NASA TECHNICAL MEMORANDUM

NASA TM X- 53385

February 1

NASA TM X-53385

EARTH ORBITAL LIFETIME PREDICTION MODEL AND PROGRAM

1- 205-453-2121 Ex. 0988

by Ann R. McNair and Edward P. Boykin
Aero-Astrodynamics Laboratory

NASA

*George C. Marshall
Space Flight Center,
Huntsville, Alabama*

TECHNICAL MEMORANDUM X- 53385

EARTH ORBITAL LIFETIME PREDICTION MODEL AND PROGRAM

By

Ann R. McNair and Edward P. Boykin

George C. Marshall Space Flight Center
Huntsville, Alabama

ABSTRACT

An earth orbital satellite lifetime deck has been developed and programmed in Fortran IV language for the IBM 7094. The deck represents the development of a sophisticated and accurate lifetime prediction technique, which includes the effect of aerodynamic drag and the nonspherical gravitational potential of the earth. The computer program can be used to predict lifetime based on only a gross description of the initial orbit and drag parameters, or based on a very exact definition of the initial orbit and detailed description of the drag parameters and their variations depending on the amount of information available. The primary factor contributing to uncertainty in lifetime predictions using this model is the atmospheric density. A very flexible model based on data from Discoverer, Gemini, and Saturn flights has been established. The primary uncertainty remaining in this model is prediction for future years of solar activity behavior and its influence on density as a function of altitude. As additional flight data and solar activity observations become available, they may readily be incorporated into the model, thus providing a rapidly changing density model which insures the best representation possible. Efforts to refine the models as presently defined and to perform pertinent studies in the lifetime area are continuing. This report represents only the present status of model definitions and defines the computer program now in use.

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

TECHNICAL MEMORANDUM X-53385

EARTH ORBITAL LIFETIME PREDICTION MODEL AND PROGRAM

By

Ann R. McNair and Edward P. Boykin

FLIGHT EVALUATION AND OPERATIONS STUDIES DIVISION
AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

TABLE OF CONTENTS

TABLE OF CONTENTS (Concluded)

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Concluded)

LIST OF TABLES

TECHNICAL MEMORANDUM X- 53385

EARTH ORBITAL LIFETIME PREDICTION MODEL AND PROGRAM

SUMMARY

An earth orbital satellite lifetime deck has been developed and programmed in Fortran IV language for the IBM 7094. The deck represents the development of a sophisticated and accurate lifetime prediction technique, which includes the effect of aerodynamic drag and the nonspherical gravitational potential of the earth. The computer program can be used to predict lifetime based on only a gross description of the initial orbit and drag parameters, or based on a very exact definition of the initial orbit and detailed description of the drag parameters and their variations depending on the amount of information available. The primary factor contributing to uncertainty in lifetime predictions using this model is the atmospheric density. A very flexible model based on data from Discoverer, Gemini, and Saturn flights has been established. The primary uncertainty remaining in this model is prediction for future years of solar activity behavior and its influence on density as a function of altitude. As additional flight data and solar activity observations become available, they may readily be incorporated into the model, thus providing a rapidly-changing density model which insures the best representation possible. Efforts to refine the models as presently defined and to perform pertinent studies in the lifetime area are continuing. This report represents only the present status of model definitions and defines the computer program in use.

SECTION I. INTRODUCTION

An extensive effort has been underway to develop a flexible and accurate orbital lifetime prediction model and a density model accurately describing the history of decayed satellites and providing the best available prediction for future satellites. This has been performed jointly by the authors and the Dynamics and Guidance Department of the Lockheed Missiles and Space Company, Huntsville Research and Engineering Center. Lockheed's work has been performed under Contracts NAS8-11148, NAS8-11121 and Task B of NASA Schedule Order No. 1, Contract NAS8-20082. Principal contributors from Lockheed are Mr. T. J.

Richards and Mr. H. F. Kilgo. This report was prepared jointly by MSFC amd LMSC, Section V being prepared by Mr. W. B. Hawkins, of Lockheed. The computer program is documented in Reference 1.

The orbital lifetime of a satellite, in the final analysis, depends solely upon two things: the initial total energy of the satellite and the time history of its rate of energy loss. Initial total energy is determined immediately either by position and velocity or by orbit elements. However, the determination of rate of energy loss is not immediately evident. Consider the external influences which act on a satellite to change its orbit: those produced by atmospheric resistance, the earth's nonspherical gravitational potential, lunar-solar gravitation, solar radiation pressure, and geomagnetic potential. Each of these act to change the shape and orientation of the orbital ellipse. Only atmospheric resistance causes a net decrease in total energy. Nevertheless, these other forces are important in determining orbital lifetime because they influence the parameters which define aerodynamic forces.

The development of sophisticated and accurate lifetime prediction and density models has been prompted by the need in the Saturn program for better estimates of lifetime and decay characteristics for earth orbital flight. These data are essential to realistic mission planning.

The basic models have been developed and a computer program written in Fortran language for the IBM 7094. The program simulates the rates of change in the orbit of a satellite and ultimately calculates the total time it remains in orbit. As presently coded, it includes the effects of aerodynamic drag and the earth's nonspherical gravitational potential. The computer program can be used to predict lifetime based on only a gross description of the initial orbit and drag parameters, or on a very exact definition of the initial orbit and detailed description of the drag parameters and their variations, depending upon the amount of information available.

Since the elements in the drag function can be input as constants or variables in a number of ways, their accuracy is limited mainly by the amount of information available to the user. These elements are atmospheric density, drag coefficient, cross-sectional area, and mass of the satellite.

Of principal significance is the atmospheric density model. It was desired to construct a model best predicting density for future years and optimally utilizing data obtained from decayed satellites and actual measurements of solar and geomagnetic activity. Consequently, any "best" density model must remain in a constant state of flux as new data and input become available.

The development of this extensive capability for earth orbital lifetime predictions is by no means completely refined. The basic model and best known inputs to date are formulated for use. A continuing effort is being made to further develop and refine the model. The following list singles out some of the more significant areas either presently being investigated or planned to be investigated. Information concerning these items may be obtained from the authors.

1. The equation used to compute the radius to the satellite as a function of a set of "mean" orbital elements was derived using a definition of "mean" elements which differs from the definition used in the transformation phase of the deck. The deck will be revised to either redefine the function or to redefine the elements to establish a consistent set.

2. The transformation between osculating and "mean" elements required before lifetime computations is indeterminant for small eccentricities. This transformation is being derived.

3. Expressions accounting for the effects of solar radiation pressure have been derived and are being programmed into the deck.

4. Expressions for solar and lunar gravity effects are being derived and will be incorporated into the deck.

5. Uncertainties in extrapolating mean solar activity predictions have been determined. The uncertainties caused by short period fluctuations in solar activity are being established to arrive at an overall uncertainty in predictions for short and long lifetimes.

6. A technique for inclusion of new solar activity data into the density model and automatic updating of the solar activity future predictions based on past and current behavior will be established.

7. Updating of the density model incorporating the latest Saturn flight data is in progress. This will result in a more accurate model in the higher altitude region near 500 km.

SECTION II. DENSITY MODEL

The carefully formulated decay equations presented in following sections for the lifetime model are of little value for application unless an accurate density model is also used. Any density model which defines density as a function of altitude alone may be in error by an order of magnitude in the 200-700 kilometer altitude range. This section is specifically devoted to discussion of the time-variant density model used in the orbital lifetime program since the model is of such primary significance.

A. Defining a Time-Dependent and Position-Dependent Density Model

For future planning, it is necessary to have the capability of accurately determining the instantaneous acceleration due to drag (for propellant seating considerations, etc.) and the amount the orbit decays during a short period of time. This presents the requirement for an accurate time-dependent and position-dependent model, whereas a less sophisticated model is usually sufficient for an accurate total lifetime prediction.

The density of the upper atmosphere (120-700 km altitude) has been shown to vary with certain indices of solar and geomagnetic activity, with local time, with season and latitude, in addition to its primary variation with altitude. Many of the relationships used in the following model were developed by H. Small in Reference 2.

To describe the variation in density due to solar and geomagnetic activity fluctuations and seasonal effect, Small defines a single parameter, S, and refers to it as a "heating parameter" or the "total heating." The heating parameter S is defined as

$$S = \overline{S}e^{g(t)}, \qquad (1)$$

where

$$\overline{S} = 25 + 0.8\overline{\overline{F}}_{10.7} + 0.4(F_{10.7} - \overline{F}_{10.7}) + 10Kp$$

$$g(t) = .025 \cos\left[2\pi\left(\frac{t - 38047.0}{365.25}\right)\right] - .06 \cos\left[4\pi\left(\frac{t - 38047.0}{365.25}\right)\right]$$

$e^{g(t)}$ = correction for seasonal effects

t = time in modified Julian days

Kp = 3-hour planetary index of geomagnetic activity

$F_{10.7}$ = daily values of the 10.7 cm solar flux

$\overline{F}_{10.7}$ = smoothed values of the 10.7 cm solar flux.

This is formed by taking the running yearly mean of $F_{10.7}$, i.e.,

$$\overline{F}_{10.7} = \frac{1}{365} \sum_{i=-182}^{182} F_{10.7}(t+i) \; .$$

The daily values of $F_{10.7}$ and Kp, which are available in Reference 3, are incorporated into the model for use in post-flight prediction (section II B deals with extrapolating these values for future predictions).

Small also points out in Reference 2 that the following relationship holds (although he apparently did not use it in formulating his model):

$$\frac{d(\ln \rho)}{d(\ln S)} = \left[3 + 2.5\left(\frac{h - 360}{240}\right) - .5\left(\frac{h - 360}{240}\right)^2\right]\left[\frac{5.6 - \cos \psi'}{6.6}\right] , \qquad (2)$$

where

$\ln \rho$ = natural log of density

$\ln S$ = natural log of S

h = altitude in km

ψ' = geocentric angle between the field point and the center of the diurnal bulge. ψ' of 75 deg represents a mean diurnal effect.

By interpreting the above as

$$\frac{\ln \rho - \ln \rho_o}{\ln S - \ln S_o} ,$$

it follows directly that

$$\rho = \rho_o \left(\frac{S}{S_o}\right)^{\left[3 + 2.5\left(\frac{h - 360}{240}\right) - .5\left(\frac{h - 360}{240}\right)^2\right]\left[\frac{5.6 - \cos\psi'}{6.6}\right]} . \quad (3)$$

To apply the above equation, all that remained was the selection of a realistic density profile (density as a function of altitude) to define ρ_o and the associated value of S_o.

Values from the above equation using various combinations of S_o and ρ_o (reference profiles) were compared to empirical density data. The results of this comparison indicated that two such combinations provide realistic density models. These are the 1959 ARDC density model with an S_o of 220 and the 1962 U. S. Standard density model with an S_o of 200.

Figures 1 and 2 show $\frac{\rho}{\rho_o}$ when $\psi' = 75°$ (mean diurnal effect)

$$\frac{\rho}{\rho_o} = \left(\frac{S}{S_o}\right)^{.81\left[3 + 2.5\left(\frac{h - 360}{240}\right) - .5\left(\frac{h - 360}{240}\right)^2\right]},$$

for various values of S and $S_o = 220$ and $S_o = 200$, respectively.

These two reference atmosphers, the 1959 ARDC and U. S. Standard 1962, are used in the program. Lifetimes may be predicted solely on these models or using these models as a base with corrections for the time frame being considered as discussed above and adjusted with data obtained from flights. The selection of a model for the base is completely arbitrary. Essentially the same density model will result for either base reference. The base model is corrected in the program for solar activity behavior and diurnal effect in the following manner:

$$\rho = \rho_o(R_i') \, D_c \left(\frac{S}{S_o}\right)^K \left\{\frac{1 + .19\left(e^{.0055 R_i'} - 1.9\right)\left(\frac{1 + \cos\psi'}{2}\right)^3}{1 + .19\left(e^{.0055 R_i'} - 1.9\right)\left(\frac{1 + \cos 75°}{2}\right)^3}\right\}, \quad (4)$$

where

$\rho_o(R_i')$ = Density of base reference atmosphere as a function of altitude R_i'. This is assumed to be a diurnal mean atmosphere.

D_c = Altitude dependent correction factor derived from satellite observations (discussed later in some detail).

S_o = Reference index of heating parameter, i. e. , the value of S to which the D_c factor is referenced.

R_i' = Field point altitude above an oblate earth

ψ' = Angle between the field point and the center of the diurnal bulge.

The heating effect on atmospheric density is altitude dependent and the density is greater on the side of the earth toward the sun. The latter effect, the diurnal bulge, is represented by the brackets

$$\left\{ \frac{1 + .19\,(e^{.0055R_i'} - 1.9)\left(\frac{1+\cos\psi'}{2}\right)^3}{1 + .19\,(e^{.0055R_i'} - 1.9)\left(\frac{1+\cos 75^\circ}{2}\right)^3} \right\}$$

and is derived in Reference 4. The formulation given above assumes that the base atmosphere represents a mean dirunal effect so that, when ψ' equals the mean value of 75 degrees the ratio becomes one (1) for any altitude (ψ' is derived below). The variation in the effect of the heating parameter with altitude and position on ρ is represented in the equation (Reference 2) by K, where

$$K = \left[3 + \frac{5}{2}\left(\frac{R_i' - 360}{240}\right) - \frac{1}{2}\left(\frac{R_i' - 360}{240}\right)^2\right]\left[\frac{5.6 - \cos\psi'}{6.6}\right]. \qquad (5)$$

This exponent K is shown in Figure 3 as a function of altitude. The angle ψ' is calculated as follows:

$$\cos\psi' = ll_B + mm_B + nn_B\,, \qquad (6)$$

(l, m, n) = direction cosines of the field point

$$l = \frac{X_s}{R_i}$$

$$m = \frac{Y_s}{R_i}$$

$$n = \frac{Z_s}{R_i} \quad .$$

R_i = Radius vector from earth center to field point. X_s, Y_s, Z_s are the space-fixed components of the position of the vehicle computed as

$$X_s = X' \cos \Omega - Y' \sin \Omega$$

$$Y_s = X' \sin \Omega + Y' \cos \Omega$$

$$Z_s = Z'$$

$$X' = X$$

$$Z' = Y \sin i$$

and

$$X = R_i \cos (\omega + v)$$

$$Y = R_i \sin (\omega + v)$$

i = orbital inclination - angle between the earth equatorial plane and the plane of the orbit

Ω = Right ascension of the ascending node-angle between the intersection of the orbital plane with the earth equatorial plane and the vernal equinox

ω = Argument of perigee-angle between the ascending node and perigee v = True anomaly-angle between perigee and the field point

(l_B, m_B, n_B) = Direction cosines of the diurnal bulge

$$l_B = \sqrt{n_s^2 + l_s^2}\ \cos RA_B \tag{7}$$

$$m_B = \sqrt{n_s^2 + l_s^2}\ \sin RA_B \tag{8}$$

$$n_B = n_s \tag{9}$$

$$\begin{pmatrix} l_s \\ m_s \\ n_s \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & -\sin \epsilon \\ 0 & \sin \epsilon & \cos \epsilon \end{pmatrix} \begin{pmatrix} \cos \lambda_s \\ \sin \lambda_s \\ 0 \end{pmatrix} \tag{10}$$

ϵ = inclination of the ecliptic = 0.4092 rad

λ_s = celestial longitude of the sun

$\lambda_s = [0.017203\ d + .0335 \sin (0.017203\ d) - 1.41]$ rad

d = number of days elapsed since Dec. 31, 1957

$$RA_s = \tan^{-1} \left(\frac{m_s}{l_s} \right)$$

$$RA_B = RA_s - \theta$$

$$\theta_{\text{Radians}} = \frac{\pi}{180} [18.5 + 30e^{K_1} + K_2\sigma + (1 - \sigma^2)]$$

if $\theta > 5$ set $\theta = 5$

$$\sigma = \frac{s - 160}{90}$$

$$K_1 = -.00567\ (R_i - 200) + e^{-.01455}(R_i^{\prime} - 200)$$

$$K_2 = 18.5 + 21.5e^{-.0315(R_i^{\prime} - 200)}\ .$$

B. Verification and Accuracy of Present Density Model From Flight Experience

The effective drag on the Saturn vehicle has been derived at MSFC primarily by the inclusion of drag as an additional unknown which is solved for in a conventional least square differential correction orbit determination program as discussed in Reference 5. The orbit correction program used included an atmospheric drag model which used the 1959 ARDC atmospheric density profile and assumed a constant ballistic factor for the satellite. The drag acceleration A_D on the satellite was calculated as

$$A_D = \frac{1}{2} D_c\ (C_D\ A/M)\ \rho_o v_e^{\,2}\ , \tag{11}$$

where C_D is the drag coefficient, A is the effective cross-sectional area, M is the satellite mass, v_e is the velocity relative to the earth, ρ_o is the reference atmospheric density at the satellite, and D_c is a constant used nominally to compensate for variation of the actual current atmospheric density from the 1959 ARDC density profile. Solutions for D_c were made for SA-5, SA-6 and SA-7 using radar tracking data in the orbit correction process described in Reference 5. Using these data in addition to Gemini and Discoverer data, a correction factor from the 1959 ARDC atmosphere was established as shown in Figure 4. These factors are referenced to an index of heating S of 100 which was applicable during the Saturn I flight time frame. Using these empirically derived solutions in the new jointly developed MSFC/LMSC lifetime program, a comparison of actual to predicted lifetime was made for 39 decayed satellites. The results of this comparison are presented in Table I.

As an indication of the accuracy of the prediction, the ratio of actual lifetime to predicted lifetime is shown in Table I for each case. This ratio

(A/P) appears to be normally distributed, having a mean value of 1.000 and a standard deviation of .082. These results indicate that the representation of solar activity as presented and the corresponding density as a function of time for the altitude region of 350 km and below is indeed valid for the time frame up to 1965. The validity for future years is only as good as the prediction of solar activity.

Similar density models are currently being investigated: one using the 59 ARDC as the reference density model, $S_o = 220$ and DC = 1.0 for all altitudes, and another using the 62 U. S. Standard as the reference density, $S_o = 200$, and DC = 1.0 for all altitudes. These yield essentially the same density as the one previously mentioned ($S_o = 100$, DC shown in Figure 4) in the 100-300 km altitude range and are currently being compared to data obtained in the 490-550 km altitude range from SA-8, SA-9, and SA-10 flights.

C. Prediction of Future Behavior of Parametric Input to the Model

Examination of the history of solar activity indicates that the present cycle need not necessarily follow a course similar to the last cycle. The $\overline{F}_{10.7}$ and $\overline{K}_p$ for the period 1958-1975, which are currently being used in the MSFC computer program, are shown in Figure 5. Figure 6 shows the heating parameter $\overline{S}$ based on the nominal values of $\overline{F}_{10.7}$ and $\overline{K}_p$ given in Figure 5. The seasonal effect $e^{g(t)}$ on S is shown in Figure 7 as a function of the time of the year. The product of these two factors yields the heating parameter S. The $\overline{F}_{10.7}$ and $\overline{K}_p$ values which occurred earlier than mid-1965 are averages of the actual recorded values, while those from mid-1965 to 1975 are based upon certain predictions and assumptions.

The extrapolation of $\overline{F}_{10.7}$ was based on the following assumptions:

1. $\overline{F}_{10.7}$ and Zurich smoothed sunspot number, R, are well correlated and the regression line is given by Reference 6 as

$$\overline{F}_{10.7} = 50 + .967\ R \qquad (\overline{F}_{10.7} > 100)$$

and

$$\overline{F}_{10} = 68 + .607\ R \qquad (F_{10.7} < 100)\ .$$

2. The beginning of the new cycle (minimum $\overline{F}_{10.7}$) was in mid-1964.

3. The new sunspot cycle was assumed to have the shape and duration of the mean of sunspot cycles 8 through 18.

4. The magnitude of the sunspot maximum, R_M, was assigned the value 150, based on the following:

(a) All predictions of R_M thus far found in the literature agree that R_M will be less than 150 (except one which indicates that $R_M < 160$).

(b) In predicting lifetimes for mission planning, it is generally better to underpredict lifetime than to overpredict lifetime.

(c) While most authors predict a value of R_M considerably lower than 150, the likelihood of this is questionable in view of the fact that the preceding cycle had the highest R_M ever recorded and natural phenomena tend not to change drastically from one occurrence to the next.

5. The time lapse of four years from minimum to maximum sunspot number for the new cycle is the same as that for the mean of cycles 8 through 18. The relationships of consideration 1 above were used to compute $\overline{F}_{10.7}$ from the R values obtained by adjusting the mean of cycles 8 through 18 by a proportionality factor which forced R_M to be 150.

The 3σ upper bound curve of $\overline{F}_{10.7}$, $\overline{F}_{10.7}$ (max), was drawn by fairing straight line segments through points computed by the following formula:

$$\overline{F}_{10.7}\,(\text{max}) = \overline{F}_{10.7} + .2(\overline{F}_{10.7} - 60) + 4(\text{year}-1964.5) \quad . \qquad (12)$$

This formula was chosen to represent the increasing uncertainty in $\overline{F}_{10.7}$ as $\overline{F}_{10.7}$ increases and as time increases. The weighting factors were chosen so as to yield

$$\overline{F}_{10.7}\,(\text{max}) = 244 \text{ at } 1968.5.$$

The 244 maximum of the previous cycle was chosen as an absolute maximum for the new cycle. The 3σ lower bound was derived similarly from the lowest recorded cycle.

TABLE I. DECAYED SATELLITE ANALYSIS

	Name	t_s (year)	h_p (km)	A (days)	P (days)	R (A/P)
1	58 DELTA 2	1959.158	207	404.0	364.0	1.11
2	58 DELTA 2	1959.725	199	197.7	179.5	1.10
3	58 ZETA	1958.966	175	33.6	33.2	1.01
4	59 GAMMA	1959.287	257	11.2	10.7	1.05
5	59 EPSILON	1959.621	215	43.4	39.9	1.09
6	59 ZETA	1959.637	218	60.7	51.6	1.18
7	59 LAMBDA	1959.889	187	108.3	110.0	0.98
8	59 EPSILON 2	1960.125	219	362.0	355.5	1.02
9	60 DELTA	1960.294	173	9.83	10.6	0.93
10	60 THETA	1960.615	256	95.0	106.6	0.89
11	60 OMICRON	1960.880	183	42.9	42.7	1.01
12	60 SIGMA	1960.960	251	107.4	113.0	0.95
13	60 TAU	1960.973	195	32.9	33.8	0.97
14	61 EPSILON	1961.135	298	525.5	575.3	0.91
15	61 ZETA	1961.146	252	422.6	460.7	0.92
16	61 LAMBDA 1	1961.272	297	372.9	384.8	0.97
17	61 LAMBDA 2	1961.321	220	391.2	429.6	0.91
18	61 XI	1961.466	224	23.2	27.8	0.83
19	61 PI	1961.537	233	133.9	143.4	0.93
20	61 ALPHA BETA	1961.745	243	27.3	27.3	1.00
21	61 ALPHA GAMMA	1961.803	234	24.9	25.0	1.00
22	61 ALPHA EPSILON	1961.855	246	394.3	413.0	0.95
23	61 ALPHA KAPPA	1961.973	248	76.8	74.6	1.03
24	62 RHO	1962.356	203	15.6	13.1	1.19
25	62 CHI	1962.435	213	20.6	19.5	1.06
26	62 ALPHA GAMMA	1962.496	209	76.2	79.6	0.96
27	62 SIGMA	1962.556	323	492.0	484.2	1.02
28	62 ALPHA ETA	1962.575	204	16.5	15.8	1.04
29	62 ALPHA THETA	1962.594	206	18.6	18.2	1.02
30	62 ALPHA KAPPA	1962.602	208	18.3	20.0	0.92
31	62 ALPHA SIGMA	1962.673	176	6.9	7.1	0.97

t_s = initial time, h_p = initial perigee altitude, A = actual lifetime, P = predicted lifetime.

TABLE I. (Concluded)

	Name	t_s (year)	h_p (km)	A (days)	P (days)	R (A/P)
32	62 ALPHA CHI	1962.728	211	56.9	56.8	1.00
33	62 BETA EPSILON	1962.797	220	29.5	26.8	1.10
34	62 BETA OMICRON	1962.865	210	20.1	20.4	0.99
35	62 BETA SIGMA	1962.928	134	3.6	3.85	0.94
36	62 BETA PHI	1962.980	200	16.1	15.15	1.06
37	64 (GEMINI)	1964.274	164	4.2	5.09	0.86
38	64 (SA-6)	1964.408	182	3.2	3.21	1.00
39	64 (SA-7)	1964.716	185	3.8	3.3	1.15

t_s = initial time, h_p = initial perigee altitude, A = actual lifetime, P = predicted lifetime.

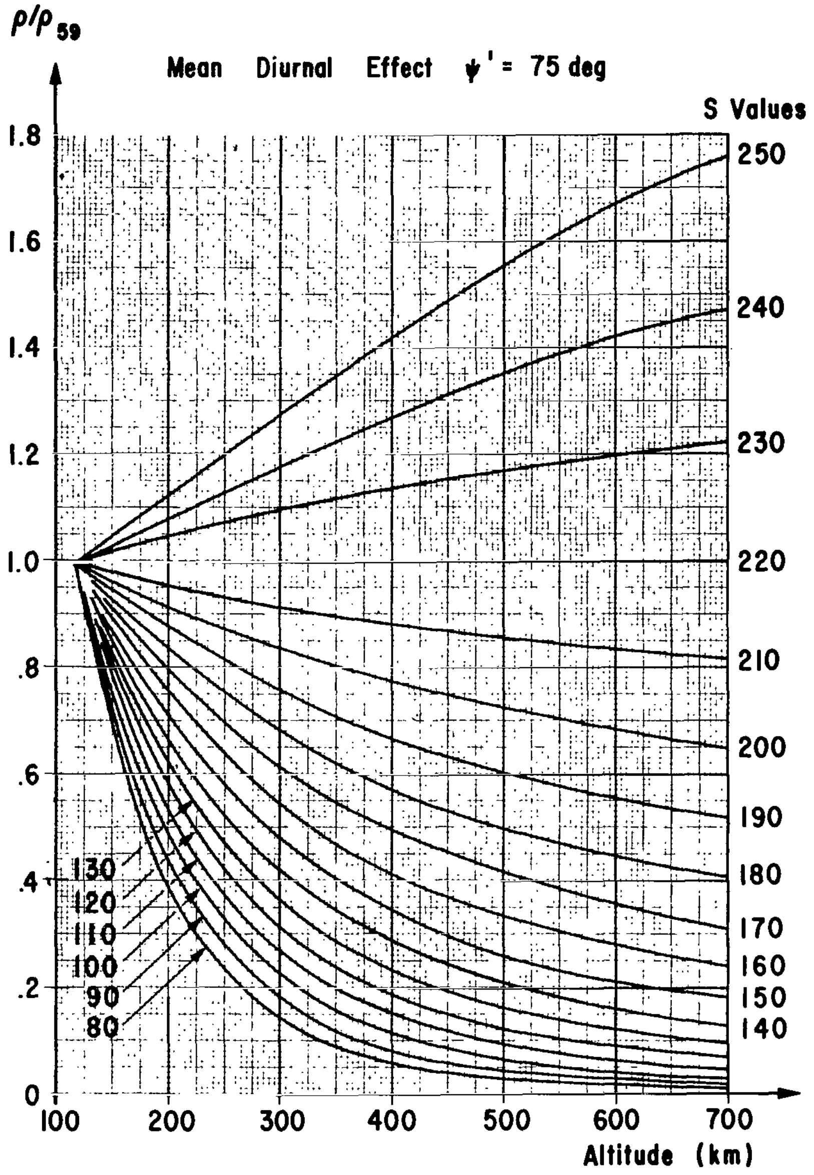


FIGURE 1. RATIO OF DENSITY FOR VARIOUS VALUES OF S TO THE 59 ARDC DENSITY

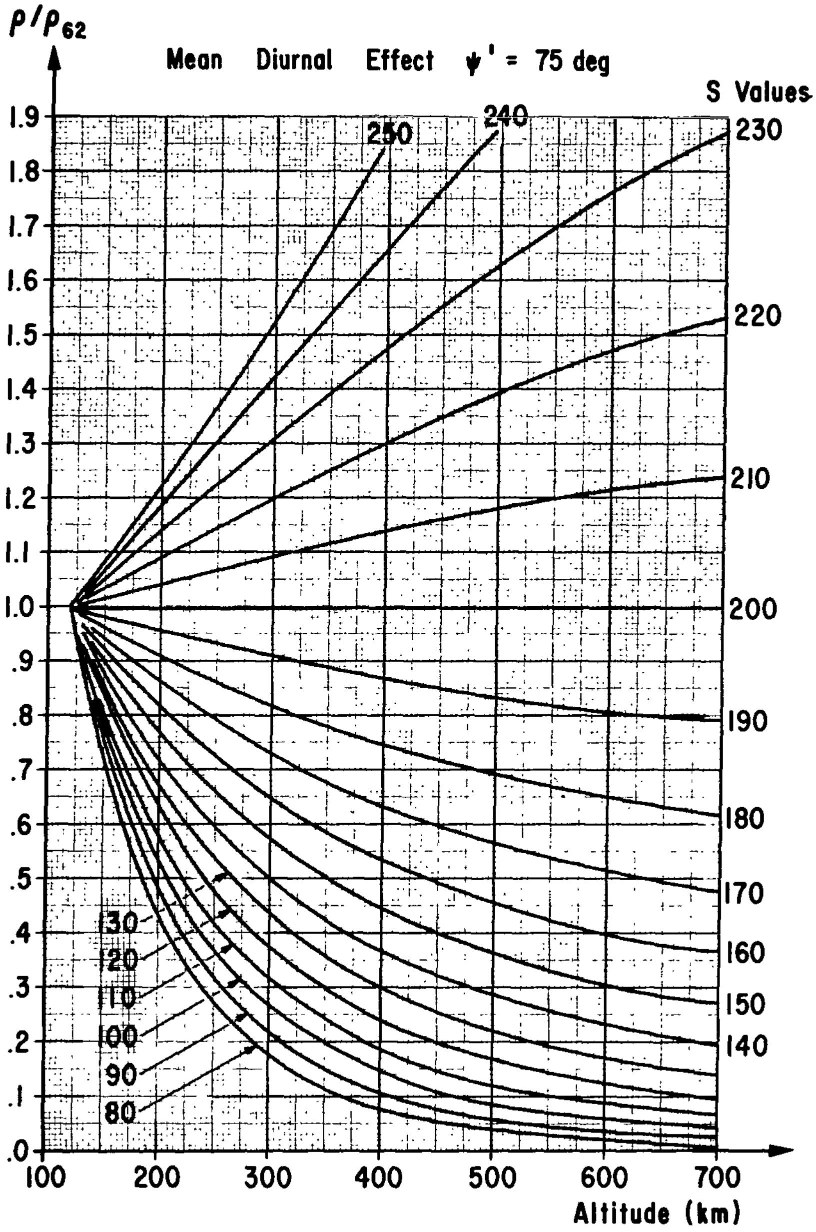


FIGURE 2. RATIO OF DENSITY FOR VARIOUS VALUES OF S TO THE 62 U. S. STANDARD DENSITY

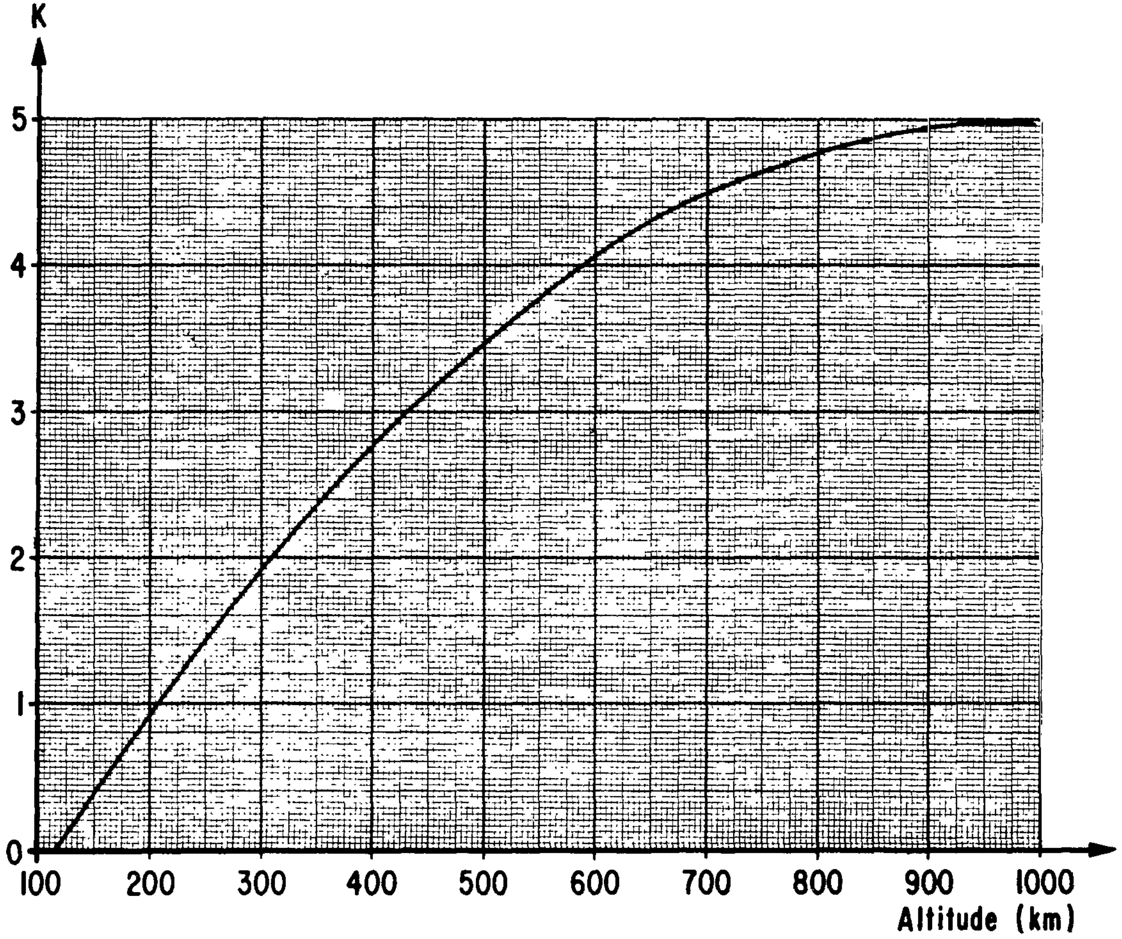


FIGURE 3. EXPONENTIAL ATMOSPHERIC WEIGHTING FACTOR (K) FOR EFFECT OF HEATING

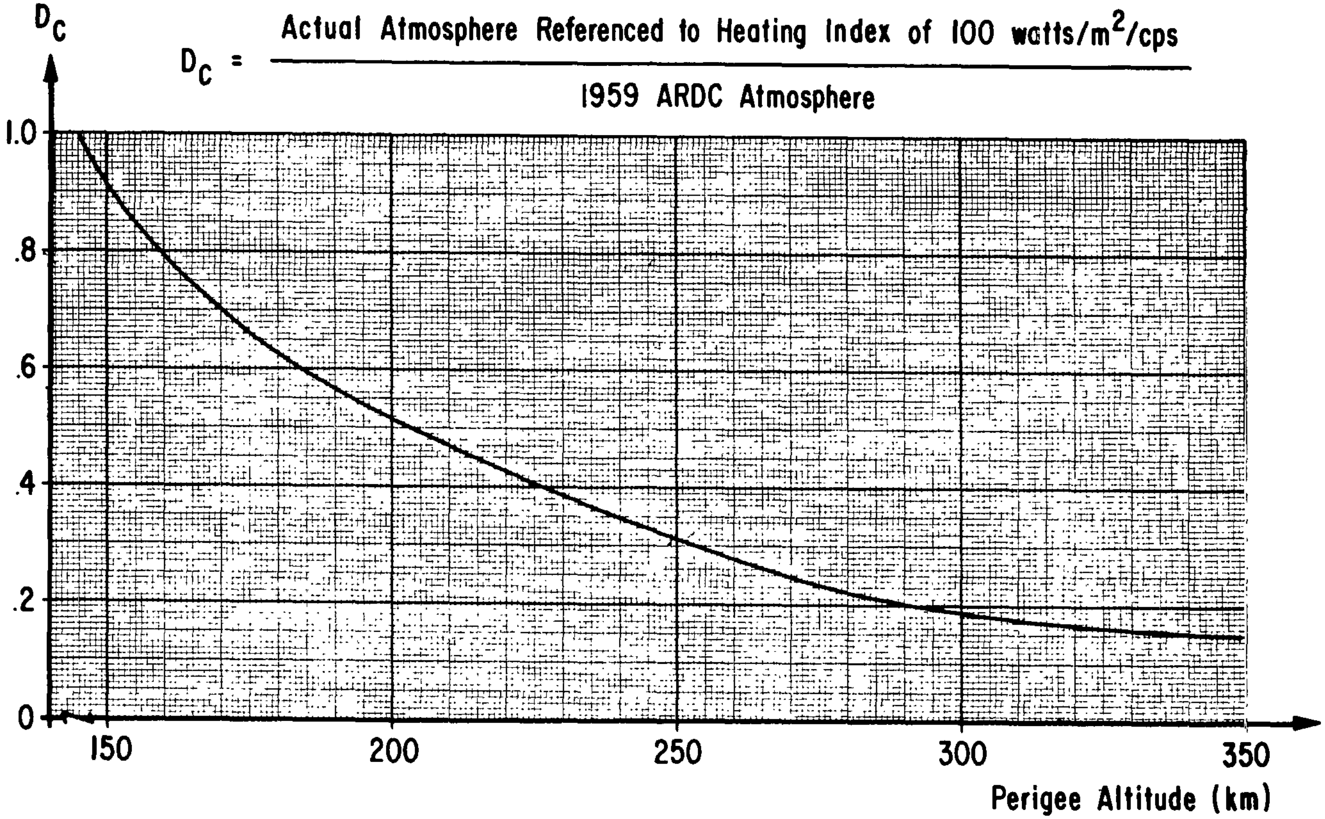


FIGURE 4. CORRECTION FACTOR FOR DENSITY

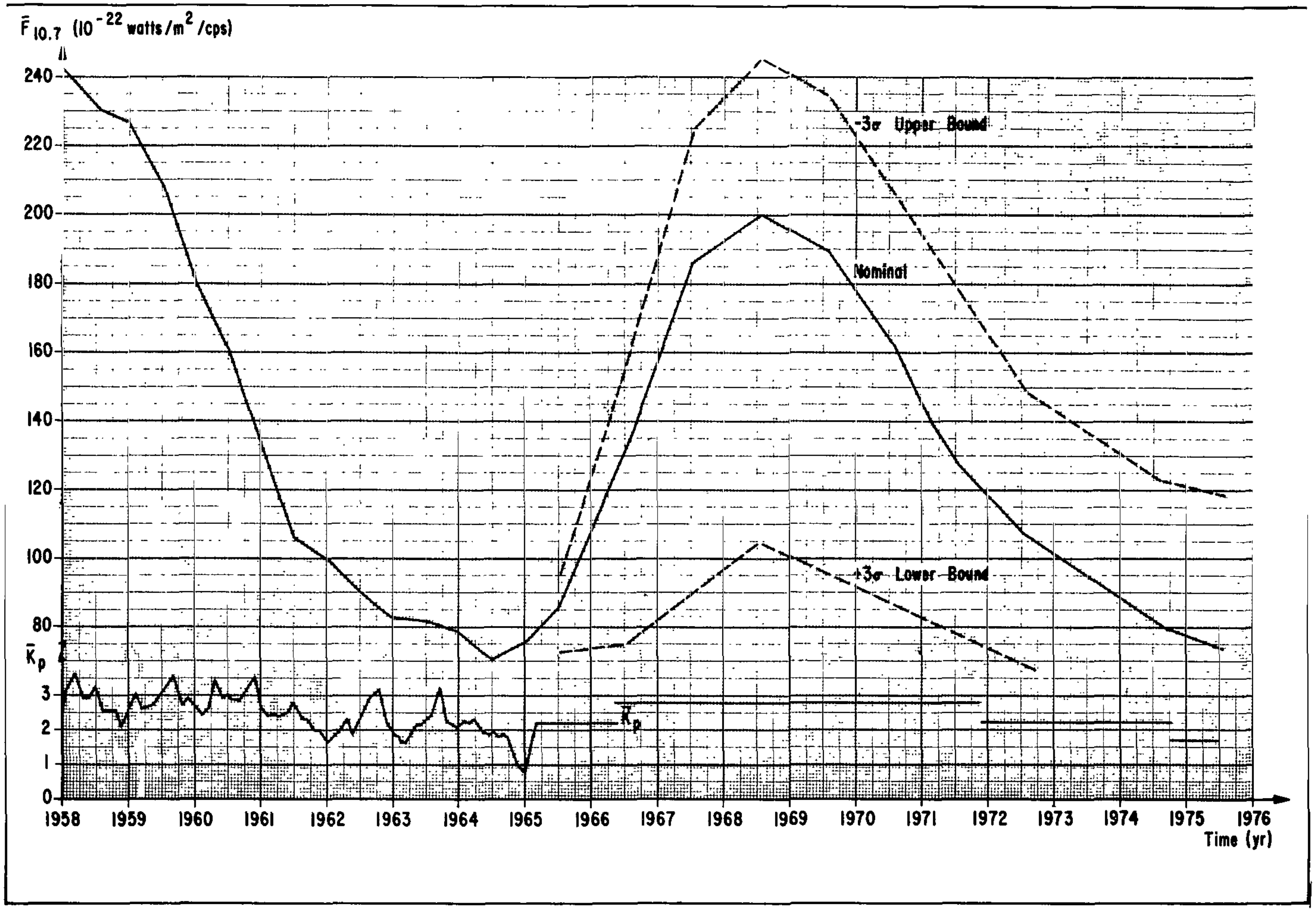


FIGURE 5. CURRENTLY DEFINED $\overline{F}_{10.7}$ AND $\overline{K}_p$ VERSUS TIME

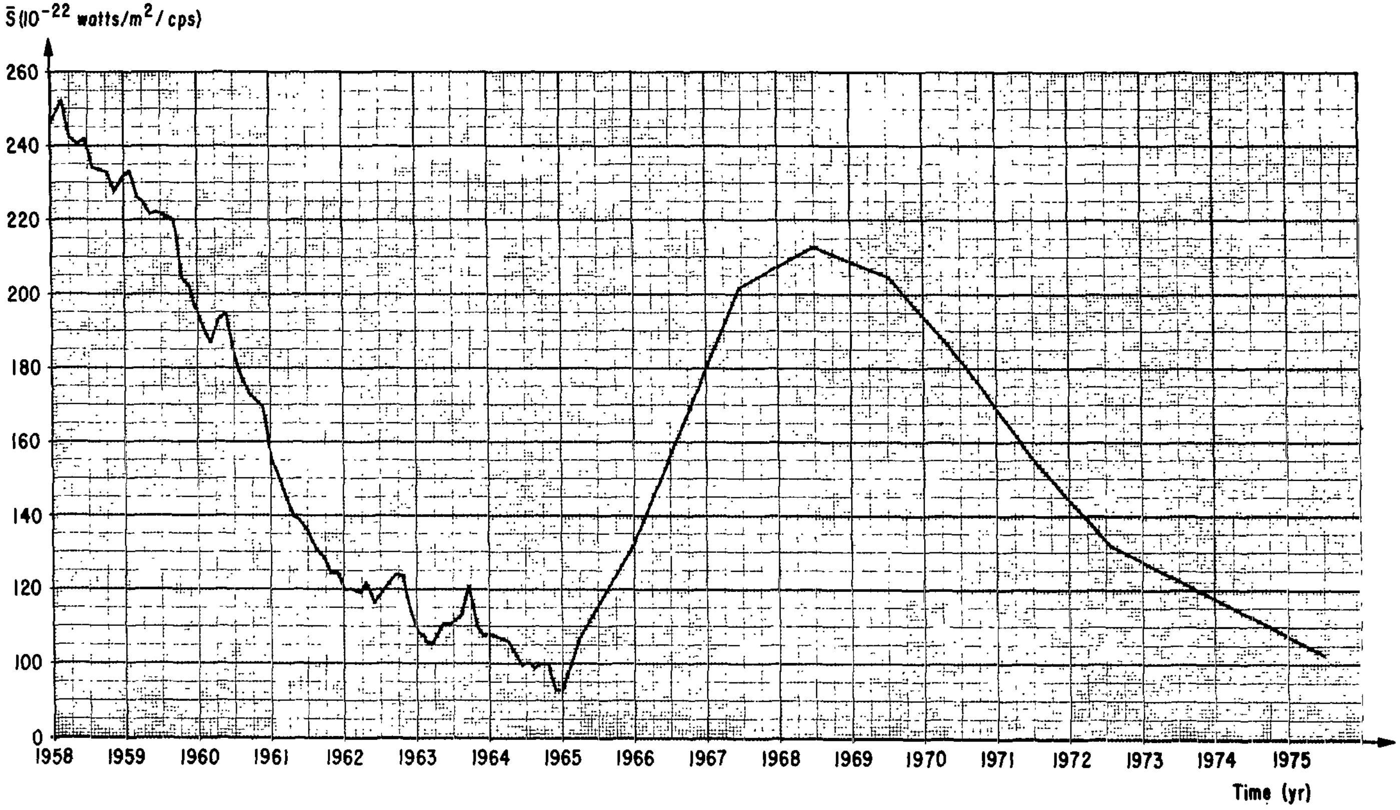


FIGURE 6. CURRENTLY DEFINED $\overline{S}$ VERSUS TIME

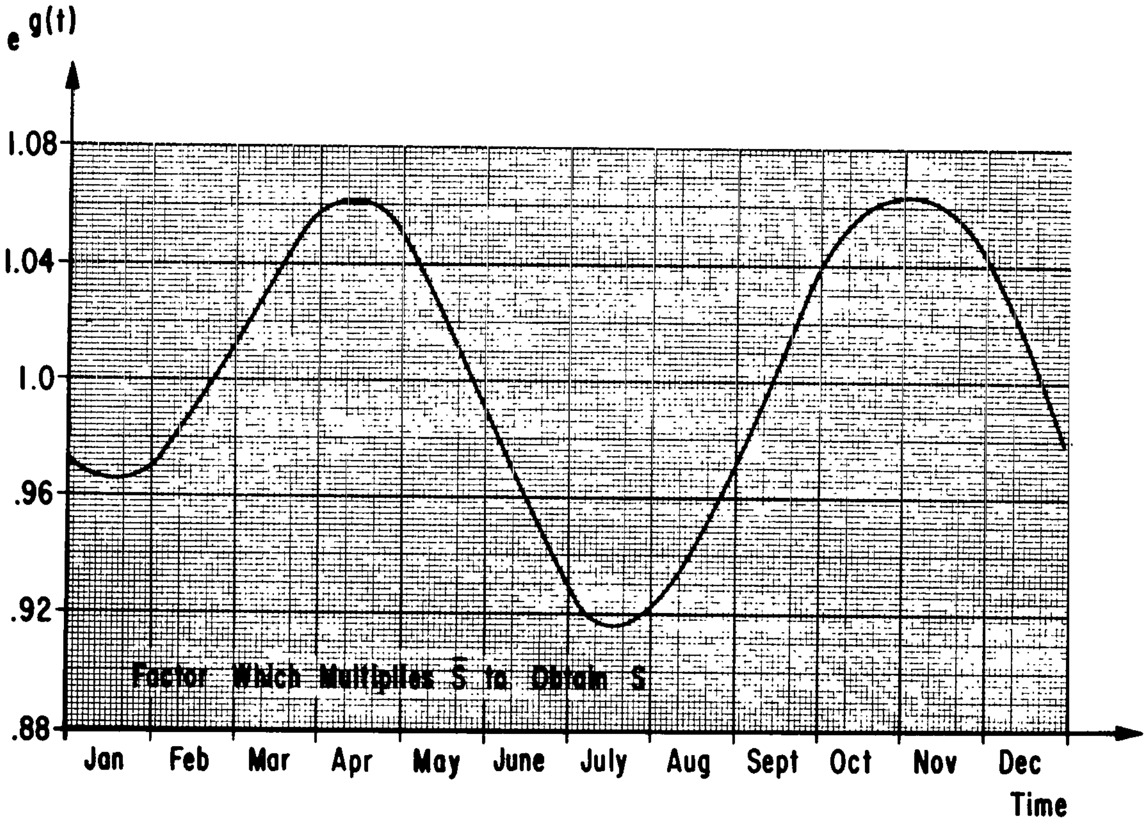


FIGURE 7. SEMIANNUAL EFFECT ON ATMOSPHERIC HEATING

III. LIFETIME PROGRAM

A. NOTATION

This list defines the equation symbols in terms of the computer input language used.

Equation Symbol	Program Symbol	Definition	Units
A_i, A_0	APO	apogee radius	km
$\dot{A}_i$	ADOT	time rate of change of apogee radius	km/day
$\dot{A}'_i$	ADOTP	$M(t)\dot{A}_i$	kg-km /day
A_{N_j}	AN	interpolated value of apogee at time T_{A_j} (j = 1...5)	km
A_{0_i}	ARAA	effective drag area of the orbiting vehicle	m^2
CD'_i	CDP	coefficient of drag as an altitude function	non-dimensional
CN_i	CNN	coefficient of drag as an attitude function	"
D	DD	coefficient of the 4th zonal harmonic of the earth's gravitational potential	"
F	F	reciprocal of the flattening of the earth (298.3)	"
$F_{10.7}$	FTEN	daily 10.7 cm solar flux	10^{-22} watts/m^2 /cycle /sec
$\bar{F}_{10.7}$	FTENB	yearly mean values of $F_{10.7}$	"
H	HH	coefficient of the 3rd zonal harmonic of the earth's gravitational potential	non-dimensional
J	JJ	coefficient of the 2nd zonal harmonic of the earth's gravitational potential	"
K	KERTH	earth's gravitational constant	km^3/sec^2

(Continued)

Equation Symbol	Program Symbol	Definition	Units
K_P	AP	daily mean values of geomagnetic index	non-dimensional
$K\rho$	XK	$\partial \log_e \rho / \partial \log_e S$	"
$M(t)$	MT	mass of orbiting vehicle at time t	kg
PD_i	PDI	anomalistic period· (time between two successive perigee passages)	min
P_i, P_0	PERI	perigee radius	km
$\dot{P}_i$	PDOT	time rate of change of perigee radius	km/day
$\dot{P}_i'$	PDOTP	$M(t)\dot{P}_i$	kg-km /day
P_{N_k}	PN	interpolated value of perigee at time T_{P_k} (k = 1...5)	km
R_{AB}	RAB	right ascension of the center of the diurnal bulge	deg
R_{AS}	RAS	right ascension of the sun	deg
R_E	AE	earth's equatorial radius	km
R_i	RI	radius to probe	km
R_i'	RIP	altitude to probe	km
RPA_i	RPAI	sub-perigee apogee earth radius	km
S	SS	current index of total heating of the atmosphere	non-dimensional
S_0	SO	reference index of total heating of the atmosphere	"

(Continued)

(Continued)

Equation Symbol	Program Symbol	Definition	Units
$T_{A_{j = 1...5}}$	INTERA	times at which apogee interpolations are performed	days
$T_{P_{k = 1...5}}$	INTERP	times at which perigee interpolations are performed	days
VP_i	VPI	earth-fixed velocity at perigee	km/sec
X_s, Y_s, Z_s	XS, YS, ZS	space-fixed ephemeris components of the position of the satellite (see Section II-B-2)	km
a_i, a_0	AI	semi-major axis of the ellipse	km
$\dot{a}_i$	SADOTI	time rate of change of semi-major axis	km/day
d	XDATE	number of days elapsed since 31 December 1957	days
e_i, e_0	EI	eccentricity	non-dimensional
i	INC	inclination	deg
l, m, n	XL, XM, XN	direction cosines of the satellite	non-dimensional
l_B, m_B, n_B	XLB, XMB, XNB	direction cosines of the center of the diurnal bulge	"
l_s, m_s, n_s	XLS, XMS, XNS	direction cosines of the vector to the sun	"
n_i, n_0	NI	mean motion	deg/day
t	TTT	universal time	hrs
t_i	TIME	time in orbit	days
t_{n_i}	REVOL	number of revolutions made by the satellite	non-dimensional

(Continued)

(Continued)

Equation Symbol	Program Symbol	Definition	Units
Δt_i	DTI	change in time for one apogee step	days
θ	THETA	sidereal time	deg
θ_ρ	XLAG	lag angle between earth-sun line and the diurnal bulge	deg
Ω_i, Ω_0	CAPW	right ascension of the ascending node	deg
$\dot{\Omega}_i, \dot{\Omega}_0$	CAPID	time rate of change of ascending node	deg/day
α_i	ALPHA	angle of attack of the satellite	deg
δA	DA	apogee integration step size	km
ε	ECLIPT	obliquity of the ecliptic	deg
λ_s	XLAMS	celestial longitude of the sun	deg
ν_i, ν_0	E	true anomaly	deg
ρ	RHO	atmospheric density	kg/m^2
ψ'	COSPP	angle between the center of the diurnal bulge and the satellite	deg
ω_e	OMEGA	rotational velocity of the earth	deg/hr
ω_i, ω_0	SMAW	argument of perigee	deg
$\dot{\omega}_i, \dot{\omega}_0$	SMAID	time rate of change of argument of perigee	deg/day

Subscripted symbols such as ω_i, Ω_i, A_i denote values at the i^{th} apogee step, whereas ω_0, Ω_0, A_0 denote initial values.

