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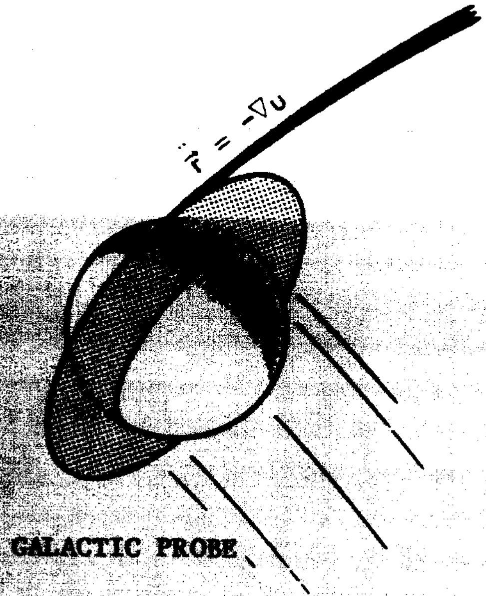

INJECTION AND MIDCOURSE CORRECTION ANALYSIS FOR THE GALACTIC PROBE

BY

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ABSTRACT

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The purpose of this paper is to provide information on the effects of injection errors on a Galactic Probe mission to Jupiter and the ability to correct these errors by performing a single attitude-restricted midcourse correction. Monte Carlo methods have been used with an injection error model and a midcourse correction error model that are assumed to be representative and probably conservative. Three possible attitude control laws as well as mission objectives are assumed. Numerical results are presented in tabular and graphical form.

LIST OF ILLUSTRATIONS

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* ( )° denotes a unit vector, i.e. $\vec{x}^{\circ} = \frac{\vec{x}}{|\vec{x}|}$

Injection Error Analysis

In order to perform an analysis of Galactic Probe injection errors, the following assumptions have been made:

1. Objectives of the mission are a flyby of Jupiter within its magnetic field and a subsequent flight to a heliocentric distance of 10 a.u.'s in a reasonable total flight time. The nominal trajectory used for this study meets these mission objectives.

2. Injection errors are similar to those assumed for the Improved Delta - X-258 vehicle. Exceptions are flight path angle and azimuth errors, which are assumed to be smaller because, in all liklihood, there will be a much smaller contribution of the kick stage to the total injection velocity for the Galactic Probe and the kick stage is the dominant source of these errors.

The nominal trajectory used for this analysis has the following characteristics:

Launch date: December 30, 1969.

Injection time: $7^h\ 5^m\ 37^s$ GMT.

Geocentric injection speed: 15.405 km/sec.

Flight time to Jupiter: 500 days.

Radius of closest approach to Jupiter: 1×10^6 km. (posigrade with respect to Jupiter's rotation about the sun).

Total flight time to 10 a.u.'s from sun: 1050 days.

This nominal trajectory is considered to be compatible with the above mission objectives and, in addition, is near optimum in terms of minimum flight time to 10 a.u. for a 500 day flight to Jupiter.

Monte Carlo methods were used to determine the effects of injection errors propagated to Jupiter. A digital computer program called McINTR (Monte Carlo-Interplanetary) was written for the purpose of studying the effects of injection errors and midcourse corrections on targetting accuracy for interplanetary trajectories. A covariance matrix of injection errors was used in conjunction with a table of normally distributed random numbers (about a zero mean) to corrupt the initial conditions for each perturbed transfer trajectory. The injection error sources used for this study were assumed to be uncorrelated, thus no off-diagonal terms appear in the injection covariance matrix. The assumed error sources and their one sigma values are as follows:

R (radial distance from earth's center)	7.027 km.
ϕ (geocentric latitude)	$.090^\circ$
λ (longitude)	$.165^\circ$
V (inertial speed)	17.67m/sec.
γ (inertial geocentric flight path angle)	$.12^\circ$
α (inertial geocentric azimuth)	$.12^\circ$

RMS errors were evaluated in terms of miss vector component ($\vec{B}\cdot\vec{T}^\circ$ and $\vec{B}\cdot\vec{R}^\circ$ as described in reference (1) in which the reference plane is Jupiter's orbital plane) deviations from the nominal aim point. Figure 1 presents a plot of the miss vector components ($\vec{B}\cdot\vec{R}^\circ$ vs. $\vec{B}\cdot\vec{T}^\circ$) for the 300 sample size Monte Carlo study. The resulting RMS errors in the miss vector components due to injection errors are as follows:

RMS ($\vec{B}\cdot\vec{T}^\circ$) error = 2.45×10^6 km.

RMS ($\vec{B}\cdot\vec{R}^\circ$) error = 0.31×10^6 km.

To provide an independent check for these results, the methods of linear error propagation were employed using the Interplanetary Error Propagation Program (reference 2). Using the same injection covariance matrix, a precision integrated trajectory (having similar characteristics as the above described nominal trajectory) and the methods of statistical error propagation with linear theory, the following RMS errors in the miss vector components resulted:

RMS ($\vec{B}\cdot\vec{T}^\circ$) error = 2.35×10^6 km.

RMS ($\vec{B}\cdot\vec{R}^\circ$) error = 0.30×10^6 km.

In order to obtain a feeling for the relative signifigance of each injection error source's contribution to the total miss vector component errors, Monte Carlo runs were performed for each error source individually, i.e. injection covariance matrix contains one diagonal element and all other diagonal and off-diagonal elements are zero. Three hundred sample size runs were used for each error source and the results are enumerated in the following table:

Individual Error Source (one sigma)	RMS ($\vec{B}\cdot\vec{T}^{\circ}$) Error	RMS ($\vec{B}\cdot\vec{R}^{\circ}$) Error
R (7.027 km.)	0.51×10^6 km.	0.01×10^6 km.
ϕ (.090°)	0.07×10^6 km.	0.07×10^6 km.
λ (.165°)	1.45×10^6 km.	0.17×10^6 km.
V (17.67m/sec)	1.61×10^6 km.	0.04×10^6 km.
γ (.12°)	1.05×10^6 km.	0.03×10^6 km.
α (.12°)	0.14×10^6 km.	0.25×10^6 km.

As more information becomes available on injection error models for launch systems of interest to Galactic Probe missions, the miss vector component errors can be scaled accordingly for each error source.

Midcourse Guidance Analysis

A major problem that arises in planning any lunar or interplanetary mission is the trade-off between injection and midcourse guidance systems in terms of cost, complexity and compatibility with mission objectives. The assumed injection errors used in this study are considered to be a conservative representation of a vehicle with a spin-stabilized solid propellant kick stage. The assumed mission objectives require flying by Jupiter in a posigrade manner (with respect to Jupiter's rotation about the sun) in order to obtain an energy "boost" to considerably shorten the flight time to 10 a.u.'s and beyond. The nominal trajectory used in this study is considered to be close to optimum in terms of the assumed mission objectives. From Figure 1 it can be seen that injection errors cause significant dispersions about the nominal aim point. It becomes readily apparent that a midcourse correction or corrections are necessary to accomplish the assumed mission objectives.

