X-643-65-430

NASA TM X-55390

INJECTION CONDITIONS FOR LUNAR TRAJECTORIES

BY
RONALD KOLENKIEWICZ
WILLIAM PUTNEY

NOVEMBER 1965

NASA

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-643-65-430

INJECTION CONDITIONS FOR LUNAR TRAJECTORIES

by

Ronald Kolenkiewicz
William Putney

November 1965

Goddard Space Flight Center
Greenbelt, Maryland

INJECTION CONDITIONS FOR LUNAR TRAJECTORIES

ABSTRACT

This paper reviews the mechanics of Earth to Moon trajectories as affected by geometric considerations and booster capabilities. It formulates and describes the equations for a computing procedure which, using the two body equations of motion, provides approximate initial injection conditions near the earth for either a fixed time of arrival at the Moon or a fixed time of flight to the Moon. Consideration of multiple circular parking orbits, arbitrary injection elevation angles, arbitrary launch site, and booster burning characteristics are also taken into account. The trajectory followed is an elliptical Earth to Moon transfer trajectory, with respect to the Earth, which intersects the Moon before apo-apsis. A digital computer program having the equations programmed in FORTRAN II is available.

CONTENTS

LIST OF ILLUSTRATIONS

INJECTION CONDITIONS FOR LUNAR TRAJECTORIES

I. INTRODUCTION

A calculating procedure, using two body equations of motion, has been developed which provides approximate injection conditions for earth to moon trajectories. The resulting injection conditions are intended for use as a first guess in generating precision trajectories. When the results obtained are used in a precision digital computer program (for example ITEM, see Reference 1) they typically produce a lunar impact. They can be further adjusted by iterating in the precision program to provide a specified miss distance for such missions as a lunar orbiter.

The procedure presented builds on and extends the work of References 2 to 4. In Reference 2 a study was made to determine the effects on lunar trajectories of some typical geometric constraints. This study concludes that application of the various constraints seriously restricts the allowable launch times during the month and day for direct-ascent launch; whereas, less serious restrictions result for the parking orbit launches. This reference also presents some of the equations for the calculation of geometrical parameters involved. Reference 3 further justifies the parking orbit type of trajectory. It also presents some of the equations necessary for matching the powered phases of the trajectory to the geometrical constraints. Reference 4 gives some general discussion on lunar trajectories.

The present paper adds the consideration of multiple circular parking orbits, arbitrary injection elevation (or flight path) angle, and booster burning characteristics. Injection conditions can be determined for either fixed time of arrival at the moon or a fixed flight time to the moon. The mechanics of earth to moon trajectories as affected by geometrical considerations and booster capabilities are reviewed, and the working equations which are used in a computer program are formulated and described.

II. NOMENCLATURE

x, y, z	Components of geocentric equatorial coordinate system (Figure 2).
$\bar{I}, \bar{J}, \bar{K}$	Unit vectors along geocentric equatorial coordinate system (Figure 2).
$\bar{r}_l$	Unit vector to launch site.
t_l	Launch time, hours.
D_l	Number of days from January 0.0 U.T. 1960 to 0.0 U.T. on the day of launch.
$\bar{r}_m$	Unit vector to the Moon.
r_{mx}, r_{my}, r_{mz}	Components of the unit position vector to the Moon.
t_m	Time of impact at the Moon, hours.
D_m	Number of days from January 0.0 U.T. 1960 to 0.0 U.T. on the day of arrival at Moon.
i	Inclination.
A	Azimuth (geocentric).
α	Right ascension.
δ	Declination.
$\bar{w}$	Unit vector normal to the orbit plane.
w_x, w_y, w_z	Components of the unit vector normal to the orbit plane (Figure 2).
η	Angle between the $\bar{K}$ and $\bar{w} \times \bar{r}_l$ vectors, see Figure 2.
Ψ	Mean longitude of Sun's apparent motion around the Earth defined by Equation (14), radians.
L	Longitude (terrestrial).

ω	Absolute rate of spin of the Earth.
T_t	Total time of flight, defined by Equation (33), hours.
R_m	Distance from the center of the Earth to the Moon.
T_{b_1}	First boost burning time, $(t_1 - t_l)$, hours.
ζ_{b_1}	First boost angle, Earth centered angle between $\overline{R}_l$ and $\overline{R}_1$.
t_1	Time of injection into parking orbit, hours.
t_{po}	Time in parking orbit, $(t_2 - t_1)$, hours.
C_1	Reciprocal of the parking orbit rate, defined in Equation (18).
ζ_{po}	Parking orbit angle, Earth centered angle between $\overline{R}_1$ and $\overline{R}_2$.
μ	Earth's gravitational parameter.
R_1	Distance from the center of the Earth to the parking orbit injection.
ξ	Angular distance (Figure 5).
ζ_{b_2}	Second boost angle, Earth centered angle between $\overline{R}_2$ and $\overline{R}_i$.
ζ_f	Translunar trajectory angle, Earth centered angle between $\overline{R}_i$ and $\overline{R}_m$.
T_{b_2}	Second boost burning time, $(t_i - t_2)$, hours.
t_i	Time of injection into translunar trajectory, hours.
t_2	Time at beginning of second boost, hours.
T_f	Time in translunar trajectory, $(t_m - t_i)$, hours.
a	Semimajor axis.
M	Mean anomaly.
e	Eccentricity.

E	Eccentric anomaly.
p	Semilatus rectum of an ellipse, $a(1 - e^2)$.
ν	True anomaly.
R_i	Distance from the center of the Earth to injection into trans-lunar trajectory.
$\tilde{R}$	Ratio of injection distance, R_i, to moon's distance, R_m.
γ_i	Injection elevation angle (See Figure 5).
V_i	Injection velocity.
V_p	Parabolic velocity.
$\tilde{V}$	Ratio of injection velocity, V_i, to parabolic velocity, V_p.
$R_1(\theta)$	Rotation matrix, defined by Equation (42).
$R_2(\theta)$	Rotation matrix, defined by Equation (43).
$R_3(\theta)$	Rotation matrix, defined by Equation (44).
Y_m	Year of lunar impact.
R_l	Distance from the center of the Earth to launch site.
h_{po}	Parking orbit altitude.
h_i	Injection altitude.
V_c	Circular velocity.
V_m	Lunar impact velocity.
R_e	Earth's radius (taken as 6378.165 km).
V_{cm}	Circular velocity at lunar distance.
T_{tA}	Equivalent to T_t defined by Equation (15).

T_{tB}	Equivalent to T_t defined by Equation (33).
ΔT_t	Total time difference, defined by Equation (38).
$\tilde{V}_1$	Minimum value of the velocity ratio, defined by Equation (34).
$\tilde{V}_2$	Maximum velocity ratio, defined by Equation (35).
$(\Delta T_t)_1$	Total time difference, Equation (38), for $\tilde{V}_1$, see Figure 6.
$(\Delta T_t)_2$	Total time difference, Equation (38), for $\check{V}_2$, see Figure 6.
$\tilde{V}_{k+1}$	Second and succeeding assumed velocity ratios, defined by Equation (39).
$\tilde{V}_f$	Final value of assumed velocity ratio, see Figure 6.
S	Parameter defined by Equation (37).
R_{mx}	x component of the moon position vector.
R_{my}	y component of the moon position vector.
R_{mz}	z component of the moon position vector.

SUBSCRIPTS

l	Launch.
1	Start of parking orbit.
2	End of parking orbit.
i	Injection.
m	Moon.
n	The orbit number.

III. GENERAL DISCUSSION OF THE PROBLEM

A. Requirements for Translunar Trajectory

In general, there are two requirements that must be met for launching a vehicle from the surface of the Earth to the vicinity of the Moon with a minimal amount of energy.

The first is that the vehicle be launched into a plane that is common to the launch site at launch time and the Moon at impact time. To launch in a plane other than this necessitates changing the plane of the trajectory after launch. This entails a reduction in the payload that will arrive in the vicinity of the Moon since additional fuel is required to accomplish this maneuver.

The second requirement is that a specific angular relationship must exist in the plane between the center of Earth and launch site and the line of centers of Earth and Moon. This comes about from the fact that there are a number of restrictions on various lunar missions such as booster capabilities, allowable time in parking orbit, and the time requirements on the mission.

The following sections will discuss the type of trajectory that must be followed to satisfy requirements on both launch plane and angular travel in the plane.

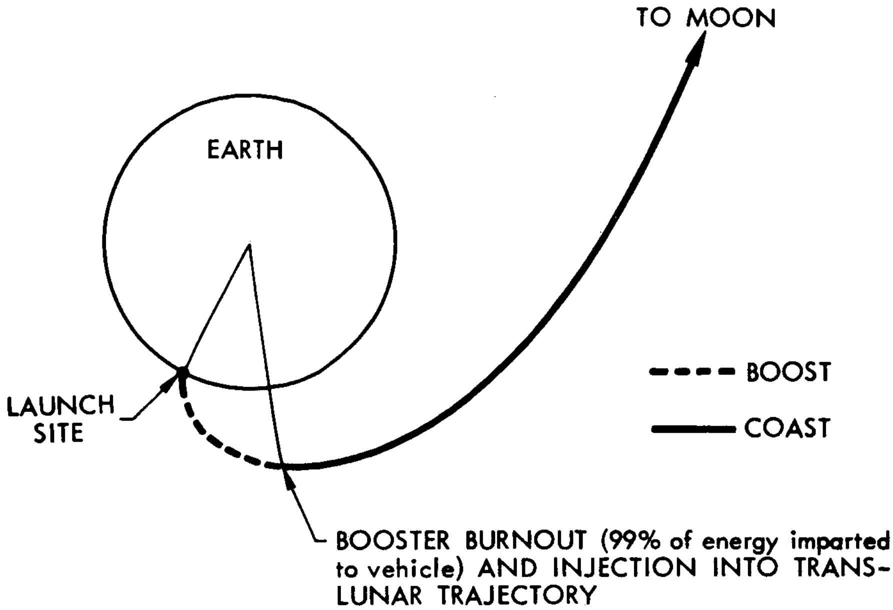
B. Launch Considerations

In general, for any given launch azimuth there are two times a day a vehicle may be launched from the Earth into a trajectory that would take it to the Moon. The exceptions occur only when the absolute value of the Moon's declination is greater or equal to the inclination of the trajectory plane; then, either none or at most one launch per day will be possible.

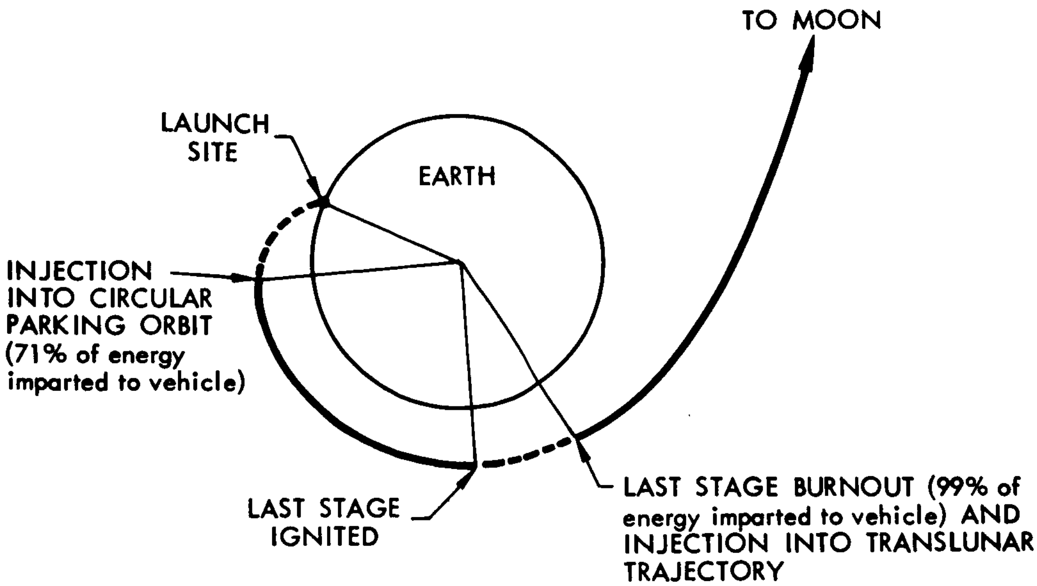
C. Boost Considerations

There are two possible methods for leaving the surface of the Earth and going to the vicinity of the Moon. The first is lunar injection from a direct launch; the second is lunar injection from a parking orbit. These trajectories are shown in Figure 1. Both the lunar injection from a direct launch as well as from a parking orbit have been discussed in previous papers (References 2 and 3).

