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ORBITAL PARAMETERS FOR THE OGO-E GEO-CORONAL HYDROGEN EXPERIMENT

BY

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ABSTRACT

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This document presents and applies methods for obtaining certain necessary orbital and spacecraft parameters for meaningful data analysis of the geo-coronal hydrogen measurement experiment (Experiment No. 22) on the fifth in Orbiting Geophysical Observatory Series, namely the OGO-E.

Techniques are presented for determining as a function of time: 1) orbit inclination with respect to the ecliptic plane, 2) the angle between the orbit's angular momentum vector and the sun's position vector, 3) the half angular dimension of the earth viewed from the satellite, 4) the solar array angle and, 5) the angle between the projection of the sun on the x-y plane of the experiment's coordinate system and the x-axis of that system.

The calculations were made using a numerical integration general purpose interplanetary trajectory program (ITEM). Consequently, the changes in the orbit due to perturbations are reflected in the calculations of the quantities mentioned above.

The results of computer runs are presented in this document.

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NOMENCLATURE

a	Semi-major axis.
a_e	Radius of the earth.
C_E	The angle between the OPEP x_E-axis and the projection of the sun vector on the OPEP x_E-y_E plane. (See Figure 2.)
e	Eccentricity.
i	Inclination.
$\bar{i}, \bar{j}, \bar{k}$	Unit vectors along the x, y, z axes of the equatorial inertial coordinate system.
$\bar{i}', \bar{j}', \bar{k}'$	Unit vectors along the x', y', z' axes of the inertial ecliptic coordinate system.
$\bar{i}_b, \bar{j}_b, \bar{k}_b$	Unit vectors along the x_b, y_b, z_b axes of the satellite fixed coordinate system.
$\bar{i}_E, \bar{j}_E, \bar{k}_E$	Unit vectors along the x_E, y_E, z_E axes of the OPEP coordinate system.
$\bar{i}_p, \bar{j}_p, \bar{k}_p$	Unit vectors along the x_p, y_p, z_p axes of the solar array coordinate system.
M	Mean anomaly.
$\bar{N}$	Unit angular momentum vector.
$\bar{r}$	Position vector of the satellite.
S'	The angle between normal to the orbit and the position vector of the sun relative to the earth.
$\bar{S}_{E_p}$	Sun vector projection on the OPEP x_E-y_E plane.
$\bar{U}_{SE}$	Unit vector from earth to sun in the x, y, z coordinate system.
x, y, z	Axes of the inertial equatorial coordinate system.
x', y', z'	Axes of the inertial ecliptic coordinate system.

x_b, y_b, z_b	Axes of the satellite fixed coordinate system.
x_E, y_E, z_E	Axes of the OPEP coordinate system.
x_p, y_p, z_p	Axes of the solar array system.

Greek Symbols

δ	Half angular dimension of the earth viewed from the satellite.
ϵ	The mean obliquity = $23^{\circ}.4437$ for 1966 (Ref. 2).
ξ	The orbit inclination with respect to the ecliptic plane.
φ_p	The solar array angle (Fig. 2).
Ψ_E	The OPEP angle, the angle between the x_E and x_b axes.
ω	Argument of perigee.
Ω	Right ascension of the ascending node.

Matrices

A	A transformation matrix defined by equation 8.
B	A transformation matrix defined by equation 9.
C	= B A, see equation 10.
X	A vector with components in the inertial equatorial coordinate system.
X_b	A vector with components in the satellite fixed coordinate system.
X_E	A vector with components in the OPEP coordinate system.
X_{E_S}	Position vector of the sun in the OPEP system.
X_S	Position vector of the sun in the inertial equatorial coordinate system.

ORBITAL PARAMETERS
FOR THE OGO-E
GEO-CORONAL HYDROGEN EXPERIMENT

INTRODUCTION

The Orbiting Geophysical Observatory (OGO) satellites form a family of satellites each designed to carry up to fifty experiments. The OGO-E is one in this series. The discussion here concerns an experiment (No. 22) on the OGO-E which will make optical measurements of the thickness of the geo-coronal hydrogen. It is theorized that hydrogen is escaping from the earth. The purpose of the experiment is to determine the gradient of the hydrogen population around the earth and also to investigate the interaction caused by extra-terrestrial effects.

This document gives methods for determining the following quantities useful in post flight data analysis:

1. The angle ξ between the ecliptic plane and the orbit plane (Fig. 1).
2. The angle S' between the normal to the orbit plane and a vector from the earth to the sun (Fig. 1).

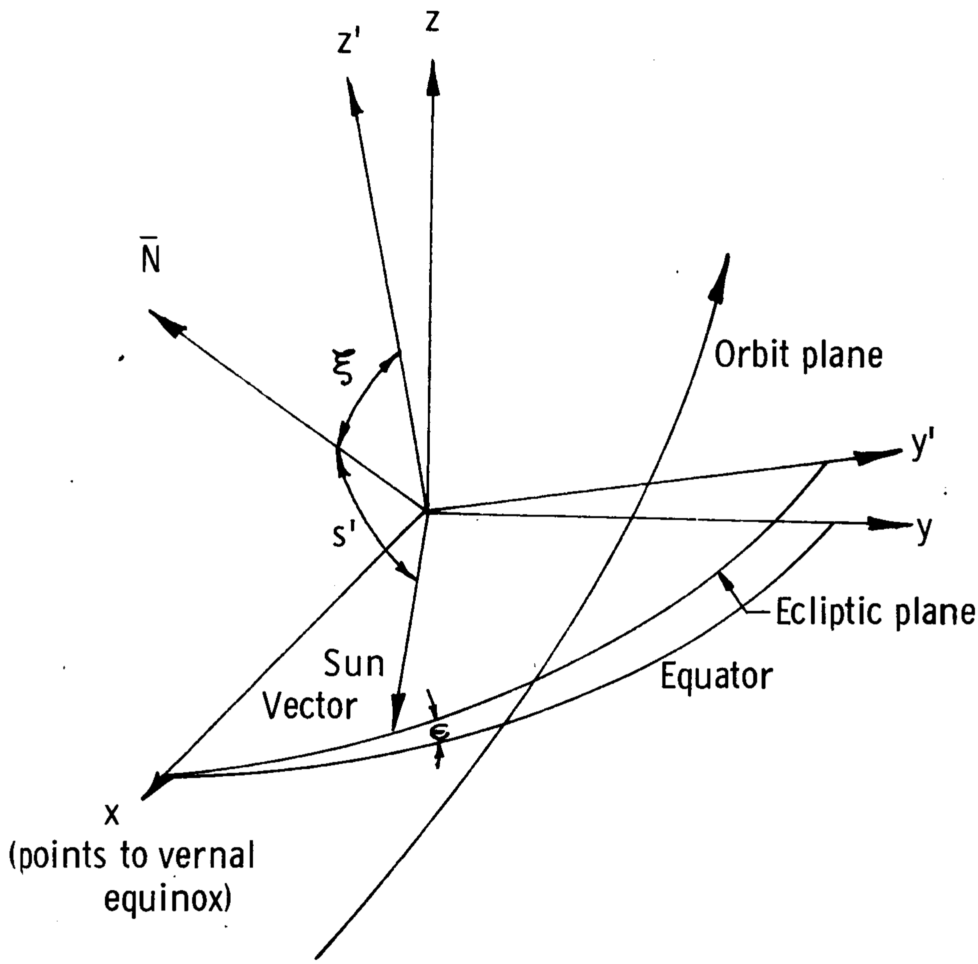

Figure 1.
Angles ξ and S'

3. The half angular dimension δ of the earth as viewed from the satellite as a function of time.

4. The solar array angle φ_p between the normal to the sun-lit side of solar array and the y_b-axis of the satellite fixed coordinate system (Fig. 2).