The following assumptions have been made in order to provide a first look at the midcourse guidance problem:

1. Spacecraft attitude will be restricted.

2. Three attitude control laws are assumed as follows:

    a. Spin-axis oriented continually toward earth.

    b. Spin-axis inertially fixed as injection velocity vector.

    c. Spin-axis oriented continually toward sun.

3. Midcourse corrections are restricted to be made parallel to the spin axis.

4. Perfect navigation system; at least 5 days are assumed to be necessary to determine the trajectory accurately.

5. Midcourse correction error model is represented by pointing errors in pitch (θ) and yaw (ψ) and an error in correction magnitude (ΔV). Assumed one sigma values of these errors are as follows:

$$\sigma_\theta = \sigma_\psi = 2^\circ$$

$$\sigma\Delta V = 5\text{m/sec.}$$

6. Guidance law corrects $(\vec{B}\cdot\vec{T}^\circ)$ based on its deviation from $(\vec{B}\cdot\vec{T}^\circ)_{NOMINAL}$ and partials derived by perturbing the nominal trajectory at the desired correction time along the nominal spin axis.

The Monte Carlo computer program, called McINTR, which was used for the injection error analysis, was also utilized for the midcourse guidance analysis. The program flows in the following manner:

1. Fly nominal injection conditions to target body (Jupiter) and store miss vector components.

2. Fly nominal to desired time of midcourse correction and add and subtract an input velocity increment (ΔV) along spin axis (specified by input attitude control law). Fly these to target body to determine average partial to be used for midcourse corrections: $\frac{\partial(\vec{B}\cdot\vec{T}^\circ)}{\partial\Delta V}$

3. Fly N trajectories having perturbed injection conditions to the target body and store the resulting miss vector components.

4. Fly each of the N perturbed trajectories to the input midcourse correction time and compute midcourse corrections based on:

$$\Delta V_i = (\vec{B}\cdot\vec{T}^\circ{}_i - \vec{B}\cdot\vec{T}_{NOM.}) \Big/ \frac{\partial(\vec{B}\cdot\vec{T}^\circ)}{\partial\Delta V}.$$

5. Apply the midcourse corrections with random errors to each of the N perturbed trajectories and fly to the target body.

6. Print out for the nominal and each perturbed trajectory with and without midcourse corrections:

a. Time to closest approach of target body.

b. Radius of closest approach.

c. $(\vec{B}\cdot\vec{T}^\circ)$

d. $(\vec{B}\cdot\vec{R}^\circ)$

e. Post encounter apohelion.

f. Post encounter inclination to ecliptic.

g. Total time to 10 a.u.'s from sun.

h. Required midcourse correction velocity (ΔV_i).

7. Provide summary printout of the following:

a. RMS $(\vec{B}\cdot\vec{T}^\circ)$ and $(\vec{B}\cdot\vec{R}^\circ)$ errors due to injection errors.

b. RMS $(\vec{B}\cdot\vec{T}^\circ)$ and $(\vec{B}\cdot\vec{R}^\circ)$ errors after midcourse corrections.

c. RMS midcourse ΔV required.

d. Partials of $(\vec{B}\cdot\vec{T}^\circ)$ and $(\vec{B}\cdot\vec{R}^\circ)$ with respect to ΔV for the nominal trajectory.

8. Printout plots of $(\vec{B}\cdot\vec{R}^\circ{}_i$ vs. $\vec{B}\cdot\vec{T}^\circ{}_i)$ with and without midcourse corrections.

A patched-conic trajectory model is employed in the program and the ephemerides of the earth and the target planet are represented by orbital elements in a heliocentric system. These features permit the luxury of using Monte Carlo techniques with large sample sizes for parametric studies with very modest digital computer time requirements. An average of about 4 trajectories to Jupiter can be flown and analyzed per second of computer time using the McINTR program. The specification of attitude control laws and guidance laws are options within the program and provision has been made for ease of incorporation of new guidance or attitude schemes as needed.

For each of the assumed attitude control laws described above, midcourse corrections were made at 5, 10, 15, 20, 30, 40 and 50 days from injection, making a total of 21 Monte Carlo runs of 300 perturbed trajectories each.

The relatively simple $(\vec{B}\cdot\vec{T}^\circ)$ guidance law was used for this study because the out-of-plane $(B\cdot R^\circ)$ sensitivity to midcourse corrections (restricted by attitude laws) was significantly lower than that of the in-plane $(\vec{B}\cdot\vec{T}^\circ)$ miss vector component. In addition, as previously noted, the RMS in-plane error at Jupiter is nearly an order of magnitude greater than the RMS out-of-plane error. Figures 2a and 2b present the partials of $(\vec{B}\cdot\vec{T}^\circ)$ and $(\vec{B}\cdot\vec{R}^\circ)$ with respect to correction ΔV for each attitude control law as a function of midcourse correction time.

In order to show the effects of midcourse corrections, Figure 3a presents an example of $(\vec{B}\cdot\vec{R}^\circ$ vs. $\vec{B}\cdot\vec{T}^\circ)$ after midcourse corrections, demonstrating the capability of correcting large targetting dispersions (see Figure 1) with single, attitude-restricted, noisy midcourse corrections using a simple one component guidance law. Figures 3b, 3c and

3d show the effects of the errors in the midcourse corrections by presenting ($\vec{B}\cdot\vec{R}^\circ$ vs. $\vec{B}\cdot\vec{T}^\circ$) plots in which the midcourse pointing and ΔV errors are zero, only the ΔV error is zero, and only the pointing errors are zero, respectively. Figures 4a, 4b, and 5 show respectively the RMS errors in ($\vec{B}\cdot\vec{T}^\circ$) after midcourse correction, the RMS errors in ($\vec{B}\cdot\vec{R}^\circ$) after correction, and the RMS midcourse ΔV required as functions of midcourse correction time for each attitude control law.

Conclusions

The results of this study are intimately tied to the assumptions that have been made. In order to meet the assumed mission objectives (flyby of Jupiter within the magnetic field and subsequent "reasonable" flight time to 10 a.u.'s), a single midcourse correction appears to be necessary and sufficient taking into account the assumed injection errors and midcourse guidance errors. It has been demonstrated that a single attitude-restricted midcourse correction using a simple guidance law (correcting only for $\vec{B}\cdot\vec{T}^\circ$ errors) will allow the mission objectives to be met. The use of this guidance law with a "noisy" midcourse correction reduces in-plane errors to tolerable levels, but does nothing to improve the out-of-plane errors (in the RMS sense). The important function of the midcourse guidance system, however, is to correct the path of the spacecraft so as to make a proper posigrade approach of Jupiter in order to obtain an energy "boost" to reduce the time to a heliocentric distance of 10 a.u.'s. This has been shown to be feasible with the ($\vec{B}\cdot\vec{T}^\circ$) guidance scheme.