In this paper attention will be focused on trajectories that are launched from the surface of the Earth and employ a parking orbit before being injected into a translunar trajectory. It will also be restricted to an elliptical Earth to Moon



(a) Lunar injection from a direct launch



(b) Lunar injection from a parking orbit

Figure 1—Trajectories for a lunar mission

transfer trajectory, with respect to the Earth, which intersects the Moon before the apo-apsis.

D. Parking Orbit Consideration

It is envisioned that capability must be provided for the parking orbit to include more than one revolution around the Earth. The purpose of this is to provide time to check out the various systems on-board the spacecraft and to establish the orbit elements so the position, time and velocity necessary for sending the spacecraft to the Moon can be calculated. In general, the spacecraft may be injected to its translunar trajectory once per revolution from its parking orbit.

IV. SOLUTION OF THE PROBLEM

A. Time of Launch into Proper Plane

As stated previously, one of the requirements that must be satisfied for a vehicle leaving the surface of the Earth and arriving in the vicinity of the Moon with a minimum amount of energy expended is that the vehicle be launched into a plane that is common to the launch site at the time of launch and the Moon at the time of impact. This plane may be defined if the declination of the launch site, the launch azimuth, the position of the Moon at the desired day and time of impact are given. In addition, the times of launch per day into this plane may be found if the longitude of the launch site and the desired day of launch are known. In this section the equations for finding the time of launch will be presented.

Consider the geocentric equatorial coordinate system, x, y, and z shown in Figure 2. Associated with these coordinates are the unit vectors $\bar{I}$, $\bar{J}$, and $\bar{K}$.

The unit vector $\bar{r}_l$ is directed toward the launch site at time of launch, t_l, on the day of launch, D_l, and the unit vector $\bar{r}_m$ is directed toward the Moon at time of impact, t_m, on the day of impact, D_m. The plane that contains both unit vectors has an inclination angle, i, at the equator and an azimuth angle, A_l, at the launch site. The launch site is defined by the right ascension, α_l, and the declination, δ_l. The unit vector $\bar{w}$ is normal to the plane defined by $\bar{r}_l$ and $\bar{r}_m$.

The following equations may be written

$$w_z = \cos i = \cos \delta_l \sin A_l \qquad (1)$$

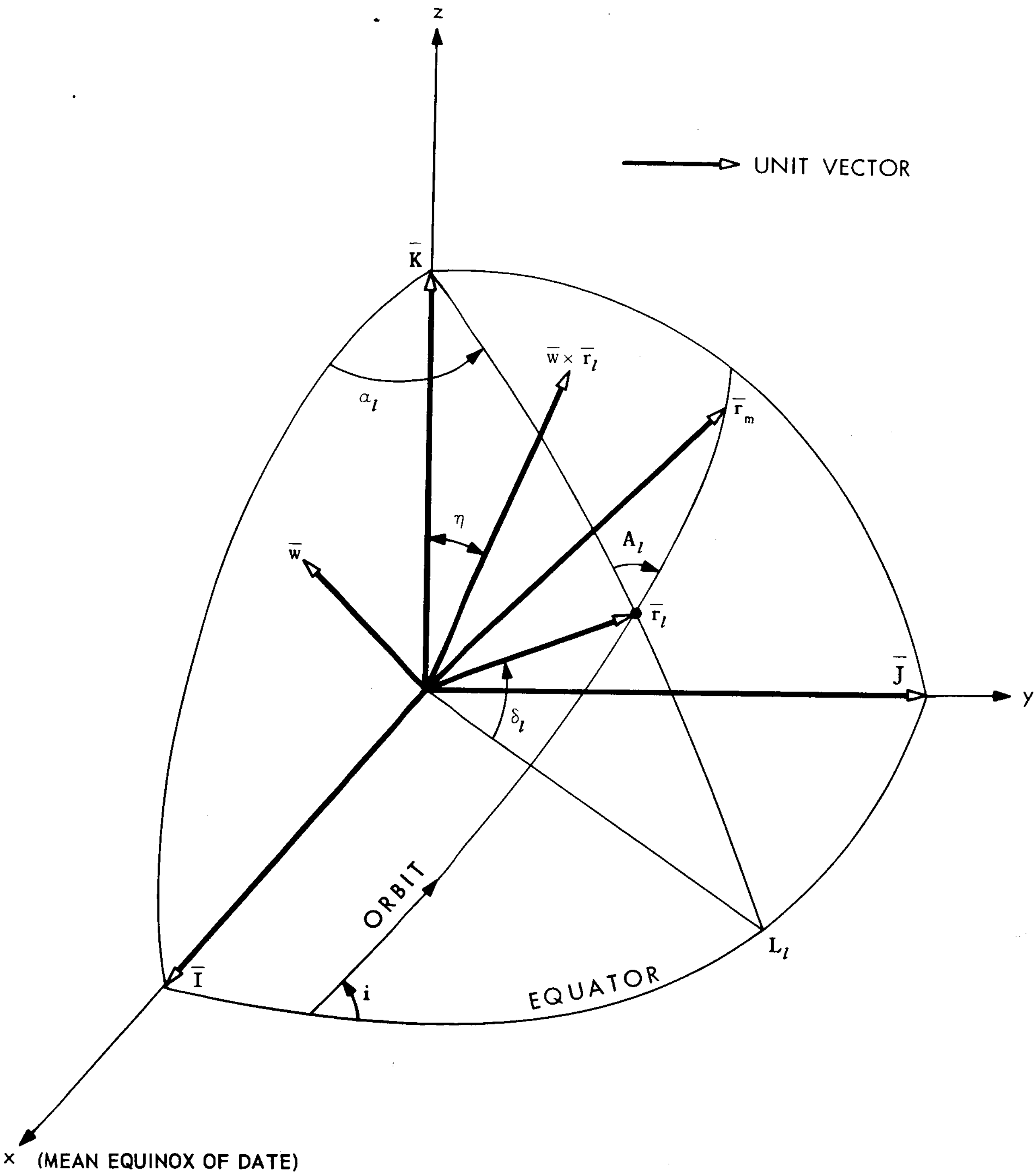


Figure 2– Coordinate system

which is derived in Appendix A

$$\bar{w} \cdot \bar{r}_m = 0 \tag{2}$$

$$\bar{w} \cdot \bar{w} = 1 \tag{3}$$

where

$$\bar{w} = \bar{I}\, w_x + \bar{J}\, w_y + \bar{K}\, w_z \tag{4}$$

and

$$\bar{r}_m = \bar{I}\, r_{mx} + \bar{J}\, r_{my} + \bar{K}\, r_{mz} \tag{5}$$

Equations (2) and (3) may be solved simultaneously (Appendix A) to yield

$$w_y = \frac{-w_z\, r_{my}\, r_{mz} \pm r_{mx}\sqrt{1 - r_{mz}^2 - w_z^2}}{r_{mx}^2 + r_{my}^2} \tag{6}$$

$$w_x = \frac{-w_y\, r_{my} - w_z\, r_{mz}}{r_{mx}} \tag{7}$$

Examining Equations (1), (6), and (7), it is seen that the launch declination, launch azimuth and position of the Moon at impact define two planes that are fixed in space. Launch into these planes is possible when the launch site on the rotating earth passes through the planes and the launch azimuth is in the plane. This occurs, in general, twice each day. Figure 3 is included to further clarify the geometry of the problem. Launch will be possible once a day or not at all depending on whether the value of the radical in Equation (6) is zero or imaginary, respectively.

Referring to Figure 2, additional equations that may be written are:

$$\bar{r}_l \cdot \bar{w} = 0 \tag{8}$$

and

$$(\bar{w} \times \bar{r}_l) \cdot \bar{K} = \cos \eta = \cos \delta_l \cos A_l \tag{9}$$

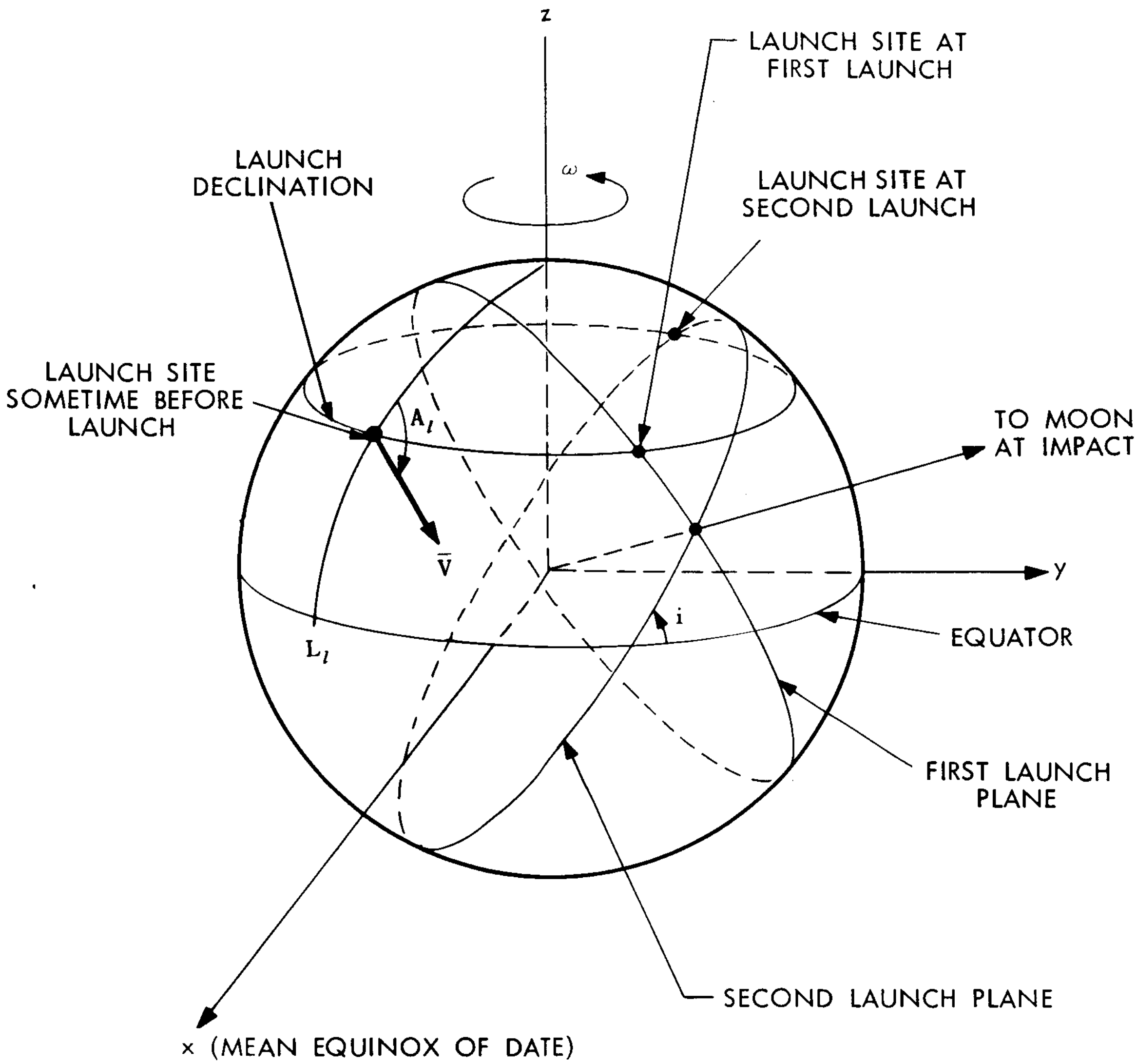


Figure 3—Geometry of the launch planes

which is derived in Appendix A where

$$\bar{r}_l = \bar{I} \cos \alpha_l \cos \delta_l + \bar{J} \sin \alpha_l \cos \delta_l + \bar{K} \sin \delta_l \tag{10}$$

