if $[\eta(0)/\eta_{max}(0)] - a$ is replaced by y and $\alpha(2t/R)^n$ is replaced by x. This equation and the values for y and x computed from the experimental data are shown in figure 2. From this figure, it is seen that the empirical equations fit the entire range of the data. The largest difference between the backscattering coefficient from experiment and that from the equations is 0.05 of the maximum backscattering coefficient.

From figure 1 (or eq. (3)), it can be seen that relative backscattering rises faster with the relative thickness for higher Z targets. This is because the higher Coulomb scattering of these metals cause more reversal of direction per penetration of the electrons into the metal. A concomitant effect is more absorption of electrons per relative thickness. Therefore, the trans-

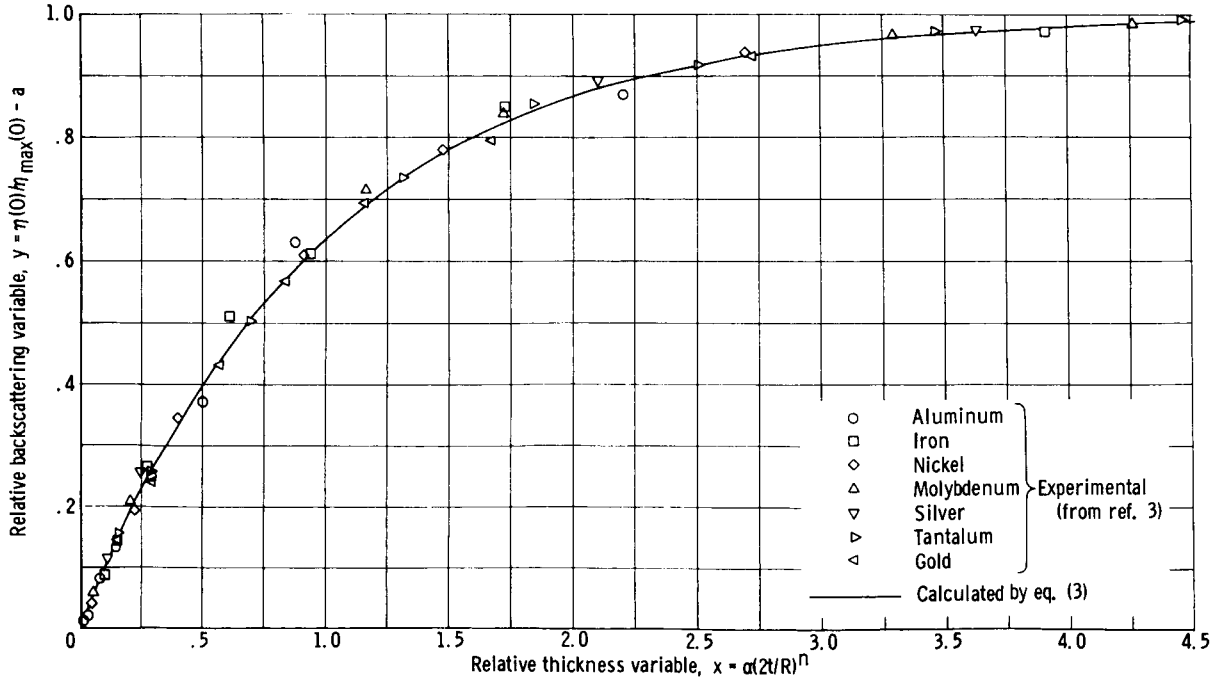

Figure 2. - Comparison of empirical equation and experimental points for relative backscattering variable as function of relative thickness variable.

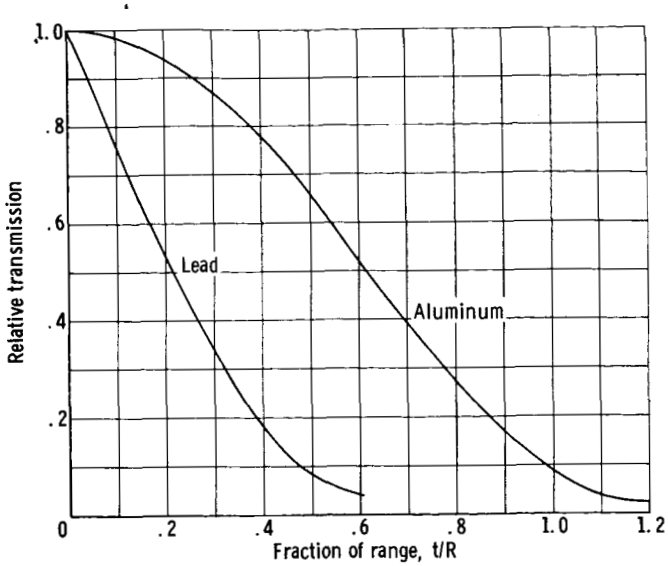

Figure 3. - Relative transmission as function of fraction of range. (Data taken from ref. 8.)

