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RESEARCH ACHIEVEMENTS REVIEW SERIES NO.15-16

RESEARCH AND DEVELOPMENT OPERATIONS
GEORGE C. MARSHALL SPACE FLIGHT CENTER
HUNTSVILLE, ALABAMA

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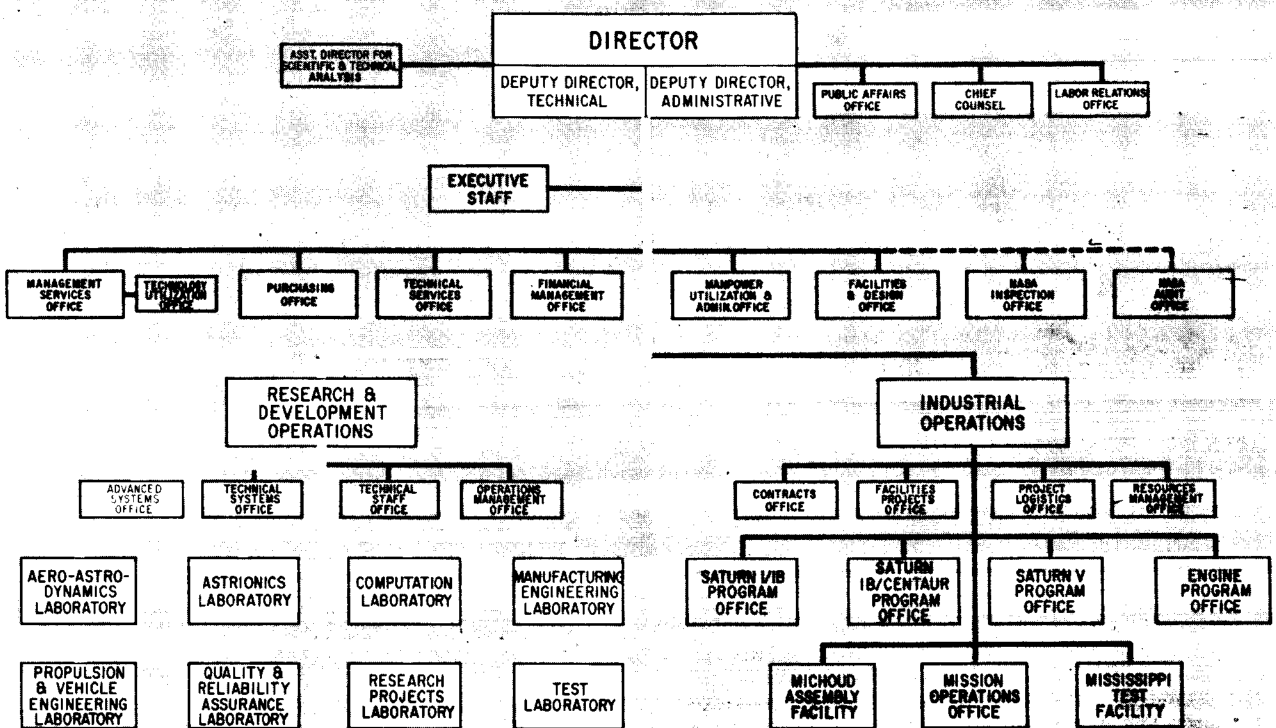

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C.

ASTRODYNAMICS-OPTIMIZATION THEORY AND GUIDANCE THEORY

RESEARCH ACHIEVEMENTS REVIEW SERIES NO.15-16

RESEARCH AND DEVELOPMENT OPERATIONS
GEORGE C. MARSHALL SPACE FLIGHT CENTER
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1965

PREFACE

In 1955, the team which has become the Marshall Space Flight Center (MSFC) began to organize a research program within its various laboratories and offices. The purpose of the program was twofold: first, to support existing development projects by research studies and second, to prepare future development projects by advancing the state of the art of rockets and space flight. Funding for this program came from the Army, Air Force, and Advanced Research Projects Agency. The effort during the first year was modest and involved relatively few tasks. The communication of results was, therefore, comparatively easy.

Today, ten years later, the double purpose of MSFC's research program is still the same. Funding for the program now comes from NASA Program Offices. The present yearly effort represents major amounts of money and hundreds of tasks. The better part of the money goes to industry and universities for research contracts. However, a substantial research effort is conducted in house at the Marshall Center by all of the laboratories. The communication of the results from this impressive research program has become a serious problem by virtue of its very voluminous technical and scientific content.

The Research Projects Laboratory, which is the group responsible for management of the consolidated research program for the Center, initiated a plan to give better visibility to the achievements of research at Marshall in a form that would be more readily usable by specialists, by systems engineers, and by NASA Program Offices for management purposes.

To initiate the plan, monthly Research Achievements Reviews have been established, repetitive over a yearly cycle, with each review covering one or two fields of research. These verbal reviews are documented in the Research Achievements Review Series.

Ernst Stuhlinger
Director, Research Projects Laboratory

These papers presented October 28, 1965

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INTRODUCTION TO RESEARCH ACHIEVEMENTS REVIEW ON ASTRODYNAMICS OPTIMIZATION THEORY AND GUIDANCE THEORY

by

Dr. E. D. Geissler*

The three subjects discussed in this review, guidance theory, optimization theory, and astrodynamics, are very closely related. They deal with trajectory shaping and are of central importance in the activities of the Aero-Astrodynamics Laboratory. Since they are strongly concerned with mathematical concepts and methods, these subjects are not easy to present in a satisfactory fashion to an audience which is a fairly mixed composition with respect to background and interests. Let it be clearly understood that these papers are directed to the nonspecialist; complete papers of particular use to specialists are available or are in preparation.

We have selected somewhat different approaches in the three papers towards the subject matter, ranging from a fairly thorough description of one particular guidance concept in the first, to a more general discussion of the status of mathematical tools in optimization theory in the second, and finally to a description of some examples of classes of trajectories in astrodynamics with primary appeal to geometrical visualization. Thus, we have not attempted to really systematically survey all activities related to the subject matter.

The conceptual development of guidance schemes at MSFC is the primary responsibility of the Aero-Astrodynamics Laboratory, whereas our Astrionics Laboratory is the primary agent for the implementation and mechanization of such schemes, i.e., the transformation of equations into a physical set of operable, functional equipment. This obviously requires very close cooperation between the two laboratories to have the full benefit of feedback of practical viewpoints and experience into the theoretical framework. This cooperation has been effective over many years in an exemplary fashion and was one important factor for the very successful accomplishment of our guidance systems in actual flights.

A fairly major change in our guidance philosophy took place at our agency a few years ago at the inception of the Saturn space vehicle program. The switchover from ballistic rockets with more limited range and more uniformly defined trajectories to space vehicles, which call for more variety of trajectory shaping, and the availability of a new computer technology, which permits rather large scale digital computations aboard a flying vehicle, induced us to deviate from the old Δ-minimum scheme that was used successfully on various vehicles like Redstone, Jupiter, Pershing, etc., and that was tailored to use analog equipment with prime emphasis on simplicity and accuracy. The new concept, which we call adaptive guidance, aims at generality in view of the many different mission geometries of multistage space vehicles, flexibility in view of frequent changes in physical characteristics of vehicles prior to flight as well as in flight, and performance optimization in the presence of major physical disturbances (e.g., engine-out cases), plus of course, accuracy of achievement of final end conditions. Various mathematical approaches are feasible toward accomplishment of these goals, and several have been explored in some detail at MSFC. Two of them, the polynomial adaptive guidance and the iterative guidance scheme, have been carried through the successful application in fullscale Saturn I earth orbital flights.

The selection of the iterative guidance scheme for the follow-on Saturn IB and Saturn V flight programs has been made based on decisive advantages in terms of flexibility with regard to changing physical characteristics, i.e., switchover to alternate mission for engine-out cases and easy adaption to a wide variety of complex three-dimensional mission profiles with a minimum of previous ground computation. A good description of both adaptive guidance systems has been given by Dr. W. Haeussermann at the August AIAA Meeting at San Francisco. Our first paper by Mr. Clyde Baker, Chief of the Astrodynamics & Guidance Theory Division, complements Dr. Haeussermann's paper by sketching the various mathematical options towards our adaptive guidance scheme and describing in somewhat more detail the particular mathematical features of the iterative guidance scheme. For more details, I refer you to the third issue of the semiannual Aero-Astrodynamics Research Report. Mr. Baker does not go into a comparison of this scheme with other similar schemes developed independently and approximately concurrently at MIT, STL, and Aerospace Corporation. Suffice it to say that we have studied these other

* Director, Aero-Astrodynamics Laboratory.

methods and have found, in spite of many similarities, some specific advantages of our iterative guidance scheme.

There is a very complex pattern of interactions between the theoretical scheme with

a. The particular mission requirements in terms of mission profile geometry and operational constraints and alternate mission requirements for engine out, etc.

b. Hardware considerations such as trade-offs between computer memory and complexity of equations and frequency of onboard computations.

c. Practical computational requirements such as need of preflight computations in view of parametric changes.

All of this demonstrates the extreme importance of a close marriage between the guidance theory development and the system development.

In view of this, a continuation of support by OART, NASA Headquarters for advanced guidance studies at MSFC, appears to us very desirable and important. We hope that this review may be of some help to underscore the value of these efforts.

With respect to the second paper by Mr. Dearman, on optimization theory, I would like to make the following observations: The application of optimization theory to trajectory shaping only is discussed in this paper, and this subject is obviously closely related to guidance theory. Most of the concepts and methods are equally applicable to other problems, in particular to control problems. Several studies related to this field are carried on by and under sponsorship of the Aero-Astrodynamics Laboratory.

An attempt has been made to describe the subject without use of equations. At the same time we did not mean to oversimplify the matter for the sake of popularization. Since optimization theory is concerned with subtle points, we cannot expect a very easy paper; however, I believe Mr. Dearman succeeded in producing a very lucid presentation on his subject. The impression may be gained from his paper that in view of the shortcomings of the present state of optimization theory, no answers can be found to many practical trajectory problems. In many cases engineering intuition and/or extensive numerical work (parametrical treatment) can substitute for more rigorous mathematical methods and produce practically acceptable optimum or near optimum solutions. The motivation for improving theory in such cases is more for reduction of computational effort and more direct assurance of optimality of a solution.

The final paper on astrodynamics by Mr. Schwaniger is probably the most acceptable one to those not familiar with the subject matter since it describes largely geometric properties of classes of trajectories. Lack of time did not permit much discussion of computational tools. While the general trend in engineering is towards more abstraction and complexity due to improvements in theory, availability of powerful computers, and increasing complexity of problems and capability for thorough optimization, there is still a need and a place for simplification especially for surveys as aids in mission synthesis. This is not only to make economical use of computers, but also to gain insight into characteristic features of solutions which may otherwise escape the attention or grasp of the investigator. There has been a creative interplay throughout the history of physical science between the intuitive approach proceeding from specific cases to generalization (inductive) and the abstract approach which typically deduces individual cases from general theory (deductive); we believe there will be a continued need for this dual approach.

The term astrodynamics has been historically used by astronomers for dynamic analysis of the motion of heavenly bodies; the prime change has been the recent emphasis of powered trajectories, i.e., bodies under the influence of forces other than gravity.

SATURN GUIDANCE CONCEPTS

N66-23082

by

Clyde D. Baker*

SUMMARY

The basic problem in space guidance is to develop some relatively simple way to compute the direction of thrust at points along a trajectory which will permit meeting the desired terminal conditions of the trajectory. The development of such a guidance law or guidance scheme usually involves some method to approximate the calculus of variations solution which maximizes payload.

Four such approximations are discussed in this paper. Three are polynomial type approximations to a closed loop steering function. The fourth method, which is actually used for Saturn guidance, is a closed form solution of the calculus of variations problem using a simplified earth model.

GLOSSARY

Adaptive guidance mode - This means that, at each point in the flight, the choice of steering angle made at that point is always the one which tends to maximize the payload delivered to the required end condition of the trajectory problem.

Iterative guidance mode -

Series reversion method -

Guidance function expansion method

Least square curve fitting -

} Four methods of obtaining adaptive guidance.

Δ minimum guidance - A guidance mode to continuously correct the steering angle to force the vehicle to stay on a predetermined trajectory.

F/m - Thrust to mass ratio

Isp - Specific Impulse - this is the ratio of thrust measured in kg to kg of propellant consumed per second.

Mixture ratio shifts - Change in propellant management during flight affecting both thrust level and mass flow of propellant.

Open loop steering - Synonymous with steering without feedback

Time-to-go - Is that time remaining before thrust cut-off in a flight.

$\tilde{\chi}$ - Principal part of angle χ such that $\tilde{\chi}$ plus an additional small angle is equal to the actual steering angle χ.

SECTION I. INTRODUCTION

This discussion deals with the research work that has been carried out for the development of guidance concepts for the Saturn vehicles. This work was motivated by the development of new mathematical techniques for maximization of payloads through optimization methods and by the development of digital computers to replace analog computers as onboard hardware. It was also obvious that space trajectories would require greater flexibility to cope with sudden changes such as engine out conditions and that more flexibility must be permitted in the selection of flight profiles.

We are primarily concerned with the iterative guidance law developed for Saturn vehicles to meet these new requirements of space-age guidance. However, we will briefly discuss some of the other techniques which were studied in parallel during the development of the iterative guidance mode.

A typical space guidance problem is that of placing a space vehicle into a specified circular orbit about the earth. This particular problem will be emphasized as a typical problem. The iterative guidance law will achieve a wide variety of other guidance tasks such as injecting a spacecraft into a specified lunar orbit or soft land a vehicle at a preselected point on the surface of the moon. The discussion will be limited to the problem of injection

* Chief, Astrodynamics and Guidance Theory Division, Aero-Astrodynamics Laboratory.

into a circular orbit about the earth, because the basic principles involved are the same for all specific applications.

This problem is illustrated in Figure 1.

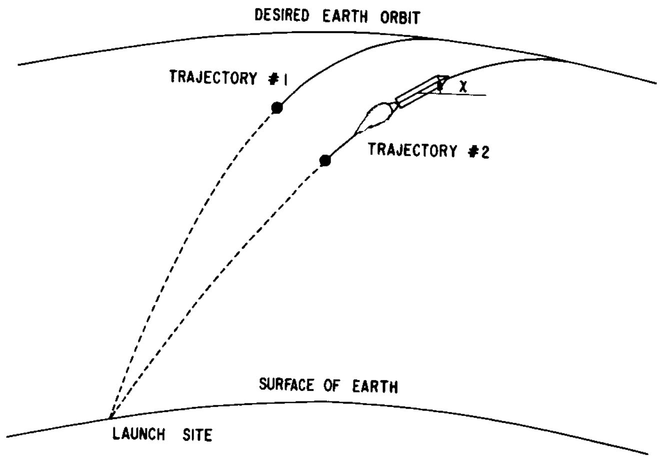

FIGURE 1. CIRCULAR EARTH ORBIT

Trajectory #1 is a nominal trajectory, and trajectory #2 is one in which some perturbation has occurred during the flight. The guidance problem in both cases is to choose the angle χ between the longitudinal axis of the vehicle and some reference direction so that the vehicle enters the desired orbit. Normally, it is additionally required that the vehicle be placed into the orbit with maximum payload or, what is equivalent, minimum burning time. Obviously, the steering angles in trajectories #1 and #2 will be different because of the different forces acting on the vehicle during the flight caused by the perturbation assumed in trajectory #2.

Guidance is active only after the vehicles are out of the atmosphere when the aerodynamic forces are zero. The flight during the first stage, shown by the dotted lines, is controlled by the autopilot, and the main concern during this portion of the flight is to keep the aerodynamic forces from destroying the vehicle. This discussion is concerned with the second stage only when guidance is active. This portion of the trajectory is shown by the solid line.