B. Program Description and Flow of Computations

The Satellite Orbit Decay and Orbital Lifetime Program has three phases: (1) a control phase that controls the sequence of events in the entire program; (2) a transformation phase that accepts input in any of eight coordinate systems and transforms to the remaining seven; and (3) a lifetime phase that computes the decay history and lifetime of the orbiting body. The deck is programmed in Fortran IV language for the IBM 7094 computer.

1. Control Phase. This phase determines the route to be taken and the values to be used in computing both the decay history and lifetime. The Control Phase first calls the input routine "MAVRIK" which reads one or more data cards for the initial case. The only card that is always required for execution of the program is the one that defines either the satellite's initial position and velocity or orbit elements in one of eight optional coordinate systems. However, the actual breakdown of these options leads to more ways of inputting these initial orbit parameters than eight. The following outline elaborates on this.

The user can input in any one of eight coordinate systems, two of which contain three coordinate "subsystems"* each:

1. Earth-fixed plumbline (position and velocity)
2. Earth-fixed ephemeris (position and velocity)
3. Space-fixed ephemeris (position and velocity)
4. Space-fixed geographic (position and velocity)
5. Earth-fixed geographic (position and velocity)
6. Platform (position and velocity)
7. Osculating orbital elements:
    (a) Semi-major axis, eccentricity, inclination, right ascension of ascending node, argument of perigee, and true anomaly (alphanumeric code OET).

---

* In effect, there are 12 coordinate systems.

(b) Semi-major axis, eccentricity, inclination, right ascension of ascending node, argument of perigee, and eccentric anomaly (OEE).

(c) Semi-major axis, eccentricity, inclination, right ascension of ascending node, argument of perigee, and mean anomaly (OEM).

8. Mean orbital elements:

(a) Same as (7) except mean elements replace osculating elements. Corresponding alphanumeric codes are MOT, MOE, and MOM.

The coordinate systems described previously are explained in detail in Section III. B. 2.

Each of the six coordinate "subsystems" in (7) and (8) above can be input in one of four optional ways:

A Apogee and perigee radius.

B Apogee and perigee radius, inclination, right ascension of ascending node, (true, eccentric, or mean) anomaly, argument of perigee, universal time, sidereal time.

C Semi-major axis and eccentricity.

D Semi-major axis, eccentricity, inclination, right ascension of ascending node, (true, eccentric, or mean) anomaly, argument of perigee, universal time, and sidereal time.

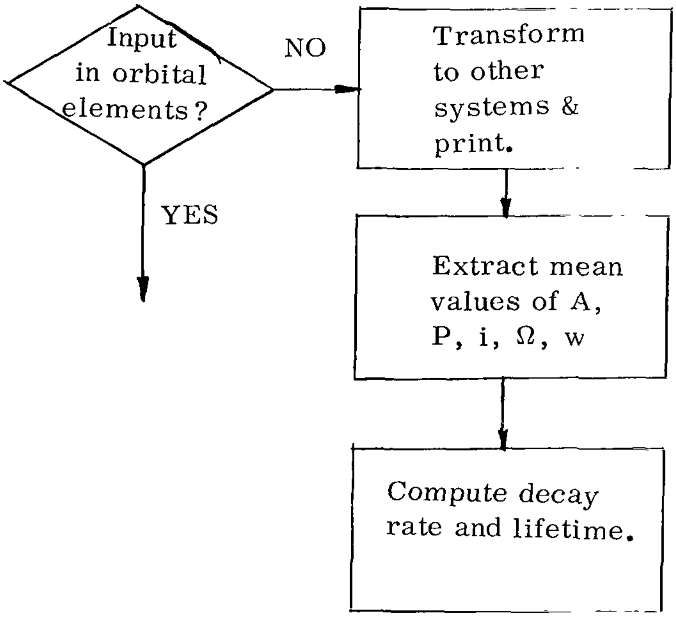
In option A and C the apogee radius, perigee radius, semi-major axis, and eccentricity are treated as mean elements whether they are mean or osculating. This is the case since the other elements required for the transformation between osculating and mean elements are not given. Nominal values of inclination, right ascension of ascending node, (true, eccentric, and mean) anomaly, argument of perigee, universal time, and sidereal time are built into the program. For orbits having elements grossly different from these, use options B or D. Options A and C should be used only if a bare minimum of information is available.

After calling "MAVRIK," the control phase logically decides whether input is position and velocity or orbit elements. If it decides that an orbital element system was not used, then the program proceeds immediately to the transformation phase; if an orbital element was used, further decision-making is required by the control phase. In all cases except options A and C of the orbital element system the program uses the transformation phase. However, when either options A or C are chosen the transformation phase is not used since it is unlikely that one would desire transformation based on "nominal" values of i, Ω, w, etc.

a. Orbital Element System Not Used. If the coordinate system input is other than orbital elements, such as earth-fixed geographic (EFG), then the following events occur:

1. Transform EFG to the remaining seven systems.
2. Print transformations.
3. Extract from mean orbital elements the following: apogee radius, perigee radius, inclination, right ascension of ascending node, and argument of perigee. These elements are then ready for use in the lifetime phase.

Logical Flow:



b. Orbital Element System Is Used. If the coordinate system input code contains the alphanumeric code for one of the Orbital Element systems, OET, OEE, OEM, MOT, MOE, MOM, then one of four input options is available. Consider the case:

```
        1       2       3       4       5       6       7       8
OET = ++. ++) ++. ++) ++. ++) ++. ++) ++. ++) ++. ++) ++. ++) ++. ++)   .
```

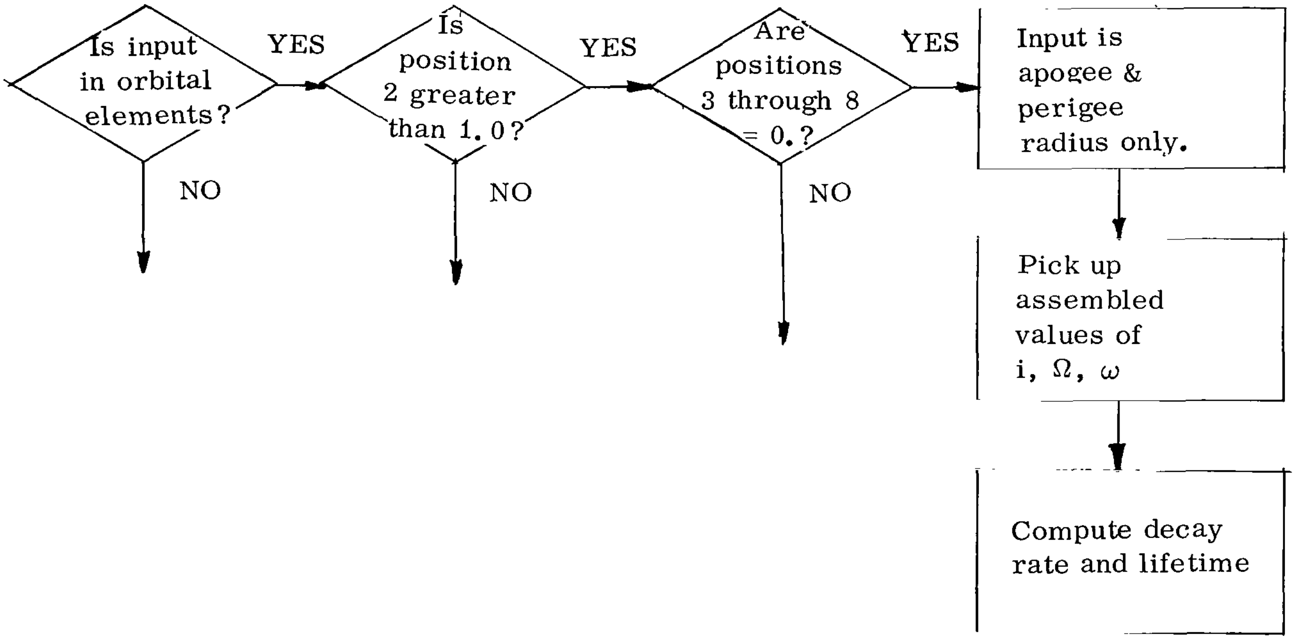
Option A

If the value loaded in position 2 of this input is greater than 1. 0, then positions 3 through 8 are tested. If all these positions contain zeroes then the input is assumed to be in the form:

OET = Apogee radius) perigee radius).

These two values along with the assembled "nominal" values for inclination, right ascension of ascending node, and argument of perigee are ready for use in the lifetime phase.

Logical Flow:



Option B

If any of positions 3 through 8 are non-zero, then the input is assumed to be in the form:

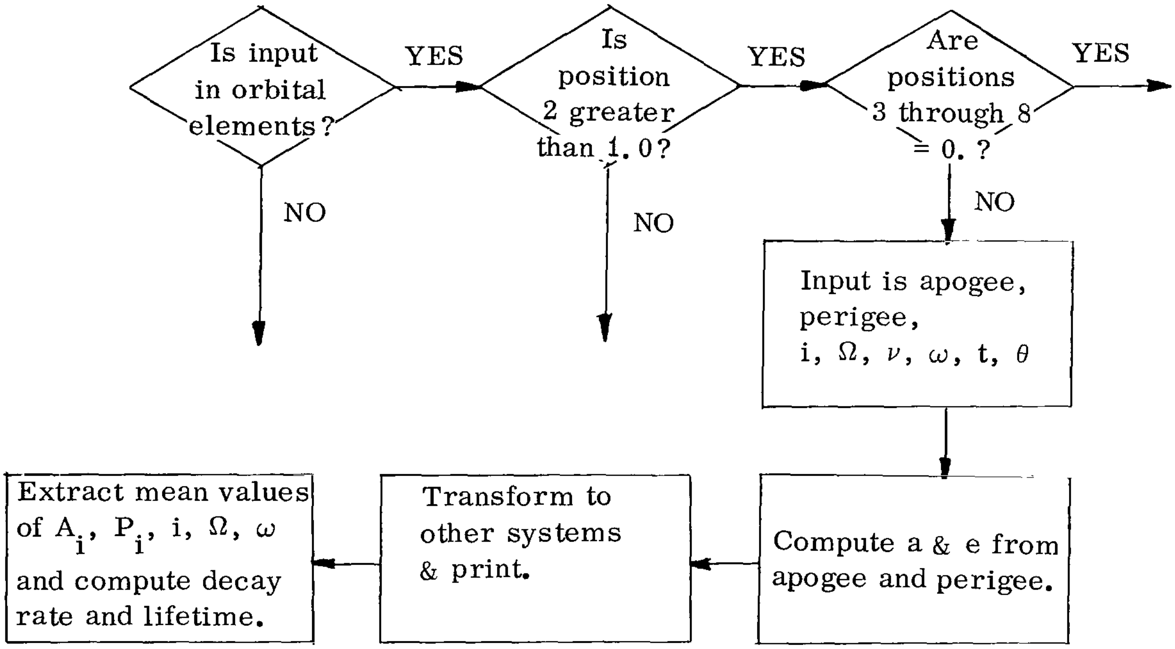
OET = apogee radius) perigee radius) inclination) right ascension of node) true anomaly) argument of perigee) universal time) sidereal time).

Semi-major axis and eccentricity are computed from the input values of apogee and perigee radius:

$$a_o = \frac{A_o + P_o}{2.} \qquad e_o = \frac{A_o - P_o}{A_o + P_o} \quad .$$

These elements are now in the format required for transformation not only to mean elements for use in the lifetime phase, but also to the elements in the remaining coordinate systems for display in the printout.

Logical Flow:



Option C

If position 2 is less than 1. 0, then positions 3 through 8 are tested. If these positions are all zero then the input is in the form:

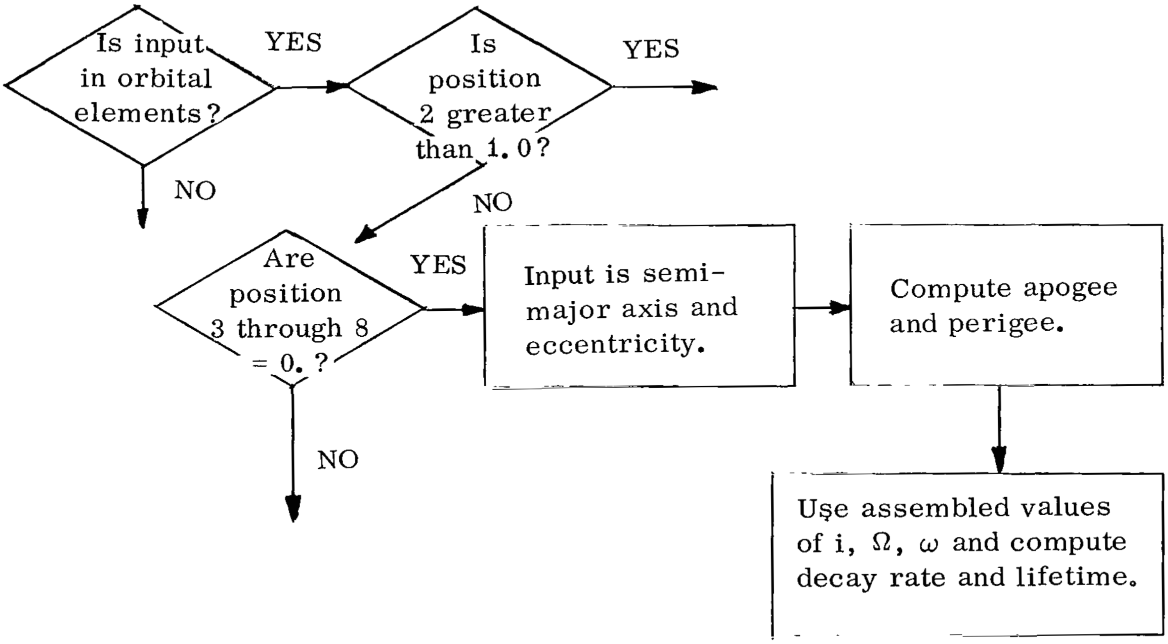
OET = semi-major axis) eccentricity).

From these two values, apogee and perigee radius are calculated:

$$A_o = a_o (1. + e_o) \qquad P_o = a_o (1. - e_o)$$

These values are used along with the assembled "nominal" values of inclination, right ascension of ascending node, and argument of perigee in the lifetime phase.

Logical Flow:



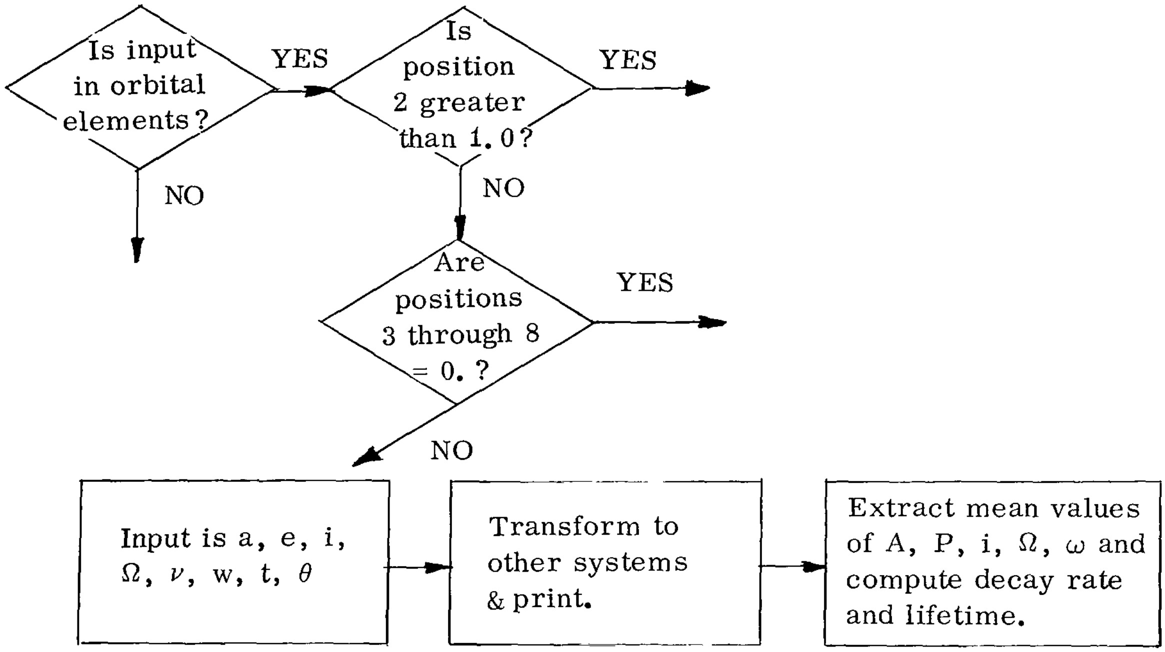
Option D

If position 2 is less then 1.0 and if any of positions 3 through 8 are nonzero then the input is in the form:

OET = semi-major axis) eccentricity) inclination) right ascension of ascending node) true anomaly) argument of perigee) universal time) sidereal time).

These elements are in the format required for transformation not only to mean elements for use in the lifetime phase, but also to the elements in the remaining coordinate systems for display in the printout.

Logical Flow:



2. Transformation Phase. The transformation phase accepts input in any one of the eight aforementioned coordinate systems, performs the required transformations, and outputs in the remaining seven. All programming is done in double precision. The transformation subroutine is a general purpose one used in other computer programs. A detailed description of this phase of the program is not included in this writeup, but may be obtained from the author. A brief description of each coordinate system follows:

a. Earth-Fixed Plumbline (EFP)

XEP, YEP, ZEP, DXEP, DYEP, DZEP

A right-handed Cartesian coordinate system with its origin at a point on the surface of the earth, specified by geodetic coordinates ϕ_o, and λ_o. The Z-axis points in the direction of the local geodetic vertical (plumbline). The X-Y plane is tangent to the geodetic ellipsoid with the X-axis pointing along a specified azimuth defined as (KAPPA) (normally the firing direction). This system is completely earth-fixed.

b. Earth-Fixed Ephemeris (EFE)

XE, YE, ZE, DXE, DYE, DZE

A right-handed geocentric Cartesian coordinate system. The Z-axis points north along the axis of rotation of the earth (through the north pole). The X-Y plane is the earth equatorial plane, and the X-axis points through the Greenwich meridian of longitude. The system is completely earth-fixed.

c. Space-Fixed Ephemeris (SFE)

XS, YS, ZS, DXS, DYS, DZS

A right-handed geocentric Cartesian coordinate system. The Z-axis points north along the axis of rotation of the earth (through the north pole). The X-Y plane is the earth equatorial plane, and the X-axis is collinear with and directed toward the vernal equinox of date (i.e., at time t). At time t = 0, the Greenwich Hour Angle equals zero.

d. Space-Fixed Geographic (SFG)

R_s, ψ_s, λ_s, V_s, α_s, ϵ_s

A right-handed system containing an earth-fixed position vector and a space-fixed velocity vector. The position vector is specified by geocentric earth-fixed spherical coordinates: radius, R_s, geocentric latitude, ψ_s, with respect to the earth equatorial plane; and geographic longitude, λ_s, measured positive eastward from the Greenwich meridian in the earth equatorial plane. The velocity vector is referenced to a fundamental plane tangent to the sphere of radius R_s, perpendicular to R_s, and with origin at the point of tangency. The velocity vector is specified by space-fixed velocity magnitude, V_s; elevation angle, ϵ_s, with respect to the fundamental plane; and azimuth α_s, the angle between the projection of the velocity vector in the fundamental plane and the north vector in that plane.

e. Earth-Fixed Geographic (EFG)

R_e, ψ_e, λ_e, V_e, α_e, ϵ_e

The right-handed (nonrectangular) system with its origin at the center of the earth. The position vector is defined by R_e, ψ_e, and λ_e which are the same as R_s, ψ_s, and λ_s in the space-fixed geographic system. The earth-fixed velocity vector is defined by V_e, the earth-fixed velocity magnitude, α_e the earth-fixed azimuth of the velocity vector, and ϵ_e the earth-fixed elevation of the velocity vector. This system is completely earth-fixed. V_e, α_e, and ϵ_e are defined the same as V_s, α_s, and ϵ_s except they are earth-fixed and not space-fixed.

f. Platform System (PLT)

XPL, YPL, ZPL, XDPL, YDPL, ZDPL

The platform coordinate system is defined such that it coincides with the earth-fixed plumbline system until the time of first motion of the vehicle. At the instant of first motion, the system becomes space-fixed and is a space-fixed plumbline system with its origin centered at the launch pad at the time of first motion. Gravitational effects on the position and velocity component at the transformation time are accounted for. The system is a right-handed rectangular coordinate system. The Y-axis points in the direction of the local geodetic vertical (plumbline). The X-Y plane is tangent to the geodetic ellipsoid and at the time of first motion, the X-axis points along a specified azimuth defined as KAPPA (normally the firing direction). The system is then completely space-fixed.

g. Osculating Orbital Elements ($\bar{\text{O}}$ET), ($\bar{\text{O}}$EM), ($\bar{\text{O}}$EE)

a, e, i, Ω, ω, (ν, E or M)

The orbital element system is defined by six elements of the two-body ellipse with the reference body being determined by the body constants used, normally those of the earth. The elements are the semi-major axis (a) of the ellipse; the eccentricity (e); the inclination of the orbital plane to the equatorial plane (i); the right ascension of the ascending node (Ω); measured eastward in the equatorial plane from the vernal equinox to the ascending node of the orbit; the argument of perigee or the angle between the ascending node and the perigee (ω); and the angular position of the satellite defined by either true (ν), eccentric (E) or mean (M) anomaly.

h. Mean Orbital Elements (M$\bar{\text{O}}$T), (M$\bar{\text{O}}$M), (M$\bar{\text{O}}$E)

$\bar{a}$, $\bar{e}$, $\bar{i}$, $\bar{\Omega}$, $\bar{\omega}$ ($\bar{\nu}$, $\bar{M}$ or $\bar{E}$)

The mean orbital elements are defined as the osculating orbital elements with either or both the long and short periodic variation due to the earth oblateness removed.

Equation defining the short-period variations in orbital elements contain trignometric functions in argument of latitude or in one of the anomalies; hence, they have periods equal to or less than one orbital period. Expressions for the long-period variations, on the other hand, contain trignometric functions in argument of perigee and hence have periods much larger than one orbital period. These long-period terms may contain some short-period terms also.

Note that universal time and sidereal time are common to all eight coordinate systems.

3. Lifetime Phase. The initial mean orbital elements A_o, P_o, i_o, Ω_o, ω_o, are input to the orbit lifetime-decay computation phase from the control phase. At the initial point the instantaneous apogee and perigee rates of change (A_o, P_o)

are evaluated using Simpson's numerical integration technique. Integration steps in true anomaly are taken from 0 to 360° , completing one orbital revolution, and the mean rates of change over the revolution evaluated. The effective drag coefficient, area, and atmospheric density are required input at each integration step. The orbital decay is thus evaluated at successive apogee steps in the following manner. For successive apogee (A) values taken in increments of dA, the corresponding perigee (P) values are determined, the rates of change $\dot{A}$ and $\dot{P}$ are evaluated and the resulting lifetime calculated using Runge-Kutta integration of $\dot{A}$ and the orbital mass function. More detailed explanations are given in the following subsections.

a. Orbital Parameter Computations

(1) Parameters that are constant at the i^{th} apogee integration step for all ν values.

Units

$$a_i = \frac{1}{2}(A_i + P_i) \qquad \text{km}$$

$$e_i = (A_i - P_i)/(A_i + P_i)$$

$$R_{PA_i} = R_E\left[1 - (\sin^2 i_o \sin^2 \omega_i)/F\right] \qquad \text{km}$$

where i_o is initial inclination.

$$n_i = (K/a_i^3)^{\frac{1}{2}}\left[1 + J\,(R_E/a_i)^2\, b_1\,(1 - e_i^2)^{-\frac{3}{2}}\right]\left[C\right] \qquad \text{deg/day}$$

where

$$b_1 = 1 - \frac{3}{2}\sin^2 i_o$$

$$C = (24)(3600)(57.2957795)$$

Units

$$\dot{\Omega}_i = -Jn_i \cos i_o \left\{ R_E / \left[a_i (1 - e_i)^2 \right] \right\}^2 \qquad \text{deg/day}$$

$$\dot{\omega}_i = \frac{1}{2} Jn_i (4 - 5 \sin^2 i_o) \left\{ R_E / \left[a_i (1 - e_i)^2 \right] \right\}^2 \qquad \text{deg/day.}$$

Compute

$$\Delta t_i = (A_i - A_{i-1}) / \dot{A}_{i-1} \; ; \qquad \text{days}$$

then

$$\Omega_i = \Omega_{i-1} + \dot{\Omega}_{i-1} \, \Delta t_i \qquad \text{deg}$$

$$\omega_i = \omega_{i-1} + \dot{\omega}_{i-1} \, \Delta t_i \quad . \qquad \text{deg}$$

At initial time,

$$\omega_i = \omega_o \; ; \quad \Omega_i = \Omega_o \; .$$

Finally, period and velocity at perigee are computed from

$$PD_i = (2\pi/60) \, (a_i^3/K)^{\frac{1}{2}} \qquad \text{min}$$

$$VP_i = K^{\frac{1}{2}} \left\{ (2/P_i) \left[1 + \frac{1}{3} J \, (R_E/P_i)^2 (1 - 3 \sin^2 i_o \sin^2 \omega_i) \right] - 1/a_i \right\}^{\frac{1}{2}} \; . \qquad \text{km/sec}$$

(2) Parameters that are variable with ν at each apogee integration step. Geocentric radius to the satellite is

Units

$$R_i = \frac{a_i(1 - e_i^2)}{1 + e_i \cos \nu)} + \frac{2JR_E^{\,2}}{3a_i\,(1 - e_i^2)} \left\{ \sin^2 i_o \left[1 - \frac{1}{2} \sin^2 (\omega_i + \nu) \right] - \frac{1}{2} \right\}$$

$$- (0.3R_E H/J) \sin i_o \left[\sin (\omega_i + \nu) - \sin (\omega_o + \nu) \right] . \qquad \text{km}$$

Altitude of the satellite is

$$R_i' = R_i - R_E \left[1 - (1/F) \sin^2 i_o \sin^2 (\omega_i + \nu) \right] . \qquad \text{km}$$

This value of R' is used in the density $\rho_o(R_i')$ and drag coefficient (C_D') calculations.

b. Ballistic Parameters Computation

(1) Drag Coefficient. The effective drag coefficient is input in two parts. (Note that the nomenclature was arbitrarily selected and may not be consistent with standard aerodynamic terminology.)

(a) The first part, C_D', is input as a function of altitude (R_i'). The input is given in table form with up to 20 values of C_D' and the associated altitude. This allows for the variation in C_D with altitude. The table is input with the highest altitude first followed by succeeding altitudes in decreasing order.

A linear interpolation is performed to determine C_D' for a specific value of P. If only one C_D' is input no interpolation is performed and the one value of C_D' is is used for all values of P. If P exceeds the first value in the table, the first value of C_D' in the table is used; if it is smaller than the last value in the table, the last value is used.

(b) The second part, C_N, is input as a function of angle of attack or time. The input is given in table form with up to 20 values of C_N and the associated angles of attack or time. Angle α

varies from 0 to 360° while time starts at smallest value. This allows for variable attitudes of the vehicle during an orbit.

1 If α is input a linear interpolation should be performed to determine C_N for a specific value of α. If only one C_N is input, this value is used for all values of α.

2 If time is input check lifetime t_{i-1}, then:

If $t_{i-1} \geq t_{N-1}$, use C_N at t_N value.

If $t_{i-1} < t_{N-1}$, use C_N at t_{N-1} value.

The times associated with the C_N's mean that discrete changes in C_N are made at these times.

The value of the coefficient of drag to be used at each integration point around the orbit is then

$$C_D = C_D' \, C_N \ .$$

(2) Drag Area

$$A_{oi} = f(\alpha, \text{ or time})$$

The effective drag area is input as a function of angle of attack or time. This then allows for variable attitude during an orbit or at some time in flight but not both. The input is given in table form with up to 20 values of area and the associated angles of attack or time.

If α is input, a linear interpolation is performed to determine A_{oi} at a specific α. If only one A_{oi} is input, this value is used for all values of α.

If time is input, check lifetime t_{i-1} at the previous integration step. Then,

if $t_{i-1} \geq t_{N-1}$, use A at t_N value;

if $t_{i-1} < t_{N-1}$, use A at t_{N-1} value.

The times associated with the A's mean that discrete changes are made in A at these time points.

(3) Special $C_D A_{oi}$ Provision

A special provision is made to compute $C_D A_{oi}$ for use in the $\dot{A}'$ and $\dot{P}'$ equations if called for by the flag. This allows for a variation in attitude during an orbit with only a specification of the number of revolutions made.

$$CDA = (\text{Sine}(N_1, t_1, \ldots N_N, t_N (\text{End}($$

where N is the number of cycles/orbit made by the orbiting vehicle input as a function of time t. A check is made of the lifetime t_{i-1}:

If $t_{i-1} \geq t_{N-1}$, use N at t_N value.

If $t_{i-1} < t_{N-1}$, use N at t_{N-1} value.

The following equation is then used to compute CDA:

$$CDA = CD_1A_{01} + (CD_2A_{02} - CD_1A_{01}) \ |\sin (N\nu + \alpha_o) | ,$$

where

$$CD_1 = C_D{}'CN_1, \ CD_2 = C_D{}'CN_2 \ldots .$$

CN_1, A_{01}, CN_2, A_{02} and $C_D{}'$ values are determined from the table functions, and α_o is taken from the angle of attack table (described below) for a true anomaly, ν, equal to zero.

(4) Angle of Attack, α

The angle of attack is input as a function of true anomaly. The input is given in table form with up to 20 values of α and the associated values of ν. If only one value of α is given, then this value should be used for all ν's.

(5) Mass Functions

$$M(t) = f(\dot{M}, t) \text{ or } f(t) \text{ or a constant}$$

The mass function $M(t)$ will be handled in three ways:

(a) Input M_o and up to 20 values each of M_N and M_{fN} in table form. Then the following calculations are made:

$$M(t_i) = M_o - \dot{M}_{N-1} t_{i-1} .$$

A check on mass at the previous time point is made:

If $M(t) > M_{fN-1}$, continue use of $\dot{M}_{N-1}$.

If $M(t) < M_{fN-1}$, change to use of $\dot{M}_N$.

This should be continued in table until $M(t) < M_{fN}$; then use $M(t) = M_{fN}$.

(b) Input up to 20 values of mass as a function of time in table form. Then, the following check is made on lifetime at the previous integration step:

If $t_{i-1} \geq t_{N-1}$, use M at t_N value.

If $t_{i-1} < t_{N-1}$, use M at t_{N-1} value.

This table allows for discrete changes to be made in M at specified time point.

(c) If mass is desired as a constant throughout the total lifetime, just the initial mass M_o is loaded.

C. Atmospheric Density Options. The provision is made to call for the use of any one of six atmospheric density models by flag. These models are 1959 ARDC, 1962 U. S. Standard, Poe, Small, Special 1959 ARDC, and Special 1962 U. S. Standard. For altitudes greater than 700 km, density is set equal to 0.

(1) 1959 ARDC and 1962 U. S. Standard Atmospheric Models. These two atmospheric density models are in subroutine form and are on the system library tape at the Computation Laboratory at Marshall Space Flight Center.

These two models differ from the standard 1959 ARDC and 1962 U. S. Standard in that they are referenced to the Patrick Air Force Base altitude rather than a mean sea level altitude.

To use this lifetime program at an installation that does not have a system library tape that originated at MSFC, either the subroutine cards must be obtained or one of the models described in the following section must be used.

(2) Poe and Small Atmospheric Models. The Poe and Small atmospheric density models (References 2 and 7) are time and position dependent since the effects of atmospheric heating are included. A subroutine for each of these models is included as a part of the Lifetime program and can be used directly.

(3) Special 1959 ARDC and Special 1962 U. S. Standard Atmospheric Models. These two models were previously discussed in detail in Section II. The models as programmed are respecified in this section. These two models are the same as (1) with the exception that atmospheric heating and diurnal bulge can be included on option. There are two options for specifying the effect of the diurnal bulge. One method is to compute the angle (ψ') between the satellite and the center of bulge as follows:

$$\cos \psi' = \ell \ell_B + m m_B + n n_B .$$

To evaluate the two sets of direction cosines, the following formulas are required:

$$\ell = \cos \Omega \cos (\omega + \nu) - \sin \Omega \cos i \sin (\omega + \nu)$$

$$m = \sin\Omega \cos (\omega + \nu) + \cos \Omega \cos i \sin (\omega + \nu)$$

$$n = \sin i \sin (\omega + \nu)$$

$$\ell_B = \sqrt{n_s^2 + \ell_s^2} \cos (RA_B)$$

$$m_B = \sqrt{n_s^2 + \ell_s^2} \sin (RA_B)$$

$$n_B = n_s ,$$

where

$$\ell_s = \cos \lambda_s \; ; \; m_s = \sin \lambda_s \cos \epsilon \; ; \; n_s = \sin \lambda_s \sin \epsilon$$

$$RA_B = [\tan^{-1} (m_s/\ell_s)] - \theta\rho$$

$$\lambda_s = .017203\, d + .0335 \sin (.017203\, d) - 1.41$$

$$\theta_\rho = (\pi/180) [18.5 + 30 \exp (K_1) + K_2 \sigma + 4(1 - \sigma^2)] \; ; \quad \theta_\rho < 5$$

$$\theta_\rho = 5 \qquad ; \quad \theta_\rho \geq 5$$

and

$$K_1 = -.00567 (R_i' - 200) + \exp [-.01455 (R_i' - 200)]$$

$$K_2 = 18.5 + 21.5 \exp [-.0315 (R_i' - 200)]$$

$$\sigma = (S - 160)/90 \; .$$

The second method for specifying the effect of the diurnal bulge is to set $\psi' = 75°$ which is simply a mean diurnal bulge.

The angle ψ' is then used in the equation

$$K_\rho = \left[3 + 2.5 \left(\frac{R_i' - 360}{240} \right) - 0.5 \left(\frac{R_i' - 360}{240} \right)^2 \right] \left[\frac{5.6 - \cos \psi'}{6.6} \right] .$$

Finally, density is computed from

$$\rho = \rho_o(R_i')D_c \left(\frac{s}{s_o}\right)^{K_\rho} \left\{\frac{1 + .19\,[\exp(.0055\,R_i - 1.9)]\,[\cos\frac{\psi'}{2}]^6}{1 + .19\,[\exp(.0055\,R_i - 1.9)]\,[\cos 37.5°]^6}\right\},$$

in which $\rho_o(R_i')$ is either the 1959 ARDC or the 1962 U. S. Standard reference density called for, and D_c is an altitude-dependent density correction factor. If $R_i' \leq 120$ km,

$$\rho = \rho_o\ (R_i')\ D_c\ .$$

However, when 120 km < R_i' < 700 km, compute

$$S = \overline{S}\exp\,[g(t)]$$

$$\overline{S} = 25 + 0.8\,\overline{F_{10.7}} + 0.4\,(F_{10.7} - \overline{F_{10.7}}) + 10K_p$$

$$g(t) = .025\cos\left[2\pi\left(\frac{d - 1843}{365.25}\right)\right] - .06\cos\left[4\pi\left(\frac{d-1843}{365.25}\right)\right]\ .$$

The yearly mean solar flux, $\overline{F_{10.7}}$, may be input in table form as a function of date (decimal year) and a linear interpolation for the current value of $\overline{F_{10.7}}$ performed. If no values of $\overline{F_{10.7}}$ are input, then $\overline{F_{10.7}}$ is computed as follows:

$$\overline{F_{10.7}} = 135 + 75\cos\left[2\pi\left(\frac{d - 136}{4090}\right)\right] + 15\cos\left[4\pi\left(\frac{d + 174}{4090}\right)\right]\ .$$

$F_{10.7}$ may also be input in table form. If no value of $F_{10.7}$ is input then $F_{10.7}$ is set equal to $\overline{F_{10.7}}$. KP is input in table form as a function of date (decimal year) and a linear interpolation performed to obtain the current value of KP.

Many of the foregoing parameters and their definitions were adopted from Reference 2.

d. Apogee and Perigee Decay Rates at the ith Apogee Integration Step. At each ith integration step the time rates of change of apogee and perigee radius (which are functions of ν) are averaged over 2π on ν. The resultant definite integrals are evaluated by Simpson's rule using fixed increments of ν. The procedure follows:*

Units

$$\dot{A}_i' = \frac{-86.4\times 10^6}{2\pi} \sqrt{\frac{a(1+e)}{K(1-e)^3}} \int_0^{2\pi} C1(1+\cos\nu)\, d\nu \qquad \text{kg-km/day}$$

$$\dot{P}_i' = \frac{-86.4\times 10^6}{2\pi} \sqrt{\frac{a(1-e)}{K(1+e)^3}} \int_0^{2\pi} C1(1-\cos\nu)\, d\nu \qquad \text{kg-km/day}$$

$$C1 = (C_{D_i})(A_{oi})(\rho_i)(1 + 2\, e_i \cos\nu + e_i^2)^{\frac{1}{2}} \quad ,$$

then

$$\dot{A}_i = \dot{A}_i'/M(t)$$

$$\dot{P}_i = \dot{P}_i'/M(t) \quad ,$$

where M(t) is taken from calculations in Section III. B. 3. b. (5).

For printout only, the time rate of change of semi-major axis is computed from

$$\dot{a}_i = \frac{1}{2}(\dot{A}_i + \dot{P}_i) \quad .$$

Finally, for use by the Runge-Kutta integration routine, the rates of change of time and perigee with respect to apogee are computed.

---

* The derivations of $\dot{A}_i'$ and $\dot{P}_i'$ are given in Section V.

$$\frac{d\,t_i}{d\,A_i} = M(t)/\dot{A}_i'$$

$$\frac{d\,P_i}{d\,A_i} = P_i'/\dot{A}_i' \quad .$$

e. Integration of Time and Perigee with Respect to Apogee. The integration scheme used to numerically determine the change in time and perigee with respect to apogee is Runge-Kutte (Reference 4).

The Fortran source listing of the Runge-Kutta routine is given in Section VI, Statements 3 to 13. However, a brief explanation of the flow of the computations performed by Runge-Kutta follows.

At the starting point for any lifetime computation, initial values of A, P, i, Ω, ω are known. Using these initial values, the initial rates of change of time and perigee with respect to apogee are computed via the equations in Sections III-3-a through III-3-d. With the information now available (A, P, dt/dA, dP/dA), the Runge-Kutta scheme is used to take a step (δA) in apogee and to arrive eventually at a solution for perigee and time at the end of this step. For the next step initial values of A, P, i, Ω, ω are available from the previous step and the whole sequence of computation is repeated as in step one. One of the main points to understand is that the Runge-Kutta scheme, for any one apogee step, is keyed to obtaining a very good value of dt/dA and dP/dA at the midpoint of the particular apogee step. Once this is obtained, values of perigee radius and time at the end of the step are evaluated immediately. Remember that δA is an exact quantity input to the program while δP and δt must be calculated. The operations performed by Runge-Kutta are as follows.

For convenience of notation let:

(1) P_i = known perigee at beginning of step.

A_i = known apogee at beginning of step.

T_i = known time at beginning of step.

δA = known step size in apogee to be taken.

$$PD = dP/dA$$

$$TD = dt/dA \quad ,$$

then

$$CP_1 = \frac{\delta A}{2} \; PD$$

$$P_i = CP_1 + P_1$$

$$A_i = A_1 + \delta A/2$$

$$CT_1 = \frac{\delta A}{2} \; TD$$

$$t_i = T_1 + CT_1$$

Compute new PD & TD using aforementioned equations in Sections III-3-a through III-3-d.

$$CP_2 = \frac{\delta A}{2} \; PD$$

$$P_i = CP_2 + P_1$$

$$CT_2 = \frac{\delta A}{2} \; TD$$

$$t_i = T_1 + CT_2$$

Compute new PD & TD as above.

$$CP_3 = \frac{\delta A}{2} \; PD$$

$$P_i = 2 \; CP_3 + P_1$$

$$A_i = A_1 + \delta A$$

$$CT_3 = \frac{\delta A}{2} TD$$

$$t_i = T_1 + 2\ CT_3$$

Compute new PD & TD as above.

$$\Delta = \frac{\delta A}{2} PD + 2\ CP_3 + 2\ CP_2 + CP_1\ /3$$

$$P_i = \Delta + P_1$$

$$\Delta_i = \frac{\delta A}{2}\ TD + 2\ CT_3 + 2\ CT_2 + CP_1\ /3$$

$$t_i = \Delta_1 + T_1$$

(2) P_i and t_i are now good at $A_i + A$

Using the latest values of P_i, A_i, t_i, PD, and TD, the above process from (1) to (2) is repeated until some cutoff criterion is met, namely, apogee, perigee or earth impact.

The lifetime t_i is converted at each apogee step to revolutions, t_{Ni} , for printing only.

$$t_{Ni} = t_{Ni-1} + \frac{(t_i - t_{i-1})\ 1440}{PD_i}$$

f. Altitude Interpolations. Although the vaules of apogee and perigee radius are computed and printed at the end of every apogee step, the user might like to know what these values are at intermediate points.

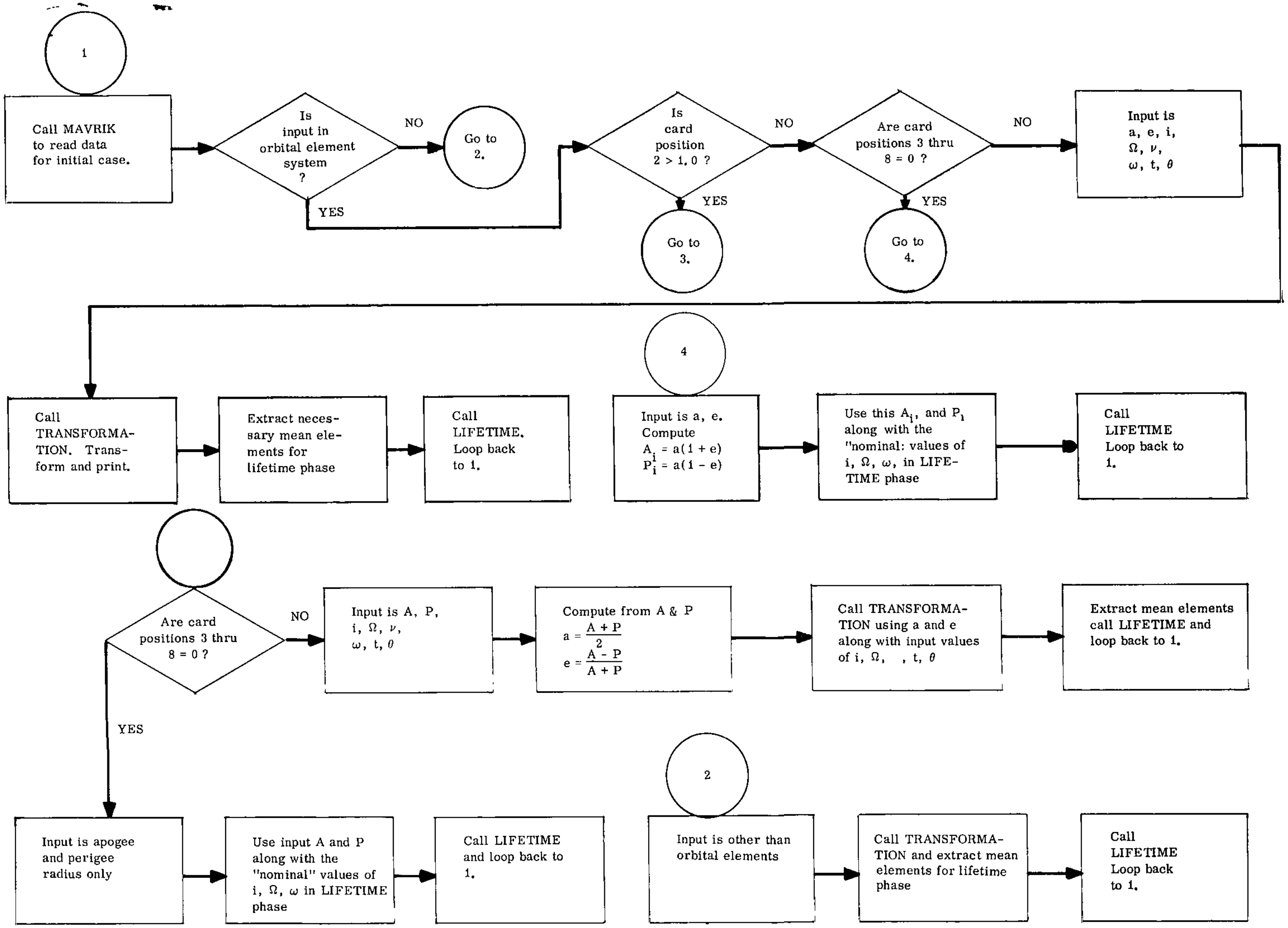
An option is available which allows the interpolation for the the printout of a maximum of five intermediate apogees and five intermediate perigees, or a total of ten extra points if the user so desires.

The equations used for interpolation are

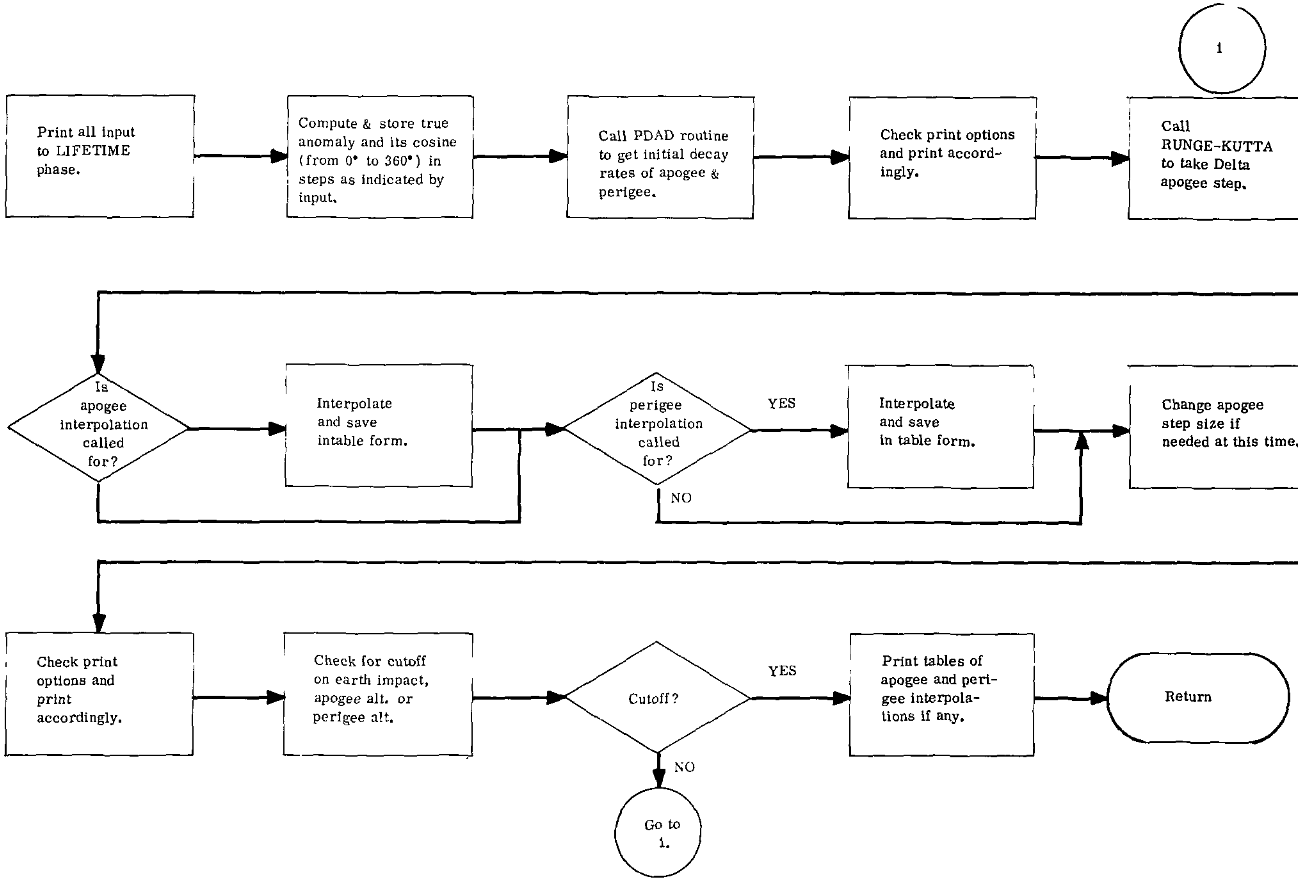
$$A_{N_j} = A_{i-1} + \frac{(T_{A_j} - t_{i-1})(A_i - A_{i-1})}{(t_i - t_{i-1})}$$

$$P_{N_k} = P_{i-1} + \frac{(T_{P_k} - t_{i-1})(P_i - P_{i-1})}{(t_i - t_{i-1})} ,$$

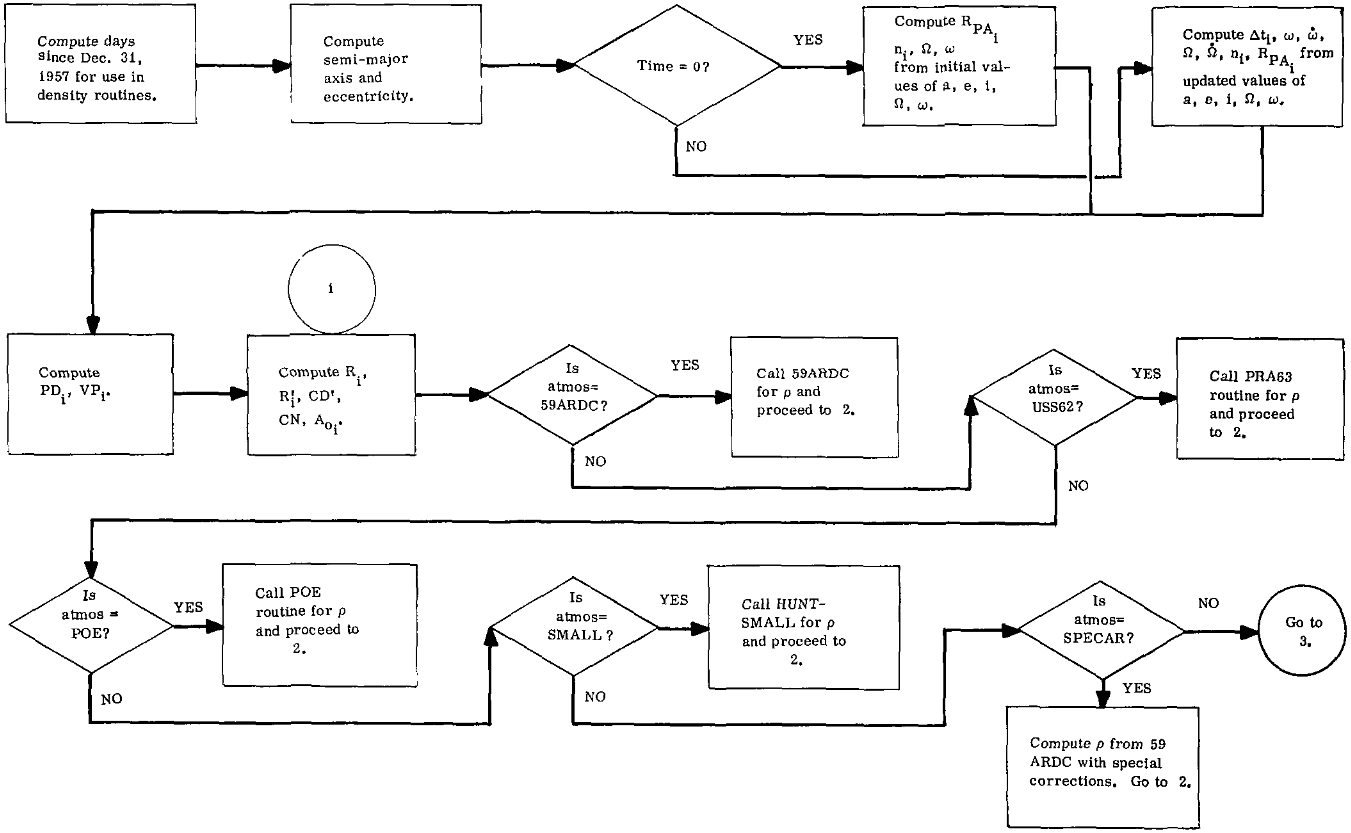
where A_{N_j} and P_{N_k} are the interpolated radii of the apogee and perigee and T_{A_j} and T_{P_k} are the times at which the interpolations are performed.



