NASA TECHNICAL NOTE

NASA TN D-3026

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UNSTEADY VISCOUS VORTEX WITH FLOW TOWARD THE CENTER

by Robert G. Deissler

Lewis Research Center
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • SEPTEMBER 1965

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SUMMARY

An analysis was made of a strong unsteady vortex that consists of an annular region, in which the flow is tangential and radial, and a core region, in which a uniform axial flow is also present. The effects of end-wall boundary layers, if present and important, were assumed to be eliminated by use of suitably rotating end walls. The Navier-Stokes equation for the tangential flow was solved numerically to carry out the analysis.

Step changes in the tangential velocity at a given radius and step changes in the radial flow are introduced for various initial tangential velocity profiles. The response of the tangential flow is then calculated as a function of radius and of time.

INTRODUCTION

Many of the vortices occurring in nature have intensities that vary with time. The formation and the decay of atmospheric vortices such as tornadoes, hurricanes, and dust devils, as well as of bathtub vortices, are examples of transient vortex motions. In addition, the random vortices in turbulent flow (if they can be identified as vortices) are unsteady.

Not a great deal of work has been done on unsteady vortex flows, and most of that done has been for no radial flow (refs. 1 to 3). Since the phenomena previously mentioned involve a concentration of vorticity by an inward radial flow, the analyses for no radial flow would not be expected to aid greatly in understanding those phenomena.

One example of unsteady flow in which radial inflow is considered, where the flow rotates in a sinusoidal manner, is given by Donaldson (ref. 4). Hamel (ref. 5) has given a general infinite series solution for an unsteady two-dimensional vortex with radial flow but no axial flow. Donaldson (ref. 4) has applied one form of that solution to a suddenly rotated porous cylinder in which the tangential flow is nearly wheel like. Some similar solutions for a transient vortex with radial but no axial flow have also been obtained

*A portion of this work was presented at the International Union of Theoretical and Applied Mechanics Symposium on Concentrated Vortex Motions in Fluids, held at the University of Michigan, Ann Arbor, July 6 to 11, 1964.

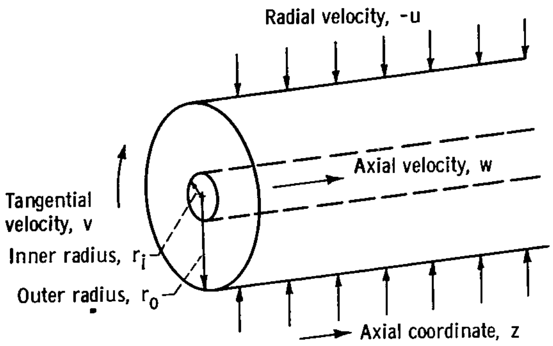


Figure 1. - Vortex model used in analysis.

(refs. 4 and 5). An investigation of vortex motion in an emptying container in which viscosity is neglected in the equations of motion is given in reference 6.

A simplified model is analyzed here in order to give some understanding of the formation and the destruction of viscous vortices with radial and axial flow. The response of the tangential flow in a vortex to sudden changes in the tangential velocity at a particular radius and to changes in the radial velocity is considered. In general, the simplest examples of these changes and combinations of these changes are analyzed numerically to give some insight into transient vortex motions. The analytical model and the method of solution are described in the ANALYSIS section.

The analysis is carried out for laminar flow. A rough estimate of the effect of turbulence might be obtained by replacing the laminar radial-flow Reynolds number in the analysis by a turbulent-flow Reynolds number as given in reference 7.

ANALYSIS

The model used for analyzing the unsteady vortex is essentially the same as that used in reference 8 and generalized in reference 7 for the steady-state vortex. A sketch of the axially symmetric vortex is given in figure 1. The tangential velocity v is a function only of radius and time. (All symbols are defined in the appendix.) The tangential and radial velocities are specified at a given reference radius. In addition, the axial velocity is specified as a function of radius. As shown in reference 7, only a linear variation of axial velocity with axial position is consistent with the assumption of a tangential velocity independent of axial position.

For the present calculations, a radially uniform axial velocity is assumed for the region near the center $0 < r < r_i$, and a zero axial velocity is assumed in most cases for the remaining annular region $r_i < r < r_o$. This type of motion might be simulated by using a rotating porous container with an exit hole at the center of one end. In addition, one example is considered in which the radial velocity at the outer radius is zero (solid wall), and the radial inflow is produced by allowing the fluid to enter the vortex uniformly at one end or by a receding free surface (eq. (9)). Step changes with time in the tangential velocity at a particular radius and in the radial velocity are allowed. The flow is assumed to be incompressible and governed by the Navier-Stokes equation for the rate of change of tangential velocity. The analysis then predicts the tangential velocity as a function of radius and of time.

Two points in connection with the model just described require some comment. In particular, the assumption that v is independent of axial position is not good near a stationary end wall in a vortex chamber. Recent work given in reference 9 has emphasized the importance of the end-wall boundary layer. The end-wall effects should be less for long vortices than for short ones, and these effects might also be reduced by extending the exit tube into the vortex chamber. For the present investigation, the end wall may be assumed to consist of concentric sections which rotate in such a way that the boundary-layer effects are eliminated.

The other point that requires discussion is the assumption that the variation of axial velocity w with r can be specified arbitrarily. Actually, that velocity is determined by the axial momentum equation and the boundary conditions, as in the analyses of references 10 and 11. Those exact solutions give considerable insight into the vortex problem, although they are restricted either to a radially uniform axial pressure gradient (ref. 10) or to vortices in which v and w are of the same order of magnitude (ref. 11). Reference 12 shows that the model used here (where the variation of w with r is specified, and v is independent of z) is consistent with the axial momentum equation, at least for steady-state vortices with strong circulation. The same conclusion is easily shown to apply to the strong transient vortices considered herein. The real axial velocity profile will be a function of the axial pressure gradient, which varies with radius in the exit hole. Inasmuch as the variation of axial pressure gradient with radius is hard to specify, it seems reasonable to assume the axial velocity to be radially uniform.