5. The angle C_E between the projection of the sun vector on the x_E- y_E plane of the orbit plane experiment package (OPEP) coordinate system and the x_E-axis of that coordinate system. The OPEP coordinate system, x_E, y_E, z_E is shown on Figure 2.

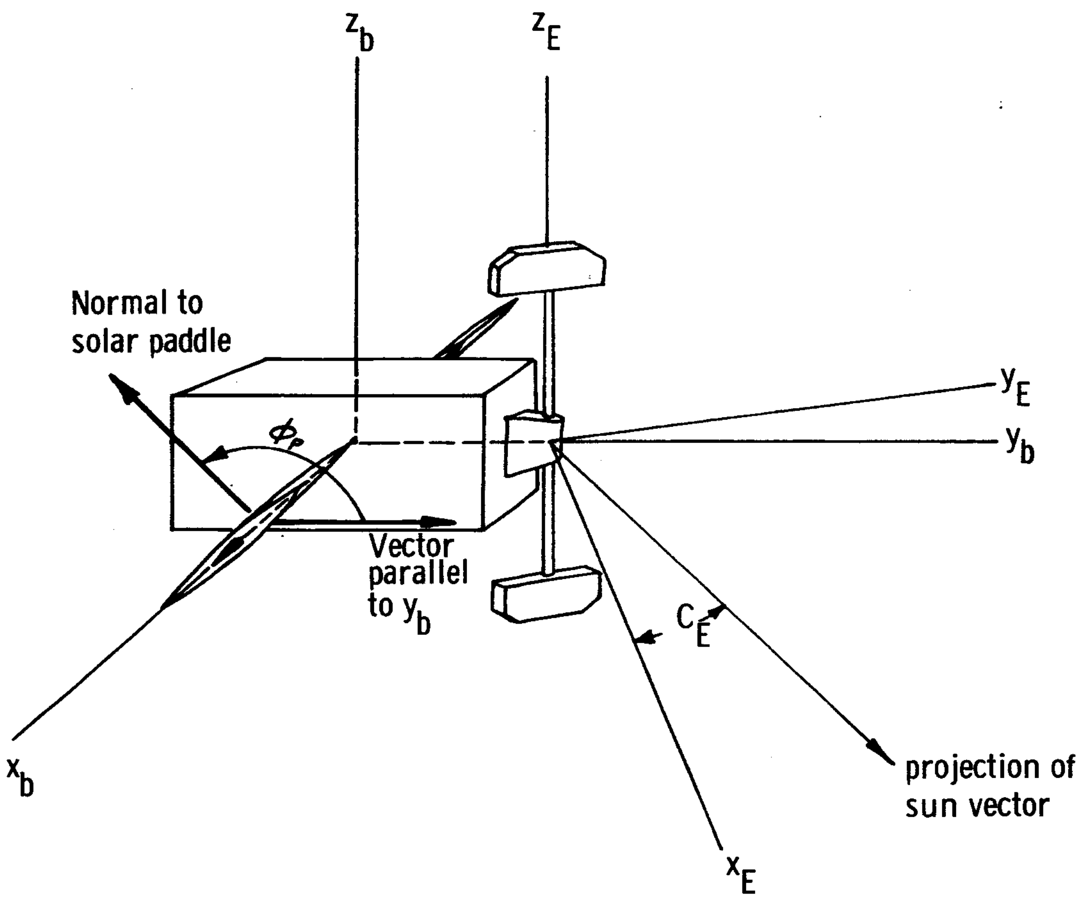

Figure 2.
Angles φ_p and C_E

I. ASSUMPTIONS

The OGO-E will be placed into a highly eccentric orbit with an apogee height approximately 148,000.0 km. The injection conditions for OGO-E are assumed to be the same as the nominal conditions for OGO-A which was launched September 4, 1964 (Ref. 1).

Table I.

OGO-E Injection Conditions

Geocentric latitude	=	$20^\circ\!.744573$ S
Longitude	=	$111^\circ\!.11923$ E
Height	=	279.2517 km
Speed	=	10.716286 km per sec
Azimuth	=	$66^\circ\!.445986$ from north
Flight path angle	=	$1^\circ\!.452111$

The osculating classical orbital elements associated with these injection conditions are shown in Table II.

Table II

Orbital Elements

Semi-major axis, a = 12.514687 earth radii

Eccentricity, e = .91666199

Inclination, i = $30\overset{\circ}{.}910496$

Argument of perigee, ω = $313\overset{\circ}{.}657536$

Right ascension of the ascending node, Ω = $195\overset{\circ}{.}74031$

Mean anomaly, M = $8\overset{\circ}{.}7160632$

Epoch = August 15, 1966 at $5^h\ 30^m$ U.T.

II. MATHEMATICAL MODEL

A. Orbit Plane Geometry

The angles ξ and S' are related to the orbit in space and the sun. These angles are considered as orbit related because the position of the spacecraft in the orbit does not enter into the calculations.

1. The Inclination with respect to the Ecliptic Plane ξ.

The first problem is to determine the angle between the normal to the ecliptic plane and the normal to the orbit plane, i.e. the inclination of the orbit with respect to the ecliptic plane. The angle can simply be written as:

$$\xi = \cos^{-1} (\overline{N} \cdot \overline{k}'). \tag{1}$$

where $\overline{N}$ = Unit angular momentum vector which is normal to the orbit.

$\overline{k}'$ = Unit vector normal to the ecliptic plane.

$$\overline{k}' = -\overline{j} \sin \epsilon + \overline{k} \cos \epsilon$$

e = 23°.44371, mean ecliptic angle for 1966 (Ref. 2).

$\overline{j}, \overline{k}$ = unit vectors along the y and z axes of the equatorial inertial coordinate system.

2. Angular Momentum - Sun Angle, S'.

The angle S' is defined by:

$$S' = \cos^{-1} (\overline{N} \cdot \overline{U}_{SE}) \tag{2}$$

where $\overline{U}_{SE}$ = The unit position vector of the sun measured from earth. The coordinates of the sun are taken from an ephemeris.

The use of $\overline{N}$ in equations (1) and (2) takes into account the motion of the orbit due to perturbations when used in a numerical integration scheme where for each integration step $\overline{N}$ is computed. Consequently, the motion $\overline{N}$ reflects both the secular and periodic perturbations. For a single orbit there is little change in $\overline{N}$; hence, it can be stated that the angles ξ and S' are dependent on the motion of the plane of the orbit. The angle S' is also dependent on the position of the sun.

B. Spacecraft Angles

1. Half Angular Dimension of the Earth, δ.

The angle, δ is easily determined from the following expression:

$$\delta = \sin^{-1} \frac{a_e}{|\overline{r}|} \qquad 0^\circ \le \delta \le 90^\circ \tag{3}$$

where a_e = 6378.165 km radius of the earth (Ref. 3).

2. The Solar Array Angle, φ_p.

The solar array angle, φ_p, is based solely on the position of the solar array with respect to the main structure on the satellite. The angle φ_p is measured counter-clockwise from the y_b-axis to the y_p-axis, as observed from the positive x_b-axis (Fig. 2). The planned range for the variation of φ_p for OGO-E is $\frac{\pi}{2} \leq \varphi_p \leq \frac{3\pi}{2}$, (Refs. 4, 5). Therefore, φ_p may be found from the formula

$$\varphi_p = \cos^{-1} (\bar{j}_p \cdot \bar{k}_b) + \frac{\pi}{2} \tag{4}$$

where $\bar{j}_p$ = Unit vector normal to solar array surface (sunlit side).