Equations (8) and (9) may be solved simultaneously (Appendix A) to yield

$$\cos \alpha_l = \frac{w_z w_x \sin \delta_l + w_y \cos \delta_l \cos A_l}{(w_z^2 - 1) \cos \delta_l} \tag{11}$$

$$\sin \alpha_l = \frac{w_y w_z \sin \delta_l - w_x \cos \delta_l \cos A_l}{(w_z^2 - 1) \cos \delta_l} \tag{12}$$

and the right ascension, α_l, at the time of launch may be found. Figure 4 shows the relationships that exist between the time and angles on the day of launch. The following equation may be written for the time of launch on the launch day.

$$t_l = \frac{\alpha_l - \Psi - L_l}{\omega} \tag{13}$$

where ω is the Earth's absolute rate of spin per mean solar hour, the angle Ψ is the mean sidereal time at Greenwich, given by the expression

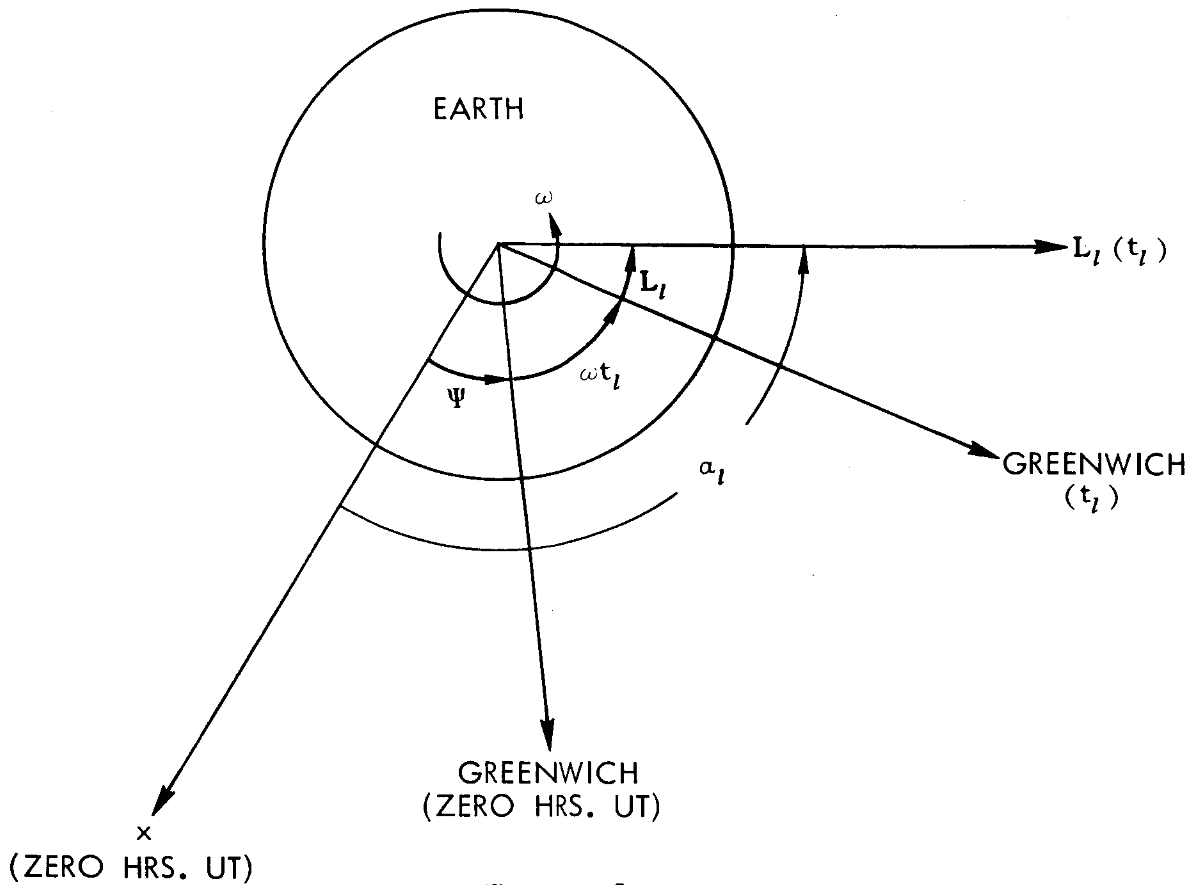
$$\Psi = 1.72218633 + 1.720279168 \times 10^{-2} D_l + .67558729 \times 10^{-5} \left(\frac{D_l}{36525}\right)^2 \tag{14}$$

where Ψ is in radians and D_l is the number of days from January 0.0 U.T. 1960 to 0.0 U.T. on the day of launch.

Equation (14) is Newcomb's expression given in Reference (5) with the units changed to radian measure and the reference time changed from Noon 1900 January 0 at the Greenwich meridian to Midnight 1960 January 0 at the Greenwich meridian (Appendix A).

Since the time of lunar impact was assumed and the time of launch calculated, the total time of the flight in hours may be obtained

$$T_t = t_m - t_l + 24 (D_m - D_l) \equiv T_{tA} \tag{15}$$



$$\alpha_l = \Psi + \omega t_l + L_l$$

Figure 4–Position of the launch site relative to Greenwich

In summary, the time of launch for a vehicle going from the surface of the Earth to the Moon without a plane change may be found if the launch declination, longitude and azimuth at the time of launch as well as the position of the Moon at the time of impact are known.

B. Motion in the Plane

Once the launch time is established, the vehicle can be launched into the proper plane that will intersect the Moon at the time of impact. Launch into the proper plane is a necessary but not a sufficient requirement for the vehicle to impact the Moon at the preselected time of impact. Figure 5 shows the geometry in the space fixed plane in which the vehicle is moving after launch. At the preselected time of impact with the Moon, t_m, the Moon passes through the plane at the position denoted by the vector $\bar{R}_m$. In order to impact the Moon, the vehicle must arrive at the proper position in the plane at exactly the right moment. Since a parking orbit approach is to be used in this paper, the problem of the in plane motion may be broken into four phases: the first boost, the parking orbit, the second boost, and the translunar trajectory. The conditions for a lunar impact are then that the sum of the times of each of these phases must be equal to the total time given by Equation (15) and the vehicle must be at the position denoted by the vector $\bar{R}_m$.

1. First Boost – The first boost phase of the problem begins at the time of launch and terminates at injection into a circular parking orbit. Both the total boost time, T_{b_1}, and the boost angle, ζ_{b_1}, will be assumed known constants. The total first boost time may be expressed by the following equation:

$$t_{b_1} = t_1 - t_l \tag{16}$$

2. Parking Orbit – The parking orbit phase of the problem begins at time of injection into the parking orbit (end of first boost) and terminates at the beginning of the second boost. Since a circular parking orbit will be assumed, the total time in the parking orbit can be expressed by the equation

$$T_{po} = t_2 - t_1 = C_1 \zeta_{po} \tag{17}$$

where C_1 is the reciprocal of the orbital rate of the parking orbit (Reference 6) given by

$$C_1 = \sqrt{\frac{R_1^3}{\mu}} \tag{18}$$

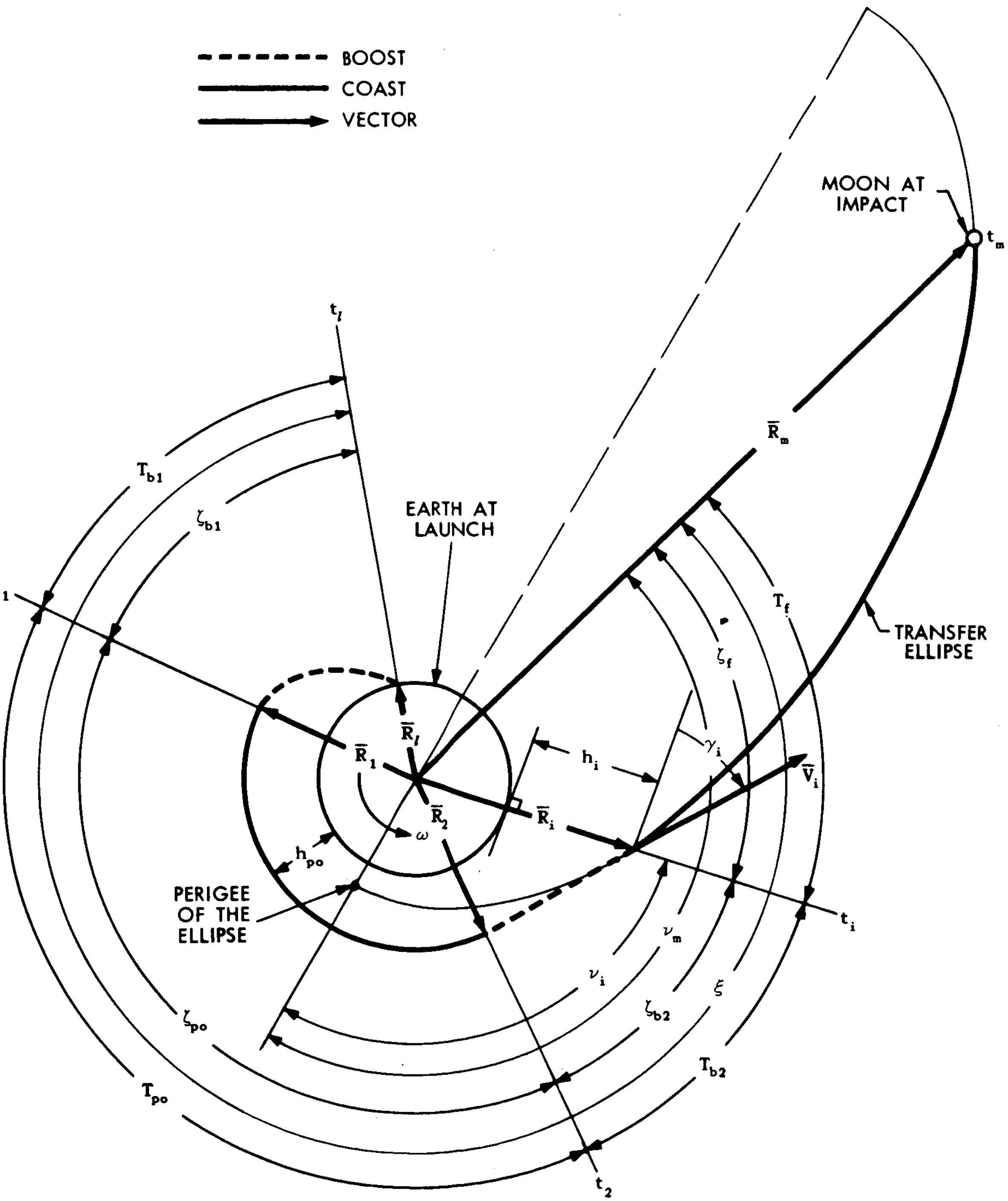


Figure 5–Geometry of the transfer ellipse

In Equation (18) μ is the Earth's gravitational parameter, R_1 is the known magnitude of the parking orbit vector, and ζ_{po} is the parking orbit angle, expressed by (See Figure 5)

$$\zeta_{po} = \xi - \zeta_{b_1} - \zeta_{b_2} - \zeta_f \tag{19}$$

in which ζ_{b_1} and ζ_{b_2} are known constants, and the angular distance, ξ, from launch to impact may be obtained from the following equation

$$\xi = \cos^{-1} (\bar{r}_l \cdot \bar{r}_m) \tag{20}$$

which may be rewritten

$$\xi = \cos^{-1} (r_{mx} \cos \alpha_l \cos \delta_l + r_{my} \sin \alpha_l \cos \delta_l + r_{mz} \sin \delta_l) \tag{21}$$

The translunar trajectory angle, ζ_f, is obtained from the expressions given in Section (4) following.

