NASA TECHNICAL
MEMORANDUM

NASA TM X-53249
April 23, 1965

NASA TM X-53249

NEAR-OPTIMUM GUIDANCE - AN ANALYSIS OF FUEL PENALTY

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ABSTRACT

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An explicit solution of the linearized equations of motion is used to obtain the deviations in end conditions, Δr and $\Delta\theta$, and the additional burning time, Δt, expressed as integrals of functions of thrust angle deviations, $\Delta\chi$. Under the constraint that $\Delta r = \Delta\theta = 0$, the calculus of variations is applied to minimize Δt. The resulting Euler-Lagrange equation evaluated at $\Delta\chi = 0$ gives a simple relationship which is used to show that Δt is a second order function of $\Delta\chi$. This function is evaluated numerically for the second stage of an early SA-6 design. Results show that, if the mission is accomplished, the thrust angle may differ by as much as 2 degrees throughout the second stage with a propellant penalty of only 34 pounds. It follows that the capability of meeting the mission is the prime requisite of a guidance function and optimality is then only a second order consideration.

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Technical Memorandum X-53249

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TECHNICAL AND SCIENTIFIC STAFF
AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

TABLE OF CONTENTS

DEFINITION OF SYMBOLS

Symbol	Definition
t	time measured on a standard trajectory
t_o	second stage ignition time on the standard trajectory
t_n	second stage cutoff time on the standard trajectory
$\dot{W}$	propellant flow rate
Δr	radius error at cutoff
Δt	deviation in second stage burning time
Δt_o	deviation in second stage ignition time from standard
ΔW	deviation in propellant consumption from standard
$\Delta\theta$	error in angle between the velocity vector and the position vector at cutoff
$\Delta\chi$	$\chi(t + \Delta t_o) - \chi_s(t)$
χ	thrust angle
χ_s	thrust angle on the standard trajectory

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SUMMARY

An explicit solution of the linearized equations of motion is used to obtain the deviations in end conditions, Δr and $\Delta\theta$, and the additional burning time, Δt, expressed as integrals of functions of thrust angle deviations, $\Delta\chi$. Under the constraint that $\Delta r = \Delta\theta = 0$, the calculus of variations is applied to minimize Δt. The resulting Euler-Lagrange equation evaluated at $\Delta\chi = 0$ gives a simple relationship which is used to show that Δt is a second order function of $\Delta\chi$. This function is evaluated numerically for the second stage of an early SA-6 design. Results show that, if the mission is accomplished, the thrust angle may differ by as much as 2 degrees throughout the second stage with a propellant penalty of only 34 pounds. It follows that the capability of meeting the mission is the prime requisite of a guidance function and optimality is then only a second order consideration.

I. INTRODUCTION

The task of constructing a guidance function to determine the thrust angle, χ, that meets a given mission under the constraint of minimum propellant consumption can be considerably simplified by first investigating the penalty for non-optimality. It is assumed that minimum propellant consumption is equivalent to minimum burning time, and it will be shown that any additional burning time required is a second order function of thrust angle deviations from optimum provided the mission is met. A numerical example is included which covers the second stage of a 100 n.m. orbital mission for an early SA-6 design. The results show that any guidance function which meets the required end conditions is acceptable even though it differs from the calculus of variations solution by as much as one or two degrees throughout the entire second stage.

II. EULER-LAGRANGE EQUATIONS

It is assumed that the calculus of variations solution is known for a given trajectory. If the thrust angle, χ, deviates slightly from optimum, it has been shown in Reference 1 that the errors in end conditions Δr and $\Delta\theta$ and the additional burning time Δt can be adequately expressed as follows:

$$\Delta r = \int_{t_o}^{t_n} f(\Delta X, t)\, dt, \tag{1}$$

$$\Delta\theta = \int_{t_o}^{t_n} g(\Delta X, t)\, dt, \tag{2}$$

$$\Delta t = \int_{t_o}^{t_n} h(\Delta X, t)\, dt, \tag{3}$$

where

$$f(\Delta X, t) = f_1(t)\,\Delta X + f_2(t)\,\Delta X^2 + \ldots\;, \tag{4}$$

$$g(\Delta X, t) = g_1(t)\,\Delta X + g_2(t)\,\Delta X^2 + \ldots\;, \tag{5}$$

$$h(\Delta X, t) = h_1(t)\,\Delta X + h_2(t)\,\Delta X^2 + \ldots\;. \tag{6}$$

Minimizing equation (3) under the constraint that equations (1) and (2) satisfy prescribed conditions leads to the Euler-Lagrange equation [2, p. 51]