$\bar{k}_b = - \dfrac{\bar{r}}{|\bar{r}|}$, $\bar{r}$ is the satellite position vector.

3. OPEP x_E-Axis - Sun Projection Angle, C_E.

The angle C_E is somewhat more complicated in that two coordinate transformations are required involving three coordinate systems. The three coordinate systems in question are:

a). $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, inertial equatorial system, where x, y, z form the right-handed coordinate system in which x points to the vernal point, z points to the north pole and y completes the set.

b) $$X_b = \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix}$$, the satellite fixed coordinate system,

where x_b, y_b, z_b form a right-handed coordinate system with the origin at the center of gravity of the satellite in which z_b points to the center of the earth from the satellite, x_b lies parallel to the solar array axis and y_b is orthogonal to x_b and z_b and points towards the orbit plane experiment package (OPEP) end of the satellite (Fig. 2).

c). $$X_E = \begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix}$$, the OPEP coordinate system,

where x_E, y_E, z_E form a right-handed coordinate system in which z_E is parallel to z_b and the x_E - y_E plane is parallel to the x_b - y_b plane and has the angle Ψ_E between the x_b and x_E axes as shown in the following figure.

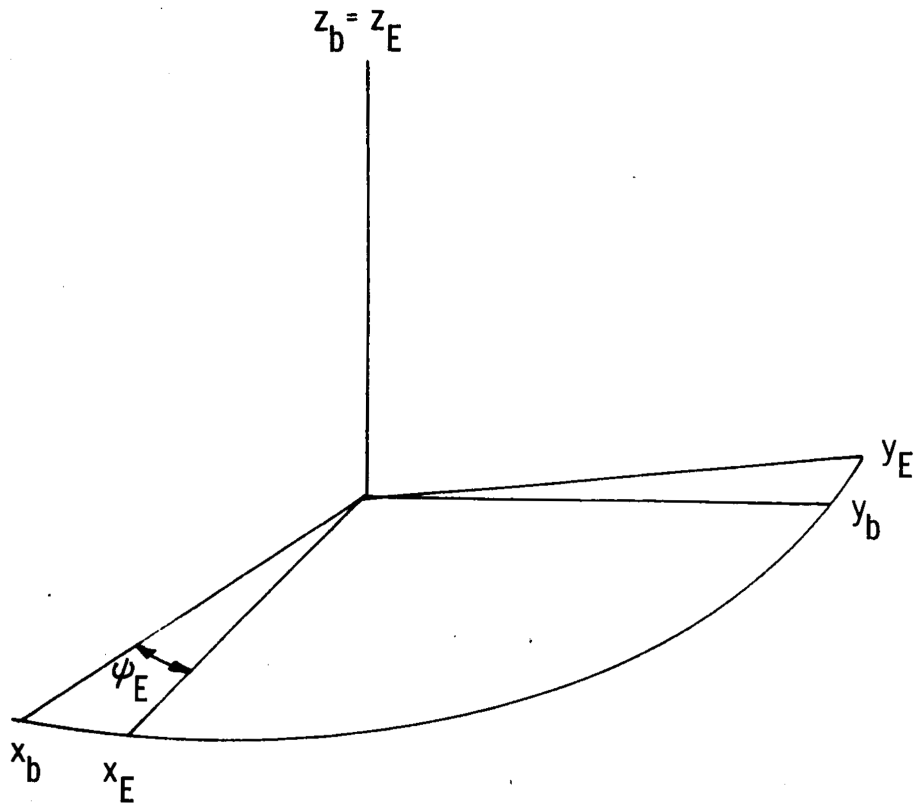

Figure 3.
Angle ψ_E

The three coordinate systems are related as follows:

$$X_b = AX \tag{5}$$

$$X_E = BX_b \tag{6}$$

Then $X_E = CX$ (7)

where A, B, C, are transformation matrices and

$$C = BA$$

In the three coordinate systems described above, let:

$\bar{i}, \bar{j}, \bar{k}$ = unit vectors along x, y, z.

$\bar{i}_b, \bar{j}_b, \bar{k}_b$ = unit vectors along x_b, y_b, z_b.

$\bar{i}_E, \bar{j}_E, \bar{k}_E$ = unit vectors along x_E, y_E, z_E.

Now, the matrix A is defined by

$$A = \begin{bmatrix} \bar{i}_b\cdot\bar{i} & \bar{i}_b\cdot\bar{j} & \bar{i}_b\cdot\bar{k} \\ \bar{j}_b\cdot\bar{i} & \bar{j}_b\cdot\bar{j} & \bar{j}_b\cdot\bar{k} \\ \bar{k}_b\cdot\bar{i} & \bar{k}_b\cdot\bar{j} & \bar{k}_b\cdot\bar{k} \end{bmatrix}. \tag{8}$$

The matrix B is:

$$B = \begin{bmatrix} \cos\Psi_E & \sin\Psi_E & 0 \\ -\sin\Psi_E & \cos\Psi_E & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{9}$$

Therefore,

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \tag{10}$$

where
$$c_{11} = (\bar{i}_b \cdot \bar{i}) \cos \Psi_E + (\bar{j}_b \cdot \bar{i}) \sin \Psi_E$$

$$c_{12} = (\bar{i}_b \cdot \bar{j}) \cos \Psi_E + (\bar{j}_b \cdot \bar{j}) \sin \Psi_E$$

$$c_{13} = (\bar{i}_b \cdot \bar{k}) \cos \Psi_E + (\bar{j}_b \cdot \bar{k}) \sin \Psi_E$$

$$c_{21} = -(\bar{i}_b \cdot \bar{i}) \sin \Psi_E + (\bar{j}_b \cdot \bar{i}) \cos \Psi_E$$

$$c_{22} = -(\bar{i}_b \cdot \bar{j}) \sin \Psi_E + (\bar{j}_b \cdot \bar{j}) \cos \Psi_E$$

$$c_{23} = -(\bar{i}_b \cdot \bar{k}) \sin \Psi_E + (\bar{j}_b \cdot \bar{k}) \cos \Psi_E$$

$$c_{31} = (\bar{k}_b \cdot \bar{i})$$

$$c_{32} = (\bar{k}_b \cdot \bar{j})$$

$$c_{33} = (\bar{k}_b \cdot \bar{k}).$$

Since the position of the sun must be defined in the OPEP coordinate system, equation (7) is used.

$$X_{E_S} = C\,X_S \tag{11}$$

$$\text{where } X_S = \begin{bmatrix} x_S \\ y_S \\ z_S \end{bmatrix}$$

and x_S, y_S, z_S are the coordinates of the sun. Hence the coordinates of the sun in the OPEP system are:

$$X_{E_S} = \begin{bmatrix} x_{E_S} \\ y_{E_S} \\ z_{E_S} \end{bmatrix}$$

In terms of vector notation the sun vector in the OPEP system is:

$$\bar{S}_E = \bar{i}_E\, x_{E_S} + \bar{j}_E\, y_{E_S} + \bar{k}_E z_{E_S} \tag{12}$$

The projection of the sun onto the OPEP x_E - y_E plane is simply:

$$\overline{S}_{E_P} = \overline{i}_E x_{E_S} + \overline{j}_E y_{E_S} \tag{13}$$

The angle C_E between the x_E axis and the projection of the sun on the OPEP x_E - y_E plane defined by:

$$C_E = \cos^{-1} \overline{i}_E \cdot \frac{\overline{S}_{E_p}}{|\overline{S}_{E_p}|} \tag{14}$$

C. <u>The Computer Program</u>

The equations in Section II were used with an existing general purpose interplanetary trajectory Encke method program (ITEM) for use on the IBM 7090 and IBM 7094 computers (Ref. 6). The perturbations accounted for in this program include the gravitational attraction of the earth, moon, sun, Jupiter, Venus, and Mars considered as point masses. Also included are the effects of the 2nd, 3rd and 4th zonal harmonics and 2nd tesseral harmonic of the earth's gravitational field. Since perigee height rises rapidly, air drag was not considered. The perigee height rise for OGO-A is about fourty-one kilometers per orbit.