3. Second Boost – The second boost begins at the time the vehicle leaves the circular parking orbit, t_2, and terminates at injection into the translunar trajectory, t_i. The total boost time, T_{b_2}, and the boost angle, ζ_{b_2}, and injection radius, R_i, will be assumed known constants. The total boost time may be expressed by the following equation.

$$T_{b_2} = t_i - t_2 \tag{22}$$

4. Translunar Trajectory – The translunar trajectory is the portion of the flight between the injection time, t_i, and the time of lunar impact, t_m. The total time of flight during the translunar trajectory, T_f, is given by the expression

$$T_f = t_m - t_i \tag{23}$$

In order to study this portion of the flight, it was assumed that the translunar trajectory could be approximated by the two-body equations which neglect the effect of the Moon gravity on the vehicle. References (2) through (4) indicate that this assumption is adequate in obtaining first order estimates of injection conditions. Utilizing the above assumption, Equations (24) through (31) may be written (Reference 6). These lead to a solution of Equation (23).

$$t_m - t_i = a\sqrt{\frac{a}{\mu}}\,(M_m - M_i) \tag{24}$$

where the mean anomaly, M , is given by Kepler's equation

$$M_j = E_j - e \sin E_j \ ; \ j = m, i \tag{25}$$

the eccentric anomaly, E, may be obtained from

$$E_j = \cos^{-1}\left(\frac{a - R_j}{ae}\right) \ ; \ j = m, i \tag{26}$$

where

$$a = \frac{p}{1 - e^2} \tag{27}$$

and

$$p = R_j\,(1 + e \cos \nu_j) \ ; \ j = m \text{ or } i \tag{28}$$

The expressions necessary to find the true anomaly, ν, and the eccentricity, e, are given by

$$\nu_m = \cos^{-1}\left[\frac{1}{e}\left(2\tilde{R}\tilde{V}^2 \cos^2 \gamma_i - 1\right)\right] \tag{29}$$

$$\nu_i = \cos^{-1}\left[\frac{1}{e}\left(2\tilde{V}^2 \cos^2 \gamma_i - 1\right)\right] \tag{30}$$

and

$$e = \left[4\tilde{V}^2\,(\tilde{V}^2 - 1) \cos^2 \gamma_i + 1\right]^{1/2} \tag{31}$$

The particular form of these equations are derived from their more familiar form in Appendix A.

Upon assuming an injection velocity ratio, $\tilde{V}$, for an injection elevation angle, γ_i, Equations (29), (30), and (31) are solved which in turn yields a solution for Equation (23).

The value of the translunar trajectory angle, ζ_f, necessary to solve Equation (19) may be found from

$$\zeta_f = \nu_m - \nu_i \tag{32}$$

The total time of the flight, T_t, can be expressed by the equation

$$T_t = T_{b_1} + T_{po} + T_{b_2} + T_f \equiv T_{tB} \tag{33}$$

This equation can be solved since T_{b_1}, T_{po}, T_{b_2} and T_f are known from Equations (16), (17), (22), and (23) respectively.

It will be noted that the total time of flight, T_f, had been previously calculated by Equation (15). Therefore, for a solution of the problem to exist, the total time of flight, T_f, in Equations (15) and (33) must be equal. If they are not equal, then another velocity ratio, $\tilde{V}$, is assumed; and the total flight times calculated again. This iteration process is continued until the total flight times in Equations (15) and (33) are equivalent. At this time the velocity ratio which yields a solution to the problem, $\tilde{V}_f$, has been found.

5. Range of Velocity Ratios and Flight Times – As previously stated, the translunar trajectory will be an ellipse, with respect to the Earth, which intersects the Moon before the apo-apsis. This type of trajectory yields a specific upper and lower limit between which the velocity ratio, $\tilde{V}$, is to be assumed.

The lower limit, $\tilde{V}_1$, may be calculated by the following equation (derived in Appendix A).

$$\tilde{V}_1 = \left[\frac{1 - \tilde{R}}{1 - (\tilde{R}\cos\gamma_i)^2}\right]^{1/2} \tag{34}$$

The upper limit, $\tilde{V}_2$, is taken to be that of parabolic flight.

$$\tilde{V}_2 = 1.0 \tag{35}$$

The velocity ratio which yields a solution to the problem, $\tilde{V}_f$, lies between these two limits. In order to get an idea of the values of these limits, Equation (34) may be solved for its minimum anticipated value. To do this some numbers consistent with the ideas previously proposed must be assumed.

Assume:

a. The earth to moon distance, R_m, varies between a maximum of 63.8 earth radii to a minimum of 55.8 earth radii. These values were obtained from Reference 7.

b. The earth injection distance, R_i, varies between 1.0 earth radii and 1.1 earth radii. These values seem adequate to cover the injection range of current boosters.

c. The injection elevation angle, γ_i, varies between zero degrees and 20 degrees. This seems to be a reasonable assumption.

Using these assumptions, Equation (34) has been evaluated and the results tabulated in Table I. From this table it is seen that the minimum anticipated velocity ratio, min $\tilde{V}_1$, is 0.990264. The range of velocity ratios that will yield a solution is, therefore, seen to lie between 0.990264 and 1.0.

Since one of the given pieces of data will be the day of launch, D_l, it is important to find what the range of flight times from injection to lunar impact, T_f, are. This may be done by utilizing the previous assumptions and corresponding velocity ratios. To calculate the maximum anticipated flight time, max T_f, Equations (23) to (31) must be evaluated using $\tilde{V}_1$.

This has been done and the results appear in Table I. From this table it is seen that the maximum anticipated flight time, max T_f, is 130.1477 hours. To calculate the minimum anticipated flight time, min T_f, the value of $\tilde{V}_2$ must be used. Since this is a parabolic flight, Equations (24) to (28) are no longer valid. Instead they are replaced by the following equations (Reference 6).

$$T_f = t_m - t_i = \frac{1}{6\sqrt{\mu}}\left[(R_i + R_m + S)^{3/2} - (R_i + R_m - S)^{3/2}\right] \tag{36}$$

which holds for parabolic flight when $\nu_m - \nu_i < \pi$ and where

$$S = \left[R_i^2 + R_m^2 - 2 R_i R_m \cos(\nu_m - \nu_i)\right]^{1/2} \tag{37}$$

The calculation is then made using Equation (35), Equations (29) to (31), and Equations (36) and (37). Results appear in Table I where it is seen that the minimum anticipated flight time, min T_f, is 44.9697 hours. The range of flight

Table I

Range of Velocity Ratios and Flight Times

Assumed				Calculated	
R_m earth radii	R_i earth radii	γ_i degrees	$\tilde{V}_2$ dimensionless	$\tilde{V}_1$ dimensionless	T_f hours
55.8	1.0	0	*	.991158	106.5595
		20	*	.991139	106.1230
	1.1	0	*	.990287	106.8411
		20	*	.990264	106.3542
63.8	1.0	0	*	.992253	129.8469
		20	*	.992240	129.3886
	1.1	0	*	.991489	130.1477
		20	*	.991472	129.6370
55.8	1.0	0	1.0	*	45.2046
		20		*	44.9697
	1.1	0		*	45.3196
		20		*	45.0562
63.8	1.0	0		*	55.0896
		20		*	54.8449
	1.1	0		*	55.2131
		20		*	54.9389

times that will yield solutions is, therefore, seen to lie between 130.1477 and 44.9697 hours. Knowing this, along with the desired day of lunar impact D_m, and the desired number of parking orbit revolutions, it is seen that solutions of the problem will exist anywhere between seven and one days before lunar impact. Therefore, a good choice for D_l to obtain all solutions would be in the range from $D_m - 7$ to $D_m - 1$.

6. <u>Obtaining Successive Velocity Ratios</u> – Having established a lower limit velocity ratio, $\tilde{V}_1$, and an upper limit velocity ratio, $\tilde{V}_2$ there remains a procedure to be followed in obtaining successive assumed velocity ratios that will ultimately lead to a solution. There are many methods by which this iteration may be done, the one described below is known as the Regula Falsi method of iteration. Calling T_t from Equation (15) T_{tA} and T_t from Equation (33) T_{tB}, the quantity ΔT_t may be calculated by

$$(\Delta T_t)_k = (T_{tB})_k - T_{tA} \; ; \; k = 1, 2, \cdots f \tag{38}$$

The object of the iteration is to yield a ΔT_t, which is a function of $\tilde{V}$, equal to zero. Figure 6 shows the procedure when ΔT_t is plotted as a function of $\tilde{V}$. After Equation (34) is used in calculating $\tilde{V}_1$, this value is used along with other knowns in calculating T_{tB}, called $(T_{tB})_1$ since $\tilde{V}_1$ was used in its calculation, by use of Equations (23) to (33). Since T_{tA} is already assumed known, and constant for this procedure; the maximum value of ΔT_t, viz. $(\Delta T_t)_1$, may be calculated by Equation (38). The upper limit, $\tilde{V}_2$, is known and another value of T_{tB}, viz. $(T_{tB})_2$, is calculated by use of Equations (29) to (33) along with Equations (36) and (37), since the trajectory is parabolic. This quantity along with the constant and known T_{tA} is used in evaluating Equation (38) for the minimum value of ΔT_t, viz. $(\Delta T_t)_2$. Through the two points on the solution curve, $[(\Delta T_t)_1, \ \tilde{V}_1]$ and $[(\Delta T_t)_2, \ \tilde{V}_2]$, a straight line (straight line 1) is constructed. The next value of the velocity ratio to be assumed, $\tilde{V}_3$, is located at the intersection of straight line 1 and $\Delta T_t = 0$. Analytically the expression yielding this new assumed velocity ratio and successive assumed velocity ratios is found to be

$$\tilde{V}_{k+1} = \frac{(\Delta T_t)_1 \tilde{V}_k - (\Delta T_t)_k \tilde{V}_1}{(\Delta T_t)_1 - (\Delta T_t)_k} \ ; \ k = 2, 3, \cdots f \tag{39}$$

The value of $\tilde{V}_3$ is then used to calculate $(T_{tB})_3$ by use of Equations (23) to (33). A new minimum value of ΔT_t, viz. $(\Delta T_t)_3$, may then be calculated by Equation (38). A new straight line (straight line 2) is constructed through the two points, $[(\Delta T_t)_1, \ \tilde{V}_1]$ and $[(\Delta T_t)_3, \ \tilde{V}_3]$, on the solution curve. The next velocity ratio to be assumed, $\tilde{V}_4$, is the intersection of straight line 2 and $\Delta T_t = 0$. Analytically this also may be found by Equation (39). The iteration continues in this manner yielding values of ΔT_t closer and closer to zero as it proceeds. When $(\Delta T_t)_k$ equals zero, the point where the solution curve intersects $\Delta T_t = 0$ has been found, and the velocity ratio yielding a solution to the problem, $\tilde{V}_f$, is known.