Three possible attitude control laws have been assumed for this study and comparisons between them will be based only on their relative advantages or disadvantages for midcourse corrections. Referring to Figure 5, it can be concluded that if midcourse corrections are made in the first 20 days the inertially fixed spin axis (along the injection velocity vector) requires a smaller RMS correction velocity than for the other two attitude control laws (spin axis along earth-probe line and sun-probe line respectively). Having the spin axis along the sun-probe line offers the advantage of a nearly constant RMS correction velocity for about 30 days after injection whereas for the other two attitude control laws the required velocity increases steadily with midcourse correction time. Beyond these conclusions it may be said, in general, that the total variation in RMS required midcourse correction velocity is in the order of only 20 percent for all of the attitude laws considered. This means that considerations of attitude control system complexity, communications requirements and other factors will probably exert a much greater influence on the selection of an attitude control law than will the midcourse correction velocity budget.

ACKNOWLEDGMENT

The author is indebted to Mr. Ralph Cromar (Bendix) for his outstanding performance in producing in a short period of time an extremely fast, efficient and utilitarian computer program (McINTR) which permitted the luxury of using Monte Carlo methods to study the injection and midcourse guidance problem in a parametric manner with very modest digital computer time requirements. The author also wishes to express appreciation for the work of Mr. James Cooley of the Mission Analysis Office in providing an independent check on the effects of injection errors on targetting errors at Jupiter using linear error propagation methods. The agreement in the results was gratifying, producing a high degree of mutual confidence in the methods and computer programs employed.

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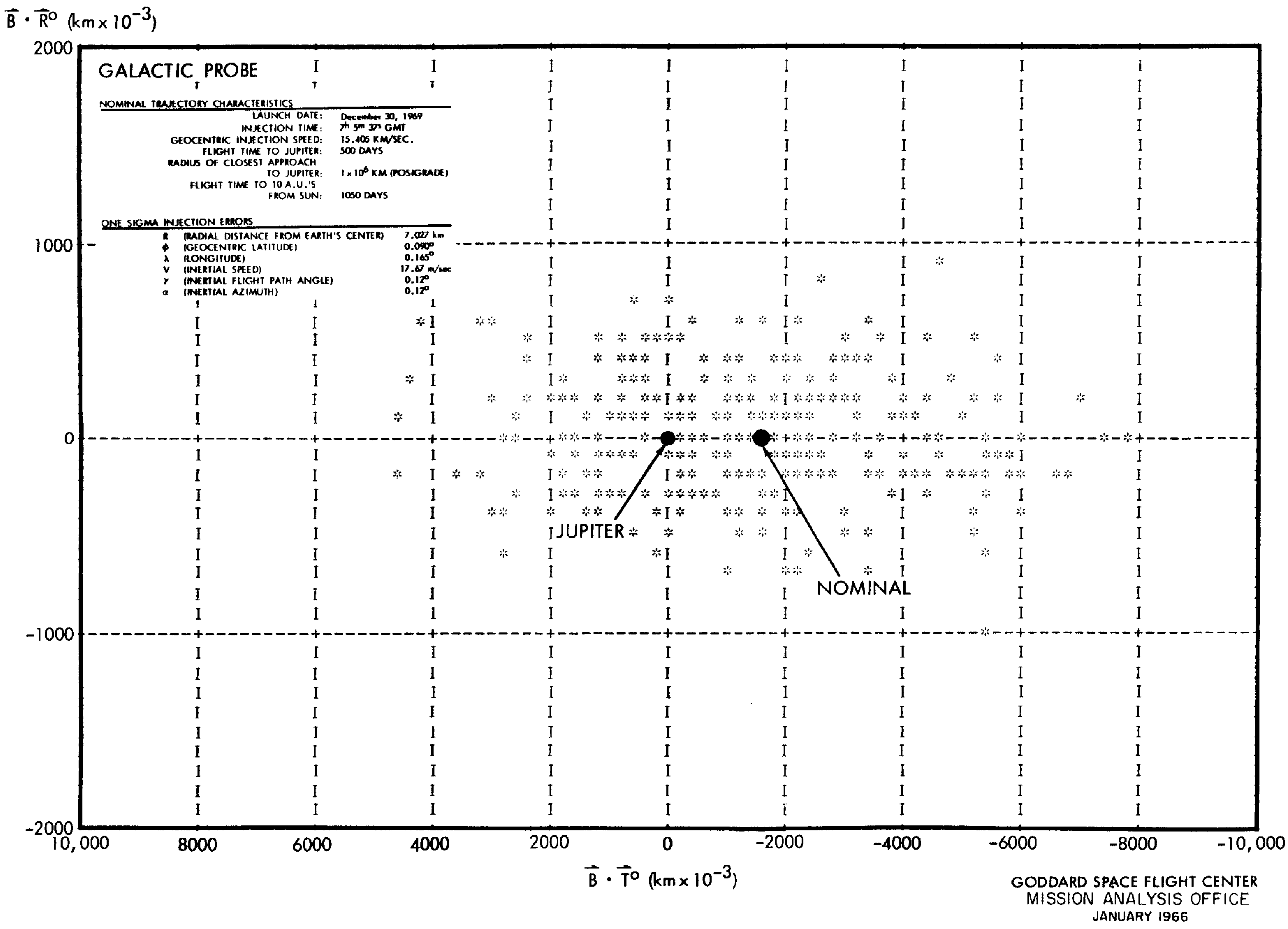

Figure 1 – $\vec{B} \cdot \vec{R}^{o}$ vs. $\vec{B} \cdot \vec{T}^{o}$ for 300 Transfer Trajectories with Random Injection Errors