III. RESULTS AND DISCUSSION

Computer runs to determine ξ, S', δ, φ_p and C_E as functions of time were made using the formulas of Section II and the following injection times:

August 7, 1966	$1^h\,45^m$	universal time
August 7, 1966	$3^h\,45^m$	universal time
August 22, 1966	$1^h\,15^m$	universal time
August 22, 1966	$3^h\,15^m$	universal time

A. Orbit Plane Geometry

1. Inclination with respect to the Ecliptic Plane, ξ.

Figures 4 and 5 show the trend of ξ for the above injection times. This angle exhibits the approximate fourteen day lunar oscillation and the bi-annual oscillation due to the solar perturbation. The mean increasing trend is due primarily to the combined long term effects of the sun and moon.

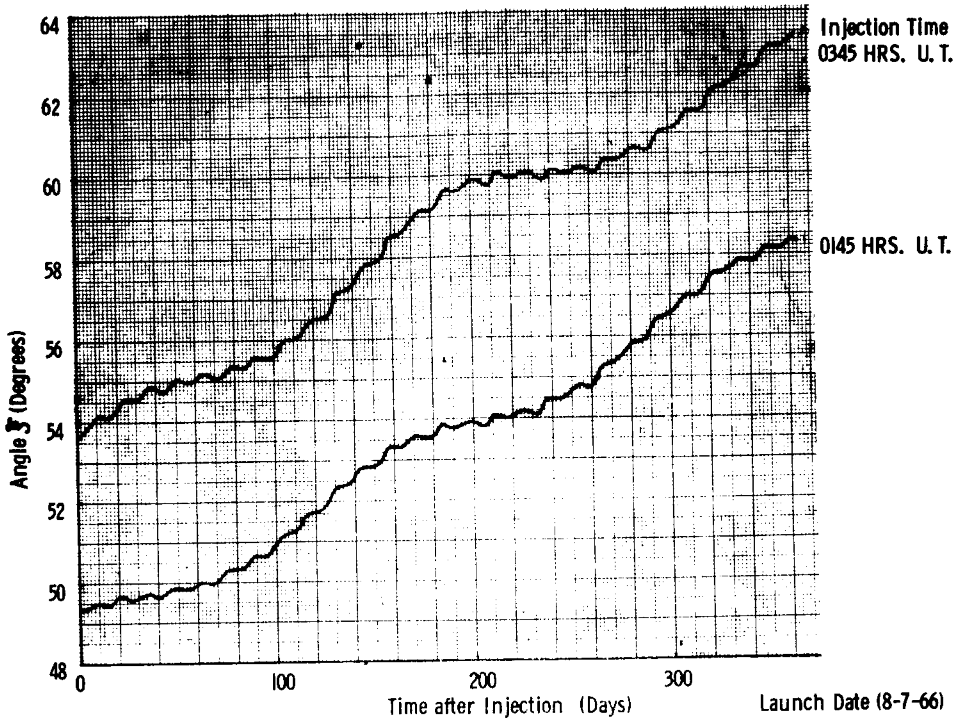

FIGURE 4 - ξ Versus Time after Injection

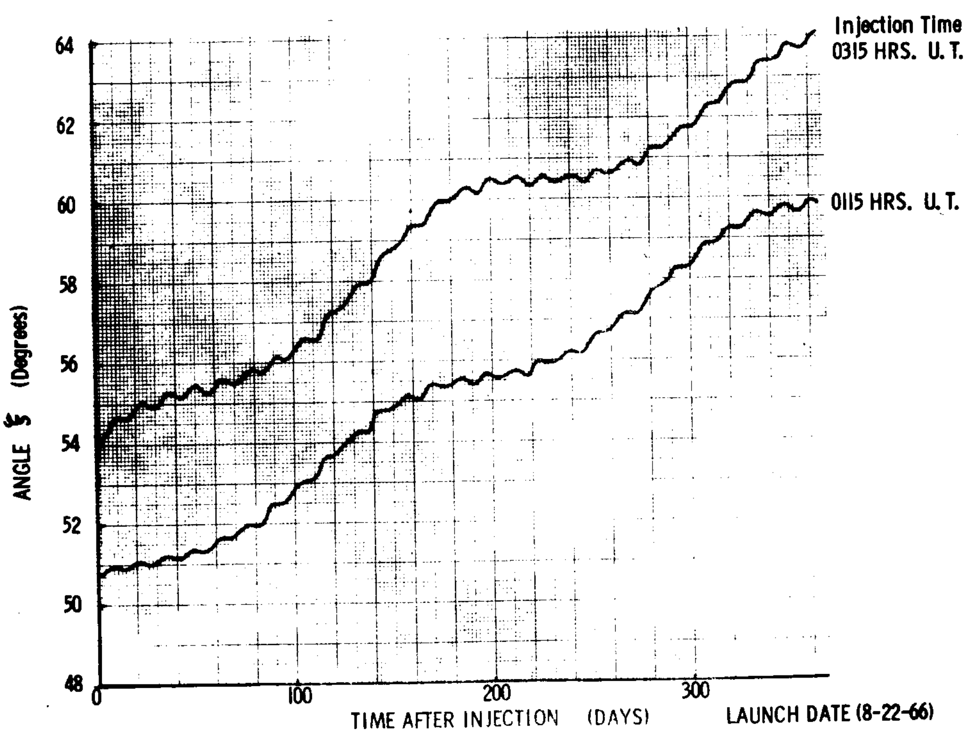

FIGURE 5 - ξ Versus Time after Injection

2. Angular Momentum - Sun Angle, S'

Since S' is mainly a function of the sun's position, the time history of S' shows a yearly periodicity. The short term periodic (about fourteen days) lunar perturbation also affects S' but these variations are in the order of tenths of a degree (about two orders of magnitude less than the average value of S') and, consequently, are not perceptible on Figures 6 and 7.