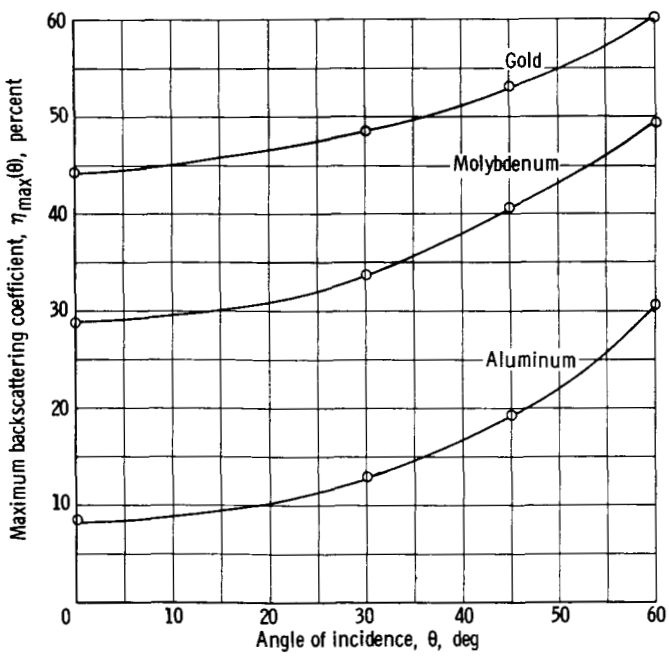

Figure 4. - Maximum backscattering coefficient at 1.2 MeV as function of angle of incidence. (From ref. 3.)

mission of electrons would be expected to be less per relative thickness for higher Z metals. This has been observed by Seliger (ref. 8). Curves of relative transmission as a function of relative thickness for 0.96-MeV electrons incident on aluminum and lead are replotted from Seliger's results in figure 3. They show that the lead transmission decreases much faster than the aluminum transmission.

Seliger also found that curves of electron transmission against relative thickness do not contain an explicit dependence on energy (ref. 8), as is the case for the relative backscattering curves presented herein. The observations on transmission were made for incident electron energies between 0.159 and 0.960 MeV.

As indicated in figure 1, the data of reference 3 do not extend down to zero relative thickness. Moreover, the experimental relative backscattering at the lowest relative thicknesses is leveling off toward a nonzero intercept (see fig. 1), particularly in the case of aluminum. Relative backscattering must, in the limit, approach zero for zero relative thickness. The empirical equations based on the data of reference 3 predict a nonzero intercept, and, therefore, are not applicable for thicknesses very close to zero (values of $2t/R \ll 0.04$).

Maximum Backscattering Coefficient as a Function of Angle of Incidence

The maximum backscattering coefficient increases with the angle of incidence of the impinging electrons. In reference 3, the dependence was measured for three metals, aluminum, molybdenum, and gold. The variation of $\eta_{max}(\theta)$ with θ is shown in figure 4 (from ref. 3) for 1.2-MeV primary electrons.

The following equation was found to describe the 1.2-MeV curves as well

TABLE II. - PARAMETER OF EQUATION (6) FOR

THE MAXIMUM BACKSCATTERING COEFFICIENT

Material	Energy, E, MeV	Range, R, mg/sq cm	B, deg^{-1}	Equation (6) value for $\eta_{max}(90)$, percent
Aluminum	0.8	309.0	3.10×10^{-2}	78.9
	1.2	517.2	3.32	81.8
	1.8	838.4	3.63	81.2
Molybdenum	0.8	340.2	1.77×10^{-2}	80.2
	1.2	569.0	1.88	80.9
	1.8	922.2	2.07	83.1
Gold	0.8	371.0	1.29×10^{-2}	82.5
	1.2	620.6	1.38	83.2
	1.8	1006.0	1.51	83.4

as the 0.8- and 1.8-MeV curves of reference 3:

$$\eta_{max}(\theta) = \eta_{max}(0) \cosh(B\theta) \tag{6}$$

For each material and energy, the parameter B is determined from the maximum backscattering coefficient at normal incidence $\eta_{max}(0)$, and the maximum backscattering coefficient at 60°, the endpoint of the data. The resulting values are shown in table II. It was found that these B values can be represented as a function of the atomic number of the material and of the energy-dependent range of the primary electrons by:

$$B = \frac{1}{\sqrt{Z}} \left(0.333 \times 10^{-4} R + 0.103\right) \tag{7}$$

The values of the maximum backscattering coefficient at 90° which are predicted by equation (6) are shown in table II. The values cluster about 82 percent and appear to be nearly independent of material and energy. This near coincidence of the projected maximum backscattering coefficients at 90° presents an alternate method (equivalent to eq. (7)) for determining the parameter B for other materials. The parameter can be evaluated to within 5 percent from

$$B = \frac{1}{90} \cosh^{-1}\left[\frac{82}{\eta_{max}(0)}\right] \tag{8}$$

Here, $\eta_{max}(0)$, the maximum backscattering coefficient at 0°, carries the dependence on atomic number and energy (or range).