The incompressible Navier-Stokes equation for the tangential velocity with axial symmetry is

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial r} - w\frac{\partial v}{\partial z} - \frac{uv}{r} + \upsilon\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right) \tag{1}$$

and the continuity equation is

$$\frac{\partial(ru)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0 \tag{2}$$

For the model considered herein, equation (1) in dimensionless form becomes

$$\frac{\partial v'}{\partial t'} = u'\frac{\partial v'}{\partial r'} + \frac{u'v'}{r'} - \frac{1}{Re}\left(\frac{\partial^2 v'}{\partial r'^2} + \frac{1}{r'}\frac{\partial v'}{\partial r'} - \frac{v'}{r'^2}\right) \tag{3}$$

where $v' = v/v_i$, $u' = u/u_i$, $r' = r/r_i$, $Re = u_i r_i/\nu$, and $t' = -(u_i/r_i)t$. The subscript i refers to values at the inner radius, and since v varies with time, v_i is taken to be the value of v at r_i for the fully developed or steady-state vortex that is considered in reference 13. Note that $u_i r_i = u_o r_o$ in the definition of the radial-flow Reynolds number, when there is no axial flow in the annular portion of the vortex. Since the radial flows considered are toward the center, Re will be negative.

Equation (3) shows that since v is independent of z, u will also be independent of z. The continuity equation then gives

$$u' = \frac{1}{r'} \tag{4}$$

for the annular region (no axial flow) and

$$u' = r' \tag{5}$$

for the core (uniform axial flow).

Solutions of equations (3) to (5) for various initial and boundary conditions would probably be possible as infinite series of product solutions. Such solutions were obtained in reference 4 for the special case of a suddenly rotated porous cylinder in which there was radial flow but no axial flow, and the tangential flow was nearly wheel like. A numerical approach with finite differences seemed simpler, however, and with that approach nothing was lost as far as understanding the physics of the flow. Equation (3) in finite difference form becomes

$$\frac{\Delta v'}{\Delta t'} = u'_r \frac{v'_{r+\Delta r} - v'_{r-\Delta r}}{2\,\Delta r'} + \frac{u'_r v'_r}{r'} - \frac{1}{Re}\left[\frac{v'_{r-\Delta r} - 2v'_r + v'_{r+\Delta r}}{(\Delta r')^2} + \frac{1}{r'}\frac{v'_{r+\Delta r} - v'_{r-\Delta r}}{2\,\Delta r'} - \frac{v'_r}{r'^2}\right] \tag{6}$$

where the subscripts r, $r+\Delta r$, and $r-\Delta r$ refer to values at particular radii, and $\Delta v'$ is the increment in v' which corresponds to the dimensionless time increment $\Delta t'$. By starting from a given initial distribution for v' as a function of r', equation (6) can be used to calculate v' at various dimensionless times and radii. In the calculations, the ratio of $\Delta t'$ to $(\Delta r')^2$ must be kept sufficiently small to ensure stability of the solution. The quantity $\Delta r'$ must also be made small enough that cutting it in half does not change the results appreciably. The calculations were done on a high speed digital computing machine. The results for the various examples are given in the section RESULTS AND DISCUSSION.

RESULTS AND DISCUSSION

The transient vortex motions considered include the following examples: (1) the decay of a fully developed vortex when the tangential velocity at a particular radius is suddenly reduced to zero, (2) the response of an initial flow without vorticity to vorticity suddenly introduced at an outer radius, (3) the concentration of an initially uniform field of vorticity by a suddenly applied radial inflow, and (4) the decay of a fully developed vortex when the radial flow is suddenly reduced to zero. In addition, a combination of two types of step changes is studied by subjecting an initial wheel flow simultaneously to a radial inflow and a zero tangential velocity at the outer radius.

Response of Initially Fully Developed Vortex to a Suddenly Applied Zero Tangential Velocity at a Particular Radius

The initial fully developed vortex is calculated from the steady-state solutions of reference 13:

$$v' = \frac{-Re}{\left(e^{-Re/2} - 1\right)} \frac{\left(r'^{Re+2} - 1\right)}{r'(2 + Re)} + \frac{1}{r'} \tag{7}$$

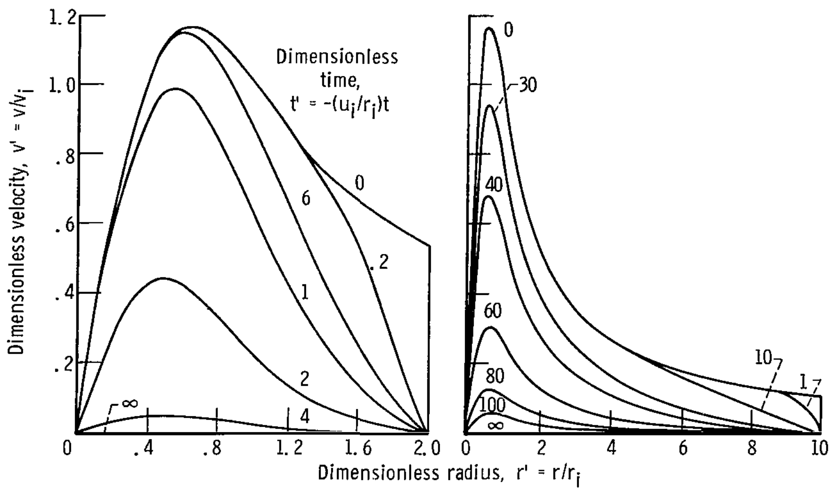
for $1 \leq r' \leq r'_o$, and

$$v' = \frac{1}{r'} \frac{1 - e^{Re r'^2/2}}{1 - e^{Re/2}} \tag{8}$$

for $0 \leq r' \leq 1$. At time $t' = 0$, the tangential velocity at a given radius is set equal to zero, so that there is a step change of velocity in time and space at that point. The variation of v' with t' and r' is then calculated from equations (4) to (6).