SECTION II. THE GUIDANCE PROBLEM

The development of a guidance concept for the problem just illustrated is somewhat unusual as an engineering problem. A typical difficulty in engineering is to find more accurate mathematical solutions for a given problem. Somewhat the reverse is true in the development of a guidance concept.

For example, by means of the calculus of variations, a precise mathematical tool exists for the calculation of the steering angles which will guide the vehicle into the desired orbit. However, there are two principal difficulties with the calculus of variations solution.

First of all, the numerical calculations required to establish the steering angles are far too complex to be carried out aboard the vehicle. The solutions of the calculus of variations equations require a computer the size of the IBM 7094, which obviously cannot be flown with the vehicle.

In the second place, the solution of the calculus of variations equations does not provide the steering law in feedback form. This last comment deserves some clarification.

The form of the solution of the calculus of variations to the problem stated is to provide the steering angles χ as a function of time. Furthermore, to obtain this solution, all physical and environmental conditions which will occur during the flight must be known before the launch takes place.

If the vehicle weight during the actual flight is different from that assumed for the determination of the steering angle, the steering will be incorrect. The same is true if the engine Isp is not nominal or winds during the first stage are different from those assumed for the guidance calculations. In fact, the steering law will be incorrect if any of the information assumed for the calculations are different from those actually encountered during the flight.

What is needed is a closed loop feedback steering law. This means that the steering angles should not be functions of time alone as the calculus of variations solution provides. The steering law should be provided as a function of position, velocity, and acceleration, i.e., onboard measurable quantities. Then if the vehicle experiences different forces during flight from those which were predicted, there is a basis to take these variations into account and to correct the steering angles accordingly.

SECTION III. FOUR GUIDANCE CONCEPTS

Four different concepts will be discussed for reducing the amount of onboard computations and obtaining a closed loop steering law. These concepts have been studied both in-house and by MSFC contractors. All four have in common an approximation to the calculus of variations solution. These four concepts, (1) series reversion method, (2) guidance function expansion, (3) least square curve fitting, and (4) iterative guidance mode, are shown schematically in Figure 2 and all are referred to as being adaptive. This simply means that at each point in the flight, the choice of the steering angle made at that point is always the one that tends to maximize the payload delivered to the required end conditions of the trajectory problem.

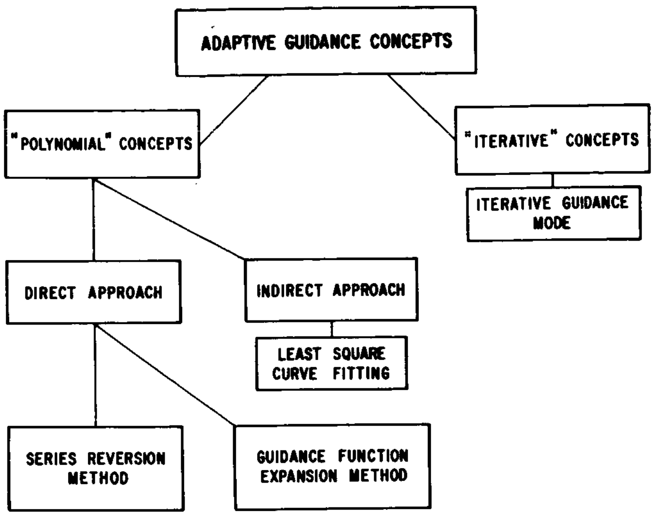

FIGURE 2. GUIDANCE CONCEPTS

Three of these concepts result in polynomial type expansions of the steering angle as functions of the current position, velocity, and acceleration of the vehicle. The fourth concept (iterative), which is actually the one used on Saturn vehicles, is an approximate explicit solution to the trajectory optimization problem.

A. CALCULUS OF VARIATIONS SOLUTION

Because all four of these concepts have their bases in the calculus of variations solution, this solution will be briefly outlined to provide a background for the modifications which have resulted in the four concepts which are of primary interest here.

To simplify the discussion of the mathematical equations, a somewhat simplified version of the original problem of injecting into a circular earth orbit will be desired. Consider then on Figure 3 the problem of the flight of a space vehicle on a flat, nonrotating earth where the gravitational vector is constant and always parallel to the y-axis. The object of the guidance system is to deliver the vehicle from the launch site shown at the origin of the x, y coordinate system to a fixed position in space with a fixed velocity. This is to be accomplished in the shortest possible time with a fixed thrust magnitude and fixed burning rate of the propellant. The angle χ as shown on the slide is the only choice to be made.

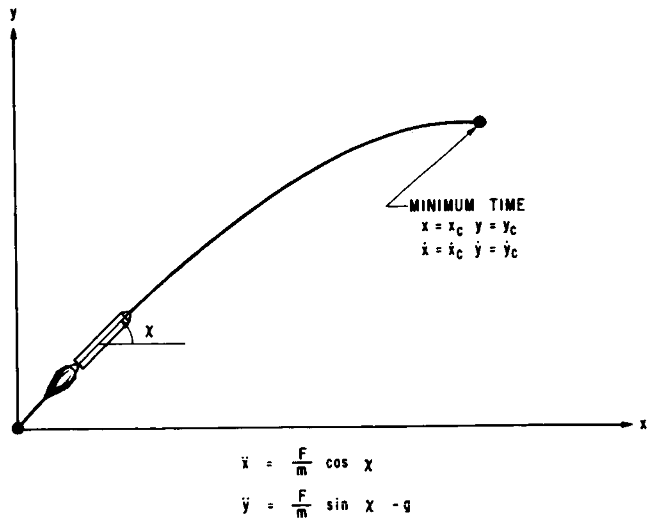

FIGURE 3. A SIMPLIFIED TRAJECTORY

The equations of motion of this simplified trajectory are shown below the trajectory. Applying the calculus of variations to this problem, the solution for χ is given in Figure 4.

$$\tan \chi(t) = \frac{P_3}{P_1} = \frac{a+bt}{1+ct}$$

a, b, c are constants

FIGURE 4. CALCULUS OF VARIATIONS SOLUTION FOR χ

Mathematically, the equation for tan χ contains four unknown quantities, a, b, c, and t_c evaluated at engine cutoff time. There are four terminal conditions to be fulfilled, two position coordinates, and two velocity coordinates. By integrating the equation in $\ddot{x}$ and $\ddot{y}$, it is then possible to obtain four equations in the four unknowns. These equations, of course, can be solved by some means to obtain values of a, b, c, and t_c. After some manipulations these values can be used to obtain the trajectory illustrated in Figure 3.

Here again, for emphasis, observe that the angle χ depends only upon time during the flight since the constants a, b, and c must be calculated before the equations of motion may be integrated. Thus the angle χ would remain the same function of time regardless of how thrust or mass flow (propellant) changed during flight. This is one undesirable aspect of the calculus of variations solution. The other undesirable feature is that a high speed computer is required to obtain the values of a, b, and c in a reasonable length of time.

Thus the calculus of variations type solution provides an open loop type guidance. The steering angle is obtained as a function of time which remains the same regardless of what disturbances occur during the flight. For disturbed flights, then, the COV solution trajectory will not provide the desired accuracy of the terminal trajectory conditions. The discussion will be directed toward describing four techniques for converting the COV solution into a true feedback system which will provide the desired accuracy when disturbances are present. The first one to be described is the series reversion method.

B. THE SERIES REVERSION METHOD

The basic idea of the series reversion method is to obtain the values of a, b, c, and t_c in terms of the instantaneous vehicle coordinates of position, velocity, and acceleration. This is illustrated on Figure 5.

$$a = f_1(x, \dot{x}, y, \dot{y}, F/m)$$

$$b = f_2(x, \dot{x}, y, \dot{y}, F/m)$$

$$c = f_3(x, \dot{x}, y, \dot{y}, F/m)$$

$$t_c = f_4(x, \dot{x}, y, \dot{y}, F/m)$$

FIGURE 5. SERIES REVERSION SOLUTION

All of the mathematical details of this method are of little interest here. Briefly stated, however, the process involves expressing the cutoff position, velocity, and time in a set of Taylor's Series involving the current position, velocity, and acceleration with the values of a, b, c, and t_c. By a simultaneous reversion of this set of Taylor's Series which involves a very considerable amount of work, it is possible to obtain the equations shown on Figure 5. Explicit forms of these equations are shown on Figure 6.

$$a = a_0 + a_1 x + a_2 y + a_3 \dot{x} + a_4 \dot{y} + a_5 x^2 + a_6 y^2 + a_7 \dot{x}^2 + a_8 \dot{y}^2 + \cdots$$

$$b = b_0 + b_1 x + b_2 y + b_3 \dot{x} + b_4 \dot{y} + b_5 x^2 + \cdots +$$

$$c = c_0 + c_1 x + c_2 y + \cdots +$$

$$t_c = t_0 + t_1 x + t_2 y + \cdots +$$

FIGURE 6. SERIES REVERSION

These equations may then be evaluated to obtain the constants a, b, c, and t_c, and the angle χ is then easily obtained from these constants.

The advantage of such a representation of the angle χ is that as the actual trajectory deviates from the expected trajectory, the values of a, b, c, and t_c change accordingly to guide the vehicle back to the desired end points and do this in the minimum amount of time. The disadvantages of such a scheme are several.

First, there is a tremendous amount of work involved in the in the calculation of the equations shown on Figure 5. In the second place, these equations must be reevaluated for each new set of terminal conditions since the actual cutoff values of position and velocity are contained as parameters in the system of equations.

Finally, the number of terms required in the Taylor's Series expansion described previously are likely to be so high that the computer storage problem becomes prohibitive.

C. THE GUIDANCE FUNCTION EXPANSION METHOD

The guidance function expansion method is in many respects similar to the series reversion technique and has essentially the same advantages and disadvantages. A brief description of this method is given leaving out most of the mathematical detail.

The basic concept again is to devise some means to calculate the values of a, b, c, and t_c as functions of the vehicle current position, velocity, and acceleration. Figure 7 shows the form of the solution of the guidance function expansion method.

$$a = a^* + \frac{\partial a}{\partial x}\Big|_{t=t^*}^{(x - x^*)} + \frac{\partial a}{\partial \dot{x}}\Big|_{t=t^*}^{(\dot{x} - \dot{x}^*)} + \frac{\partial a}{\partial y}\Big|_{t=t^*}^{(y - y^*)} + \frac{\partial a}{\partial \dot{y}}\Big|_{t=t^*}^{(\dot{y} - \dot{y}^*)} + \frac{\partial a}{\partial t}\Big|_{t=t^*}^{(t - t^*)}$$

$$b = b^* + \frac{\partial b}{\partial x}\Big|_{t=t^*}^{(x - x^*)} + \cdot \cdot \cdot +$$

$$c = c^* + \frac{\partial c}{\partial x}\Big|_{t=t^*}^{(x - x^*)} + \cdot \cdot \cdot +$$

$$t_c = t^* + \frac{\partial t_c}{\partial x}\Big|_{t=t^*}^{(t - t_\chi)} + \cdot \cdot \cdot +$$

FIGURE 7. GUIDANCE FUNCTION EXPANSION SOLUTION

Observe that this method also involves an expansion of the values of a, b, c, and t_c into a Taylor's Series. However, in this case, the expansion is carried out about some preselected point on a nominal optimal trajectory. This point is identified by the coordinates x^*, $\dot{x}^*$, y^*, $\dot{y}^*$, and t^*. There are several variations of the guidance function expansion method which selected several points about which to carry out the series expansion, or in one case, the reference point is allowed to move continuously along the curve.

Much work has been devoted to the study of the first two methods just described, but neither has ever been carried out to the point of obtaining an actual steering equation. The number of terms required in the Taylor's Series is likely to be so large as to be prohibitive.

D. THE LEAST SQUARE CURVE FIT

The least square curve fitting technique is probably the simplest of the four methods conceptually and will now be explained. The equation which is desired as an end product of the curve fitting technique is shown on Figure 8. Here, the idea is to express the steering angle χ directly in terms of the position, velocity, and acceleration, together with products of these coordinates.

$$\chi = a_0 + a_1\dot{x} + a_2\dot{y} + a_3 x + a_4 y + a_5 (F/m) + a_6\dot{x}^2 + a_7\dot{y}^2 + a_8 x^2 + a_9 y^2 + a_{10}(F/m)^2 + a_{11}\dot{x}\dot{y} + a_{12}\dot{x}x + a_{13}\dot{x}y + a_{14}\dot{x}(F/m) + a_{15}\dot{y}x + a_{16}\dot{y}y + \cdots +$$

FIGURE 8. THE LEAST SQUARE CURVE FITTING

The coefficients of this polynomial, a_i, ($i = 1, 2, 3 \ldots 16, \ldots$) are obtained by precalculating optimal trajectories with various types of disturbances deliberately introduced into the trajectories. Then from a sampling of the steering angles associated with the values of the coordinates on each of these trajectories, a least square curve fit is made which minimizes the sum of the squares of the errors in the angle χ at each of the points selected in the curve fit.

This method appears to be more of an art than a science for several reasons. A number of choices must be made somewhat arbitrarily. These include the number of terms to include in the equation, the actual terms which will appear in the equation, the number of trajectories which should be precalculated to include in the least square curve fitting process, and finally some choice must be made of the points to be utilized from a given trajectory. There has not been a clear answer, and there is still none as to how these choices should be made. Some people have been successful in making these selections so that adequate steering functions have been computed by this process. The flights of SA-6 and SA-7 were guided by steering functions generated in this manner, and the accuracy obtained was certainly satisfactory.

Perhaps the biggest fallacy in the technique, however, lies in the tacit assumption that the polynomial which produces the least sum of squares of errors in expressing the angle χ as a function of the coordinates also provides the best steering law. Since this is not true, an additional constraint was placed on the curve fitting, that the partial derivatives of the angle χ with respect to the coordinates in the curve fit would match the partial derivatives of the angle χ with respect to the coordinates obtained directly from the calculus of variations.

This constraint vastly improved the steering law when guidance was applied to a single stage only.

However, the additional complexities imposed upon the curve fitting by multiple stages and the introduction of the step mixture ratio shift, together with the requirements of continuously variable launch azimuths, caused the decision to drop the polynomial curve fits in favor of the iterative guidance mode which will now be discussed.

E. THE ITERATIVE GUIDANCE MODE

Because the iterative guidance mode has been successfully flown on SA-8, 9, and 10 and shows promise of providing the accuracy, performance optimization, and flexibility for future Saturn vehicles, it will be discussed in considerably more detail than the previous three. The basic problem remains to devise some means to express the constants a, b, c, and t_c as functions of the current coordinates of the vehicle. In the three previously described techniques, these constants were obtained by series representations. The methods differed principally in the way in which the coefficients of the series were derived. The iterative guidance techniques solves for these values of a, b, c, and t_c in a basically different way.

To illustrate the iterative guidance concept, consider once again the problem of placing a vehicle in a desired circular orbit around the earth. The concern will only be that the circular orbit is obtained, and no conditions will be placed upon the exact point at which the vehicle enters the circular orbit. Removal of the constraint on the point at which injection occurs makes possible some simplification of the guidance law to obtain this orbit. In this case the guidance law can be written as shown in Figure 9.

$$\tan \chi = a + bt$$

$$\chi \approx A + Bt$$

FIGURE 9. ITERATIVE GUIDANCE LAW FOR CIRCULAR ORBIT

This law is only an approximation even for the flat earth model, but it is an excellent approximation. A further simplification is made that the angle χ itself may be expressed as a linear function of time where this linear function is different from the linear law obtained for tan χ.

This expression for $\chi = (A + Bt)$ may now be substituted into the equation of motion as shown on Figure 10. These equations appear somewhat complicated, but there are only two essential simple facts that should be noted about these equations. First of all, they contain only three unknowns. These are A, B, and t_c. All other information is either known or measured during flight. The second significant thing to note is that there are only three conditions to be satisfied at injection. These are $\dot{x}_c$, $\dot{y}_c$, and y_c. The fourth coordinate x_c has been eliminated by eliminating the constraint on the range at which injection takes place.