The fit of equations (6) and (7) can be seen by plotting $\eta_{max}(\theta)/\eta_{max}(0) = \cosh x$ and evaluating the data points for $\eta_{max}(\theta)/\eta_{max}(0)$

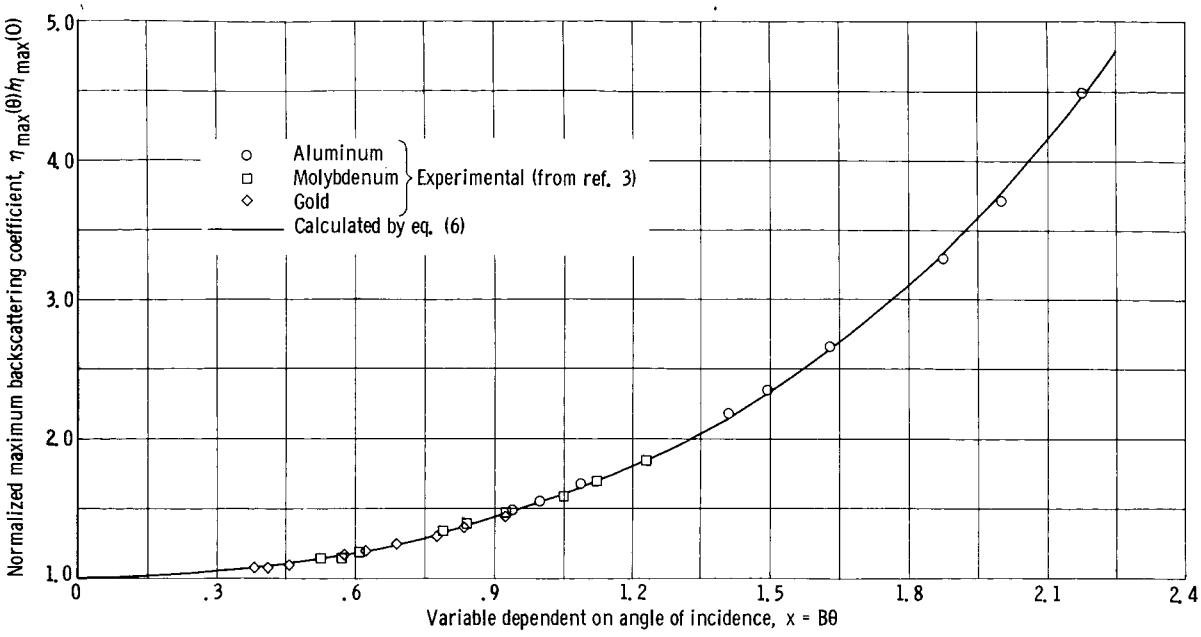

Figure 5. - Comparison of empirical equation and experimental points for normalized maximum backscattering coefficient as function of variable dependent on angle of incidence.

and $x = B\theta$. This plot in figure 5 shows that the equations give $\eta_{max}(\theta)/\eta_{max}(0)$ to within 2.5 percent of the measured value.

Equations (3) and (6) have been shown to fit the experimental data only in the energy range 0.6 to 1.8 MeV, but it is reasonable to expect that they can be extended to both somewhat higher and lower energies. Their use at higher and lower energies requires knowledge of the maximum backscattering coefficient at normal incidence. A graph of this coefficient against energy is presented