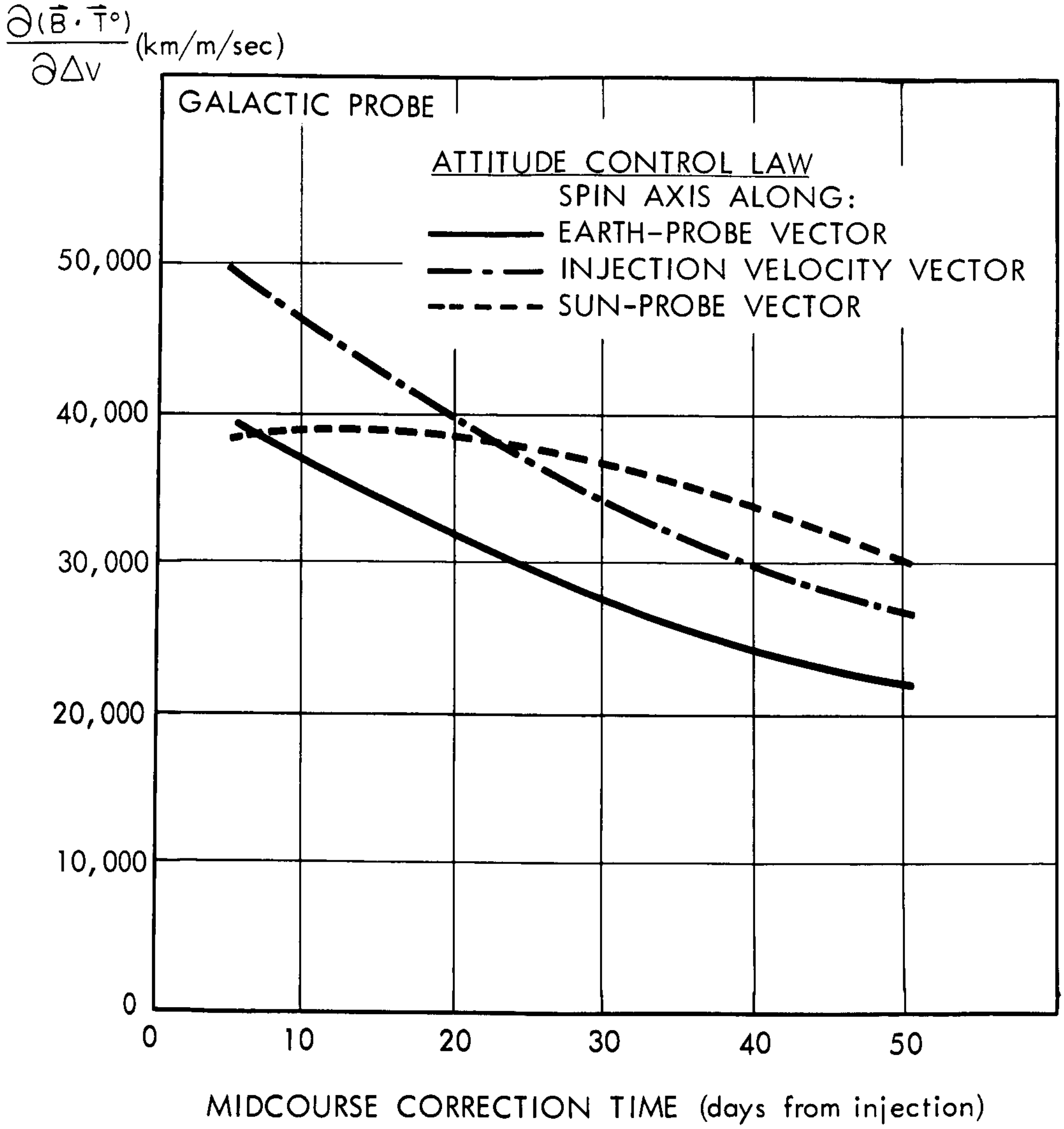

Figure 2a - $\frac{\partial(\vec{B}\cdot\vec{T}^\circ)}{\partial\Delta v}$ vs Midcourse Correction Time for Three Attitude Control Laws

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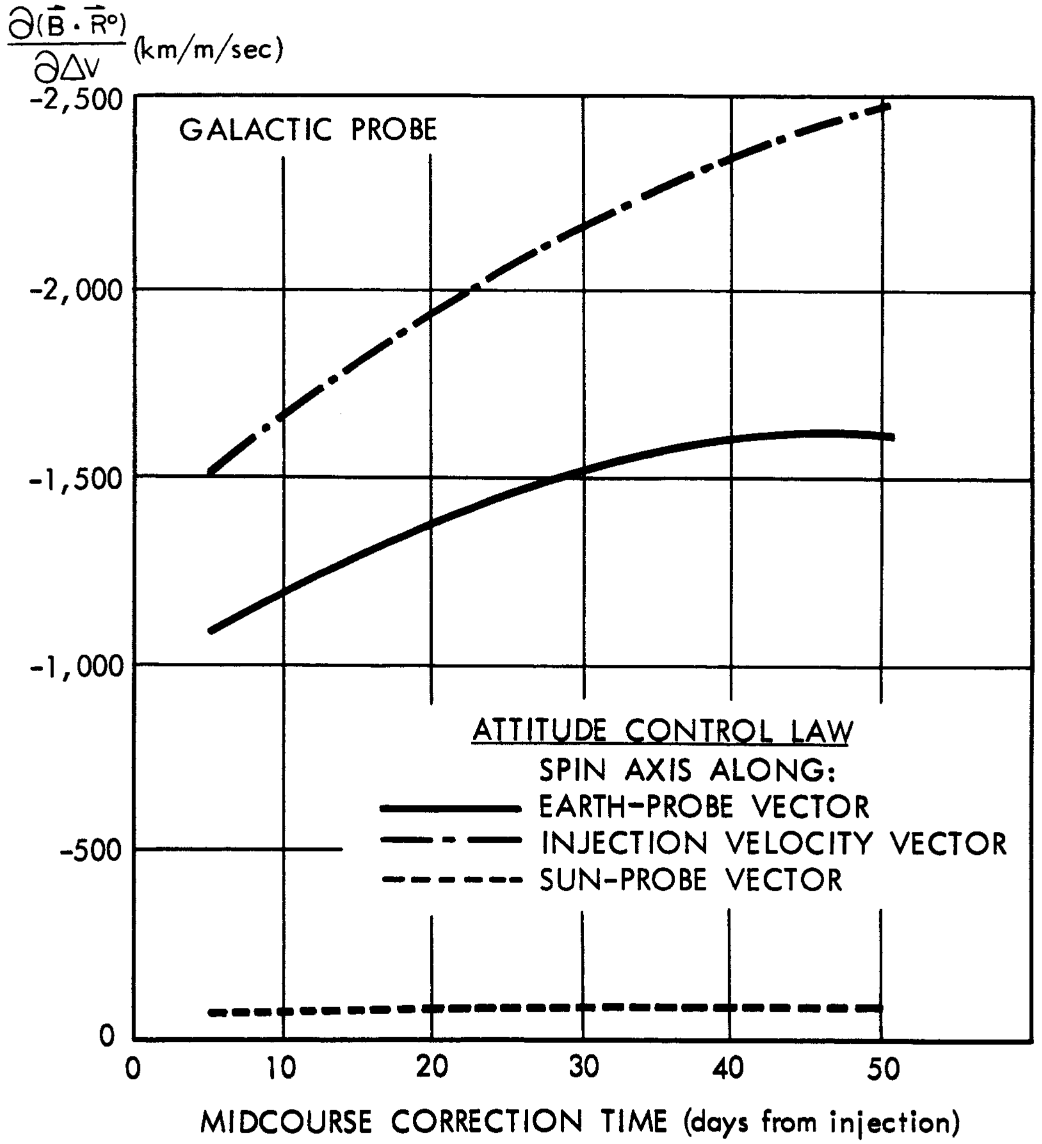

Figure 2b - $\frac{\partial(\vec{B}\cdot\vec{R}^{o})}{\partial\Delta v}$ vs Midcourse Correction Time for Three Attitude Control Laws