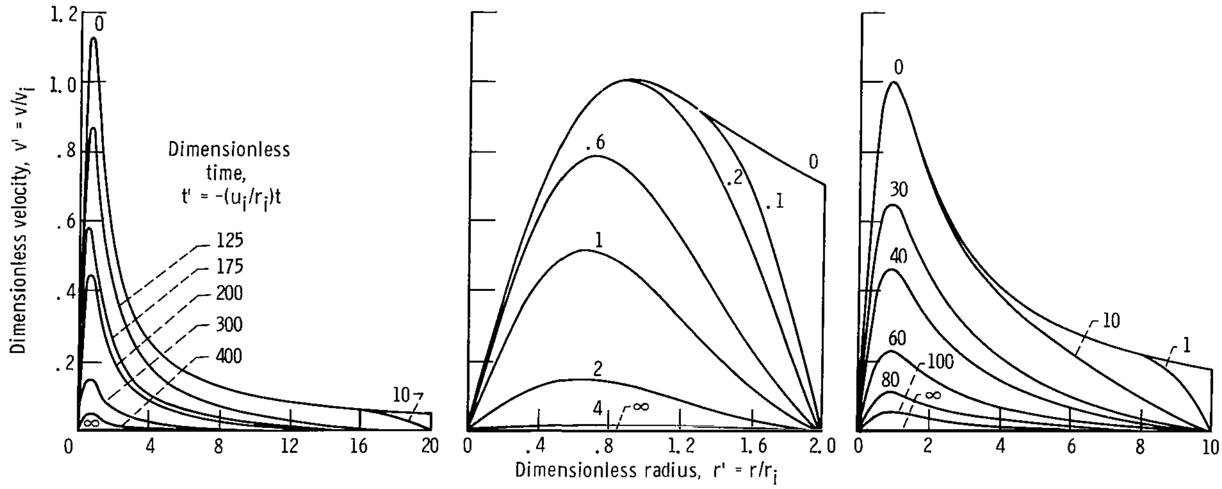
Figure 2 (p. 6) shows how the effect of a zero tangential velocity at r'_o quickly propagates out into the vortex like a growing boundary layer and soon begins to reduce the tangential velocities at all radii. After an initial adjustment period, the shape of the vortex remains similar as it decays until it is finally destroyed. This similarity for large values of time is possible because of the linearity of the governing differential equation. Because of its linearity, equation (3) has the product solution

$$v' = f(r') \cdot e^{-at'}$$



(a) Radial-flow Reynolds number, -6; ratio of outer to inner radius, 2.

(b) Radial-flow Reynolds number, -6; ratio of outer to inner radius, 10.



(c) Radial-flow Reynolds number, -6; ratio of outer to inner radius, 20.

(d) Radial-flow Reynolds number, -3; ratio of outer to inner radius, 2.

(e) Radial-flow Reynolds number, -3; ratio of outer to inner radius, 10.

Figure 2. - Response of vortex to suddenly applied zero tangential velocity at outer edge.

The effects of radius ratio r_o/r_i and of radial-flow Reynolds number Re are illustrated by the curves in figure 2. The curves for the smaller radius ratios decay much more rapidly than those for the larger radius ratios for the same value of u_i/r_i. For instance, the curves for $r_o/r_i = 10$ and $t' = 100$ show about the same degree of decay as those for $r_o/r_i = 2$ and a t' of 3 or 4. Increasing the radius ratio by a factor of 5 increased the dimensionless decay time by a factor of at least 25. The effect of negatively increasing the radial-flow Reynolds number Re is also to increase the decay time, although the effect of increasing Re from -3 to -6 is not large.

The foregoing comparison of decay times for various values of r_o/r_i was made on the basis that u_i/r_i remains constant. If u_o/r_o rather than u_i/r_i remains constant, the results are considerably different. A dimensionless time t'_o defined as $-(u_o/r_o)t$

is related to t' by $t'_o = t'(u_o/u_i)/(r_o/r_i) = t'/(r_o/r_i)^2$ (eq. (4)). The results in figure 2 show that for the same u_o/r_o and Re, the decay times for the various radius ratios are about the same. For example, for Re = -6 and $t'_o = -(u_o/r_o)t \approx 1$, the decay is about 95 percent complete for all three radius ratios. Thus, the unexpected result is produced that the decay time for a vortex of given r_o and u_o (or u_o/r_o) is about the same regardless of the initial shape, where r_o/r_i determines the initial shape.