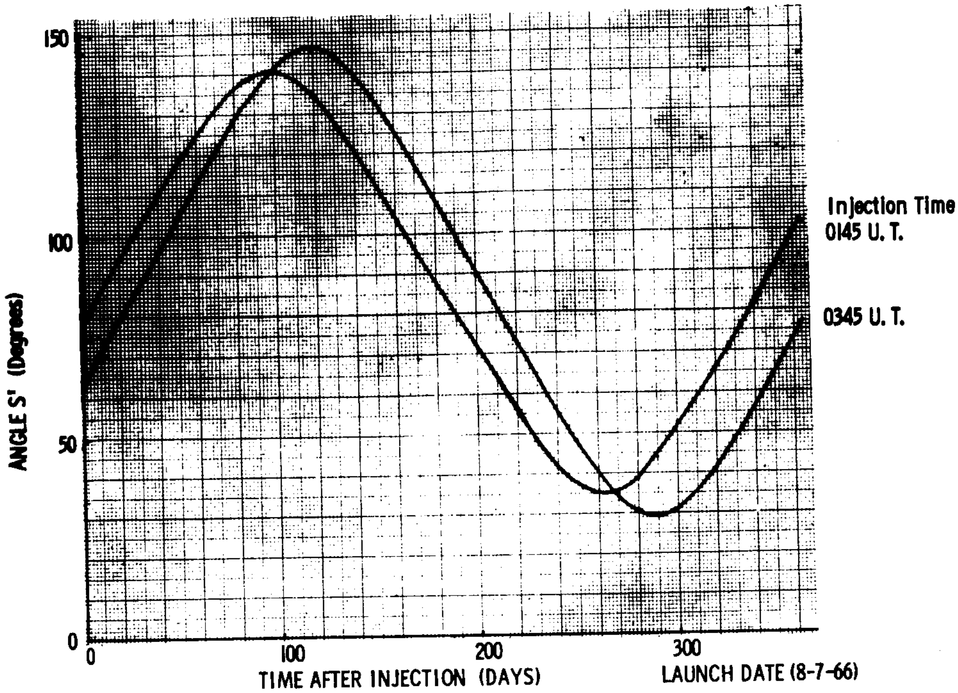

FIGURE 6- S' Versus Time After Injection

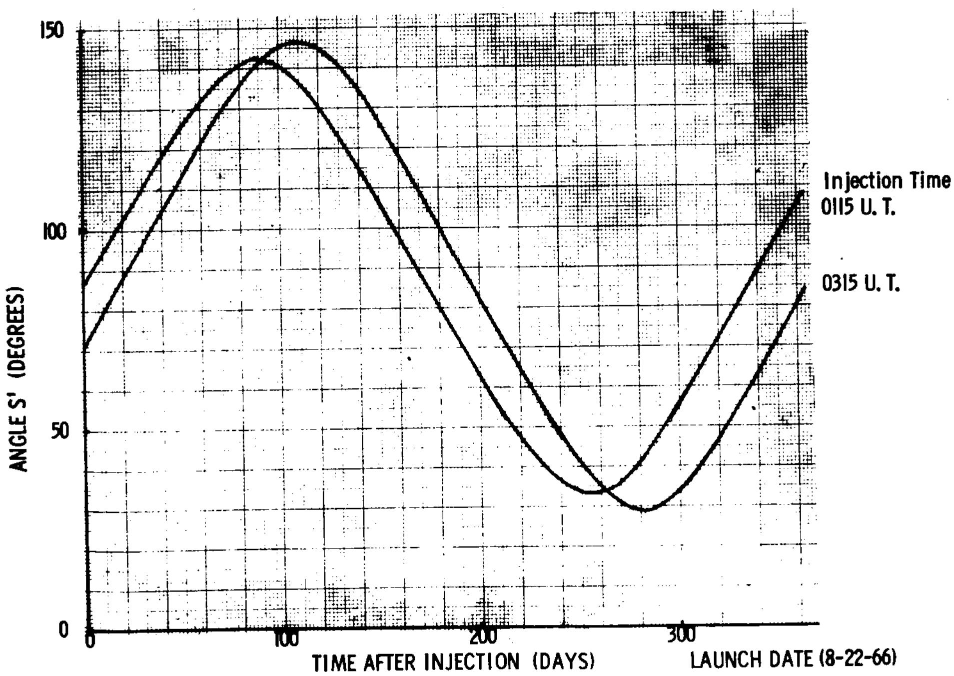

FIGURE 7 - S' Versus Time after Injection

B. Spacecraft Angles

1. Half Angular Dimension of the Earth, δ.

The angle δ is nearly periodic since it is a function solely of the distance of the spacecraft from the earth. The time history for the first 150 hours after injection is essentially the same for all the injection dates and times shown on page 15. See Figure 8.

After almost one year of flight the time history of δ is again shown for 150 hours (Figure 9). These results are based on an injection into orbit time of August 7, 1966, at $3^h\ 45^m$ universal time. The results shown on Figure 9 are typical of results for different injection dates and times.