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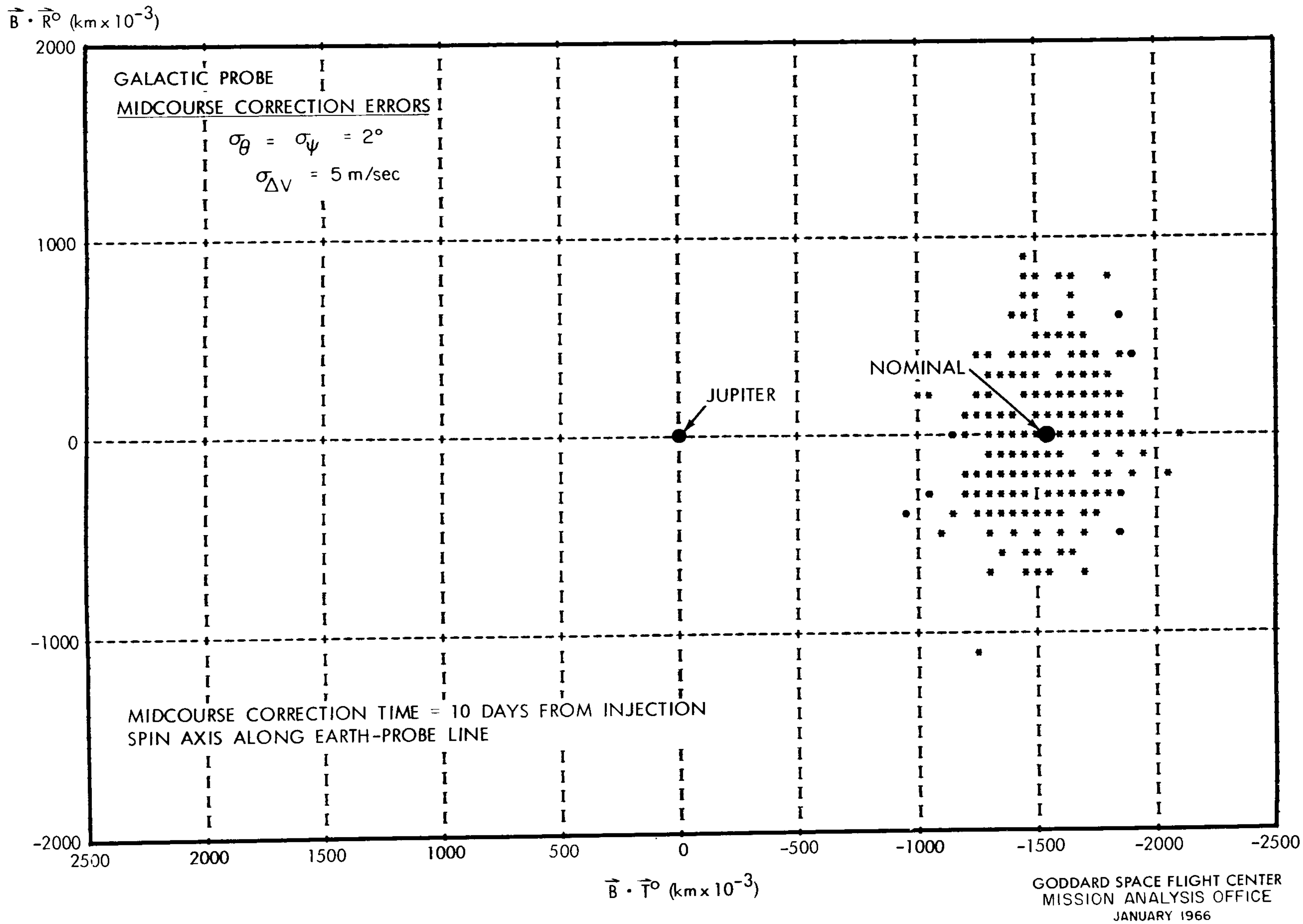

Figure 3a - $\vec{B} \cdot \vec{R}^{o}$ vs. $\vec{B} \cdot \vec{T}^{o}$ For 300 Perturbed Transfer Trajectories with Midcourse Corrections

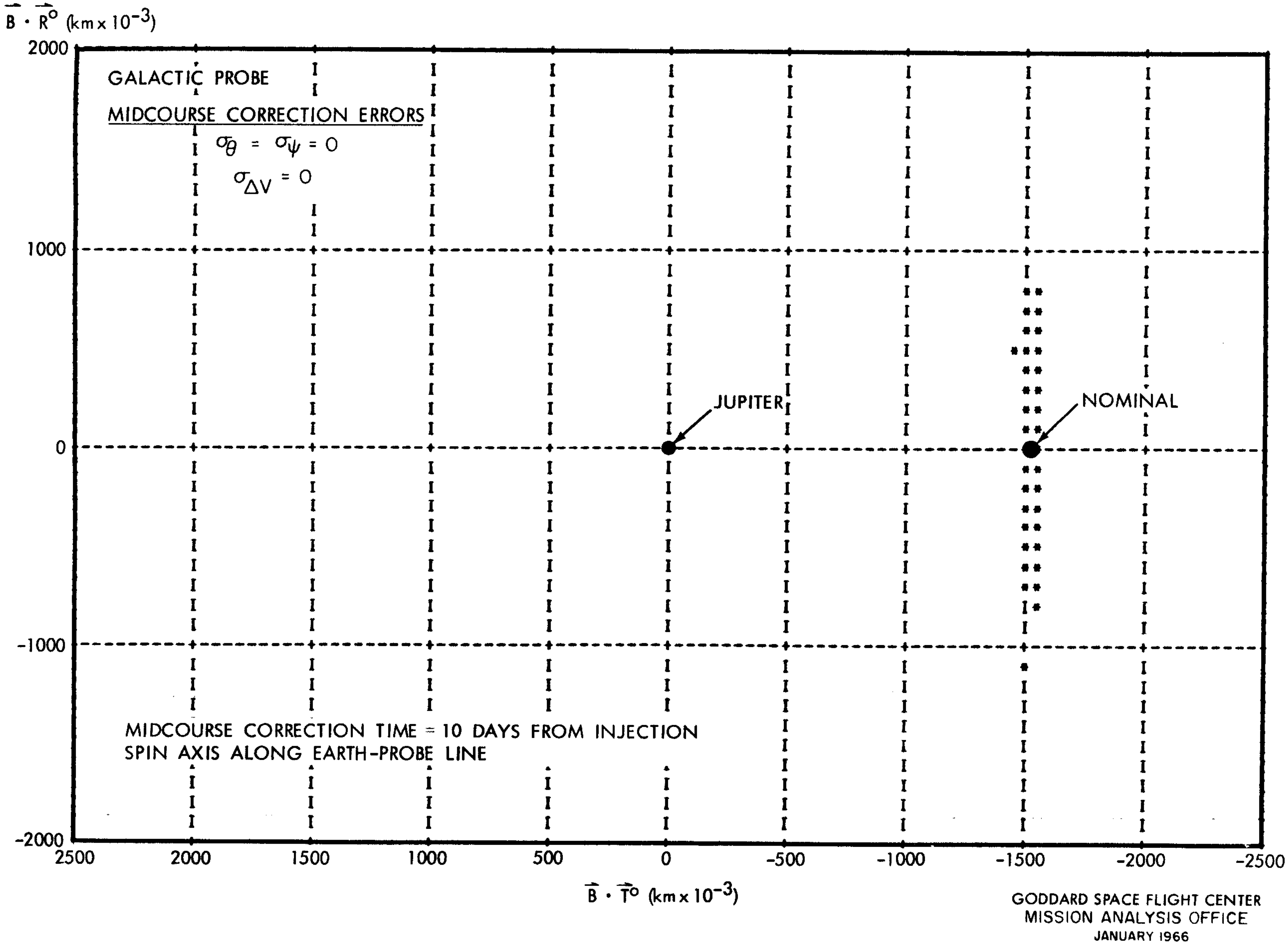

Figure 3b – $\vec{B} \cdot \vec{R}^o$ vs. $\vec{B} \cdot \vec{T}^o$ For 300 Perturbed Transfer Trajectories with Midcourse Corrections