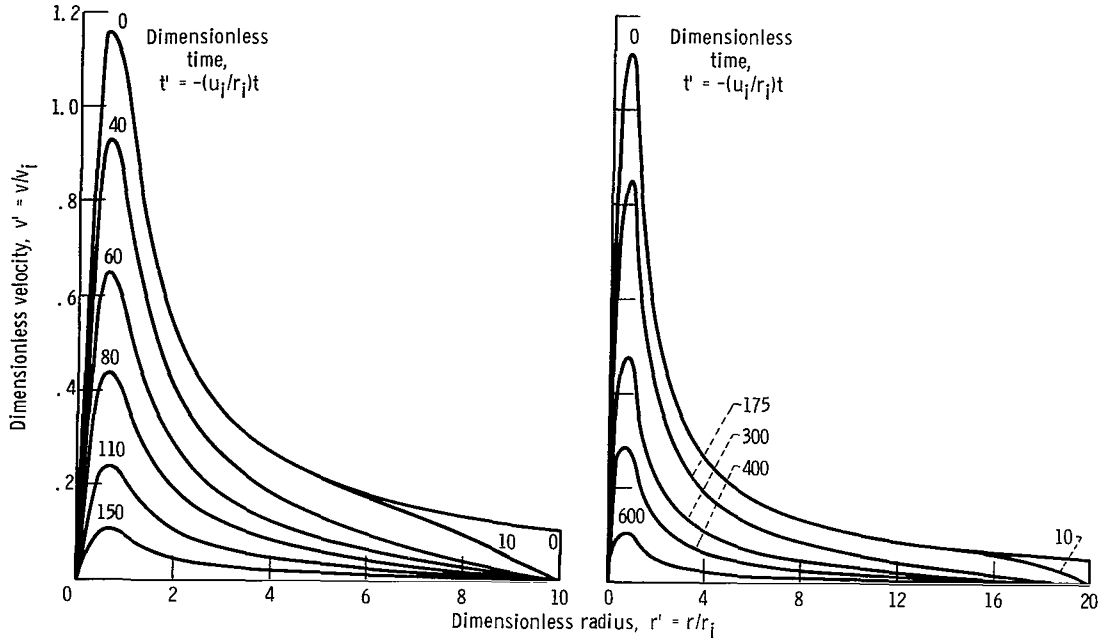
The vortex considered in figure 2 might be simulated with a rotating container with a porous cylindrical wall through which the fluid enters. The fluid would exit through a central hole in an end of the container. That case is now compared with the case in which the radial inflow is produced by allowing the fluid to enter the vortex uniformly at one end, rather than through the cylindrical wall. The same type of flow could be produced approximately in an emptying container of liquid in which the radial inflow comes from the changing height of a free surface. For those examples, if u is again independent of z, it is found from the continuity equation that equation (4) should be replaced by

$$u' = \frac{r_o'^2 - r'^2}{r'(r_o'^2 - 1)} \tag{9}$$

for the annular region. Equation (9) satisfies the continuity equation (eq. (2)) if $\partial w/\partial z$ is independent of r and also satisfies the boundary conditions that $u = 0$ at r_o and $u = u_i$ at r_i. For the core region, equation (5) is still applicable. Although $\partial w/\partial z$ is uniform within either the core or the annular region, its value is different in the two regions, being positive in the core and negative in the annulus.

The transient vortex in which u is zero at r_o but not at other radii is illustrated in figure 3. In order to obtain the effect of the change in boundary condition for u at the outer radius on the transient motion for the same initial conditions, equations (7) and (8) are again used for the initial conditions. Comparison of the curves in figure 3 (p. 8) with those in figure 2 shows that the trends are similar, but that the decay times are longer in figure 3. Longer decay times result because the radial velocity is zero at the outer edge of the vortex in figure 3, so that it takes longer for the effects of the zero tangential velocity at the outer edge to be brought into the vortex than it does in figure 2, where the fluid can come in through a porous cylindrical wall.

The results in figures 2 and 3 show that, when the tangential velocity at the outer edge of a vortex is zero, the vortex will eventually be destroyed. In a very large container a small residual vorticity may of course remain because of the effect of Earth rotation. An initially strong vortex in a symmetrical nonrotating container, however, will necessarily decay to a large extent. The reason that such a vortex may appear to be steady state (as in an emptying bathtub) is evidently that the decay times are very long. From the defini-



(a) Ratio of outer to inner radius, 10.

(b) Ratio of outer to inner radius, 20.

Figure 3. - Response of fully developed vortex to suddenly applied zero tangential velocity at outer radius. Radial-velocity is zero at outer radius and is given by equation (9) in place of equation (4) in annular region. Radial-flow Reynolds number, -6.