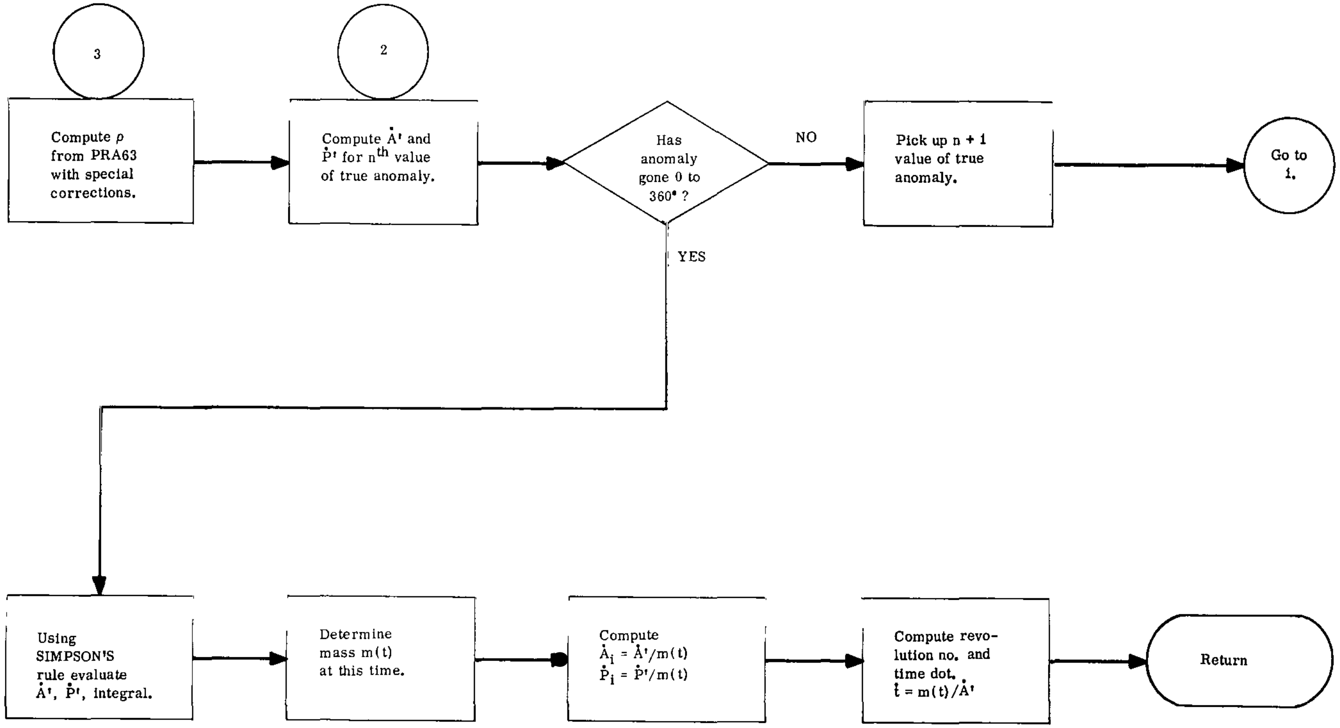
Block Flow of Control Phase



Block Flow of "Life" Routine



Block Flow of "PDAD" Routine



Block Flow of "PDAD" Routine (Cont'd)

1. Description of Input Routine (MAVRIK) Used By The Orbit Decay and Orbital Lifetime Program

The MAVRIK input routine is unique to the Computation Laboratory at MSFC. This routine allows flexibility of input and is therefore well suited for engineering work.

Single precision floating point numbers containing up to 10 digits (sign, decimal, and exponent excluded) may be input. Such numbers always contain a decimal point, are terminated by a comma, and may be input in one of several forms. For example, the number 6378.168 may be input as follows:

6378.165, 6.378165E+03, 6.378165+03,

6.378165+3, 6378165.E-3, 637816.5-2,

Integers may also contain up to 10 digits (sign excluded). Integers contain no decimal points and are terminated by commas as follows: 6378, 100, 71658,.

Double precision numbers may contain up to 16 digits (sign, decimal, and exponent excluded). These numbers always contain a decimal point, are terminated by a right hand parenthesis, and may be input in one of several forms. For example, the number 6378.165987654321 may be input as:

6378.165987654321) .6378165987654321D+04)

637.8165987654321D+1) 637816.5987654321D-2)

Alphanumeric information, as used in this program, may contain up to 6 characters. Each piece of information is enclosed by left hand parenthesis and is left-adjusted in machine storage. Examples of alphanumeric input follows:

ON CARD	IN STORAGE
(ALPHA(	ALPHAb
(MASS(	MASSbb
(bMASS(	bMASSb

All input is loaded by code name followed by an equality sign. For example: MASS = 74812., ATMOS = (ARDC(. A complete listing of all input codes is given in the following tables.

2. Description of Parameters That May Be Input to The Lifetime Phase

Input Code	Equation Symbol	Unit & Direction	Nominal Value	Definition
	***** BALLISTIC PARAMETERS *****			
AREA=(***(++. ++, ++. ++, ... (END(	A_o	m^2	(ALPHA (1., 360., (END(	Table of effective drag area values as a function of either angle of attack or time. *** specifies whether area is a function of (ALPHA(or (TIME(. The first value after*** is the dependent variable, AREA, followed the independent variable; either ALPHA in degrees or TIME in hours. The table may contain up to 20 values of area (with the corresponding 20 values of independent variable).
ATTACK=++. ++, ++. ++, ... (END(	α	deg	1., 360., (END(	Table of angle of attack as a function of true anomaly. First table value is the dependent variable, α, followed by the independent variable, true anomaly, in degrees. Table may contain up to 20 values of α (with the corresponding 20 values of true anomaly.

Input Code	Equation Symbol	Unit & Direction	Nominal Value	Definition
CDPRIM=++. ++, ++. ++, . . . (END(	CD'	non-dimensional	1., 0., (END(	Table of drag coefficients as a function of perigee altitude. First value is dependent variable, CD, followed by independent variable, perigee altitude, in kilometers. Table may contain up to 20 values of CD with corresponding values of perigee altitude.
CDA=(***(++. ++, ++. ++, . . . (END(	N	cycles/orbit		When this table is input the quantity CDA is computed as some function of AREA, ATTACK, COPRIM, and N. ***indicates the sine function (SINE(. Other functions could be added at a later time. The first number in the table is N, the number of cycles/orbit made by the orbiting body. The second number in the table is time in hours at which the next value of N should be used. Table may contain up to 20 values of N with the corresponding times.

Input Code	Equation Symbol	Unit & Direction	Nominal Value	Definition
MASS=(***(++. ++, ++. ++, . . . (END(	m(t)	see definition	(CON(1.,	Orbiting mass function. There are three methods for specifying mass. Method one. ***specifies (RATE(, the first value in the table is an initial mass (M_o) in kilograms, the second value is a mass decay rate ($\dot{m}$) in kilograms/day, and the third value is a final mass (M_f) in kilograms. If the mass decays to (M_f) then the next decay rate in the table is used along with a new M_f. If an (END(is found in the table the last M_f is used as a constant mass until the run is completed. Table may contain up to 20 values of $\dot{m}$ and corresponding M_f. Method two. *** specifies (TIME(, the first value in the table is an initial mass (M) in kilograms, and the second value in the table is a time (t) in hours to change to a succeeding mass (m). If an (END(is found in the table the last M is used as a constant mass until the run is completed. Table may contain up to 20 values of M and corresponding times.

Input Code	Equation Symbol	Unit & Direction	Nominal Value	Definition
MASS (cont.)				Method three. ***specifies (CON(, the first and only value in the table is the mass in kilograms to be used throughout the entire run. For this option no (END(is needed in the table.
CN=(***(++. ++, ++. ++, . . . , (END(	C_N	non-dimensional	(ALPHA(1., 360., (END(	Table of normal force coefficients as a function of either angle of attack or time. ***specifies whether cn is a function of (ALPHA(or (TIME(. The first value in the table is the dependent variable, C_N followed by the independent variable, either α in degrees or time in hours. The table may contain up to 20 values of C_N with the corresponding independent variable.

Input Code	Equation Symbol	Unit & Direction	Nominal Value	Definition
***** DENSITY PARAMETERS *****				
DATE=++. ++, ++.++, ++. ++,		month, day year		Calendar date to be used in density calculation when special density is desired.
ATMOS=(***(	ρ	kg/m^3	(ARDC(	Flag indicating atmosphere to be used. Options available are: (ARDC(1959 ARDC Atmosphere. USSTD(1962 US Standard Atmosphere. (POE(LMSC Atmosphere Routine (SMALL(LMSC Atmosphere Routine (SPECAR(Modified 1959 ARDC Atmosphere (SPECUS(Modified 1962 US Standard Atmosphere
CORREC=++. ++, ++. ++,... (END(	DC	non-dimensional	1., 0., (END(	Table of density correction factors DC as a function of perigee altitude in km. First value in table is DC, second value is perigee altitude. Table may contain up to 50 values of DC and corresponding altitude.

Input Code	Equation Symbol	Unit & Direction	Nominal Value	Definition
DIURNL=(***(	ψ'	deg	(MEAN(	Flag indicating diurnal (bulge) effect desired in special density calculations. ***=(MEAN(indicates a mean diurnal effect where lag angle, ψ', is set equal to 75°. This effect corresponds to 9:00 a.m. local time for a lag angle of 30°. ***=(NORMAL(indicates computation of diurnal effect as given by special density equations.
ECLIPT=++.++,		deg	23.4436	Obliquity of the ecliptic. Used in computing ψ'.
KP=++.++, ++.++, ... (END(	KP	k_p	2.5, 2000., (END(	Table of monthly mean values of geomagnetic index. Table is input as a function of decimal year. Up to 50 KP's and corresponding years may be loaded.

Input Code	Equation Symbol	Unit & Direction	Nominal Value	Definition
KP (cont.)				Note: In the literature K_p is listed as 0_o, 0+, 1-, 1_o1+, . . ., 9-, 9_o; however loaded values are specified as a decimal number, e.g. 8.7.
FTEN=++.++, ++.++, ... (END(	$F_{10.7}$	10^{-22}watts/ m^2/cycle/ sec	0.	Table of monthly mean values of $F_{10.7}$ These values will be available only for post flight predictions. The values are loaded in the order $F_{10.7}$ decimal year, F_{10}, decimal year etc. Up to 50 values of $F_{10.7}$ and corresponding years may be loaded.
FTENB=++.++, ++.++, ... (END(	$\overline{F}_{10.7}$	10^{-22}watts/ m^2/cycle/ sec	SEE BELOW	Table of yearly mean values of $F_{10.7}$ These values are loaded in the order $\overline{F}_{10.7}$ decimal year, $\overline{F}_{10.7}$ decimal year, etc. Up to 50 values of $\overline{F}_{10.7}$ and corresponding years may be loaded. The assembled, nominal table is given below.

Nominal Value of $\overline{F_{10}}.7$

243.6, 1958., 230.7, 1958.5, 226.5, 1959., 208.9, 1959.5, 180.5, 1960., 161., 1960.5, 130.8, 1961., 104.8, 1961.5, 99.3, 1962., 89.7, 1962.5, 382.7, 1963., 80.8, 1963.5, 77.9, 1964., 70., 1964.5, 75., 1965., 80., 1965.5, 131., 1966.5, 186., 1967.5, 200., 1968.5, 190., 1969.5, 163., 1970.5, 142., 1971., 128., 1971.5, 108., 1972.5, 94., 1973.5, 81., 1974.5, 75., 1975., 75., 1975.5, (END(

Input Code	Equation Symbol	Unit & Direction	Nominal Value	Definition
SO=++. ++,	S_o	non-dimensional	220.0	Reference value of total heating used in the computation of special 59 ARDC and 62STD densities. Normally the 100. value will be used but if a D_c curve derived for some other heating value is loaded then SO must be changed to that total heating value.
SA=++. ++,	ss	non-dimensional	0.	Reference value of total heating used in LMSC Hunt Small density computation. If no SA is loaded, program automatically computes this value.
		***** SPECIAL INPUT *****		
PRINT=(***(			(NORMAL(	Flag denoting type of output desired. (NORMAL(, (SHORT(, (DETAIL(are the options available. Complete description of each option is given in the output section of this report.

Input Code	Equation Symbol	Unit & Direction	Nominal Value	Definition
INTERA=++.++, ++.++, ...	A_{N_j}	days		Table of up to 5 times for which an interpolation for apogee will be performed. Apogee values stored in table form to be output at end of run.
INTERP=++.++, ++.++, ...	P_{N_K}	days		Table of up to 5 times for which an interpolation for perigee will be performed. Perigee values stored in table form to be output at end of run.
CUTOFF=(***(++.++,		km from earth center	(I(6378.166,	Flag denoting method of terminating run. *** indicates which parameter cutoff will be made on. Three options are available: (A(apogee radius, (P(perigee radius, (I(earth impact. ++.++ is the radius at which cutoff is desired.
DANOM=++.++,	$\delta\nu$	deg.	10.0	Integration step size in true anomaly to be used in Simpson's rule integration around orbit. Must be a multiple of 360.

Input Code	Equation Symbol	Unit & Direction	Nominal Value	Definition
DAPOGE=++.++, ++.++, ...	δA	km	See table at right.	Integration step size in apogee used in RUNGE-KUTTA integration. The step size is a function of apogee radius and is loaded in the form of δA, A (km), δA, A(km), etc. Up to 5 step sizes and the corresponding apogee radius may be loaded. The nominal, assembled table is: -5., 6778., -10., 6578., -20., 0.,
	***** CONSTANTS *****			
F=++. ++,	f	non-dimensional	298.3	Reciprocal of the flattening of the earth.

Description of Parameters That May Be Input to The Transformation Phase.

Input Code	Equation Symbol	Unit & Direction	Nominal Value	Definition
The following constants, KERTH through DD, are input to the Transformation phase in double precision and are carried to the Lifetime phase in single precision. These constants are common to both phases.				
KERTH=++. ++)	K	km^3/sec^2	398603.2	Earth gravitation constant.
AE=++. ++)	R_e	km	6378.165	Mean equatorial radius of the earth.
AO=++. ++)	AO	km	6378.165	Semi-major axis of Fischer ellipsoid
BO=++. ++)	BO	km	6356.784	Semi-minor axis of Fischer ellipsoid
OMEGA=++. ++)	ω_e	deg/hr	15.04106705	Rotational velocity of the earth.
JJ=++. ++)	J	non-dimensional	.162345D-2	Coefficient of the 2nd zonal harmonic of the earth's gravitational potential.
HH=++. ++)	H	"	.575D-5	Coefficient of the 3rd zonal harmonic of the earth's gravitational potential.
DD=++. ++)	D	"	.7875D-5	Coefficient of the 4th zonal harmonic of the earth's gravitational potential.
PHIO=++. ++)	Φ	deg. n	28.5	Geodetic latitude of launch pad.
LAMDO=++. ++)	λ	deg. w	80.5	Longitude of launch pad.

Input Code	Equation Symbol	Unit & Direction	Nominal Values	Definition
KAPPA=++. ++)	α_F	deg. e of n	105.	Launch Azimuth
XG=++. ++)	XG	km	0.	X-component of position due to gravitational acceleration at transformation time.
YG=++. ++)	YG	km	0.	Y-component of position due to gravitational acceleration at transformation time.
ZG=++. ++)	ZG	km	0.	Z-component of position due to gravitational acceleration at transformation time.
XDG=++. ++)	XDG	km/sec	0.	X-component of velocity due to gravitational acceleration at transformation time.
YDG=++. ++)	YDG	km/sec	0.	Y-component of velocity due to gravitational acceleration at transformation time.
ZDG=++. ++)	ZDG	km/sec	0.	Z-component of velocity due to gravitational acceleration at transformation time.
TFM=++. ++)	t_{fm}	hrs	0.	Universal time of first motion.

Coordinate Systems That May Be Input to the Transformation Phase

Input Code	Order of Input	Unit & Direction	Definition
EFE=++. ++)++. ++). . . .)	XE	km	Earth-fixed ephemeris system in order: position vector, velocity vector, universal time, sidereal time.
	YE	km	
	ZE	km	
	$\dot{X}E$	km/sec	
	$\dot{Y}E$	km/sec	
	$\dot{Z}E$	km/sec	
	t	hours	
	θ	deg	
EFP=++. ++)++. ++). . . .)	XEP	km	Earth-fixed plumbline system in order: position vector, velocity vector, universal time, sidereal time.
	YEP	km	
	ZEP	km	
	$\dot{X}EP$	km/sec	
	$\dot{Y}EP$	km/sec	
	$\dot{Z}EP$	km/sec	
	t	hours	
	θ	deg	
SFE=++. ++)++. ++). . . .)	XS	km	Space-fixed ephemeris system in order: position vector, velocity vector, universal time, sidereal time.
	YS	km	
	ZS	km	
	$\dot{X}S$	km/sec	
	$\dot{Y}S$	km/sec	
	$\dot{Z}S$	km/sec	
	t	hours	
	θ	deg	
SFG=++. ++)++. ++). . . .)	Rs	km	Space-fixed geographic system in order: range, geocentric latitude, longitude, velocity, azimuth, geocentric elevation.
	ψs	deg	
	λs	deg	
	Vs	km/sec	
	αs	deg	
	εs	deg	
	t	hours	
	θ	deg	

Input Code	Order of Input	Unit & Direction	Definition
EFG=++. ++)++. ++)....)	Re	km	Earth-fixed geographic system in order: range, geocentric latitude, longitude, velocity, azimuth, geocentric elevation.
	ψe	deg	
	λe	deg	
	Ve	km/sec	
	αe	deg	
	εe	deg	
	t	hours	
	θ	deg	
OET=++. ++)++. ++)...)	A	km	Osculating orbital element system in order: semi-major axis, eccentricity, inclination, ascending node, anomaly, argument of perigee, universal time, sidereal time. Anomaly may be input in any of three ways: True (OET), Mean (OEM), Eccentric (OEE).
	E	non-dimensional	
OEM=++. ++)++. ++)...)	I	deg	
	Ω	deg	
OEE=++. ++)++. ++0...)	V, M, E	deg	
	ω	deg	
	t	hours	
	θ	deg	
MOT=++. ++)++. ++)...)	A	km	Mean orbital element system in order: semi-major axis, eccentricity, inclination, ascending node, anomaly, argument of perigee, universal time, sidereal time. Anomaly may be input in any of three ways: True (MOT), Mean (MOM), Eccentric (MOE).
	E	non-dimensional	
MOM=++. ++)++. ++)...)	I	deg	
	Ω	deg	
MOE=++. ++)++. ++)...)	V, M, E	deg	
	ω	deg	
	t	hours	
	θ	deg	
PLT=++. ++)++. ++)...)	Xpl	km	Platform coordinate system in order: position vector, velocity vector, universal time, sidereal time.
	Ypl	km	
	Zpl	km	
	$\dot{X}$pl	km/sec	
	$\dot{Y}$pl	km/sec	
	$\dot{Z}$pl	km/sec	
	t	hours	
	θ	deg	

Description of IOMET Flag Controlling Addition or Subtraction of Long and Short Periodic Terms in Orbital Element Transformations

INPUT CODE	IOMET=(***((***(
NOMINAL VALUE	IOMET=(SHORT((NOLONG(

If input is given in any system other than mean orbital elements, then the following conditions apply.

IOMET = (SHORT((NOLONG(	Short-period terms are subtracted.
IOMET = (NOSHRT((LONG(	Long-period terms are subtracted.
IOMET = (NOSHRT((NOLONG(	No terms are subtracted.
IOMET = (SHORT((LONG(	Both short and long-period terms are subtracted.

If input is in mean elements the above conditions apply with the exception that the periodic terms are added instead of subtracted.

D. Computer Program Output Options

There are three formates for output available in the Lifetime program. They are: short, normal, and detail. The following input cards to "MAVRIK" produced a sample of each of the three types of output.

```
CN       = (ALPHA(2., 360., (END(

DATE     = 4., 8., 1964.,

AREA     = (ALPHA(26.04, 360., (END(

MASS     = (CON(6040.,

MOT      = 6618.567) .01211) 32.68) 81.4) 101.809) 60.407)

ATMOS    = (ARDC(

PRINT    = (SHORT(/

PRINT    = (DETAIL(/

PRINT    = (NORMAL(/
```

1. Short Output. (See computer output on following page.)

```
                EARTH ØRBIT TRANSFØRMATIØN

PHIØ   =    0.28500000000000000D 02  DEG     LAMDØ =   0.80500000000000000D 02  DEG       AZFIR =   0.10500000000000000D 03  DEG
A      =    0.63781650000000000D 04   KM     B     =   0.63567839999999999D 04   KM       ØMEGA =   0.15041067050000000D 02  DEG/HR
RADIUS=     0.63781650000000000D 04   KM     KERTH =   0.39860320000000000D 06  KM3/SEC2   J    =   0.16234500000000000D-02
H      =    0.57500000000000000D-05          D     =   0.78750000000000000D-05            XG    =   0.                               KM
YG     =    0.                        KM     ZG    =   0.                        KM       XDG   =   0.                               KM/SEC
YDG    =    0.                     KM/SEC    ZDG   =   0.                     KM/SEC      TFM   =   0.                               HØURS

INPUT  IN MØT SYSTEM SHØRT  NØLØNG
PØSITIØN (KM)    VELØCITY (KM/SEC)    ANGLES (DEG)
EFP
XE     =  -0.30370778834733387D 04      YE     =  -0.10461426284968821D 04      ZE     =   0.25059899243364800D 04
XDE    =    0.59572460659288640 01      YDE    =    0.16205054871575630 01      ZDE    =   0.39827149189795430 01

PLT
XPL    =  -0.30370778834733385D 04      YPL    =  -0.10461426284968822D 04      ZPL    =   0.25059899243364790D 04
XDPL   =    0.59796869036366780 01      YDPL   =    0.18500678161846280 01      ZDPL   =   0.41057415253642900 01

EFE
XEP    =  -0.26302375979548400D 04      YEP    =  -0.59862995727152910 04       ZEP    =   0.10896894766029310 04
XDEP   =    0.53181203429086350 01      YDEP   =  -0.31610772620431630 01       ZDEP   =  -0.39625808812588630 01

SFE
XS     =  -0.26302375979548400D 04      YS     ■  -0.59862995727152910 04       ZS     =   0.10896894766029310 04
XDS    =    0.57546481970670240 01      YDS    =  -0.33528772146875550 01       ZDS    =  -0.39625808812588630 01
TIME   =  -0.                           THETA =   -0.

SFG
R      =    0.66288276151522970 04      GCLAT =    0.94615942917960260 01      LØNG   =   0.24628045601763250 03
VS     =    0.77498263676869540 01      AZVS  =    0.12135935423781510 03      ELVS   =   0.68844204099386370 00

EFG
R      =    0.66288276151522970 04      GCLAT =    0.94615942917960260 01      LØNG   =   0.24628045601763250 03
VE     =    0.73468946282618750 01      AZVE  =    0.12329496702450730 03      ELVE   =   0.72620079920493150 00

ØØE
AXIS   =    0.66209031696896410 04      ECCEN =    0.12074748215571460-01      INC    =   0.32615834466220470 02
ASNØD  =    0.81376057355369100 02      ARGP  =    0.65866222236963070 02      TANØM  =   0.96376689682992690 02
EANØM  =    0.95688659639330660 02      MANØM =    0.95000234656844450 02

MØE
AXIS   =    0.66185669999999990 04      ECCEN =    0.12110000000000000D-01      INC    =   0.32599999999999990 02
ASNØD  =    0.81399999999999990 02      ARGP  =    0.60406999999999990 02      TANØM  =   0.10180900000000000D 03
EANØM  =    0.10112897369097720 03      MANØM =    0.10044816955907160 03

AUXILIARY CALCULATIØNS
APØGEE =    0.32268390642332060 03   KM     PERIGEE=    0.16279243095595980 03   KM      PERIØD =   0.89358132140913800 02   MIN
RANGE  =    0.40580341323003530 04   KM     ALT    =    0.25124323092168190 03   KM      TPITCH =   0.89386929603982360 02   DEG
ECV    =  -0.46419844888672120-02 KM/SEC    EEV    =  -0.32166479449530370 01 KM/SEC     MAPØGEE=   0.32055284636999840 03    KM
MPERIGE=    0.16025115362999940 03   KM     MPERIØD=    0.89310841611512520 02   MIN     DARGP  =   0.11179298298297090 02   DEG/DAY
DASNØD =  -0.73935526963860700 01   DEG/DAY
```

```
                              ***** EARTH CØNSTANTS *****

EARTH SECØND HARMØNIC 0.16234499E-02      EARTH THIRD HARMØNIC 0.57500000E-05      EARTH FØURTH HARMØNIC 0.78749999E-05
EARTH GRAVITATIØNAL CØNSTANT (KILØMETERS CUBED/SECØNDS SQUARED) 0.39860319E 06
EQUATØRIAL RADIUS (KILØMETERS) 0.63781650E 04      ELLIPTICITY 0.29830000E 03

                              ***** BALLISTIC PARAMETERS *****

ANGLE ØF ATTACK FUNCTIØN
ALPHA(DEGREES)          ANØMALY(DEGREES)
 0.                      0.36000000E 03

CØEFFICIENT ØF DRAG FUNCTIØN
CN                      ALPHA(DEGREES)
 0.20000000E 01          0.36000000E 03

CDPRIME                 PERIGEE(KILØMETERS)
 1.00000000E 00          0.

EFFECTIVE DRAG AREA FUNCTIØN
AREA(METERS SQUARED)       ALPHA(DEGREES)
 0.26040000E 02             0.36000000E 03

MASS CØNSTANTS
INITIAL MASS(KILØGRAMS) 0.60399999E 04

                              ***** DENSITY PARAMETERS *****

MØNTH= 0.40E 01      DAY= 0.80E 01      YEAR 0.1964E 04      DAYS ELAPSED SINCE DEC. 31 , 1957 0.22899999E 04
1959 ARDC ATMØSPHERE

DENSITY CØRRECTIØN
DC                      PERIGEE(KILØMETERS)
 0.12000000E-00          0.50000000E 03
 0.13000000E-00          0.40000000E 03
 0.14200000E-00          0.34999999E 03
 0.18400000E-00          0.30000000E 03
 0.22000000E-00          0.27999999E 03
 0.27500000E-00          0.26000000E 03
 0.30400000E-00          0.25000000E 03
 0.34000000E-00          0.23999999E 03
 0.38500000E-00          0.23000000E 03
 0.42500000E-00          0.22000000E 03
 0.47000000E-00          0.20999999E 03
 0.52000000E 00          0.20000000E 03
 0.56500000E 00          0.19000000E 03
 0.62000000E 00          0.18000000E 03
 0.70000000E 00          0.16999999E 03
 0.80000000E 00          0.16000000E 03
 0.84000000E 00          0.15499999E 03
 0.86000000E 00          0.15000000E 03
 1.00000000E 00          0.14500000E 03
 1.00000000E 00          0.

KP                      YEAR
 0.23400000E 01          0.19580000E 04
 0.24800000E 01          0.19588000E 04
 0.18900000E 01          0.19588999E 04
 0.25099999E 01          0.19589999E 04
 0.29700000E 01          0.19591000E 04
 0.24199999E 01          0.19592000E 04
 0.25600000E 01          0.19593000E 04
 0.25900000E 01          0.19594000E 04
```

```
 0.29200000E 01         0.19594999E 04
 0.31599999E 01         0.19596000E 04
 0.34999999E 01         0.19597000E 04
 0.24600000E 01         0.19598000E 04
 0.28900000E 01         0.19599000E 04
 0.25699999E 01         0.19599999E 04
 0.23199999E 01         0.19601000E 04
 0.24400000E 01         0.19602000E 04
 0.34199999E 01         0.19603000E 04
 0.27899999E 01         0.19604000E 04
 0.29200000E 01         0.19605000E 04
 0.27100000E 01         0.19605999E 04
 0.27500000E 01         0.19606999E 04
 0.32299999E 01         0.19608000E 04
 0.34400000E 01         0.19609000E 04
 0.24900000E 01         0.19610000E 04
 0.22700000E 01         0.19611000E 04
 0.23199999E 01         0.19611999E 04
 0.22899999E 01         0.19613000E 04
 0.23999999E 01         0.19614000E 04
 0.26899999E 01         0.19615000E 04
 0.22600000E 01         0.19616000E 04
 0.21799999E 01         0.19616999E 04
 0.18499999E 01         0.19617999E 04
 0.19199999E 01         0.19619000E 04
 0.14900000E 01         0.19620000E 04
 0.17299999E 01         0.19621000E 04
 0.18099999E 01         0.19622000E 04
 0.23100000E 01         0.19622999E 04
 0.16000000E 01         0.19623999E 04
 0.21799999E 01         0.19625000E 04
 0.26199999E 01         0.19626000E 04
 0.29399999E 01         0.19627000E 04
 0.30800000E 01         0.19628000E 04
 0.20100000E 01         0.19628999E 04
 0.17500000E 01         0.19620000E 04
 0.17200000E 01         0.19631000E 04
 0.15400000E 01         0.19631200E 04
 0.15099999E 01         0.19632100E 04
 0.18600000E 01         0.19632900E 04
 0.20800000E 01         0.19633800E 04
 0.20599999E 01         0.19634600E 04
 0.22300000E 01         0.19635399E 04
 0.23499999E 01         0.19636200E 04
 0.32600000E 01         0.19637100E 04
 0.21999999E 01         0.19637900E 04
 0.20200000E 01         0.19638800E 04
 0.19800000E 01         0.19639600E 04
 0.20599999E 01         0.19640400E 04
 0.22099999E 01         0.19641200E 04
 0.21600000E 01         0.19642100E 04
 0.22499999E 01         0.19642900E 04
 0.18099999E 01         0.19643800E 04
 0.17299999E 01         0.19644599E 04
 0.18900000E 01         0.19645400E 04
 0.16900000E 01         0.19646200E 04
 0.17800000E 01         0.19647100E 04
 0.16700000E 01         0.19647899E 04
 0.90000000E 00         0.19648800E 04
 0.77000000E 00         0.19649599E 04
 0.24999999E 01         0.19650000E 04
 0.24999999E 01         0.20000000E 04

FTEN                    YEAR
 0.                      0.
```

```
FIENB                  YEAR
 0.24360000E 03         0.19580000E 04
 0.23070000E 03         0.19585000E 04
 0.22650000E 03         0.19589999E 04
 0.20890000E 03         0.19594999E 04
 0.18049999E 03         0.19599999E 04
 0.16100000E 03         0.19605000E 04
 0.13079999E 03         0.19610000E 04
 0.10479999E 03         0.19615000E 04
 0.99300000E 02         0.19620000E 04
 0.89699999E 02         0.19625000E 04
 0.82699999E 02         0.19630000E 04
 0.80800000E 02         0.19634999E 04
 0.77899999E 02         0.19639999E 04
 0.70000000E 02         0.19645000E 04
 0.75000000E 02         0.19650000E 04
 0.87000000E 02         0.19655000E 04
 0.13100000E 03         0.19665000E 04
 0.18600000E 03         0.19675000E 04
 0.20000000E 03         0.19684999E 04
 0.19000000E 03         0.19695000E 04
 0.16300000E 03         0.19705000E 04
 0.14200000E 03         0.19710000E 04
 0.12800000E 03         0.19715000E 04
 0.10800000E 03         0.19724999E 04
 0.94000000E 02         0.19735000E 04
 0.80999999E 02         0.19745000E 04
 0.75000000E 02         0.19750000E 04
 0.75000000E 02         0.19755000E 04

DIURNAL   MEAN

                        ***** SPECIAL EVENTS *****

EARTH IMPACT CUTØFF

                        ***** INITIAL CØNDITIØNS *****

SHØRT PRINTØUT

ANØMALY STEP(DEGREES) 0.99999999E 01

APØGEE STEPS(KM)              PERIGEE RADIUS(KM)
 -0.49999999E 01                0.67780000E 04
 -0.99999999E 01                0.65780000E 04
 -0.20000000E 02                0.
  0.                            0.
  0.                            0.
```

```
APOGEE,PERIGEE,MAJOR AXIS,AND EARTH RADIUS(KM)
APOGEE,PERIGEE,MAJOR AXIS RATES(KM/DAY) MASS(KG)
ASCENDING NODE,ARGUMENT OF PERIGEE(DEG)
NODE,PERIGEE REGRESSION RATES(DEG/DAY)
PERIGEE VELOCITY(KM/SEC)
ORBITAL PERIOD(MIN)
LIFETIME SPENT(ORBIT AND DAY)
RHO(KG/M3), EI(UNITLESS), RIPERG AND RIPAPG(KM)

-------------------------------------------------------------------------------------------------
A        0.66987178E 04   P         0.65384161E 04   AXIS    0.66185670E 04   RADIUS 0.63734720E 04
ADOT    -0.20172297E 02   PDOT     -0.40069964E 01   AXIDOT-0.12089644E 02   MASS    0.60399999E 04
NODE     0.81400000E 02   ARGP      0.60406999E 02   DNODE  -0.73800376E 01   DARGP   0.11164444E 02
VPERIG   0.78563969E 01   PERIOD    0.89310846E 02   ORBIT   0.               TIME    0.
RHO      0.10019261E-08   EI        0.12110000E-01   RIPERG  0.16281848E 03   RIPAPG  0.32312018E 03

-------------------------------------------------------------------------------------------------
A        0.63687178E 04   P         0.63691773E 04   AXIS    0.63689475E 04   RADIUS  0.63724373E 04
ADOT    -0.45995754E 11   PDOT     -0.46000738E 11   AXIDOT-0.45998245E 11   MASS    0.60399999E 04
NODE     0.51174578E 02   ARGP      0.10612672E 03   DNODE  -0.84419879E 01   DARGP   0.12769567E 02
VPERIG   0.79116516E 01   PERIOD    0.84306248E 02   ORBIT   0.60625867E 02   TIME    0.37282697E 01
RHO      0.12254324E 01   EI       -0.36066525E-04   RIPERG-0.59685668E 01   RIPAPG-0.57626952E 01
```

```
                    EARTH ØRBIT TRANSFØRMATIØN

PHIØ   =  0.28500000000000000D 02  DEG      LAMDØ =  0.80500000000000000D 02  DEG      AZFIR =  0.10500000000000000D 03  DEG
A      =  0.63781650000000000D 04  KM       B     =  0.63567839999999990D 04  KM       ØMEGA =  0.15041067050000000D 02  DEG/HR
RADIUS=   0.63781650000000000D 04  KM       KERTH =  0.39860320000000000D 06  KM3/SEC2  J    =  0.16234500000000000D-02
H      =  0.57500000000000000D-05           D     =  0.78750000000000000D-05           XG    =  0.                         KM
YG     =  0.                       KM       ZG    =  0.                       KM       XDG   =  0.                         KM/SEC
YDG    =  0.                       KM/SEC   ZDG   =  0.                       KM/SEC   TFM   =  0.                         HØURS

INPUT IN MØT SYSTEM SHØRT  NØLØNG
PØSITIØN (KM)    VELØCITY (KM/SEC)    ANGLES (DEG)
EFP
XE     =  -0.30370778834733870 04     YE     =  -0.10461426284968210 04     ZE     =  0.25059899243364800 04
XDE    =   0.59572480659288640 01     YDE    =   0.16205054871575630 01     ZDE    =  0.39827149189795430 01

PLT
XPL    =  -0.30370778834733850 04     YPL    =  -0.10461426284968220 04     ZPL    =  0.25059899243364790 04
XDPL   =   0.59796869036366780 01     YDPL   =   0.18500678161846280 01     ZDPL   =  0.41057415253642900 01

EFE
XEP    =  -0.26302375979548400 04     YEP    =  -0.59862995727152910 04     ZEP    =  0.10896894766029310 04
XDEP   =   0.53181203429086350 01     YDEP   =  -0.31610772620431630 01     ZDEP   = -0.39625808812588630 01

SFE
XS     =  -0.26302375979548400 04     YS     =  -0.59862995727152910 04     ZS     =  0.10896894766029310 04
XDS    =   0.57546481970670240 01     YDS    =  -0.33528772146875550 01     ZDS    = -0.39625808812588630 01
TIME   =  -0.                         THETA  =  -0.

SFG
R      =   0.66288276151522970 C4     GCLAT  =   0.94615942917960260 01     LØNG   =  0.24628045601763250 03
VS     =   0.77498263676869540 01     AZVS   =   0.12135935423781510 03     ELVS   =  0.68844204099386370 00

EFG
R      =   0.66288276151522970 04     GCLAT  =   0.94615942917960260 01     LØNG   =  0.24628045601763250 03
VE     =   0.73468946282618750 01     AZVE   =   0.12329456702450730 03     ELVE   =  0.72620079920493150 00

ØØE
AXIS   =   0.66209031696896410 04     ECCEN  =   0.12074748215571460-01     INC    =  0.32615834466220470 02
ASNØD  =   0.81376057355369100 02     ARGP   =   0.65866222236963070 02     TANØM  =  0.96376689682992690 02
EANØM  =   0.95688659639330660 02     MANØM  =   0.95000234656844450 02

MØE
AXIS   =   0.66185669999999990 04     ECCEN  =   0.12110000000000000D-01     INC    =  0.32599999999999990 02
ASNØD  =   0.81399999999999990 C2     ARGP   =   0.60406999999999990 02     TANØM  =  0.10180900000000000D 03
EANØM  =   0.10111289736909772D 03    MANØM  =   0.10044816955907160 03

AUXILIARY CALCULATIØNS
APØGEL  =  0.32268390842332060 03  KM       PERIGEE=  0.16279243095595980 C3  KM       PERIØD =  0.89359132140913800 02  MIN
RANGE   =  0.40580341323003530 04  KM       ALT    =  0.25124323092168190 03  KM       TPITCH =  0.39386929503982360 02  DEG
ECV     = -0.46419844888672120-02  KM/SEC   EEV    = -0.32166479449530370 01  KM/SEC   MAPØGEE=  0.32055284536999640 03  KM
MPERIGE=   0.16025115362999940 03  KM       MPERIØD=  0.89310841611512520 C2  MIN      DARGP  =  0.11179238298297090 02  DEG/DAY
CASNØD  = -0.73935526963860700 01  DEG/DAY
```

```
                      ***** EARTH CONSTANTS *****

EARTH SECØND HARMØNIC 0.16234499E-02    EARTH THIRD HARMØNIC 0.57500000E-05    EARTH FØURTH HARMØNIC 0.78749999E-05
EARTH GRAVITATIØNAL CØNSTANT (KILØMETERS CUBED/SECØNDS SQUARED) 0.39860319E 06
EQUATØRIAL RADIUS (KILØMETERS) 0.63781650E 04     ELLIPTICITY 0.29830000E 03

                      ***** BALLISTIC PARAMETERS *****

ANGLE ØF ATTACK FUNCTIØN
ALPHA(DEGREES)       ANØMALY(DEGREES)
 0.                   0.36000000E 03

CØEFFICIENT ØF DRAG FUNCTIØN
CN                   ALPHA(DEGREES)
 0.20000000E 01       0.36000000E 03

CDPRIME              PERIGEE(KILØMETERS)
 1.00000000E 00       0.

EFFECTIVE DRAG AREA FUNCTIØN
AREA(METERS SQUARED)    ALPHA(DEGREES)
 0.26040000E 02          0.36000000E 03

MASS CØNSTANTS
INITIAL MASS(KILØGRAMS) 0.60399999E 04

                      ***** DENSITY PARAMETERS *****

MØNTH= 0.40E 01     DAY= 0.80E 01     YEAR 0.1964E 04     DAYS ELAPSED SINCE DEC. 31 , 1957 0.22899999E 04
1959 ARDC ATMØSPHERE

DENSITY CØRRECTIØN
DC                   PERIGEE(KILØMETERS)
 0.12000000E-00       0.50000000E 03
 0.13000000E-00       0.40000000E 03
 0.14200000E-00       0.34999999E 03
 0.18400000E-00       0.30000000E 03
 0.22000000E-00       0.27999999E 03
 0.27500000E-00       0.26000000E 03
 0.30400000E-00       0.25000000E 03
 0.34000000E-00       0.23999999E 03
 0.38500000E-00       0.23000000E 03
 0.42500000E-00       0.22000000E 03
 0.47000000E-00       0.20999999E 03
 0.52000000E 00       0.20000000E 03
 0.56500000E 00       0.19000000E 03
 0.62000000E 00       0.18000000E 03
 0.70000000E 00       0.16999999E 03
 0.80000000E 00       0.16000000E 03
 0.84000000E 00       0.15499999E 03
 0.86000000E 00       0.15000000E 03
 1.00000000E 00       0.14500000E 03
 1.00000000E 00       0.

KP                   YEAR
 0.23400000E 01       0.19580000E 04
 0.24800000E 01       0.19588000E 04
 0.18900000E 01       0.19588999E 04
 0.25099999E 01       0.19589999E 04
 0.29700000E 01       0.19591000E 04
 0.24199999E 01       0.19592000E 04
 0.25600000E 01       0.19593000E 04
 0.25900000E 01       0.19594000E 04
```

```
0.29200000E 01        0.19594999E 04
0.31599999E 01        0.19596000E 04
0.34999999E 01        0.19597000E 04
0.24600000E 01        0.19598000E 04
0.28900000E 01        0.19599000E 04
0.25699999E 01        0.19599999E 04
0.23199999E 01        0.19601000E 04
0.24400000E 01        0.19602000E 04
0.34199999E 01        0.19603000E 04
0.27899999E 01        0.19604000E 04
0.29200000E 01        0.19605000E 04
0.27100000E 01        0.19605999E 04
0.27500000E 01        0.19606999E 04
0.32299999E 01        0.19608000E 04
0.34400000E 01        0.19609000E 04
0.24900000E 01        0.19610000E 04
0.22700000E 01        0.19611000E 04
0.23199999E 01        0.19611999E 04
0.22899999E 01        0.19613000E 04
0.23999999E 01        0.19614000E 04
0.26899999E 01        0.19615000E 04
0.22600000E 01        0.19616000E 04
0.21799999E 01        0.19616999E 04
0.18499999E 01        0.19617999E 04
0.19199999E 01        0.19619000E 04
0.14900000E 01        0.19620000E 04
0.17299999E 01        0.19621000E 04
0.18099999E 01        0.19622000E 04
0.23100000E 01        0.19622999E 04
0.16000000E 01        0.19623999E 04
0.21799999E 01        0.19625000E 04
0.26199999E 01        0.19626000E 04
0.29399999E 01        0.19627000E 04
0.30800000E 01        0.19628000E 04
0.20100000E 01        0.19628999E 04
0.17500000E 01        0.19620000E 04
0.17200000E 01        0.19631000E 04
0.15400000E 01        0.19631200E 04
0.15099999E 01        0.19632100E 04
0.18600000E 01        0.19632900E 04
0.20800000E 01        0.19633800E 04
0.20599999E 01        0.19634600E 04
0.22300000E 01        0.19635399E 04
0.23499999E 01        0.19636200E 04
0.32600000E 01        0.19637100E 04
0.21999999E 01        0.19637900E 04
0.20200000E 01        0.19638800E 04
0.19800000E 01        0.19639600E 04
0.20599999E 01        0.19640400E 04
0.22099999E 01        0.19641200E 04
0.21600000E 01        0.19642100E 04
0.22499999E 01        0.19642900E 04
0.18099999E 01        0.19643800E 04
0.17299999E 01        0.19644599E 04
0.18900000E 01        0.19645400E 04
0.16800000E 01        0.19646200E 04
0.17800000E 01        0.19647100E 04
0.16700000E 01        0.19647899E 04
0.90000000E 00        0.19648800E 04
0.77000000E 00        0.19649599E 04
0.24999999E 01        0.19650000E 04
0.24999999E 01        0.20000000E 04

FTEN                  YEAR
 0.                    0.
```

```
FTENB                        YEAR
 0.24360000E 03               0.19580000E 04
 0.23070000E 03               0.19585000E 04
 0.22650000E 03               0.19589999E 04
 0.20890000E 03               0.19594999E 04
 0.18049999E 03               0.19599999E 04
 0.16100000E 03               0.19605000E 04
 0.13079999E 03               0.19610000E 04
 0.10479999E 03               0.19615000E 04
 0.99300000E 02               0.19620000E 04
 0.89699999E 02               0.19625000E 04
 0.82699999E 02               0.19630000E 04
 0.80800000E 02               0.19634999E 04
 0.77899999E 02               0.19639999E 04
 0.70000000E 02               0.19645000E 04
 0.75000000E 02               0.19650000E 04
 0.87000000E 02               0.19655000E 04
 0.13100000E 03               0.19665000E 04
 0.18600000E 03               0.19675000E 04
 0.20000000E 03               0.19684999E 04
 0.19000000E 03               0.19695000E 04
 0.16300000E 03               0.19705000E 04
 0.14200000E 03               0.19710000E 04
 0.12800000E 03               0.19715000E 04
 0.10800000E 03               0.19724999E 04
 0.94000000E 02               0.19735000E 04
 0.80999999E 02               0.19745000E 04
 0.75000000E 02               0.19750000E 04
 0.75000000E 02               0.19755000E 04

DIURNAL    MEAN

                                  ***** SPECIAL EVENTS *****

EARTH IMPACT CUTOFF

                                  ***** INITIAL CONDITIONS *****

DETAIL PRINTOUT

ANOMALY STEP(DEGREES) 0.99999999E 01

APOGEE STEPS(KM)                   PERIGEE RADIUS(KM)
-0.49999999E 01                      0.67780000E 04
-0.99999999E 01                      0.65780000E 04
-0.20000000E 02                      0.
 0.                                  0.
 0.                                  0.
```

```
APØGEE,PERIGEE,MAJØR AXIS,AND EARTH RADIUS(KM)
APØGEE,PERIGEE,MAJØR AXIS RATES(KM/DAY) MASS(KG)
ASCENDING NØDE,ARGUMENT ØF PERIGEE(DEG)
NØDE,PERIGEE REGRESSIØN RATES(DEG/DAY)
PERIGEE VELØCITY(KM/SEC)
ØRBITAL PERIØD(MIN)
LIFETIME SPENT(ØRBIT AND DAY)
RHØ(KG/M3), EI(UNITLESS), RIPERG AND RIPAPG(KM)

A       0.66987178E 04   P       0.65384161E 04   AXIS    0.66185670E 04   RADIUS 0.63734720E 04
ADØT   -0.20172292E 02   PDØT   -0.40069964E 01   AXIDØT-0.12089644E 02   MASS   0.60399999E 04
NØDE    0.81400000E 02   ARGP    0.60406999E 02   DNØDE  -0.73808376E 01   DARGP  0.11164444E 02
VPERIG 0.78563969E 01   PERIØD 0.89310846E 02   ØRBIT   0.                TIME   0.
RHØ     0.10019261E-08   EI      0.12110000E-01   RIPERG 0.16281848E 03   RIPAPG 0.32312018E 03

A       0.66887178E 04   P       0.65363847E 04   AXIS    0.66125512E 04   RADIUS 0.63729899E 04
ADØT   -0.21854386E 02   PDØT   -0.45366886E 01   AXIDØT-0.13195537E 02   MASS   0.60399999E 04
NØDE    0.77741100E 02   ARGP    0.65941543E 02   DNØDE  -0.74041695E 01   DARGP  0.11199737E 02
VPERIG 0.78550523E 01   PERIØD 0.89189111E 02   ØRBIT   0.76963580E 01   TIME   0.47668843E-00
RHØ     0.10763352E-08   EI      0.11518484E-01   RIPERG 0.16103314E 03   RIPAPG 0.31368445E 03

A       0.66787178E 04   P       0.65342648E 04   AXIS    0.66064913E 04   RADIUS 0.63726118E 04
ADØT   -0.23836359E 02   PDØT   -0.51580800E 01   AXIDØT-0.14497220E 02   MASS   0.60399999E 04
NØDE    0.74353144E 02   ARGP    0.71066251E 02   DNØDE  -0.74277838E 01   DARGP  0.11235456E 02
VPERIG 0.78538398E 01   PERIØD 0.89066537E 02   ØRBIT   0.14785457E 02   TIME   0.91516171E 00
RHØ     0.11652293E-08   EI      0.10932652E-01   RIPERG 0.15911090E 03   RIPAPG 0.30412115E 03

A       0.66687178E 04   P       0.65320580E 04   AXIS    0.66003878E 04   RADIUS 0.63723329E 04
ADØT   -0.26187366E 02   PDØT   -0.58897937E 01   AXIDØT-0.16038579E 02   MASS   0.60399999E 04
NØDE    0.71236986E 02   ARGP    0.75779830E 02   DNØDE  -0.74516790E 01   DARGP  0.11271600E 02
VPERIG 0.78527560E 01   PERIØD 0.88943137E 02   ØRBIT   0.21272498E 02   TIME   0.13158407E 01
RHØ     0.12720751E-08   EI      0.10352410E-01   RIPERG 0.15705157E 03   RIPAPG 0.29444018E 03

A       0.66587178E 04   P       0.65297680E 04   AXIS    0.65942429E 04   RADIUS 0.63721424E 04
ADØT   -0.29008224E 02   PDØT   -0.67591456E 01   AXIDØT-0.17883664E 02   MASS   0.60399999E 04
NØDE    0.68391461E 02   ARGP    0.80084042E 02   DNØDE  -0.74758501E 01   DARGP  0.11308163E 02
VPERIG 0.78517914E 01   PERIØD 0.88818957E 02   ØRBIT   0.27161858E 02   TIME   0.16790955E 01
RHØ     0.14009906E-08   EI      0.97774551E-02   RIPERG 0.15486291E 03   RIPAPG 0.28465612E 03

A       0.66487178E 04   P       0.65273999E 04   AXIS    0.65880588E 04   RADIUS 0.63720265E 04
ADØT   -0.32431474E 02   PDØT   -0.78004355E 01   AXIDØT-0.20115954E 02   MASS   0.60399999E 04
NØDE    0.65814312E 02   ARGP    0.83982302E 02   DNØDE  -0.75002896E 01   DARGP  0.11345130E 02
VPERIG 0.78509342E 01   PERIØD 0.88694043E 02   ØRBIT   0.32462037E 02   TIME   0.20055498E 01
RHØ     0.15571679E-08   EI      0.92074110E-02   RIPERG 0.15255591E 03   RIPAPG 0.27478613E 03

A       0.66387178E 04   P       0.65249590E 04   AXIS    0.65818384E 04   RADIUS 0.63719703E 04
ADØT   -0.36582107E 02   PDØT   -0.90565055E 01   AXIDØT-0.22819306E 02   MASS   0.60399999E 04
NØDE    0.63501654E 02   ARGP    0.87480488E 02   DNØDE  -0.75249887E 01   DARGP  0.11382490E 02
VPERIG 0.78501709E 01   PERIØD 0.88568456E 02   ØRBIT   0.37189028E 02   TIME   0.22962075E 01
RHØ     0.17472959E-08   EI      0.86418749E-02   RIPERG 0.15014362E 03   RIPAPG 0.26484796E 03

A       0.66287178E 04   P       0.65224504E 04   AXIS    0.65755841E 04   RADIUS 0.63719590E 04
ADØT   -0.41668416E 02   PDØT   -0.10584679E 02   AXIDØT-0.26126547E 02   MASS   0.60399999E 04
NØDE    0.61444640E 02   ARGP    0.90591979E 02   DNØDE  -0.75499392E 01   DARGP  0.11420231E 02
VPERIG 0.78494892E 01   PERIØD 0.88442247E 02   ØRBIT   0.41366296E 02   TIME   0.25528480E 01
RHØ     0.19801785E-08   EI      0.80804481E-02   RIPERG 0.14763916E 03   RIPAPG 0.25485864E 03

A       0.66187178E 04   P       0.65198807E 04   AXIS    0.65692993E 04   RADIUS 0.63719793E 04
ADØT   -0.48006608E 02   PDØT   -0.12472527E 02   AXIDØT-0.30237567E 02   MASS   0.60399999E 04
NØDE    0.59632730E 02   ARGP    0.93332718E 02   DNØDE  -0.75751306E 01   DARGP  0.11458337E 02
VPERIG 0.78488757E 01   PERIØD 0.88315480E 02   ØRBIT   0.45016393E 02   TIME   0.27768318E 01
RHØ     0.22675510E-08   EI      0.75226466E-02   RIPERG 0.14505602E 03   RIPAPG 0.24483331E 03
```

```
A      0.66087178E 04   P      0.65172561E 04   AXIS   0.65629869E 04   RADIUS 0.63720200E 04
ADØT  -0.55973225E 02   PDØT  -0.14833832E 02   AXIDØT-0.35403528E 02   MASS   0.60399999E 04
NØDE   0.58054795E 02   ARGP   0.95719543E 02   DNØDE -0.76005523E 01   DARGP  0.11496790E 02
VPERIG 0.78483187E 01   PERIØD 0.88188218E 02   ØRBIT  0.48174605E 02   TIME   0.29701240E 01
RHØ    0.26253967E-08   EI     0.69680004E-02   RIPERG 0.14240667E 03   RIPAPG 0.23478466E 03

A      0.65987178E 04   P      0.65145827E 04   AXIS   0.65566503E 04   RADIUS 0.63720719E 04
ADØT  -0.66103096E 02   PDØT  -0.17819200E 02   AXIDØT-0.41961148E 02   MASS   0.60399999E 04
NØDE   0.56696904E 02   ARGP   0.97773523E 02   DNØDE -0.76261927E 01   DARGP  0.11535574E 02
VPERIG 0.78478073E 01   PERIØD 0.88060527E 02   ØRBIT  0.50869267E 02   TIME   0.31349111E 01
RHØ    0.30758083E-08   EI     0.64160102E-02   RIPERG 0.13970275E 03   RIPAPG 0.22472332E 03

A      0.65887178E 04   P      0.65118655E 04   AXIS   0.65502916E 04   RADIUS 0.63721281E 04
ADØT  -0.79264738E 02   PDØT  -0.21702515E 02   AXIDØT-0.50483626E 02   MASS   0.60399999E 04
NØDE   0.55543222E 02   ARGP   0.99518611E 02   DNØDE -0.76520441E 01   DARGP  0.11574678E 02
VPERIG 0.78473331E 01   PERIØD 0.87932458E 02   ØRBIT  0.53137720E 02   TIME   0.32734323E 01
RHØ    0.36504593E-08   EI     0.58663261E-02   RIPERG 0.13695270E 03   RIPAPG 0.21465692E 03

A      0.65787178E 04   P      0.65091074E 04   AXIS   0.65439127E 04   RADIUS 0.63721834E 04
ADØT  -0.96654691E 02   PDØT  -0.26856437E 02   AXIDØT-0.61755563E 02   MASS   0.60399999E 04
NØDE   0.54577845E 02   ARGP   0.10097887E 03   DNØDE -0.76781022E 01   DARGP  0.11614094E 02
VPERIG 0.78468911E 01   PERIØD 0.87804039E 02   ØRBIT  0.55017389E 02   TIME   0.33880452E 01
RHØ    0.43957919E-08   EI     0.53187103E-02   RIPERG 0.13416217E 03   RIPAPG 0.20459143E 03

A      0.65687178E 04   P      0.65063073E 04   AXIS   0.65375125E 04   RADIUS 0.63722347E 04
ADØT  -0.12024515E 03   PDØT  -0.33940001E 02   AXIDØT-0.77092577E 02   MASS   0.60399999E 04
NØDE   0.53783459E 02   ARGP   0.10218047E 03   DNØDE -0.77043711E 01   DARGP  0.11653829E 02
VPERIG 0.78464800E 01   PERIØD 0.87675259E 02   ØRBIT  0.56546763E 02   TIME   0.34811621E 01
RHØ    0.53830783E-08   EI     0.47732574E-02   RIPERG 0.13133202E 03   RIPAPG 0.19453058E 03

A      0.65487178E 04   P      0.65005544E 04   AXIS   0.65246361E 04   RADIUS 0.63723277E 04
ADØT  -0.20129748E 03   PDØT  -0.59263927E 02   AXIDØT-0.13028070E 03   MASS   0.60399999E 04
NØDE   0.52502015E 02   ARGP   0.10411882E 03   DNØDE -0.77576034E 01   DARGP  0.11734350E 02
VPERIG 0.78457771E 01   PERIØD 0.87416356E 02   ØRBIT  0.58709557E 02   TIME   0.36124563E 01
RHØ    0.86214397E-08   EI     0.36908918E-02   RIPERG 0.12552423E 03   RIPAPG 0.17441961E 03

A      0.65287178E 04   P      0.64944185E 04   AXIS   0.65115681E 04   RADIUS 0.63723896E 04
ADØT  -0.39824621E 03   PDØT  -0.12856697E 03   AXIDØT-0.26340659E 03   MASS   0.60399999E 04
NØDE   0.51731255E 02   ARGP   0.10528468E 03   DNØDE -0.78121534E 01   DARGP  0.11816864E 02
VPERIG 0.78453854E 01   PERIØD 0.87153862E 02   ØRBIT  0.59921826E 02   TIME   0.36858271E 01
RHØ    0.15900792E-07   EI     0.26337263E-02   RIPERG 0.11935028E 03   RIPAPG 0.15434387E 03

A      0.65087178E 04   P      0.64875169E 04   AXIS   0.64981174E 04   RADIUS 0.63724229E 04
ADØT  -0.11186879E 04   PDØT  -0.41531130E 03   AXIDØT-0.76699963E 03   MASS   0.60399999E 04
NØDE   0.51338927E 02   ARGP   0.10587813E 03   DNØDE -0.78688589E 01   DARGP  0.11902638E 02
VPERIG 0.78456540E 01   PERIØD 0.86883955E 02   ØRBIT  0.60463034E 02   TIME   0.37184814E 01
RHØ    0.39444161E-07   EI     0.16313130E-02   RIPERG 0.11242608E 03   RIPAPG 0.13430102E 03

A      0.64887178E 04   P      0.64790406E 04   AXIS   0.64838792E 04   RADIUS 0.63724351E 04
ADØT  -0.70584279E 04   PDØT  -0.36614551E 04   AXIDØT-0.53599416E 04   MASS   0.60399999E 04
NØDE   0.51198246E 02   ARGP   0.10609093E 03   DNØDE -0.79295006E 01   DARGP  0.11994366E 02
VPERIG 0.78473248E 01   PERIØD 0.86598551E 02   ØRBIT  0.60608292E 02   TIME   0.37272170E 01
RHØ    0.18488389E-06   EI     0.74624963E-03   RIPERG 0.10393823E 03   RIPAPG 0.11428174E 03

A      0.64687178E 04   P      0.64654747E 04   AXIS   0.64670962E 04   RADIUS 0.63724370E 04
ADØT  -0.13953651E 06   PDØT  -0.11068825E 06   AXIDØT-0.12511238E 06   MASS   0.60399999E 04
NØDE   0.51175778E 02   ARGP   0.10612491E 03   DNØDE -0.80017868E 01   DARGP  0.12103708E 02
VPERIG 0.78536133E 01   PERIØD 0.86262540E 02   ØRBIT  0.60625134E 02   TIME   0.37282259E 01
RHØ    0.26460587E-05   EI     0.25074344E-03   RIPERG 0.90364990E 02   RIPAPG 0.94273559E 02

A      0.64487178E 04   P      0.64478089E 04   AXIS   0.64482634E 04   RADIUS 0.63724373E 04
ADØT  -0.34856556E 07   PDØT  -0.32746775E 07   AXIDØT-0.33801666E 07   MASS   0.60399999E 04
NØDE   0.51174630E 02   ARGP   0.10612664E 03   DNØDE -0.90839223E 01   DARGP  0.12227948E 02
VPERIG 0.78633704E 01   PERIØD 0.85886906E 02   ØRBIT  0.60625828E 02   TIME   0.37282673E 01
RHØ    0.64216545E-04   EI     0.70479095E-04   RIPERG 0.72692382E 02   RIPAPG 0.74266540E 02
```

```
A       0.64287178E 04   P        0.64284948E 04   AXIS    0.64286062E 04   RADIUS 0.63724373E 04
ADØT   -0.36040654E 08   PDØT    -0.35568516E 08   AXIDØT-0.35804585E 08   MASS   0.60399999E 04
NØDE    0.51174583E 02   ARGP     0.10612672E 03   DNØDE -0.81708138E 01   DARGP  0.12359383E 02
VPERIG  0.78752603E 01   PERIØD   0.85493578E 02   ØRBIT   0.60625864E 02   TIME   0.37282695E 01
RHØ     0.72545402E-03   EI       0.17350844E-04   RIPERG  0.53371032E 02   RIPAPG 0.54259277E 02
----------------------------------------------------------------------------------------------------
A       0.64087178E 04   P        0.64086128E 04   AXIS    0.64086653E 04   RADIUS 0.63724373E 04
ADØT   -0.57522210E 09   PDØT    -0.57563592E 09   AXIDØT-0.57542901E 09   MASS   0.60399999E 04
NØDE    0.51174579E 02   ARGP     0.10612672E 03   DNØDE -0.82601909E 01   DARGP  0.12494577E 02
VPERIG  0.78874360E 01   PERIØD   0.85096096E 02   ØRBIT   0.60625867E 02   TIME   0.37282697E 01
RHØ     0.10391000E-01   EI       0.81952714E-05   RIPERG  0.33481811E 02   RIPAPG 0.34251953E 02
----------------------------------------------------------------------------------------------------
A       0.63887178E 04   P        0.63887154E 04   AXIS    0.63887166E 04   RADIUS 0.63724373E 04
ADØT   -0.12913613E 11   PDØT    -0.12713984E 11   AXIDØT-0.12813799E 11   MASS   0.60399999E 04
NØDE    0.51174578E 02   ARGP     0.10612672E 03   DNØDE -0.83508641E 01   DARGP  0.12631731E 02
VPERIG  0.78996825E 01   PERIØD   0.84699079E 02   ØRBIT   0.60625867E 02   TIME   0.37282697E 01
RHØ     0.24382363E-00   EI       0.18629493E-06   RIPERG  0.13577026E 02   RIPAPG 0.14244629E 02
----------------------------------------------------------------------------------------------------
A       0.63687178E 04   P        0.63691773E 04   AXIS    0.63689475E 04   RADIUS 0.63724373E 04
ADØT   -0.45995754E 11   PDØT    -0.46000738E 11   AXIDØT-0.45998245E 11   MASS   0.60399999E 04
NØDE    0.51174578E 02   ARGP     0.10612672E 03   DNØDE -0.84419879E 01   DARGP  0.12769567E 02
VPERIG  0.79116516E 01   PERIØD   0.84306248E 02   ØRBIT   0.60625867E 02   TIME   0.37282697E 01
RHØ     0.12254324E 01   EI      -0.36066525E-04   RIPERG-0.59685668E 01   RIPAPG-0.57626952E 01
----------------------------------------------------------------------------------------------------
A       0.63687178E 04   P        0.63691773E 04   AXIS    0.63689475E 04   RADIUS 0.63724373E 04
ADØT   -0.45995754E 11   PDØT    -0.46000738E 11   AXIDØT-0.45998245E 11   MASS   0.60399999E 04
NØDE    0.51174578E 02   ARGP     0.10612672E 03   DNØDE -0.84419879E 01   DARGP  0.12769567E 02
VPERIG  0.79116516E 01   PERIØD   0.84306248E 02   ØRBIT   0.60625867E 02   TIME   0.37282697E 01
RHØ     0.12254324E 01   EI      -0.36066525E-04   RIPERG-0.59685668E 01   RIPAPG-0.57626952E 01
```

```
          EARTH ORBIT TRANSFORMATION

PHIO   =   0.2850000000000000D 02  DEG     LAMDO =   0.8050000000000000D 02  DEG      AZFIR =   0.1050000000000000D 03  DEG
A      =   0.6378165000000000D 04  KM      B     =   0.6356783999999999D 04  KM       OMEGA =   0.1504106705000000D 02  DEG/HR
RADIUS=    0.6378165000000000D 04  KM      KERTH =   0.3986032000000000D 06  KM3/SEC2  J    =   0.1623450000000000D-02
H      =   0.5750000000000000D-05          D     =   0.7875000000000000D-05            XG   =   0.                      KM
YG     =   0.                      KM      ZG    =   0.                         KM     XDG   =   0.                     KM/SEC
YDG    =   0.                      KM/SEC  ZDG   =   0.                         KM/SEC TFM   =   0.                      HOURS

INPUT IN MKT SYSTEM SHORT  NOLONG
POSITION (KM)    VELOCITY (KM/SEC)    ANGLES (DEG)
EFP
XE     =  -0.3037077883473387D 04     YE     =  -0.1046142628496821D 04     ZE     =   0.2505989924336480D 04
XDE    =   0.5957246065928864D 01     YDE    =   0.1620505487157563D 01     ZDE    =   0.3982714918979543D 01

PLT
XPL    =  -0.3037077883473385D 04     YPL    =  -0.1046142628496822D 04     ZPL    =   0.2505989924336479D 04
XDPL   =   0.5979686903636678D 01     YDPL   =   0.1850067816184628D 01     ZDPL   =   0.4105741525364290D 01

EFE
XEP    =  -0.2630237597954840D 04     YEP    =  -0.5986299572715291D 04     ZEP    =   0.1089689476602931D 04
XDEP   =   0.5318120342908635D 01     YDEP   =  -0.3161077262043163D 01     ZDEP   =  -0.3962580881258863D 01

SFE
XS     =  -0.2630237597954840D 04     YS     =  -0.5986299572715291D 04     ZS     =   0.1089689476602931D 04
XDS    =   0.5754648197067024D 01     YDS    =  -0.3352877214687555D 01     ZDS    =  -0.3962580881258863D 01
TIME   =  -0.                         THETA  =  -0.

SFG
R      =   0.6628827615152297D 04     GCLAT  =   0.9461594291796026D 01     LONG   =   0.2462804560176325D 03
VS     =   0.7749826367686954D 01     AZVS   =   0.1213593542378151D 03     ELVS   =   0.6884420409938637D 00

EFG
R      =   0.6628827615152297D 04     GCLAT  =   0.9461594291796026D 01     LONG   =   0.2462804560176325D 03
VE     =   0.7346894628261875D 01     AZVE   =   0.1232949670245073D 03     ELVE   =   0.7262007992049315D 00

OOE
AXIS   =   0.6620903169689641D 04     ECCEN  =   0.1207474821557146D-01     INC    =   0.3261583446622047D 02
ASNOD  =   0.8137605735536910D 02     ARGP   =   0.6586622223696307D 02     TANOM  =   0.9637668968299269D 02
EANOM  =   0.9568865963933066D 02     MANOM  =   0.9500023465684445D 02

MOE
AXIS   =   0.6618566999999999D 04     ECCEN  =   0.1211000000000000D-01     INC    =   0.3259999999999999D 02
ASNOD  =   0.8139999999999999D 02     ARGP   =   0.6040699999999999D 02     TANOM  =   0.1018090000000000D 03
EANOM  =   0.1011289736909772D 03     MANOM  =   0.1004481695590716D 03

AUXILIARY CALCULATIONS
APOGEE =   0.3226839084233206D 03   KM       PERIGEE=  0.1627924309559598D 03   KM     PERIOD =   0.8935813214091380D 02  MIN
RANGE  =   0.4058034132300353D 04   KM       ALT    =  0.2512432309216819D 03   KM     TPITCH =   0.8938692960398236D 02  DEG
ECV    =  -0.4641984480867217D-02 KM/SEC     ELV    = -0.3216647944953037D 01 KM/SEC   MAPOGEE=   0.3205528463699984D 03   KM
MPERIGE=   0.1602511536299994D 03   KM       MPERIOD=  0.8931084161151252D 02  MIN     DARGP  =   0.1117929829829709D 02  DEG/DAY
CASNOD =  -0.7393552696386070D 01  DEG/DAY
```

```
                    ***** EARTH CØNSTANTS *****

EARTH SECØND HARMØNIC 0.16234499E-02     EARTH THIRD HARMØNIC 0.57500000E-05     EARTH FØURTH HARMØNIC 0.78749999E-05
EARTH GRAVITATIØNAL CØNSTANT (KILØMETERS CUBED/SECØNDS SQUARED) 0.39860319E 06
EQUATØRIAL RADIUS (KILØMETERS) 0.63781650E 04     ELLIPTICITY 0.29830000E 03

                    ***** BALLISTIC PARAMETERS *****

ANGLE ØF ATTACK FUNCTIØN
ALPHA(DEGREES)      ANØMALY(DEGREES)
 0.                  0.36000000E 03

CØEFFICIENT ØF DRAG FUNCTIØN
CN                  ALPHA(DEGREES)
 0.20000000E 01      0.36000000E 03

CDPRIME             PERIGEE(KILØMETERS)
 1.00000000E 00      0.

EFFECTIVE DRAG AREA FUNCTIØN
AREA(METERS SQUARED)   ALPHA(DEGREES)
 0.26040000E 02         0.36000000E 03

MASS CØNSTANTS
INITIAL MASS(KILØGRAMS) 0.60399999E 04

                    ***** DENSITY PARAMETERS *****

MØNTH= 0.40E 01     DAY= 0.80E 01     YEAR 0.1964E 04     DAYS ELAPSED SINCE DEC. 31 , 1957 0.22899999E 04
1959 ARDC ATMØSPHERE

DENSITY CØRRECTIØN
DC                  PERIGEE(KILØMETERS)
 0.12000000E-00      0.50000000E 03
 0.13000000E-00      0.40000000E 03
 0.14200000E-00      0.34999999E 03
 0.18400000E-00      0.30000000E 03
 0.22000000E-00      0.27999999E 03
 0.27500000E-00      0.26000000E 03
 0.30400000E-00      0.25000000E 03
 0.34000000E-00      0.23999999E 03
 0.38500000E-00      0.23000000E 03
 0.42500000E-00      0.22000000E 03
 0.47000000E-00      0.20999999E 03
 0.52000000E 00      0.20000000E 03
 0.56500000E 00      0.19000000E 03
 0.62000000E 00      0.18000000E 03
 0.70000000E 00      0.16999999E 03
 0.80000000E 00      0.16000000E 03
 0.84000000E 00      0.15499999E 03
 0.86000000E 00      0.15000000E 03
 1.00000000E 00      0.14500000E 03
 1.00000000E 00      0.

KP                  YEAR
 0.23400000E 01      0.19580000E 04
 0.24800000E 01      0.19588000E 04
 0.18900000E 01      0.19588999E 04
 0.25099999E 01      0.19589999E 04
 0.29700000E 01      0.19591000E 04
 0.24199999E 01      0.19592000E 04
 0.25600000E 01      0.19593000E 04
 0.25900000E 01      0.19594000E 04
```

```
 0.29200000E 01           0.19594999E 04
 0.31599999E 01           0.19596000E 04
 0.34999999E 01           0.19597000E 04
 0.24600000E 01           0.19598000E 04
 0.28900000E 01           0.19599000E 04
 0.25699999E 01           0.19599999E 04
 0.23199999E 01           0.19601000E 04
 0.24400000E 01           0.19602000E 04
 0.34199999E 01           0.19603000E 04
 0.27899999E 01           0.19604000E 04
 0.29200000E 01           0.19605000E 04
 0.27100000E 01           0.19605999E 04
 0.27500000E 01           0.19606999E 04
 0.32299999E 01           0.19608000E 04
 0.34400000E 01           0.19609000E 04
 0.24900000E 01           0.19610000E 04
 0.22700000E 01           0.19611000E 04
 0.23199999E 01           0.19611999E 04
 0.22899999E 01           0.19613000E 04
 0.23999999E 01           0.19614000E 04
 0.26899999E 01           0.19615000E 04
 0.22600000E 01           0.19616000E 04
 0.21799999E 01           0.19616999E 04
 0.18499999E 01           0.19617999E 04
 0.19199999E 01           0.19619000E 04
 0.14900000E 01           0.19620000E 04
 0.17299999E 01           0.19621000E 04
 0.18099999E 01           0.19622000E 04
 0.23100000E 01           0.19622999E 04
 0.16000000E 01           0.19623999E 04
 0.21799999E 01           0.19625000E 04
 0.26199999E 01           0.19626000E 04
 0.29399999E 01           0.19627000E 04
 0.30800000E 01           0.19628000E 04
 0.20100000E 01           0.19628999E 04
 0.17500000E 01           0.19620000E 04
 0.17200000E 01           0.19631000E 04
 0.15400000E 01           0.19631200E 04
 0.15099999E 01           0.19632100E 04
 0.18600000E 01           0.19632900E 04
 0.20800000E 01           0.19633800E 04
 0.20599999E 01           0.19634600E 04
 0.22300000E 01           0.19635399E 04
 0.23499999E 01           0.19636200E 04
 0.32600000E 01           0.19637100E 04
 0.21999999E 01           0.19637900E 04
 0.20200000E 01           0.19638800E 04
 0.19800000E 01           0.19639600E 04
 0.20599999E 01           0.19640400E 04
 0.22099999E 01           0.19641200E 04
 0.21600000E 01           0.19642100E 04
 0.22499999E 01           0.19642900E 04
 0.18099999E 01           0.19643800E 04
 0.17299999E 01           0.19644599E 04
 0.18900000E 01           0.19645400E 04
 0.16800000E 01           0.19646200E 04
 0.17600000E 01           0.19647100E 04
 0.16700000E 01           0.19647899E 04
 0.90000000E 00           0.19648800E 04
 0.77000000E 00           0.19649599E 04
 0.24999999E 01           0.19650000E 04
 0.24999999E 01           0.20000000E 04

FTEN                     YEAR
 0.                       0.
```

```
 FIENB                    YEAR
  0.24360000E 03           0.19580000E 04
  0.23070000E 03           0.19585000E 04
  0.22650000E 03           0.19589999E 04
  0.20890000E 03           0.19594999E 04
  0.18049999E 03           0.19599999E 04
  0.16100000E 03           0.19605000E 04
  0.13079999E 03           0.19610000E 04
  0.10479999E 03           0.19615000E 04
  0.99300000E 02           0.19620000E 04
  0.89699999E 02           0.19625000E 04
  0.82699999E 02           0.19630000E 04
  0.80800000E 02           0.19634999E 04
  0.77899999E 02           0.19639999E 04
  0.70000000E 02           0.19645000E 04
  0.75000000E 02           0.19650000E 04
  0.87000000E 02           0.19655000E 04
  0.13100000E 03           0.19665000E 04
  0.18600000E 03           0.19675000E 04
  0.20000000E 03           0.19684999E 04
  0.19000000E 03           0.19695000E 04
  0.16300000E 03           0.19705000E 04
  0.14200000E 03           0.19710000E 04
  0.12800000E 03           0.19715000E 04
  0.10800000E 03           0.19724999E 04
  0.94000000E 02           0.19735000E 04
  0.80999999E 02           0.19745000E 04
  0.75000000E 02           0.19750000E 04
  0.75000000E 02           0.19755000E 04

DIURNAL    MEAN

                                  ***** SPECIAL EVENTS *****

EARTH IMPACT CUT0FF

                                  ***** INITIAL C0NDITI0NS *****

SH0RT PRINT0UT

AN0MALY STEP(DEGREES) 0.99999999E 01

AP0GEE STEPS(KM)                 PERIGEE RADIUS(KM)
-0.49999999E 01                    0.67780000E 04
-0.99999999E 01                    0.65780000E 04
-0.20000000E 02                    0.
 0.                                0.
 0.                                0.
```

```
APØGEE,PERIGEE,MAJØR AXIS,AND,EARTH RADIUS(KM)
APØGEE,PERIGEE,AND MAJØR AXIS RATES(KM/DAY)
LIFETIME SPENT(ØRBIT AND DAY)

A        0.66987178E 04   P        0.65384161E 04   AXIS    0.66185670E 04   RADIUS 0.63734720E 04
ADØT    -0.20172292E 02   PDØT    -0.40069964E 01   AXIDØT-0.12089644E 02   ØRBIT  0.                TIME     0.

A        0.66887178E 04   P        0.65363847E 04   AXIS    0.66125512E 04   RADIUS 0.63729899E 04
ADØT    -0.21854386E 02   PDØT    -0.45366886E 01   AXIDØT-0.13195537E 02   ØRBIT  0.76963580E 01   TIME     0.47668843E-00

A        0.66787178E 04   P        0.65342648E 04   AXIS    0.66064913E 04   RADIUS 0.63726118E 04
ADØT    -0.23836359E 02   PDØT    -0.51580800E 01   AXIDØT-0.14497220E 02   ØRBIT  0.14785457E 02   TIME     0.91516171E 00

A        0.66687178E 04   P        0.65320580E 04   AXIS    0.66003878E 04   RADIUS 0.63723329E 04
ADØT    -0.26187366E 02   PDØT    -0.58897937E 01   AXIDØT-0.16038579E 02   ØRBIT  0.21272498E 02   TIME     0.13158407E 01

A        0.66587178E 04   P        0.65297680E 04   AXIS    0.65942429E 04   RADIUS 0.63721424E 04
ADØT    -0.29008224E 02   PDØT    -0.67591456E 01   AXIDØT-0.17883684E 02   ØRBIT  0.27161858E 02   TIME     0.16790955E 01

A        0.66487178E 04   P        0.65273999E 04   AXIS    0.65880588E 04   RADIUS 0.63720265E 04
ADØT    -0.32431474E 02   PDØT    -0.78004355E 01   AXIDØT-0.20115954E 02   ØRBIT  0.32462037E 02   TIME     0.20055498E 01

A        0.66387178E 04   P        0.65249590E 04   AXIS    0.65818384E 04   RADIUS 0.63719703E 04
ADØT    -0.36582107E 02   PDØT    -0.90565055E 01   AXIDØT-0.22819306E 02   ØRBIT  0.37189028E 02   TIME     0.22962875E 01

A        0.66287178E 04   P        0.65224504E 04   AXIS    0.65755841E 04   RADIUS 0.63719590E 04
ADØT    -0.41668416E 02   PDØT    -0.10584679E 02   AXIDØT-0.26126547E 02   ØRBIT  0.41366296E 02   TIME     0.25528480E 01

A        0.66187178E 04   P        0.65198607E 04   AXIS    0.65692993E 04   RADIUS 0.63719793E 04
ADØT    -0.48006608E 02   PDØT    -0.12472527E 02   AXIDØT-0.30239567E 02   ØRBIT  0.45018393E 02   TIME     0.27768318E 01

A        0.66087178E 04   P        0.65172561E 04   AXIS    0.65629869E 04   RADIUS 0.63720200E 04
ADØT    -0.55973225E 02   PDØT    -0.14833832E 02   AXIDØT-0.35403528E 02   ØRBIT  0.48174605E 02   TIME     0.29701240E 01

A        0.65987178E 04   P        0.65145827E 04   AXIS    0.65566503E 04   RADIUS 0.63720719E 04
ADØT    -0.66103096E 02   PDØT    -0.17819200E 02   AXIDØT-0.41961148E 02   ØRBIT  0.50869267E 02   TIME     0.31349111E 01

A        0.65887178E 04   P        0.65118655E 04   AXIS    0.65502916E 04   RADIUS 0.63721281E 04
ADØT    -0.79264738E 02   PDØT    -0.21702515E 02   AXIDØT-0.50483626E 02   ØRBIT  0.53137720E 02   TIME     0.32734323E 01

A        0.65787178E 04   P        0.65091074E 04   AXIS    0.65439127E 04   RADIUS 0.63721834E 04
ADØT    -0.96654691E 02   PDØT    -0.26856437E 02   AXIDØT-0.61755563E 02   ØRBIT  0.55017389E 02   TIME     0.33880452E 01

A        0.65687178E 04   P        0.65063073E 04   AXIS    0.65375125E 04   RADIUS 0.63722347E 04
ADØT    -0.12024515E 03   PDØT    -0.33940001E 02   AXIDØT-0.77092577E 02   ØRBIT  0.56546763E 02   TIME     0.34811621E 01

A        0.65487178E 04   P        0.65005544E 04   AXIS    0.65246361E 04   RADIUS 0.63723277E 04
ADØT    -0.20129748E 03   PDØT    -0.59263927E 02   AXIDØT-0.13028070E 03   ØRBIT  0.58709557E 02   TIME     0.36124563E 01

A        0.65287178E 04   P        0.64944185E 04   AXIS    0.65115681E 04   RADIUS 0.63723896E 04
ADØT    -0.39824621E 03   PDØT    -0.12856697E 03   AXIDØT-0.26340659E 03   ØRBIT  0.59921826E 02   TIME     0.36858271E 01

A        0.65087178E 04   P        0.64875169E 04   AXIS    0.64981174E 04   RADIUS 0.63724229E 04
ADØT    -0.11186879E 04   PDØT    -0.41531130E 03   AXIDØT-0.76699963E 03   ØRBIT  0.60463034E 02   TIME     0.37184814E 01

A        0.64887178E 04   P        0.64790406E 04   AXIS    0.64838792E 04   RADIUS 0.63724351E 04
ADØT    -0.70584279E 04   PDØT    -0.36614551E 04   AXIDØT-0.53599416E 04   ØRBIT  0.60608292E 02   TIME     0.37272170E 01

A        0.64687178E 04   P        0.64654747E 04   AXIS    0.64670962E 04   RADIUS 0.63724370E 04
ADØT    -0.13953651E 06   PDØT    -0.11068825E 06   AXIDØT-0.12511238E 06   ØRBIT  0.60625134E 02   TIME     0.37282259E 01

A        0.64487178E 04   P        0.64478089E 04   AXIS    0.64482634E 04   RADIUS 0.63724373E 04
ADØT    -0.34856556E 07   PDØT    -0.32746775E 07   AXIDØT-0.33801666E 07   ØRBIT  0.60625828E 02   TIME     0.37282673E 01
```

```
A      0.64287178E 04    P       0.64284948E 04   AXIS    0.64286062E 04   RADIUS 0.63724373E 04
ADOT  -0.36040654E 08    PDOT   -0.35568516E 08   AXIDOT-0.35804585E 08   ORBIT  0.60625864E 02   TIME    0.37282695E 01

A      0.64087178E 04    P       0.64086128E 04   AXIS    0.64086653E 04   RADIUS 0.63724373E 04
ADOT  -0.57522210E 09    PDOT   -0.57563592E 09   AXIDOT-0.57542901E 09   ORBIT  0.60625867E 02   TIME    0.37282697E 01
-------------------------------------------------------------------------------------------------------------------------
A      0.63887178E 04    P       0.63887154E 04   AXIS    0.63887166E 04   RADIUS 0.63724373E 04
ADOT  -0.12913613E 11    PDOT   -0.12713984E 11   AXIDOT-0.12813799E 11   ORBIT  0.60625867E 02   TIME    0.37282697E 01

A      0.63687178E 04    P       0.63691773E 04   AXIS    0.63689475E 04   RADIUS 0.63724373E 04
ADOT  -0.45995754E 11    PDOT   -0.46000738E 11   AXIDOT-0.45998245E 11   ORBIT  0.60625867E 02   TIME    0.37282697E 01
-------------------------------------------------------------------------------------------------------------------------
A      0.63687178E 04    P       0.63691773E 04   AXIS    0.63689475E 04   RADIUS 0.63724373E 04
ADOT  -0.45995754E 11    PDOT   -0.46000738E 11   AXIDOT-0.45998245E 11   ORBIT  0.60625867E 02   TIME    0.37282697E 01
```