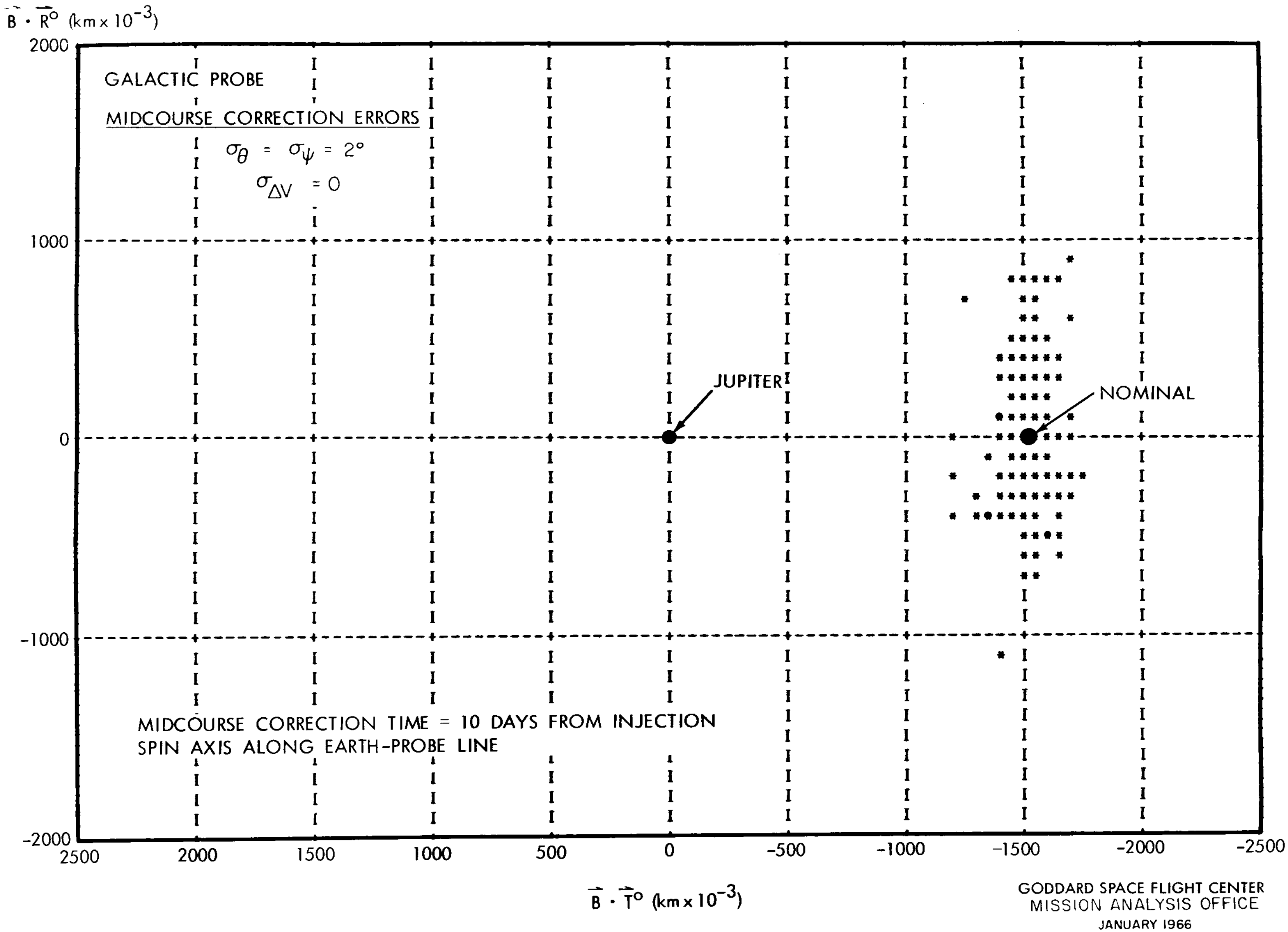

Figure 3c - $\vec{B} \cdot \vec{R}^o$ vs. $\vec{B} \cdot \vec{T}^o$ For 300 Perturbed Transfer Trajectories with Midcourse Corrections