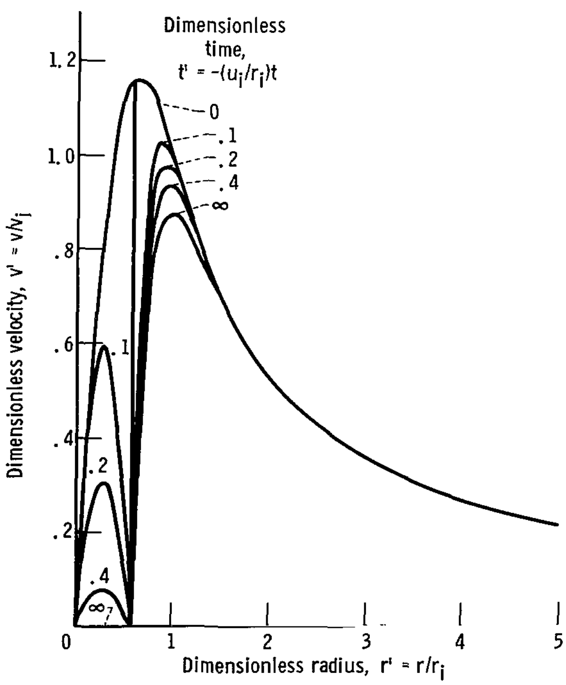
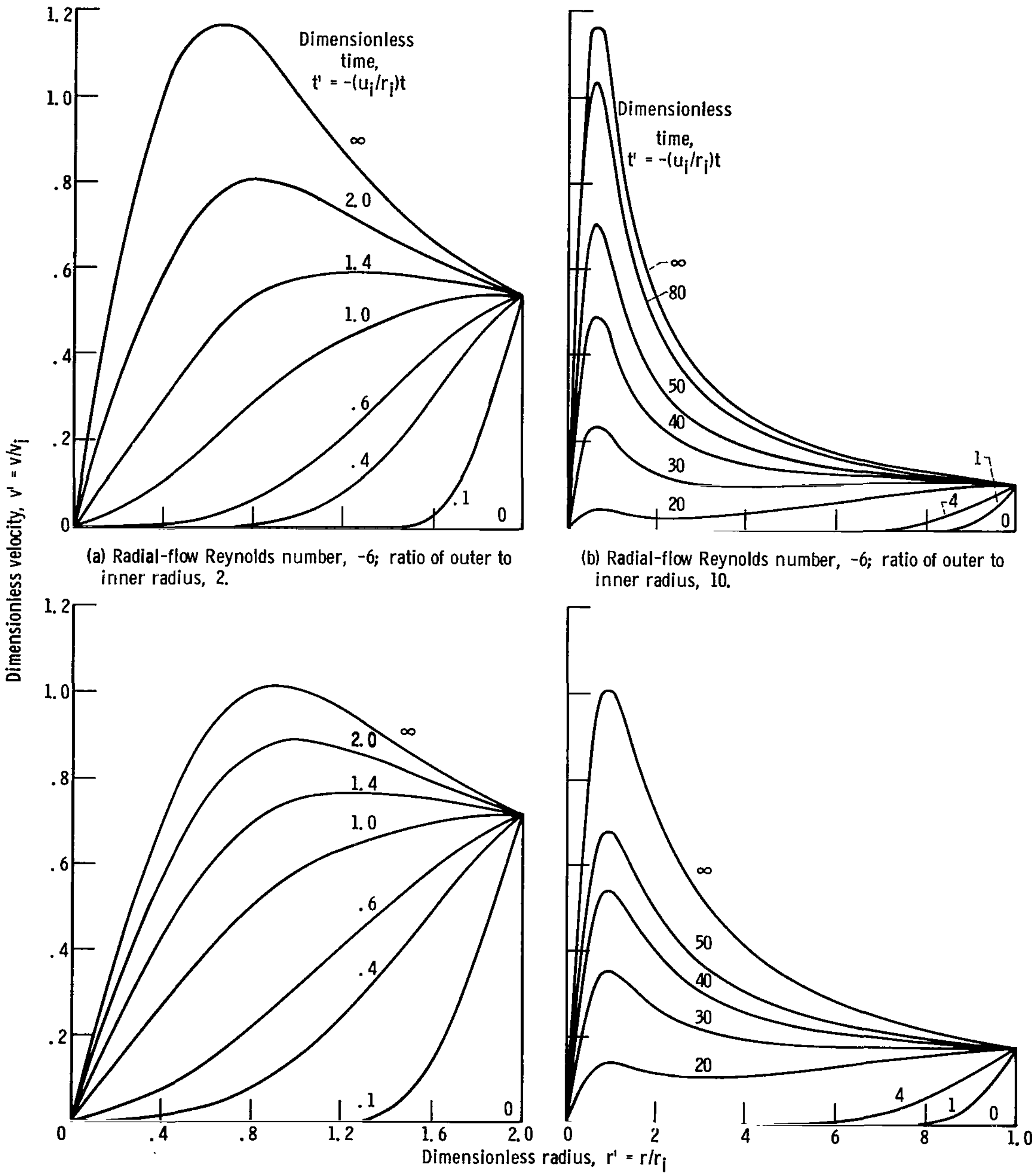


Figure 4. - Response of vortex to zero tangential velocity at point where initial tangential velocity was maximum. Radial-flow Reynolds number, -6.

tions of t' and Re, $t = -t' r_i^2/(\mathrm{Re}\,\nu)$, or in terms of $t'_o \equiv -(u_o/r_o)t = t'/(r_o/r_i)^2$, as discussed earlier in this section, $t = -t'_o r_o^2$ (Reν). For the vortices in figure 3 and typical values for r_o (e.g., $r_o \sim 6$ in.), the decay times may be longer than it takes to empty the vortex container, even if ν is replaced by a turbulent viscosity, which is several times as large as the molecular kinematic viscosity (ref. 8).

In the results obtained thus far, the tangential velocity is reduced to zero at the outer edge of the vortex. Determination of the response of a vortex to a zero tangential velocity at a smaller radius, say at the radius where the initial tangential velocity is a maximum, may be of interest in connection with the possible destruction of atmospheric or other vortices. Results are presented in figure 4. The radial velocity in the annular region is given by equa-



(a) Radial-flow Reynolds number, -6; ratio of outer to inner radius, 2.

(b) Radial-flow Reynolds number, -6; ratio of outer to inner radius, 10.

(c) Radial-flow Reynolds number, -3; ratio of outer to inner radius, 2.

(d) Radial-flow Reynolds number, -3; ratio of outer to inner radius, 10.

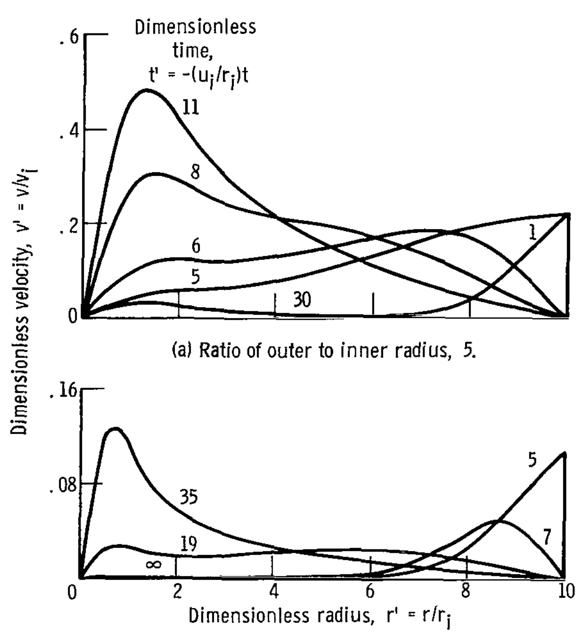
Figure 5. - Growth of vortex produced by vorticity introduced at outer radius.

tion (4) rather than equation (9) in this and in all the succeeding examples. The inner portion of the vortex is quickly dissipated; however, only a small part of the outer portion of the vortex is removed, and the rest remains; that is, the portion of a vortex outside the radius where a zero tangential velocity is applied is affected relatively slightly and quickly adjusts to a new steady-state shape.