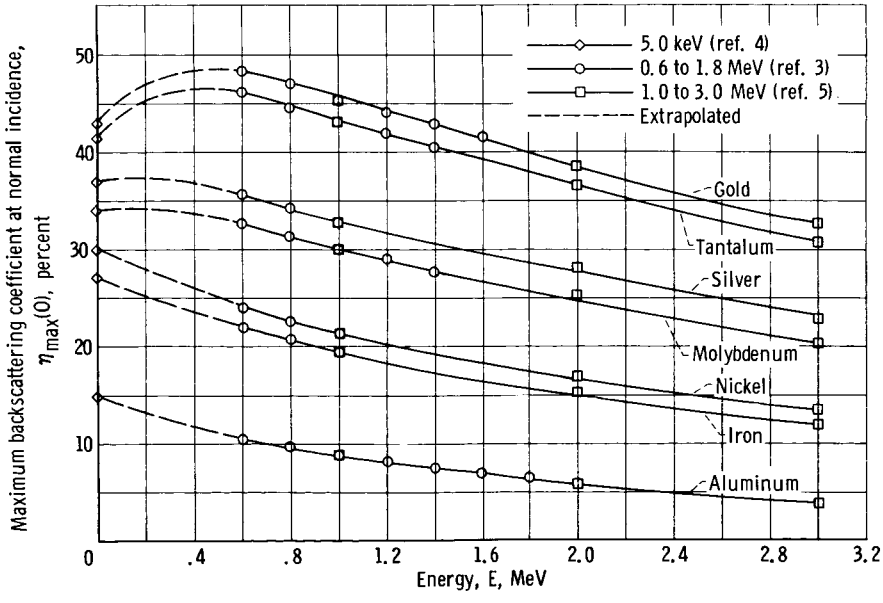

Figure 6. - Maximum backscattering coefficient at normal incidence as function of energy.

in figure 6. The data of reference 3 are combined with measurements by Palluel (ref. 4) and values from the crossplots of Wright and Trump (ref. 5). Where comparable, the values of reference 5 were slightly lower than those of reference 3. (The backscattering coefficients, expressed in percent, differed by between 1.2 and 2.8 percent.) Since corrections for secondary emission from the collecting electrode were not applied in reference 5, it is the most likely reason for the difference. Accordingly, the values of reference 5 have been corrected in absolute value in figure 6 by requiring coincidence with the data of reference 3 at 1.0 MeV.

Relative Backscattering at Angular Incidence for Arbitrary Thickness

For angular incidence of the primary electrons, the relative backscattering $\eta(\theta)/\eta_{max}(\theta)$ is again dependent only on the relative thickness and not explicitly on the electron energy. This is shown in figure 7 for molybdenum (data taken from ref. 3) at angles of 30°, 45°, and 60°. The curve for relative backscattering at normal incidence ($\theta = 0$) is shown for comparison. The relative backscattering increases faster with relative thickness the larger the angle of incidence. Molybdenum is the only material for which angular data with the target thickness less than $R/2$ is available.

It is seen from figure 7 that for molybdenum the curves at various angles of incidence have the same form as the one for relative backscattering at normal incidence. The curves can be fit by a generalization of equation (3) simply by multiplying n by a function of the angle of incidence

$$\frac{\eta(\theta)}{\eta_{max}(\theta)} = 1 - e^{-\alpha(2t/R)^{nC(\theta)}} + a \qquad (9)$$

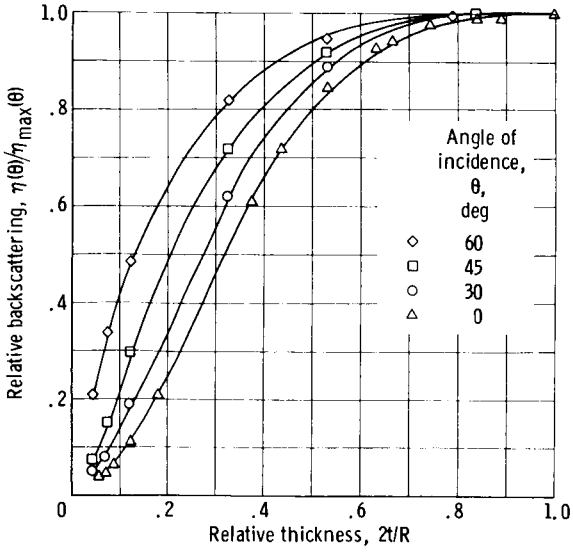

Figure 7. - Relative backscattering at angles of incidence as function of relative thickness for molybdenum. (Data taken from ref. 3.)

TABLE III. - PARAMETER OF EQUATION (9) FOR RELATIVE BACKSCATTERING AT ANGULAR INCIDENCE

[Material, molybdenum.]

Angle of incidence, θ, deg	$C(\theta)$
0	1.00
30	.825
45	.716
60	.551

The values for $C(\theta)$ are determined as in the case of determining n at normal incidence and appear in table III. The values of α, n, and a are as before (see table I, p. 4).

If $(2t/R)^{C(\theta)}$ is set equal to x and if a, n, and α are given their values for molybdenum in equation (9), the following equation results:

$$\frac{\eta(\theta)}{\eta_{max}(\theta)} = 1.005 - e^{-6.05x^{1.94}}$$

This equation is plotted in figure 8, and the data points evaluated at $\eta(\theta)/\eta_{max}(\theta)$ and x are compared to it. The empirical equation gives the relative backscattering for molybdenum to within 0.02 of the maximum backscattering coefficient at the given angle, according to the figure. A plot of $C(\theta)$