SECTION IV: GRAPHIC METHOD FOR LIFETIME PREDICTION

A lifetime model has been presented for making reasonably accurate predictions of orbital lifetime. Primary factors influencing earth orbital satellite lifetimes have been included. A method has been devised to provide a means of graphically predicting lifetime based on this model. This graphic technique allows a quick prediction of lifetime independent of computer runs.

Two sets of normalized lifetime curves have been generated, one using the 1959 ARDC density model as the basic reference and the other using the 1962 U. S. Standard atmospheric density model. These sets are presented in Figures 8, 9, and 10 for 1959 ARDC and 11, 12, and 13 for the 1962 U. S. Standard. Figures 9, 10, 12, and 13 are blown up altitude regions of Figures 8 and 11. A mean diurnal bulge and constant values of one (1) for vehicle mass, area and drag coefficients are assumed. The normalized lifetime read from these charts is L_1. For both references the formula for computing lifetime is then:

$$\text{Lifetime} = L_1 \left[\frac{m}{C_D A}\right] (f_{i,\omega}) (f_d)$$

where

L_1 = normalized lifetime which is a function of apogee and perigee altitudes

m = orbiting mass in kg

C_D = orbital drag coefficient

A = effective drag area in square meters

$f_{i,\omega}$ = correction factor to the normalized reference for initial orbital inclination and argument of perigee. The normalized reference assumes values of

$i = 30°$ $\qquad \omega = 180°$

f_d = correction factor to the normalized reference for the actual calendar dates the satellite is in orbit and the initial perigee altitude. This correction is required to account for the variation of density with solar and geomagnetic activity which varies with time. The value of of f_d for future years is based upon current predictions of these effects.

The step-wise procedure to be followed in predicting lifetime is as follows:

1. Compute perigee and apogee altitudes in km. Use the earth radius defined at the sub perigee point in computing altitude (see Table 2). If the i and ω necessary to define this radius are not known, use the equatorial earth radius of 6378.165 km in computing altitude.

2. For the specified perigee altitude read the L_1 corresponding to the given apogee altitude, interpolating between lines of constant apogee altitude if required. Depending on altitude region of interest use Figures 8, 9, or 10 for ARDC reference and Figures 11, 12, or 13 for 1962 U. S. Standard.

3. Compute A, effective area. For attitude stabilized vehicles this area is the surface projection on a plane perpendicular to the direction of vehicle motion. Projected areas are computed as follows:

Nose-on ($\alpha = 0°$)	Broadside ($\alpha = 90°$)
Cone $A = \frac{\pi D^2}{4}$	$A = DL/2$
Cylinder $A = \frac{\pi D^2}{4}$	$A = DL$

A = projected area (m^2)

D = vehicle diameter (m)

L = vehicle length (m)

α = angle of attack

For random tumbling bodies A is computed as one-fourth the total surface area.

4. Determine drag coefficient, C_D, from Figure 14. These data are extracted from Reference 8. The following values are for a 200 km altitude:

Cone, nose-on = 2.06

Cylinder broadside = 2.2

Tumbling Body = 2.18

5. Compute the ratio M/C_DA in $\frac{kg}{m^2}$

6. To obtain total lifetime in days based on 1959 ARDC or 1962 U. S. Standard reference, multiply M/C_DA by the value read from the graph to obtain L_2. ($i = 30°$, $\omega = 180°$)

$$L_2 = L_1 \, M/C_DA$$

If something more than the apogee and perigee values are known about the orbit or a prediction for later launch dates is desired, using the 1959 ARDC or 1962 U. S. Standard predictions computed in step 6 continue with the following steps.

7. For given values of i and ω read $f(i,\omega)$ in Figure 15, interpolating between lines of constant inclination if necessary. Note that the figure consists of two sets of curves, one for use when $L_2 < 30$ days and the other for use when $L_2 \geq 30$ days. Multiply L_2 by $f(i, \omega)$ to obtain L_3.

$$L_3 = L_2 \, f(i,\omega)$$

L_3 is then lifetime in days for proper inclination and argument of perigee. Omit this step if i and ω are not defined and the mean radius was used in step 1.

8. For the given launch date and perigee altitude enter Figure 16 or Figure 17 for the 1959 or 1962 atmospheres respectively and obtain an average value of the ordinate for the given perigee altitude over the interval of time from the launch date (year and fraction) over the time (in years) corresponding to L_3 days. The same lifetime will be obtained regardless of which base is used. This average is f_d which is multiplied by L_3 to obtain the first lifetime estimate $L_3(1)$.

$$L_3(1) = L_3 f_d$$

These f_d correction factors are current only for the date of this publication. As new data become available and the solar activity prediction is updated, this f_d factor must be updated.

9. Step 8 is repeated using $L_3(1)$ instead of L_3 to perform the average. However, when the new value of f_d is obtained it is multiplied by L_3 not $L_3(1)$ to obtain $L_3(2)$.

$$L_3(2) = L_3 f_d$$

Depending upon the accuracy desired and the variations in successive values of f_d, step 8 may be repeated until $L_3(n)$ is obtained which differs insignificantly from $L_3(n-1)$.

Primary uncertainties associated with these predictions are a 25% uncertainty in C_D and uncertainty in prediction of f_d. The latter is a function of time and altitude. To obtain three sigma lifetime values for f_d uncertainty, Figures 18 and 19 or 20 and 21 should be used for the 1959 or 1962 atmospheres respectively in the same manner as given in step 8 for Figures 16 and 17 and nominal lifetime. Three sigma curves are given only for future dates as there is no prediction uncertainty associated with past solar activity behavior.

To assess the accuracy of the graphic technique, a total of 103 comparisons was made between lifetimes predicted using the computer deck and lifetimes predicted using the graphic technique. It was found that a maximum of three iterations as specified in step 9 was sufficient for convergence. For short lifetimes only one iteration was necessary. The ratio of the program value and the graphic value was computed for all cases. The results of this yielded a mean and standard deviation of 1.01 and .15 respectively. From the results of this comparison no systematic errors seemed to be present in the graphic technique.

As well as total lifetime, time spent in decaying from one circular altitude to another can also be obtained by subtracting the respective lifetime values. For elliptical orbits, the time spent in decaying from one apogee to another can be found in a like manner if the perigee could be assumed fixed. However, this assumption could cause a significant error since the perigee will in actuality experience a decay whose effect cannot be neglected.

TABLE II. EARTH RADIUS AS A FUNCTION OF INCLINATION (i) AND ARGUMENT OF PERIGEE (ω)

ω i	0 180 360	15 165 195 345	30 150 210 330	45 135 225 315	60 120 240 300	75 105 255 285	90 270
15	6378.2	6378.1	6377.9	6377.5	6377.1	6376.8	6376.7
30	6378.2	6377.9	6376.9	6375.5	6374.2	6373.2	6372.8
45	6378.2	6377.5	6375.5	6372.8	6370.2	6368.2	6367.5
60	6378.2	6377.1	6374.2	6370.2	6366.1	6363.3	6362.1
75	6378.2	6376.8	6373.2	6368.2	6363.3	6359.6	6358.2
90	6378.2	6376.7	6372.8	6367.5	6362.1	6358.2	6356.8

Computed from

$$R_E = 6378.165 \left[1 - \frac{1}{298.3} (\sin^2 i \sin^2 \omega) \right]$$

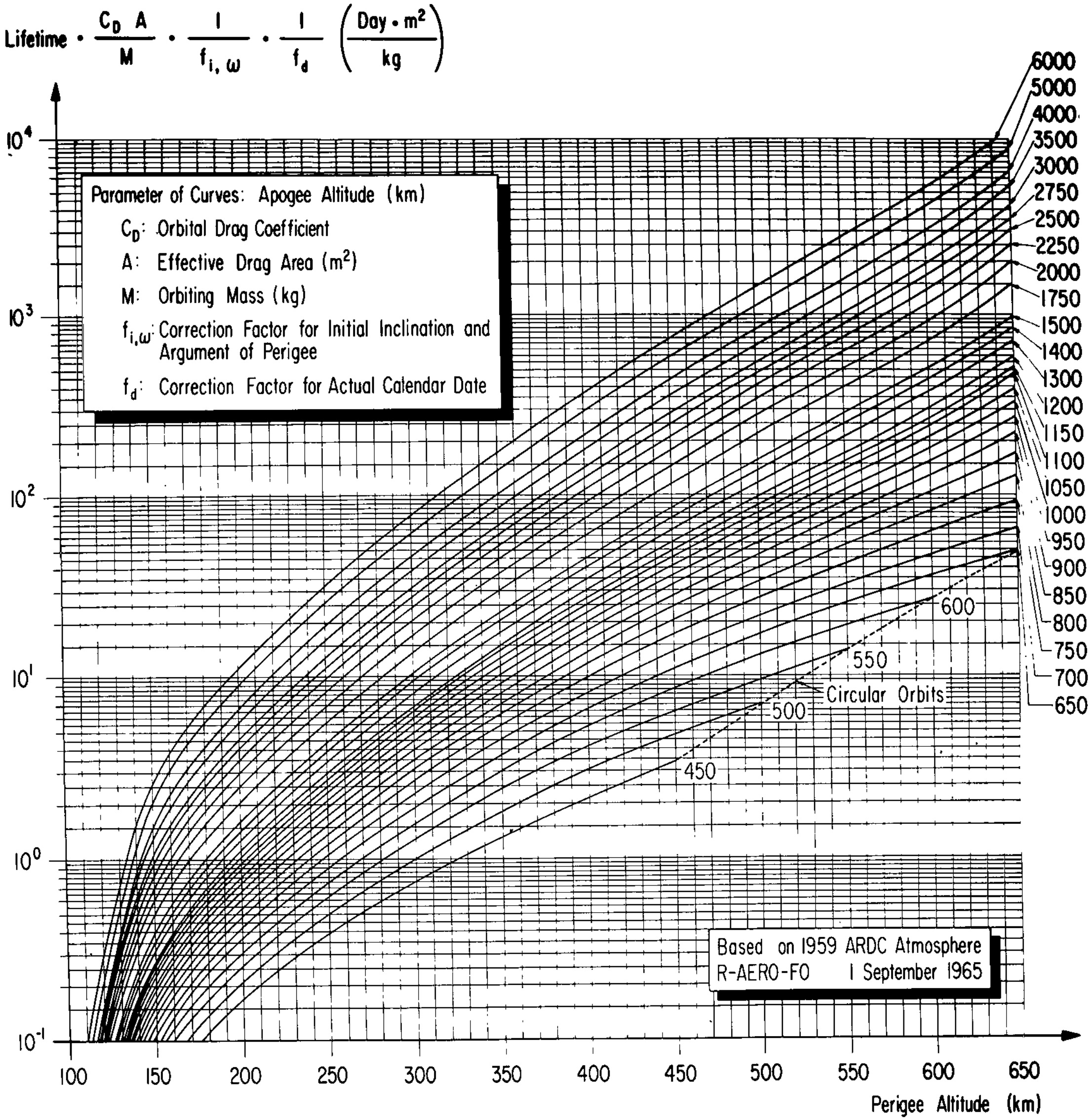
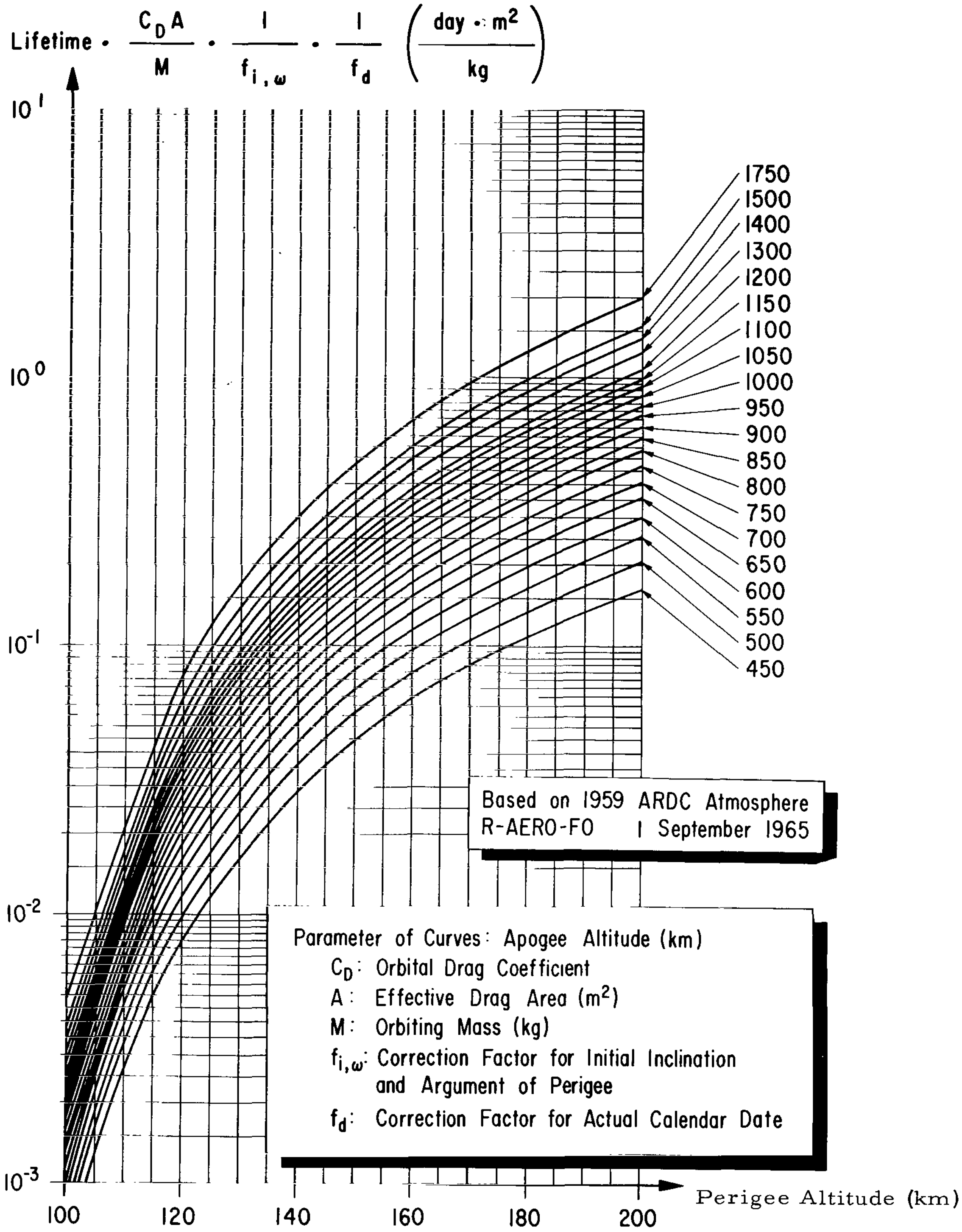


FIGURE 8. EARTH ORBITAL LIFETIME
Perigee: 100 - 650 km
Apogee: 450 - 600 km



Based on 1959 ARDC Atmosphere
R-AERO-FO 1 September 1965

Parameter of Curves: Apogee Altitude (km)
C_D: Orbital Drag Coefficient
A: Effective Drag Area (m^2)
M: Orbiting Mass (kg)
$f_{i,\omega}$: Correction Factor for Initial Inclination and Argument of Perigee
f_d: Correction Factor for Actual Calendar Date