Response of Initial Flow Without Vorticity to Vorticity Introduced at Outer Radius

The next example is the inverse of that given in figure 2 (p. 6). The growth of a vortex produced by vorticity introduced at the outer radius in an initially radial and axial flow is calculated. Evidently, the criterion of Lewellen (ref. 12), that the ratio of tangential to sink velocity should be large in order for the model used here to be consistent with the axial momentum equation, will not be satisfied for small duration of time. Since the ratio of tangential to sink velocity is not a parameter in this analysis, that ratio can, however, be made as large as desired (except at $t = 0$), and the time for which the criterion is not satisfied can consequently be made as short as desired.

Results are presented in figure 5 (p. 9), where the initial tangential velocities are zero except at the outer radius. The effect of the applied tangential velocity at the outer radius first propagates throughout the flow field. Then the vortex grows until it becomes fully developed. The trends with radius ratio and radial-flow Reynolds number Re are similar to those in figure 2. Also, the dimensionless time required to grow a fully developed vortex for a particular radius ratio and Reynolds number turns out to be about the same as that which was required to destroy it in figure 2. For instance, comparison of the curves for Re = -6 and $t' = 80$ shows that the vortex in figure 5(b) is about 90 percent developed at a t' of 80, whereas that in figure 2(b) is about 90 percent dissipated. This result does not seem to be intuitively obvious, since the shapes of the changing vortices in the two examples are not the same.



(a) Ratio of outer to inner radius, 5.

(b) Ratio of outer to inner radius, 10.

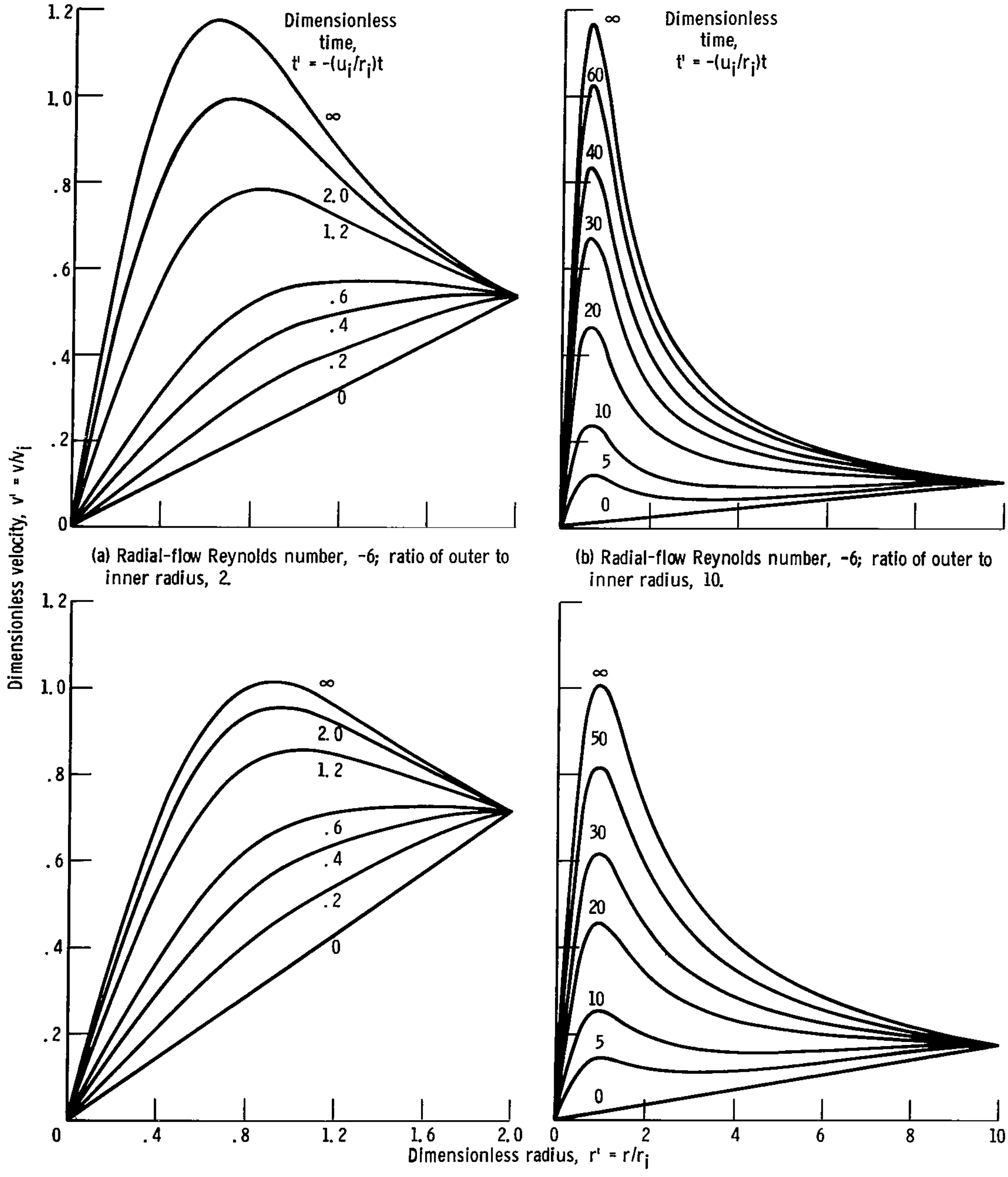
Figure 6. - Formation and decay of vortex resulting from pulse of vorticity. Radial-flow Reynolds number, -6; dimensionless length of vorticity pulse $\Delta t'$, 5.