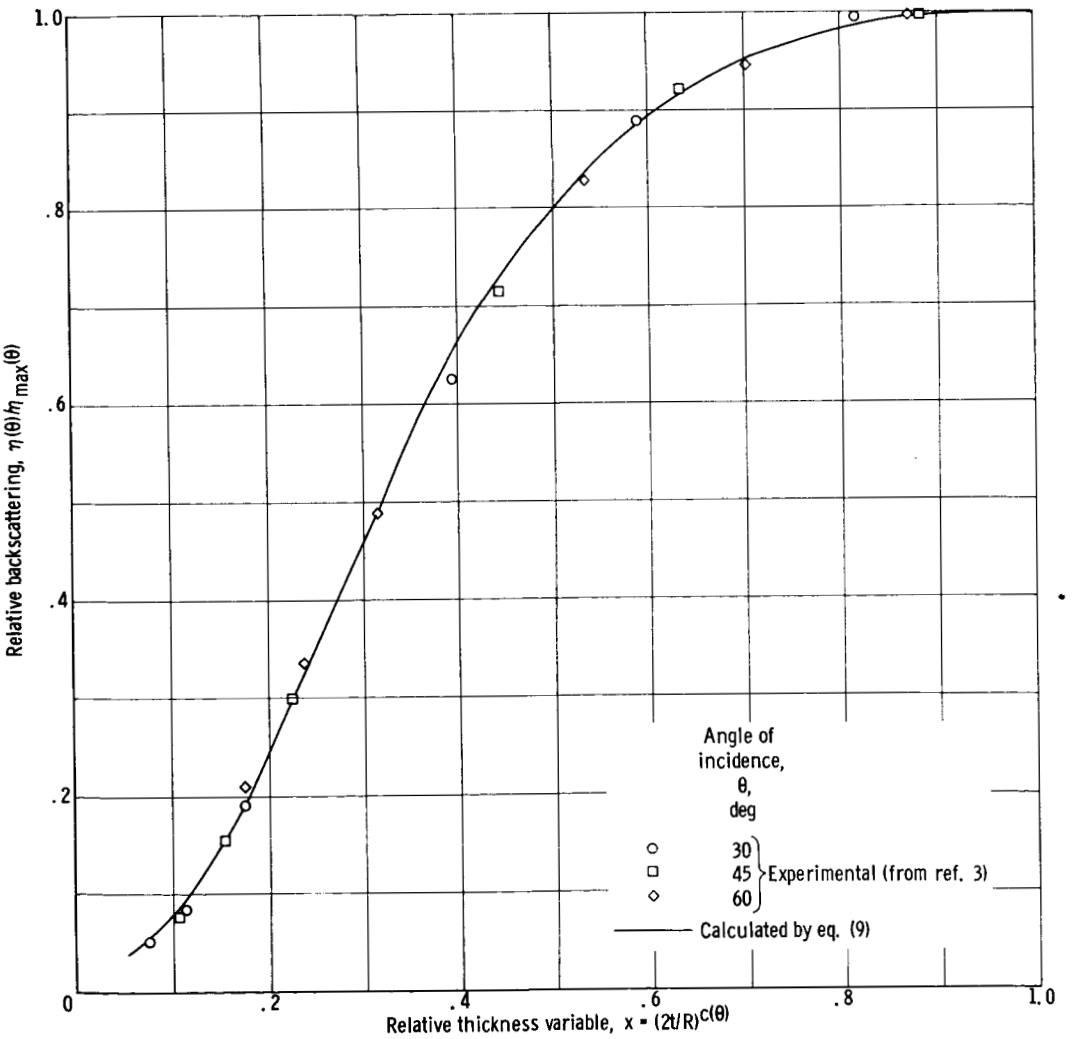

Figure 8. - Comparison of empirical equation and experimental data for relative backscattering at angles of incidence as function of relative thickness variable for molybdenum.

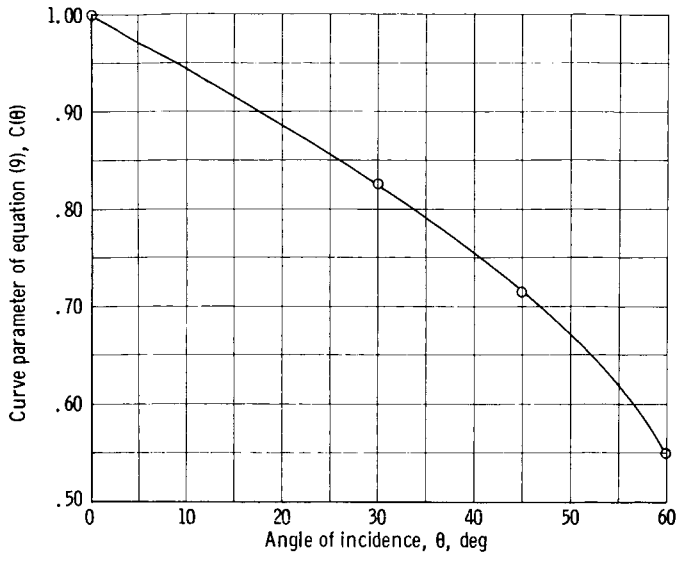

Figure 9. - Curve parameter of equation (9) as function of angle of incidence for molybdenum.

for molybdenum is shown in figure 9 to facilitate interpolation to other angles of incidence.