FIGURE 9. EARTH ORBITAL LIFETIME
Perigee: 100 - 200 km
Apogee: 450 - 1750 km

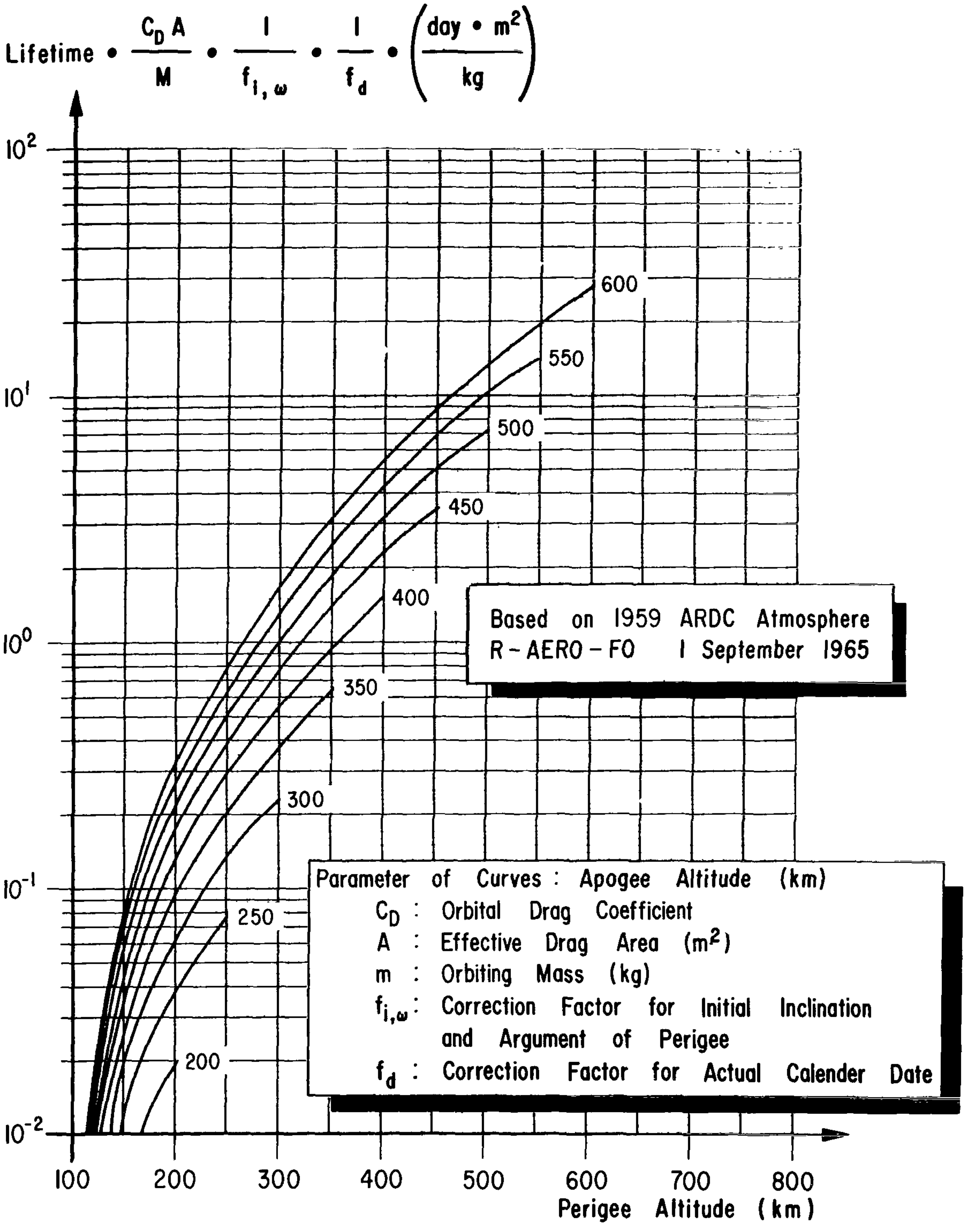


FIGURE 10. EARTH ORBITAL LIFETIME
Perigee: 100 - 600 km
Apogee: 200 - 600 km

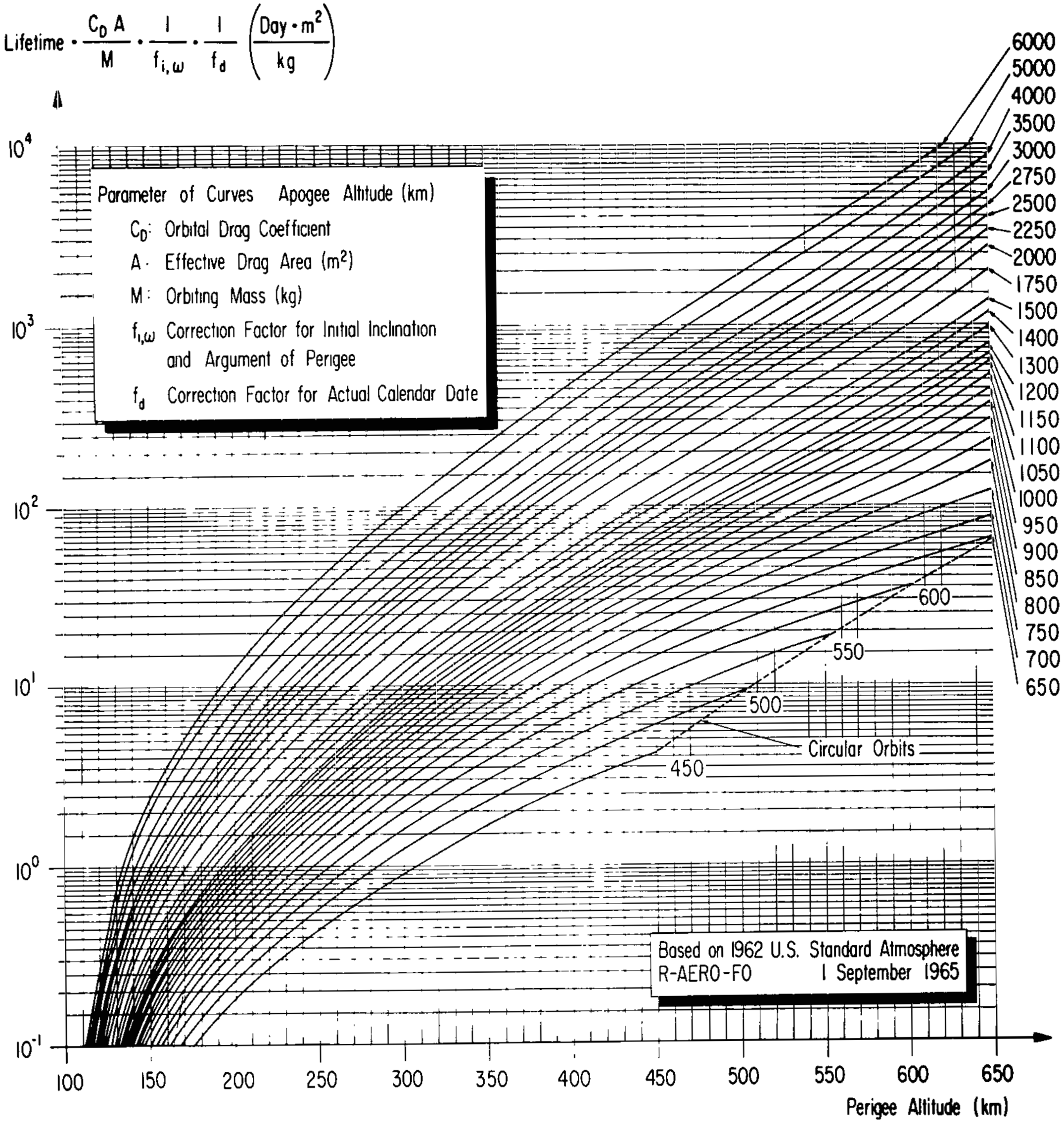


FIGURE 11. EARTH ORBITAL LIFETIME
Perigee: 100 - 650 km
Apogee: 450 - 6000 km

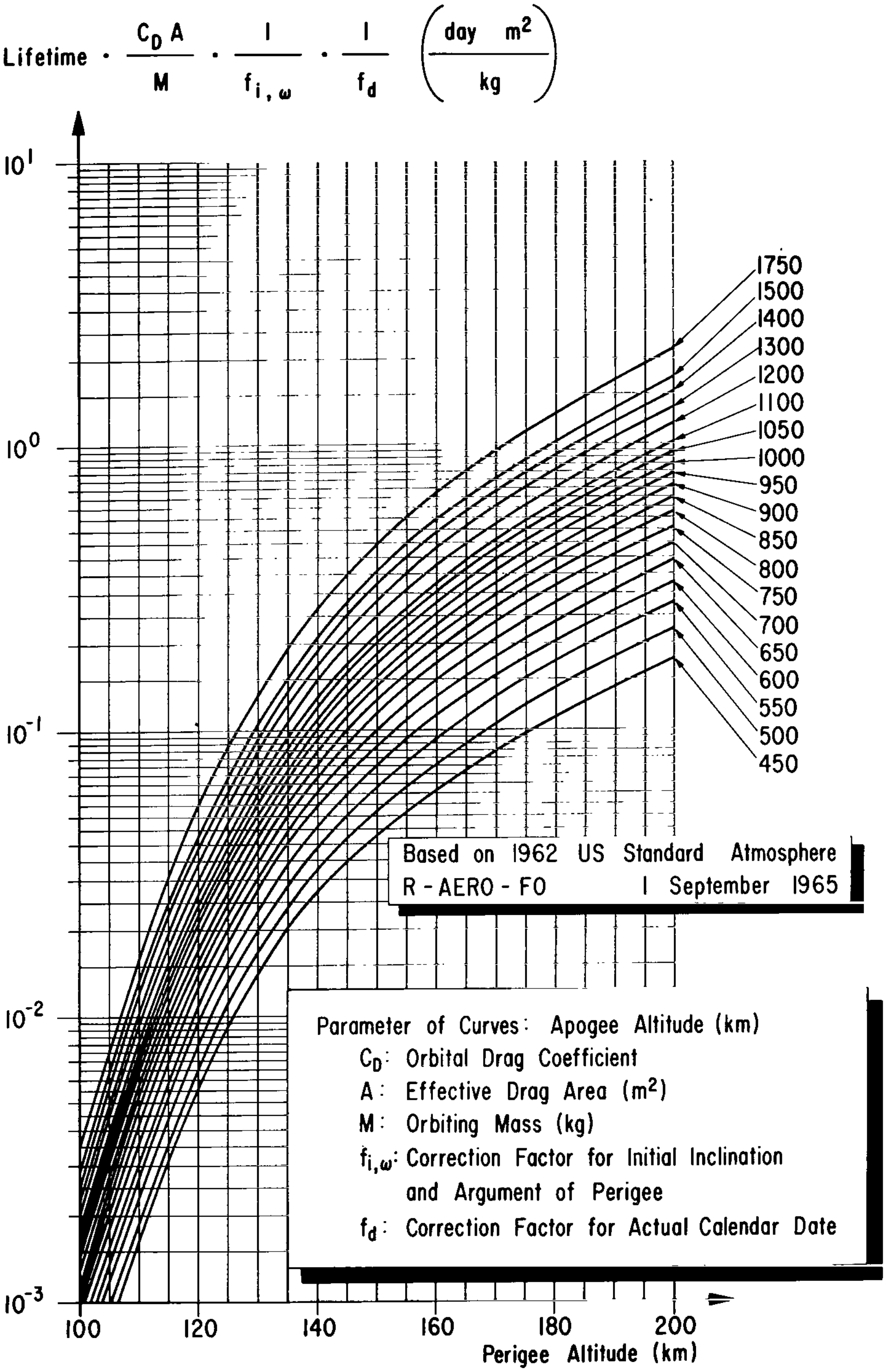


FIGURE 12. EARTH ORBITAL LIFETIME
Perigee: 100 - 200 km
Apogee: 450 - 1750 km

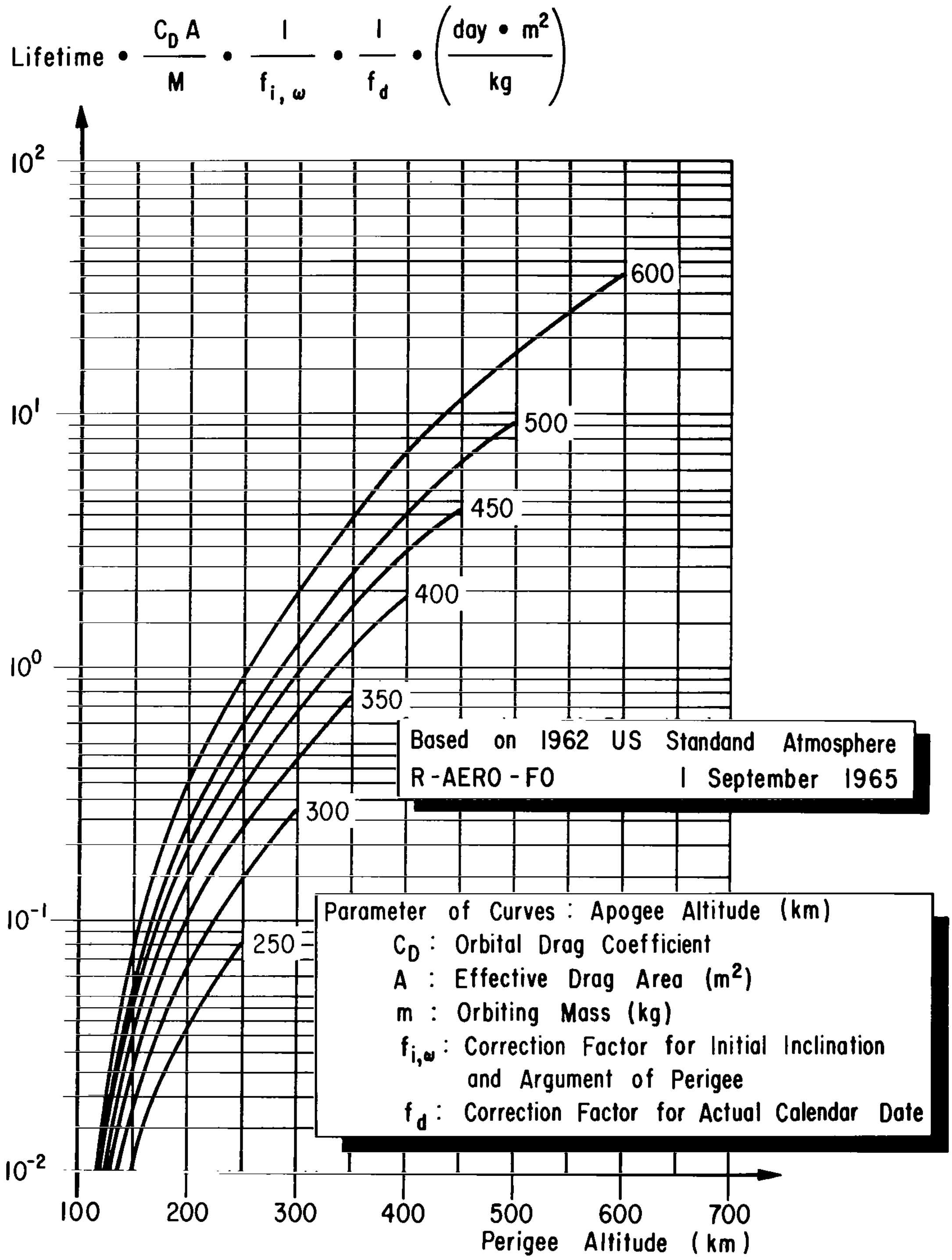


FIGURE 13. EARTH ORBITAL LIFETIME
Perigee: 100 - 600 km
Apogee: 250 - 600 km

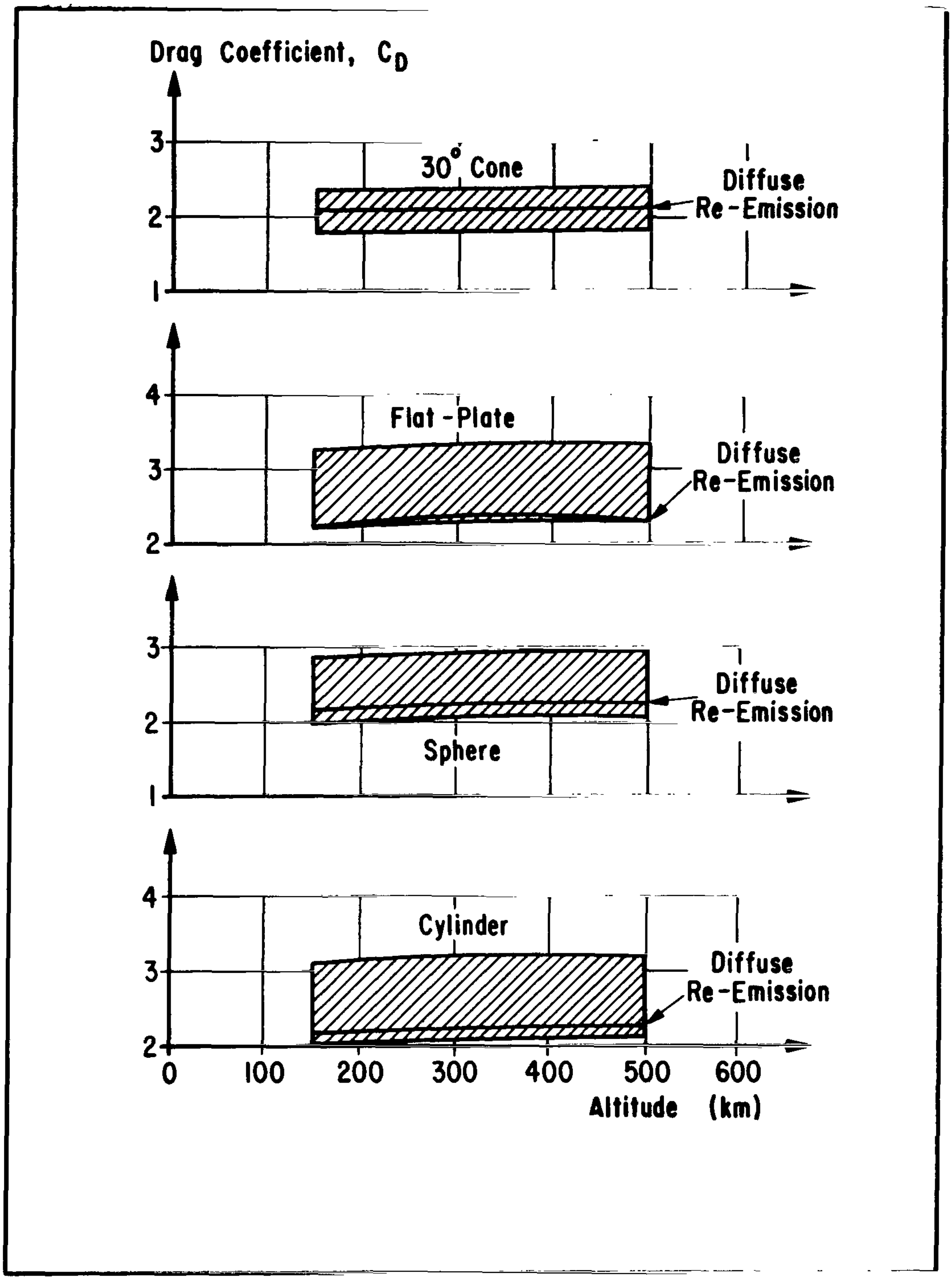


FIGURE 14. ORBITAL DRAG COEFFICIENT

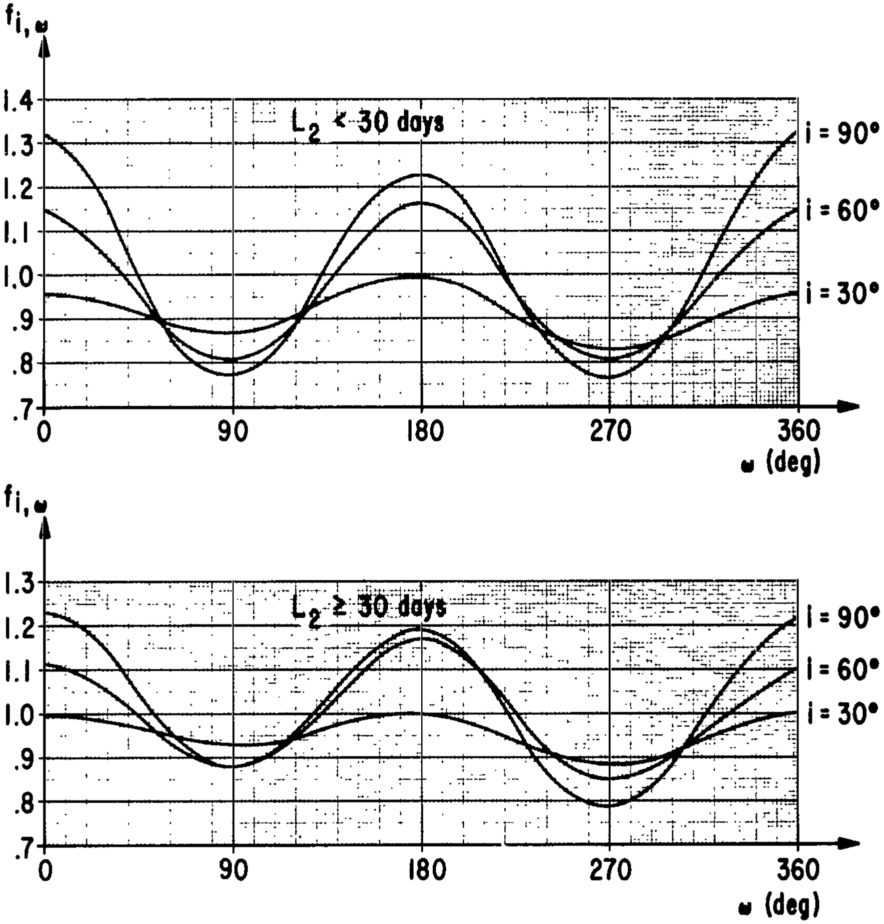


FIGURE 15. $f_{i,\ \omega}$ CORRECTION FACTOR

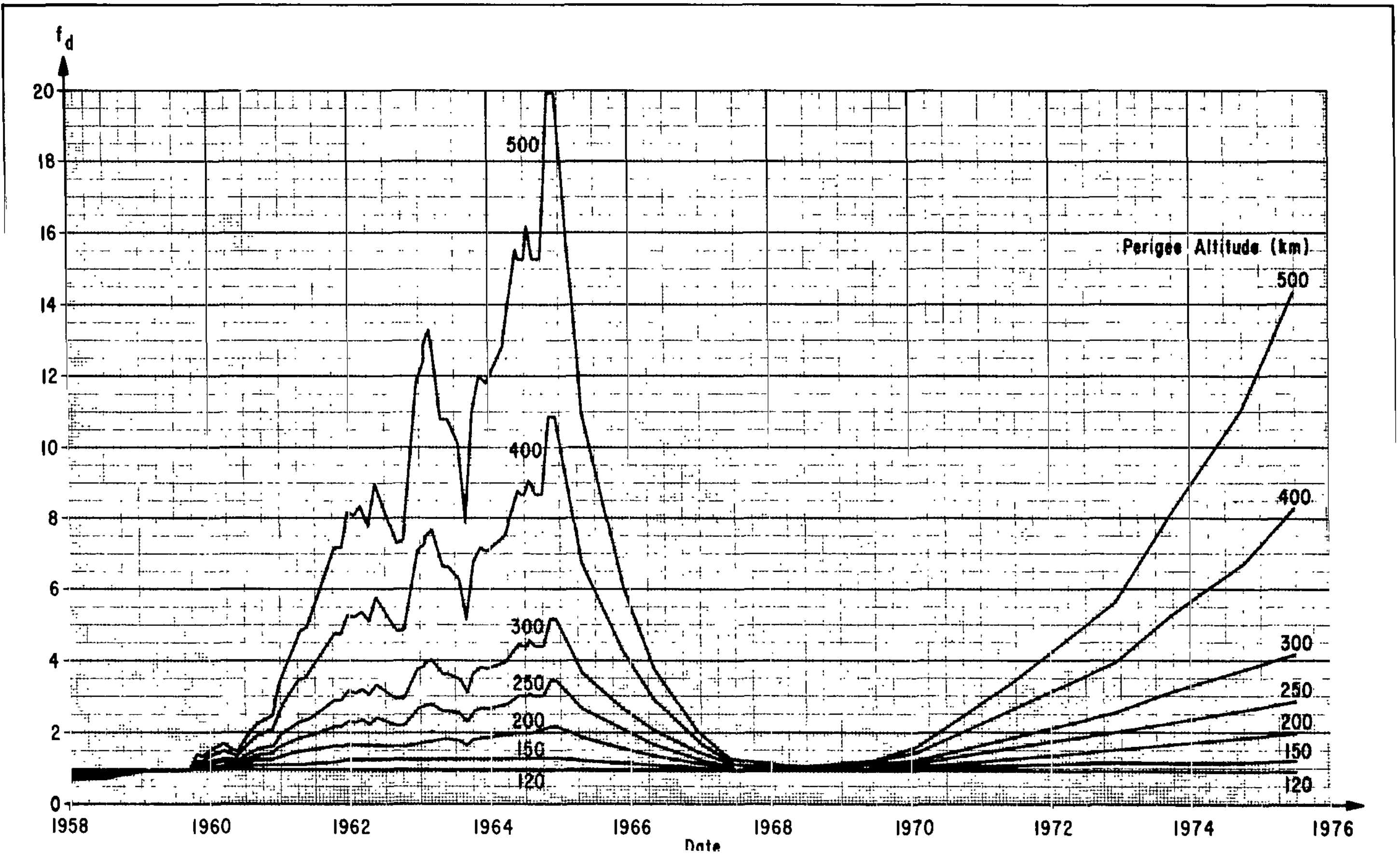


FIGURE 16. f_d CORRECTION FACTOR FOR NOMINAL LIFETIME: 1959 ARDC REFERENCE

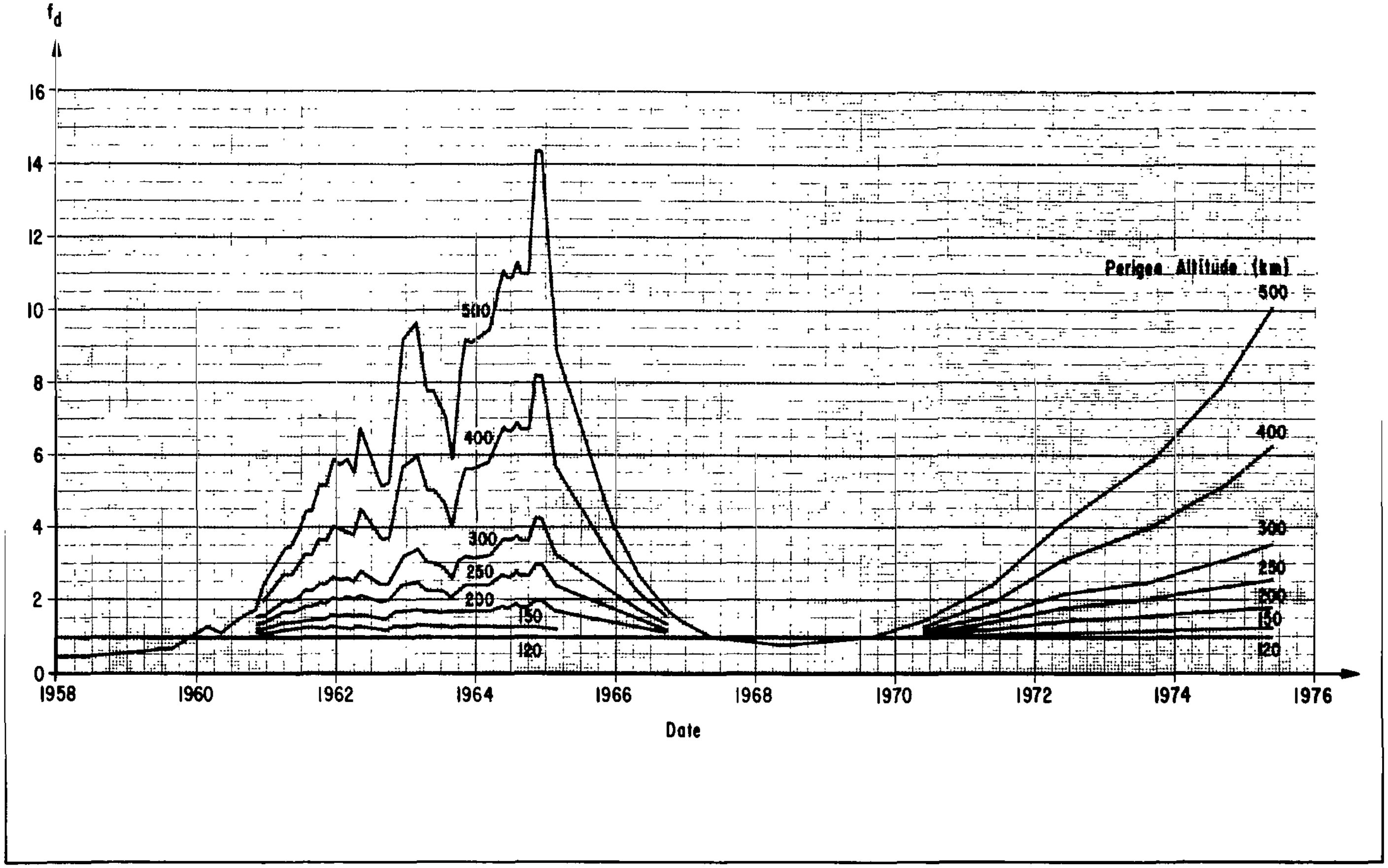


FIGURE 17. f_d CORRECTION FACTOR FOR NOMINAL LIFETIME: 1962 U. S. STANDARD REFERENCE

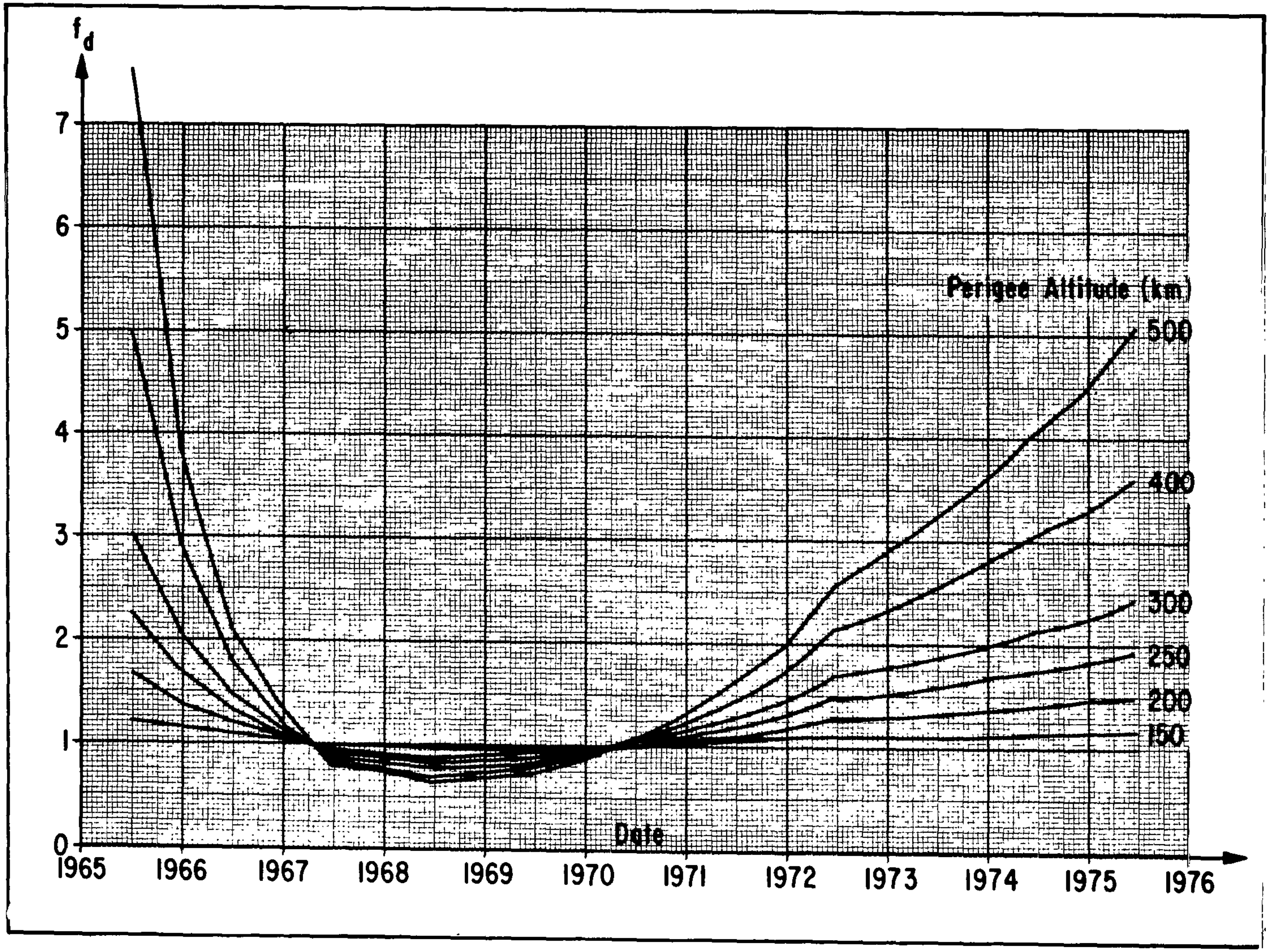


FIGURE 18. f_d CORRECTION FACTOR FOR -3σ LIFETIME: 1959 ARDC REFERENCE

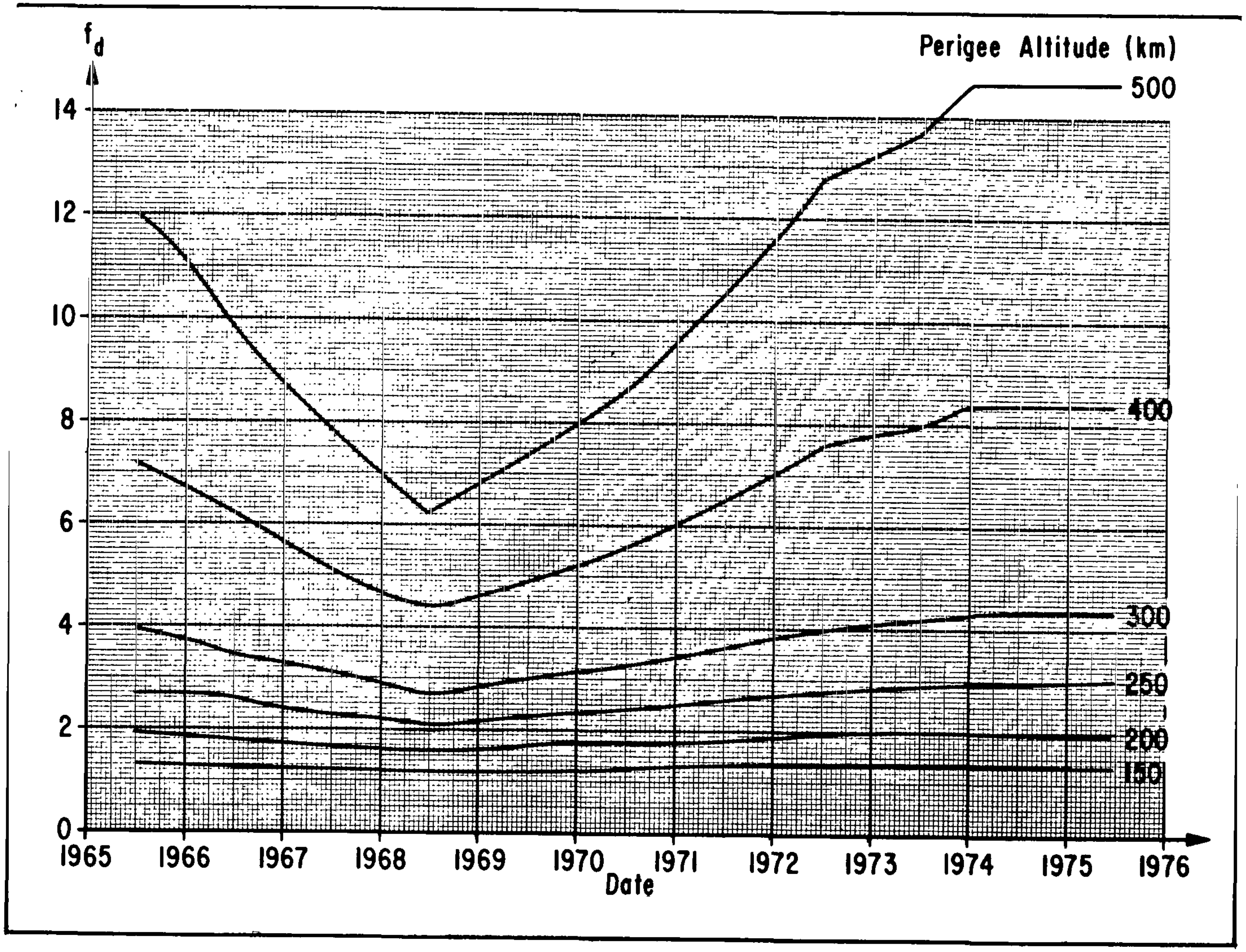


FIGURE 19. f_d CORRECTION FACTOR FOR $+3\sigma$ LIFETIME: 1959 ARDC REFERENCE

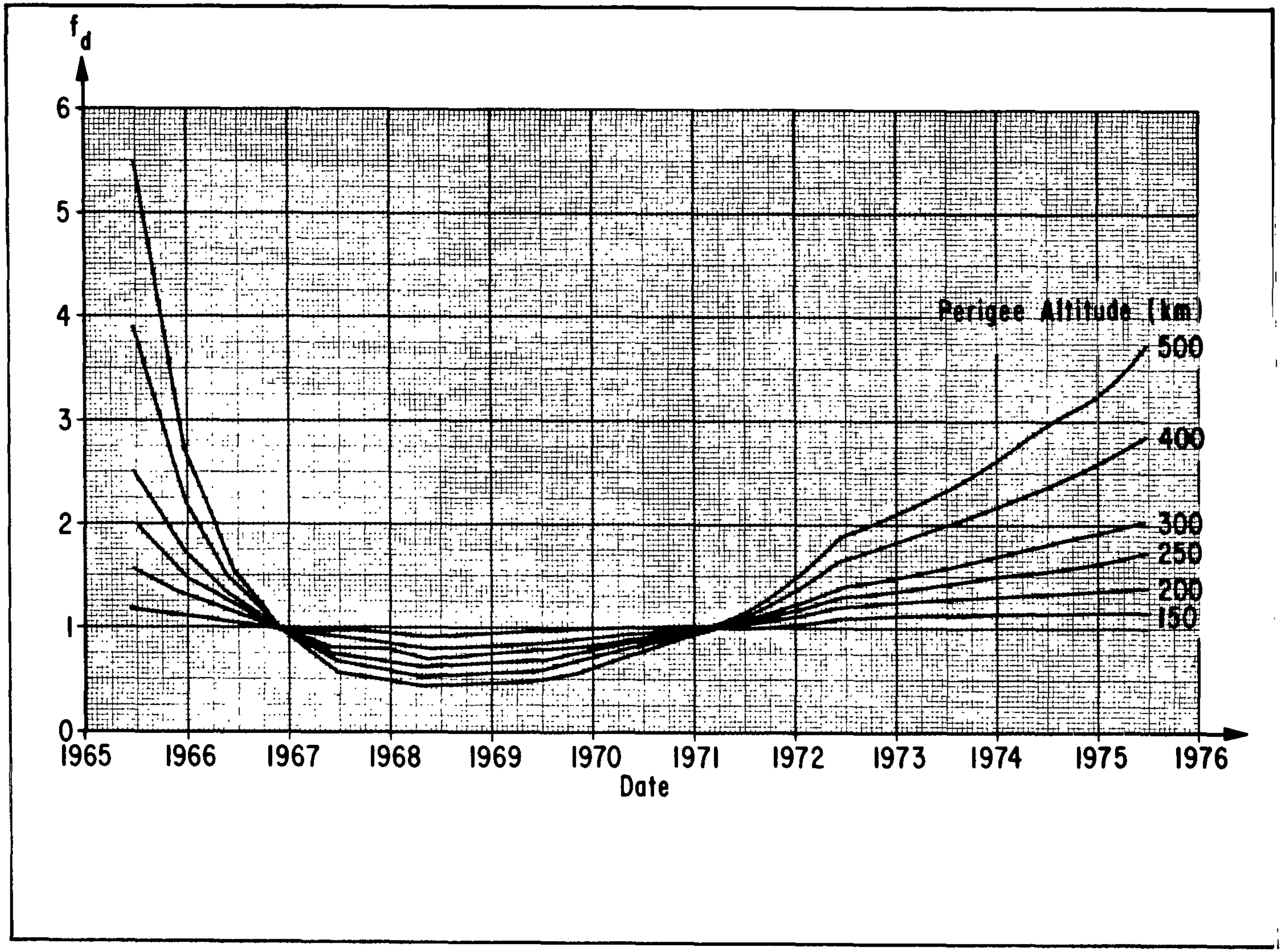


FIGURE 20. f_d CORRECTION FACTOR FOR -3σ LIFETIME: 1962 U. S. STANDARD REFERENCE

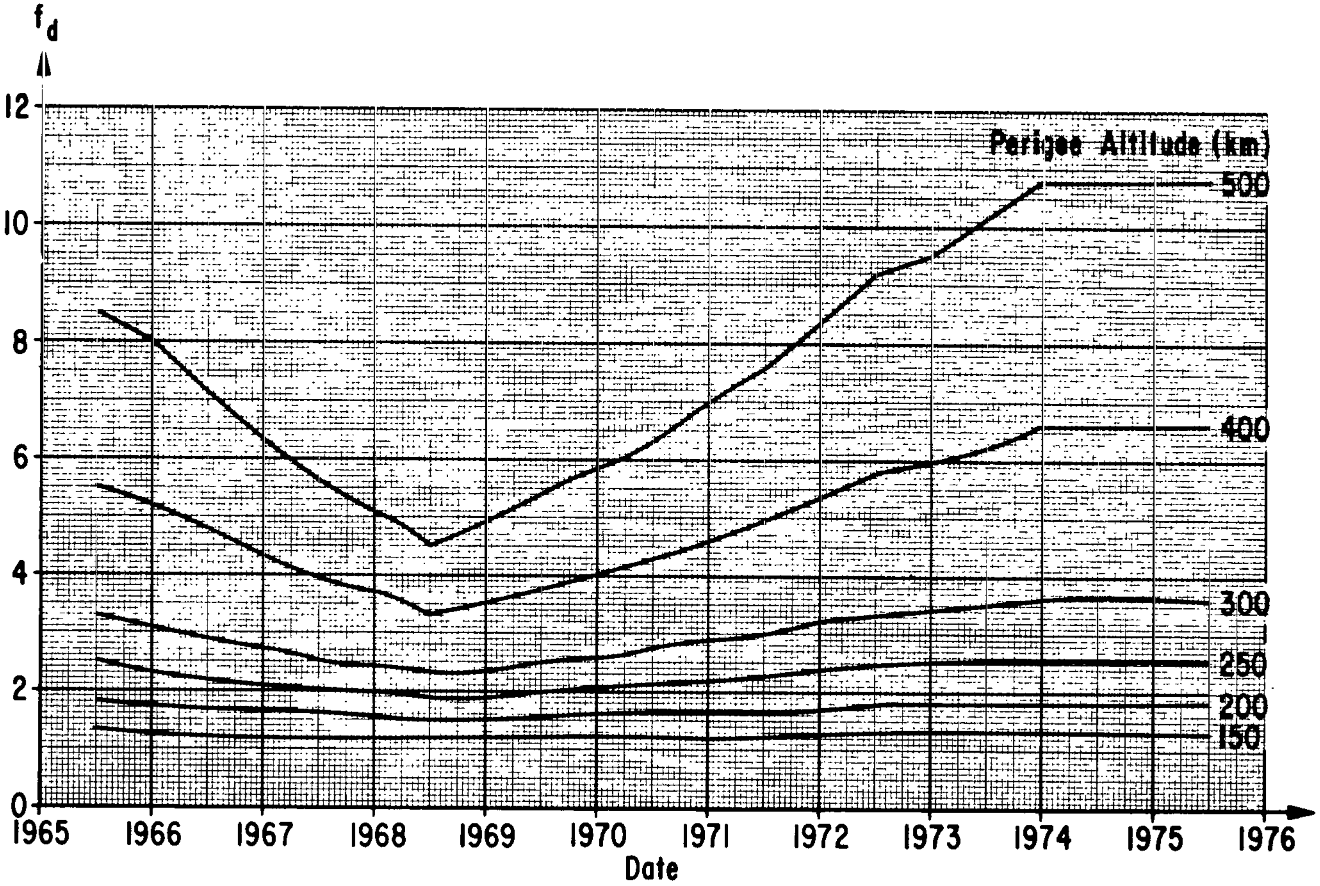
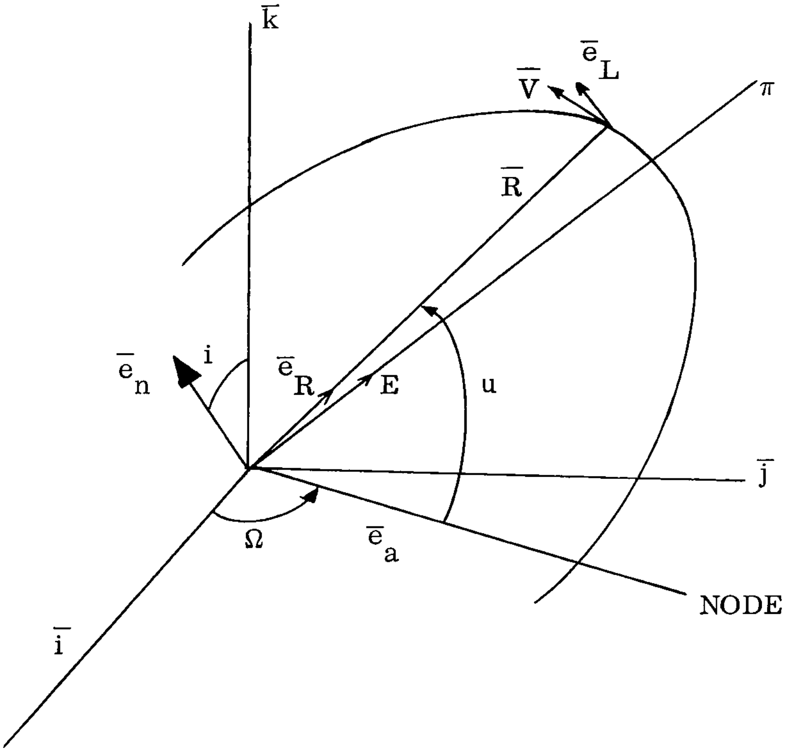


FIGURE 21. f_d CORRECTION FACTOR FOR $+3\sigma$ LIFETIME: 1962 U. S. STANDARD REFERENCE

APPENDIX A. DERIVATION OF THE DECAY EQUATIONS

$$\frac{d\lambda}{dt} = f(\lambda) \cdot \bar{\bar{\mathfrak{F}}}$$

A. Definitions



From the above geometry:

$$\cos i = \bar{e}_h \cdot \bar{k}$$

$$\bar{e}_R = \bar{e}_L \times \bar{e}_h$$

$$\cos \Omega = \bar{e}_\Omega \cdot \bar{i}$$

$$\cos u = \bar{e}_\Omega \cdot \bar{e}_R$$

$$\sin \Omega = \bar{e}_\Omega \cdot \bar{j}$$

$$\sin u = \bar{e}_\Omega \times \bar{e}_R \cdot \bar{e}_h$$

$$\bar{e}_\Omega = \frac{\bar{k} \times \bar{e}_h}{\sin i} = \frac{\bar{k} \times \bar{h}}{h \sin i}$$

$$\bar{e}_R = \frac{\bar{R}}{R}$$

$$\bar{e}_h = \frac{\bar{R} \times \bar{V}}{|\bar{R} \times \bar{V}|}$$

$$\begin{pmatrix} \bar{e}_R \\ \bar{e}_L \\ \bar{e}_h \end{pmatrix} = M \begin{pmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{pmatrix}$$

$$\begin{pmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{pmatrix} = M^T \begin{pmatrix} \bar{e}_R \\ \bar{e}_L \\ \bar{e}_h \end{pmatrix}$$

The transformation matrix M is given by

$$M = \begin{pmatrix} \cos u \cos \Omega - \sin u \cos i \sin \Omega & \cos u \sin \Omega + \sin u \cos i \cos \Omega & \sin u \sin i \\ -\sin u \cos \Omega - \cos u \cos i \sin \Omega & -\sin u \sin \Omega + \cos u \cos i \cos \Omega & \cos u \sin i \\ \sin i \sin \Omega & -\sin i \cos \Omega & \cos i \end{pmatrix}$$

$$\bar{h} \equiv \bar{R} \times \bar{V}$$

$$h \equiv R^2 \frac{d\theta}{dt}$$

$$\bar{e} \equiv \frac{\bar{V} \times (\bar{R} \times \bar{V})}{\mu} - \bar{e}_R$$

$$e = |\bar{e}|$$

$$\sin \omega = \frac{(\bar{e}_\Omega \times \bar{e}) \cdot \bar{e}_h}{e}$$

$$\cos \omega = \frac{\bar{e}_\Omega \cdot \bar{e}}{e}$$

$$A = e \cos \omega = \bar{e}_\Omega \cdot \bar{e}$$

$$B = e \sin \omega = \bar{e}_\Omega \times \bar{e} \cdot \bar{e}_h$$

$$p = \frac{\bar{h} \cdot \bar{h}}{\mu} = \frac{h^2}{\mu}$$

$$\bar{R} = \frac{p\bar{e}_R}{1 + e \cos (u - \omega)}$$

$$\bar{V} = \sqrt{\frac{\mu}{p}} \{ [1 + e \cos (u - \omega)] \, \bar{e}_R + e \sin (u - \omega) \, \bar{e}_L \}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} R \cdot \bar{i} \\ R \cdot \bar{j} \\ R \cdot \bar{k} \end{pmatrix} = \frac{p}{1 + \cos \theta} \begin{pmatrix} \bar{e}_R \cdot \bar{i} \\ \bar{e}_R \cdot \bar{j} \\ \bar{e}_R \cdot \bar{k} \end{pmatrix} \qquad \theta = u - \omega$$

$$\psi \equiv 1 + e \cos \theta \equiv 1 + A \cos u + B \sin u$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \bar{V} \cdot \bar{i} \\ \bar{V} \cdot \bar{j} \\ \bar{V} \cdot \bar{k} \end{pmatrix} = \sqrt{\frac{\mu}{p}} \left\{ [1 + e \cos \theta] \begin{pmatrix} \bar{e}_L \cdot \bar{i} \\ \bar{e}_L \cdot \bar{j} \\ \bar{e}_L \cdot \bar{k} \end{pmatrix} + e \sin \theta \begin{pmatrix} \bar{e}_R \cdot \bar{i} \\ \bar{e}_R \cdot \bar{j} \\ \bar{e}_R \cdot \bar{k} \end{pmatrix} \right\}$$

Consider now $\bar{R}$ to be "radius" and $\bar{V}$ to be "velocity." In Newtonian motion the two are related to some "force" $\bar{F}$ as follows:

$$\overline{V} = \frac{d\overline{R}}{dt}$$

or in alternate form:

$$\frac{d\overline{V}}{dt} = \frac{\overline{F}}{M} \qquad \text{or} \ \frac{d\overline{V}}{dt} = \frac{-\mu\overline{R}}{R^3} + \overline{\mathcal{F}}$$

Since the definitions express the orbital elements most directly as functions of $\overline{h}$ and $\overline{e}$, their derivatives are facilitated with expressions for $\frac{d\overline{h}}{dt}$ and $\frac{d\overline{e}}{dt}$.

$$\frac{d\overline{h}}{dt} = \frac{d}{dt}\ (\overline{R} \times \overline{V}) \ = \frac{d\overline{R}}{dt} \ \times \overline{V} + \overline{R} \times \frac{d\overline{V}}{dt}$$

$$= \overline{V} \times \overline{V} + \overline{R} \times \left(\overline{\mathcal{F}} - \frac{\mu\overline{R}}{R^3} \right)$$

$$= \overline{R} \times \left(\overline{\mathcal{F}} - \frac{\mu\overline{R}}{R^3} \right)$$

$$= \overline{R} \times \overline{\mathcal{F}} - \overline{R} \times \frac{\mu\overline{R}}{R^3}$$

$$\frac{d\overline{h}}{dt} = \overline{R} \times \overline{\mathcal{F}}$$

$$\mu \frac{d\overline{e}}{dt} = \frac{d}{dt}\ (\overline{V} \times \overline{h}) \ - \mu \frac{d}{dt}\ \overline{e}_R$$

$$= \frac{d\overline{V}}{dt} \times \ \overline{h} + \overline{V} \times \frac{d\overline{h}}{dt} \ - \mu \frac{d}{dt} \left(\frac{\overline{R}}{R} \right)$$

$$= \frac{d\bar{V}}{dt} \times \bar{h} + \bar{V} \times \frac{d\bar{h}}{dt} - \frac{\mu}{R^2} \left[R\frac{d\bar{R}}{dt} - \bar{R}\frac{dR}{dt} \right]$$

$$= \frac{d\bar{V}}{dt} \times \bar{h} + \bar{V} \times (\bar{R} \times \bar{\mathfrak{F}}) - \frac{\mu}{R^3} \left(\bar{R} \times \frac{d\bar{R}}{dt} \right) \times \bar{R}$$

$$= \frac{d\bar{V}}{dt} \times \bar{h} + \bar{V} \times (\bar{R} \times \bar{\mathfrak{F}}) - \frac{\mu}{R^3} \; (\bar{h} \times \bar{R})$$

$$= \left(\frac{d\bar{V}}{dt} + \frac{\mu\bar{R}}{R^3} \right) \times \bar{h} + \bar{V} \times (\bar{R} \times \bar{\mathfrak{F}})$$

$$= \bar{\mathfrak{F}} \times (\bar{R} \times \bar{V}) + \bar{V} \times (\bar{R} \times \bar{\mathfrak{F}})$$

1. The derivation of $\frac{di}{dt}$

$$\cos i = \bar{e}_h \cdot \bar{k}$$

$$-\sin i \frac{di}{dt} = \bar{k} \cdot \frac{d\bar{e}_h}{dt} + \bar{e}_h \cdot \frac{d\bar{k}}{dt} = \bar{k} \cdot \frac{d\bar{e}_h}{dt} + 0$$

$$= \frac{1}{h^3} \left(\bar{h} \times \frac{d\bar{h}}{dt} \times \bar{h} \cdot \bar{k} \right)$$

$$= \frac{1}{h^3} \left(\bar{h} \times \frac{d\bar{h}}{dt} \right) \cdot (-\bar{e}_\Omega \; h \sin i)$$

$$\frac{di}{dt} = \frac{1}{h^2} \left(\bar{h} \times \frac{d\bar{h}}{dt} \right) \cdot e_\Omega$$

$$\frac{di}{dt} = \frac{d\bar{h}}{dt} \cdot \frac{(\bar{e}_\Omega \times \bar{h})}{h^2}$$

$$= (\bar{R} \times \bar{\mathfrak{F}}) \cdot \frac{(\bar{e}_\Omega \times \bar{e}_h)}{h}$$

$$= \frac{R}{h} (\bar{e}_R \times \bar{e}_h) \times \bar{e}_R \cdot \bar{\mathfrak{F}}$$

$$\frac{di}{dt} = \frac{R}{h} \cos u \; (\bar{e}_h \cdot \bar{\mathfrak{F}})$$

2. The derivation of $\frac{d\Omega}{dt}$

$$\cos \Omega = \bar{e}_\Omega \cdot \bar{i}$$

$$-\sin \Omega \frac{d\Omega}{dt} = \bar{i} \cdot \frac{d\bar{e}_\Omega}{dt} + 0$$

$$\frac{d\bar{e}_\Omega}{dt} = \frac{d}{dt} \left(\frac{\bar{k} \times \bar{h}}{h \sin i} \right)$$

$$= \frac{\bar{k}}{h \sin i} \times \frac{d\bar{h}}{dt} - \frac{\bar{k} \times \bar{h}}{h^2 \sin^2 i} \frac{d}{dt} (h \sin i)$$

$$= \frac{\bar{k}}{h \sin i} \times \frac{dh}{dt} - \frac{\bar{e}_\Omega}{h \sin i} \frac{d}{dt} (\bar{k} \times \bar{h} \cdot \bar{e}_\Omega)$$

$$= \frac{1}{h \sin i} \left[\left(\bar{k} \times \frac{d\bar{h}}{dt} \right) - \bar{e}_\Omega \left\{ (\bar{k} \times \bar{h}) \cdot \frac{d\bar{e}_\Omega}{dt} + \bar{e}_\Omega \cdot \frac{d}{dt} (\bar{k} \times \bar{h}) \right\} \right]$$

$$= \frac{1}{h \sin i}\left[\bar{k}\times\frac{d\bar{h}}{dt} - \bar{e}_\Omega\left\{0 + \bar{e}_\Omega\cdot\bar{k}\times\frac{d\bar{h}}{dt}\right\}\right]$$

$$\frac{d\bar{e}_\Omega}{dt} = \frac{1}{h \sin i}\left[\bar{k}\times\frac{d\bar{h}}{dt} - \bar{e}_\Omega\left(\bar{k}\times\frac{d\bar{h}}{dt}\cdot\bar{e}_\Omega\right)\right]$$

$$\frac{d\bar{e}_\Omega}{dt} = \frac{1}{h \sin i}\left[e_\Omega\times\left(\bar{k}\times\frac{d\bar{h}}{dt}\right)\right]\times\bar{e}_\Omega$$

$$-\sin\Omega\,\frac{d\Omega}{dt} = \frac{d\bar{e}_\Omega}{dt}\cdot\bar{i}$$

$$= \frac{1}{h \sin i}\left[\bar{e}_\Omega\times\left(\bar{k}\times\frac{d\bar{h}}{dt}\right)\right]\times\bar{e}_\Omega\cdot\bar{i}$$

$$= \frac{1}{h \sin i}\left[\bar{e}_\Omega\times\left(\bar{k}\times\frac{d\bar{h}}{dt}\right)\right]\cdot\bar{e}_\Omega\times\bar{i}$$

$$= \frac{1}{h \sin i}\left[\bar{e}_\Omega\times\left(\bar{k}\times\frac{d\bar{h}}{dt}\right)\right]\cdot(-k\sin\Omega)$$

$$\frac{d\Omega}{dt} = \frac{1}{h \sin i}\left[\bar{e}_\Omega\times\left(\bar{k}\times\frac{d\bar{h}}{dt}\right)\right]\cdot\bar{k}$$

$$= \frac{\bar{k}\times\bar{e}_\Omega}{h \sin i}\cdot\bar{k}\times\frac{d\bar{h}}{dt}$$

$$= \frac{(\bar{k}\times\bar{e}_\Omega)}{h \sin i}\times\bar{k}\cdot\frac{d\bar{h}}{dt}$$

$$= \frac{\bar{e}_\Omega}{h \sin i}\cdot(\bar{k}\times\bar{\mathfrak{F}})$$

$$\frac{d\Omega}{dt} = \frac{\overline{e}_\Omega}{h \sin i} \cdot (R\, \overline{e}_R \times \overline{\mathfrak{F}})$$

$$= \frac{R\, \overline{e}_\Omega}{h \sin i} \cdot (\overline{e}_R \times \overline{\mathfrak{F}})$$

$$= \frac{R}{h \sin i} (\overline{e}_\Omega \times \overline{e}_R) \cdot \overline{\mathfrak{F}}$$

$$\frac{d\Omega}{dt} = \frac{R}{h \sin i} \sin u \, (\overline{e}_h \cdot \overline{\mathfrak{F}})$$

3. The derivation of $\frac{dp}{dt}$

$$\frac{dp}{dt} = \frac{d}{dt}\left(\frac{\overline{h} \cdot \overline{h}}{\mu}\right)$$

$$= \frac{2}{\mu}\, \overline{h} \cdot \frac{d\overline{h}}{dt}$$

$$= \frac{2}{\mu}\, \overline{h} \cdot (\overline{R} \times \overline{\mathfrak{F}})$$

$$= \frac{2}{\mu}\, h\, \overline{e}_h \cdot (R\, \overline{e}_R \times \overline{\mathfrak{F}})$$

$$\frac{dp}{dt} = \frac{2hR}{\mu}\, \overline{e}_h \cdot \overline{e}_R \times \overline{\mathfrak{F}}$$

$$= \frac{2hR}{\mu}\, \overline{e}_h \times \overline{e}_R \cdot \overline{\mathfrak{F}}$$

$$\frac{dp}{dt} = \frac{2hR}{\mu} \, (\bar{e}_h \cdot \bar{\mathcal{F}})$$

$$\frac{1}{p}\frac{dp}{dt} = \frac{2R}{h} \, (\bar{e}_h \cdot \bar{\mathcal{F}})$$