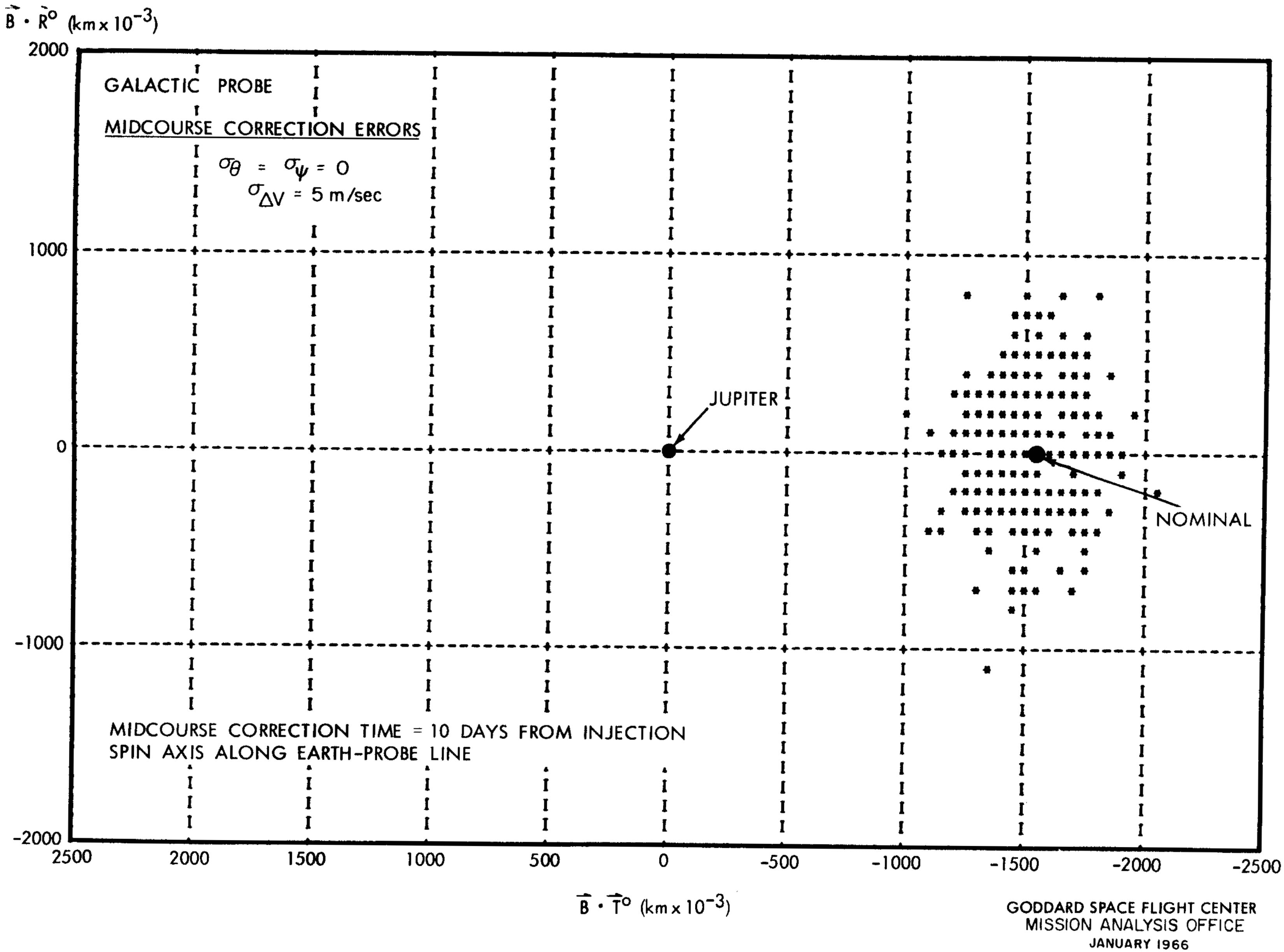

Figure 3d – $\vec{B} \cdot \vec{R}^{o}$ vs. $\vec{B} \cdot \vec{T}^{o}$ For 300 Perturbed Transfer Trajectories with Midcourse Corrections

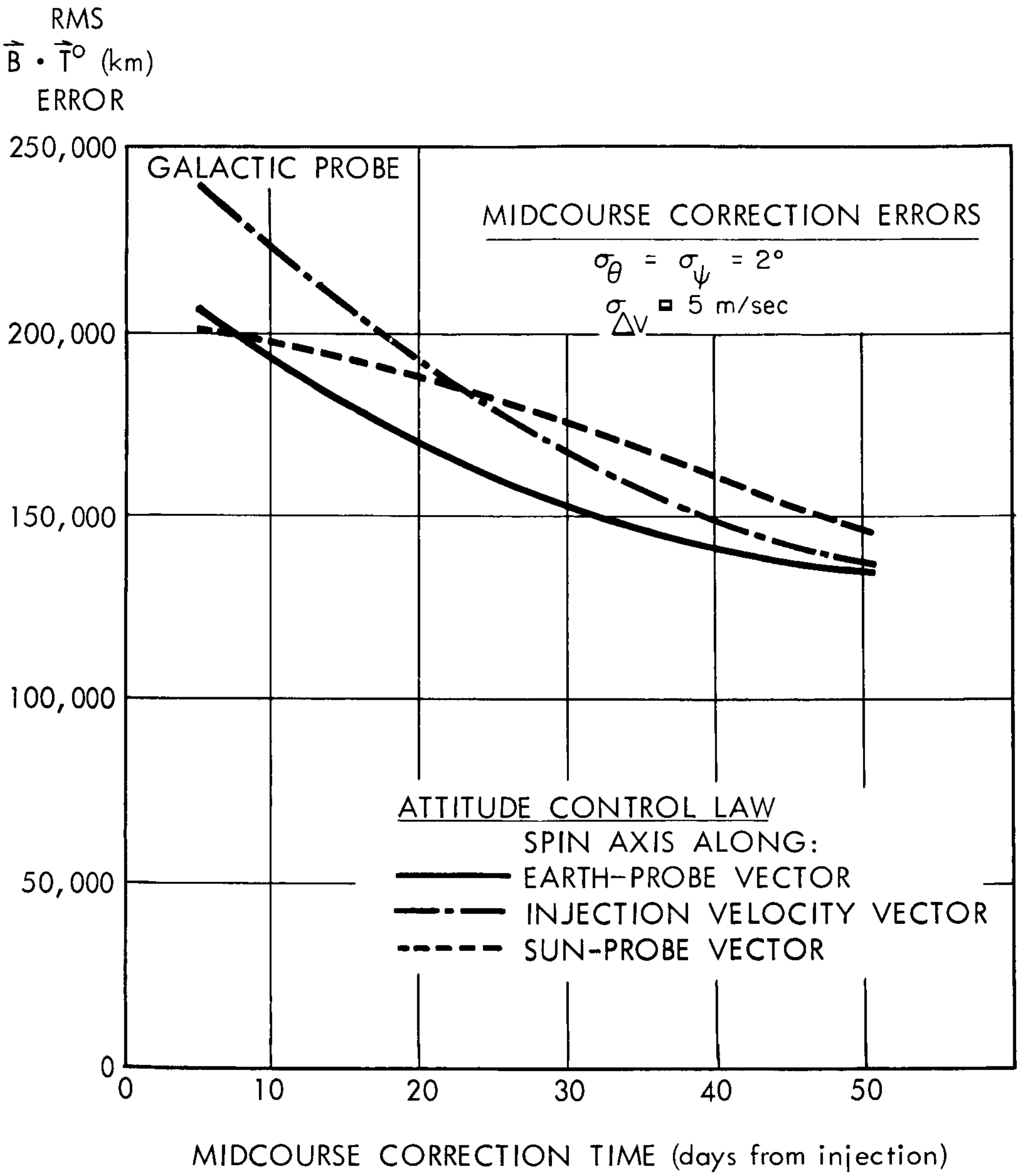

Figure 4a - RMS ($\vec{B} \cdot \vec{T}^{o}$) Errors After Midcourse Correction vs. Midcourse Correction Time for Three Attitude Control Laws