Equation (9) has been shown to fit the data for molybdenum. For other materials, the parameter $C(\theta)$ cannot be determined from existing data. However, equation (9) can be used for other materials by assuming that $C(\theta)$ is independent of the atomic number. Then, by using the α and n of equations (4) and (5) and the a of table I, as well as the $C(\theta)$ for molybdenum, the relative backscattering at angular incidence can be approximated for materials from aluminum to gold by equation (9). More data are needed to determine the accuracy of this procedure.

Backscattering From the Collector of a Beta Radioisotope Cell

The preceding empirical equations can be used to calculate the effect of electron backscattering from the collector upon the performance of a beta radioisotope electric generator. The beta cell (refs. 1 and 2) is a power generation concept in which a central electrode, the emitter, is coated with a radioisotope which emits electrons that travel across a vacuum gap to an insulated outer electrode, the collector. The kinetic energy of the electrons is converted to electric potential energy by building up a high voltage between the two electrodes. Any backscattering of electrons impinging on the collector back to the emitter reduces the efficiency of the cell. Collector secondary electrons (almost always less than 50 eV in energy) also reduce cell efficiency. Secondary electrons, however, can be suppressed by a screen with a moderate bias. Such a suppressor screen would not work for the high-energy backscattered electrons.

An upper limit for the fraction of backscattered electrons can be calculated by assuming that all the electrons backscattered from the collector travel back to the emitter. To find this upper limit a knowledge of the distribution in angle and energy of the impinging electrons is required. This information is calculated in reference 2 for a spherical, cerium 144 beta cell design, the model assumed herein. Details of the use of the empirical equations for the spherical cell calculations appear in appendix B. Of course, a completely rigorous and detailed calculation of the effects of backscattering would require consideration of additional factors not covered in appendix B, such as backscattering from the emitter surfaces themselves.

The cylindrical design of reference 2 could also have been used as a model

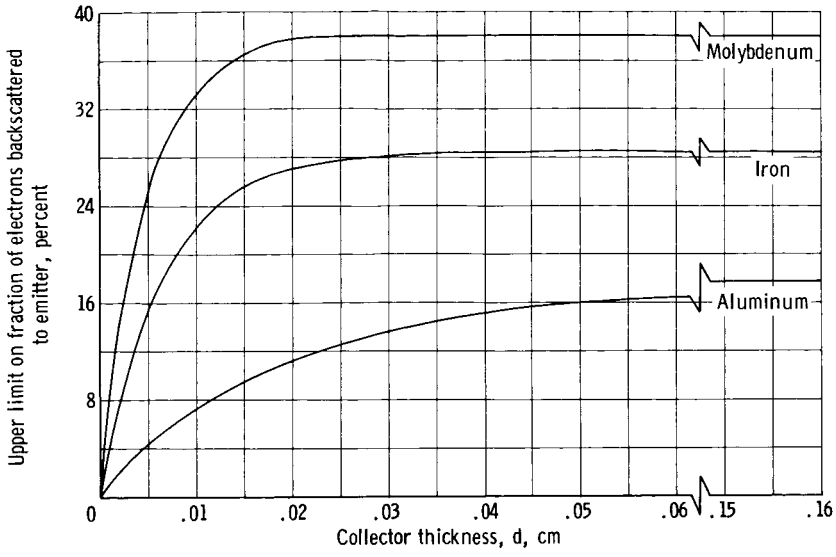

Figure 10. - Upper limit for electrons backscattered from collector of spherical, beta cell as function of collector thickness.

for the present calculation. Backscattering from the collector to the emitter would be similar because the higher average energy of the impinging electrons would reduce the backscattering, while their higher average angle of incidence would increase it.

Calculated results for the spherical model are shown in figure 10. The upper limit on the fraction of electrons backscattered from collector to emitter over total number of electrons hitting the collector is plotted against the collector thickness for three materials. This upper limit increases very rapidly with the collector thickness and is more than one-half the maximum value at a thickness of 0.015 centimeter (0.006 in.) for all three materials. For aluminum, the upper limit of particles backscattered has the lowest value and the slowest rate of increase with thickness.