4. The derivation of $\dfrac{dA}{dt}$

$$A = \bar{e}_\Omega \cdot \bar{e}$$

$$\frac{dA}{dt} = \bar{e} \cdot \frac{d\bar{e}_\Omega}{dt} + \bar{e}_\Omega \frac{d\bar{e}}{dt}$$

$$= \bar{e} \cdot \left[\frac{1}{h \sin i} \, \bar{e}_\Omega \times \left(\bar{k} \times \frac{d\bar{h}}{dt} \right) \right] \times \bar{e}_\Omega + \bar{e}_\Omega \cdot \frac{d\bar{e}}{dt}$$

$$= \bar{e} \cdot \left[\frac{d\Omega}{dt} \bar{k} \right] \times \bar{e}_\Omega + \bar{e}_\Omega \cdot \frac{d\bar{e}}{dt}$$

$$= \frac{d\Omega}{dt} \bar{k} \cdot (\bar{e}_\Omega \times \bar{e}) + \bar{e}_\Omega \cdot \frac{d\bar{e}}{dt}$$

$$= \frac{d\Omega}{dt} \, \bar{k} \cdot B\bar{e}_h + \bar{e}_\Omega \cdot \frac{d\bar{e}}{dt}$$

$$= B \frac{d\Omega}{dt} \, (\bar{R} \cdot \bar{e}_h) + \bar{e}_\Omega \cdot \frac{d\bar{e}}{dt}$$

$$\frac{dA}{dt} = B \frac{d\Omega}{dt} \cos i + \bar{e}_\Omega \cdot \frac{d\bar{e}}{dt}$$

$$\frac{d\overline{e}}{dt} = \frac{1}{\mu} \{\overline{\mathcal{F}} \times (\overline{R} \times \overline{V}) + \overline{V} \times (\overline{R} \times \overline{\mathcal{F}})\}$$

$$\overline{e}_\Omega \cdot \frac{d\overline{e}}{dt} = \frac{\overline{e}_\Omega}{dt} \cdot \{\overline{R}\,(\overline{V} \cdot \overline{\mathcal{F}}) - \overline{V}\,(\overline{R} \cdot \overline{\mathcal{F}}) + \overline{R}\,(\overline{V} \cdot \overline{\mathcal{F}}) - \overline{\mathcal{F}}\,(\overline{V} \cdot \overline{R})\}$$

$$\overline{e}_\Omega = \overline{e}_R \cos u - \overline{e}_L \sin u$$

$$\overline{e}_\Omega \cdot \overline{e}_R = \cos u$$

$$\overline{e}_\Omega \cdot \overline{e}_L = -\sin u$$

$$\overline{R} \cdot \overline{e}_\Omega = R \cos u$$

$$\overline{V} \cdot \overline{e}_\Omega = \frac{-\mu}{h} [(1 + A \cos u + B \sin u)\, \overline{e}_L + (A \sin u - B \cos u)\, \overline{e}_R] \cdot \overline{e}_\Omega$$

$$= \frac{\mu}{h} [\,(1 + A \cos u + B \sin u)\, \overline{e}_L + (A \sin u - B \cos u) \overline{e}_R] \cdot [\overline{e}_R \cos u - \overline{e}_L \sin u]$$

$$= \frac{\mu}{h} [-(1 + A \cos u + B \sin u) \sin u + (A \sin u - B \cos u) \cos u]$$

$$= \frac{\mu}{h} [-\sin u - A \sin u \cos u - B \sin^2 u + A \sin u \cos u - B \cos^2 u]$$

$$\overline{V} \cdot \overline{e}_\Omega = \frac{\mu}{h} [B + \sin u]$$

$$\overline{V} \cdot \overline{R} = \frac{\mu}{h} [(1 + A \cos u + B \sin u)\, \overline{e}_L + (A \sin u - B \cos u)\, \overline{e}_R] \cdot R\, \overline{e}_R$$

$$\overline{V} \cdot \overline{R} = \frac{\mu R}{h} [A \sin u - B \cos u]$$

$$\bar{e} \cdot \frac{d\bar{e}}{dt} = \frac{1}{\mu}\{2(\bar{V} \cdot \bar{\mathcal{F}})\,(\bar{R} \cdot e_{\Omega}) - (\bar{R} \cdot \bar{\mathcal{F}})\,(\bar{V} \cdot \bar{e}_{\Omega}) - (\bar{\mathcal{F}} \cdot \bar{e}_{\Omega})\,(\bar{V} \cdot \bar{R})\}$$

$$= \frac{1}{\mu}\{2R\cos u(\bar{V} \cdot \bar{\mathcal{F}}) + \frac{\mu}{h}(B + \sin u)\,(\bar{R} \cdot \bar{\mathcal{F}}) - \frac{\mu R}{h}[A\sin u - B\cos u](\bar{\mathcal{F}} \cdot \bar{e}_R)\}$$

$$= \frac{1}{\mu}\{2R\cos u\,\bar{V} + \frac{\mu}{h}(B + \sin u)\,R\bar{e}_R - \frac{\mu}{h}R\,[A\sin u - B\cos u]\,\bar{e}_{\Omega}\} \cdot \bar{\mathcal{F}}$$

$$= \frac{1}{\mu}\{\frac{2\mu R}{h}\cos u\,[1 + A\cos u + B\sin u]\,\bar{e}_L + \frac{2\mu R}{h}\cos u\,[A\sin u - B\cos u]\,\bar{e}_R$$

$$-\frac{\mu R}{h}[B + \sin u]\bar{e}_R - \frac{\mu R}{h}[A\sin u - B\cos u][\bar{e}_R\cos u - e_L\sin u]\} \cdot \bar{\mathcal{F}}$$

$$= \frac{R}{h}\{(2\cos u\,[1 + A\cos u + B\sin u] + [A\sin u - B\cos u]\,\sin u)\,\bar{e}_L$$

$$+ (2[A\sin u - B\cos u]\,\cos u + [B + \sin u] - [A\sin u - B\cos u]\,\cos u)\bar{e}_R\} \cdot \bar{\mathcal{F}}$$

$$= \frac{R}{h}\{(2\psi\cos u + A\sin^2 u - B\sin u\cos u)\bar{e}_L + (A\sin u\cos u - B\cos^2 u$$

$$+ \sin u)\,\bar{e}_R\} \cdot \bar{\mathcal{F}}$$

$$\bar{e} \cdot \frac{d\bar{e}}{dt} = \frac{R}{h}\{[A + (1 + \psi)\,\cos u]\bar{e}_L + \psi\sin u\,\bar{e}_R\} \cdot \bar{\mathcal{F}}$$

$$\frac{dA}{dt} = B\frac{d\Omega}{dt}\cos i + \bar{e}_{\Omega} \cdot \frac{d\bar{e}}{dt}$$

$$\frac{dA}{dt} = B\frac{d\Omega}{dt}\cos i + \frac{R}{h}\{[A + (1 + \psi)\,\cos u]\bar{e}_L + \psi\sin u\,\bar{e}_R\} \cdot \bar{\mathcal{F}}$$

5. The derivation of $\frac{dB}{dt}$

$$B = \bar{e}_\Omega \times \bar{e} \cdot \bar{e}_h$$

$$\frac{dB}{dt} = \frac{d}{dt}(\bar{e}_\Omega \times \bar{e}) \cdot \bar{e}_h + \frac{d\bar{e}_h}{dt} \cdot (\bar{e}_\Omega \times \bar{e})$$

$$= \left[\frac{d\bar{e}_\Omega}{dt} \times \bar{e} + \bar{e}_\Omega \times \frac{d\bar{e}}{dt}\right] \cdot \bar{e}_h + \frac{d\bar{e}_h}{dt} \cdot (\bar{e}_\Omega \times \bar{e})$$

$$= \bar{e}_h \cdot \frac{d\bar{e}_\Omega}{dt} \times \bar{e} + \bar{e}_h \cdot \bar{e}_\Omega \times \frac{d\bar{e}}{dt} + \frac{d\bar{e}_h}{dt} \cdot (\bar{e}_\Omega \times \bar{e})$$

$$\frac{dB}{dt} = \bar{e}_h \cdot \frac{d\bar{e}_\Omega}{dt} \times \bar{e} + \bar{e}_h \cdot \bar{e}_\Omega \times \frac{d\bar{e}}{dt} + \frac{d\bar{e}_h}{dt} \cdot B\,\bar{e}_h$$

$$\frac{d\bar{e}_\Omega}{dt} = \frac{d\Omega}{dt}\,\bar{k} \times \bar{e}_\Omega$$

$$\frac{dB}{dt} = \bar{e}_h \cdot \left(\frac{d\Omega}{dt}\,\bar{k} \times \bar{e}_\Omega\right) \times \bar{e} + \bar{e}_h \cdot \bar{e}_\Omega \times \frac{d\bar{e}}{dt}$$

$$= \frac{d\Omega}{dt}(\bar{k} \times \bar{e}_\Omega) \times \bar{e} \cdot \bar{e}_h + \bar{e}_h \cdot \bar{e}_\Omega \times \frac{d\bar{e}}{dt}$$

$$= \frac{d\Omega}{dt}(\bar{k} \times \bar{e}_\Omega) \cdot (\bar{e} \times \bar{e}_h) + \bar{e}_h \cdot \bar{e}_\Omega \times \frac{d\bar{e}}{dt}$$

$$= \frac{d\Omega}{dt}\left[(\bar{k} \cdot \bar{e})(\bar{e}_\Omega \cdot \bar{e}_h) - (\bar{k} \cdot \bar{e}_h)(\bar{e}_\Omega \cdot \bar{e})\right] + \bar{e}_h \cdot \bar{e}_\Omega \times \frac{d\bar{e}}{dt}$$

$$= -\frac{d\Omega}{dt}\ (\overline{k} \cdot \overline{e}_h)(\overline{e}_\Omega \cdot \overline{e}) + \overline{e}_h \cdot \overline{e}_\Omega \times \frac{d\overline{e}}{dt}$$

$$\frac{dB}{dt} = -\frac{d\Omega}{dt} A \cos i + \overline{e}_h \cdot \overline{e}_\Omega \times \frac{d\overline{e}}{dt}$$

$$\frac{dB}{dt} = -\frac{d\Omega}{dt} A \cos i \quad \overline{e}_\Omega \cdot \frac{d\overline{e}}{dt} \times \overline{e}_h$$

$$\frac{dB}{dt} = -\frac{d\Omega}{dt}\ A \cos i + \overline{e}_h \times \overline{e}_\Omega \cdot \frac{d\overline{e}}{dt}$$

$$\overline{e}_\Omega = \overline{e}_R \cos u - \overline{e}_L \sin u$$

$$\overline{e}_h \times \overline{e}_\Omega = \overline{e}_h \times [\overline{e}_R \cos u - \overline{e}_L \sin u]$$

$$= \overline{e}_h \times \overline{e}_R \cos u - \overline{e}_h \times \overline{e}_L \sin u$$

$$\overline{e}_h \times \overline{e}_\Omega = \overline{e}_L \cos u + \overline{e}_R \sin u$$

$$\overline{e}_h \times \overline{e}_\Omega \cdot \frac{d\overline{e}}{dt} = [\overline{e}_L \cos u + \overline{e}_R \sin u] \cdot \frac{d\overline{e}}{dt}$$

$$= \overline{e}_L \cdot \frac{d\overline{e}}{dt} \cos u + \overline{e}_R \cdot \frac{d\overline{e}}{dt} \sin u$$

$$\frac{d\overline{e}}{dt} = \frac{1}{\mu}\ [\, 2\overline{R}\ (\overline{V} \cdot \mathcal{F}) - \overline{V}(\overline{R} \cdot \mathcal{F}) - \mathcal{F}(\overline{V} \cdot \overline{R})\,]$$

$$\overline{e}_L \cdot \frac{d\overline{e}}{dt} = \frac{1}{\mu}\ [2(\overline{V} \cdot \mathcal{F})(\overline{R} \cdot \overline{e}_L) - (\overline{R} \cdot \mathcal{F})(\overline{V} \cdot \overline{e}_L) - (\overline{V} \cdot \overline{R})(\mathcal{F} \cdot \overline{e}_L)]$$

$$= \frac{1}{\mu} [2\bar{V}(\bar{R} \cdot \bar{e}_L) - \bar{R}(\bar{V} \cdot \bar{e}_L) - (\bar{V} \cdot \bar{R})e_L] \cdot \bar{\mathcal{F}}$$

$$\bar{R} \cdot \bar{e}_L = 0$$

$$\bar{R} \cdot \bar{V} = \frac{\mu R}{h} [A\sin u - B\cos u]$$

$$\bar{V} \cdot \bar{e}_L = \frac{\mu}{h} \{[1 + A\cos u + B\sin u]\bar{e}_L + [A\sin u - B\cos u]\bar{e}_R\} \cdot \bar{e}_L$$

$$\bar{V} \cdot \bar{e}_L = \frac{\mu}{h} [1 + A\cos u + B\sin u]$$

$$\bar{e}_R \cdot \frac{d\bar{e}}{dt} = \frac{1}{\mu} \{2(\bar{V} \cdot \bar{\mathcal{F}})\ (\bar{R} \cdot \bar{e}_R) - (\bar{R} \cdot \bar{\mathcal{F}})\ (\bar{V} \cdot \bar{e}_R) - (\bar{V} \cdot \bar{R})\ (\bar{\mathcal{F}} \cdot \bar{e}_R)\}$$

$$= \frac{1}{\mu} \{2\bar{V}(R \cdot \bar{e}_R) - \bar{R}(\bar{V} \cdot \bar{e}_R) - (\bar{V} \cdot R)\bar{e}_R\} \cdot \bar{\mathcal{F}}$$

$$\bar{V} \cdot \bar{e}_R = \frac{\mu}{h} \{[1 + A\cos u + B\sin u]\bar{e}_L + [A\sin u - B\cos u]\bar{e}_R\} \cdot \bar{e}_R$$

$$\bar{V} \cdot e_R = \frac{\mu}{h} [A\sin u - B\cos u]$$

$$\bar{e}_L \cdot \frac{d\bar{e}}{dt} \cos u + \bar{e}_R \cdot \frac{d\bar{e}}{dt} \sin u = \frac{1}{\mu} \cos u \left\{-\frac{\mu}{h} \bar{R} [1 + A\cos u + B\sin u]\right.$$

$$\left. - \frac{\mu R}{h} [A\sin u - B\cos u]\bar{e}_L\right\} \cdot \bar{\mathcal{F}} + \frac{1}{\mu} \sin u \left\{2R\bar{V} - \frac{\mu R}{h} [A\sin u - B\cos u]\right.$$

$$\left. - \frac{\mu R}{h} [A\sin u - B\cos u]\bar{e}_R\right\} \cdot \bar{\mathcal{F}}$$

$$\bar{e}_L \cdot \frac{d\bar{e}}{dt} \cos u + \bar{e}_R \cdot \frac{d\bar{e}}{dt} \sin u = \frac{R}{h} \{ -[\psi] \cos u \, \bar{e}_R - [A \sin u - B \cos u] \cos u \, \bar{e}_L$$

$$+ 2 \sin u \, \psi \, \bar{e}_L + [A \sin u - B \cos u] \sin u \, \bar{e}_R$$

$$- [A \sin u - B \cos u] \sin u \, \bar{e}_R - [A \sin u - B \cos u] \sin u \, \bar{e}_R \} \cdot \bar{\mathcal{F}}$$

$$\bar{e}_L \cdot \frac{d\bar{e}}{dt} \cos u + \bar{e}_R \cdot \frac{d\bar{e}}{dt} \sin u = \frac{R}{h} \{ (-\psi \cos u + [A \sin u - B \cos u] \sin u$$

$$- [A \sin u - B \cos u] \sin u) \, \bar{e}_R + (-[A \sin u - B \cos u] \cos u$$

$$+ 2\psi \sin u \, \bar{e}_L) \quad \cdot \bar{\mathcal{F}}$$

$$\bar{e}_L \cdot \frac{d\bar{e}}{dt} \cos u + \bar{e}_R \cdot \frac{d\bar{e}}{dt} \sin u = \frac{R}{h} \{ -\psi \cos u \, \bar{e}_R + (-[A \sin u - B \cos u] \cos u$$

$$+ 2\psi \sin u) \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

$$\bar{e}_L \cdot \frac{d\bar{e}}{dt} \cos u + \bar{e}_R \cdot \frac{d\bar{e}}{dt} \sin u$$

$$= \frac{R}{h} \{ -\psi \cos u \, \bar{e}_R + (-A \sin u \cos u + B \cos^2 u + 2\psi \sin u) \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

$$= \frac{R}{h} \{ -\psi \cos u \, \bar{e}_R + (-\sin u \cos u + B - B \sin^2 u + 2\psi \sin u) \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

$$= \frac{R}{h} \{ -\psi \cos u \, \bar{e}_R + [\sin u \, (-\cos u - B \sin u + 2\psi) + B] \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

$$= \frac{R}{h} \{ -\psi \cos u + [(1 + \psi) \sin u + B] \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

$$\frac{dB}{dt} = -\frac{d\Omega}{dt} A\cos i + \bar{e}_h \times \bar{e}_\Omega \cdot \frac{d\bar{e}}{dt}$$

$$= -\frac{d\Omega}{dt} A\cos i + \bar{e}_L \cdot \frac{d\bar{e}}{dt} \cos u + \bar{e}_R \cdot \frac{d\bar{e}}{dt} \sin u$$

$$\frac{dB}{dt} = -\frac{d\Omega}{dt} A\cos i + \frac{R}{h} \left\{ -\psi \cos u\, \bar{e}_R + \left[(1+\psi)\sin u + B \right] \bar{e}_L \right\} \cdot \bar{\mathcal{F}}$$

6. The derivation of $\frac{de}{dt}$

$$\frac{de}{dt} = \frac{d}{dt} (\bar{e} \cdot \bar{e})^{\frac{1}{2}}$$

$$= \frac{1}{e} \bar{e} \cdot \frac{d\bar{e}}{dt}$$

$$= \frac{1}{\mu e} \left[\frac{\bar{V} \times (\bar{R} \times \bar{V})}{\mu} - \bar{e}_R \right] \cdot \left[\bar{\mathcal{F}} \times (\bar{R} \times \bar{V}) + \bar{V} \times (\bar{R} \times \bar{\mathcal{F}}) \right]$$

$$= \frac{1}{\mu e} \left\{ \left[\frac{(\bar{V} \cdot \bar{V})\bar{R} - (\bar{V} \cdot \bar{R})\bar{V}}{\mu} - \bar{e}_R \right] \cdot \left[2(\bar{V} \cdot \bar{\mathcal{F}})\bar{R} - (\bar{R} \cdot \bar{F})\bar{V} - (\bar{V} \cdot \bar{R})\bar{\mathcal{F}} \right] \right\}$$

$$= \frac{1}{\mu e} \left\{ \frac{(\bar{V} \cdot \bar{V})\bar{R}}{\mu} \cdot \left[2(\bar{V} \cdot \bar{\mathcal{F}})\bar{R} - (\bar{R} \cdot \bar{\mathcal{F}})\bar{V} - (\bar{V} \cdot \bar{R})\bar{\mathcal{F}} \right] \right.$$

$$- \frac{(\bar{V} \cdot \bar{R})}{\mu} \bar{V} \cdot \left[2(\bar{V} \cdot \bar{\mathcal{F}})\bar{R} - (\bar{R} \cdot \bar{\mathcal{F}})\bar{V} - (\bar{V} \cdot \bar{R})\bar{\mathcal{F}} \right]$$

$$\left. - \bar{e}_R \cdot \left[2(\bar{V} \cdot \bar{\mathcal{F}})\bar{R} - (\bar{R} \cdot \bar{\mathcal{F}})\bar{V} - (\bar{V} \cdot \bar{R})\bar{\mathcal{F}} \right] \right\}$$

$$\begin{aligned} &= \frac{1}{\mu e}\left\{\frac{(\overline{V}\cdot\overline{V})}{\mu}\;[2R^2\overline{V} - 2\overline{R}\;(\overline{V}\cdot\overline{R})] - \frac{(\overline{V}\cdot\overline{R})}{\mu}\;[(\overline{V}\cdot\overline{R})\overline{V} - (\overline{V}\cdot\overline{V})\overline{R}]\right. \\ &\qquad \left. - [2\;(\overline{R}\cdot\overline{e}_R)\overline{V} - (\overline{V}\cdot\overline{e}_R)\overline{R} - (\overline{V}\cdot\overline{R})\overline{e}_R]\right\}\mathcal{F} \end{aligned}$$

$$\overline{V} = \frac{\mu}{h}\{[1 + e\cos\theta]\;\overline{e}_L + e\sin\theta\;\overline{e}_R\}$$

$$\overline{R} = R\,\overline{e}_R$$

$$(\overline{V}\cdot\overline{R}) = \frac{\mu R}{h}\,e\sin\theta$$

$$(\overline{V}\cdot\overline{V}) = \frac{\mu^2}{h^2}\;\{[1 + e\cos\theta]^2 + e^2\sin^2\theta\}$$

$$= \frac{\mu^2}{h^2}\;\{1 + e^2 + 2e\cos\theta\}$$

$$(\overline{V}\cdot\overline{e}_R) = \frac{\mu}{h}\,e\sin\theta$$

$$\begin{aligned} \frac{de}{dt} &= \frac{1}{\mu e}\left\{\frac{\mu}{h^2}\;[1 + e^2 + 2e\cos\theta]\;\left[2R^2\,\frac{\mu}{h}\;([1 + e\cos\theta]\overline{e}_L + e\sin\theta\;\overline{e}_R)\right.\right. \\ &\qquad \left. - \frac{2R^2\mu}{h}\,e\sin\theta\;\overline{e}_R\right] - \frac{Re\sin\theta}{h}\;\left[\frac{\mu Re\sin\theta}{h}\,\frac{\mu}{h}\;([1 + e\cos\theta]\overline{e}_L + e\sin\theta\overline{e}_R)\right. \\ &\qquad \left.\left. - \frac{\mu^2}{h^2}\;R[1 + e^2 + 2e\cos\theta]\overline{e}_R\right] - \frac{2\mu R}{h}\;([1 + e\cos\theta]\overline{e}_L + e\sin\theta\;\overline{e}_R)\right. \end{aligned}$$

$$+ \frac{\mu R}{h} e\sin\theta\, \bar{e}_R + \frac{\mu R}{h} e\sin\theta\, \bar{e}_R \Bigg\} \cdot \bar{\mathcal{F}}$$

$$= \frac{R}{h} \Bigg\{ \frac{2\mu R}{eh^2} [1 + e^2 + 2e\cos\theta][(\psi \bar{e}_L + e\sin\theta\, \bar{e}_R) - e\sin\theta\, \bar{e}_R]$$

$$- \frac{\mu R e\sin\theta}{h^2} [e\sin\theta\, (\psi \bar{e}_L + e\sin\theta\, \bar{e}_R) - (1 + e^2 + 2e\cos\theta)\, \bar{e}_R]$$

$$- \frac{2}{e} (\psi \bar{e}_L + e\sin\theta\, \bar{e}_R) + 2\sin\theta\, \bar{e}_R \Bigg\} \cdot \bar{\mathcal{F}}$$

$$\frac{de}{dt} = \frac{R}{h} \Bigg\{ \frac{2}{\psi e} [1 + e^2 + 2e\cos\theta][(\psi \bar{e}_L + e\sin\theta\, \bar{e}_R) - e\sin\theta\, \bar{e}_R]$$

$$- \frac{\sin\theta}{\psi} [e\sin\theta(\psi\, \bar{e}_L + e\sin\theta\, \bar{e}_R) - [1 + e^2 + 2e\cos\theta]\, \bar{e}_R]$$

$$- \frac{2}{e} (\psi\, \bar{e}_L + e\sin\theta\, \bar{e}_R) + 2\sin\theta\, \bar{e}_R \Bigg\} \cdot \bar{\mathcal{F}}$$

$$= \frac{R}{h} \Bigg\{ \left[\frac{2}{\psi e} [1 + e^2 + 2e\cos\theta]\, \psi - \frac{e\sin^2\theta}{\psi}\, \psi - \frac{2\psi}{e} \right] \bar{e}_L$$

$$+ \left[-\frac{e^2\sin^3\theta}{\psi} + \frac{\sin\theta}{\psi} [1 + e^2 + 2e\cos\theta] \right] \bar{e}_R \Bigg\} \cdot \bar{\mathcal{F}}$$

$$= \frac{R}{h} \Bigg\{ \left[\frac{2}{e} [1 + e^2 + 2e\cos\theta] - e\sin^2\theta - \frac{2}{e} [1 + e\cos\theta] \right] \bar{e}_L$$

$$+ \frac{\sin\theta}{\psi} \left[- e^2\sin^2\theta + [1 + e^2 + 2e\cos\theta] \right] \bar{e}_R \Bigg\} \cdot \bar{\mathcal{F}}$$

$$= \frac{R}{h} \left\{ [2e + 2\cos\theta - e\sin^2\theta]\bar{e}_L + \left[-\frac{e\sin^2\theta}{\psi} \right.\right.$$

$$\left.\left. + \frac{\sin\theta}{\psi} \; [1 + e^2 + 2e\cos\theta] \right] \bar{e}_R \right\} \cdot \bar{\mathfrak{F}}$$

$$\frac{de}{dt} = \frac{R}{h} \left\{ [e + (1 + \psi)\cos\theta]\; \bar{e}_L + \psi\sin\theta\; \bar{e}_R \right\} \cdot \bar{\mathfrak{F}}$$

7. The derivation of $\frac{d\omega}{dt}$

$$\cos\omega = \frac{\bar{e}_\Omega \cdot \bar{e}}{e}$$

$$-\sin\omega \frac{d\omega}{dt} = \frac{1}{e} \frac{d}{dt} (\bar{e}_\Omega \cdot \bar{e}) - \frac{1}{e^2} (\bar{e}_\Omega \cdot \bar{e}) \frac{de}{dt}$$

$$-\sin\omega \frac{d\omega}{dt} = \frac{1}{e} \frac{dA}{dt} - \frac{\cos\omega}{e} \frac{de}{dt}$$

$$-e\sin\omega \frac{d\omega}{dt} = \frac{dA}{dt} - \cos\omega \frac{de}{dt}$$

$$= B \frac{d\Omega}{dt} \cos i + \frac{R}{h} \{ [A + (1 + \psi)\cos u]\; \bar{e}_L + \psi \sin u\, \bar{e}_R \} \cdot \bar{\mathfrak{F}}$$

$$-\cos\omega \frac{R}{h} \{ \psi\sin\theta\; \bar{e}_R + [e + (1 + \psi)\cos\theta]\; \bar{e}_L \} \cdot \bar{\mathfrak{F}}$$

$$\frac{d\omega}{dt} = \frac{R}{he} \cot\omega \{ \psi \sin\theta \, \bar{e}_R + [e + (1 + \psi) \cos\theta] \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

$$- \frac{d\Omega}{dt} \cos i - \frac{R}{he} \csc\omega \{\psi \sin u \, \bar{e}_R + [A + (1 + \psi) \cos u] \, \bar{e}_L\} \cdot \bar{\mathcal{F}}$$

$$\frac{d\omega}{dt} = - \frac{d\Omega}{dt} \cos i + \frac{R}{he} \{ \psi \sin\theta \cot\omega \, \bar{e}_R + \cot\omega \, [e + (1 + \psi) \cos\theta] \, \bar{e}_L$$

$$- \psi \sin u \csc\omega \, \bar{e}_R - \csc\omega \, [A + (1 + \psi) \cos u] \, \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

$$\frac{d\omega}{dt} = - \frac{d\Omega}{dt} \cos i + \frac{R}{he} \{ [\psi \sin\theta \cot\omega - \psi \sin u \csc\omega] \, \bar{e}_R$$

$$+ \left(\cot\omega \, [e + (1 + \psi) \cos\theta] - \csc\omega \, [A + (1 + \psi) \cos u \right) \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

$$\frac{d\omega}{dt} = - \frac{d\Omega}{dt} \cos i + \frac{R}{he} \{ -\psi \cos\theta \, \bar{e}_R + (1 + \psi) \sin\theta \, \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

8. A derivation of $\frac{dr_p}{dt}$

$$\frac{dr_p}{dt} = \frac{d}{dt} \quad \frac{p}{1 + e}$$

$$= \frac{1}{1 + e} \frac{dp}{dt} - \frac{p}{(1 + e)^2} \frac{de}{dt}$$

$$= 2 \frac{1}{1 + e} \frac{Rp}{h} (\bar{e}_L \cdot \bar{\mathcal{F}}) - \frac{p}{(1 + e)^2} \frac{R}{h} \{ \psi \sin\theta \, \bar{e}_R +$$

$$+ [e + (1 + \psi) \cos\theta] \, \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

$$= \frac{R}{h} \frac{p}{(1 + e)^2} \{ 2(1 + e)(\bar{e}_L \cdot \bar{\mathcal{F}}) - \psi \sin\theta \, \bar{e}_R - [e$$

$$+ (1 + \psi) \cos\theta] \, \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

$$= \frac{R}{h} \frac{p}{(1 + e)^2} \{ (2(1 + e) - [e + (1 + \psi) \cos\theta] \Big) \bar{e}_L - \psi \sin\theta \, \bar{e}_R \} \cdot \bar{\mathcal{F}}$$

$$\frac{dr_p}{dt} = \frac{R}{h} \frac{p}{(1 + e)^2} \{ -\psi \sin\theta \, \bar{e}_R + [2(1 - \cos\theta) + e \sin^2\theta \,] \, \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

9. A derivation of $\frac{dr_a}{dt}$

$$\frac{dr_a}{dt} = \frac{d}{dt} \left(\frac{p}{1 - e} \right)$$

$$= \frac{1}{1 - e} \frac{dp}{dt} + \frac{p}{(1 - e)^2} \frac{de}{dt}$$

$$= \frac{2}{1 - e} \frac{pR}{h} (\bar{e}_L \cdot \bar{\mathcal{F}}) + \frac{p}{(1 - e)^2} \frac{R}{h} \{ [e + (1 + \psi) \cos\theta] \, \bar{e}_L$$

$$+ \psi \sin\theta \, \bar{e}_R \} \cdot \bar{\mathcal{F}}$$

$$= \frac{pR}{h(1 - e)^2} \{ 2(1 - e)\, \bar{e}_L + [e + (1 + \psi) \cos\theta]\bar{e}_L + \psi \sin\theta\, \bar{e}_R \} \cdot \bar{\mathcal{F}}$$

$$= \frac{pR}{h(1 - e)^2} \{ [2(1 - e) + e + (1 + \psi) \cos\theta]\, \bar{e}_L + \psi \sin\theta\, \bar{e}_R \} \cdot \bar{\mathcal{F}}$$

$$= \frac{pR}{h(1 - e)^2} \{ [(2 - e) + (1 + \psi) \cos\theta]\, \bar{e}_L + \psi \sin\theta\, \bar{e}_R \} \cdot \bar{\mathcal{F}}$$

$$\frac{dr_a}{dt} = \frac{pR}{h(1 - e)^2} \{ \psi \sin\theta\, \bar{e}_R + [2(1 + \cos\theta) - e\sin^2\theta]\, \bar{e}_L \} \cdot \bar{\mathcal{F}}$$

B. The description and substitution of $\bar{\mathcal{F}}$ in the decay equations $\frac{d\lambda}{dt}$

Define $\bar{\mathcal{F}}$ as follows:

$$\bar{\mathcal{F}} = \bar{\mathcal{F}}_G + \bar{\mathcal{F}}_D + \bar{\mathcal{F}}_S$$

Where $\bar{\mathcal{F}}_G$ is the gravitational perturbing force due to earth oblatness
$\bar{\mathcal{F}}_D$ is the perturbing force due to drag
$\bar{\mathcal{F}}_S$ is the perturbing force due to solar radiation pressure

$$\bar{\mathcal{F}}_G = \frac{\mu}{R^2} \sum_{N=2}^{3} J_N \left(\frac{R_{eq}}{R}\right)^N \{ (N+1)\, P_N\, \bar{e}_R - P_N' [\cos i\, \bar{e}_h + \sin i \cos u\, \bar{e}_L] \}$$

R_{eq} = earth's equatorial radius

R, β are the geocentric radius and latitude of the satellite

$P_N(\sin\beta)$ is the N^{th} Legendre polynomial

J_N are empirically determined constants

$$P_N' = \frac{\partial}{\partial \sin\beta}\,(P_N \sin\beta)$$

$$\bar{\mathfrak{F}}_D = -\frac{\mu\rho B}{p}\,[1 + e^2 + 2e\cos\theta]^{\frac{1}{2}}\left[[1 + e\cos\theta]\,\bar{e}_L + e\sin\theta\,\bar{e}_R\right]$$

$$B = \frac{C_D A}{2m}$$

A = reference area

m = mass of satellite

C_D = drag coefficient

$$\bar{\mathfrak{F}}_S = -\frac{A}{m}\,Pr\,[L\,\bar{e}_R + M\,\bar{e}_L + N\,\bar{e}_h]$$

$$\begin{pmatrix} L \\ M \\ N \end{pmatrix} = \begin{bmatrix} \cos u \cos\Omega - \sin u \cos i \sin\Omega & \cos u \sin\Omega + \sin u \cos i \cos\Omega & \sin u \sin i \\ -\sin u \cos\Omega - \cos u \cos i \sin\Omega & -\sin u \sin\Omega + \cos u \cos i \cos\Omega & \cos u \sin i \\ \sin i \sin\Omega & -\sin i \cos\Omega & \cos i \end{bmatrix} \begin{bmatrix} l_S \\ m_S \\ n_S \end{bmatrix}$$

l_S, m_S, n_S are the direction cosines of the sun

P_R = solar radiation pressure

A reference area

$$P_2 = \frac{1}{2}\,(3\sin^2\beta - 1) = \frac{1}{2}\,(3\sin^2 i\,\sin^2 u - 1)$$

$$P_3 = \frac{1}{2}\,(5\sin^3\beta - 3\sin\beta) = \frac{1}{2}\,(5\sin^3 i\,\sin^3 u - 3\sin i\,\sin u)$$

$$P_2' = 3\sin\beta = 3\sin i\,\sin u$$

$$P_3' = \frac{3}{2}\,(5\sin^2\beta - 1) = \frac{3}{2}\,(5\sin^2 i\,\sin^2 u - 1)$$

1. Substitution of the above expression for $\mathfrak{F}$ in the $\frac{d\lambda}{dt}$

$$\frac{di}{dt} = \frac{R}{h}\cos u\,(\bar{\mathfrak{F}}\cdot\bar{e}_h)$$

$$\frac{di}{d\theta} = \frac{R^3}{h^2}\cos u\,(\bar{\mathfrak{F}}\cdot\bar{e}_h)$$

$$\frac{di}{d\theta} = \frac{R^3}{h^2}\cos u\,(\bar{\mathfrak{F}}_G + \bar{\mathfrak{F}}_D + \bar{\mathfrak{F}}_S)\cdot\bar{e}_h$$

$$\bar{\mathfrak{F}}_G\cdot\bar{e}_h = -\frac{\mu}{R^2}\sum_{N=2}^{3} J_N \left(\frac{R_{eq}}{R}\right)^N P_N'\cos i$$

$$\bar{\mathfrak{F}}_D\cdot\bar{e}_h = 0$$

$$\bar{\mathfrak{F}}_S\cdot\bar{e}_h = -\frac{A}{m}\,P_R N$$

$$\frac{di}{d\theta} = \frac{R^3}{h^2} \cos u \left\{ - \frac{\mu}{R^2} \cos i \sum_{N=2}^{3} J_N \left(\frac{R_{eq}}{R} \right)^N P'_N - \frac{A}{m} P_R N \right\}$$

$$= \frac{R^3}{h^2} \cos u \left\{ - \frac{\mu}{R^2} \cos i \left[J_2 \left(\frac{R_{eq}}{R} \right)^2 P'_2 + J_3 \left(\frac{R_{eq}}{R} \right)^3 P'_3 \right] - \frac{A}{m} P_R N \right\}$$

$$= \frac{R^3}{h^2} \cos u \left\{ - \frac{\mu}{R^2} \cos i \left[3J_2 \left(\frac{R_{eq}}{R} \right)^2 \sin i \sin u + \frac{3}{2} J_3 \left(\frac{R_{eq}}{R} \right)^3 (5 \sin^2 i \sin^2 u - 1) \right] - \frac{A}{m} P_r N \right\}$$

$$= - \frac{3\mu J_2}{Rh^2} \cos u \sin u \cos i \sin i R_{eq}^2 - \frac{15}{2} \frac{\mu J_3}{h^2 R^2} \cos u \sin^2 u \sin^2 i \cos i R_{eq}^3$$

$$+ \frac{3}{5} \frac{\mu}{h^2 R^2} J_3 \cos u \cos i R_{eq}^3 - \frac{P_r AN}{mh^2} R^3 \cos u$$

$$\frac{di}{d\theta} = - \frac{3J_2}{p^2} R_{eq}^2 \cos i \sin i \cos u \sin u \, [1 + e\cos\theta]$$

$$= \frac{15}{2p^3} J_3 R_{eq}^3 \sin^2 i \cos i \cos u \sin^2 u \, [1 + e\cos \ ]^2$$

$$+ \frac{3}{2} \frac{J_3}{p^3} R_{eq}^3 \cos i \cos u \, [1 + e\cos\theta]^2 - \frac{ANR^3}{mh^2} \cos u \, P_r$$

Define $\tilde{i}$ so that $\tilde{i} = i - \Delta i$ where Δi is the purely periodic portion of i. A characteristic of any periodic function is that its derivative is also periodic. The only portion of the above expression that is not necessarily periodic is the term involving solar radiation pressure, P_r. I. e. P_r is assumed to have a constant positive value in sunlight and a value of zero in the earth's shadow.

The process of removing the periodic portions of the $\frac{d\lambda}{dt}$ is done by averaging in the following way:

$$\frac{d\tilde{\lambda}}{dt} = \frac{1}{4\pi^2} \int_o^{2\pi} \int_o^{2\pi} \left[\frac{d\lambda}{dt} \right]_G d\theta \, d\omega$$

It is easily shown that terms containing $\sin^N u$, $\cos^N u$, $\sin^N u \cos^N u$, N ODD will vanish over $[o, 2\pi]$ when averaged in the above manner. This aid then makes the integration a process of inspection.

$$\frac{d\tilde{i}}{d\theta} = - \frac{ANR^3 P_r \cos u}{mh^2}$$

$$\frac{d\tilde{i}}{dt} = \frac{di}{d\theta} \frac{\mu^{\frac{1}{2}}}{a^{\frac{3}{2}}}$$

$$\frac{d\tilde{i}}{dt} = - \frac{ANR^3 P_r \cos u}{m\mu a (1 - e^2)} \frac{\mu^{\frac{1}{2}}}{a^{\frac{3}{2}}}$$

$$\frac{d\tilde{i}}{dt} = - \frac{ANR^3 P_r \cos u}{m\mu^{\frac{1}{2}} a^{\frac{5}{2}} (1 - e^2)}$$

2. Derivation of $\frac{d\tilde{\Omega}}{dt}$

$$\frac{d\Omega}{dt} = \frac{R}{h} \frac{\sin u}{\sin i} (\bar{\mathcal{F}} \cdot \bar{e}_h)$$

$$\frac{d\Omega}{d\theta} = \frac{R^3}{h^2} \frac{\sin u}{\sin i} (\bar{\mathcal{F}} \cdot \bar{e}_h)$$

$$\frac{d\Omega}{d\theta} = \frac{R^3}{h^2}\,\frac{\mathrm{sinu}}{\mathrm{sini}}\,(\bar{\mathfrak{F}}_G + \bar{\mathfrak{F}}_D + \bar{\mathfrak{F}}_S)\cdot \bar{e}_h$$

$$\left.\begin{array}{l} \bar{\mathfrak{F}}_G \cdot \bar{e}_h \\[1em] \bar{\mathfrak{F}}_D \cdot \bar{e}_h \\[1em] \bar{\mathfrak{F}}_S \cdot \bar{e}_h \end{array}\right\} \quad \text{are the same expressions as given in } \frac{di}{dt}$$

On substitution then the expression for $\frac{d\Omega}{d\theta}$ becomes

$$\frac{d\Omega}{d\theta} = \frac{R^3}{h^2}\,\frac{\sin u}{\sin i}\left\{-\frac{\mu}{R^2}\cos i\left[3J_2\left(\frac{R_{eq}}{R}\right)^2 \sin i \sin u + \frac{3}{2}J_3\left(\frac{R_{eq}}{R}\right)^3 (5\sin^2 i\,\sin^2 u - 1)\right] - \frac{A}{m}\,P_r N\right\}$$

$$\frac{d\Omega}{d\theta} = \frac{3J_2 R_{eq}^{\;2}}{p^2}\cos i\,\sin^2 u\;[1 + e\cos\theta] - \frac{15J_3}{2p^3}\,R_{eq}^{\;3}\sin i\,\cos i\,\sin^3 u\,[1 + e\cos\theta]^2 + \frac{3}{2}\,\frac{J_3}{p^3}\,R_{eq}^{\;3}\csc i\,\sin u\,[1 + e\cos\theta]^2 - \frac{ANR^3}{mh^2 \sin}\sin u\,P_r$$

Averaging as before, only the first term of the above expression will remain since it contains a secular term. The term involving solar radiation pressure is retained

$$\frac{d\Omega}{d\theta} = -\frac{3}{p^2}\,J_2\,R_{eq}^{\;2}\cos i\left\{\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\right\} - \frac{ANR^3 \sin u\,P_r}{mh^2 \sin i}$$

$$\frac{d\tilde{\Omega}}{d\theta} = -\frac{3}{2}\frac{J_2}{p^2} R_{eq}{}^2 \cos i - \frac{AMR^3 \sin u\ P_r}{mh^2 \sin i}$$

$$\frac{d\tilde{\Omega}}{d\theta} = -\frac{3}{2}\bar{J}_2 \cos i - \frac{ANR^3 \sin u\ P_r}{mh^2 \sin i} \quad , \qquad \bar{J}_2 = J_2 \left(\frac{R_{eq}}{p}\right)^2$$

$$\frac{d\tilde{\Omega}}{dt} = -\frac{3}{2}\frac{\mu^{\frac{1}{2}}}{a^{\frac{3}{2}}} \bar{J}_2 \cos i - \frac{ANR^3 \sin u}{m\mu^{\frac{1}{2}} a^{\frac{5}{2}} (1 - e^2) \sin i}$$

3. Derivation of $\frac{d\tilde{\omega}}{dt}$

$$\frac{d\omega}{dt} = -\frac{d\Omega}{dt}\cos i + \frac{R}{eh}\left\{ -\psi\cos\theta\ \bar{e}_R + (1+\psi)\sin\theta\ \bar{e}_L \right\} \cdot \bar{\mathcal{F}}$$

$$\frac{d\omega}{dt} = -\frac{d\Omega}{dt}\cos i + \frac{R}{eh}\left\{ -\psi\cos\theta\ \bar{e}_R + (1+\psi)\sin\theta\ \bar{e}_L \right\} \cdot (\bar{\mathcal{F}}_G + \bar{\mathcal{F}}_D + \bar{\mathcal{F}}_S)$$

$$\bar{e}_R \cdot \bar{\mathcal{F}}_G = \frac{\mu}{R^2} \sum_{N=2}^{3} J_N \left(\frac{R_{eq}}{R}\right)^N (N+1)\ P_N$$

$$\bar{e}_L \cdot \bar{\mathcal{F}}_G = -\frac{\mu}{R^2} \sum_{N=2}^{3} J_N \left(\frac{R_{eq}}{R}\right)^N P_N' \sin i \cos u$$

$$\bar{e}_R \cdot \bar{\mathcal{F}}_D = -\frac{\mu e \rho B}{p} [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} \sin\theta$$

$$\bar{e}_L \cdot \bar{\mathcal{F}}_D = -\frac{\mu \rho B}{p} [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} [1 + e\cos\theta]$$

$$\overline{e}_R \cdot \overline{\mathcal{F}}_S = - \frac{A}{m} P_r L$$

$$\overline{e}_L \cdot \overline{\mathcal{F}}_S = - \frac{A}{m} P_R m$$

$$\frac{d\omega}{dt} = - \frac{d\Omega}{dt} \cos i - \frac{R}{eh} \psi \cos\theta \left(\overline{e}_R \cdot \overline{\mathcal{F}}_G + \overline{e}_R \cdot \overline{\mathcal{F}}_D + \overline{e}_R \cdot \overline{\mathcal{F}}_S \right)$$

$$+ \frac{R}{eh} (1 + \psi) \sin\theta \left(\overline{e}_L \cdot \overline{\mathcal{F}}_G + \overline{e}_L \cdot \overline{\mathcal{F}}_D + \overline{e}_L \cdot \overline{\mathcal{F}}_S \right)$$

$$\frac{d\omega}{dt} = - \frac{d\Omega}{dt} \cos i - \frac{R\psi\cos\theta}{eh} \left\{ \frac{\mu}{R^2} \sum_{N=2}^{3} J_N \left(\frac{R_{eq}}{R} \right)^N (N+1) P_N \right.$$

$$\left. - \frac{\mu e \rho B}{p} [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} \sin\theta - \frac{A}{m} P_r L \right\} +$$

$$+ \frac{R(1 + \psi) \sin\theta}{eh} \left\{ - \frac{\mu}{R^2} \sum_{N=2}^{3} J_N \left(\frac{R_{eq}}{R} \right)^N P_N' \sin i \cos u \right.$$

$$\left. - \frac{\mu \rho B}{p} [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} [1 + e\cos\theta] - \frac{A}{m} P_r M \right\}$$

$$\sum_{N=2}^{3} J_N \left(\frac{R_{eq}}{R} \right)^{N+1} (N + 1) P_N = 3J_2 \left(\frac{R_{eq}}{R} \right)^2 \left(\frac{1}{2} \right) (3\sin^2 i \sin^2 u - 1)$$

$$+ 4J_3 \left(\frac{R_{eq}}{R}\right)^3 \left(\frac{1}{2}\right) (5\sin^3 i \sin^3 u - 5\sin i \sin u)$$

$$= \frac{9}{2} J_2 \left(\frac{R_{eq}}{R}\right)^2 \sin^2 i \sin^2 u - \frac{3}{2} J_2 \left(\frac{R_{eq}}{R}\right)^2 + 10J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin^3 i \sin s^3 u$$

$$- 6J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin i \sin u$$

$$\sum_{N=2}^{3} J_N \left(\frac{R_{eq}}{R}\right)^N P'_N \sin i \cos u = 3J_2 \left(\frac{R_{eq}}{R}\right)^2 \sin^2 i \sin u \cos u$$

$$+ \frac{15}{2} J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin^3 i \sin^2 u \cos u - \frac{3}{2} J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin i \cos u$$

$$\frac{d\omega}{d\theta} = \frac{d\omega}{dt} \frac{dt}{d\theta} = \frac{R^2}{h} \frac{d\omega}{dt}$$

$$\frac{d\omega}{d\theta} = - \frac{d\Omega}{d\theta} \cos i - \frac{\mu R^3 \psi \cos\theta}{eh^2} \frac{1}{R^2} \left[\frac{9}{2} J_2 \left(\frac{R_{eq}}{R}\right)^2 \sin^2 i \sin^2 u - \frac{3}{2} J_2 \left(\frac{R_{eq}}{R}\right)^2\right.$$

$$\left. + 10J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin^3 i \sin^3 u - 6J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin i \sin u\right]$$

$$\left. - \frac{e\rho B}{p} [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} \sin\theta - \frac{AP_r L}{m\mu}\right\}$$

$$+ \frac{\mu R^3 (1 + \psi)}{eh^2} \sin\theta \left\{\frac{1}{R^2} \left[-3J_2 \left(\frac{R_{eq}}{R}\right)^2 \sin^2 i \sin u \cos u\right.\right.$$

$$\left. - \frac{15}{2} J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin^3 i \sin^2 u \cos u + \frac{3}{2} J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin i \cos u\right]$$

$$- \frac{\rho B}{p} [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} [1 + e\cos\theta] - \frac{AP_r M}{\mu m} \Bigg\}$$

$$\frac{d\omega}{d\theta} = - \frac{d\Omega}{d\theta} \cos i - \frac{\cos\theta}{e} \Bigg\{ \frac{9}{2} J_2 \left(\frac{R_{eq}}{R}\right)^2 \sin^2 i \sin^2 u - \frac{3}{2} J_2 \left(\frac{R_{eq}}{R}\right)^2$$

$$+ 10J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin^3 i \sin^3 u - 6 J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin i \sin u$$

$$- \frac{e\rho BR^2}{p} [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} \sin\theta - \frac{AP_r LR^2}{m\mu} \Bigg\}$$

$$+ \frac{\sin\theta}{\psi e} \Bigg\{ - 3J_2 \left(\frac{R_{eq}}{R}\right)^2 \sin^2 i \cos u - \frac{15}{2} J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin^3 i \sin^2 u \cos u$$

$$+ \frac{3}{2} J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin i \cos u - \frac{\rho BR^2}{p} [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} [1 + e\cos\theta]$$

$$- \frac{AP_r MR^2}{\mu m} \Bigg\} + \frac{\sin\theta}{e} \Bigg\{ -3J_2 \left(\frac{R_{eq}}{R}\right)^2 \sin^2 i \sin u \cos u$$

$$- \frac{15}{2} J_2 \left(\frac{R_{eq}}{R}\right)^3 \sin^3 i \sin^2 u \cos u + \frac{3}{2} J_3 \left(\frac{R_{eq}}{R}\right)^3 \sin i \cos u$$

$$- \frac{\rho BR^2}{p} [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} [1 + e\cos\theta] - \frac{AP_r MR^2}{\mu m} \Bigg\}$$

For convenience the effects due to the three perturbing forces will be considered separately below. For $\left[\frac{d\omega}{d\theta}\right]_G$ only the first, second and third terms of the expression remain, and become

$$\left[\frac{d\tilde{\omega}}{d\theta}\right]_G = \frac{3}{2p^2} J_2 R_{eq}^{\ 2} \cos^2 i - \frac{9}{4p^2} J_2 R_{eq}^{\ 2} \sin^2 i + \frac{3}{2p^2} J_2 R_{eq}^{\ 2}$$

$$\left[\frac{d\tilde{\omega}}{d\theta}\right]_G = \frac{3}{2} \bar{J}_2 \cos^2 i - \frac{9}{4} \bar{J}_2 \sin^2 i + \frac{3}{2} J_2$$

$$\left[\frac{d\tilde{\omega}}{d\theta}\right]_G = \frac{3}{4} \bar{J}_2 \left[4 - 5\sin^2 i\right]$$

$$\frac{d\tilde{\omega}}{dt} = \frac{\mu^{\frac{1}{2}}}{a^{\frac{3}{2}}} \frac{d\tilde{\omega}}{d\theta}$$

$$\left[\frac{d\tilde{\omega}}{dt}\right]_G = \frac{3\mu^{\frac{1}{2}}}{4a^{\frac{3}{2}}} \bar{J}_2 \left[4 - 5\sin^2 i\right]$$

The effects due to drag become

$$\left[\frac{d\omega}{d\theta}\right]_D = \frac{\rho BR^2 \sin\theta}{ep} \left[1 + e^2 + 2e\cos\theta\right]^{\frac{1}{2}} \left\{e\cos\theta - 1 - (1 + e\cos\theta)\right\}$$

$$\left[\frac{d\omega}{d\theta}\right]_D = \frac{\rho BR^2 \sin\theta}{ep} \left[1 + e^2 + 2e\cos\theta\right]^{\frac{1}{2}} (-2)$$

$$\left[\frac{d\omega}{d\theta}\right]_D = -\frac{2\rho BR^2}{ep} \left[1 + e^2 + 2e\cos\theta\right]^{\frac{1}{2}} \sin\theta$$

$$\left[\frac{d\omega}{dt}\right]_D = -\frac{2\rho B\mu^{\frac{1}{2}} a^{\frac{1}{2}} (1 - e^2)^2}{e\,a(1 - e^2)} \left[1 + e^2 + 2e\cos\theta\right]^{\frac{1}{2}} \sin\theta$$

$$\left[\frac{d\omega}{dt}\right]_D = -\frac{2\rho B\mu^{\frac{1}{2}}(1-e^2)}{ea^{\frac{1}{2}}}\,[1+e^2+2e\cos\theta]^{\frac{1}{2}}\sin\theta$$

$\left[\frac{d\omega}{dt}\right]_D$ will vanish over $[o, 2\pi]$ if we consider $\rho = \rho\,(R)$ to be an even function of θ. Thus $\left[\frac{d\omega}{dt}\right]_D = 0.$

The effects of solar radiation pressure remain

$$\left[\frac{d\omega}{d\theta}\right]_S = \frac{AP_r LR^2\cos\theta}{\mu me} - \frac{AP_r MR^2\sin\theta}{\mu m\psi e} - \frac{AP_r MR^3\sin\theta}{\mu me}$$

$$\left[\frac{d\omega}{d\theta}\right]_S = \frac{AP_r R^2}{\mu me}\left[L\cos\theta - \frac{M\sin\theta}{1+e\cos\theta} - M\sin\theta\right]$$

$$\left[\frac{d\omega}{dt}\right]_S = \frac{AP_r R^2}{\mu^{\frac{1}{2}} a^{\frac{3}{2}} me}\left[L\cos\theta - \frac{M\sin\theta}{1+e\cos\theta} - M\sin\theta\right]$$

$$\frac{d\tilde{\omega}}{dt}_S = \frac{3}{4}\,\frac{\mu^{\frac{1}{2}}}{a^{\frac{3}{2}}}\,J_2\,[4-5\sin^2 i] + \frac{AP_r R^2}{m\mu^{\frac{1}{2}}a^{\frac{3}{2}}\,e}\left[L\cos\theta - \frac{M\sin\theta}{1+e\cos\theta} - M\sin\theta\right]$$