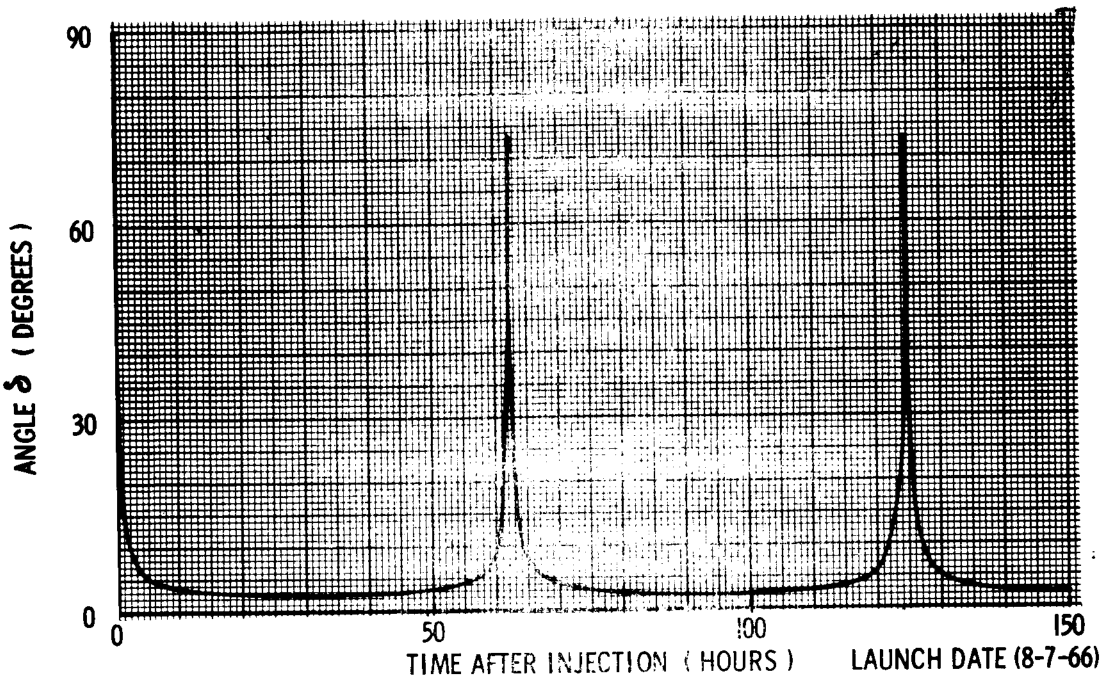

FIGURE 8 - δ Versus Time after Injection

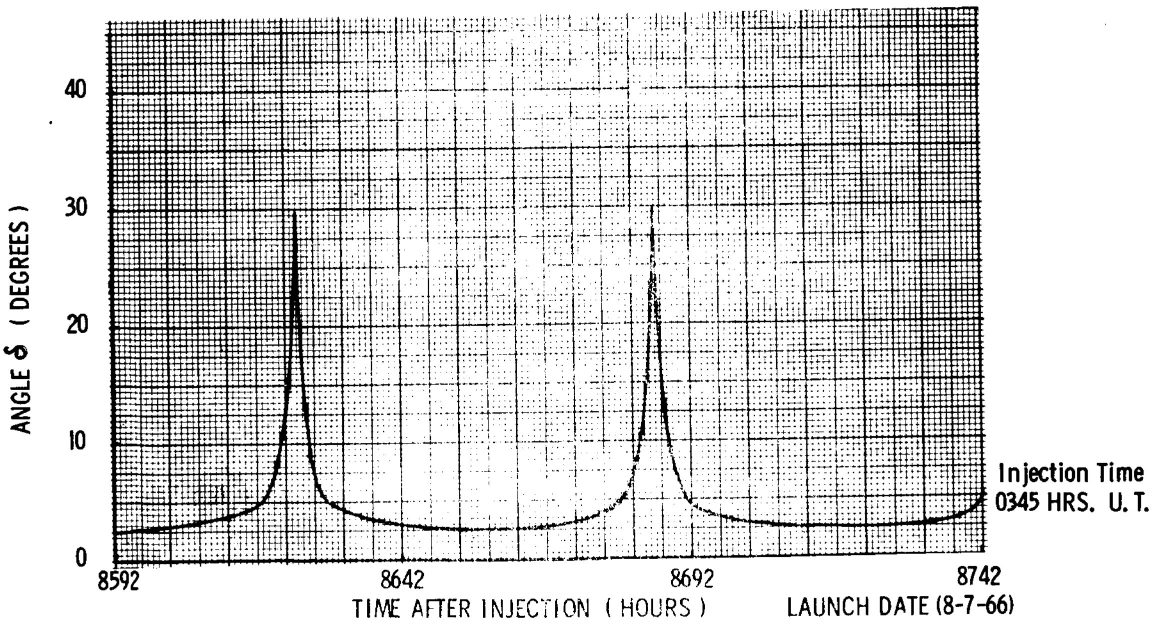

FIGURE 9 - δ Versus Time after Injection

2. The Solar Array Angle, φ_p.

Figures 10 and 11 show the behavior of the solar array angle φ_p for the first 150 hours in orbit for the injection into orbit times shown on page 15. The different time histories are due to the different spacecraft-sun geometry.

Figures 12 and 13 show the behavior of the solar array angle φ_p after almost a year in orbit.