7. Initial Injection Conditions – If a solution to the problem exists, a value for $\tilde{V}_f$ is found; and it is then possible to calculate the initial injection conditions for a translunar trajectory. These conditions can be obtained in either cartesian coordinates ($x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i$) or polar coordinates ($L_i, \delta_i, R_i, V_i, A_i, \gamma_i$).

The cartesian coordinates may be found by the following equations.

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = R_3(-\alpha_l) R_2(\delta_l) R_1(A_l - \pi/2) R_3(-\zeta_{b_1} - \zeta_{po} - \zeta_{b_2}) \begin{bmatrix} R_i \\ 0 \\ 0 \end{bmatrix} \tag{40}$$

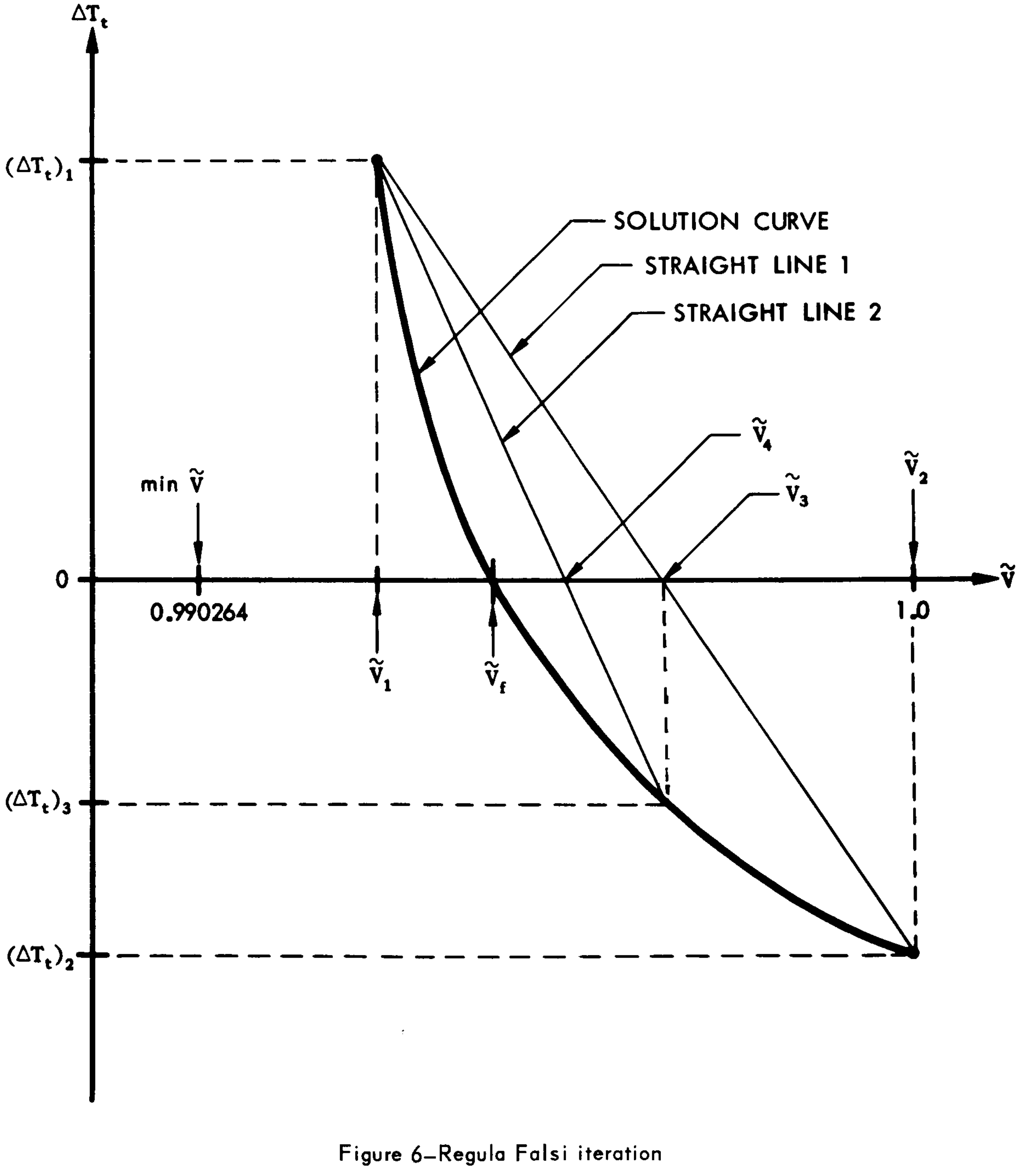


Figure 6–Regula Falsi iteration

and

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{bmatrix} = R_3(-\alpha_l)\, R_2(\delta_l)\, R_1(A_l - \pi/2)\, R_3(-\zeta_{b_1} - \zeta_{po} - \zeta_{b_2} - \pi/2 + \gamma_i) \begin{bmatrix} V_i \\ 0 \\ 0 \end{bmatrix} \tag{41}$$

where the rotations $R_1(\theta)$, $R_2(\theta)$, and $R_3(\theta)$ are defined as

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \tag{42}$$

$$R_2(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \tag{43}$$

$$R_3(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{44}$$

The injection velocity, V_i, may be obtained since the final velocity ratio, $\tilde{V}_f$, is known, and the parabolic velocity at injection, V_p, is given (Reference 6) as

$$V_p = \sqrt{\frac{2\mu}{R_i}} \tag{45}$$

Therefore

$$V_i = \tilde{V}_f\, V_p \tag{46}$$

In polar coordinates three of the quantities R_i , V_i , and γ_i are known by products of the solution, and the remaining quantities may be found by the equations .

$$L_i = \tan^{-1}\left(\frac{y_i}{x_i}\right) - (\Psi + \omega t_l) - \omega(t_i - t_l) \tag{47}$$

$$\delta_i = \tan^{-1}\left(\frac{z_i}{\sqrt{x_i^2 + y_i^2}}\right) \tag{48}$$

and

$$A_i = \tan^{-1}\left[\frac{R_i(x_i\dot{y}_i - y_i\dot{x}_i)}{y_i(y_i\dot{z}_i - z_i\dot{y}_i) - x_i(z_i\dot{x}_i - x_i\dot{z}_i)}\right] \tag{49}$$

8. Solutions for Successive Orbits – The problem as outlined thus far will yield a solution for the first revolution of the parking orbit around the earth. To obtain a solution for successive orbits, the following technique is used.

Figure 7 is a sketch of the geometry for the solutions in the plane of the trajectory. The position of injection for the first parking orbit solution is indicated by the vector $(\bar{R}_i)_1$ and the position of injection for the second parking orbit solution by vector $(\bar{R}_i)_2$. Note that the second position occurs after remaining in the parking orbit somewhat more than one full revolution in inertial space. This comes about from a combination of facts.

a. The time of launch from the surface of the earth to the time of lunar impact remains the same.

b. The injection elevation angle, γ_i , is the same.

c. The moon is essentially in the same position since the time for one parking orbit ($\approx$ 1.5 hrs.) does not allow for the moon to move very much (moon's motion $\approx$ 12°/day).

d. Since more time is spent in the earth parking orbit as the number of parking orbits is increased, it is evident that the time in the translunar trajectory must decrease in order for a. to be true.

The above facts necessitate a higher energy translunar trajectory which goes through essentially the same point indicated by the vector $\bar{R}_m$. The only

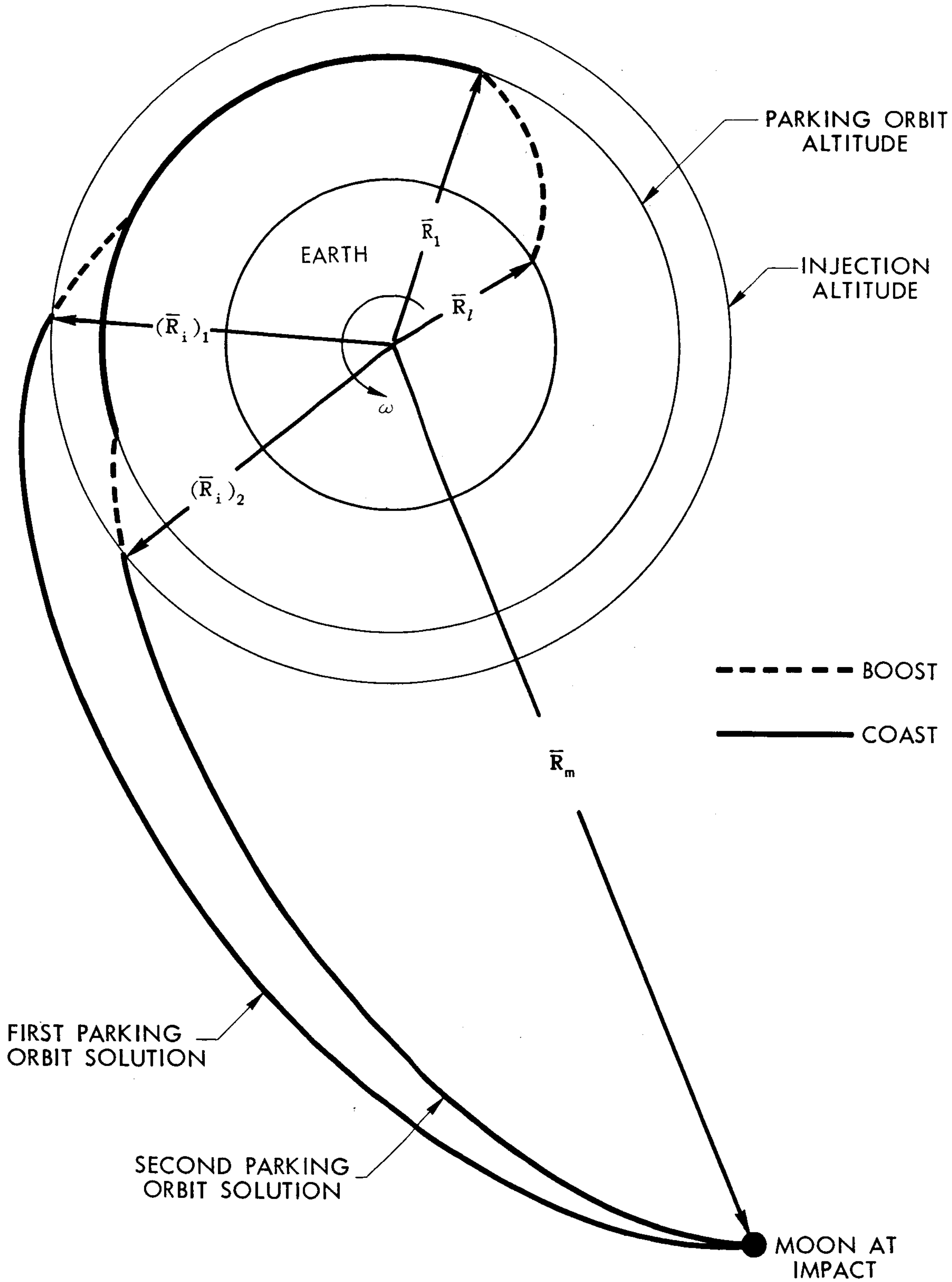


Figure 7–Successive parking orbit solutions

way this can be accomplished is by rotating the position injection vector from $(\bar{R}_i)_1$ to $(\bar{R}_i)_2$ in a counter-clockwise sense as shown in Figure 7.