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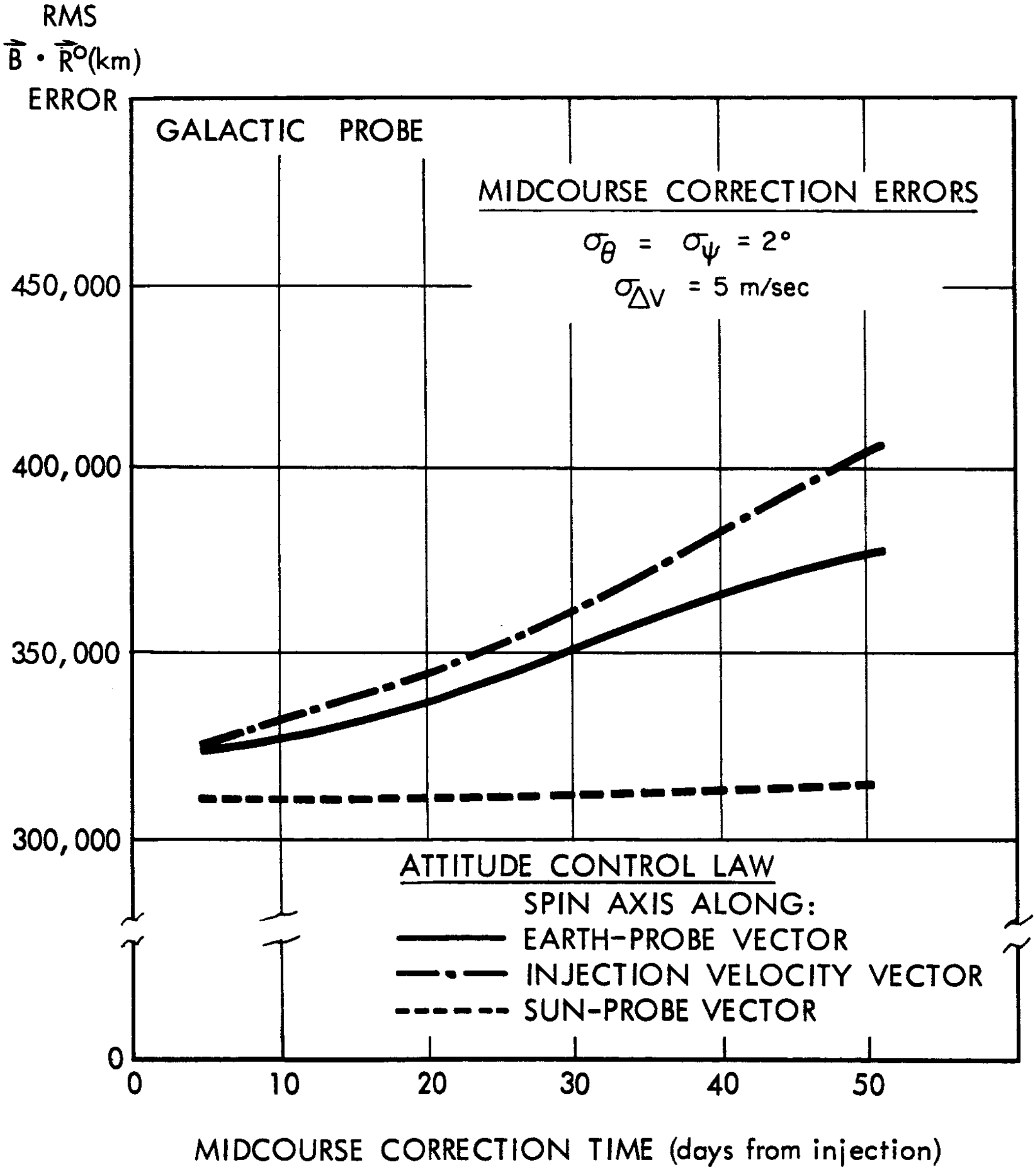

Figure 4b - RMS ($\vec{B} \cdot \vec{R}^o$) Errors After Midcourse Correction vs. Midcourse Correction Time for Three Attitude Control Laws

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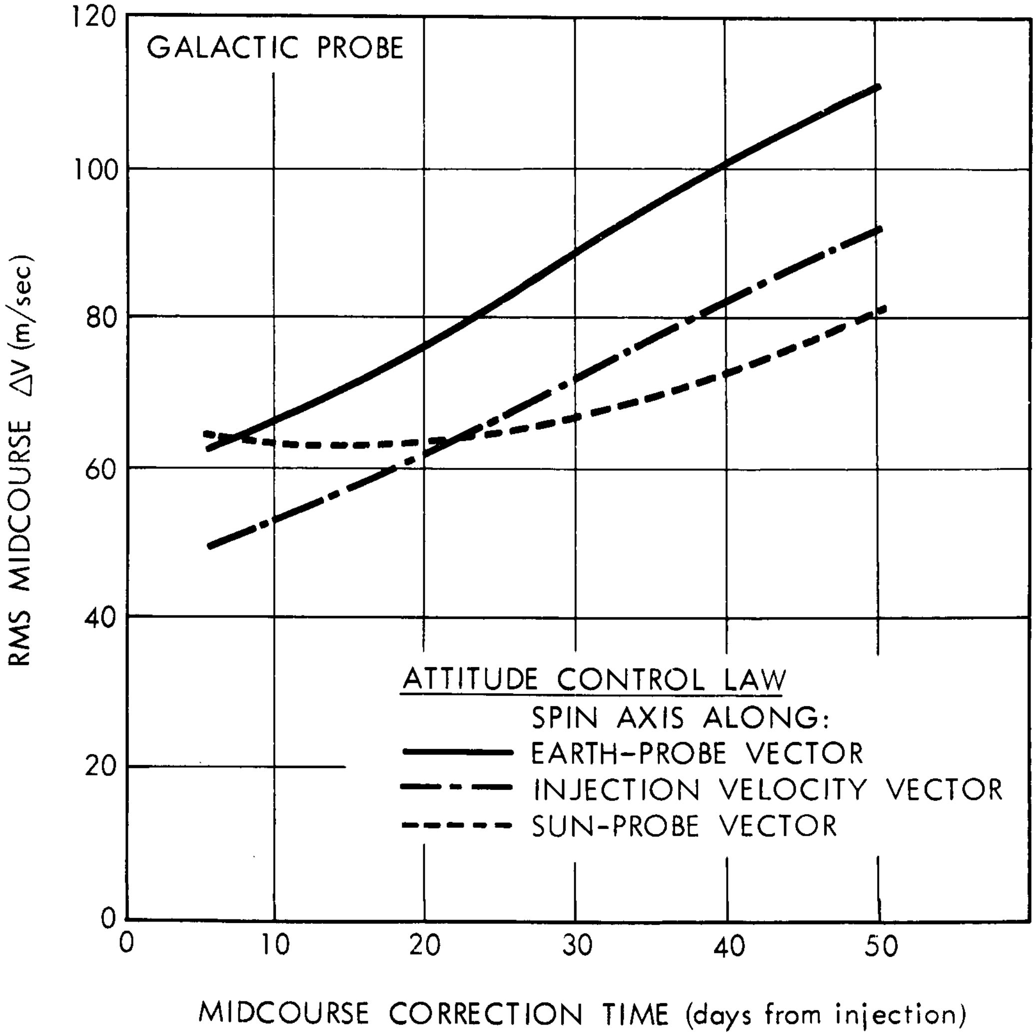

Figure 5 - RMS Required Midcourse Correction ΔV vs. Midcourse Correction Time for Three Attitude Control Laws

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