$$\frac{\partial f^*}{\partial \Delta X} - \frac{d}{dt}\frac{\partial f^*}{\partial \Delta \dot{X}} = 0 \tag{7}$$

where

$$f^* = h + \lambda_1 f + \lambda_2 g. \tag{8}$$

Since f^* is independent of $\dot{\Delta X}$, equation (7) degenerates to the following:

$$\frac{\partial f^*}{\partial \Delta X} = \frac{\partial h}{\partial \Delta X} + \lambda_1 \frac{\partial f}{\partial \Delta X} + \lambda_2 \frac{\partial g}{\partial \Delta X} = 0,$$

which yields the following necessary condition:

$$(h_1 + \lambda_1 f_1 + \lambda_2 g_1) + 2(h_2 + \lambda_1 f_2 + \lambda_2 g_2)\, \Delta X + \ldots = 0. \qquad (9)$$

Since the standard was assumed to be a calculus of variations solution, equation (9) is satisfied for $\Delta X = 0$ and the following relationship is determined:

$$h_1 + \lambda_1 f_1 + \lambda_2 g_1 = 0. \qquad (10)$$

Table I shows the numerical values obtained for h_1, f_1, and g_1. The following values of λ_1 and λ_2 were obtained by the method of least squares.

λ_1 = -.08734 sec/km

λ_2 = .8589 sec/deg.

These values satisfy equation (10) within the numerical accuracy with which h_1, f_1, and g_1 were determined. This is illustrated in Table I where the quantity $h_1 + \lambda_1 f_1 + \lambda_2 g_1$ is shown.

III. SECOND ORDER EFFECT

From equation (10), it follows that

$$h_1 \Delta X + \lambda_1 f_1 \Delta X + \lambda_2 g_1 \Delta X = 0$$

TABLE I

t (sec)	f_1 (10^{-2} km/deg sec)	g_1 (10^{-2}/sec)	h_1 (10^{-2}/deg)	$h_1 + \lambda_1 f_1 + \lambda_2 g_1$ (10^{-2}/deg)
160	-3.680	.0454	-.3610	-.0006
200	-3.738	.0536	-.3730	-.0005
240	-3.772	.0628	-.3836	-.0002
280	-3.774	.0732	-.3927	-.0002
320	-3.738	.0850	-.3996	-.0001
360	-3.654	.0986	-.4038	.0000
400	-3.507	.1144	-.4044	.0002
440	-3.280	.1330	-.4005	.0002
480	-2.944	.1554	-.3903	.0003
520	-2.458	.1828	-.3715	.0002
560	-1.758	.2177	-.3405	.0000
600	- .727	.2643	-.2907	-.0002

λ_1 = -.08734 sec/km

λ_2 = .8589 sec/deg.

and

$$\int_{t_o}^{t_n} h_1 \Delta X \, dt = -\lambda_1 \int_{t_o}^{t_n} f_1 \Delta X \, dt - \lambda_2 \int_{t_o}^{t_n} g_1 \Delta X \, dt. \qquad (11)$$

From equations (4) and (5) together with equations (1) and (2), the following relationships are determined:

$$- \lambda_1 \int_{t_o}^{t_n} f_1 \Delta X \, dt = -\lambda_1 \Delta r + \lambda_1 \int_{t_o}^{t_n} [f_2 \Delta X^2 + \ldots] \, dt,$$

and

$$- \lambda_2 \int_{t_o}^{t_n} g_1 \Delta X \, dt = - \lambda_2 \, \Delta\theta + \lambda_2 \int_{t_o}^{t_n} [g_2 \Delta X^2 + \ldots] \, dt.$$

Substituting these values into equation (11) yields

$$\int_{t_o}^{t_n} h_1 \Delta X \, dt = - \lambda_1 \Delta r - \lambda_2 \Delta\theta + \int_{t_o}^{t_n} [(\lambda_1 f_2 + \lambda_2 g_2) \, \Delta X^2 + \ldots] \, dt.$$