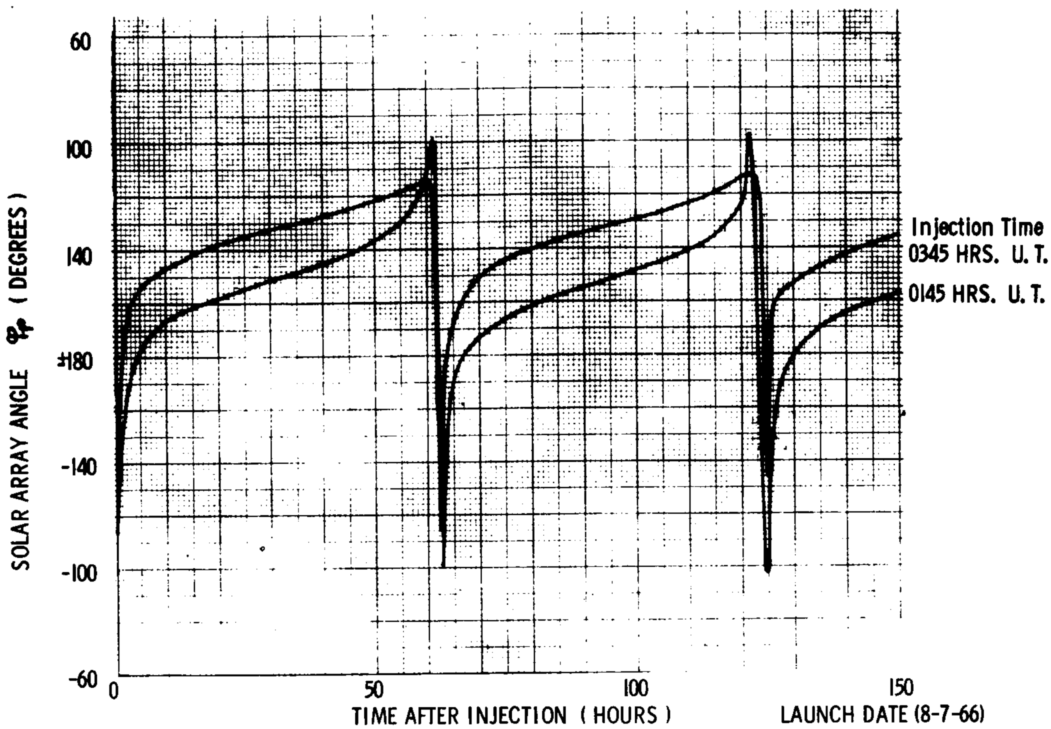

FIGURE 10 - φ_p Versus Time after Injection

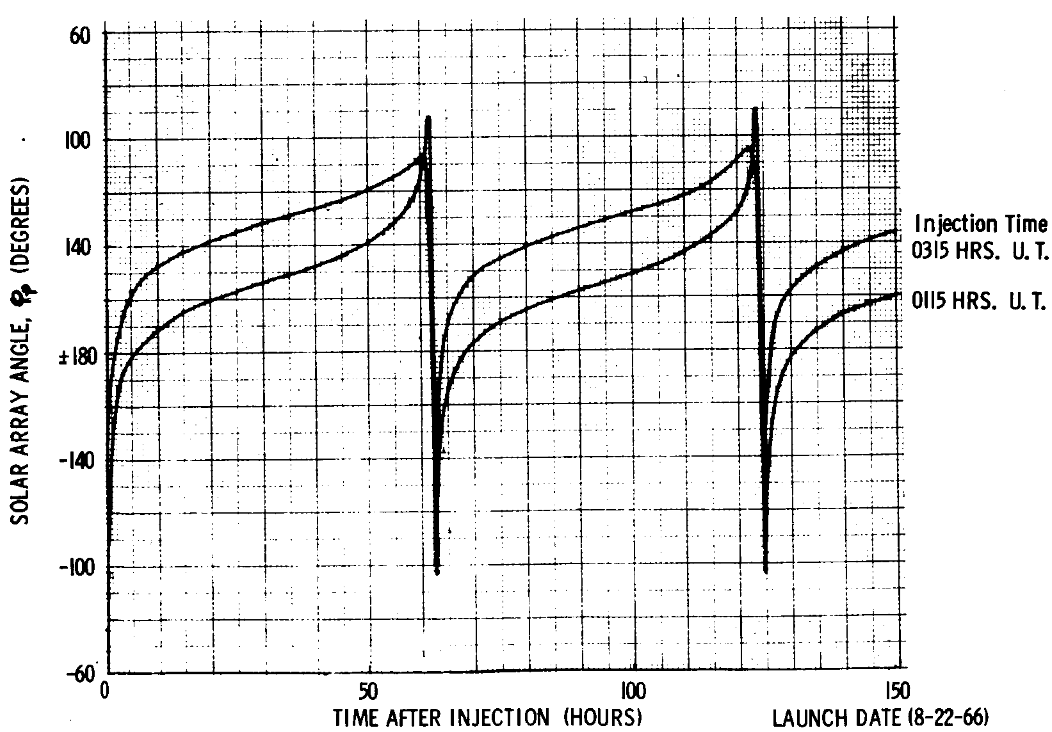

FIGURE 11 - φ_p Versus Time after Injection

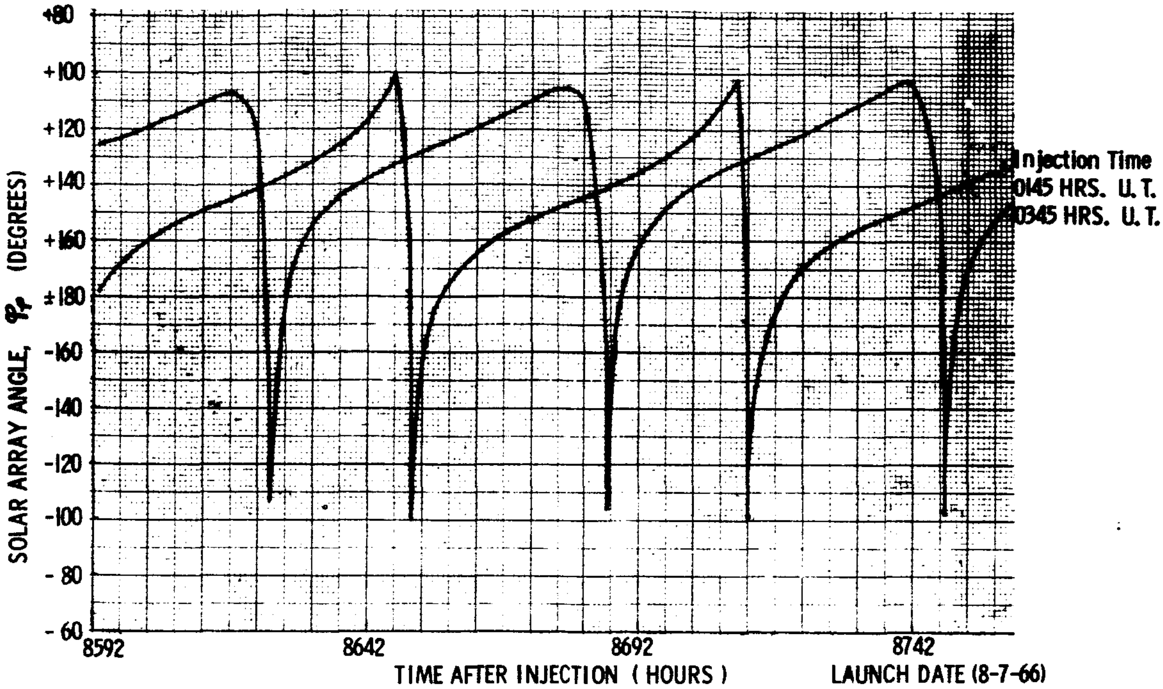

FIGURE 12 - φ_p Versus Time after Injection

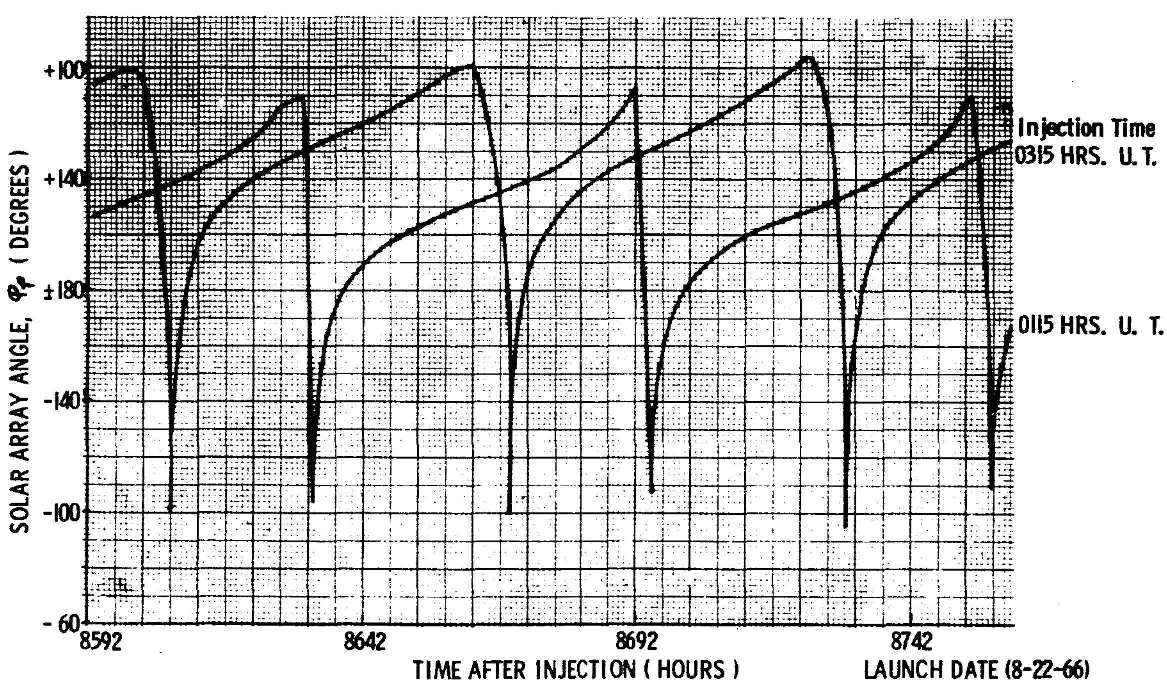

FIGURE 13 - φ_p Versus Time from Injection

3. OPEP x_E-Axis - Sun Projection Angle, C_E.

Figures 14 and 15 show the behavior of C_E for the first 150 hours in orbit for the injection into orbit times shown on page 15. As in the case of the solar array angle the difference in the time histories of C_E for different injection times is due to the different spacecraft-sun geometry.

Figures 16 and 17 show the behavior of C_E after almost a year in orbit.