This means that if some relatively simple way can be found to solve these three equations in three unknowns, the guidance problem will be solved. The

$$\ddot{x} = \frac{F}{m} \cos (A + Bt)$$

$$\ddot{y} = \frac{F}{m} \sin (A + Bt) - g$$

$$\dot{x}_c = \int_0^{t_c} \frac{F}{m} \cos (A + Bt)\, dt + \dot{x}_0$$

$$\dot{y}_c = \int_0^{t_c} \frac{F}{m} \sin (A + Bt)\, dt - g t_c + \dot{y}_0$$

$$x_c = \int_0^{t_c}\int_0^{t} \cos (A + Bt)\, dt^2 + \dot{x}_0 t_c + x_0$$

$$y_c = \int_0^{t_c}\int_0^{t} \sin (A + Bt)\, dt^2 - \frac{g t_c^2}{2} + \dot{y}_0 t_c + y_0$$

FIGURE 10. EQUATIONS OF MOTION AND THEIR INTEGRALS

remainder of the discussion will be concerned with the problem of solving these equations with an indication of what impact on accuracy and performance results from the necessary simplification made to solve the equations.

In order to solve the equations, one additional bit of information from the calculus of variations is useful. This fact is that if the only conditions to be fulfilled at cutoff time are velocity conditions, that is, only $\dot{x}_c$ and $\dot{y}_c$ are prescribed at cutoff time, then the calculus of variations states that the steering angle under these conditions is a constant.

This fact has two important consequences. As a first step in the solution of the three simultaneous equations, it is possible to satisfy the velocity conditions with a constant steering angle which makes the integration of the equations of motion trivial. Thus the solution for required velocity has temporarily been separated from the problem of calculating the required altitude, and the equations of motion have been greatly simplified.

The equations of motion for a fixed steering angle together with their first integrals are shown on Figure 11. The last two equations can be solved for χ as shown on Figure 12. It should be noted that at this point in the solution, the value of t_c is not known in the equations for tan χ.

The next step in the solution is to satisfy the additional condition that y_c equals the required value. The value of $\tilde{\chi}$ is the principal part of the original angle $\chi = A + Bt$. Let this be rewritten as shown in Figure 13.

$$\ddot{x} = \frac{F}{m} \cos \tilde{\chi} \qquad m = m_0 - \dot{m}_t$$

$$\ddot{y} = \frac{F}{m} \sin \tilde{\chi} - g$$

$$\dot{x}_c = -\frac{F}{\dot{m}} \ln (m_0 - \dot{m}\, t_c) \cos \tilde{\chi} + \dot{x}_0$$

$$\dot{y}_c = -\frac{F}{\dot{m}} \ln (m_0 - \dot{m}\, t_c) \sin \tilde{\chi} - g\, t_c + \dot{y}_0$$

FIGURE 11. CONSTANT STEERING ANGLE EQUATIONS

$$\tan \tilde{\chi} = \frac{\dot{y}_c + g t_c - \dot{y}_0}{\dot{x}_c - \dot{x}_0}$$

FIGURE 12. CONSTANT STEERING ANGLE χ

$$\chi = \tilde{\chi} - K_1 + K_2 t$$

FIGURE 13. SEPARATED FORM OF χ

This expression is substituted into the equations of motion to obtain the equations as shown on Figure 14.

$$\ddot{x} = \frac{F}{m} \cos (\tilde{\chi} - K_1 + K_2 t)$$

$$\ddot{y} = \frac{F}{m} \sin (\tilde{\chi} - K_1 + K_2 t)$$

FIGURE 14. EQUATIONS OF MOTION

By making the assumption that $-K_1 + K_2\, t$ is a small angle so that the sine of the angle is equal to the angle and that the cosine of this angle is equal to one, these equations may be integrated in closed form. These closed form solutions are indicated on Figure 15. These equations satisfy all of the terminal conditions.

$$\dot{x}_c = f_1 (K_1, K_2, t_c, \dot{x}, \dot{y}, x, y, F/m)$$

$$\dot{y}_c = f_2 (K_1, K_2, t_c, \dot{x}, \dot{y}, x, y, F/m)$$

$$x_c = f_3 (K_1, K_2, t_c, \dot{x}, \dot{y}, x, y, F/m)$$

$$y_c = f_4 (K_1, K_2, t_c, \dot{x}, \dot{y}, x, y, F/m)$$

FIGURE 15. CLOSED FORM SOLUTIONS FOR $\dot{x}_c$, $\dot{y}_c$, x_c, y_c

These equations appear to be essentially the same as similar equations which have appeared in the description of some of the previous guidance concepts. They are in principle very different because algebraic representations can be obtained for each of them. This means that they do not have to be resolved when the terminal conditions are changed. In the previous guidance concepts these representations were given in numerical form only.

The significance of the equations of Figure 15 is that their solutions can be programmed on a relatively small computer and that the values of K_1, K_2, t_c, and χ can be solved as functions of the current vehicle coordinates. Figure 16 shows the explicit form of the equations on Figure 15.

$$\tau = V_e / \frac{F}{m} \qquad A_1 = V_e \ln \left[1 - (T/\tau)\right]$$

$$A_2 = A_1 \tau - V_e T$$

$$A_3 = -A_2 + T A_1$$

$$A_4 = A_3 \tau - \left[(V_e T^2)/2\right]$$

$$\phi_T = (A_3 + V_0 T)/\eta_T$$

$$\xi_0 = X_0 \cos (\phi_0 + \phi_T) - y_0 \sin (\phi_0 + \phi_T) \quad \text{and similarly for velocity}$$

$$\eta_0 = X_0 \sin (\phi_0 + \phi_T) + y_0 \cos (\phi_0 + \phi_T)$$

$$g^* = \tfrac{1}{2} (g_0 + g_T) \qquad \phi^* = \tfrac{1}{2} \phi_T$$

$$\begin{cases} \Delta V_0^{\,2} = (\dot{\xi}_T - \dot{\xi}_0 - g^* T \sin \phi^*)^2 + (\dot{\eta}_T - \dot{\eta}_0 + g^* T \cos \phi^*)^2 & \text{Solve for } T \\ \Delta V_0 = -V_e \ln \left[1 - (T/\tau)\right] & \end{cases}$$

$$\bar{\chi}_\xi = \tan^{-1} \left[(\dot{\eta}_T - \dot{\eta}_0 + g^* T \cos \phi^*)/(\dot{\xi}_T - \dot{\xi}_0 - g^* T \sin \phi^*)\right]$$

$$P = A_3 \cos \bar{\chi}_\xi \qquad Q = A_4 \cos \bar{\chi}_\xi$$

$$R = \eta_T - \eta_0 - \dot{\eta}_0 T + \tfrac{1}{2} g^* T^2 \cos \phi^* - A_3 \sin \bar{\chi}_\xi$$

$$K_1 = (A_2 R)/(A_1 Q - A_2 P) \qquad K_2 = (A_1 R)/(A_1 Q - A_2 P)$$

$$\chi_\xi = \bar{\chi}_\xi - (K_1 - K_2 t) \qquad \chi = \chi_\xi - \phi_0 - \phi_T$$

Check velocity or T for cutoff. Stop computation of K_1 and K_2 when T becomes small. (T as shown in these equations = t_c, time of cutoff, used in text.)

FIGURE 16. ITERATIVE GUIDANCE EQUATIONS

1. Accuracy Considerations. Some explanation must now be given of the accuracy to be expected of the iterative guidance concept. Many approximations have been made to arrive at the closed form solutions of the equations presented. First of all, the earth model chosen was a flat, nonrotating earth. After that several small angle approximations were made to simplify the integration of the equations of motion. With the aid of Figure 17, some of the effects of these simplifications will be interpreted.

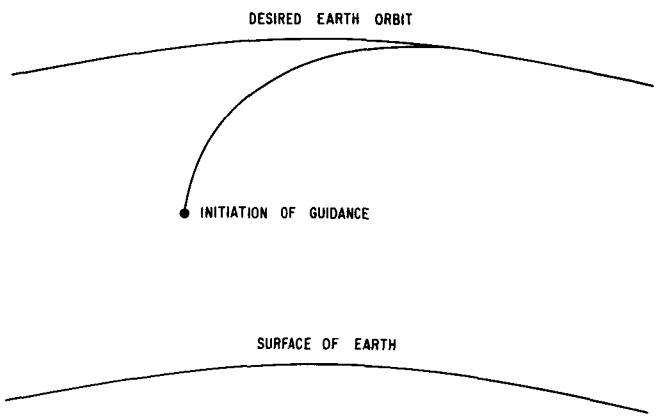

FIGURE 17. ITERATIVE GUIDANCE MODE COMPUTATIONS

At the initiation of guidance as shown in the figure, an initial value of the steering angle χ is computed. This angle is not precisely correct because of the many simplifications. But this angle is used to guide the vehicle for some brief interval of time, say one or two seconds. Then a recomputation of the angle χ is made based upon the coordinates at the later time. This process is repeated at short time intervals throughout the remainder of the flight. As time progresses, the assumptions become more accurate until at the instant of cutoff they are exact. Thus, the scheme is a self-correcting process for the errors committed by the simplifications and it is also self-correcting for any perturbations which may occur during the flight. This is true because the problem is resolved at each computation step without regard to what has happened in the past. Final accuracy of the terminal conditions is assured by this self-correcting feature.

2. Optimality. While the very nature of the process insures the desired accuracy of the system, maximizing the payload by this technique is not guaranteed. For example, it is necessary to correct for the assumption of constant magnitude and constant direction of the gravity vector.

This compensation is made by introducing into the equations of motion a weighted average of the gravity magnitude and direction between the current point on the flight and the final point. It is also necessary to rotate the coordinate system so that one axis coincides with the local vertical at the cutoff point. This is done to avoid introducing difficulties into the equations of motion by awkward end conditions if the coordinate system is not rotated.

Some additional accuracy problems arise when this concept is extended to cover multiple guided stages and the programmed mixture ratio shifts. These problems are more in the nature of minor annoyances because solutions have always been found so that the payload loss of the IGM as compared to the strict calculus of variations solution is negligible.

3. Stability and Error Analysis. The partial derivatives of attitude with respect to the state variables are the most significant criteria for stability and accuracy. The F/M derivative is small during the entire flight, eliminating this usually rather noisy measurement as trouble source. However, as the trajectory optimization is based on a predicted relation of the future thrust profile for a stage to the instantaneously measured value, any major thrust change will cause a performance loss.

The other derivatives start at low values and increase approximately inversely proportional to the time-to-go (for velocity errors) or its square (for displacement). The tightening of the guidance loop toward the end of flight is very desirable as it keeps residual errors small. However, it creates a potential stability problem. This problem was eliminated without causing a significant error by stopping computation of the steering equations at a given time-to-go (e.g., T = 20 seconds) and flying open loop. A better method is to freeze the time-to-go at a minimum value and continue guidance.

The low guidance gains at early flight make the system very tolerant to major disturbances, noise, and time lags during this phase.

Guidance scheme errors for realistic variations of initial conditions (Fig. 18) are very small. The effects of performance variations, changes in air density, and winds are equally insignificant.

A time lag of five seconds from measurement to steering command causes no error and no loss of weight in orbit. A 40-sec lag caused 3-km altitude error and 11 percent payload loss.

Periodic thrust fluctuations with a maximum amplitude of 65 percent of nominal and periods of 5 to 100 seconds create no serious stability problem.

INITIAL STAGE VARIABLE				PAYLOAD LOSS	INJECTION ERRORS		
Δx_1	$\Delta \dot{x}_1$	Δy_1	$\Delta \dot{y}_1$		Altitude	Velocity	Path Angle
km	m/s	km	m/s	percent	m	m/s	degrees
2.7	0	0	0	.11	.1	0	.001
0	143	0	0	.32	.13	−.04	0
0	− 57	0	0	.07	.1	.01	.001
0	0	1.0	0	.11	.1	0	.001
0	0	0	78	.14	.1	−.01	0
0	0	0	−80	.11	.1	0	.001

FIGURE 18. ITERATIVE GUIDANCE MODE ACCURACY & PERFORMANCE

4. Present and Future Guidance Research. The iterative guidance mode as just described provides accuracy, optimality, and flexibility for the examples shown on Figure 19.

1. Single Stage to Orbit
2. Multiple Stages to Orbit
3. Earth - Moon Guidance
4. Earth - Mars Guidance
5. Plane Change Capability
6. Three Dimensional Guidance
7. Flexibility for Alternate Guidance & Abort

FIGURE 19. ITERATIVE GUIDANCE ACHIEVEMENTS

Actual calculations have demonstrated its capability for placing a space vehicle into earth orbit by a single guided stage to orbit; it is also successful with two stages to orbit including a step mixture ratio shift which essentially becomes three guided stages to orbit.

The iterative guidance mode has been used successfully also to guide trajectories out of earth orbit to the moon and also to guide to the planets. This is an impressive list of accomplishments for a guidance concept, and others could be related concerning flexibility to change to alternate mission after launch and to handle abort situations which may occur during flight.

However, the intent at this point is to describe the guidance situation for which the iterative guidance mode has not been demonstrated adequately and to indicate what efforts are being made to provide satisfactory guidance for these cases.

As a general comment, it may be pointed out that iterative guidance mode does not perform satisfactorily at present when any one of the following three conditions shown in Figure 20 is encountered.

1. Low Thrust to Weight Ratio
2. Large Central Angle
3. Rendezvous

FIGURE 20. NECESSITIES OF NEW OR IMPROVED METHODS

a. The first limitation occurs when thrust to weight ratio is less than a few tenths of one g. Under these conditions the approximations which are made in the derivation of IGM become so inaccurate at this thrust level that satisfactory performance is not attained. Thus new concepts are being sought for low thrust interplanetary flights where the thrust levels are of the order of a small fraction of one g.

b. The second limitation of the IGM occurs when the central arc, i.e., the angle at the center of the earth between the point of beginning of guidance and termination of guidance, exceeds approximately thirty degrees. Starting from earth orbit with low thrust trajectories, it is sometimes necessary to spiral around the earth several times before the velocity of the space vehicle reaches essentially escape velocity. IGM so far has not proved adequate for these cases.

c. A third category which must be studied carefully is a trajectory where all coordinates are specified at the terminal end of guidance as in rendezvous. IGM has not yet been shown adequate for such cases. It may be possible to modify the iterative scheme to handle all three of these problem areas. Until such modifications are made, efforts will be directed toward studying other concepts together with attempts to modify the iterative concept.

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RESEARCH ACHIEVEMENTS IN OPTIMIZATION TECHNIQUES

by

C. C. Dearman*

SUMMARY

This paper is a survey of the work that has been done at MSFC in attempting to compute optimal, i.e., minimum-fuel, multistage trajectories for space missions. In the early studies, attempts at trajectory optimization were confined to single-stage trajectories. The calculus of variations was the optimization technique principally employed in these studies as well as in the more complex problems which followed. With the discovery of the work by C. H. Denbow, who theoretically, at least, solved a large class of variational problems, work was begun on computing fuel-minimizing, Apollo type, multistage trajectories. While Denbow's extension of the classical theory of the calculus of variations provided for the optimization of multistage trajectories, it demanded that all state variables be continuous. Therefore, it was not satisfactory for solving realistic multistage trajectories in which some state variables, for example, the mass, are discontinuous functions of time at those points of vehicle stage separation where the large mass of the burned-out stages are detached from the vehicle. Therefore, it was necessary to extend Denbow's work to include discontinuous variables. This was done, but only necessary conditions for an optimum were found. At present, there have been discovered no sufficient conditions for the case involving discontinuous state variables as have been found for the Denbow problem with continuous state variables, and unless sufficient conditions are satisfied by the trajectory it cannot be said to be fuel minimizing. This, briefly, is the state of the theory at present.