4. Derivation of $\frac{d\tilde{r}_p}{dt}$

$$\frac{dr_p}{dt} = -\frac{r_p^{\ 2}}{p}\,\frac{R}{h}\,\{\psi\sin\theta\,\bar{e}_R - [2(1-\cos\theta)+e\sin^2\theta]\,\bar{e}_L\}\cdot\bar{\mathcal{F}}$$

$$\frac{dr_p}{d\theta} = -\frac{r_p^{\ 2}}{p}\,\frac{R^3}{h^2}\,\{\psi\sin\theta\,\bar{e}_R\cdot(\bar{\mathcal{F}}_G+\bar{\mathcal{F}}_D+\bar{\mathcal{F}}_S) - [2(1-\cos\theta)$$

$$+ e\sin^2\theta]\ \bar{e}_L \cdot (\bar{\mathcal{F}}_G + \bar{\mathcal{F}}_D + \bar{\mathcal{F}}_S)\ \}$$

$$\left[\frac{dr_p}{d\theta}\right]_G = -\frac{r_p^{\ 2}R^3}{ph^2}\left\{\psi\sin\theta\,(\bar{e}_R\cdot\bar{\mathcal{F}}_G) - [2(1-\cos\theta) + e\sin^2\theta]\ \bar{e}_L\cdot\bar{\mathcal{F}}_G\right\}$$

$$\begin{aligned}
\left[\frac{dr_p}{d\theta}\right]_G = -\frac{\mu r_p^{\ 2}R}{ph^2}\Bigg\{ & \sin\theta\left[\frac{9}{2}J_2\left(\frac{R_{eq}}{R}\right)^2 \sin^2 i\ \sin^2 u - \frac{3}{2}J_2\left(\frac{R_{eq}}{R}\right)^2\right. \\
& \left. + 10J_3\left(\frac{R_{eq}}{R}\right)^3 \sin^3 i\ \sin^3 u - 6J_3\left(\frac{R_{eq}}{R}\right)^3 \sin i\ \sin u\right] \\
& + \left[2(1-\cos\theta) + e\sin^2\theta\right]\left[3J_2\left(\frac{R_{eq}}{R}\right)^2 \sin^2 i\ \sin u\cos u\right. \\
& \left. + \frac{15}{2}J_3\left(\frac{R_{eq}}{R}\right)^3 \sin^3 i\ \sin^2 u\cos u\right]\Bigg\}
\end{aligned}$$

On integration of the above expression for $\left[\frac{dr_p}{d\theta}\right]_G$ all terms will vanish due to the odd powers of the trigonometric terms involving u

$$\left[\frac{dr_p}{dt}\right]_D = -\frac{r_p^{\ 2}R}{ph}\left\{\psi\sin\theta\,(\bar{e}_R\cdot\bar{\mathcal{F}}_D) - [2(1-\cos\theta) + e\sin^2\theta](\bar{e}_L\cdot\bar{\mathcal{F}}_D)\right\}$$

The drag effects $\left[\frac{dr_p}{dt}\right]_D$ are

$$\left[\frac{dr_p}{dt}\right]_D = -\frac{\mu r_p{}^2 R\rho B}{hp^2}\,[1 + e^2 + 2e\cos\theta]^{\frac{1}{2}}\{-\psi e\sin^2\theta + [2(1 - \cos\theta) + e\sin^2\theta]\}$$

$$\left[\frac{dr_p}{dt}\right]_D = -\frac{\mu r_p{}^2 R\psi\rho B}{hp^2}\,[1 + e^2 + 2e\cos\theta]^{\frac{1}{2}}[2(1 - \cos\theta)]$$

$$\left[\frac{dr_p}{dt}\right]_D \quad -\frac{2\mu r_p{}^2\rho B}{hp}\,[1 + e^2 + 2e\cos\theta]^{\frac{1}{2}}[1 - \cos\theta]$$

$$\left[\frac{dr_p}{dt}\right]_D = -\frac{2\mu^{\frac{1}{2}} r_p{}^2\rho B}{a^{\frac{3}{2}}(1 - e^2)^{\frac{3}{2}}}\,[1 + e^2 + 2e\cos\theta]^{\frac{1}{2}}[1 - \cos\theta]$$

$$\left[\frac{dr_p}{dt}\right]_S = -\frac{r_p{}^2 R}{ph}\,\{\psi\sin\theta\,(\bar{e}_R\cdot\bar{\mathcal{F}}_S) - [2(1 - \cos\theta) + e\sin^2\theta]\,(\bar{e}_L\cdot\bar{\mathcal{F}}_S)\}$$

$$\left[\frac{dr_p}{dt}\right]_S = -\frac{r_p{}^2 R}{ph}\left\{-\frac{\psi\sin\theta AP_r L}{m} + [2(1 - \cos\theta) + e\sin^2\theta]\,\frac{AP_r M}{m}\right\}$$

$$\left[\frac{dr_p}{dt}\right]_S \quad -\frac{r_p{}^2 AP_r}{\mu^{\frac{1}{2}}a^{\frac{1}{2}}(1 - e^2)^{\frac{1}{2}}\,m}\left\{-L\sin\theta + \frac{[2(1 - \cos\theta) + e\sin^2\theta]M}{[1 + e\cos\theta]}\right\}$$

Hence the final expression for $\frac{d\tilde{r}_p}{dt}$ is

$$\frac{d\tilde{r}_p}{dt} = -\frac{2\mu^{\frac{1}{2}}\, r_p{}^2 \rho B\, [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}}}{a^{\frac{3}{2}}\, (1 - e^2)^{\frac{3}{2}}}\, [1 - \cos\theta]$$

$$- \frac{r_p{}^2 A P_r}{m\mu^{\frac{1}{2}}\, a^{\frac{1}{2}}\, (1 - e^2)^{\frac{1}{2}}} \left\{ -L\sin\theta + \frac{[2(1 - \cos\theta) + e\sin^2\theta]M}{1 + \cos\theta} \right\}$$

5. Derivation of $\frac{d\tilde{r}_a}{dt}$

$$\frac{dr_a}{dt} = \frac{r_a{}^2 R}{hp} \{ \psi\sin\theta\, \bar{e}_R + [2(1 + \cos\theta) - e\sin^2\theta]\, \bar{e}_L \} \cdot \bar{\mathfrak{F}}$$

$$\left[\frac{dr_a}{dt}\right]_G = \frac{r_a{}^2 R}{hp} \{\psi\sin\theta(\bar{e}_R \cdot \bar{\mathfrak{F}}_G) + [2(1 + \cos\theta) - e\sin^2]\ (\bar{e}_L \cdot \bar{\mathfrak{F}}_G)\}$$

Since the above expression is identical to $\left[\frac{dr_p}{d\theta}\right]_G$ insofar as trigonometric terms are involved, it vanishes over $[o, 2\pi]$ in the same manner.

$$\left[\frac{dr_a}{dt}\right]_D = -\frac{\mu r_a{}^2 R\rho B}{hp^2}\, [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} \{\psi e\sin^2\theta + [2(1 + \cos\theta) - e\sin^2\theta]\psi\}$$

$$\left[\frac{dr_a}{dt}\right]_D = -\frac{\mu r_a{}^2 \rho B}{hp}\, [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}}\, [2(1 + \cos\theta)]$$

$$\left[\frac{dr_a}{dt}\right]_D = - \frac{2\mu^{\frac{1}{2}} r_a{}^2 \rho B}{a^{\frac{3}{2}} (1 - e^2)^{\frac{3}{2}}} [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} [1 + \cos\theta]$$

$$\left[\frac{dr_a}{dt}\right]_S = \frac{r_a{}^2 R}{hp} \{\psi \sin\theta \; (\overline{e}_R \cdot \overline{\mathcal{F}}_S) + [2(1 + \cos\theta) - e\sin^2\theta] \; (\overline{e}_L \cdot \overline{\mathcal{F}}_S)\}$$

$$\left[\frac{dr_a}{dt}\right]_S = \frac{r_a{}^2 R}{hp} \left\{ \psi \sin\theta \left(- \frac{AP_r L}{m} \right) + [2(1 + \cos\theta) - e\sin^2\theta] \left(- \frac{AP_r M}{m} \right) \right\}$$

$$\left[\frac{dr_a}{dt}\right]_S = - \frac{r_a{}^2 AP_r}{m\mu^{\frac{1}{2}} a^{\frac{1}{2}} (1 - e^2)^{\frac{1}{2}}} \left\{ L\sin + \frac{M[2(1 + \cos\theta) - e\sin^2\theta]}{[1 + e\cos\theta]} \right\}$$

Hence the final expression for $\frac{d\tilde{r}_a}{dt}$ is

$$\frac{d\tilde{r}_a}{dt} \quad - \frac{2\mu^{\frac{1}{2}} r_a{}^2 \rho B}{a^{\frac{3}{2}} (1 - e^2)^{\frac{3}{2}}} [1 + e^2 + 2e\cos\theta]^{\frac{1}{2}} [1 + \cos\theta]$$

$$- \frac{r_a{}^2 AP_r}{m\mu^{\frac{1}{2}} a^{\frac{1}{2}} (1 - e^2)^{\frac{1}{2}}} \left\{ L\sin\theta + \frac{M[2(1 + \cos\theta) - e\sin^2\theta]}{1 + e\cos\theta} \right\}$$