The reduction in efficiency of the beta cell due to collector backscattering, as a percent of the calculated efficiency is simply equal to the fraction of particles backscattered (ref. 2). The efficiency of the beta cell considered is 26.0 percent according to reference 2. For a 0.010-centimeter aluminum collector, as an example, the upper limit on the fraction of particles backscattered to the emitter is 7.4 percent. Therefore, the upper limit on the reduction of calculated efficiency is 7.4 percent times 26.0 percent or 1.9 percent. Backscattering of electrons from the collector can, therefore, reduce the efficiency in this case at worst from 26.0 percent to about 24.1 percent. Similarly, for a thick aluminum collector (greater than 0.150 cm), taking into account, in addition, the dependency on the number of sum intervals used in the evaluation (see appendix B), the backscattering can reduce the efficiency at most from 26.0 percent to about 20.9 percent. For the particular proposed design considered, then, electron backscattering from an aluminum collector would cause only moderate efficiency losses.

CONCLUDING REMARKS

Empirical equations for electron backscattering coefficients were found as functions of metal thickness and primary electron angle of incidence. Backscattering coefficients for normal incidence with an arbitrary thickness material, and for angular incidence with a material thicker than half range, are accurately given for a wide range of atomic numbers. Moreover, backscattering coefficients for angular incidence and an arbitrary thickness material are given for the same range of atomic numbers to the approximation that the molybdenum relative backscattering dependence on angle of incidence is identical with that of other metals.

The empirical equations were applied to a spherical, beta radioisotope cell of specific design. An upper limit on the fraction of electrons incident on the collector that are backscattered to the emitter was computed by utilizing previously calculated angle and energy distributions (ref. 2). Values of the limit were lower for aluminum collectors than for iron or molybdenum collectors by a factor of at least $1\frac{1}{2}$ or 2, respectively. With aluminum, the cell efficiency can be reduced by collector backscattering from 26.0 percent to about 20.9 percent for a thick collector and from 26.0 percent to about 24.1 percent for a 0.010-centimeter collector.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, April 12, 1965.

APPENDIX A

SYMBOLS

A	atomic weight
a	parameter in relative backscattering equation
B	parameter in maximum backscattering coefficient equation, deg^{-1}
C	parameter in relative backscattering equation
d	thickness, cm
E	kinetic energy of incident electron, MeV
$\overline{E}$	average kinetic energy of incident electron, MeV
E_i	kinetic energy of electron at midpoint of i^{th} interval, MeV
F_i	fraction of electrons in i^{th} interval
n	parameter in relative backscattering equation
R	practical range of electron, mg/sq cm
t	thickness, mg/sq cm
x	independent variable
y	dependent variable
Z	atomic number
α	parameter in relative backscattering equation
η	backscattering coefficient (ratio of electrons backscattered to electrons incident), percent
η_{max}	maximum backscattering coefficient, percent
θ	angle of incidence, deg
$\overline{\theta}$	average angle of incidence, deg
θ_i	angle of incidence at midpoint of i^{th} interval, deg

APPENDIX B

CALCULATION OF BACKSCATTERING FROM THE COLLECTOR FOR A SPHERICAL, BETA RADIOISOTOPE CELL

An upper limit for the fraction of particles backscattered from the collector to the emitter for a spherical, beta cell can be obtained by assuming that all the collector backscattered electrons return to the emitter. The fraction of collector backscattered electrons, given the angle and energy distribution of impinging electrons, can be determined as a function of collector thickness and material. It is necessary to assume

(1) That the empirical equations developed in this report apply over the incident electron energy range of 0 to 2.1 MeV

(2) That for materials other than molybdenum, the parameter $C(\theta)$ of equation (9) is the same as the $C(\theta)$ for molybdenum

The angle and energy distributions of impinging electrons for a specific spherical, cerium 144 beta cell design are given in reference 2. (The complete cell design is given in that reference also.) For the calculation, the impinging electrons are broken up into five energy groups between 0 and 2.1 MeV. The fraction of electrons in each group is shown in table IV along with the average energy $\bar{E}$ and the average angle of incidence $\bar{\theta}$ of the electrons. The average energy for an electron group is defined as

$$\bar{E} = \frac{\sum_i F_i E_i}{\sum_i F_i} \tag{B1}$$

with the sum over 0.1-MeV energy intervals from the minimum to the maximum energy of the group. In equation (B1), E_i is the energy at the midpoint of each interval and F_i the fraction of electrons in the interval. Likewise, for a given energy group,

$$\bar{\theta} = \frac{\sum_i F_i \theta_i}{\sum_i F_i} \tag{B2}$$

where the sum is over 5.0-degree angle-of-incidence intervals from 0 to 90 degrees and θ_i is the angle of incidence at the midpoint of each interval. For

TABLE IV. - ELECTRONS INCIDENT ON THE COLLECTOR

OF A SPHERICAL, CERIUM 144 BETA CELL

Energy range, E, MeV	Fraction of electrons in energy range, percent	Average energy, $\bar{E}$, MeV	Average angle, $\bar{\theta}$, deg
0 to 0.4	16.25	0.275	51.0
0.4 to .8	34.65	.598	40.5
.8 to 1.2	26.89	.987	33.0
1.2 to 1.6	16.06	1.38	29.2
1.6 to 2.1	6.14	1.76	25.9

a collector of given material and given thickness, the fraction of electrons backscattered can be calculated for each energy group from the information in table IV, and the final result achieved by summing over the five energy groups.