In attempting to calculate fuel-minimizing multistage trajectories on a digital computer, several difficulties have arisen which have prevented computation of the trajectories. These difficulties seem to stem from the introduction of the additional differential equations and new variables, called Lagrange multipliers, required by the theory. The problem is being investigated both in-house and by outside contractors.

Conducted simultaneously with the studies in multistage trajectory optimization were the studies in low-thrust trajectories, optimal orbital transfers, and optimal reentry trajectories. In the first two of these studies, the calculus of variations techniques proved to be unsuccessful, and other optimizing methods were employed. All of these studies, however, are in the exploratory stage, and much work remains to be done.

RESEARCH ACHIEVEMENTS IN OPTIMIZATION TECHNIQUES

The research in optimization techniques and its applications to problems of space flight at Marshall Space Flight Center has been largely geared to attempts to determine optimal guidance schemes and optimal control laws for space vehicles. The problems in optimization relative to the development of optimal control laws were discussed at the Research Achievements Review Series No. 3 which was held last April. Research in optimization techniques as it relates to space vehicle guidance is the subject of this review. This relation to guidance has two important aspects that motivate the research. These are (1) the development of optimal guidance schemes, and (2) the testing of nonoptimal guidance by providing optimal trajectories as standards for comparison. Both aspects demand the existence of a capability for generating optimal, that is, minimum-fuel trajectories, in both perturbed and unperturbed cases.

In the first aspect, the development of optimal guidance functions, it appears necessary that a suitable sampling of perturbed optimal trajectories meeting all constraints and boundary conditions and satisfying the equations of motion be generated as a preliminary but essential step. The development of the guidance equations from the sampling of perturbed optimal trajectories and, indeed, the selection of the sampling itself, are very difficult problems and do not properly belong in a discussion on optimization techniques even though they are significant motivating factors in the discovery and use of these techniques. In the second aspect, the testing of nonoptimal guidance procedures, the amount of fuel required for nonoptimal guidance in a perturbed trajectory can be compared with the strictly minimum-fuel trajectory subjected to the same perturbations, thereby possibly forming the basis for a judgement

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as to the adequacy of the nonoptimal guidance procedure.

The research in optimization of trajectories may be categorized as shown in Figure 1.

1. **Multistage (Apollo - type) Trajectories**
2. **Low Thrust Trajectories**
3. **Impulsive Orbital Transfer**
4. **Reentry Trajectories**
5. **Supporting Research Activities**

FIGURE 1. RESEARCH IN TRAJECTORY OPTIMIZATION

Mathematically, low-thrust trajectory optimization, impulsive orbital transfer, and reentry trajectory optimization are special cases of the problem in the first category, and the solution of this problem, at least theoretically, implies the solution of the others. However, research in these categories was carried on simultaneously with research in problems in the first category because the considerable difficulties encountered with problems in the first category did not portend an early solution. Also, it was thought that study of the less general problems might shed some light on the multistage problem.

The fifth category, supporting research activities, has been confined largely to studies in celestial mechanics and its application to the motion of spacecraft in cislunar and solar space and to methods of obtaining approximate solutions of the two-point boundary value problem of which the optimal trajectory problem is but a special case.

Figure 2 shows the principal contractors who have worked in the areas of trajectory optimization that we have just discussed.

It is clear from the foregoing that the research in optimization has been motivated almost entirely by the necessity for solving intensely practical problems of immediate and lasting interest in the space program.

When work was first begun several years ago by what is now called the Astrodynamics and Guidance Theory Division of the Aero-Astrodynamics Laboratory on the problem of generating optimal guidance functions for missions involving multistage trajectories, a survey was made of existing optimization techniques in an attempt to discover the technique that would be most practicable for the solution of trajectory optimization problems. The survey led to the selection of the calculus of variations as the most likely candidate for solving the problems. Other techniques are known under such names as the Pontryagin maximum principle, the method of steepest descent, and dynamic programming. The calculus of variations, however, had enjoyed a long and fruitful history of development, and because of this it was the most mature and most highly refined of all known methods.

AREA OF RESEARCH	CONTRACTORS
1. Calculus of Variations	Vanderbilt U., Southern Illinois U., Republic Aviation, Grumman Aircraft, Hayes International, General Electric, Auburn U., Analytical Mechanics Assoc., United Aircraft, Martin-Marietta, Lockheed Aircraft
2. Low Thrust Trajectories	Aeronutronics (Ford), Grumman Aircraft
3. Impulsive Orbital Transfer	North American Aviation, United Aircraft
4. Reentry Trajectories	Auburn U., Raytheon
5. Supporting Research a. Celestial Mechanics b. Multivariant Approx. c. Two-Point Boundary Value Problem	U. of Kentucky, U. of North Carolina, Northeast Louisiana S. C., Georgia Institute of Tech., Chrysler Corporation Missile Division, U. of Wisconsin

FIGURE 2. CONTRACTOR PARTICIPATION IN TRAJECTORY OPTIMIZATION RESEARCH

In this country, the work of G. A. Bliss and his students in the calculus of variations at the University of Chicago Department of Mathematics was available in the form of published books and scores of research papers. Also, in the long history of the development of calculus of variations, many practical applications of the theory had appeared in numerous publications; these were considerably in excess of the number of applications of the other optimizing techniques. Thus, it was that major emphasis, both in-house and among outside contractors employed for the purpose of assisting in the solution of the problem, was given to exploiting the capabilities of the calculus of variations. However, "a foot was kept in the door of the other techniques," so to speak, by assigning some studies in them to contractors and to maintaining a relatively small in-house effort in these techniques as well. Because the principal effort was in exploiting the techniques of calculus of variations, it is this technique that will be emphasized in this review.

Let us consider the general problem of directing a space vehicle from a prescribed initial state to a prescribed terminal state. Let us further suppose that some steering law is available. The state of a space vehicle is considered to be known at any given time t when certain defining parameters are known. For the purposes of discussion here the state of the vehicle will be considered to be known if its position coordinates, velocity components, thrust, mass, and burning rate are known at the given time. Thus, in what is usually called the "state space," nine coordinates are required to define the state of the vehicle at any time. It is to be emphasized at this point that this particular choice of stage coordinates is not necessarily the best choice. It is entirely possible that another choice would make the problem more tractable and the computational aspects much easier to perform. The determination of the best choice of state variables is under continual study.

As a consequence of the number of variables chosen to define the instantaneous state of the vehicle, the equations of motion, considered as a system of first-order differential equations, are nine in number. These equations must be solved simultaneously in order to obtain a solution curve or trajectory in state space. Perhaps it should be mentioned that the solution is obtained by numerical means by a digital computer and not by analytical methods. The attainment of the solution by analytical methods would be, of course, the more desirable, but no one as yet has been able to do this except for simplified cases.

Any solution curve or trajectory must pass through the initial point in state space and must satisfy terminal conditions and numerous constraints as well. Figure 3 illustrates a solution of the equations of motion.

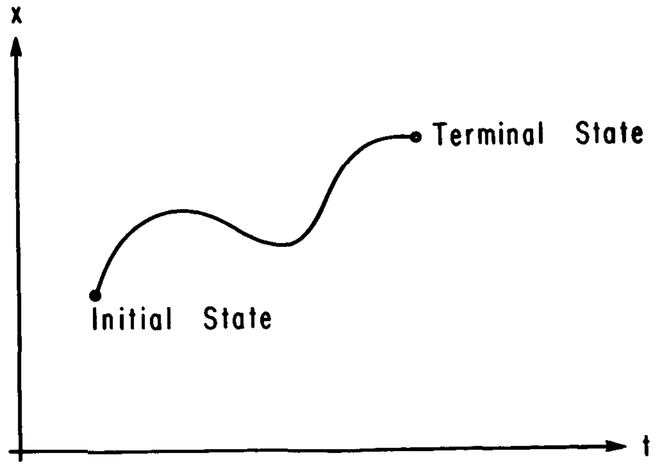

FIGURE 3. A SOLUTION CURVE IN STATE SPACE

Now, if it is required that the solution curve be the optimal or minimum-fuel trajectory, then some optimizing technique must be employed to insure this requirement. As will be discussed in more detail in the sequel, the requirement of optimality introduces additional differential equations and new variables called Lagrange multipliers which must be solved for simultaneously with the nine equations of motion, and "there's the rub," as we shall see.

It was clear from the beginning of the studies in trajectory optimization that the problem of optimization of an arbitrarily general multistage trajectory that began from launch on earth and ended upon return to earth would be far too complex a problem to attack initially. It was decided to concentrate on the problem of optimizing only that part of a multistage trajectory whose initial point is just outside the earth's atmosphere and whose terminus is a prescribed lunar orbit as shown in Figure 4. This trajectory represents a part of an Apollo type trajectory.

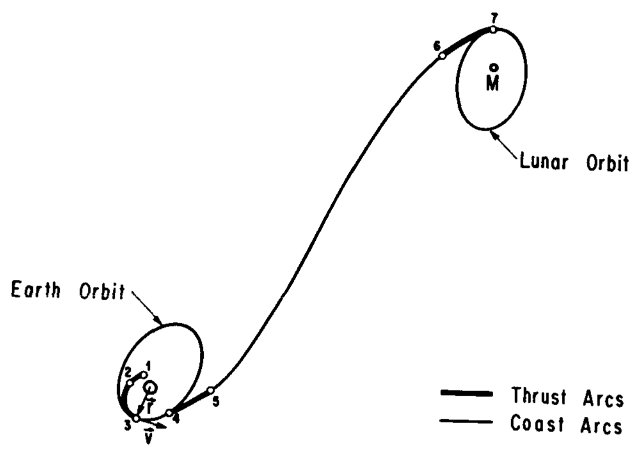

FIGURE 4. A MULTISTAGE TRAJECTORY

No effort was made in the early studies to compute a calculus of variations (COV) trajectory for the atmospheric portion of the flight. Even when this computation is done now, certain simplifications, such as exponential atmosphere, are usually made which probably cannot be tolerated in realistic simulations. Even when the simplifications are not made, as in the deck recently compiled by R. E. Burns of the Aero-Astrodynamics Laboratory, only necessary conditions are satisfied; this means that the resulting COV trajectory is not guaranteed to be fuel minimizing.

In retrospect it appears that the problem of optimizing that portion of the Apollo type trajectory illustrated in Figure 4 is itself too complicated a problem for a beginning effort. Some less ambitious program may be decided upon to provide a simpler problem. Dr. Jan Andrus of General Electric Company and Dr. M. G. Boyce of Vanderbilt University, who have been working for us for several months on this trajectory optimization problem, are rather of the opinion that simpler problems should now be attacked in an effort to learn how to overcome some of the difficulties that have been encountered in our attempts to solve the more complex multistage trajectory problem.

Let us now describe the multistage trajectory to which we have applied the techniques of the COV. Assume that point 1 in Figure 4 represents the termination of an S-1C stage above the earth's surface and that at this point all state variables are known. The point is also presumed to be outside the earth's atmosphere so that the entire space vehicle is there subject only to thrust and gravitational forces. At point 2 assume that the S-II stage is detached and that the S-IVB stage engines are ignited. These separating and igniting actions in the beginning studies were assumed to occur simultaneously. Of considerable significance in the evolution of the optimization problem is that at point 2 the mathematical model of the motion of the vehicle was based on the assumption that there existed discontinuities in the state variables of thrust, burning rate, and mass because of the separation sequence that occurred at that point. It is the assumption of discontinuities in these three state variables that has been the source of many problems, some of which have not yet been solved.

At point 3 the vehicle enters an essentially elliptical orbit of a specified shape and size. This requires that the distance of the point from the earth's center, the velocity of the vehicle at the point, and the angle between the radius vector and velocity vector be specified. The location of point 3 on the ellipse, the orientation of the plane of the ellipse relative to the coordinate system used and the orientation of the ellipse within the plane are not specified. In all studies thus far undertaken, point 3 has been chosen as the perigee of the ellipse thereby fixing the angle between the radius vector r and velocity vector v at 90°. The remaining unspecified factors, the orientation of the plane of the ellipse and the orientation of the ellipse within the plane, are to be determined by the optimization procedure. Because a coast arc is entered at point 3, the thrust and burning rate are reduced to zero. If, as it has been assumed, this is done instantaneously, discontinuities appear again in these two variables.

The optimization process is used to determine point 4 and the time t_4 of the space vehicle's arrival there when thrust is again initiated. The time of flight from point 4 to point 7, where the vehicle enters the lunar orbit, is specified. This is done by initiating thrust at point 4 such that the time lapse between points 4 and 7 is equal to the specified time. This condition of specified flight duration is imposed to prevent the lunar transfer trajectory from extending over an excessively long time. It may be possible for a space vehicle in a minimum-fuel trajectory without a time constraint to coast for years before entering the desired lunar orbit.

At point 5, the burned out S-IVB stage of the vehicle is separated and the thrusting and burning rate both die out to zero. The vehicle is now on a free flight lunar transfer trajectory. At point 6, a retro-thrust maneuver is initiated so that the vehicle may be placed into the prescribed lunar orbit. At point 7, the specified lunar orbit is entered.

It will be noted in the physical description of the problem just given that the trajectory consists of several parts or stages and that at time points defining the terminus of some of the stages, and simultaneously the origin of the stages following, it is assumed that there exist discontinuities in some of the state variables. The existence of the several stages and the assumption of discontinuities in some of the state variables provided considerable obstacles in the use of the classical theory of the calculus of variations in optimizing multistage trajectories. For in the classical theory, it is demanded that there exist only one stage, i.e., that the function to be minimized depend on the coordinates of the end-points of the trajectory and not on the coordinates of any intermediate points. The classical theory further requires that the state variables be continuous functions of time and have continuous derivatives at least up to a specified order. Thus, the beginning efforts in applying the calculus of variations to trajectory optimization were necessarily confined to attempts to optimize a single stage, because it was only for this that the necessary mathematical tools were known at that time to be available.

The optimization of the one-stage trajectory is an example of the classical problem of Mayer in the calculus of variations and was first recognized as such by P. Cicala and A. Miele in 1956. It is enlightening to observe that the problem of Mayer is a special case of the more general problem of Bolza. Because the Bolza problem may be transformed into the Mayer problem and vice versa, one hears the trajectory optimization problem frequently referred to as a Bolza problem, as a Mayer problem, or as a Bolza-Mayer problem. But regardless of whether the problem is formulated as a problem of Bolza or as a problem of Mayer it presents similar difficulties when attempts are made to solve it. At best we can, at the present time, do no better than to find a numerical solution on the digital computer. An analytical solution will probably always elude us.

However, the basic theory of the classical Bolza (or Mayer) problem may be considered to be almost complete. By almost complete, we mean that a set of necessary conditions for an optimum has been found that, when suitably modified, forms a set of conditions sufficient to guarantee an optimum at least when compared with a certain class of solutions. The necessary conditions are the multiplier rule-a corollary of which is the famous Euler-Lagrange differential equations, the condition of Weierstrass, the condition of Clebsch, and the Jacobi or so-called fourth necessary condition.

It is important to observe that these four necessary conditions for the problem of Bolza are only loosely analogous to the conditions bearing the same names for the simpler problems in two-dimensional state space. Any attempt, therefore, to illustrate these conditions for the problem of Bolza by use of geometric diagrams in the plane would lead to confusion and erroneous impressions.

It may be deduced from the basic theory of the calculus of variations that every minimum-fuel, flyable trajectory must satisfy these conditions. However, it must be noted, since these conditions are only necessary, that there may exist nonminimizing trajectories that also satisfy them. Therefore, a COV trajectory which satisfies only some or all of the necessary conditions is not necessarily a minimum-fuel trajectory. About the most that can be said for such a trajectory is that it is a promising candidate for a minimum-fuel trajectory. For this discussion, the important fact is that slight modification of the last three of these conditions transform the necessary conditions into a set of sufficient conditions.