Finally, consider the formation and decay of a vortex resulting from a pulse of vorticity at the outer radius. The model is the same as that in the preceding case except that, here, the vorticity introduced at $t' = 0$ is cut off at $t' = 5$, or $\Delta t' = 5$. Figure 6 shows results for radius ratios of 5 and 10, and an Re of -6. For both ratios, the tangential velocities near the center continue to rise for a considerable length of time after the cutoff time, that is, after a t' of 5. After cutoff, a hump forms near the outer edge. As time goes on, the outer hump dies out, and another hump near the center begins to grow, reaches a maximum, as shown in figure 6, and finally decays. The maximum height of the center hump is smaller for the larger r_o/r_i, the length of the vorticity pulse being the same. This trend seems to occur because the vorticity pulse had farther to travel with the larger r_o/r_i, and thus had more time to decay. For the smaller r_o/r_i, the maximum height of the center hump is about twice the height of the initial pulse at the outer edge.

Concentration of Initially Uniform Field of Vorticity by Suddenly Applied Radial Inflow

The preceding cases considered the effect of a change in tangential velocity at a given radius in a vortex on the tangential velocities at other radii. This section and the following one consider the effect of a sudden change in radial flow on the tangential velocities. The present example might, for instance, be related to the growth of a tornado. In order for a tornado to form, a widespread area of weak vorticity must initially be present, as in a cyclone. Then a radial inflow must occur to concentrate the vorticity. The same conditions apply to the formation of a vortex in an emptying container. If there is no vorticity present in the fluid, there will, of course, be no vortex formed when the drain plug is removed, if the container is symmetrical. Shear-flow turbulence also involves a concentration of vorticity at various random locations in a shear field. The required radial inflows are produced by stretching vortex filaments in the flow.

Figure 7 (p. 12) shows the effect of a sudden radial inflow on an initially uniform field of vorticity. The initial uniform field of vorticity is a wheel flow ($v'/v'_o = r'/r'_o$). A uniform field of vorticity could also be obtained in a shear flow in which the velocity gradient is uniform. A radial inflow caused by removing fluid at some location would then produce a concentration of vorticity much as for an initial wheel flow, except that the flow would not be axially symmetric. The curves in figure 7 show how the initial wheel flow changes with time and finally becomes a fully developed vortex. The effects of r_o/r_i and Reynolds number on the development of the vortex are similar to those in the preceding figures; the dimensionless growth times are longer for the larger radius ratios and Reynolds numbers. This example is very much like that in figure 5; the difference is that the initial tangential profile is linear in figure 7, whereas in figure 5 the initial tangential velocity is zero except at r'_o, where there is a step increase in v. Thus, the dimensionless growth times in figure 7 are shorter because the boundary layer, which grows from the outer radius in figure 5, has in a sense already developed at $t' = 0$ in



(a) Radial-flow Reynolds number, -6; ratio of outer to inner radius, 2.

(b) Radial-flow Reynolds number, -6; ratio of outer to inner radius, 10.

(c) Radial-flow Reynolds number, -3; ratio of outer to inner radius, 2.

(d) Radial-flow Reynolds number, -3; ratio of outer to inner radius, 10.

Figure 7. - Growth of vortex from initial wheel flow produced by radial inflow.

figure 7. The curve for $t' = 20$ in figure 5(b) (p. 9) corresponds approximately to that for $t' = 0$ in figure 7(b). If, then, 20 is subtracted from the values of t' on the curves in figure 5(b) (for $t' > 20$), those curves correspond closely to the curves in figure 7(b).

The effect of Earth rotation on the results for this case can be taken into account by adding $\Omega \sin \gamma$ to the initial angular velocity of the fluid relative to the Earth, where Ω is the angular velocity of the Earth and γ is the latitude. If the initial vorticity of the fluid relative to the Earth is zero, the fluid will still rotate at the rate $\Omega \sin \gamma$ relative to stationary coordinates, and a weak vortex may be formed when a radial inflow is present. This effect was studied experimentally by Shapiro (ref. 14).

Decay of Fully Developed Vortex When Radial Flow is Suddenly Reduced to Zero

This example is the inverse of the preceding one and is related to the decay of a tornado if the radial flow is reduced to zero or to the decay of the vortex in a rotating container if a plug is inserted in the exit hole. A vortex filament in turbulent flow will also decay when its radial inflow goes to zero.

Results are given in figure 8. The initial fully developed vortex decays with time and approaches wheel flow for large times. The decay is much faster than in figure 2 (p. 6), because the radial inflow, which is absent in figure 8, tends to preserve the initial vortex shape in figure 2.