To find solutions for successive orbits, the procedure is to subtract $(n - 1)\ 2\pi C_1$ from the total time given by Equation (15) or

$$(T_t)_n = t_m - t_l + 24\ (D_m - D_l) - (n-1)\ 2\pi C_1 \equiv (T_{tA})_n \tag{50}$$

where n is the number of the orbit in which the solution is to be found. In all other respects the problem remains the same.

V. REFERENCES

1. Shaffer, F., Squires, R. K., Wolf, H., "Interplanetary Trajectory Encke Method (ITEM) Program," Goddard Space Flight Center X-640-63-71, May 1963.

2. Tolson, Robert H., "Effects of some typical geometrical constraints of lunar trajectories," NASA Technical Note D-938, August 1961.

3. Clarke, Victor Jr., "Design of Lunar and Interplanetary Ascent Trajectories," JPL Technical Report No. 32-30, March 1962.

4. Sedov, L. I., "Orbits of Cosmic Rockets Toward the Moon," ARS Journal, Vol. 30, No. 1, January 1960.

5. "Explanatory Supplement to the Astronomical Ephemeris and The American Ephemeris and Nautical Almanac," London: Her Majesty's Stationery Office, 1961.

6. Ehricke, K. A., "Space Flight I. Environment and Celestial Mechanics," Princeton, New Jersey: D. Van Nostrand Company, Inc., 1960.

7. Woolston, Donald S., "Declination, Radial Distance, and Phases of the Moon for the Years 1961 to 1971 for use in Trajectory Considerations," NASA Technical Note D-911, August 1961.

Appendix A

DERIVATIONS

1. Derivation of Equation (1): Refer to Figure A1, and use the law of sines

$$\frac{\sin(90° - i)}{\sin A_l} = \frac{\sin(90° - \delta_l)}{\sin 90°}$$

$$\therefore w_z = \cos i = \cos \delta_l \sin A_l \tag{1}$$

2. Simultaneous solution of Equations (2) and (3) to yield Equations (6) and (7):

$$\bar{w} \cdot \bar{r}_m = 0 \tag{2}$$

$$\bar{w} \cdot \bar{w} = 1 \tag{3}$$

$$\bar{w} \cdot \bar{r}_m = w_x r_{mx} + w_y r_{my} + w_z r_{mz} = 0$$

$$\bar{w} \cdot \bar{w} = w_x^2 + w_y^2 + w_z^2 = 1$$

$$\therefore w_x = \frac{-(w_y r_{my} + w_z r_{mz})}{r_{mx}} \tag{7}$$

$$w_y^2 = 1 - w_x^2 - w_z^2 = 1 - \left(\frac{w_y r_{my} + w_z r_{mz}}{r_{mx}}\right)^2 - w_z^2$$

Which upon simplification becomes

$$(r_{mx}^2 + r_{my}^2) w_y^2 + (2w_z r_{my} r_{mz}) w_y + (w_z^2 r_{mx}^2 + w_z^2 r_{mz}^2 - r_{mx}^2) = 0$$

Solving this quadratic equation in w_y yields

$$w_y = \frac{-w_z\, r_{my}\, r_{mz} \pm r_{mx}\sqrt{1 - r_{mz}^2 - w_z^2}}{r_{mx}^2 + r_{my}^2} \tag{6}$$

3. Derivation of Equation (9): Refer to Figure A2, it is seen that

$$(\bar{w} \times \bar{r}_l) \cdot \bar{K} = \cos\ \eta$$

The direction cosines of $\bar{K}$ yield

$$\cos^2 i + \cos^2 \eta + \cos^2(90 - \delta_l) = (\bar{K})^2 = 1$$

Substituting in Equation (1) for the first term, transposing, and simplifying

$$\cos^2 \eta = 1 - \cos^2 \delta_l \sin^2 A_l - \sin^2 \delta_l = \cos^2 \delta_l \cos^2 A_l$$

Upon taking the square root of both sides

$$\cos \eta = \cos \delta_l \cos A_l$$

where the positive value is considered only if launch is in the northern hemisphere ($\pi/2 > \delta_l > 0$) and eastward ($\pi > A_l > 0$)

$$\therefore (\bar{w} \times \bar{r}_l) \cdot \bar{K} = \cos \eta = \cos \delta_l \cos A_l \tag{9}$$

4. Simultaneous solution of Equations (8) and (9) yield Equations (11) and (12).

$$\bar{r}_l \cdot \bar{w} = 0 \tag{8}$$

$$(\bar{w} \times \bar{r}_l) \cdot \bar{K} = \cos \delta_l \cos A_l \tag{9}$$

Since

$$\bar{r}_l = \bar{I} \cos \alpha_l \cos \delta_l + \bar{J} \sin \alpha_l \cos \delta_l + \bar{K} \sin \delta_l \tag{10}$$

Performing the vector operations yields

$$\bar{r}_l \cdot \bar{w} = \cos \alpha_l \cos \delta_l w_x + \sin\alpha_l \cos \delta_l w_y + \sin \delta_l w_z = 0 \quad \text{(A1)}$$

$$(\bar{w} \times \bar{r}_l) \cdot \bar{K} = \sin \alpha_l \cos \delta_l w_x - \cos \alpha_l \cos \delta_l w_y = \cos \delta_l \cos A_l \quad \text{(A2)}$$

Upon substituting (A2) into the second term of (A1)

$$\cos \alpha_l \cos \delta_l w_x + \frac{w_y}{w_x} (\cos \alpha_l \cos \delta_l w_y + \cos \delta_l \cos A_l) + \sin \delta_l w_z = 0$$

Which upon transposing and solving for $\cos \alpha_l$ yields

$$\cos \alpha_l = \frac{w_z w_x \sin \delta_l + w_y \cos \delta_l \cos A_l}{(w_z^2 - 1) \cos \delta_l} \quad (11)$$

Upon substituting (A1) into the second term of (A2)

$$\sin \alpha_l \cos \delta_l w_x - \frac{w_y}{w_x} (-\sin \alpha_l \cos \delta_l w_y - \sin \delta_l w_z) = \cos \delta_l \cos A_l$$

which upon transposing and solving for $\sin \alpha_l$ yields

$$\sin \alpha_l = \frac{w_y w_z \sin \delta_l - w_x \cos \delta_l \cos A_l}{(w_z^2 - 1) \cos \delta_l} \quad (12)$$

5. Changing the reference time and units of measure in Newcomb's expression.

Greenwich mean siderial time at zero hours universal time on successive dates are computed from Newcomb's expression (Reference 5).

$$\Psi_o = 6^h\, 38^m\, 45\overset{s}{.}836 + 8640184\overset{s}{.}542\ T + 0\overset{s}{.}0929\ T^2 \quad \text{(A3)}$$

where for any date, T denotes the number of Julian centuries of 36525 days which have elapsed since noon on 1900 January 0.0 at the Greenwich meridian.

This equation may be written as

$$\Psi_o = \Psi_{1900} + AT + BT^2$$

To shift the reference time from January 0.5, 1900 to January 0.0, 1960

$$\Psi_o = \Psi_{1900} + A(T + T_1) + B(T + T_1)^2 = \Psi_{1900} + AT + BT^2 + (A + 2BT)\,T_1 + BT_1^2$$

where T is the time in Julian centuries between January 0.5, 1900 and January 0.0, 1960 and T_1 is the time in Julian centuries after January 0.0, 1960. The value of T in this case is

$$T = \frac{365 \times 60 + 13.5}{36525} = .599958932 \text{ Julian centuries}$$

$$\therefore \Psi_o = \Psi_{1960} + [(8640184.542 + 2(.0929)(.599958932)]\ T_1 + .0929\ T_1^2$$

and the Equation (A3) becomes

$$\Psi_o = 6^h\ 34^m\ 41\overset{s}{.}762 + 8640184\overset{s}{.}653\ T_1 + 0\overset{s}{.}0929\ T_1^2 \qquad \text{(A4)}$$

where T_1 denotes the number of Julian centuries which have elapsed since 1960 January 0.0.

The units of Equation (A4) may be changed to yield

$$\Psi = 1.72218633 + 1.720279168 \times 10^{-2}\ D_l + 0.67558729 \times 10^{-5}\left(\frac{D_l}{36525}\right)^2 \qquad (14)$$

where Ψ is in radians and D_l is the number of days from January 0.0 U.T. 1960.

6. Equations (29), (30), and (31) may be obtained in the following manner. Starting with the basic equations (Reference 6).

$$p = R\left(\frac{V}{V_c}\right)^2 \cos^2\gamma \qquad \text{(A5)}$$

$$p = a(1 - e^2) \tag{A6}$$

$$V^2 = \mu\left(\frac{2}{R} - \frac{1}{a}\right) \tag{A7}$$

$$\cos \nu = \frac{1}{e}\left(\frac{p}{R} - 1\right) \tag{A8}$$

Equating p in Equations (A5) and (A6) and solve for e^2

$$e^2 = 1 - \frac{1}{a}\left[R\left(\frac{V}{V_c}\right)^2 \cos^2 \gamma\right]$$

Substitute for 1/a using Equation (A7)

$$e^2 = 1 - \left[\frac{2}{r} - \frac{V^2}{\mu}\right] \left[R\left(\frac{V}{V_c}\right)^2 \cos^2 \gamma\right]$$

Equation (A7) also implies for parabolic velocity (i.e. $a = \infty$).

$$V_p = \sqrt{2}\, V_c = \sqrt{\frac{2\mu}{R}} \tag{A9}$$

Substitute this relationship in and solve for e yields

$$e = \left[4\, \tilde{V}^2 \left(\tilde{V}^2 - 1\right) \cos^2 \gamma_i + 1\right]^{1/2} \tag{31}$$

Combining Equations (A5), (A9), and (A8) yields

$$\cos \nu_i = \frac{1}{e}\left(2\,\widetilde{V}^2 \cos^2 \gamma_i - 1\right) \tag{30}$$

Applying Equation (A5) for earth injection distances and lunar distances yields the results

$$p = R_m \left(\frac{V_m}{V_{cm}}\right)^2 \cos^2 \gamma_m = R_i \left(\frac{V_i}{V_c}\right)^2 \cos^2 \gamma_i$$

Substituting p/R_m from above results along with (A9) into Equation (A8) yields

$$\nu_m = \cos^{-1}\left[\frac{1}{e}\left(2\widetilde{R}\,\widetilde{V}^2 \cos^2 \gamma_i - 1\right)\right] \tag{29}$$

7. Derivation of the lower limit for assumed velocity ratios.

Equation (35) for the lower limit, $\widetilde{V}_1$, is obtained in the following manner. Upon substitution of Equation (31) into Equation (29) and squaring yields

$$\cos^2 \nu_m = \frac{4\,\widetilde{R}^2\,\widetilde{V}^4 \cos^2 \gamma_i - 4\,\widetilde{R}\,\widetilde{V}^2 + 1}{4\,\widetilde{V}^2\left(\widetilde{V}^2 - 1\right)\cos^2 \gamma_i + 1} \tag{A10}$$