It is rather common practice to designate the four necessary conditions for the problem of Bolza by the Roman numerals I, II, III, IV as shown in Figure 5. If a trajectory is an optimum trajectory, it must satisfy these conditions. The modifications of the conditions that transform them into a set of sufficient conditions are designated by the symbols I, II', III', IV'. Any trajectory that satisfies all of these conditions is an optimum trajectory.

1.	The Multiplier Rule (Corollary - Euler - Lagrange Differential Equations)	I
2.	The Condition of Weierstrass	II
3.	The Condition of Clebsch	III
4.	The Jacobi or Fourth Necessary Condition	IV

All Optimal Trajectories Must Satisfy All the Necessary Conditions.

FIGURE 5. NECESSARY CONDITIONS IN THE CALCULUS OF VARIATIONS

Now, it is possible to calculate a COV trajectory on a digital computer by invoking only some of the necessary conditions. All necessary conditions are not required for the computation. However, to guarantee that the computed trajectory is indeed an optimum trajectory, it must satisfy all four sufficient conditions. Even if the sufficient conditions are satisfied, however, there is no assurance that the trajectory satisfying them is unique. There may be other trajectories which use no more fuel and satisfy the same conditions. If a set of conditions could be found which were at once both necessary and sufficient, then the trajectory satisfying them would be optimum at least among all trajectories of its class and lying in some neighborhood of it. For the trajectory problem, which, as we have mentioned before, is an example of the problem of Bolza, there have been discovered no conditions that are both necessary and sufficient.

In the evolution of attempts to find an optimum trajectory, it was considered expedient to employ the easier to invoke necessary conditions for COV trajectory calculation rather than to use the more severe and more numerous sufficiency conditions. The experience and knowledge gained thereby could later be used to apply the sufficiency conditions if that should appear to be desirable. However, up to this time no serious attempt has been made to invoke sufficient conditions even for the single stage case. Instead, concentration of the multistage problem has been made in the hope that a least a set of necessary conditions could be determined from which a solution curve could be found that would satisfy them. There was always the strong conviction that physical considerations would be sufficient to guarantee that this solution curve was indeed a minimum-fuel trajectory, at least among all trajectories that satisfied the physical conditions of the problem if not among all mathematically possible trajectories. When it appeared that the single stage problem was essentially solved, it was decided to attack the multistage problem with the assumption of continuity of state variables even though discontinuities, as mentioned before, in some of them had been assumed to exist in what was then believed to be the more realistic mathematical model. Several of the contractors who had been working on other aspects of trajectory optimization were asked to begin investigating the multistage trajectory problem. When the problem was discussed with Dr. Boyce of Vanderbilt University, one of the contractors, several months after the work with some other contractors had been initiated, he pointed out that, effectively, the multistage problem with continuous state variables had been investigated and solved by C. H. Denbow and that his results had been published in his doctoral dissertation at the University of Chicago in 1937. Not only had Denbow found necessary conditions for a minimum in the general problem of Bolza for the multistage case with state variables continuous, but he had found a set of sufficiency conditions as well.

Immediately work was begun on extending the work of Denbow to include the case with discontinuities in the state variables. This investigation was advanced considerably by the work of Dr. R. W. Hunt, a consultant for the division and professor of mathematics at Southern Illinois University. Hunt applied Denbow's methods to a Mayer formulation of the multistage problem and permitted discontinuities in the state variables and constraints at the staging points. However, Hunt's extension required that the times of staging be fixed, but not necessarily known. Hunt obtained three necessary conditions which the minimum-fuel trajectory must satisfy, but no set of sufficient conditions. The necessary conditions for the case he treated are analogues to the multiplier rule, the Weierstrass condition, and the Clebsch condition, respectively.

M. G. Boyce and J. L. Linnstaedter of Vanderbilt University Department of Mathematics further extended the work of Denbow and Hunt to include control variables, finite equation conditions, and inequality constraints. Boyce and Linnstaedter also obtained necessary conditions for their more general problem, but no sufficient conditions.

This is the state of the theory at present. Although much has been done toward the solution of finding the minimum-fuel trajectory for multistage type missions, much remains to be done. For example, for the classical problem of Bolza, many necessary conditions have been found, but for its generalizations by Hunt or Boyce and Linnstaedter, only three necessary conditions have been obtained. Since these three conditions have not resolved some serious difficulties in attempts to compute by digital simulation a trajectory satisfying them, it appears that further efforts should be made to obtain other necessary conditions analogous to those already obtained for the classical problem of Bolza, which, when used instead of or possibly in conjunction with the necessary conditions already known, will resolve some of the difficulties presently encountered. In addition, it is of paramount importance that serious efforts be started on the development of a set of sufficient conditions because the satisfaction by the solution curve of necessary conditions does not guarantee that it is an optimum solution. If a solution curve satisfies sufficiency conditions, however, it is indeed an optimum, at least when compared to other trajectories in a certain neighborhood that are flyable and satisfy the imposed constraints and boundary conditions.

Although these suggestions for the direction which further research should take have assumed the existence of discontinuities in the state variables, mass, thrust, and burning rate, the continuous variable approach to obtaining a minimum-fuel trajectory still has some attractive qualities. Because the thrust and burning rate are not physically discontinuous functions of time, it might be more realistic to obtain a continuous approximation of their rapid decreases and increases at the staging points where engine cutoff and reignition occur. The only physical discontinuity, that of mass, could be approximated by a very rapidly decreasing function. Then all state variables could be considered as continuous throughout the trajectory and the work of Denbow, modified for the trajectory problem, could be used. As mentioned previously, in Denbow's work, necessary conditions as well as sufficient conditions have been found. The trajectory obtained by satisfying the sufficient conditions would be at least one minimum-fuel trajectory for the imposed conditions. Guidance functions could then be derived around this trajectory as the nominal.

Thus, by way of summary, it may be said that if a sufficiently accurate mathematical model of the physical system is obtained by considering all state variables as continuous functions of time, although some of them may be very steeply increasing or decreasing at certain staging points, or physically discontinuous at these points, then the basic mathematical theory sufficient to guarantee a minimum-fuel multistage trajectory is available for use. If it is not adequate to consider all state variables as continuous functions of time, but to take as discontinuous at certain staging points the steeply rising or falling variables such as thrust and burning rate and to treat the physical discontinuous mass as mathematically discontinuous, then only necessary conditions are available for use. In this case, finding sufficient conditions may be a matter of much importance. Some in-house work is being done on this problem now.

So far, we have not discussed several difficulties which have been encountered in attempting to compute minimum-fuel trajectories on a digital computer. Basically, the difficulties arise because of the introduction of additional differential equations which result from applying the methods of the calculus of variations. Among these new differential equations, as we have mentioned, are some which introduce new variables which we call Lagrange multipliers. These new variables result, of course, from invoking the multiplier rule. If you recall, we stated that the state variables at point 1 (Fig. 4) on the trajectory, the initial point, were known. With the introduction of the Lagrange multipliers, however, we add new variables whose values at point 1 are not known. They must be determined if we are to direct the vehicle in an optimum manner to point 2. In fact, not only must these multipliers be known at point 1, but they must be determined at every integration time step along the trajectory when thrust is being applied. This statement, of course, implies that the multipliers are intimately related to the vehicle's pitch and yaw angles, as indeed they are. In fact, the relations of the multipliers to the pitch and yaw angles are through simple trigonometric expressions.

In early attempts to calculate multistage trajectories, only two stages were used. In one effort the boost stage was the first stage; from boost burnout to a specified earth orbit was the second stage. No attempt was made in the early efforts to apply the calculus of variations to the boost stage. Instead, a zero-lift trajectory was calculated from the initiation of tilt, ten seconds from liftoff, until booster burnout. From that point a COV second stage trajectory to the specified orbit was calculated. Of course, a different tilt program for the booster might result in a trajectory which, overall, uses less fuel than the trajectory originally calculated. To try to find a better overall trajectory, a family of boost trajectories was generated by using different kicks at tilt initiation. From the burnout point for each of the boost trajectories, a COV second stage trajectory to the specified earth orbit was calculated. Then, by interpolation, the kick which would initiate the tilt program of the booster that would result in the best overall trajectory was determined. The result was not a proven minimum fuel trajectory, but at least it was better than any member of the trajectory family.

Simultaneously with the study just described, attempts were made to compute two-stage COV trajectories with both stages lying outside the earth's atmosphere. Continuity of all state variables was assumed, and no attempt initially was made to satisfy any necessary conditions except the Euler-Lagrange differential equations. While the state variables were taken to be continuous throughout both stages, it was not known whether the Lagrange multipliers were also continuous especially at the staging points. It was intuitively felt that the multipliers were continuous throughout both stages, and COV trajectories were calculated with this assumption. The question of multiplier continuity was answered for continuous state variables by M. G. Boyce of Vanderbilt University.

In late 1962, Boyce applied some necessary conditions to a simplified multistage problem. He assumed that all state variables were continuous and avoided the vexing problem of discontinuity in the mass at staging points by assuming that the mass was a known function of time; as such it was not a state variable. He further assumed that a fuel minimizing trajectory existed and that it was the unique solution to the equations of motion. With these assumptions, of course, Boyce could then declare that the trajectory which he obtained from invoking only necessary conditions must be a fuel minimizing trajectory. Boyce's principal contribution, however, was his proof that the Lagrange multipliers were continuous not only throughout each stage but at the staging points as well. Because his proof is valid for any finite number of stages, it represents a significant contribution.

In the complex trajectory illustrated in Figure 4, the determination of precise values of the initial Lagrange multipliers made heavy demands on the analyst's experience, ingenuity, and ability to communicate successfully with whatever gods have control of such matters. But so much spadework has been done in the past that now, with the experience gained, it is not too time consuming to calculate at least reasonable first approximations of them. An iterative procedure called the "differential correction scheme," formulated as a part of the contract requirements by G. N. Nomicos of Republic Aviation, is then employed to find more nearly precise values of the Lagrange multipliers at point 1. Having found the Lagrange multipliers and having been given the state variables which define point 1, the computation can be begun which, hopefully, will result in an optimum trajectory with the correct retrothrust maneuver at point 6.

Unfortunately, difficulties, the nature of which are not fully understood, have arisen which thus far have prevented the successful computation of the trajectory from point 1 all the way to point 7. The difficulties, whatever they may be, prevent the retrothrust maneuver at point 6. Thus far, no trajectory, proven to be a minimum-fuel multistage trajectory from point 1 to point 7, has been computed. Investigations and studies are being made from both in-house and by private contractors to locate the difficulties.

The trouble seems to lie in the convergence properties of the iterative scheme for finding the initial values of the Lagrange multipliers to high precision. The scheme simply will not converge to any value unless the first approximations of the multipliers in some instances, but not all, are very, very close to their exact values. In other cases, the scheme converges to a value which is not acceptable because its subsequent use does not permit attainment of the objective or desired terminal state for the stage. That is to say, the terminal state is extremely sensitive to changes in the initial values of the Lagrange multipliers. If these multipliers at point 1 are not known with great precision, the desired terminal state cannot be attained.

A hopeful remedy seems to lie in the use of another iterative process, the so-called Newton-Raphson method. It appears at present that this scheme will converge for rougher first approximations than the differential correction scheme that is now in use. But whether it will converge to sufficiently accurate values of the initial values of the multipliers is as yet not known.

The remarks just made might imply that the situation relating to multistage trajectory computation is in a sad plight, indeed. Be assured that this is not the case at all. We can calculate quite satisfactory multistage trajectories for any desired missions. While these trajectories may not be strictly minimum-fuel trajectories, they nevertheless require the expenditure of less fuel than is available for use. The purpose of optimization is to find that trajectory, if it exists, which will use the least possible amount of fuel to accomplish the mission. The savings in fuel over nonoptimum trajectories could possibly be converted into payload. We must be prepared, however, for the possibility that the savings will be negligible. But we will never know whether they are or not until successful optimization of the trajectories has been achieved.

Conducted simultaneously with the studies in multistage trajectory optimization are studies in the optimization of low-thrust trajectories. The goal of the low-thrust trajectory optimization project is the development of techniques and computer programs for determination of minimum-time or minimum-fuel trajectories for both geocentric orbital transfer and interplanetary rendezvous and flyby operations. Successful low-thrust trajectory optimization techniques would be essential in the generation of optimal guidance schemes. The problem of optimizing low-thrust trajectories was attacked by classical variational methods but with little success. The principal difficulties are again those of the two-point boundary value problem arising out of attempts to find numerical solutions of the Euler-Lagrange equations. The use of the method of gradients, a technique employing successive approximations, one of the so-called direct methods of the calculus of variations, has been explored by several contractors, especially by H. K. Hinz and his associates at Grumman Aircraft. This is an attempt to circumvent some of the difficulties of the two-point boundary value problem. Of course these difficulties are also inherent in optimizing multistage trajectories but they are somewhat heightened in the case of low-thrust trajectories which may spiral about the earth many hundreds of times before departing into the transfer trajectory. The length of time involved is so great that large accumulations of round-off and truncation errors are made. A second difficulty associated with the use of the successive approximations techniques, which seemingly must be employed, is the need to store control variables as functions of time. If the functions are rapidly changing the amount of computer storage required may become prohibitive. A third difficulty, already encountered in the discussion of multistage trajectories, is the extreme sensitivity of terminal conditions to changes in initial values of the Lagrange multipliers. In an attempt to surmount this last difficulty in a relatively simple problem, H. K. Hinz and his associates at Grumman Aircraft considered the specific problem of determining the optimum thrust steering program that would minimize the time to transfer between coplanar circular orbits in a central force field. Both orbits encircle a single body. Since they considered the thrust magnitude as fixed, minimum transfer time was equivalent to minimum fuel consumption. The use of the generalized Newton-Raphson method of successive approximations permitted the computation of optimum thrust steering programs for progressively increasing values of final time for trajectories up to the final time for 21 revolutions about the earth. But for transfers involving 21.5 revolutions or more, the method did not converge to an accuracy of four significant figures of the Lagrange multipliers. Higher precision integration schemes seem to offer the best hope of obtaining convergence to more significant figures.

The third area of research in trajectory optimization is in impulsive orbital transfers. The orbital transfer in the multistage trajectory we have been discussing is nonimpulsive. The aim of this research is to attempt to gain knowledge in this simplified transfer problem that would help in the understanding of nonimpulsive transfers.

The impulsive thrust orbital transfer is, of course, an idealization. In the two-impulse case, for example, there is one instantaneous thrust to get from the initial orbit onto the transfer trajectory and a second instantaneous thrust to get into the terminal orbit as illustrated in Figure 6.

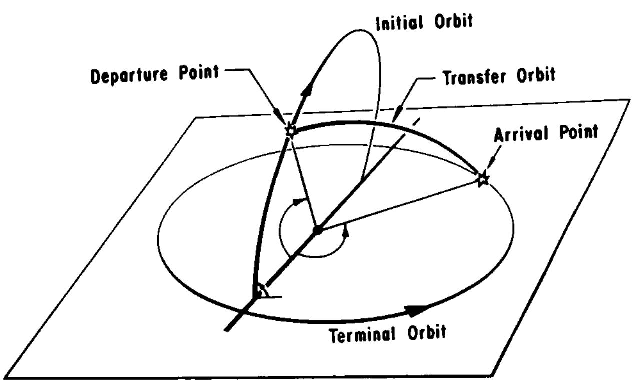

FIGURE 6. TWO-IMPULSE ORBITAL TRANSFER

Investigations have been made using one, two, and three impulse transfers. Because mathematical formulation of the problem leads to expressions which, except for special cases, are analytically intractable, the studies were made largely by numerical methods.

Dr. D. F. Bender and his associates at North American Aviation, Inc., are responsible for the majority of the numerical work that has been done by contractors in impulsive orbital transfers. They have done some important analytical work as well. F. W. Gobetz of United Aircraft has contributed to the problem of optimum low-thrust orbital transfers.