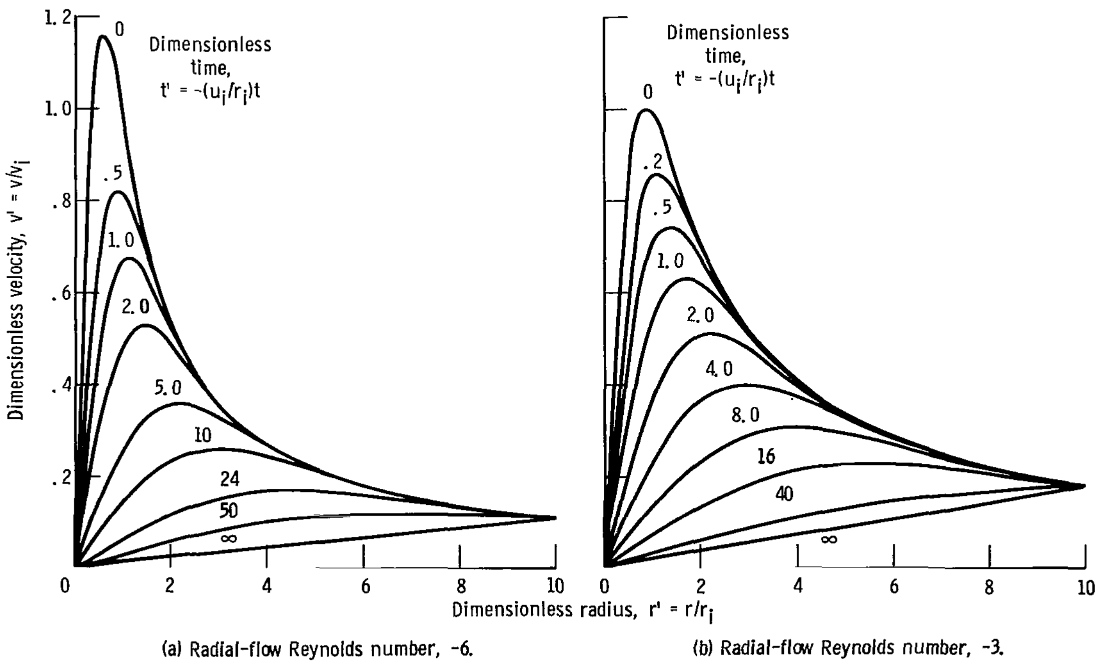


Figure 8. - Decay of fully developed vortex to wheel flow when radial inflow is reduced to zero. Ratio of outer to inner radius, 10.

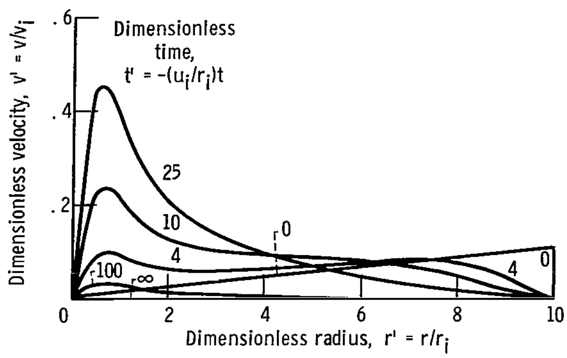


Figure 9. - Initial wheel flow simultaneously subjected to radial inflow and zero tangential velocity at outer radius. Radial-flow Reynolds number, -6; ratio of outer to inner radius, 10.

Initial Wheel Flow Simultaneously Subjected to Radial Inflow and Zero Tangential Velocity at the Outer Radius

The two types of step changes considered in in the preceding sections are combined in this example. The novel feature is that both a growth and a decay of a vortex are obtained by step changes introduced at one time as illustrated in figure 9 for an r_o/r_i of 10 and a Reynolds number of -6. The vortex first begins to grow because of the radial inflow, as in figure 7. This growth continues until the boundary layer due to the zero tangential velocity at the outer radius penetrates the inner region. At that time the vortex begins to decay, as in figure 2 (p. 6), and this decay continues until the tangential velocities are zero at all radii. This phenomenon is somewhat like the growth and decay of the vortex in a nonrotating container when the fluid contains initial vorticity, and the plug in the exit hole is removed.

CONCLUDING REMARKS

If the tangential velocity at the outer edge of a fully developed vortex with radial flow is suddenly reduced to zero, a boundary layer from the outer edge first propagates into the vortex. After the initial adjustment period, the shape of the vortex remains similar until the tangential velocities at all radii are zero. The growth time for a vortex produced by vorticity suddenly introduced at the outer radius of a nonrotating flow is about the same as the decay time for the fully developed vortex just mentioned.

The growth and decay times for a vortex increase with increasing radius ratio and radial-flow Reynolds number if u_i/r_i is held constant. If, however, u_o/r_o is held constant, the growth and decay times are not greatly affected by radius ratio (or fully developed profile shape). The decay times for a vortex in which the radial inflow is produced by allowing the fluid to enter the vortex uniformly at one end are longer than those obtained when the flow comes in through a porous cylindrical wall. The portion of a vortex outside the radius where a zero tangential velocity is applied is affected relatively slightly by the change and quickly approaches a new steady-state shape. If a pulse of vorticity is applied at the outer edge of a nonrotating radial inflow, a vortex begins to grow, continues to grow for some time after the vorticity is cut off, and then decays.

The growth of a vortex from an initial wheel flow with a suddenly applied inward radial velocity is similar to that produced by vorticity introduced at the outer edge of a nonrotating flow, except that the growth times are somewhat shorter for the initial wheel flow. This shorter growth time results because the boundary layer which grows from the outer edge in the initially nonrotating flow is in a sense already developed when flow is started from a wheel flow. The time required for a fully developed vortex to decay to a wheel flow when the radial velocity is reduced to zero is shorter than that required when the decay is produced by a zero tangential velocity at the outer radius. The radial flow in the latter example tends to preserve the fully developed shape. If a wheel flow is simultaneously subjected to a radial inflow and a zero tangential velocity at the outer radius, the resulting vortex first grows and then decays as the boundary layer from the outer radius reaches the inner portions of the vortex.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 20, 1965.

APPENDIX - SYMBOLS

Re radial-flow Reynolds number, $u_i r_i/\upsilon$

r radial coordinate

r' r/r_i

t time

t' dimensionless time, $-(u_i/r_i)t$

t'_o $-(u_o/r_o)t$

u radial velocity

u' u/u_i

v tangential velocity

v' v/v_i

v_i fully developed tangential velocity at r_i

w axial velocity

z axial coordinate

υ kinematic viscosity

Subscripts:

i inner radius (fig. 1)

o outer radius (fig. 1)

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NASA-Langley, 1965 E-2938

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