This, together with equations (3) and (6), gives the following expression for Δt.

$$\Delta t = -\lambda_1 \Delta r - \lambda_2 \Delta\theta + \int_{t_o}^{t_n} [(h_2 + \lambda_1 f_2 + \lambda_2 g_2) \, \Delta X^2 + \ldots] \, dt. \qquad (12)$$

If the required end conditions are met, $\Delta r = \Delta\theta = 0$. Then,

$$\Delta t = \int_{t_o}^{t_n} [(h_2 + \lambda_1 f_2 + \lambda_2 g_2) \Delta X^2 + \dots] \, dt, \tag{13}$$

and the additional burning time Δt resulting from non-optimum X is a second order function of ΔX. The sufficient condition that t is a local minimum is that $h_2 + \lambda_1 f_2 + \lambda_2 g_2 > 0$, $t_o \leqq t \leqq t_n$.

IV. NUMERICAL RESULTS

Table II shows the numerical values obtained for h_2, f_2, g_2 and $(h_2 + \lambda_1 f_2 + \lambda_2 g_2)$. In addition, the quantity $h_2^*(t_i) = (h_2 + \lambda_1 f_2 + \lambda_2 g_2) \Delta t_i$ is shown in Table III. This quantity can be used to obtain a good approximation to the integral of the second ordered term in equation (13) by the following summation.

$$\Delta t = \sum_{i=1}^{12} h_2^*(t_i) \, \Delta X^2(t_i). \tag{14}$$

This expression, in turn, has the following upper bound:

$$\Delta t \leqq \Delta X_m^2 \sum_{i=1}^{12} h_2^*(t_i), \tag{15}$$

which, for this example, is

$$\Delta t \leqq .0409 \quad \Delta X_m^2, \tag{16}$$

where ΔX_m is the maximum value of $|\Delta X(t)|$, $t_o \leqq t \leqq t_n$.

The propellant loss ΔW can be expressed as follows:

$$\Delta W = \dot{W} \Delta t.$$

For this example, $\dot{W} = 208$ lbs/sec, which yields the following bound:

$$\Delta W \leqq 8.51 \, \Delta X_m^2 \ (\text{lbs/deg}^2). \tag{17}$$

TABLE II

t (sec)	f_2 (10^{-3}km/deg^{2}sec)	g_2 (10^{-5}/deg sec)	h_2 (10^{-4}/deg^2)	$(h_2 + \lambda_1 f_2 + \lambda_2 g_2)$ (10^{-4}/deg^2)
160	-.3831	.8899	.2168	.6278
200	-.3516	.8798	.2606	.6433
240	-.3194	.8663	.3094	.6628
280	-.2863	.8482	.3641	.6870
320	-.2524	.8239	.4260	.7172
360	-.2175	.7915	.4969	.7548
400	-.1819	.7483	.5789	.8021
440	-.1458	.6907	.6755	.8622
480	-.1096	.6131	.7915	.9399
520	-.0742	.5074	.9344	1.0428
560	-.0408	.3602	1.1167	1.1832
600	-.0118	.1485	1.3604	1.3834

$\lambda_1 = -.08734$

$\lambda_2 = .8589$ sec/deg

TABLE III

t_i (sec)	Δt_i (sec)	$h_2^*(t_i)$ (10^{-2} sec/deg^2)
160	33.19	.2084
200	40	.2573
240	40	.2651
280	40	.2748
320	40	.2869
360	40	.3019
400	40	.3208
440	40	.3449
480	40	.3760
520	40	.4171
560	40	.4733
600	40.68	.5628

$$\sum_{i=1}^{12} h_2^*(t_i) = 4.0893 \times 10^{-2} \text{ sec/deg}^2$$

V. CONCLUSIONS

Equation (17) shows that the propellant penalty from non-optimum guidance is less than 34 pounds even if the thrust angle deviates from optimum by as much as 2 degrees throughout the second stage. This clearly indicates that, if a guidance function can be determined which meets the mission within satisfactory limits and predicts χ within one or two degrees of the optimum value, there is very little to be gained by expending much effort to further reduce the prediction error in χ.

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