Because backscattering varies with the angle of incidence, the use of the average angle of incidence $\bar{\theta}$ for an entire energy group can lead to error. A check on the calculation method was made using the case of a thick aluminum collector. It was found that dividing the incident electrons into angle-of-incidence groups and using average values for each of these groups produces only small changes in the final results. For a thick aluminum collector, backscattered fraction totals 17.8 percent with a single angle-of-incidence group and 19.1 and 19.5 percent with three and six angle-of-incidence groups, respectively. The beta cell efficiency is reduced by the electron backscattering correspondingly, from 26.0 (no backscattering) to 21.4 percent with one group and to 21.0 and 20.9 percent with three and six groups, respectively. Therefore, the efficiency calculation appears to converge to a value of about 20.9 percent for a thick aluminum collector, and the use of only a single angle-of-incidence group is justified.

TABLE V. - ELECTRONS BACKSCATTERED FROM COLLECTOR

OF A SPHERICAL, CERIUM 144 BETA CELL

Collector material	Collector thickness, d, cm	Fraction of total incident electrons backscattered from collector, percent					
		$0 < E \leq 0.4$ MeV	$0.4 < E \leq 0.8$ MeV	$0.8 < E \leq 1.2$ MeV	$1.2 < E \leq 1.6$ MeV	$1.6 < E \leq 2.1$ MeV	Total
Aluminum	0.005	2.84	1.00	0.30	0.11	0.03	4.3
	.010	4.42	2.25	.51	.16	.04	7.4
	.020	4.71	4.75	1.18	.32	.07	11.0
	.030	4.71	6.27	1.95	.53	.11	13.6
	.060	4.71	6.83	3.55	1.24	.27	16.6
	.120	4.71	6.83	3.82	1.85	.53	17.7
Iron	0.001	2.99	0.94	0.22	0.08	0.02	4.2
	.004	6.39	5.41	1.28	.34	.07	13.5
	.010	6.39	10.22	4.11	1.26	.26	22.2
	.025	6.39	10.67	6.72	3.13	.85	27.8
	.054	6.39	10.67	6.72	3.52	1.19	28.5
Molybdenum	0.0005	2.78	0.90	0.18	0.06	0.01	3.9
	.002	7.02	5.23	1.26	.35	.07	13.9
	.004	7.51	10.08	3.28	.98	.20	22.0
	.010	7.51	14.03	7.85	3.07	.74	33.2
	.025	7.51	14.03	9.55	5.12	1.65	37.9
	.044	7.51	14.03	9.55	5.20	1.81	38.1

An example of the calculation for a thin aluminum collector using an average value of the angle of incidence for an energy group follows. The average energy for the group of particles with energies between 1.2 and 1.6 MeV is 1.38 MeV (from table IV). By equation (1), the average range for this average energy is 613 milligrams per square centimeter. The maximum backscattering coefficient at normal incidence for this average energy is 7.5 percent according to figure 6 (p. 7). The maximum backscattering coefficient at the average angle of incidence, 29.2°, is given by equation (6) as 11.5 percent. Now consider a specific collector thickness of 0.030 centimeter, which is a relative thickness, $2t/R$, of 0.264. For this relative thickness the relative backscattering is determined from equation (9), with values of n, α, and a for aluminum taken from table I (p. 4) and the value of $C(29.2^{\circ})$ from figure 9 (p. 10). The relative backscattering is computed as 0.287. The backscattering coefficient is therefore 0.287 times 11.5 percent or 3.31 percent. The fraction of electrons backscattered is the percentage times the fraction of electrons incident in this energy group, which is 16.06 percent (from table IV). Therefore, 0.53 percent of the total particles incident on the collector are backscattered because of the energy group 1.2 to 1.6 MeV. Upon considering the other energy groups and summing, the total fraction of electrons backscattered from the collector is 13.6 percent for this case.

For other representative thicknesses, table V gives, for each energy range, the fraction of total incident electrons that are backscattered from the collector. Electrons with incident energies between 0.4 and 0.8 MeV make the largest contribution to the backscattering. Table V also gives the total fraction backscattered, that is, the sum over the energy groups. With the assumption that all of these electrons return to the emitter, figure 10 (p. 11) is a plot of the upper limit of the fraction of electrons backscattered to the emitter as a function of collector material and thickness for the given design.

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