However, all of these studies in orbital transfer, whether impulsive or low-thrust, were made with mathematical models that represented considerable simplifications of the physical model that actually exists. Their usefulness is therefore limited to design studies and for suggesting modes of attack on the more realistic problems. The present stage of this research is still somewhat exploratory; it has not progressed to the point where valid conclusions may be made that would be helpful in multistage trajectory optimization.

The fourth area of trajectory optimization which has been studied is the atmospheric reentry trajectory wherein the reentry vehicle is subject only to gravitational and aerodynamic forces.

Because no thrusting, except for control jets, is employed, attention was directed toward minimization of the accumulated gravitational forces on the vehicle's occupants. In mathematical form, this means the minimization of the integral of the square of the total aerodynamic acceleration. The optimization analysis which results from this formulation of the problem may be treated as a problem of Lagrange in the classical calculus of variations with fixed end-points or as a Pontryagin fixed end-point problem. The fixed end-points are, of course, the initial point of the reentry trajectory on the edge of the earth's atmosphere and the known and fixed terminal point on the earth's surface. It is assumed that the reentry vehicle's control system is capable of directing the vehicle to the desired landing point. Studies in this area thus far by out-of-house contractors have been done mainly by W. A. Shaw and his associates at Auburn University and by Blanton and Muzyka of Raytheon Corporation. They have treated the problem both as a Lagrange problem and, therefore, used the methods of the classical calculus of variations and as a Pontryagin fixed end-point problem and used Pontryagin's maximum principle as the optimization technique. Results from each of the methods are identical, but both approaches were taken to determine whether one offered any computational advantage over the other. The answer appears to be in the negative.

In these beginning studies the most vexing problem in trajectory optimization, that of determining the initial values of the Lagrange multipliers, was sidestepped by assuming that the multipliers were known. The Euler-Lagrange equations were formulated and solved simultaneously with the equations of motion, and a trajectory was obtained which satisfied them and certain specified constraints. Such a trajectory, of course, may not be the optimal trajectory since it satisfies only one necessary condition. Studies could be continued to determine whether the trajectory which satisfies this one necessary condition also satisfies sufficiency conditions, several sets of which are available for the single-stage two fixed point problem.

In summary, then, if the study of the problem of multistage trajectory optimization is to be continued, it is essential that a primary effort be directed toward determining a means for finding the initial values of the Lagrange multipliers to much greater precision than seems to be possible using presently available methods. If this problem can be successfully resolved, certainly a multistage trajectory can be found that satisfied some necessary conditions. If, additionally, we are eventually able to show that this trajectory also satisfies a set of sufficient conditions, we can with certainty say that a minimum-fuel trajectory has been found.

It appears that the best way to accomplish these aims for the realistic multistage problem is through attacking much simpler trajectory optimization problems. Having solved these, the realistic problem can be approached, it is to be hoped, by adding additional constraints one at a time.

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ASTRODYNAMICS RESEARCH AT MSFC

by

Arthur J. Schwaniger*

SUMMARY

The term astrodynamics and the nature of that field are very briefly discussed. Then a brief coverage of some of the research projects in astrodynamics is given. These include earth-moon trajectories, interplanetary trajectories, and the various models used for trajectory studies. In the area of earth-moon transits, Apollo type transits and periodic orbits studied for possible Pegasus orbits are discussed in some detail. In the area of interplanetary flight, reference is made only to the specific publications available and the types of problems being studied. The trajectory models used are primarily the restricted three-body model and for interplanetary studies a matched-conic model. The use of precision models using complete ephemeris data and the current approach to such a model are mentioned. Finally, some of the future areas of effort are given.

GLOSSARY

Perisel - On a trajectory, the point of closest approach to the center of the moon.

Barycenter - The center of mass of two bodies in a trajectory model.

MEP - Moon-earth orbit plane.

Perigee Belt - The locus of perigee points of a class of earth-moon transits.

Perisel Belt - The locus of perisel points of a class of earth-moon transits.

Vertex - The point representing the region at which a family of transits converges.

Class of Transits - All transits having a common perigee radius, perisel radius, and transit time between these points.

Family of Transits - Transits, all of one class, which have perigee on a straight line segment of the perigee belt or perisel on a straight line segment of the perisel belt.

Periodic Orbit - A trajectory which periodically repeats itself.

Transition Orbit - A trajectory which includes both near elliptical motion around earth and near elliptical motion around the moon with one or more transitions between the two.

Cislunar - On this side of the moon or between earth and moon, or sometimes more generally in the vicinity of earth and moon.

Flyby Transit - A trajectory which passes near one of the celestial bodies, but does not stop or remain any appreciable time near the body.

Swing-By Transit - A trajectory which passes near a celestial body and utilizes the bending by its gravitational attraction to be directed to another celestial body.

Central Force Field - Usually a gravitational field at all points of which the force is directed toward one central point.

Libration Point - A point at which a body tends to remain stationary due to cancelling effects of gravitational and centrifugal forces at that point.

Launch Azimuth - the direction measured in a horizontal plane at launch of the projection of the intended flight path on that plane. The reference is usually the north direction.

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Conic Flight - Flight in a central force field; it is shaped like a plane section of a cone. The focus of the conic is located at the center of force.

Central Angle - The angle measured at the center of earth between any two position vectors of a space vehicle's flight path.

Inertial Space - The space associated with an interial, or in other words stationary, reference frame.

Rotating Frame - A reference frame or coordinate system which rotates.

Restricted Model or Restricted Three-Body Model - A model of a gravitational system of two massive bodies which revolve in circles about their common center of mass and a third body of negligible mass which moves in that gravitational system.

Three-Body Problem - The problem of motion of three massive bodies under mutual gravitational attraction.

SECTION I. INTRODUCTION

Astrodynamics is the treatment of problems in celestial mechanics as they apply to contemporary space flight. Classical celestial mechanics has dealt with the description of orbits in various gravitational models primarily in terms of application to the natural bodies in the solar system or simply for academic reasons. It has relied on the resources of higher mathematics in describing motion in the system and has had only observation of natural bodies by which to test its results. Astrodynamics deals with the determination of flight paths for propelled and unpropelled spacecraft and with the matching of flight paths to booster flight characteristics.

In addition to the standard methods of celestial mechanics, astrodynamics uses high speed computers to evaluate numerically many of the previously unsolved equations of celestial mechanics. Astrodynamics studies also seek simplified concepts for a better understanding of space problems and their rapid solution.

In any exploration effort the choice of a path that satisfies as many of the exploratory missions requirements as possible is one of the most basic problems. The work of astrodynamics in providing a thorough description of the paths available to the experimenter, therefore, is of primary importance to the mission.

The following is a brief review of some research efforts that have been made and are being made in astrodynamics at the Marshall Space Flight Center.

Even with the availability of our huge, very high speed computers the problem of representing all possible trajectories by computing large quantities of exact trajectories, which incorporate a precise representation of the gravitational fields of the solar system, is virtually impossible. Trajectories that are very nearly correct can be, and are, calculated for specific applications to well defined missions; however, the computer time required for these calculations is prohibitively large and the character of the trajectory so complicated by the complexity of the real solar system that it is not feasible to exclusively utilize these calculations in providing the general description of trajectories that is necessary for the planning of a mission. The aim of the Astrodynamicist, therefore, is to provide approximate descriptions of trajectory behaviors that are adequate for general mission planning, and to gain accurate descriptions of trajectory behavior by refinement of the approximate descriptions to continually bring them into closer agreement with the precise results of complicated approaches. Thus extensive use is made of approximate models.

SECTION II. RESTRICTED THREE-BODY MODEL STUDIES

A. APOLLO TYPE TRANSITS

Much of our effort in the study of trajectories in earth-moon space has utilized the restricted three-body model. This model is pictured in the first figure.

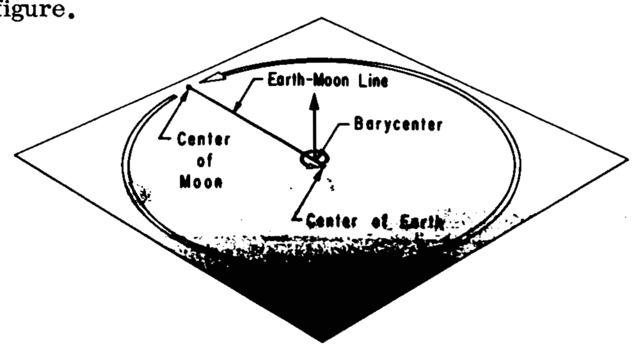


FIGURE 1. RESTRICTED THREE-BODY MODEL

The earth and moon are assumed to revolve in a plane, which we will designate the earth-moon plane for a plane of reference, and they move in circles about their center of mass, the barycenter. The plane of motion, referred to as the MEP or moon-earth plane, and the line connecting the centers of earth and moon are usually used as references. The use of the earth-moon line implies a rotating coordinate system. The results of our studies of trajectories applicable to Apollo type missions have been published in a series of reports entitled, "Lunar Flight Study Series" and have in a sense been summarized in another larger report, "A Comprehensive Astrodynamic Exposition and Classification of Earth-Moon Transits." A very brief review of the main points of the report can be given as follows. A class of earth-moon transits is defined by three parameters: The radial distance of close approach to the center of earth (radius of perigee), the radial distance of close approach to the center of the moon (radius of perisel), and the time of transit between these points. Under this classification all transits within a given class (specified radius of perigee, radius of perisel, and time of transit) have a near circular band of perigee positions from which departure is made. This is illustrated for several classes in Figure 2. The radii of perigee and perisel are constant over the classes shown, with only transit time variable.

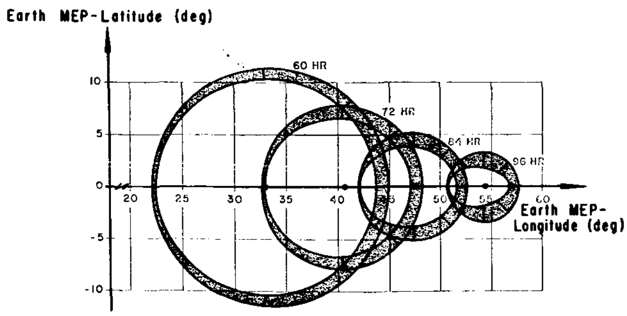


FIGURE 2. LOCI OF ALL POSSIBLE PERIGEES FOR CLASSES $C(T_i$, 6,555 KM, 1,923 KM).
T_i = 60, 72, 84 and 96 HR

At the moon the locations of perisel points of a class also form an annular belt, (all points in this belt have the same radial distance from the center of the moon) as shown in Figure 3. It is also found that all transits of a class depart from a point in the departure belt in a predetermined radial direction. (To visualize the transit it is noted that the trajectory of a given class of transits is horizontal at the perigee belt and the direction of the trajectory is generally away from the center of the perigee belt.) The arrival at the moon is in a direction generally toward the center of the perisel arrival belt. Subclasses or families of trajectories are identified by

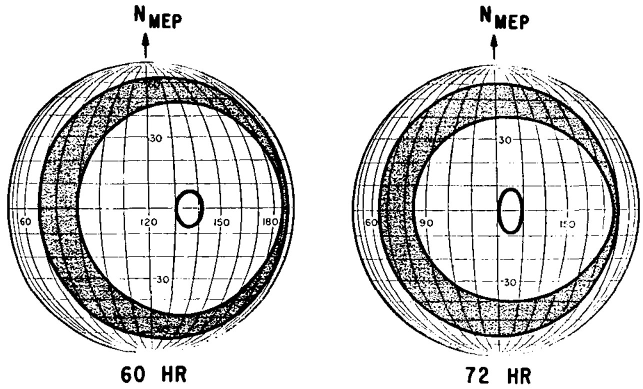


FIGURE 3. SURVEY OF ARRIVAL AREAS AT MOON FOR TRANSIT CLASSES C(60 HR, 6,555 KM, 1,923 KM) AND C(72 HR, 6,555 KM, 1,923 KM)

considering those transits whose perigees lie within the belt width on a line from the central point in the belt. Such a subclass or family is shown in Figure 4.

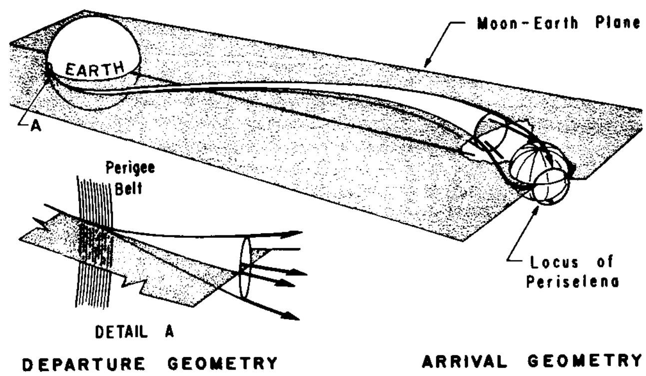


FIGURE 4. FAMILY OF TRANSITS ENVELOPING MOON

The fixed parameters of the family are the classifying parameters, flight time, radius of perigee, radius of perisel, and the horizontal path angles associated with perigee and perisel. Depending on the placement of perigee as stated above and the remaining parameters (velocity magnitude and azimuthal direction) at perigee a family of trajectories can envelope the moon from all directions. The trajectories pass horizontally through the perisel belt toward the center of the belt. The region of convergence of all trajectories of a family near the center of the perisel belt is called the vertex and this region is small enough to be considered a point. The perisel points of the family lie on a near circular locus within the perisel belt for the family class.

Each class of transits is in fact composed of such families, the perigee belt being composed of the perigee line segments and the perisel belt the locus of all perisel circles. The vertices of the families within a class also form an annular locus within the perisel belt. If the true orientation of the earth's and moon's equators is superimposed on the MEP system, geographical coordinates can then be used in the design of a lunar mission. An example is shown graphically in Figure 5.

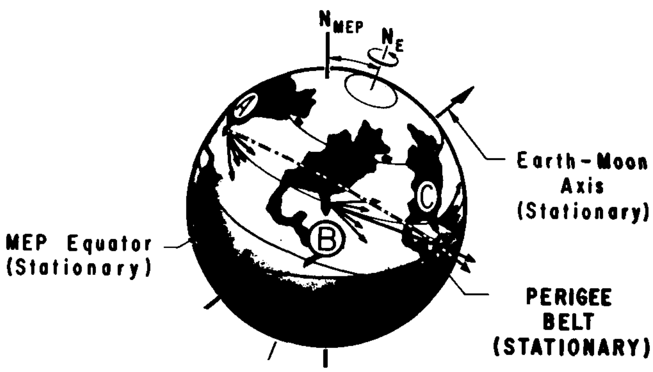


FIGURE 5. DEVELOPMENT OF LAUNCH WINDOW BY RELATING TRUE EARTH AXIS AND EQUATOR TO MEP-REFERENCE SYSTEM

Positions A, B, and C represent three successive times in the day at which launch is considered. The belt of perigee points that will produce a flight of desired time and close approach distance to the moon is indicated. The arrows indicate the spread of launch azimuths that are acceptable from range safety considerations. One must then determine when, if at all, the launch position on the earth is such that the azimuthal aiming direction, and the powered flight central angle will place perigee on the perigee belt with proper direction away from the center of the belt. One can see that several possibilities for the flight may be available in this case, particularly if a parking orbit is used to extend the arc of flight (central angle) from liftoff to the perigee of the transit.

To place perigee of the trajectory on the indicated perigee belt with direction away from the center of the belt, the flight must pass over the center of the belt. The earliest launch time at which this can be done while staying within range safety restriction of launch azimuth is represented by point A. From this launch time the azimuth of launch is at the northern range safety limit, and the central angle of flight to the perigee location is so large that a coast phase in a parking orbit will be required to place perigee at the specified point. The flight originating at the time represented by point B has a much smaller central angle so that little or no coast in parking orbit is required. The latest time at which launch is possible will occur when the central angle from the launch point to the perigee belt becomes smaller than the central angle of flight with no parking orbit, or when the most southerly azimuth acceptable for range safety is reached. On Figure 5 the most southerly azimuth angle shown at location C does not provide a trajectory that will pass through the perigee belt and therefore a launch is not possible.