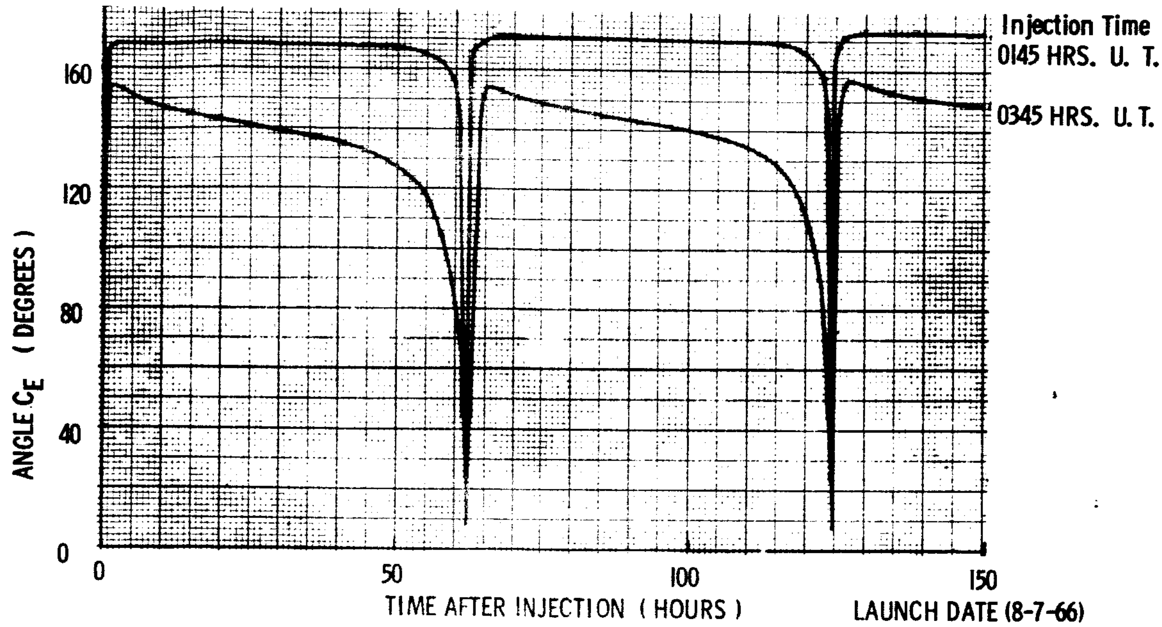

FIGURE 14 - C_E Versus Time after Injection

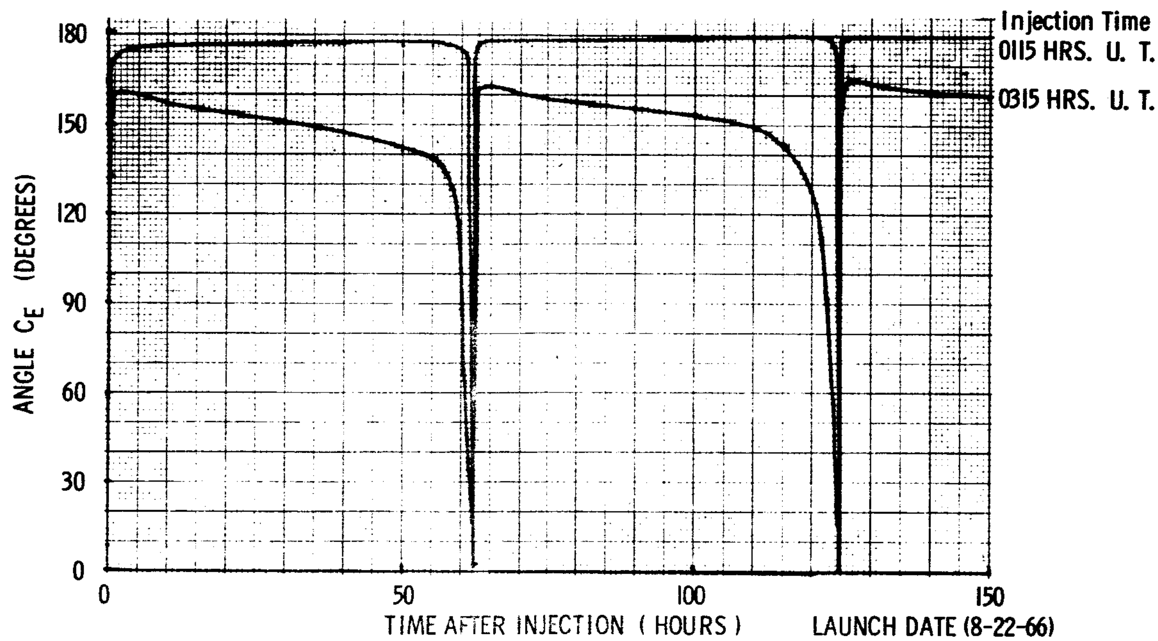

FIGURE 15 - C_E Versus Time after Injection

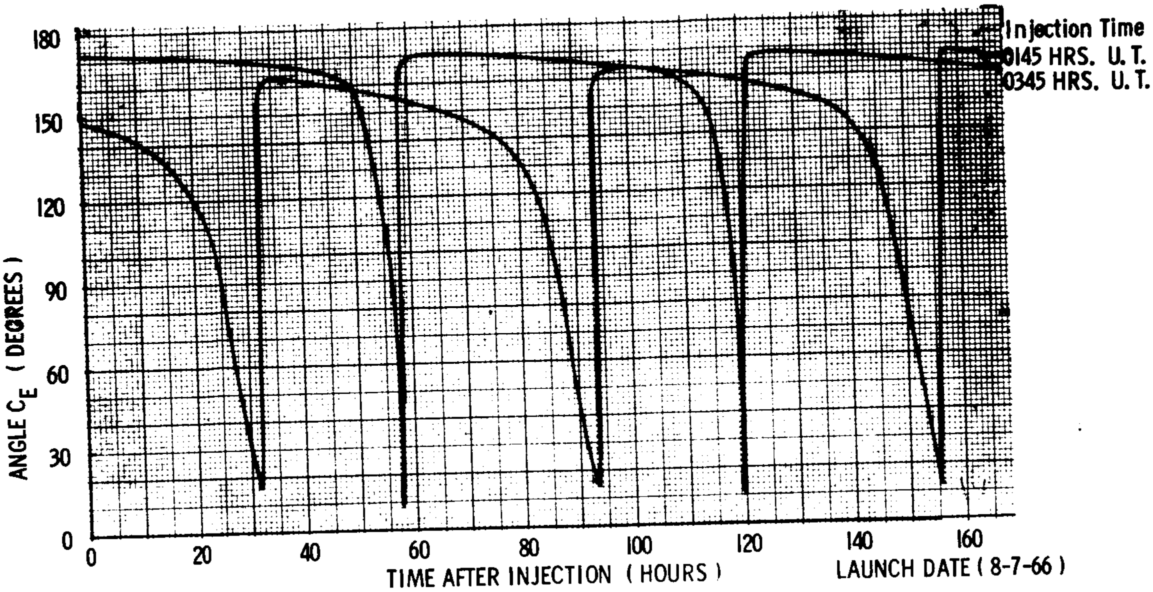

FIGURE 16 - C_E Versus Time after Injection

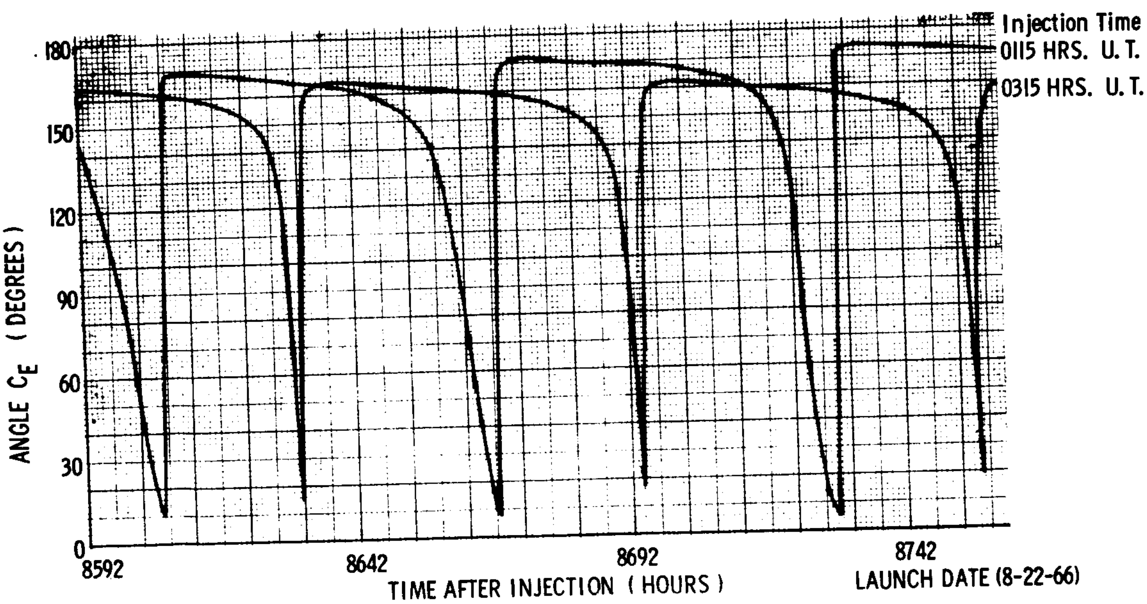

FIGURE 17 - C_E Versus Time after Injection

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