The limiting case occurs when $\cos^2 \nu_m = 1$ which corresponds to a lunar impact at the apo-apsis of the translunar ellipse. Making this substitution and solving for the velocity ratio Equation (A10) becomes

$$\widetilde{V}_1 = \left[\frac{1 - \widetilde{R}}{1 - \left(\widetilde{R}\cos \gamma_i\right)^2}\right]^{1/2} \tag{34}$$

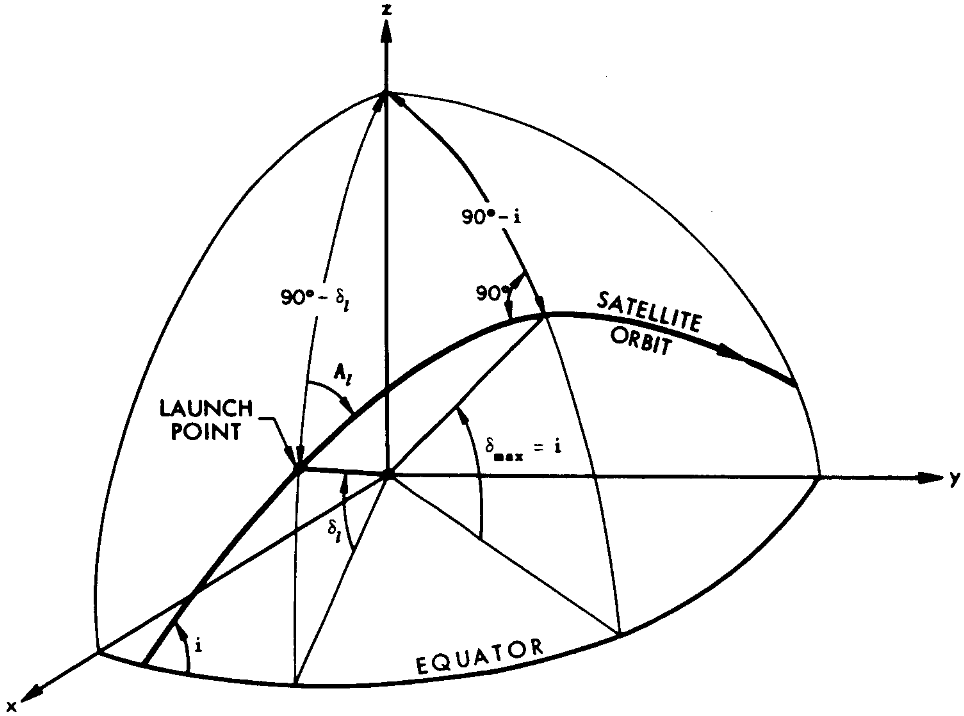


Figure A-1—Geometry of the orbit

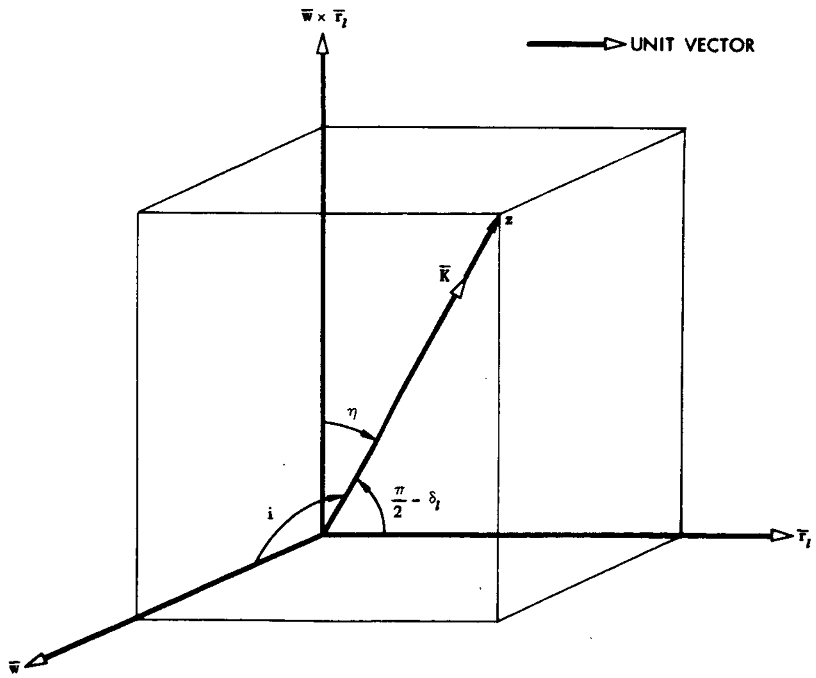


Figure A-2—Geometry describing the angle η

APPENDIX B

CALCULATING PROCEDURE

A. Data Required

In order to solve the problem of a lunar impact from a given launch site, various data are required before the calculation may begin. A summary of these input data and its source are given below.

1. The first boost are ζ_{b_1} : Refer to Figure 5. This is a characteristic of the booster to be used in the mission. It is usually available or may be obtained from data in reports giving the booster characteristics.

2. The first boost time, T_{b_1} : Refer to Figure 5. The same remarks in Item 1 apply here.

3. Parking orbit altitude, $h_{po} = R_1 - R_e$. The altitude at which the first boost injects its payload into a circular orbit. The same remarks in Item 1 apply here.

4. The second boost arc, ζ_{b_2} : Refer to Figure 5. This is a characteristic of the booster used to boost the vehicle from circular parking velocity to translunar injection velocities. This information may be obtained from data in reports giving the booster characteristics.

5. The second boost time, T_{b_2} : Refer to Figure 5. The same remarks in Item 4 apply here.

6. Injection altitude, $h_i = R_i - R_e$. The altitude at which the second boost injects the vehicle into its translunar trajectory. The same remarks in Item 4 apply here.

7. Injection elevation angle, γ_i : Refer to Figure 5. This angle is a function of the particular guidance system employed during the second boost.

8. Launch azimuth, A_i. The desired launch azimuth at launch site. Depends upon range safety and tracking requirements.

9. Parking orbit revolutions. The number of parking orbit revolutions for which solutions are desired.

10. Launch days(s), D_l. The number of days before impact that the launch is to take place. For the elliptical trajectories described this is from 7 to 1 days before D_m.

    Flight time, T_f, the time of the flight from injection to impact may be given instead of D_l.

11. Lunar impact time, Y_m, D_m, t_m. The year, day and time it is desired to impact the moon.

B. Calculating Steps

A flow diagram indicating the solution of the problem is given in Figure B1. This figure is a guide to the solution of the problem and indicates the general flow from equation to equation in the text. Steps in the calculating procedure which may be used in conjunction with this figure are given as follows:

1. Given a year, Y_m, day, D_m, and time, t_m, it is desired to impact the Moon. The position of the Moon in geocentric equatorial coordinates, R_{mx}, R_{my}, R_{mz} may be obtained from an ephemeris of the Moon. The resultant of these coordinates, R_m, as well as the components of the unit vector, r_{mx}, r_{my}, r_{mz}, are then calculated.

2. Given the launch site declination, δ_l, and a launch azimuth, A_l, components, w_x, w_y, w_z of the unit vector $\bar{w}$ normal to and defining the plane of motion from the surface of the Earth to the Moon are obtained from Equations (1), (6), and (7). Depending upon the value under the radical in Equation (6) the unit vector $\bar{w}$ will have two values, one value, or be imaginary. These results indicate for the launch azimuth, A_l, launch into the plane is possible twice a day, once a day, or not at all. If a real value of $\bar{w}$ exists the calculating procedure is continued. If two real values of $\bar{w}$ exist, use one of them at a time.

3. The right ascension at launch, α_l, may be calculated by Equations (11) and (12). A desired day of launch, D_l, is given (i.e. D_l is an integer in the range of $D_m - 7$ to $D_m - 1$) and the angle Ψ is calculated by Equation (14). Since the launch longitude, L_l, and the Earth's spin rate, ω, are knowns, the time of launch after zero hour U.T. on the day of launch, D_l, can be calculated from Equation (13). The day of launch, D_l, and the time of launch, t_l, as well as the day of impact, D_m, and time of impact, t_m, are now all knowns; the total time of the flight, T_t, can be calculated from Equation (15). This time is designated as T_{tA}.

4. A value of the injection angle, γ_i , is given, and the injection radius, R_i , can be calculated from the given injection altitude, h_i , and Earth's radius R_e . A velocity ratio, $\widetilde{V}_1$, calculated by Equation (34) is now obtained.

5. Using this velocity ratio, the eccentricity, e , as well as the true anomalies to the Moon, ν_m , and to the injection point, ν_i , are calculated by Equations (31), (29), and (30) respectively. The translunar trajectory angle, ζ_f , may now be calculated from Equation (32); and the angular distance, ξ , is calculated by Equation (21). Since the booster burning angles, ζ_{b_1} and ζ_{b_2} , are given, the parking orbit angle, ζ_{po} , can be calculated from Equation (19).

   A circular parking orbit was considered in this analysis; and therefore, the parking orbit radius, R_1 , is constant and is obtained from the given parking orbit altitude, h_{po}, and Earth's radius. The inverse orbital rate of the parking orbit, C_1 , is defined by Equation (18). This is used in calculating the time in parking orbit, T_{po} , given by Equation (17).

6. Since the velocity ratio yields an elliptical flight path, the calculation continues as follows: The semilatus rectum, p , and the semimajor axis, a , may be calculated by Equations (28) and (27) respectively. Having these quantities the eccentric anomalies to the Moon, E_m , and to the injection point, E_i , can be calculated by Equation (26); and the mean anomalies to the Moon, M_m, and to the injection point, M_i , can be calculated by Kepler's equation, viz. Equation (25). The time difference on the translunar trajectory between injection and lunar impact, $t_m - t_i$, can be calculated by Equation (24). This time difference, called T_f, is the same as that defined by Equation (23).

7. The boost times, T_{b_1} and T_{b_2} , are both given and are used with T_{po} and T_f to calculate the total time of the flight, T_t , from Equation (33). This time is designated as T_{tB}. In step (3) above, it is seen that the total time of flight, $T_t = T_{tA}$, has already been calculated by another method, viz. Equation (15). The difference in these times, ΔT_t , is given by Equation (38).

8. The particular ΔT_t as calculated by Equation (38) for $\widetilde{V}_1$ from Equation (34) is designated, since $k = 1$, as $(\Delta T_t)_1$ on Figure 6. If $(\Delta T_t)_1$ is less

than zero, the solution of the problem requires that the transfer trajectory intersect the Moon after apo-apsis and the calculation is concluded. A new value of the launch day, D_l , greater than before is used, and the calculation is reinitiated at step (3), Equation (14). If $(\Delta T_t)_1$ is greater than zero, continue.

9. The next velocity ratio used, $\tilde{V}_2$, is given by Equation (35). Step (5) is then repeated. Since the velocity ratio corresponds to a parabolic trajectory, the parameter, S , is calculated by Equation (37) and is used in calculating the total time from injection to lunar impact, T_f , given by Equation (36). Step (7) is then repeated. The particular ΔT_t as calculated by Equation (38) for $\tilde{V}_2$ from Equation (35) is designated, since $k = 2$, as $(\Delta T_t)_2$ on Figure 6. If $(\Delta T_t)_2$ is greater than zero, the solution of the problem requires a velocity ratio greater than parabolic; and the calculation is concluded. A new value of the launch day, D_l , less than before is used, and the calculation is reinitiated at step (3), Equation (14). If $(\Delta T_t)_2$ is less than zero, a new velocity ratio, $\tilde{V}_3$, must be calculated.