B. IMAGE TRANSITS IN THE RESTRICTED MODEL

Another feature of the material is that it can be readily interpreted for moon-to-earth flights. This is possible because of image or reflection principles inherent in the restricted three-body model. Detailed explanation of these principles and their application would be too lengthy for this presentation. However, a brief explanation of the basic principle is in order here because it will be referred to again in connection with other projects.

This principle can be intuitively understood as illustrated in Figure 6.

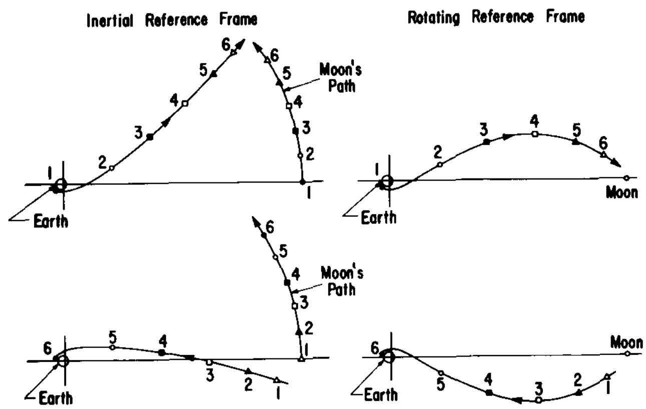


FIGURE 6. IMAGE OR REFLECTION TRAJECTORIES IN THE RESTRICTED THREE-BODY MODEL

Consider the motion of a vehicle toward the moon in inertial space as illustrated on the upper left. At the start, the spacecraft is at point 1 on its path and the moon at point 1 on its orbit. The moon moves counterclockwise as viewed from the north with successive locations of the bodies occurring as indicated by the numbers. This motion appears in the rotating frame as illustrated at upper right. If such motion, indicated in the inertial frame, can occur, then the same paths of the two bodies can also occur with the reverse motion, with the moon's motion clockwise and the vehicle moving toward earth. This is equivalent to viewing the motion from the southerly direction. If this reverse motion is viewed, however, from what would now be the north direction so that the moon's motion again appears counterclockwise, the path appears as indicated at lower left. This path is clearly an image of the first but the motion is from moon to earth rather than from earth to moon.

In the rotating frame illustrated at the lower right, the image path is a reflection on the earth-moon line of the original path and the direction is reversed. Although this explanation has been limited to two dimensions, it can be extended to three dimensions with the result that all data concerning earth to moon transits can be transformed to represent moon-to-earth transits.

Since Apollo type missions were being considered, the flight times concerned with in the survey are not very large. During such time intervals the moon moves only about one seventh of a revolution or less around the barycenter, and the circular arc representing the moon's motion in the model is a sufficiently accurate approximation of any segment of the true more elliptical orbit. The action of the sun's gravity over this time period is also negligible for the approximation desired. One should not infer, however, that the restricted problem finds no other application. Another of the projects being pursued at this center is the study of periodic orbits in the restricted three-body problem. Poincare's justification for the study of periodic orbits in the restricted three-body model was that such orbits represent one of the very few openings to the solution of the restricted problem. This fact still provides one point of justification for their study. Dr. Arenstorff of the Computation Laboratory has given an analytic proof for the existence of the periodic orbits even when they encompass both earth and moon and their motion is highly different from circular.

C. PERIODIC ORBITS

A systematic generation of periodic orbits on the computer has been initiated in an attempt to uncover a pattern or patterns by which the nature and classification of the orbits will be more fully understood. To generate periodic orbits we again use the reflection on the earth-moon line. Because of this reflection, a trajectory that crosses the earth-moon line perpendicular to it will continue with the path after the crossing being a reflection of the path before the crossing. It follows that if a trajectory has two perpendicular crossings of the earth-moon line it closes on itself and is periodic in the rotating system. To determine a periodic orbit the calculation is started on the earth-moon line with direction perpendicular to it. Figure 7 shows the trajectory geometry that develops for a typical case.

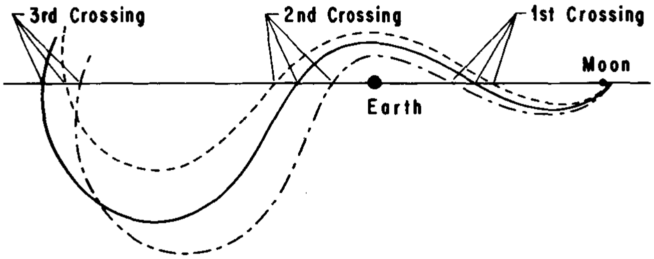


FIGURE 7. DEVELOPMENT OF PERPENDICULAR CROSSING OF EARTH-MOON LINE

The magnitude of the velocity vector is then adjusted until trajectories are generated whose velocity component along the earth-moon line changes direction at at given crossing. In Figure 7 the third crossing is chosen. Zero velocity component along the line, at the time of crossing, indicates perpendicular crossing. The periodic orbit is then determined by isolating, within the capability of the computing scheme being used, the trajectory with zero velocity along the earth-moon line at the time of crossing. The task of classifying all orbits is considerably complicated by the fact that the orbits can have many different basic shapes and further that some of these are extremely complicated shapes. Some examples of the less complicated orbit shapes are shown in Figures 8 and 9.

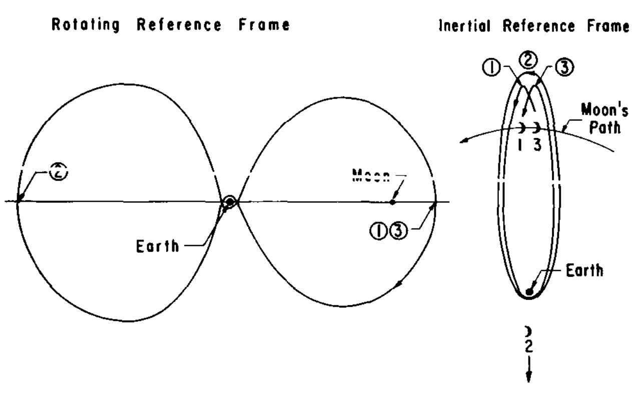


FIGURE 8. A PERIODIC ORBIT IN THE RESTRICTED THREE-BODY MODEL

In these the orbit is shown in both rotating and inertial coordinates. In the rotating frame the orbit is seen looping from the vicinity of earth through space with one loop passing around the moon. In the inertial system the orbit is nearly elliptical with the

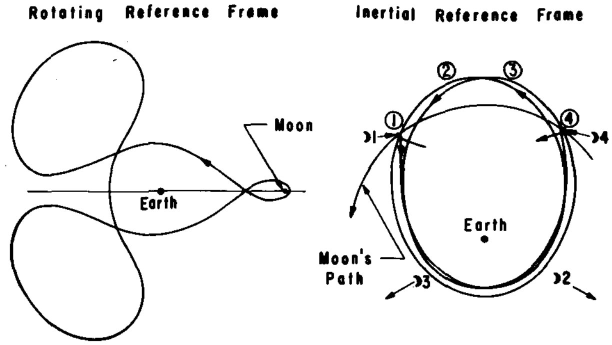


FIGURE 9. A PERIODIC ORBIT IN THE RESTRICTED THREE-BODY MODEL

ellipse being highly distorted by lunar gravity at times when the position of the moon is near that of the small body on the orbit. Some examples of the more exotic shapes are given in Figures 10 and 11.

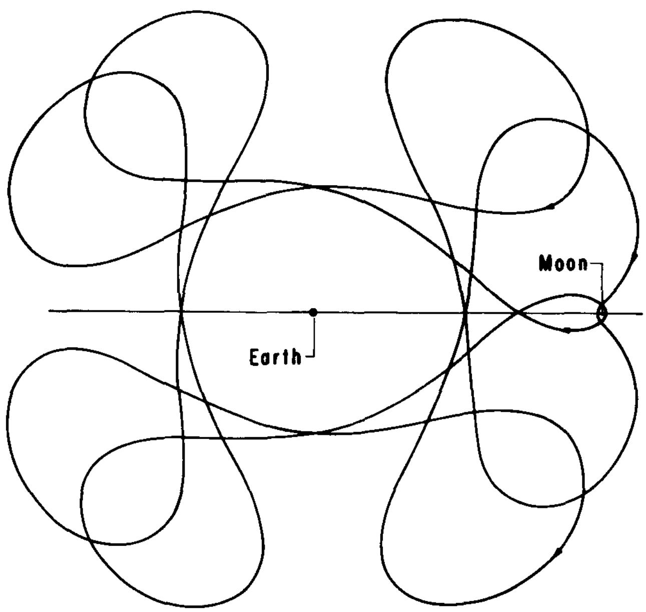


FIGURE 10. A PERIODIC ORBIT IN THE RESTRICTED THREE-BODY MODEL

Another kind of orbit that occurs in the restricted model is shown in Figure 12. This kind of orbit, called a transition orbit, was discovered by M. C. Davidson of Computation Laboratory. This orbit is, for several revolutions, nearly an ellipse about the earth. On one revolution, however, the orbit passes near the libration point between earth and moon. This

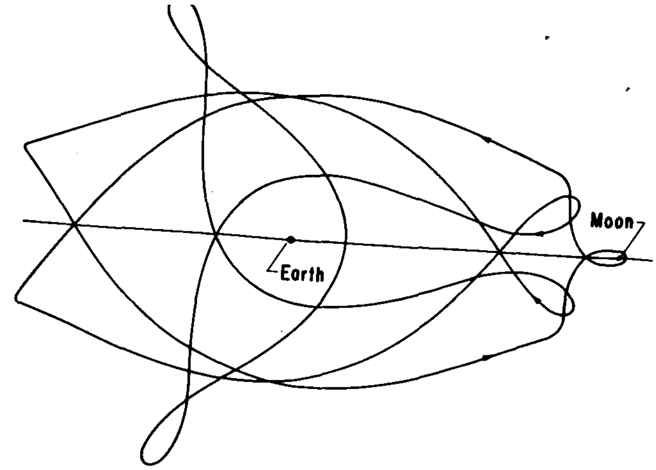


FIGURE 11. A PERIODIC ORBIT IN THE RESTRICTED THREE-BODY MODEL

is a point at which the centrifugal force on a body moving with the rotating earth-moon system, combined with the force of the moon's gravity, is just balanced by the force of earth's gravity. As it passes this point, the orbit is distorted and eventually is temporarily captured by the moon. Its motion now becomes essentially elliptic around the moon until, after several revolutions there, it comes back to near the libration point and is temporarily recaptured by earth.

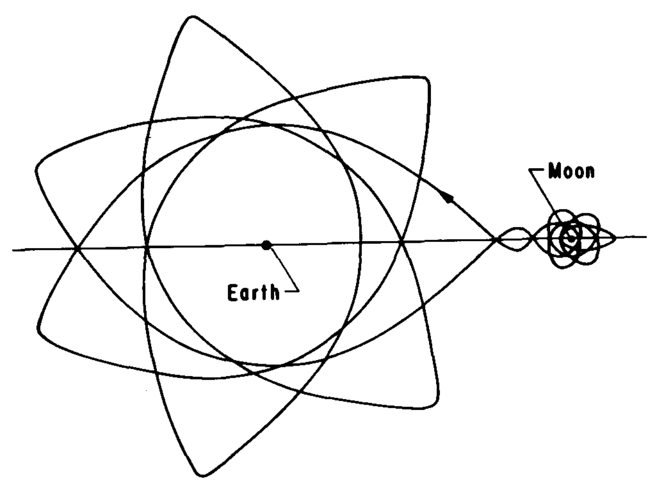


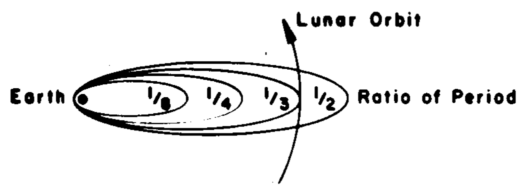
FIGURE 12. A TRANSITION ORBIT IN COORDINATE SYSTEM ROTATING WITH EARTH-MOON SYSTEM

SECTION III. PERIODIC AND NEAR PERIODIC ORBIT APPLICATIONS

In general the periodic orbits of the restricted model will not reappear as periodic orbits if the moon's path is deviated from circular to one more representative of its real path or if the gravitational attraction of the sun is introduced in the model. Some of them, however, may appear that are near periodic for at least a limited time. The possibility of application of the near periodic orbits to many space exploration missions gives another justification for the study of periodic orbits in the restricted model. Such orbits that pass at least a few times near both earth and moon have obvious value for photographic missions and missions to measure and evaluate the nature of the cislunar space. If any of these orbits can be maintained over long periods with reasonably small orbit correction, they may even be utilized for ferry vehicles that shuttle between points close to earth and close to the moon during each orbit period and can be used by men during the major portion of the trip from earth to moon and back.

When an actual mission application is to be considered, for which a long period orbit is required or for which the orbit must meet certain specifications over long periods of time, the effects of the sun's gravity and the elliptical shape of the moon's orbit must be considered. Such a mission was recently investigated. It was proposed that a Pegasus type payload be placed in an orbit that would provide for determination of the density of micrometeoroids in cislunar space. No midcourse guidance or propulsion after insertion was to be provided. With this restriction it soon becomes apparent that the orbit cannot approach very near to the moon and continue on a repeating path, due to the varying distance and velocity of moon relative to earth. Therefore it was decided to study only periodic or near periodic orbits, which essentially avoid encounter with the moon. Two efforts were made to find orbits to satisfy this and similar proposals. The first effort, done in-house, was an investigation of the periodic orbit that passes near the earth twice each month making two loops in space with one of these loops encompassing the moon as seen in the rotating frame. This is the orbit that was shown in Figure 8. We note that although the orbit does encompass the moon its closest approach distance to the moon is almost one fourth the earth-moon distance. The second effort, which was done by Lockheed, was a study of orbits that were near periodic and that reached to various distances from earth while avoiding the moon as much as possible and not looping around the moon. In both these cases the problem, then, is to determine what orbits are available that will approximately repeat themselves in cislunar space over a specified mission duration and what opportunities are available to launch the spacecraft into these orbits. Although the proposed orbit layouts avoid encounter of the moon and spacecraft, these very high apogee orbits are nevertheless highly perturbed by both the moon and sun. These perturbations can easily cause the perigee of the orbit to decrease so far that the spacecraft will collide with the earth early in its lifetime, even on its first return to earth. On the other hand, if the proper orientation of the orbit is chosen relative to the moon's and sun's positions the perturbations will increase the perigee radius. It was found that to gain an understanding of the perturbations due to sun and moon and their positioning relative to the spacecraft, the effects of each body had to be studied separately.

The orbits investigated in the Lockheed effort were classified by the ratio of the number of revolutions of the moon to the number of revolutions of the spacecraft. Four of these are represented in Figure 13. Several intermediate ratios not shown here were also included. The one-eighth ratio orbit was the smallest considered and the one-half ratio the largest.