10. The next velocity ratio to be assumed may be calculated by Equation (39). Steps (5), (6), (7), and (10) are repeated in that order and are continued to be repeated until the value of ΔT_t given by Equation (38) is as close to zero as desired. When this occurs, a velocity ratio which has yielded a solution to the problem, $\tilde{V}_f$, has been obtained.

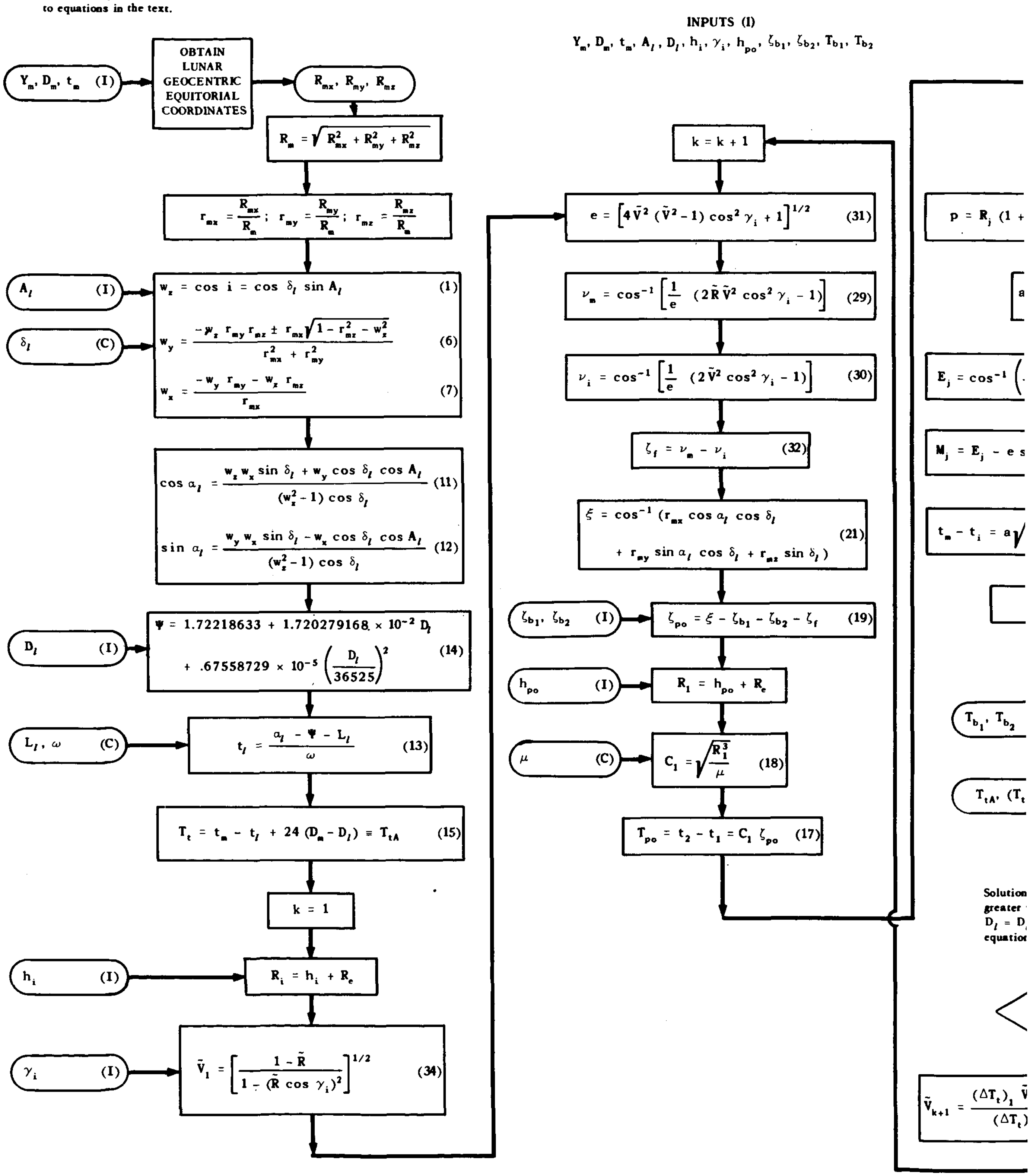
11. At this point several things may be calculated depending upon the particular purpose of making the calculation. Two possibilities come to mind, and to distinguish them one will be called Option A and the other Option B.

    Option A: In this case it is assumed the purpose for making the calculation is to obtain all the possible injection conditions that will yield a lunar impact at a given desired time for given launch azimuth, booster conditions, and injection elevation angle if the parking orbit is allowed to orbit the earth several (say n) times. For this option n was assumed to be equal to one through steps 10. The value of n is now increased to two, and Equation (50) is used in place of Equation (15), the entire calculation being repeated from step 4. This value of n is increased as many times as desired repeating the calculation from step 4. Having done this, a solution has been found for each revolution of the parking orbit (i.e. n of them). Assuming two values of w_y were obtained when Equation (6) was solved (see step 2), there remains now to repeat the calculation from steps (3) to (10) for the second possible value of w_y . Having done this (for $n = 1$), the values of n are increased again one at a time as was previously done. Now the two possible solutions per day for each parking orbit revolution have been found, yielding 2n possible solutions. The results thus far have been for only one assumed launch day, D_l. Since lunar impact at the desired time is possible, as previously explained, for launch times between 44.9697 and 130.1477 hours before lunar impact, more launch days must be investigated. To be sure to include all possible cases, the launch days to be investigated are between $D_m - 7$ and $D_m - 1$, or a total of 6 days. Doing this for n revolutions in the parking orbit for each day is seen to yield a possible 12 n solutions per launch azimuth per elevation angle that will impact the moon at the same time.

Option B: In this case it is assumed the purpose for making the calculation is to obtain initial injection conditions that will have a given desired flight time, T_f, from injection to lunar impact. Lunar impact will be near the desired time, but not at it. Solutions will be obtained in the first parking orbit for a given launch azimuth, booster conditions, and injection elevation angle. The calculation is made as indicated in steps 1 through 10, and a solution is obtained. At this point the desired flight time $(T_f)_D$, is compared with flight time, T_f, that was the solution to the problem, and their difference, $\Delta T_f = (\Delta T_f)_D - T_f$, is obtained. This time difference is algebraically added to the original lunar impact time (thus having the effect of moving the moon), and steps 1 through 10 are again repeated, yielding a new solution. Again a time difference ΔT_f, is obtained, and the process is repeated until this time difference is zero. A solution during the first revolution of the parking orbit having a given time of flight from injection to impact has thus been found. This solution will yield a lunar impact near (say within a day) the original impact time desired. The original impact time is then utilized with the second value of w_y obtained when Equation (6) was solved, and the entire process of Option B is repeated. The final results will yield a total of two solutions, having the same desired flight time, in the first parking orbit for a given launch azimuth, injection elevation angle, and boost conditions that impact the moon close to the desired impact time.

12. Having found the final velocity ratio, $\tilde{V}_f$, all the by-products necessary for finding the injection vector components in both cartesian coordinates $(x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i)$ or polar coordinates $(L_i, \delta_i, R_i, V_i, A_i, \gamma_i)$ may be found. Parabolic velocity, V_p, is found using Equation (45) which then enables the calculation of V_i from $\tilde{V}_f$, Equation (46). All the necessary data is now known to calculate rotation matrices, Equations (42), (43), and (44), which are used in obtaining the cartesian position components and velocity components from Equations (40) and (41) respectively. Using these components the remaining unknown polar coordinate components, viz. L_i, δ_i, and A_i, may be calculated by use of Equation (47), (48), and (49) respectively.

Note: Numbers in parenthesis refer to equations in the text.



#1

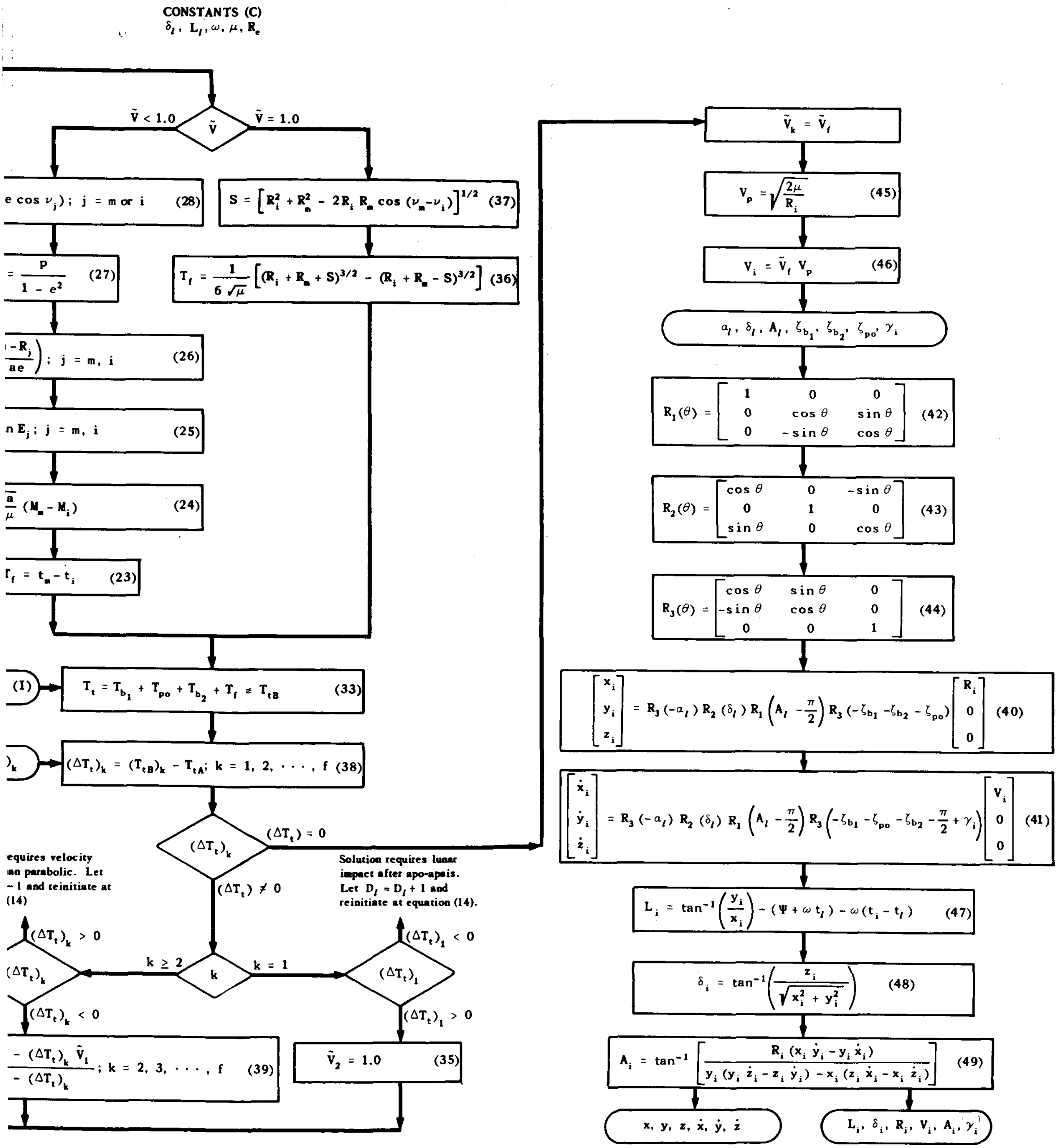


Figure B-1—Flow diagram indicating the solution of the problem.

#2