RATIO T_M/T	APOGEE (km)	PERIGEE VELOCITY $V - V_E$ (m/sec)
1/8	185,000	-194
1/4	297,000	-123
1/3	362,000	-98
1/2	476,000	-66

FIGURE 13. ORBIT TYPES INVESTIGATED

In both efforts the displacement of perigee due to the sun alone was found to be essentially a function of only the ratio, and the orientation of the major axis of the orbit relative to the direction of the sun. The effect of orientation is shown in Figure 14. The orbit maintains a nearly space-fixed orientation. The displacement increases from the time the orbit major axis is aligned in the direction of the earth-sun line until the major axis is perpendicular to the earth-sun line. This time interval is one-fourth of a year. Then perigee displacement decreases until the major axis is again aligned in the earth-sun direction. This implies then, that if a flight is initiated with the major axis in a similar direction as the earth-sun line the sun's gravitational perturbation will increase the length of the major orbital axis and therefore moves the perigee between its initial height and

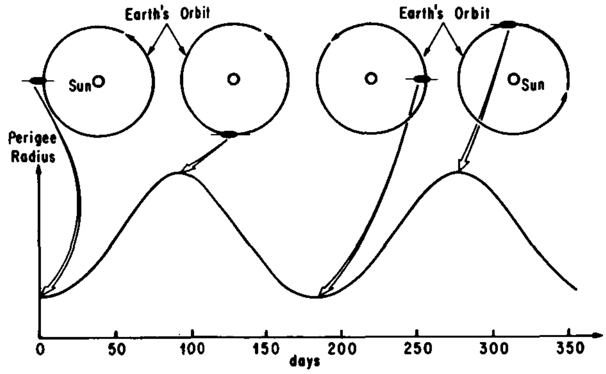


FIGURE 14. NATURE OF SUN'S PERTURBATION OF PERIGEE RADIUS AS A FUNCTION OF SPACECRAFT ORBIT ORIENTATION

a definite greater height. On the other hand, if the original alignment is perpendicular to the earth-sun line, the sun's effect produces perigees always lower than or equal to the initial perigee height. The effect of ratio of the orbit is shown in Figure 15, where it is seen that only the magnitude of the displacement is decreased as ratio decreases. The position of the spacecraft in its orbit at any time does not greatly affect the perturbation history since the direction of the sun changes only very little during any one revolution of the spacecraft in the orbit.

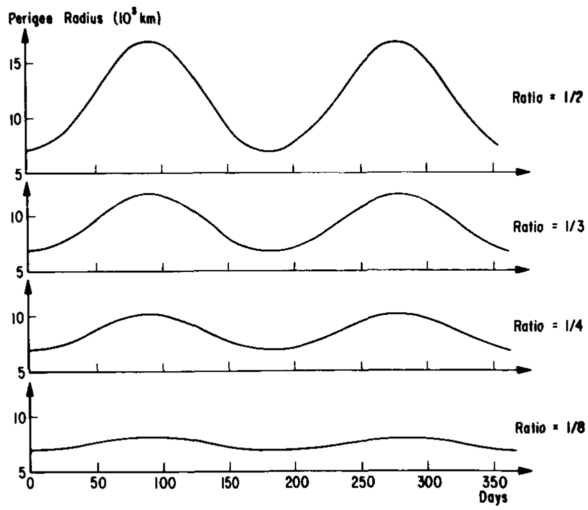


FIGURE 15. EFFECT OF ORBIT RATIO ON SUN'S PERTURBATION OF PERIGEE

In determining the effects of the moon on the spacecraft orbit, both the position of the spacecraft at any time relative to the position of the moon and the alignment of the spacecraft orbit relative to the major axis of the moon's orbit must be considered. As an example of this, we refer to the Lockheed effort. By the definition of the study, the spacecraft position is chosen so as to maintain large distances between moon and spacecraft. This is done by keeping the major axis of the orbit as far as possible from the earth-moon line at the times when the spacecraft is at apogee. Figure 16 illustrates this for two of the ratios by indicating the positions of the moon at the times of apogee of the spacecraft's orbit.

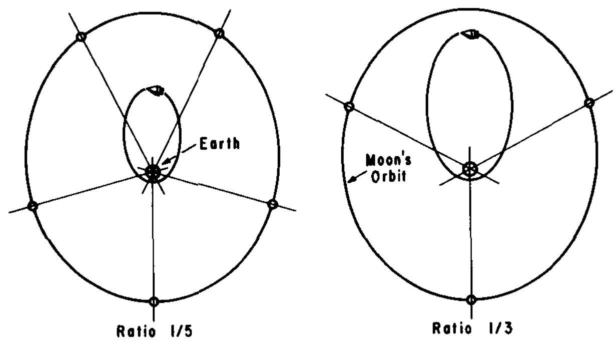


FIGURE 16. ORIENTATION OF ORBIT TO MAINTAIN DISTANCE FROM MOON (Positions of Moon Indicated at Times of Spacecraft Apogee)

The effect of the ellipticity of the moon's orbit appears as a function of the angle between the major axes of the moon's orbit and the spacecraft orbit as illustrated in Figure 17.

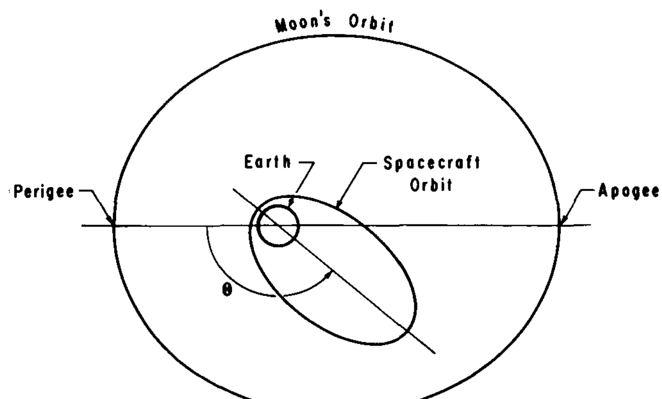


FIGURE 17. PRIMARY PARAMETER AFFECTING PERTURBATION OF SPACECRAFT ORBIT DUE TO ELLIPTICITY OF MOON'S ORBIT

The change of perigee radius as a function of the angle, θ, is illustrated in Figure 18, for the ratio one-half orbit which avoids the moon.

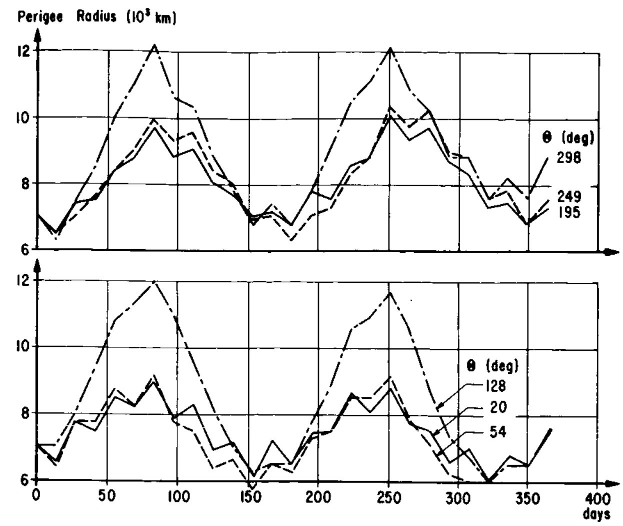


FIGURE 18. EFFECT OF RELATIVE ORIENTATION OF MAJOR AXES OF MOON'S ORBIT AND SPACECRAFT ORBIT

It is observed that for this case the lunar perturbation of perigee is generally always upward. The magnitude of the perturbation, however, is dependent on the angle, θ, with maximum upward displacement of perigee occurring for θ in the neighborhood of 120 degrees or 300 degrees.

Once the separate perturbations of the sun and the moon are known, the combined effect of the two on the spacecraft can be very well approximated by a simple addition of the two separate perturbation curves.

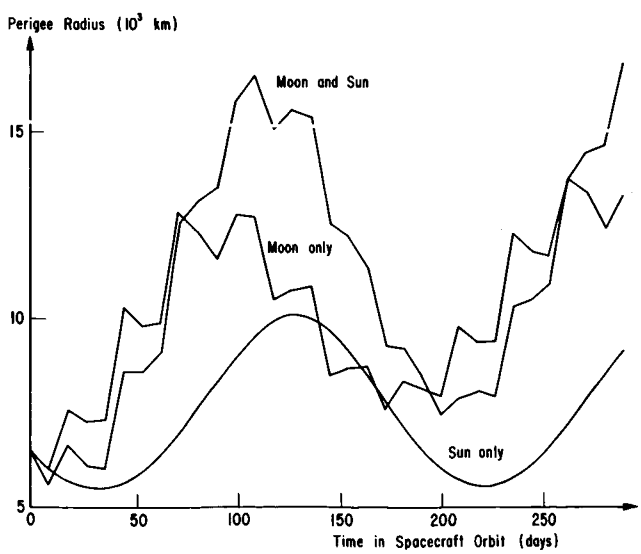


FIGURE 19. COMPARISON OF PERTURBATIONS BY SUN AND MOON SEPARATELY WITH PERTURBATION BY SUN AND MOON TOGETHER

The lower curve in Figure 19 is perigee history with only the sun acting on the orbit. The next higher curve is perigee history with only the moon acting. The third curve is the perigee history due to the actual effect of both bodies acting together. It is easily seen that this curve will be very well approximated by addition of the other two.

The launch windows for a mission using orbits of the type discussed here, then, must be chosen at times such that the orientation of the moon's orbit, the spacecraft orbit, and the position of the sun will produce upward or zero displacement of the perigee over the time interval desired.

SECTION IV. INTERPLANETARY TRANSITS

The major effort in astrodynamics research here, as in the national space program, is in the areas of lunar and cislunar orbits and the discussion has therefore been devoted primarily to these efforts. Nevertheless, a great deal of effort has been already made and is being continued into the areas of interplanetary trajectory study, and this deserves mention here. A "Study of Manned Interplanetary Flyby Missions to Mars and Venus" was recently completed by Advanced Projects Study Branch. The report contains an "in-depth" mission analysis study of manned interplanetary flyby missions to Mars or Venus during the 1970s using Apollo technology and hardware wherever possible. Because the planetary orbits are inclined to the earth's orbit, even though only by a few degrees, the trajectory geometry changes appreciably over long time periods. The time interval that must be studied to cover a representative number of all possible Earth-Mars transits is about 15 years. The opportunities to fly reasonably short flight time, low energy transits will occur each 2.2 years. For Earth-to-Venus transits the interval to be studied is about 8 years, and the applicable transit opportunities are available each 1.6 years. Work is being continued to expand the above study to complete these cycles.

Another project now in progress is the study of so-called swing-by trajectories for Earth-to-Mars flyby transits by way of Venus. This type of transit goes first to the vicinity of Venus, where that planet's gravitational attraction is used to turn the trajectory to Mars. It offers as the main advantage a much lower reentry velocity on the return trip to Earth than that occurring on a direct flight to Mars.

These and almost all studies of interplanetary flight use a "matched-conic" model to approximate the trajectory. This model assumes that flight between two planets can be represented by a combination of three segments of conic flight or flight in a central force field. This model is illustrated in two dimensions in Figure 20.

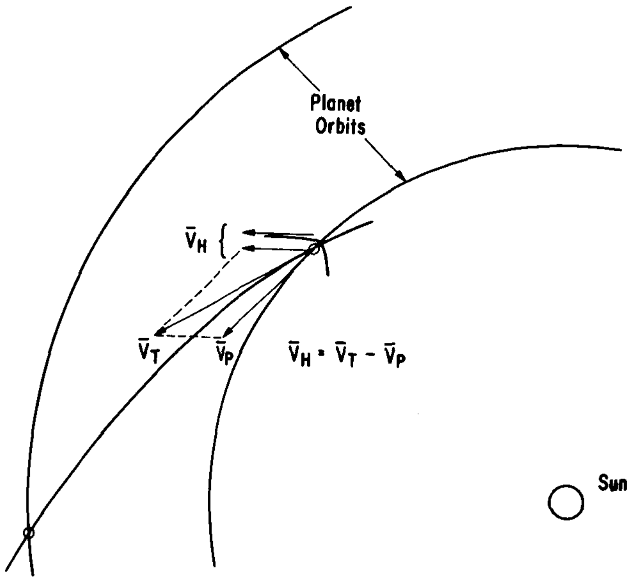


FIGURE 20. DETERMINATION OF HYPERBOLIC EXCESS VELOCITY VECTOR, $\bar{V}_H$

The trajectory in the region sufficiently distant from the planet so that the planet's gravitational attractions are negligible is taken to be an ellipse connecting the centers of the two planets and having the sun at one focus. The trajectory near the planet then is taken to be a hyperbola with the planet at one focus. It is then assumed that, at a distance so far from the planet that its gravitational attraction is negligible, the velocity vector ($\bar{V}_H$ on Figure 20) on this hyperbola is equal to the difference between the velocity vector relative to the sun ($\bar{V}_T$ on Figure 20) of the ellipse at its encounter with the planet and the velocity vector relative to the sun ($\bar{V}_P$ on Figure 20) of the planet at that time. This vector difference, therefore, is called the hyperbolic excess velocity vector and is used to define a hyperbola that represents the orbit near the planet.

The assumptions made in the construction of this model may seem rather gross; however, transits calculated in this manner are actually remarkably good approximations to the correct solution. On the other hand, when detailed study of a specific transit or family of transits is needed, for example, to determine guidance accuracy requirements to accomplish a mission, the exact equations of motion incorporating precise representation of the significant solar bodies, must be solved. The same is true when final analysis of earth-moon flights is necessary.

SECTION V. PRECISION TRAJECTORY PROGRAM

Neither the representation of the solar system or the solution of the equations of motion of a small body can be obtained except by numerical methods. The problem of assuring accuracy of these numerical solutions is another area of effort here. Dr. Hans Sperling of Aero-Astrodynamics Laboratory is now developing a computer program with which we hope to be able to compute realistic trajectories with the precision of the computation assured for 12 to 16 digits even over long transit times. To do this the method presently being considered incorporates integration of the differential equations of motion for all of the bodies to be included in the model as well as those for the spacecraft. The initial conditions for the integration of the equations are obtained from the best known ephemerides of the solar system. Integration of the motion of the model eliminates the problems of uncertain error magnitude introduced when ephemeris data are incorporated into a program by numerical interpolations from tabulated data. The entire system of equations is solved by a numerical integration technique using successive power series expansions.

At present these techniques have been employed in a computer program which includes four finite bodies and one massless body in the model. Three of the finite bodies are treated as point masses or homogeneous spheres. One oblateness term is incorporated in the gravitational function for the fourth body. This program is operational and can be used for some applications; however, many additional features such as triaxial shape of the moon, more oblateness terms for earth, and radiation pressure from the sun must be added to achieve the accuracy necessary for many projects. Work is continuing in this direction.

Only the highlights of the projects discussed were given here. Most of the details are available in the publications mentioned and in other published material from the various organizations involved.

SECTION VI. FUTURE EFFORTS

Future problems of astrodynamics include the natural continuation of the projects discussed plus several new areas. Perhaps the most urgent future problems are those concerning Voyager flights. Considerable detailed analysis of trajectory characteristics will be required in the layout of the actual flight to be chosen. Launch and injection windows will be determined, equations of cutoff conditions for the booster flight will be required, and methods of midcourse correction maneuvers will be formulated.

In addition the search for more comprehensive description and classification of all orbits in the restricted three-body model will be continued. Investigation of the possibility of utilizing three-body trajectory behavior in interplanetary flight, in particular, simulating a transition orbit in the Mars-sun system or Jupiter-Sun system and thereby forming a round trip trajectory that includes several orbits of the target planet, has been initiated. Methods of evaluating the three-body trajectory behavior under the influence of perturbations of the solar system are also being continued. The mastery of problems such as these will be necessary before large-scale exploration of the solar system is possible.

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Astrodynamics-Optimization Theory
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By E. D. Geissler, Clyde Baker, C. C. Dearman, and Arthur Schwaniger

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"In January of this year NASA directed that the international system of units should be considered the preferred system of units, and should be employed by the research centers as the primary system in all reports and publications of a technical nature, except where such use would reduce the usefulness of the report to the primary recipients. During the conversion period the use of customary units in parentheses following the SI units is permissible, but the parenthetical usage of conventional units will be discontinued as soon as it is judged that the normal users of the reports would not be particularly inconvenienced by the exclusive use of SI units."

The International System of Units (SI Units) has been adopted by the U. S. National Bureau of Standards (see NBS Technical News Bulletin, Vol. 48, No. 4, April 1964).

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