APPENDIX B

COMPUTER PROGRAM SOURCE LISTING

```
     KILGØ  HPC                                                  06/18/65
         EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT    -   INTERNAL FØRMULA NUMBER(S)

 1001 BETA=TAN2PI(CL,CM)-TAN2PI(CLS,CMS)-180.*GAMMA/PI                          110
 2002 IF(BETA) 2000,2001,2001                                                   111
 2000 BETA=BETA+360.                                                            112
      GØ TØ 2002                                                                113
 2001 CØNTINUE                                                                  114
  700 AMP=HKM/4000.+(.91+.44*SIG+.38*SIG2)*EXP (-(2.-HKM/(405.+143.*SIG)
     1)**2)                                                                     115
  710 AMPS=.245+.0425*SIG-.0625*SIG2                                            116
      U=AMP*(-.08*EXP (-((BETA-250.)/55.)**2)+AMPS*EXP (-((BETA-135.)/
     134.)**2))+AMP*4.E-6*BETA                                                  117
      FACT=1.+(1.-CN**2)*U                                                      118
      RHØA=RHØA*FACT                                                            119
 3000 RETURN                                                                    120
      END                                                                       121
```

```
     KILGØ  TANZPI                                               06/18/65
         EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT    -   INTERNAL FØRMULA NUMBER(S)

      FUNCTIØN TAN2PI(X,Y)
C     TAN2PI=ARCTAN(Y/X)                                          0985
C     TAN2PI  EQUAL  ØR  LESS  THAN  2PI                          0986
C     TAN2PI  EQUAL  ØR  GREATER  THAN  ZERØ                      0987
      RADDEG=57.2957795                                           0988     1
      IF(Y)1,2,3                                                  0989     2
    2 IF(X)5,4,6                                                  0990     3
    4 TAN2PI=1.0E+30                                              0991     4
      GØ TØ 20                                                    0992     5
    5 TAN2PI=180.0                                                0993     6
      GØ TØ 20                                                    0994     7
    6 TAN2PI=0.0                                                  0995     8
      GØ TØ 20                                                    0996     9
    1 IF(X)7,8,9                                                  0997    10
    7 TAN2PI=180.+RADDEG*ATAN(Y/X)                                        11
      GØ TØ 20                                                    0999    12
    8 TAN2PI=270.0                                                09991   13
      GØ TØ 20                                                    0999B   14
    9 TAN2PI=360.+RADDEG*ATAN(Y/X)                                        15
      GØ TØ 20                                                    0999C   16
    3 IF(X)7,10,11                                                0999E   17
   10 TAN2PI=90.0                                                 0999F   18
      GØ TØ 20                                                    0999I   19
   11 TAN2PI=RADDEG*ATAN(Y/X)                                             20
   20 RETURN                                                      0999H   21
      END                                                         0999J   22
```

```
 REAL IØMET,MASS,INTERA,INTERP
 DIMENSIØN FTENB(153),FTEN(153),AP(153)
 DIMENSIØN          MAT1(9),MAT2(9),MAT3(9),EFP(8),EFE(8),SFE(8),
1SFG(8),EFG(8),ØET(8),ØEM(8),ØEE(8),MØT(8),MØM(8),MØE(8),AMAT(9),
1BMAT(9),CMAT(9),A1MAT(9),W1MAT(9),WMAT(9),TEMP(20),IØMET(2),
1SYIC(13),PLT(8),WDØMAT(9),WDMAT(9),WSUB1(9),WSUB1T(9)
 DIMENSIØN                                  CDPRIM(25),
1ATTACK(45),CN(45),AREA(45),MASS(25),INTERA(6),INTERP(6),
1DAPØGE(10),CDA(50),CUTØFF(2),DATE(3),CØRREC(110)
 DØUBLE PRECISIØN KERTH,KAPPA,PHIØ,LAMDØ,      AØ,BØ,ØMEGA,AE,
1JJ,HH,DD,TTT,THETA,RRR,PSI,
1LAMD,VE,ALPV,EV,XS,YS,ZS,XDS,YDS,ZDS,AXIS,ECCEN,INC,
1ASNØD,ARGP,ANØM,ECANØM,MNANØM,XE,YE,ZE,XDE,YDE,ZDE,XEP,YEP,ZEP,
1XDEP,YDEP,ZDEP,XP,YP,ZP,VP,ALPP,EP,RPHIØ,RLAMDØ,RØMEGA,RPSIØ,RO,
1BETA,AMAT,BMAT,CMAT,A1MAT,WMAT,W1MAT,RKAPPA,RTHETA,XEC,YEC,ZEC,
1TEMP,RRRE,PSIE,LAMDE,VEE,ALPVE,EVE,MAT1,MAT2,MAT3,EFP,EFE,SFE,SFG,
1EFG,ØET,ØEM,ØEE,MØT,MØM,MØE,A1,A2,A3,C1,C3,LR,CØSE,SR,SN,C2V,
1B1,B2,B3,AXISB,ECCB,INCB,ASNB,ARGB,ANMB,ECAB,MNAB,
1PITCH,RANGE,ALT,ECV,EEV,PERIGE,APØGEE,PERIØD,MVAPØG,MVPERG,
1MVPERD,BØMEG,LØMEG,PLT,XG,YG,ZG,XDG,YDG,ZDG,TFM
1,XPL,YPL,ZPL,XDPL,YDPL,ZDPL,WDØMAT,WDMAT,WSUB1,WSUB1T
 DØUBLE PRECISIØN TØLX,DPR,PIE
 CØMMØN/HBLK/XKERTH,XAØ,XBØ,XAE,XJJ,XHH,XDD,XDANØM,XF,XPRINT,
1XATMØS,XDIURN,XXLAG,XMVA,XMVP,XCDPM(25),XMASS(25),
2XATTK(45),XCN(45),XAREA(45),XINTA(6),XINTP(6),XDAPØ(10),
3XCDA(50),XCUT(2),XXDAT(3),XCØR(110),XIN,XASN,XARG
4,XECLPT,XAMPR,         XFTENB(153),XFTEN(153),XAP(153),XSA,XSR
 CØMMØN/BLK/KERTH,AØ,BØ,AE,JJ,HH,DD,TTT,RRR,PSI,LAMD,VE,ALPV,
1EV,XS,YS,ZS,XDS,YDS,ZDS,AXIS,ECCEN,INC,ASNØD,ARGP,ANØM,ECANØM,
1MNANØM,XE,YE,ZE,XDE,YDE,ZDE,XEP,YEP,ZEP,XDEP,YDEP,ZDEP,XP,YP,ZP,
1VP,ALPP,EP,RPHIØ,RLAMDØ,RØMEGA,RPSIØ,RO,BETA,AMAT,BMAT,CMAT,
1LAMDØ,KAPPA,ØMEGA,PHIØ,THETA,XG,YG,ZG,XDG,YDG,ZDG,TFM,
1XPL,YPL,ZPL,XDPL,YDPL,ZDPL,WDØMAT,WDMAT,WSUB1,WSUB1T,
1A1MAT,WMAT,W1MAT,RKAPPA,RTHETA,XEC,YEC,ZEC,TEMP,RRRE,PSIE,LAMDE,
1VEE,ALPVE,EVE,MAT1,MAT2,MAT3,A1,A2,A3,C1,C3,LR,CØSE,SR,SN,C2V,
1B1,B2,B3,AXISB,ECCB,INCB,ASNB,ARGB,ANMB,ECAB,MNAB,
1SHØRT,XLØNG
 DATA IØMET/6HSHØRT /,IØMET(2)/6HNØLØNG/
 DATA CN/6HALPHA /,CN(2)/1./,CN(3)/360./,CN(4)/6HEND    /,
1CDPRIM/1./,CDPRIM(2)/0./,CDPRIM(3)/6HEND    /,ATTACK/0./,
2ATTACK(2)/360./,ATTACK(3)/6HEND    /,AREA/6HALPHA /,AREA(2)
3/1./,AREA(3)/360./,AREA(4)/6HEND    /,MASS/6HCØN    /,MASS(2)
4/1./,MASS(3)/0./,DAPØGE/-20./,DAPØGE(2)/0./,PRINT/6HNØRMAL/,
5ATMØS/6HARDC  /,DIURNL/6HMEAN  /,ECLIPT/23.4436/
 DATA(CØRREC(I),I=1,41)/
1 .12,500.,.13,400.,.142,350.,.184,300.,.22,280.,.275,260.,
1.304,250.,.34,240.,.385,230.,.425,220.,.47,210.,.52,200.,.565,
1190.,.62,180.,.7,170.,.8,160.,.84,155.,.86,150.,1.,145.,1.,0.,
16HEND    /
 DATA (DATE(I),I=1,3)/9.,18.,1964./,SA/0./
 DATA SØ/100./,AMPR/0./
 DATA XG/0.D0/,YG/0.D0/,ZG/0.D0/,XDG/0.D0/,YDG/0.D0/,
1ZDG/0.D0/,TFM/0.D0/,XLAG/0./
 DATA(DAPØGE(I),I=1,10)/-5.,6778.,-10.,6578.,-20.,0.,0.,0.,0.,0./
 DATA KERTH/398603.2D0/
```

```
   KILGØ  CNTRL                                                            06/18/65
         EXTERNAL FØRMULA NUMBER    -    SØURCE STATEMENT    -   INTERNAL FØRMULA NUMBER(S)

      1KAPPA/105.D0/,PHIØ/28.5D0/,LAMDØ/80.5D0/,AØ/6378.165D0/,
      2BØ/6356.784D0/,ØMEGA/15.04106705D0/,AE/6378.165D0/,
      3JJ/.00162345D0/,HH/.0000575D0/,DD/.0000C7875D0/,
      4DANØM/10./,F/298.3/,
      8                                         PRINT/6HNØRMAL/,
      9INC/30./,WØ/180./,CAPØ/180./
       DATA(AP(I),I=1,141)/
      12.34,1958.,2.48,1958.8,1.89,1958.9,2.51,1959.,2.97,1959.1,2.42,
      21959.2,2.56,1959.3,2.59,1959.4,2.92,1959.5,3.16,1959.6,3.50,1959.7
      3,2.46,1959.8,2.89,1959.9,2.57,1960.,2.32,1960.1,2.44,1960.2,3.42,1
      1960.3,2.79,
      11960.4,2.92,1960.5,2.71,1960.6,2.75,1960.7,3.23,1960.8,3.44,1960.9
      1,2.49,1961.,2.27,1961.1,2.32,1961.2,2.29,1961.3,2.40,1961.4,2.69,1
      1961.5,2.26,
      11961.6,2.18,1961.7,1.85,1961.8,1.92,1961.9,1.49,1962.,1.73,1962.1,
      11.81,1962.2,2.31,1962.3,1.60,1962.4,2.18,1962.5,2.62,1962.6,2.94,1
      1962.7,3.08,
      11962.8,2.01,1962.9,1.75,1962.,1.72,1963.1,1.54,1963.12,1.51,1963.2
      11,1.86,1963.29,2.08,1963.38,2.06,1963.46,2.23,1963.54,2.35,1963.62
      1,3.26,
      11963.71,2.20,1963.79,2.02,1963.88,1.98,1963.96,2.06,1964.04,2.21,
      11964.12,2.16,1964.21,2.25,1964.29,1.81,1964.38,1.73,1964.46,1.89,
      11964.54,1.68,1964.62,1.78,1964.71,1.67,1964.79,.90,1964.88,.77,196
      14.96,2.5,1965.,2.5,2000.,6HEND   /
       DATA(RTENB(I),I=1,57)/
      1243.6,1958.,230.7,1958.5,226.5,1959.,208.9,1959.5,180.5,1960.,
      2161.,1960.5,130.8,1961.,104.8,1961.5,99.3,1962.,89.7,1962.5,
      382.7,1963.,80.8,1963.5,77.9,1964.,70.,1964.5,75.,1965.,
      487.,1965.5,131.,1966.5,186.,1967.5,200.,1968.5,190.,1969.5,
      5163.,1970.5,142.,1971.,128.,1971.5,108.,1972.5,94.,1973.5,
      681.,1974.5,75.,1975.,75.,1975.5,
      46HEND    /
       DATA (FTEN(I),I=1,3)/0.,0.,6HEND    /
       DØ 1 I=1,8                                                        ,1
       EFP(I)=-0.                                                        ,2
       EFE(I)=-0.                                                        ,3
       SFE(I)=-0.                                                        ,4
       SFG(I)=-0.                                                        ,5
       PLT(I)=-0.                                                        ,6
       EFG(I)=-0.                                                        ,7
       ØET(I)=-0.                                                        ,8
       ØEM(I)=-0.                                                        ,9
       ØEE(I)=-0.                                                        ,10
       MØT(I)=-0.                                                        ,11
       MØM(I)=-0.                                                        ,12
 1     MØE(I)=-0.                                                        ,13    ,14
 75    CALL MAVRIK(IERR,5HKERTH,KERTH,5HKAPPA,KAPPA,4HPHIØ,PHIØ,
      15HLAMDØ,LAMDØ,2HAØ,AØ,2HBØ,BØ,5HØMEGA,ØMEGA,2HAE,AE,
      12HJJ,JJ,2HHH,HH,2HDD,DD,5HIØMET,IØMET,3HEFP,EFP,3HEFE,EFE,
      13HSFE,SFE,3HSFG,SFG,3HEFG,EFG,3HØET,ØET,3HØEM,ØEM,
      13HØEE,ØEE,3HMØT,MØT,3HMØM,MØM,3HMØE,MØE,3HPLT,PLT,2HXG,XG,
      12HYG,YG,2HZG,ZG,3HXDG,XDG,3HYDG,YDG,3HZDG,ZDG,3HTFM,TFM,
      15HDANØM,DANØM,1HF,F,6HCDPRIM,CDPRIM,6HATTACK,ATTACK,
      12HCN,CN,4HAREA,AREA,4HMASS,MASS,6HINTERA,INTERA,
      16HINTERP,INTERP,6HCAPØGE,CAPØGE,3HCDA,CDA,5HPRINT,PRINT,
      16HCUTØFF,CUTØFF,4HDATE,DATE,5HATMØS,ATMØS,6HCØRREC,CØRREC,
```

```
       KILGØ  CNTRL                                                       06/18/65 .
           EXTERNAL FØRMULA NUMBER   -    SØURCE STATEMENT    -   INTERNAL FØRMULA NUMBER(S)

     16HDIURNL,DIURNL,3HLAG,XLAG,4HFTEN,FTEN,5HFTENB,FTENB,2HKP,AP,
     16HECLIPT,ECLIPT,2HSA,SA,2HSØ,SØ,4HAMPR,AMPR)                                 ,15
      IF(IERR)76,77,76                                                             ,16
76    WRITE(6,78)                                                                  ,17    ,18
78    FØRMAT(1H017HCARD FØRMAT ERRØR)
      CALL PDUMP                                                                   ,19
      GØTØ75                                                                       ,20
77    CØNTINUE                                                                     ,21
      IF(ØET.EQ.-0.)GØ TØ 4                                                        ,22    ,23    ,24
      IF(ØET(2).GT.1.)GØ TØ 3                                                      ,25    ,26    ,27
      DØ 2 I=3,8                                                                   ,28
      IF(ØET(I).NE.-0.)GØ TØ 200                                                   ,29    ,30    ,31
2     CØNTINUE                                                                     ,32    ,33
      MVAPØG=ØET*(1.+ØET(2))                                                       ,34
      MVPERG=ØET*(1.-ØET(2))                                                       ,35
      SWIT=1.                                                                      ,36
      GØ TØ 100                                                                    ,37
3     DØ 300 I=3,8                                                                 ,38
      IF(ØET(I).NE.-0.)GØ TØ 301                                                   ,39    ,40    ,41
300   CØNTINUE                                                                     ,42    ,43
      MVAPØG=ØET                                                                   ,44
      MVPERG=ØET(2)                                                                ,45
      SWIT=1.                                                                      ,46
      GØ TØ 100                                                                    ,47
301   TEMP=(ØET+ØET(2))/2.                                                         ,48
      ØET(2)=(ØET-ØET(2))/(ØET+ØET(2))                                             ,49
      ØET=TEMP                                                                     ,50
      GØTØ200                                                                      ,51
4     IF(ØEM.EQ.-0.)GØ TØ 7                                                        ,52    ,53    ,54
      IF(ØEM(2).GT.1.)GØTØ 6                                                       ,55    ,56    ,57
      DØ 5 I=3,8                                                                   ,58
      IF(ØEM(I).NE.-0.)GØTØ 200                                                    ,59    ,60    ,61
5     CØNTINUE                                                                     ,62    ,63
      MVAPØG=ØEM*(1.+ØEM(2))                                                       ,64
      MVPERG=ØEM*(1.-ØEM(2))                                                       ,65
      SWIT=1.                                                                      ,66
      GØ TØ 100                                                                    ,67
6     DØ 302 I=3,8                                                                 ,68
302   CØNTINUE                                                                     ,69    ,70
      MVAPØG=ØEM                                                                   ,71
      MVPERG=ØEM(2)                                                                ,72
      IF (ØEM(I).NE.-0.) GØ TØ 303                                                 ,73    ,74    ,75
      SWIT=1.                                                                      ,76
      GØ TØ 100                                                                    ,77
303   TEMP=(ØEM+ØEM(2))/2.                                                         ,78
      ØEM(2)=(ØEM-ØEM(2))/(ØEM+ØEM(2))                                             ,79
      ØEM=TEMP                                                                     ,80
      GØTØ200                                                                      ,81
7     IF(ØEE.EQ.-0.)GØ TØ 10                                                       ,82    ,83    ,84
      IF(ØEE(2).GT.1.)GØTØ 9                                                       ,85    ,86    ,87
      DØ 8 I=3,8                                                                   ,88
      IF(ØEE(I).NE.-0.)GØTØ200                                                     ,89    ,90    ,91
8     CØNTINUE                                                                     ,92    ,93
      MVAPØG=ØEE*(1.+ØEE(2))                                                       ,94
      MVPERG=ØEE*(1.-ØEE(2))                                                       ,95
      SWIT=1.                                                                      ,96
```

```
       KILGØ  CNTRL                                                         06/18/65
           EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT   -   INTERNAL FØRMULA NUMBER(S)

       GØTØ100                                                           ,97
 9     DØ 304 I=3,8                                                      ,98
       IF(ØEE(I).NE.-0.)GØTØ305                                          ,99    ,100   ,101
 304   CØNTINUE                                                          ,102   ,103
       MVAPØG=ØEE                                                        ,104
       MVPERG=ØEE(2)                                                     ,105
       SWIT=1.                                                           ,106
       GØTØ100                                                           ,107
 305   TEMP=(ØEE+ØEE(2))/2.                                              ,108
       ØEE(2)=(ØEE-ØEE(2))/(ØEE+ØEE(2))                                  ,109
       ØEE=TEMP                                                          ,110
       GØTØ200                                                           ,111
 10    IF(MØT.EQ.-0.)GØ TØ 13                                            ,112   ,113   ,114
       IF(MØT(2).GT.1.)GØ TØ 12                                          ,115   ,116   ,117
       DØ 11 I=3,8                                                       ,118
       IF(MØT(I).NE.-0.)GØTØ200                                          ,119   ,120   ,121
 11    CØNTINUE                                                          ,122   ,123
       MVAPØG=MØT*(1.+MØT(2))                                            ,124
       MVPERG=MØT*(1.-MØT(2))                                            ,125
       SWIT=1.                                                           ,126
       GØTØ100                                                           ,127
 12    DØ 306 I=3,8                                                      ,128
       IF(MØT(I).NE.-0.)GØTØ307                                          ,129   ,130   ,131
 306   CØNTINUE                                                          ,132   ,133
       MVAPØG=MØT                                                        ,134
       MVPERG=MØT(2)                                                     ,135
       SWIT=1.                                                           ,136
       GØTØ100                                                           ,137
 307   TEMP=(MØT+MØT(2))/2.                                              ,138
       MØT(2)=(MØT-MØT(2))/(MØT+MØT(2))                                  ,139
       MØT=TEMP                                                          ,140
       GØTØ200                                                           ,141
 13    IF(MØM.EQ.-0.)GØTØ 16                                             ,142   ,143   ,144
       IF(MØM(2).GT.1.)GØTØ15                                            ,145   ,146   ,147
       DØ14 I=3,8                                                        ,148
       IF(MØM(I).NE.-0.)GØTØ200                                          ,149   ,150   ,151
 14    CØNTINUE                                                          ,152   ,153
       MVAPØG=MØM*(1.+MØM(2))                                            ,154
       MVPERG=MØM*(1.-MØM(2))                                            ,155
       SWIT=1.                                                           ,156
       GØTØ100                                                           ,157
 15    DØ 308 I=3,8                                                      ,158
       IF(MØM(I).NE.-0.)GØTØ309                                          ,159   ,160   ,161
 308   CØNTINUE                                                          ,162   ,163
       MVAPØG=MØM                                                        ,164
       MVPERG=MØM(2)                                                     ,165
       SWIT=1.                                                           ,166
       GØTØ100                                                           ,167
 309   TEMP=(MØM+MØM(2))/2.                                              ,168
       MØM(2)=(MØM-MØM(2))/(MØM+MØM(2))                                  ,169
       MØM=TEMP                                                          ,170
       GØTØ200                                                           ,171
 16    IF(MØE.EQ.-0.)GØTØ 200                                            ,172   ,173   ,174
       IF(MØE(2).GT.1.)GØTØ 18                                           ,175   ,176   ,177
       DØ 17 I=3,8                                                       ,178
       IF(MØE(I).NE.-0.)GØTØ200                                          ,179   ,180   ,181
```

```
       KILGØ  CNTRL                                                    06/18/65
            EXTERNAL FØRMULA NUMBER    -    SØURCE STATEMENT    -   INTERNAL FØRMULA NUMBER(S)

 17    CØNTINUE                                                             ,182   ,183
       MVAPØG=MØE*(1.+MØE(2))                                               ,184
       MVPERG=MØE*(1.-MØE(2))                                               ,185
       SWIT=1.                                                              ,186
       GØTØ100                                                              ,187
 18    DØ 310 I=3,8                                                         ,188
       IF(MØE(I).NE.-0.)GØTØ311                                             ,189   ,190   ,191
 310   CØNTINUE                                                             ,192   ,193
       MVAPØG=MØE                                                           ,194
       MVPERG=MØE(2)                                                        ,195
       SWIT=1.                                                              ,196
       GØTØ100                                                              ,197
 311   TEMP=(MØE+MØE(2))/2.                                                 ,198
       MØE(2)=(MØE-MØE(2))/(MØE+MØE(2))                                     ,199
       MØE=TEMP                                                             ,200
       GØTØ200                                                              ,201
 200   SWIT=0.                                                              ,202
       CALL TRFM(KERTH,KAPPA,PHIØ,LAMDØ,AØ,BØ,ØMEGA,AE,JJ,HH,
      1DD,IØMET,EFP,EFE,SFE,SFG,EFG,ØET,ØEM,ØEE,MØT,MØM,MØE,
      1PLT,XG,YG,ZG,XDG,YDG,ZDG,TFM,MVAPØG,MVPERG,INCB,ASNB,ARGB)           ,203
       MVAPØG=MVAPØG+AE                                                     ,204
       MVPERG=MVPERG+AE                                                     ,205
 100   XKERTH=KERTH                                                         ,206
       XAMPR=AMPR                                                           ,207
       XAØ=AØ                                                               ,208
       XBØ=BØ                                                               ,209
       XAE=AE                                                               ,210
       XJJ=JJ                                                               ,211
       XHH=HH                                                               ,212
       XSA=SA                                                               ,213
       XSR=SØ                                                               ,214
       XDD=DD                                                               ,215
       XDANØM=DANØM                                                         ,216
       XF=F                                                                 ,217
       XPRINT=PRINT                                                         ,218
       XATMØS=ATMØS                                                         ,219
       XDIURN=DIURNL                                                        ,220
       XXLAG=XLAG                                                           ,221
       XMVA=MVAPØG                                                          ,222
       XMVP=MVPERG                                                          ,223
       DØ 50 I=1,25                                                         ,224
       XCDPM(I)=CDPRIM(I)                                                   ,225
 50    XMASS(I)=MASS(I)                                                     ,226   ,227
       XECLPT=ECLIPT                                                        ,228
       DØ 60 I=1,153                                                        ,229
       XFTENB(I)=FTENB(I)                                                   ,230
       XFTEN(I)=FTEN(I)                                                     ,231
 60    XAP(I)=AP(I)                                                         ,232   ,233
       DØ 51 I=1,45                                                         ,234
       XATTK(I)=ATTACK(I)                                                   ,235
       XCN(I)=CN(I)                                                         ,236
 51    XAREA(I)=AREA(I)                                                     ,237   ,238
       DØ 52 I=1,6                                                          ,239
       XINTA(I)=INTERA(I)                                                   ,240
 52    XINTP(I)=INTERP(I)                                                   ,241   ,242
       DØ 53 I=1,10                                                         ,243
```

```
         KILGØ  CNTRL                                              06/18/65
             EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT   -   INTERNAL FØRMULA NUMBER(S)

   53    XDAPØ(I)=DAPØGE(I)                                          ,244   ,245
         DØ 54 I=1,50                                                ,246
   54    XCDA(I)=CDA(I)                                              ,247   ,248
         XCUT=CUTØFF                                                 ,249
         XCUT(2)=CUTØFF(2)                                           ,250
         DØ 55 I=1,3                                                 ,251
   55    XXDAT(I)=DATE(I)                                            ,252   ,253
         DØ 56 I=1,110                                               ,254
   56    XCØR(I)=CØRREC(I)                                           ,255   ,256
         IF(SWIT)57,57,58                                            ,257
   57    XIN=INCB                                                    ,258
         XASN=ASNB                                                   ,259
         XARG=ARGB                                                   ,260
         GØ TØ 59                                                    ,261
   58    XIN=INC                                                     ,262
         XASN=CAPØ                                                   ,263
         XARG=WØ                                                     ,264
   59    CALL LIFE                                                   ,265
         GØTØ 75                                                     ,266
         END                                                         ,267
```

```
KILGØ  LIFE                                                              06/18/65
    EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT   -   INTERNAL FØRMULA NUMBER(S)

     SUBRØUTINE LIFE
     REAL JJ,KERTH,INTA,INTP,MASS,MT,INTERA,INTERP,INC,NI
     DIMENSIØN DATE(3), XMØNTH(12)
     DIMENSIØN CØRREC(110),SØLAR(301)
     DIMENSIØN TN(6),INTERA(6),TN1(6),INTERP(6),
    1CØSE(365),E(365),CDPRIM(25),AREA(45),CN(45),
    2ATTACK(45),STEPA(365),STEPP(365),MASS(25)
     DIMENSIØN FTENB(153),FTEN(153),AP(153)
     CØMMØN /ARCC/TEMPT,TEMK,PRESS,PØMM,
    1PSPØ,RHØ,RHØSRØ,VISC,VISCSL,KVISC,VS,G
     DATA DPR/57.2957795/,           PI/3.14159291/,SNBCI/6HSINE  /
    1,BCITIM/6HTIME  /
     DATA AID/6HA     /,PID/6HP     /,DETID/6HDETAIL/,
    1SHØID/6HSHØRT /
     DATA ENDID/6HEND   /
     DATA (XMØNTH(I),I=1,12)/31.,28.,31.,30.,31.,30.,31.,31.,
    130.,31.,30.,31./
     DATA ARDID/6HARDC  /,USSID/6HUSSTD /,DINØR/6HNØRMAL/,
    1DIMM/6HMEAN  /
     DATA AT1/6HARDC  /,AT2/6HUSSTD /,AT3/6HPØE   /,AT4/6HSMALL /,
    1AT5/6HSPEOAR/
     CØMMØN/HBLK/XKERTH,XAØ,XBØ,XAE,XJJ,XHH,XCD,XDANØM,XF,XPRINT,
    1XATMØS,XDIURN,XXLAG,XMVA,XMVP,XCDPM(25),XMASS(25),
    2XATTK(45),XCN(45),XAREA(45),XINTA(6),XINTP(6),XDAPØ(10),
    3XCDA(50),XCUT(2),XXDAT(3),XCØR(110),XIN,XASN,XARG
    4,XECLPT,XAMPR,        XFTENB(153),XFTEN(153),XAP(153),XSA,XSR
     CØMMØN/CLK/APØ,PER1,TIME,AE,SINI,BØ,F,JJ,KERTH,
    1CØSI,    JCNT,CDPRIM,       AREA,ATTACK,       CN,
    2MASS,ADØT,PDØT,PDØAC,TIMED
    3,     CØSE,E,DAØ2,PER,TIME1,HH,DEØ3
    4,CAPDM1,CAPID,SMAM1,ADTM1,APØM1,SMAW,SMADM1,SMAID,CAPM1,CAPW
    5,CAPØ,DATE,XMØNTH,FTENB,AP
    6,INTERA,INTERP,DD,DANØM,DAPØGE(10),CDA(50)
    7,PRINT,     CUTØFF(2),                       AI,RPAI,SADØTI
    8,REVØL,MT,VPI,PDI
    9,PN(6),AN(6),REV1
    1,CØRREC,SØLAR,ATMØS,FTEN,DIURNL,XLAG,RHØXX,SØ,SA
    1,EI,RIPP,RIPA,AMPR
     DØ 10 I=1,365                                                        ,1
     CØSE(I)=0.                                                           ,2
     E(I)=0.                                                              ,3
     STEPA(I)=0.                                                          ,4
  10 STEPP(I)=0.                                                          ,5      ,6
     REV1=0.                                                              ,7
     REVØL=0.                                                             ,8
     TIME1=0.                                                             ,9
     DØ 104 I=1,5                                                         ,10
     INTERA(I)=0.                                                         ,11
 104 INTERP(I)=0.                                                         ,12     ,13
     DD=0.                                                                ,14
     TIME=0.                                                              ,15
     DØ 7000 I=4,110                                                      ,16
7000 CØRREC(I)=0.                                                         ,17     ,18
     DØ 857 I=1,153                                                       ,19
     FTENB(I)=XFTENB(I)                                                   ,20
```

```
      EXTERNAL FØRMULA NUMBER  -  SØURCE STATEMENT  -  INTERNAL FØRMULA NUMBER(S)

      FTEN(I)=XFTEN(I)                                              ,21
857   AP(I)=XAP(I)                                                  ,22    ,23
      ECLIPT=XECLPT                                                 ,24
      AMPR=XAMPR                                                    ,25
      SA=XSA                                                        ,26
      SØ=XSR                                                        ,27
      KERTH=XKERTH                                                  ,28
      AØ=XAØ                                                        ,29
      BØ=XBØ                                                        ,30
      AE=XAB                                                        ,31
      JJ=XJJ                                                        ,32
      HH=XHH                                                        ,33
      DD=XDD                                                        ,34
      DANØM=XDANØM                                                  ,35
      F=XF                                                          ,36
      PRINT=XPRINT                                                  ,37
      ATMØS=XATMØS                                                  ,38
      DIURNL=XDIURN                                                 ,39
      XLAG=XXLAG                                                    ,40
      AØØ=XMVA                                                      ,41
      PØØ=XMVP                                                      ,42
      DØ 850 I=1,25                                                 ,43
      CDPRIM(I)=XCDPM(I)                                            ,44
850   MASS(I)=XMASS(I)                                              ,45    ,46
      DØ 851 I=1,45                                                 ,47
      ATTACK(I)=XATTK(I)                                            ,48
      CN(I)=XCN(I)                                                  ,49
851   AREA(I)=XAREA(I)                                              ,50    ,51
      DØ 852 I=1,6                                                  ,52
      INTERA(I)=XINTA(I)                                            ,53
852   INTERP(I)=XINTP(I)                                            ,54    ,55
      DØ 853 I=1,10                                                 ,56
853   DAPØGB(I)=XDAPØ(I)                                            ,57    ,58
      DØ 854 I=1,50                                                 ,59
854   CDA(I)=XCDA(I)                                                ,60    ,61
      CUTØFF=XCUT                                                   ,62
      CUTØFF(2)=XCUT(2)                                             ,63
      DØ 855 I=1,3                                                  ,64
855   DATE(I)=XXCAT(I)                                              ,65    ,66
      DØ 856 I=1,11C                                                ,67
856   CØRREC(I)=XCØR(I)                                             ,68    ,69
      INC=XIN                                                       ,70
      CAPØ=XASN                                                     ,71
      WØ=XARG                                                       ,72
3015  CØNTINUE                                                      ,73
2700  FØRMAT(1H0,1A6/)
      WRITE(6,30)                                                   ,74    ,75
  30  FØRMAT(1H1,25X,27H***** EARTH CØNSTANTS *****)
      WRITE(6,31)JJ,HH,DD,KERTH,AE,F                                ,76    ,77    ,78
  31  FØRMAT(1H021HEARTH SECØND HARMØNICE15.8,5X,
     120HEARTH THIRD HARMØNICE15.8,5X,
     221HEARTH FØURTH HARMØNICE15.8/,
     31H 40HEARTH GRAVITATIØNAL CØNSTANT (KILØMETERS,
     423H CUBED/SECØNDS SQUARED)E15.8/,
     51H 30HEQUATØRIAL RADIUS (KILØMETERS)E15.8,5X,
     611HELLIPTICITYE15.8/)
```

```
      WRITE(6,1400)                                                   ,79   ,80
 1400 FØRMAT(1H0,25X,32H***** BALLISTIC PARAMETERS *****)
      IF(CDA-SNBCI)32,33,32                                           ,81
   33 WRITE(6,34)                                                     ,82   ,83
   34 FØRMAT(1H011HSPECIAL CDA)
      WRITE(6,1100)                                                   ,84   ,85
 1100 FØRMAT(1H 17HREV(CYCLES/ØRBIT),5X,10HTIME(DAYS))
      KK=2                                                            ,86
 1103 IF(CDA(KK)-ENDID)1101,1102,1101                                 ,87
 1101 KK=KK+2                                                         ,88
      GØTØ1103                                                        ,89
 1102 KK=KK-2                                                         ,90
      DØ 1104 I=1,KK,2                                                ,91
 1104 WRITE(6,1105)CDA(I+1),CDA(I+2)                                  ,92   ,93   ,94   ,95
 1105 FØRMAT(1H E15.8,5X,E15.8)
   32 WRITE(6,35)                                                     ,96   ,97
   35 FØRMAT(1H024HANGLE ØF ATTACK FUNCTIØN)
      WRITE(6,36)                                                     ,98   ,99
   36 FØRMAT(1H 14HALPHA(DEGREES),6X,
     116HANØMALY(DEGREES))
      KK=2                                                            ,100
  702 IF(ATTACK(KK-1)-ENDID)700,701,700                               ,101
  700 KK=KK+2                                                         ,102
      GØ TØ 702                                                       ,103
  701 KK=KK-2                                                         ,104
      DØ 38 I=1,KK,2                                                  ,105
      WRITE(6,37)ATTACK(I),ATTACK(I+1)                                ,106  ,107  ,108
   37 FØRMAT(1H E15.8,5X,E15.8)
   38 CØNTINUE                                                        ,109  ,110
      WRITE(6,39)                                                     ,111  ,112
   39 FØRMAT(1H028HCØEFFICIENT ØF DRAG FUNCTIØN)
      KK=2                                                            ,113
  705 IF(CN(KK)-ENDID)703,704,703                                     ,114
  703 KK=KK+2                                                         ,115
      GØ TØ 705                                                       ,116
  704 CØNTINUE                                                        ,117
      IF(CN-BCITIM)40,41,40                                           ,118
   40 WRITE(6,42)                                                     ,119  ,120
   42 FØRMAT(1H 2HCN,18X,14HALPHA(DEGREES))
      GØ TØ 44                                                        ,121
   41 WRITE(6,43)                                                     ,122  ,123
   43 FØRMAT(1H 2HCN,18X,10HTIME(DAYS))
   44 KK=KK-2                                                         ,124
      DØ 46 I=1,KK,2                                                  ,125
      WRITE(6,45)CN(I+1),CN(I+2)                                      ,126  ,127  ,128
   45 FØRMAT(1H E15.8,5X,E15.8)
   46 CØNTINUE                                                        ,129  ,130
      WRITE(6,47)                                                     ,131  ,132
   47 FØRMAT(1H07HCDPRIME,13X,
     119HPERIGEE(KILØMETERS))
      KK=2                                                            ,133
  708 IF(CDPRIM(KK-1)-ENDID)706,707,706                               ,134
  706 KK=KK+2                                                         ,135
      GØ TØ 708                                                       ,136
  707 KK=KK-2                                                         ,137
      DØ 49 I=1,KK,2                                                  ,138
```

```
      WRITE(6,48)CDPRIM(I),CDPRIM(I+1)                                   ,139  ,140  ,141
   48 FØRMAT(1H E15.8,5X,E15.8)
   49 CØNTINUE                                                           ,142  ,143
      WRITE(6,50)                                                        ,144  ,145
   50 FØRMAT(1H028HEFFECTIVE DRAG AREA FUNCTIØN)
      KK=2                                                               ,146
  711 IF(AREA(KK)-ENDID)709,710,709                                      ,147
  709 KK=KK+2                                                            ,148
      GØ TØ 711                                                          ,149
  710 CØNTINUE                                                           ,150
      IF(AREA-BOITIM)51,52,51                                            ,151
   51 WRITE(6,53)                                                        ,152  ,153
   53 FØRMAT(1H 20HAREA(METERS SQUARED),3X,
     114HALPHA(DEGREES))
      GØ TØ 55                                                           ,154
   52 WRITE(6,54)                                                        ,155  ,156
   54 FØRMAT(1H 20HAREA(METERS SQUARED),3X,
     110HTIME(DAYS))
   55 CØNTINUE                                                           ,157
      KK=KK-2                                                            ,158
      DØ 57 I=1,KK,2                                                     ,159
      WRITE(6,56)AREA(I+1),AREA(I+2)                                     ,160  ,161  ,162
   56 FØRMAT(1H E15.8,8X,E15.8)
   57 CØNTINUE                                                           ,163  ,164
      WRITE(6,58)                                                        ,165  ,166
   58 FØRMAT(1H014HMASS CØNSTANTS)
      IF(MASS-BOITIM)59,60,59                                            ,167
   59 WRITE(6,61)MASS(2)                                                 ,168  ,169  ,170
   61 FØRMAT(1H 23HINITIAL MASS(KILØGRAMS)E15.8/)
      IF(MASS(3))62,70,62                                                ,171
   62 WRITE(6,63)                                                        ,172  ,173
   63 FØRMAT(1H 23HMASS DECAY RATE(KG/DAY),3X,
     114HFINAL MASS(KG))
      KK=2                                                               ,174
  714 IF(MASS(KK+1)-ENDID)712,713,712                                    ,175
  712 KK=KK+2                                                            ,176
      GØ TØ 714                                                          ,177
  713 KK=KK-2                                                            ,178
      DØ 200 I=1,KK,2                                                    ,179
      WRITE(6,64)MASS(I+2),MASS(I+3)                                     ,180  ,181  ,182
   64 FØRMAT(1H E15.8,11X,E15.8)
  200 CØNTINUE                                                           ,183  ,184
      GØ TØ 70                                                           ,185
   60 WRITE(6,65)                                                        ,186  ,187
   65 FØRMAT(1H 15HMASS(KILØGRAMS),3X,
     110HTIME(DAYS))
      KK=2                                                               ,188
  717 IF(MASS(KK)-ENDID)715,716,715                                      ,189
  715 KK=KK+2                                                            ,190
      GØ TØ 717                                                          ,191
  716 KK=KK-2                                                            ,192
      DØ 202 I=1,KK,2                                                    ,193
      WRITE(6,66)MASS(I+1),MASS(I+2)                                     ,194  ,195  ,196
   66 FØRMAT(1H E15.8,5X,E15.8)
  202 CØNTINUE                                                           ,197  ,198
   70 CØNTINUE                                                           ,199
```

```
       KILGO  LIFE                                                   06/18/65
           EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

      WRITE(6,1401)                                                           ,200  ,201
 1401 FORMAT(1H0,25X,30H***** DENSITY PARAMETERS *****)
      DELYR=DATE(3)-1957.                                                     ,202
      XDAYS=DELYR*365.                                                        ,203
      XLEAP=DELYR/4.                                                          ,204
      I=XLEAP                                                                 ,205
      XLEAP=I                                                                 ,206
      XDAYS=XDAYS+XLEAP                                                       ,207
      K=DATE(1)                                                               ,208
      YDAYS=0.                                                                ,209
      K=K-1                                                                   ,210
      DO 1700 I=1,K                                                           ,211
 1700 YDAYS=XMONTH(I)+YDAYS                                                   ,212  ,213
      XDAYS=XDAYS+YDAYS+DATE(2)                                               ,214
      IF(DATE(1)-2.)1702,1701,1701                                            ,215
 1701 XLPE=DATE(3)-1956.                                                      ,216
 1704 XLPE=XLPE-4.                                                            ,217
      IF(XLPE)1702,1703,1704                                                  ,218
 1703 XDAYS=XDAYS+1.                                                          ,219
 1702 XDAYS=XDAYS-365.                                                        ,220
      WRITE(6,1705)DATE,XDAYS                                                 ,221  ,222  ,223
 1705 FORMAT(1H06HMONTH=E9.2,5X,4HDAY=E9.2,5X,4HYEARE11.4,5X,
     133HDAYS ELAPSED SINCE DEC. 31 , 1957E15.8)
 1402 IF(ATMOS-AT1)6002,6000,6002                                             ,224
 6000 WRITE(6,6001)                                                           ,225  ,226
 6001 FORMAT(1H 20H1959 ARDC ATMOSPHERE)
      GOTO 6016                                                               ,227
 6002 IF(ATMOS-AT2)6005,6003,6005                                             ,228
 6003 WRITE(6,6004)                                                           ,229  ,230
 6004 FORMAT(1H 29H1962 U.S. STANDARD ATMOSPHERE)
      GOTO 6016                                                               ,231
 6005 IF(ATMOS-AT3)6008,6006,6008                                             ,232
 6006 WRITE(6,6007)                                                           ,233  ,234
 6007 FORMAT(1H 21HPOE ATMOSPHERE (LMSC))
      GOTO 6016                                                               ,235
 6008 IF(ATMOS-AT4)6011,6009,6011                                             ,236
 6009 WRITE(6,6010)                                                           ,237  ,238
 6010 FORMAT(1H 28HHUNT SMALL ATMOSPHERE (LMSC))
      GOTO 6016                                                               ,239
 6011 IF(ATMOS-AT5)6014,6012,6014                                             ,240
 6012 WRITE(6,6013)                                                           ,241  ,242
 6013 FORMAT(1H 28HSPECIAL 1959 ARDC ATMOSPHERE)
      GOTO 6016                                                               ,243
 6014 WRITE(6,6015)                                                           ,244  ,245
 6015 FORMAT(1H 37HSPECIAL 1962 U.S. STANDARD ATMOSPHERE)
 6016 CONTINUE                                                                ,246
      WRITE(6,1412)                                                           ,247  ,248
 1412 FORMAT(1H018HDENSITY CORRECTION)
      WRITE(6,1413)                                                           ,249  ,250
 1413 FORMAT(1H 2HDC,18X,19HPERIGEE(KILOMETERS))
      KK=2                                                                    ,251
 1416 IF(CORREC(KK-1)-ENDID)1414,1415,1414                                    ,252
 1414 KK=KK+2                                                                 ,253
      GOTO 1416                                                               ,254
 1415 KK=KK-2                                                                 ,255
      DO 1417 I=1,KK,2                                                        ,256
```

```
     KILGØ  LIFE                                                   06/18/65
          EXTERNAL FØRMULA NUMBER  -   SØURCE STATEMENT    -   INTERNAL FØRMULA NUMBER(S)

       WRITE(6,1418)CØRREC(I),CØRREC(I+1)                              ,257  ,258  ,259
 1418  FØRMAT(1H E15.8,5X,E15.8)
 1417  CØNTINUE                                                        ,260  ,261
       WRITE(6,1465)                                                   ,262  ,263
 1465  FØRMAT(1H02HKP,18X,4HYEAR)
       KK=3                                                            ,264
 1452  IF(AP(KK-2)-ENDID)1450,1451,1450                                ,265
 1450  KK=KK+2                                                         ,266
       GØTØ1452                                                        ,267
 1451  KK=KK-3                                                         ,268
       DØ 1453 I=1,KK,2                                                ,269
       WRITE(6,1454)AP(I),AP(I+1)                                      ,270  ,271  ,272
 1454  FØRMAT(1H E15.8,5X,E15.8)
 1453  CØNTINUE                                                        ,273  ,274
       WRITE(6,1455)                                                   ,275  ,276
 1455  FØRMAT(1H04HFTEN,16X,4HYEAR)
       KK=3                                                            ,277
 1458  IF(FTEN(KK-2)-ENDID)1456,1457,1456                              ,278
 1456  KK=KK+2                                                         ,279
       GØTØ1458                                                        ,280
 1457  KK=KK-3                                                         ,281
       DØ 1459 I=1,KK,2                                                ,282
       WRITE(6,1454)FTEN(I),FTEN(I+1)                                  ,283  ,284  ,285
 1459  CØNTINUE                                                        ,286  ,287
       WRITE(6,1460)                                                   ,288  ,289
 1460  FØRMAT(1H05HFTENB,15X,4HYEAR)
       KK=3                                                            ,290
 1463  IF(FTENB(KK-2)-ENDID)1461,1462,1461                             ,291
 1461  KK=KK+2                                                         ,292
       GØTØ1463                                                        ,293
 1462  KK=KK-3                                                         ,294
       DØ 1464 I=1,KK,2                                                ,295
       WRITE(6,1454)FTENB(I),FTENB(I+1)                                ,296  ,297  ,298
 1464  CØNTINUE                                                        ,299  ,300
       IF(DIURNL-DINØR)1422,1420,1422                                  ,301
 1420  WRITE(6,1421)                                                   ,302  ,303
 1421  FØRMAT(1H016HDIURNAL    NØRMAL)
       GØTØ1427                                                        ,304
 1422  CØNTINUE                                                        ,305
 1423  WRITE(6,1424)                                                   ,306  ,307
 1424  FØRMAT(1H014HDIURNAL    MEAN)
 1427  CØNTINUE                                                        ,308
       WRITE(6,1429)                                                   ,309  ,310
 1429  FØRMAT(1H0,25X,26H***** SPECIAL EVENTS *****)
       IF(CUTØFF-AID)71,72,71                                          ,311
   72  WRITE(6,73)CUTØFF(2)                                            ,312  ,313  ,314
   73  FØRMAT(1H017HAPØGEE CUTØFF(KM)E15.8)
       GØ TØ 80                                                        ,315
   71  IF(CUTØFF-PID)74,75,74                                          ,316
   75  WRITE(6,76)CUTØFF(2)                                            ,317  ,318  ,319
   76  FØRMAT(1H018HPERIGEE CUTØFF(KM)E15.8)
       GØ TØ 80                                                        ,320
   74  WRITE(6,77)                                                     ,321  ,322
   77  FØRMAT(1H019HEARTH IMPACT CUTØFF)
   80  CØNTINUE                                                        ,323
       IF(INTERA)83,83,82                                              ,324
```

```
     KILGØ  LIFE                                                                                  06/18/65
         EXTERNAL FØRMULA NUMBER  -    SØURCE STATEMENT     -   INTERNAL FØRMULA NUMBER(S)

   82 WRITE(6,84)                                                                                  ,325  ,326
 84    FØRMAT(1H034HTIME FØR APØGEE INTERPØLATIØN(DAY))
      DØ 86 I=1,5                                                                                  ,327
      WRITE(6,85)INTERA(I)                                                                         ,328  ,329  ,330
   85 FØRMAT(1H E15.8)
  86  CØNTINUE                                                                                     ,331  ,332
   83 IF(INTERP)87,87,88                                                                           ,333
   88 WRITE(6,89)                                                                                  ,334  ,335
 89    FØRMAT(1H035HTIME FØR PERIGEE INTERPØLATIØN(DAY))
      DØ 91 I=1,5                                                                                  ,336
      WRITE(6,90)INTERP(I)                                                                         ,337  ,338  ,339
   90 FØRMAT(1H E15.8)
   91 CØNTINUE                                                                                     ,340  ,341
 87   CØNTINUE                                                                                     ,342
      WRITE(6,1430)                                                                                ,343  ,344
 1430 FØRMAT(1H0,25X,30H***** INITIAL CØNDITIØNS *****)
      IF(PRINT-DETIC)92,93,92                                                                      ,345
  93  WRITE(6,94)                                                                                  ,346  ,347
   94 FØRMAT(1H015HDETAIL PRINTØUT)
      GØ TØ 100                                                                                    ,348
   92 IF(PRINT-SHØIC)95,96,95                                                                      ,349
   96 WRITE(6,97)                                                                                  ,350  ,351
   97 FØRMAT(1H014HSHØRT PRINTØUT)
      GØ TØ 100                                                                                    ,352
   95 WRITE(6,98)                                                                                  ,353  ,354
   98 FØRMAT(1H015HNØRMAL PRINTØUT)
  100 CØNTINUE                                                                                     ,355
      WRITE(6,81)DANØM                                                                             ,356  ,357  ,358
   81 FØRMAT(1H021HANØMALY STEP(DEGREES)E15.8)
      WRITE(6,101)                                                                                 ,359  ,360
  101 FØRMAT(1H016HAPØGEE STEPS(KM),13X,
    118HPERIGEE RADIUS(KM))
      DØ 103 I=1,10,2                                                                              ,361
      WRITE(6,102)DAPØGE(I),DAPØGE(I+1)                                                            ,362  ,363  ,364
  102 FØRMAT(1H E15.8,15X,E15.8)
  103 CØNTINUE                                                                                     ,365  ,366
      TEMM=0.                                                                                      ,367
      TEMP=360./DANØM                                                                              ,368
      RADDE=DANØM/DPR                                                                              ,369
      I=TEMP                                                                                       ,370
      JCNT=I+1                                                                                     ,371
      K=2                                                                                          ,372
    4 IF(I)2,2,3                                                                                   ,373
    3 I=I-1                                                                                        ,374
      TEMM=RADDE+TEMM                                                                              ,375
      CØSE(K)=CØS(TEMM)                                                                            ,376
      E(K)=TEMM*DPR                                                                                ,377
      K=K+1                                                                                        ,378
      GØ TØ 4                                                                                      ,379
    2 CØSE=1.                                                                                      ,380
      E=0.                                                                                         ,381
      DE03=RADDE/3.                                                                                ,382
      SINI=SIN(INC/DPR)                                                                            ,383
      CØSI=CØS(INC/DPR)                                                                            ,384
      KA =1                                                                                        ,385
      KP=1                                                                                         ,386
```

```
      KILGO  LIFE                                                   06/18/65
         EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

       I=1                                                                 ,387
  302  IF(AOO-DAPOGE(I+1))300,301,301                                      ,388
  300  I=I+2                                                               ,389
       GO TO 302                                                           ,390
  301  DA=DAPOGE(I)                                                        ,391
       DAO2=DA/2.                                                          ,392
       PER=POO                                                             ,393
       PER1=ROO                                                            ,394
       APO=AOO                                                             ,395
       CALL PDAD                                                           ,396
       ADTM1=ADOT                                                          ,397
       IF(PRINT-DETIC)105,106,105                                          ,398
  106  WRITE(6,150)                                                        ,399   ,400
       CALL PRINTT                                                         ,401
  150  FORMAT(1H146HAPOGEE,PERIGEE,MAJOR AXIS,AND EARTH RADIUS(KM)/,
      11X,48HAPOGEE,PERIGEE,MAJOR AXIS RATES(KM/DAY) MASS(KG)/,
      21X,39HASCENDING NODE,ARGUMENT OF PERIGEE(DEG)/,
      31X,38HNODE,PERIGEE REGRESSION RATES(DEG/DAY)/,
      41X,24HPERIGEE VELOCITY(KM/SEC)/,
      51X,19HORBITAL PERIOD(MIN)/,
      61X,29HLIFETIME SPENT(ORBIT AND DAY)/,
      71X,47HRHO(KG/M3), EI(UNITLESS), RIPERG AND RIPAPG(KM)/)
       GOTO 108                                                            ,402
  105  IF(PRINT-SHOIC)107,106,107                                          ,403
  107  WRITE(6,109)                                                        ,404   ,405
  109  FORMAT(1H129HAPOGEE,PERIGEE,MAJOR AXIS,AND,
      117H,EARTH RADIUS(KM)/,1X,
      243HAPOGEE,PERIGEE,AND MAJOR AXIS RATES(KM/DAY)/,1X,
      329HLIFETIME SPENT(ORBIT AND DAY))
       WRITE(6,110)APO,PER1,AI,RPAI,ADOT,PDOT,SADOTI,REVOL,TIME            ,406   ,407   ,408
  110  FORMAT(1H06HA     E15.8,3X,6HP     E15.8,3X,6HAXIS  E15.8,3X,
      16HRADIUSE15.8/,1X,6HADOT  E15.8,3X,6HPDOT  E15.8,3X,
      26HAXIDOTE15.8,3X,6HORBIT E15.8,3X,6HTIME  E15.8/)
  108  CONTINUE                                                            ,409
 3001  CALL RK                                                             ,410
 3021  IF(INTERA(KA))400,400,401                                           ,411
  401  IF(TIME-INTERA(KA))400,402,402                                      ,412
  402  AN(KA)=(APO-DA)+(INTERA(KA)-TIME1)*(APO-(APO-DA))/(TIME-TIME1)      ,413
       KA=KA+1                                                             ,414
       GOTO3021                                                            ,415
  400  IF(INTERP(KP))403,403,404                                           ,416
  404  IF(TIME-INTERP(KP))403,405,405                                      ,417
  405  PN(KP)=PER+(INTERP(KP)-TIME1)*(PER1-PER)/(TIME-TIME1)               ,418
       KP=KP+1                                                             ,419
       GOTO400                                                             ,420
  403  CONTINUE                                                            ,421
       I=1                                                                 ,422
  305  IF(APO-DAPOGE(I+1))303,304,304                                      ,423
  303  I=I+2                                                               ,424
       GO TO 305                                                           ,425
  304  DA=DAPOGE(I)                                                        ,426
       DAO2=DA/2.                                                          ,427
       PER=PER1                                                            ,428
       TIME1=TIME                                                          ,429
       IF(PRINT-DETIC)111,112,111                                          ,430
  111  IF(PRINT-SHOIC)113,114,113                                          ,431
```

```
    KILGØ  LIFE                                                        06/18/65
         EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT   -   INTERNAL FØRMULA NUMBER(S)

 113 WRITE(6,110)APØ,PER1,AI,RPAI,ADØT,PDØT,SADØTI,REVØL,TIME          ,432   ,433   ,434
     GØ TØ 114                                                         ,435
 112 CALL PRINTT                                                       ,436
 114 IF(CUTØFF-AID)115,116,115                                         ,437
 116 IF(APØ-CUTØFF(2))130,130,3001                                     ,438
 115 IF(CUTØFF-PID)117,118,117                                         ,439
 118 IF(PER1-CUTØFF(2))130,130,3001                                    ,440
 117 IF(PER1-AB)130,130,3001                                           ,441
 130 IF(PRINT-DETID)131,132,131                                        ,442
 131 IF(PRINT-SHØID)133,132,133                                        ,443
 132 CALL PRINTT                                                       ,444
     GØ TØ 3000                                                        ,445
 133 WRITE(6,110)APØ,PER1,AI,RPAI,ADØT,PDØT,SADØTI,REVØL,TIME          ,446   ,447   ,448
     GØ TØ 3000                                                        ,449
3000 IF(INTERA)500,500,501                                             ,450
 501 WRITE(6,502)                                                      ,451   ,452
 502 FØRMAT(1H033HAPØGEE ALTITUDE INTERPØLATIØN(KM),3X,
    112HTIME IN DAYS)
     KA=KA-1                                                           ,453
     DØ 504 I=1,KA                                                     ,454
     WRITE(6,503)AN(I),INTERA(I)                                       ,455   ,456   ,457
 503 FØRMAT(1H0E15.8,5X,E15.8)
 504 CØNTINUE                                                          ,458   ,459
 500 IF(INTERP)505,505,506                                             ,460
 506 WRITE(6,507)                                                      ,461   ,462
 507 FØRMAT(1H034HPERIGEE ALTITUDE INTERPØLATIØN(KM),3X,
    112HTIME IN DAYS)
     KP=KP-1                                                           ,463
     DØ 509 I=1,KP                                                     ,464
     WRITE(6,508)PN(I),INTERP(I)                                       ,465   ,466   ,467
 508 FØRMAT(1H0E15.8,5X,E15.8)
 509 CØNTINUE                                                          ,468   ,469
 505 CØNTINUE                                                          ,470
     TIME=0.                                                           ,471
     TIME1=0.                                                          ,472
     REVØL=0.                                                          ,473
     REV1=0.                                                           ,474
     RETURN                                                            ,475
3020 STØP                                                              ,476
     END                                                               ,477
```

```
       SUBRØUTINE PDAD
       REAL JJ,KERTH,INTA,INTP,MASS,MT,INTERA,INTERP,INC,NI
       DIMENSIØN DATE(3), XMØNTH(12)
       DIMENSIØN FTENB(153),FTEN(153),AP(153)
       DIMENSIØN CØRREC(110),SØLAR(301)
       DIMENSIØN PR(16)
       DIMENSIØN TN(6),INTERA(6),TN1(6),INTERP(6),
      1CØSE(365),E(365),CDPRIM(25),AREA(45),CN(45),
      2ATTACK(45),STEPA(365),STEPR(365),MASS(25)
       CØMMØN /ARDC/TEMPT,TEMK,PRESS,PØMP,
      1PSPØ,RHØ,RHØSRØ,VISC,VISCSL,KVISC,VS,G
       DATA DPR/57.2957795/,           PI/3.14159291/,SNBCI/6HSINE   /
      1,BCITIM/6HTIME   /
       DATA DINØR/6HNØRMAL/,DIMN/6HMEAN  /,DINØN/6HNØNE  /
       DATA AID/6HA      /,PID/6HP      /,DETID/6HDETAIL/,
      1SHØID/6HSHØRT /
       DATA ENDID/6HEND    /
       DATA AT1/6HARDC  /,AT2/6HUSSTD /,AT3/6HPØE   /,AT4/6HSMALL /,
      1AT5/6HSPECAR/
       CØMMØN/CLK/APØ,PER1,TIME,AE,SINI,WØ,F,JJ,KERTH,
      1CØSI,    JCNT,CDPRIM,        AREA,ATTACK,        CN,
      2MASS,ADØT,PDØT,PDØAD,TIMED
      3,    CØSE,E,DAØ2,PER,TIME1,HH,DEØ3
      4,CAPDM1,CAPID,SMAM1,ADTM1,APØM1,SMAW,SMADM1,SMAID,CAPM1,CAPW
      5,CAPØ,DATE,XMØNTH,FTENB,AP
      6,INTERA,INTERP,DD,DANØM,DARØGE(10),CDA(50)
      7,PRINT,    CUTØFF(2),                    AI,RPAI,SADØTI
      8,REVØU,MT,VPI,PDI
      9,PN(6),AN(6),REVI
      1,CØRREC,SØLAR,ATMØS,FTEN,DIURNL,XUAG,RHØXX,SØ,SA
      1,EI,RIPP,RIPA,AMPR
       DELYR=DATE(3)-1957.                                        ,1
       XDAYS=DELYR*365.                                           ,2
       XLEAP=DELYR/4.                                             ,3
       I=XLEAP                                                    ,4
       XLEAP=I                                                    ,5
       XDAYS=XDAYS+XLEAP                                          ,6
       K=DATE(1)                                                  ,7
       YDAYS=0.                                                   ,8
       K=K-1                                                      ,9
       DØ 1706 I=1,K                                              ,10
  1706 YDAYS=XMØNTH(I)+YDAYS                                      ,11    ,12
       XDAYS=XDAYS+YDAYS+DATE(2)                                  ,13
       IF(DATE(1)-2.)1702,1701,1701                               ,14
  1701 XLPE=DATE(3)-1956.                                         ,15
  1704 XLPE=XLPE-4.                                               ,16
       IF(XLPE)1702,1703,1704                                     ,17
  1703 XDAYS=XDAYS+1.                                             ,18
  1702 XDAYS=XDAYS-365.                                           ,19
       AI=(APØ+PER1)/2.                                           ,20
       EI=(APØ-PER1)/(APØ+PER1)                                   ,21
       IF(TIME)10,10,11                                           ,22
    10 RPAI=AE*(1.-(SINI*SINI*SIN(WØ/DPR)**2/F))                  ,23
       B1=1.-3.*SINI*SINI/2.                                      ,24
       TEMP=1.-EI*EI                                              ,25
```

```
   KILGØ  PDAD                                                        06/18/65
       EXTERNAL FØRMULA NUMBER   -    SØURCE STATEMENT     -   INTERNAL FØRMULA NUMBER(S)

        TEMP1=SQRT(TEMP)                                                        ✓26
        NI=1.+(JJ*AE/AI*AE/AI*B1/(TEMP*TEMP1))                                  ✓27
        NI=SQRT(KERTH/(AI*AI*AI))*NI*24.*3600.*DPR                              ✓28
        SMAID=JJ*NI*(AE/(AI*TEMP))**2*(2.-5.*SINI*SINI/2.)                      ✓29
        CAPID=-JJ*NI*(AE/(AI*TEMP))**2*CØSI                                     ✓30
        CAPM1=CAPØ                                                              ✓31
        SMAM1=WØ                                                                ✓32
        SMAW=WØ                                                                 ✓33
        CAPW=CAPØ                                                               ✓34
        APØM1=APØ                                                               ✓35
        CAPDM1=CAPID                                                            ✓36
        SMADM1=SMAID                                                            ✓37
        GØ TØ 12                                                                ✓38
     11 DTI=(APØ-APØM1)/ADTM1                                                   ✓39
        IF(AMPR)468,468,469                                                     ✓40
    468 SINTA=0.                                                                ✓41
        SINTP=0.                                                                ✓42
        SINTW=0.                                                                ✓43
        GØTØ470                                                                 ✓44
    469 CALL SØRAP(XDAYZ,AI,EI,CAPW,SMAW,INC,AMPR,KERTH,SDW,SDRA,SDRP)          ✓45
        SINTA=(2.*SDRA-SDRP)/3./5.729578                                        ✓46
        SINTP=SDRP/3./5.729578                                                  ✓47
        SINTW=SDW*10./3.                                                        ✓48
   470  SMAW=SMAM1+DTI*(SMADM1+SINTW)                                           ✓49
   2102 IF(SMAW-360.)2100,2101,2101                                             ✓50
   2101 SMAW=SMAW-360.                                                          ✓51
        GØTØ 2102                                                               ✓52
   2100 SMAW=SMAW                                                               ✓53
        CAPW=CAPM1+CAPDM1*DTI                                                   ✓54
   2105 IF(CAPW-360.)2103,2104,2104                                             ✓55
   2104 CAPW=CAPW-360.                                                          ✓56
        GØTØ2105                                                                ✓57
   2103 CAPW=CAPW                                                               ✓58
        B1=1.-3.*SINI*SINI/2.                                                   ✓59
        TEMP=1.-EI*EI                                                           ✓60
        TEMP1=SQRT(TEMP)                                                        ✓61
        NI=1.+(JJ*AE/AI*AE/AI*B1/(TEMP*TEMP1))                                  ✓62
        NI=SQRT(KERTH/(AI*AI*AI))*NI*24.*3600.*DPR                              ✓63
        CAPID=-JJ*NI*(AE/(AI*TEMP))**2*CØSI                                     ✓64
        SMAID=JJ*NI*(AE/(AI*TEMP))**2*(2.-5.*SINI*SINI/2.)                      ✓65
        RPAI=AE*(1.-(SINI*SINI*SIN(SMAW/DPR)**2/F))                             ✓66
     12 CØNTINUE                                                                ✓67
        PDI=2.*PI/60.*SQRT(AI*AI*AI/KERTH)                                      ✓68
        VPI=SQRT(KERTH)*SQRT(2./PER1*(1.-JJ/3.*AE
       1/PER1*AE/PER1*(3.*SINI*SINI*SIN(SMAW/DPR)*SIN
       2(SMAW/DPR)-1.))-1./AI)                                                  ✓69
        L=JCNT                                                                  ✓70
        J=1                                                                     ✓71
   1502 IF(L)1500,1500,1501                                                     ✓72
   1501 L=L-1                                                                   ✓73
        RI=AI*(1.-EI*EI)/(1.+EI*CØSE(J))+
       1(2./3.*JJ/AI*AE/(1.-EI*EI)*AE)*((SINI*SINI)*(1.-
       2SIN((SMAW+E(J))/DPR)**2/2.)-.5)-3./10.*HH/
       3JJ*AE*SINI*(SIN((SMAW+E(J))/DPR)-SIN((WØ
       4+E(J))/DPR))                                                            ✓74
        RIP=RI-AE*(1.-SINI*SINI*
```

```
        EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT   -   INTERNAL FØRMULA NUMBER(S)

      1SIN((SMAW+E(J))/DPR)**2/F)                                    /75
       IF(E(J))1900,1900,1901                                        /76
 1900  RIPP=RIP                                                      /77
       GØTØ1903                                                      /78
 1901  IF((ABS(E(J)-180.))-.001)1902,1902,1903                       /79
 1902  RIPA=RIP                                                      /80
 1903  CØNTINUE                                                      /81
       IF(RIP)80,/81,81                                              /82
   80  RIP=0.                                                        /83
   81  CØNTINUE                                                      /84
       I=1                                                           /85
   28  IF(RIP-CDPRIM(I+1))29,30,31                                   /86
   29  I=I+2                                                         /87
       IF(I-24)28,28,7001                                            /88
   31  IF(I-1)30,/30,32                                              /89
   32  CDP=CDPRIM(I-2)+(CDPRIM(I)-CDPRIM(I-2))*(RIP-
      1CDPRIM(I-1))/(CDPRIM(I+1)-CDPRIM(I-1))                        /90
       GØ TØ 26                                                      /91
   30  CDP=CDPRIM(I)                                                 /92
   26  IF(CDA-SNBCI)100,/101,100                                     /93
  101  I=2                                                           /94
 1103  IF(TIME-CDA(I+1))1100,1100,/1102                              /95
 1102  I=I+2                                                         /96
       IF(I-44)1103,1103,7001                                        /97
 1100  CDAX=CDA(I)                                                   /98
       CDAREA=CDP*CN(2)*AREA(2)+(CDP*CN(4)*AREA(4)
      1-CDP*CN(2)*AREA(2))*(ABS(SIN((CDAX  *E(J)+ATTACK)/
      2DPR)))                                                        /99
       GØ TØ 500                                                     /100
  100  I=1                                                           /101
  203  IF(E(J)-ATTACK(I+1))200,201,202                               /102
  202  I=I+2                                                         /103
       IF(I-44)203,203,7001                                          /104
  200  IF(I-1)201,201/204                                            /105
  204  ALPHA=ATTACK(I-2)+(ATTACK(I)-ATTACK(I-2))
      1*(E(J)-ATTACK(I-1))/(ATTACK(I+1)-
      2ATTACK(I-1))                                                  /106
       GØ TØ 205                                                     /107
  201  ALPHA=ATTACK(I)                                               /108
  205  IF(CN-BCITIM)34,33,34                                         /109
   34  I=2                                                           /110
   38  IF(ALPHA-CN(I+1))35,36/37                                     /111
   37  I=I+2                                                         /112
       IF(I-44)38,38,7001                                            /113
   35  IF(I-2)36,/36,39                                              /114
   39  CNN=CN(I-2)+(CN(I)-CN(I-2))*(ALPHA-
      1CN(I-1))/(CN(I+1)-CN(I-1))                                    /115
       GØ TØ 40                                                      /116
   36  CNN=CN(I)                                                     /117
       GØ TØ 40                                                      /118
   33  I=2                                                           /119
   53  IF(TIME-CN(I+1))50,51,52                                      /120
   52  I=I+2                                                         /121
       IF(I-44)53,53,7001                                            /122
   50  IF(I-2)51,/51,54                                              /123
   54  CNN=CN(I)                                                     /124
```

```
      GØ TØ 40                                                  ✓125
   51 CNN=CN(I)                                                 ✓126
   40 CD=CDP*CNN                                                ✓127
      IF(AREA-BCITIM)60,61,60                                   ✓128
   60 I=2                                                       ✓129
   65 IF(ALPHA-AREA(I+1))62,63,64                               ✓130
   64 I=I+2                                                     ✓131
      IF(I-44)65,65,7001                                        ✓132
   62 IF(I-2)63,63,66                                           ✓133
   66 ARAA=AREA(I-2)+(AREA(I)-AREA(I-2))*(ALPHA-
     1AREA(I-1))/(AREA(I+1)-AREA(I-1))                          ✓134
      GØ TØ 70                                                  ✓135
   63 ARAA=AREA(I)                                              ✓136
      GØ TØ 70                                                  ✓137
   61 I=2                                                       ✓138
   74 IF(TIME-AREA(I+1))71,72,73                                ✓139
   73 I=I+2                                                     ✓140
      IF(I-44)74,74,7001                                        ✓141
   71 IF(I-2)72,72,75                                           ✓142
   75 ARAA=AREA(I)                                              ✓143
      GØ TØ 70                                                  ✓144
   72 ARAA=AREA(I)                                              ✓145
   70 CDAREA=ARAA*CD                                            ✓146
500    IF(RIP-700.)300,301,301                                  ✓147
301    RHØ=0.                                                   ✓148
       GØTØ 503                                                 ✓149
300    IF(ATMØS-AT1)302,303,302                                 ✓150
303    CALL ARDC59(RIP*1.E3)                                    ✓151
       GØ TØ 503                                                ✓152
302    IF(ATMØS-AT2)304,305,304                                 ✓153
305    PR(1)=RIP*1.E3                                           ✓154
       CALL PRA63(PR,ERRØR)                                     ✓155
       RHØ=PR(6)/9.81                                           ✓156
       GØ TØ 503                                                ✓157
304    IF(ATMØS-AT3)306,307,306                                 ✓158
307    X=RI *CØS((SMAW+E(J))/DPR)                               ✓159
       Y=RI *SIN((SMAW+E(J))/DPR)                               ✓160
       XP=X                                                     ✓161
       YP=Y*SQRT(1.-SINI**2)                                    ✓162
       ZP=Y*SINI                                                ✓163
       TEMP=CØS(CAPW/DPR)                                       ✓164
       TEMQ=SIN(CAPW/DPR)                                       ✓165
       XS=XP*TEMP-YP*TEMQ                                       ✓166
       YS=XP*TEMQ+YP*TEMP                                       ✓167
       ZS=ZP                                                    ✓168
       RS=SQRT(XS*XS+YS*YS+ZS*ZS)                               ✓169
       IF(ATMØS-AT3)350,351,350                                 ✓170
351    D=XDAYS+TIME                                             ✓171
       CALL PØEAT(RIP,RHØ,D,XS,YS,ZS,RS)                        ✓172
       RHØ=RHØ*515.7/9.81                                       ✓173
       GØTØ 503                                                 ✓174
350    D=XDAYS+TIME+36203.                                      ✓175
       CALL SMATMS(XS,YS,ZS,SA,D,RHØ,RIP)                       ✓176
       RHØ=RHØ/9.81                                             ✓177
       GØ TØ 503                                                ✓178
306    IF(ATMØS-AT4)308,307,308                                 ✓179
```

```
      KILGØ  PDAD                                                        06/18/65
          EXTERNAL FØRMULA NUMBER  -   SØURCE STATEMENT    -   INTERNAL FØRMULA NUMBER(S)

308   IF(ATMØS-AT5)310,311,310                                                    ,180
311   CALL ARDC59(RIP*1.E3)                                                       ,181
      RHØ=RHØ*9.81                                                                ,182
      GØTØ 312                                                                    ,183
310   PR(1)=RIP*1.E3                                                              ,184
      CALL PRA63(PR,ERRØR)                                                        ,185
      RHØ=PR(6)                                                                   ,186
312   I=1                                                                         ,187
316   IF(CØRREC(I)-ENDID)345,346,345                                              ,188
346   DCC=CØRREC(I-2)                                                             ,189
      GØTØ318                                                                     ,190
345   IF(RIP-CØRREC(I+1 ))313,314,315                                             ,191
313   I=I+2                                                                       ,192
      IF(I-42)316,316,7001                                                        ,193
315   IF(I-1)314,314,317                                                          ,194
317   DCC=CØRREC(I-2)+(CØRREC(I)-CØRREC(I-2))*(RIP-
     1CØRREC(I-1))/(CØRREC(I+1)-CØRREC(I-1))                                      ,195
      GØ TØ 318                                                                   ,196
314   DCC=CØRREC(I)                                                               ,197
318   IF(RIP-120.)319,319,320                                                     ,198
319   RHØ=RHØ*DCC/9.81                                                            ,199
      GØ TØ 503                                                                   ,200
320   YERR=1958.+(TIME+XDAYS)/365.24                                              ,201
      XDAYZ=XDAYS+TIME+36204.                                                     ,202
      IF(FTENB)322,321,322                                                        ,203
 321  CØNTINUE                                                                    ,204
      TEMP=(XDAYZ-36030.)/4090.*4.*PI                                             ,205
      TEMO=CØS(TEMP)*15.                                                          ,206
      TEMP=(XDAYZ-36340.)/4090.*2.*PI                                             ,207
      TEMR=CØS(TEMP)*75.                                                          ,208
      FTENBX=135.+TEMR+TEMO                                                       ,209
      GØTØ4206                                                                    ,210
 322  I=1                                                                         ,211
4205  IF(FTENB(I)-ENDID)4201,4200,4201                                            ,212
4200  FTENBX=FTENB(I-2)                                                           ,213
      GØTØ 4206                                                                   ,214
4201  IF(FTENB(I+1)-YERR)4204,4203,4202                                           ,215
4204  I=I+2                                                                       ,216
      GØTØ4205                                                                    ,217
4203  FTENBX=FTENB(I)                                                             ,218
      GØTØ 4206                                                                   ,219
4202  IF(I-1)4203,4203,5200                                                       ,220
5200  FTENBX=FTENB(I-2)+(YERR-FTENB(I-1))*(FTENB(I)-FTENB(I-2))/
     1(FTENB(I+1)-FTENB(I-1))                                                     ,221
4206  IF(FTEN)4207,4208,4207                                                      ,222
4208  FTENX=FTENBX                                                                ,223
      GØ TØ 4215                                                                  ,224
4207  I=1                                                                         ,225
4214  IF(FTEN(I)-ENDID)4209,4210,4209                                             ,226
4210  FTENX=FTEN(I-2)                                                             ,227
      GØTØ 4215                                                                   ,228
4209  IF(FTEN(I+1)-YERR)4211,4212,4213                                           ,229
4211  I=I+2                                                                       ,230
      GØ TØ 4214                                                                  ,231
4212  FTENX=FTEN(I)                                                               ,232
      GØTØ 4215                                                                   ,233
```

```
4213 IF(I-1)4212,4212,5201                                            √234
5201 FTENX=FTEN(I-2)+(YERR-FTEN(I-1))*(FTEN(I)-FTEN(I-2))/
    1(FTEN(I+1)-FTEN(I-1))                                             √235
4215 IF(AP)4216,4217,4216                                              √236
4217 APX=0.                                                            √237
     GØTØ343                                                           √238
4216 I=1                                                               √239
4223 IF(AP(I)-ENDIC)4218,4219,4218                                     √240
4219 APX=AP(I-2)                                                       √241
     GØTØ343                                                           √242
4218 IF(AP(I+1)-YERR)4220,4221,4222                                    √243
4220 I=I+2                                                             √244
     GØTØ4223                                                          √245
4221 APX=AP(I)                                                         √246
     GØTØ343                                                           √247
4222 IF(I-1)4221,4221,5202                                             √248
5202 APX=AP(I-2)+(YERR-AP(I-1))*(AP(I)-AP(I-2))/
    1(AP(I+1)-AP(I+1))                                                 √249
343  SBAR=25.+.8*FTENBX+.4*(FTENX-FTENBX)+APX*10.                      √250
     TEMP=(XDAYZ-38047.)/365.25                                        √251
     TEMQ=.06*CØS(4.*PI*TEMP)                                          √252
     TEMR=.025*CØS(2.*PI*TEMP)                                         √253
     GTT=TEMR-TEMQ                                                     √254
     SS=SBAR*EXP (GTT)                                                 √255
     IF(DIURNL-DINØR)400,403,400                                       √256
400  CØNTINUE                                                          √257
402  CØSPP=CØS(75./DPR)                                                √258
     GØTØ 404                                                          √259
403  CØNTINUE                                                          √260
     TEMP=(SS-160.)/90.                                                √261
     TEMR=-.00567*(RIP-200.)+EXP(-.01455*(RIP-200.))                   √262
     TEMQ=18.5+21.5*EXP(-.0315*(RIP-200.))                             √263
     TEMS=(18.5+30.*EXP(TEMR)+TEMQ*TEMR+4.*(1.-TEMP*TEMP))/DPR         √264
     IF(TEMS-5.)510,510,511                                            √265
511  TEMS=5.                                                           √266
510  XLAG=TEMS*DPR                                                     √267
     XLAMS=.017203*XDAYZ+.0335*SIN(.017203*XDAYZ)-1.41                 √268
     TEMP=CØS(XLAMS)                                                   √269
     TEMQ=SIN(XLAMS)                                                   √270
     XLS=TEMP                                                          √271
     TEMR=CØS(ECLIPT/DPR)                                              √272
     TEMS=SIN(ECLIPT/DPR)                                              √273
     XMS=TEMR*TEMQ                                                     √274
     XNS=TEMS*TEMQ                                                     √275
     RAS=ATAN2(XMS,XLS)                                                √276
     RAB=RAS-XLAG/DPR                                                  √277
     XLB=SQRT(XNS*XNS+XLS*XLS)*CØS(RAB)                                √278
     XMB=SQRT(XNS*XNS+XLS*XLS)*SIN(RAB)                                √279
     XNB=XMS                                                           √280
     X=RI *CØS((SMAW+E(J))/DPR)                                        √281
     Y=RI *SIN((SMAW+E(J))/DPR)                                        √282
     XP=X                                                              √283
     YP=Y*SQRT(1.-SINI**2)                                             √284
     ZP=Y*SINI                                                         √285
     TEMP=CØS(CAPW/DPR)                                                √286
     TEMQ=SIN(CAPW/DPR)                                                √287
```

```
       KILGØ  PDAC                                                          06/18/65
           EXTERNAL FØRMULA NUMBER  -  SØURCE STATEMENT  -  INTERNAL FØRMULA NUMBER(S)

        XS=XP*TEMP-YP*TEMQ                                                      ,288
        YS=XP*TEMQ+YP*TEMP                                                      ,289
        ZS=ZP                                                                   ,290
        XL=XS/RI                                                                ,291
        XM=YS/RI                                                                ,292
        XN=ZS/RI                                                                ,293
        CØSPP=XL*XLB+XM*XMB+XN*XNB                                              ,294
 404    XK=(3.+2.5*((RIP-360.)/240.)-.5*((RIP-360.)/240.)**2)*
       1((5.6-CØSPP)/6.6)                                                       ,295
 405    TEMP=(EXP (.0055*RIP)-.19)*.19                                          ,296
        XMLTP= (1.+(TEMP)*((1.+CØSPP)/2.)**3)/
       1(1.+(TEMP)*((1.+CØS(75./DPR))/2.)**3)                                   ,297
 406    RHØ=RHØ*DCC*((SS/SØ)**XK)     *XMLTP/9.81                               ,298
        GØ TØ 503                                                               ,299
 503    RHØ=RHØ*9.81                                                            ,300
        RHØXX=RHØ                                                               ,301
        TERM=CDAREA*RHØ*(SQRT(1.+2.*EI*CØSE(J)
       1+EI*EI)/((1.+EI*CØSE(J))**2))                                           ,302
        STEPA(J)=TERM*(1.+CØSE(J))                                              ,303
        STEPP(J)=TERM*(1.-CØSE(J))                                              ,304
        J=J+1                                                                   ,305
        GØ TØ 1502                                                              ,306
 1500   INTA=0.                                                                 ,307
        INTP=0.                                                                 ,308
        TERM1=-86.4E6*SQRT(KERTH)/6.2831858*SQRT(AI)                            ,309
        L=JCNT-1                                                                ,310
        M=1                                                                     ,311
 1506   IF(L)1504,1504,1505                                                     ,312
 1505   L=L-2                                                                   ,313
        INTA=INTA+STEPA(M)+STEPA(M+2)+STEPA(M+1)*4.                             ,314
        INTP=INTP+STEPP(M)+STEPP(M+2)+STEPP(M+1)*4.                             ,315
        M=M+2                                                                   ,316
        GØ TØ 1506                                                              ,317
 1504   CØNTINUE                                                                ,318
        INTA=INTA*DEØ3                                                          ,319
        INTP=INTP*DEØ3                                                          ,320
        ADØTP=TERM1*(INTA*(1.+EI)**2-SINTA)                                     ,321
        PDØTP=TERM1*(INTP*(1.-EI)**2-SINTP)                                     ,322
        IF(MASS-BCITIM)1601,1600,1601                                           ,323
 1601   IF(MASS(3))1603,1602,1603                                               ,324
 1602   MT=MASS(2)                                                              ,325
        GØ TØ 1700                                                              ,326
 1603   IF(TIME)1602,1602,1604                                                  ,327
 1604   I=3                                                                     ,328
 1607   IF(MT-MASS(I+1))1605,1606,1606                                          ,329
 1606   MT=MASS(2)-MASS(I)*TIME                                                 ,330
        GØ TØ 1700                                                              ,331
 1605   I=I+2                                                                   ,332
        IF(MASS(I)-ENDID)1,2,1                                                  ,333
 2      MT=MASS(I-1)                                                            ,334
        GØTØ 1700                                                               ,335
 1      IF(I-24)1607,1607,7001                                                  ,336
 1600   I=2                                                                     ,337
 1803   IF(TIME-MASS(I+1))1800,1801,1802                                        ,338
 1802   I=I+2                                                                   ,339
        IF(MASS(I)-ENDID)3,4,3                                                  ,340
```

```
        KILGØ  PDAD                                              06/18/65
              EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT   -   INTERNAL FØRMULA NUMBER(S)

   3    IF(I-24)1803,1803,7001                                                  ,341
   4    MT=MASS(I-2)                                                            ,342
        GØTØ 1700                                                               ,343
 1800   IF(I-2)1801,1801,1804                                                   ,344
 1804   MT=MASS(I)                                                              ,345
        GØ TØ 1700                                                              ,346
 1801   MT=MASS(I)                                                              ,347
 1700   ADØT=ADØTP/MT                                                           ,348
        PDØT=RDØTP/MT                                                           ,349
        SADØTI=(ADØT+PDØT)/2.                                                   ,350
        PDØAD=PDØT/ADØT                                                         ,351
        TIMED=MT/ADØTP                                                          ,352
        REVØL=REV1+(TIME-TIME1)*1440./PD)                                       ,353
        RETURN                                                                  ,354
 7001   WRITE(6,7002)                                                           ,355   ,356
        CALL DUMP                                                               ,357
        STØP                                                                    ,358
 7002   FØRMAT(1H020HTABLE VALUE EXCEEDED)
        END                                                                     ,359
```

```
      SUBROUTINE PRINTT
      REAL JJ,KERTH,INTA,INTP,MASS,MT,INTERA,INTERP,INC,NI
      DIMENSION DATE(3), XMONTH(12)
      DIMENSION FTENB(153),FTEN(153),AP(153)
      DIMENSION C3RREC(11C),SOLAR(3C1)
      DIMENSION TN(6),INTERA(6),TN1(6),INTERP(6),
     1COSE(365),E(365),CDPRIM(25),AREA(45),CN(45),
     2ATTACK(45),STEPA(365),STEPR(365),MASS(25)
      COMMON /ARDC/TEMPT,TEMK,PRESS,POMP,
     1PSPO,RHO,RHOSRO,VISC,VISCSL,KVISC,VS,G
      DATA DPR/57.2957795/,            PI/3.14159291/,SNBCI/6HSINE   /
     1,BCITIM/6HTIME   /
      DATA AID/6HA      /,PID/6HP      /,DETID/6HDETAIL/,
     1SHOID/6HSHORT /
      DATA BNDID/6HEND    /
      COMMON/CLK/APO,PER1,TIME,AE,SINI,WO,F,JJ,KERTH,
     1COSI,    JCNT,CDPRIM,        AREA,ATTACK,         CN,
     2MASS,ADOT,PDOT,PDOAC,TIMED
     3,    COSE,E,DAO2,PER,TIME1,HH,DEO3
     4,CAPDM1,CAPID,SMAM1,ADTM1,APOM1,SMAW,SMADM1,SMAID,CAPM1,CAPW
     5,CAPO,DATE,XMONTH,FTENB,AP
     6,INTERA,INTERP,DD,CANOM,DAROGE(10),CDA(5C)
     7,PRINT,    CUTOFF(2),                  AI,RPAI,SADOTI
     8,REVOL,MT,VPI,PDI
     9,PN(6),AN(6),REV1
     1,CORREC,SOLAR,ATMOS,FTEN,DIURNL,XLAG,RHOXX,SO,SA
     1,EI,RIPP,RIPA,AMPR
      WRITE(6,151)APO,PER1,AI,RPAI,ADOT,PDOT,SADOTI,
     1MT,CAPW,SMAW,CAPID,SMAID,VPI,PDI,REVOL,TIME,RHOXX
     1,EI,RIPP,RIPA                                                    ,1     ,2     ,3
      RETURN                                                           ,4
  151 FORMAT(1HO6HA      E15.8,3X,6HP      E15.8,3X,6HAXIS  E15.8,3X,
     16HRADIUSE15.8/,1X,6HADOT  E15.8,3X,6HPDOT  E15.8,3X,
     26HAXIDOTE15.8,3X,6HMASS  E15.8/,1X,6HNODE  E15.8,3X,
     36HARGP  E15.8,3X,6HDNODE E15.8,3X,6HDARGP E15.8/,1X,
     46HVPERIGE15.8,3X,6HPERIODE15.8,3X,6HORBIT E15.8,3X,
     56HTIME  E15.8/,1X,6HRHO   E15.8,3X,6HEI    E15.8,3X,
     16HRIPERGE15.8,3X,6HRIPAPGE15.8/)
      END
```

```
KILGO  RK                                                                  06/18/65
    EXTERNAL FORMULA NUMBER  -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

       SUBROUTINE RK
       REAL JJ,KERTH,INTA,INTP,MASS,MT,INTERA,INTERP,INC,NI
       DIMENSION CATE(3), XMONTH(12)
       DIMENSION FTENB(153),FTEN(153),AP(153)
       DIMENSION CORREC(11C),SOLAR(3C1)
       DIMENSION TN(6),INTERA(6),TN1(6),INTERP(6),
      1COSE(365),E(365),CCPRIM(25),AREA(45),CN(45),
      2ATTACK(45),STEPA(365),STEPP(365),MASS(25)
       COMMON /ARCC/TEMPT,TEMK,PRBSS,POMP,
      1PSPO,RHO,RHOSRO,VISC,VISCSL,KVISC,VS,G
       DATA DPR/57.2957795/,          PI/3.14159291/,SNBCI/6HSINE  /
      1,BCITIM/6HTIME  /
       DATA AID/6HA     /,PID/6HP     /,DETID/6HDETAIL/,
      1SHOID/6HSHORT /
       DATA BNDID/6HEND   /
       COMMON/CLK/APO,PER1,TIME,AE,SINI,XO,F,JJ,KERTH,
      1COSI,   JCNT,CDPRIM,       AREA,ATTACK,       CN,
      2MASS,ADOT,PDOT,PDOAC,TIMED
      3,    GOSE,E,DAO2,PER,TIME1,HH,DEO3
      4,CAPDM1,CAPID,SMAM1,ADTM1,APOM1,SMAW,SMACM1,SMAID,CAPM1,CAPW
      5,CAPO,DATE,XMONTH,FTENB,AP
      6,INTERA,INTERP,DD,DANOM,DAROGE(10),CDA(5C)
      7,PRINT,    CUTOFF(2),                AI,RPAI,SADOTI
      8,REVOL,MT,VPI,PCI
      9,PN(6),AN(6),REV1
      1,CORREC,SOLAR,ATMOS,FTEN,DIURNL,XLAG,RHOXX,SO,SA
      1,EI,RIPP,RIPA,AMPR
       IF(ADOT)1,1,2                                                       1
1      DAO2=-ABS(CAO2)                                                     2
       GOTO3                                                               3
2      DAO2=ABS(DAO2)                                                      4
3      CONTINUE                                                            5
       CK1=DAO2*PCOAC                                                      6
       PER1=PER+CK1                                                        7
       APO=APO+DAO2                                                        8
       CK1X=DAO2*TIMED                                                     9
       TIME=TIME1+CK1X                                                    10
       CALL PDAD                                                          11
       IF(ADOT)4,4,5                                                      12
4      DAO2=-ABS(CAO2)                                                    13
       GOTO6                                                              14
5      DAO2=ABS(DAO2)                                                     15
6      CONTINUE                                                           16
       CK2=DAO2*PCOAC                                                     17
       PER1=PER+CK2                                                       18
       CK2X=DAO2*TIMED                                                    19
       TIME=TIME1+CK2X                                                    20
       CALL PDAD                                                          21
       IF(ADOT)7,7,8                                                      22
7      DAO2=-ABS(CAO2)                                                    23
       GOTO9                                                              24
8      DAO2=ABS(DAO2)                                                     25
9      CONTINUE                                                           26
       CK3=DAO2*PCOAC                                                     27
       PER1=CK3+CK3+PER                                                   28
```

```
        KILGØ  RK                                                     06/18/65
            EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT   -   INTERNAL FØRMULA NUMBER(S)

        APØ=APØ+DAØ2                                                         $29
        CK3X=DAØ2*TIMED                                                      $30
        TIME=CK3X+CK3X+TIME1                                                 $31
        CALL RØAD                                                            $32
        IF(ADØT)10,10,11                                                     $33
  10    DAØ2=-ABS(DAØ2)                                                      $34
        GØTØ12                                                               $35
  11    DAØ2=ABS(DAØ2)                                                       $36
  12    CØNTINUE                                                             $37
        DELT=((DAØ2*PCØAD)+CK3+CK2+CK1+CK2+CK3)/3.                           $38
        PER1=DELT+PER                                                        $39
        TIME=(((DAØ2*TIMED)+CK3X+CK2X+CK1X+CK2X
       1+CK3X)/3.)+TIME1                                                     $40
        CALL PØAD                                                            $41
        IF(ADØT)13,13,14                                                     $42
  13    DAØ2=-ABS(DAØ2)                                                      $43
        GØTØ15                                                               $44
  14    DAØ2=ABS(DAØ2)                                                       $45
  15    CØNTINUE                                                             $46
        ADTM1=ADØT                                                           $47
        SMADM1=SMAID                                                         $48
        CAPDM1=CAPID                                                         $49
        SMAM1=SMAW                                                           $50
        CAPM1=CAPW                                                           $51
        APØM1=APØ                                                            $52
        REV1=REVØU                                                           $53
        RETURN                                                               $54
        END                                                                  $55
```

```
       KILGØ  PØEAT                                                           06/18/65
          EXTERNAL FØRMULA NUMBER    -    SØURCE STATEMENT    -    INTERNAL FØRMULA NUMBER(S)

C      HG=GEØMETRIC ALTITUDE                                                 0406
C      N=LENGTH ØF TABLE                                                     0407
C      HB=ALTITUDE BASE                                                      0408
C      ATMØSPHERE DATA LØØKUP-GSK                                            0409
C      RHØB=BASE DENSITY                                                     0412
C      GIVEN HG,N,HB(1) TØ HB(N),TMB(1) TØ TMB(N),GLMB(1) TØ                 0417
C      GLMB(N),RHØB(1) TØ RHØB(N)                                            0418
C      CØMPUTE T,RHØ,P,V                                                     0419
C      R=GEØCENTRIC DISTANCE ØF THE FIELD PØINT                              0420
C      CLS,CMS,CNS=DIRECTIØN CØSINES ØF SUN                                  0421
C      CL,CM,CN=DIRECTIØN CØSINES ØF FIELD PØINT                             0422
C      AVERAGE LØNGITUDINAL LAG ØF DIURNAL BULGE=.55 RADIANS                 0423
C      SEPS,CEPS=SINE AND CØSINE ØF THE INCLINATIØN ØF THE ECLIPTIC          0424
C      PSIP=GEØCENTRIC ANGLE BETWEEN DIURNAL BULGE AND FIELD PØINT           0425
C                                                                            0426
       SUBRØUTINE PØEAT(HG,RHØ,D,X,Y,Z,R)
       DIMENSIØN HB(18),TMB(18),GLMB(18),RHØB(18)                            0428
       IF(N-12)500,10,500                                                            ,1
   500 CØNTINUE                                                              0430    ,2
       N=12                                                                  0431    ,3
       HB(1)=0.                                                              0432    ,4
       HB(2)=36089.239                                                       0433    ,5
       HB(3)=82020.997                                                       0434    ,6
       HB(4)=154199.475                                                      0435    ,7
       HB(5)=173884.514                                                      0436    ,8
       HB(6)=259186.352                                                      0437    ,9
       HB(7)=295275.591                                                      0438    ,10
       HB(8)=344488.189                                                      0439    ,11
       HB(9)=524934.383                                                      0440    ,12
       HB(10)=557742.782                                                     0441    ,13
       HB(11)=656167.979                                                     0442    ,14
       HB(12)=2296587.93                                                     0443    ,15
       TMB(1)=518.69                                                         0444    ,16
       TMB(2)=389.988                                                        0445    ,17
       TMB(3)=389.988                                                        0446    ,18
       TMB(4)=508.788                                                        0447    ,19
       TMB(5)=508.788                                                        0448    ,20
       TMB(6)=298.188                                                        0449    ,21
       TMB(7)=298.188                                                        0450    ,22
       TMB(8)=406.188                                                        0451    ,23
       TMB(9)=2386.188                                                       0452    ,24
       TMB(10)=2566.188                                                      0453    ,25
       TMB(11)=2836.188                                                      0454    ,26
       TMB(12)=5986.188                                                      0455    ,27
       GLMB(1)=-3.56616E-3                                                   0456    ,28
       GLMB(2)=0.                                                            0457    ,29
       GLMB(3)=1.646592E-3                                                   0458    ,30
       GLMB(4)=0.                                                            0459    ,31
       GLMB(5)=-2.46888E-3                                                   0460    ,32
       GLMB(6)=0.                                                            0461    ,33
       GLMB(7)=2.19456E-3                                                    0462    ,34
       GLMB(8)=1.09728E-2                                                    0463    ,35
       GLMB(9)=5.4864E-3                                                     0464    ,36
       GLMB(10)=2.7432E-3                                                    0465    ,37
       GLMB(11)=1.92024E-3                                                   0466    ,38
```

```
     KILGØ  PØEAT                                                    06/18/65
        EXTERNAL FØRMULA NUMBER  -  SØURCE STATEMENT  -   INTERNAL FØRMULA NUMBER(S)

          GLMB(12)=1.92024E-3                                   0467      ,39
          RHØB(1)=2.3769E-3                                     0468      ,40
          RHØB(2)=7.0547E-4                                     0469      ,41
          RHØB(3)=7.7615E-5                                     0470      ,42
          RHØB(4)=2.8829E-6                                     0471      ,43
          RHØB(5)=1.3964E-6                                     0472      ,44
          RHØB(6)=4.1123E-8                                     0473      ,45
          RHØB(7)=4.2560E-9                                     0474      ,46
          RHØB(8)=2.2243E-10                                    0475      ,47
          RHØB(9)=1.8477E-12                                    0476      ,48
          RHØB(10)=1.3397E-12                                   0477      ,49
          RHØB(11)=6.1161E-13                                   0478      ,50
          RHØB(12)=4.468E-16                                    0479      ,51
       10 PI=3.14159265                                         0480      ,52
          HG=HG*3280.833                                                  ,53
          T1=0.017203*D                                         0481      ,54
          TENBE=2.30258509                                      0482      ,55
          SLAMB=T1+0.0335*SIN (T1)-1.410                                  ,56
          SEPS=SIN (.4092)                                                ,57
          CEPS=CØS (.4092)                                                ,58
          CLS=CØS (SLAMB)                                                 ,59
          SSLAMB=SIN (SLAMB)                                              ,60
          CMS=SSLAMB*CEPS                                       0488      ,61
          CNS=SSLAMB*SEPS                                       0489      ,62
        1 HNM=HG/6076.1003                                      0490      ,63
          H=(20855531.*HG)/(20855531.+HG)                       0491      ,64
      100 DØ 111 I=1,N                                          0492      ,65
          IF(H-HB(I))121,111,111                                0493      ,66
      111 CØNTINUE                                              0494      ,67    ,68
      200 T=TMB(N)+GLMB(N)*(H-HB(N))                            0495      ,69
          GØ TØ 50                                              0496      ,70
      121 IF(I-1)123,122,123                                    0497      ,71
      122 I=2                                                   0498      ,72
      123 IF(GLMB(I-1))131,141,131                              0499      ,73
      141 T=TMB(I-1)                                            0500      ,74
          RHØ=RHØB(I-1)*EXP (-(H-HB(I-1))*32.1740485/(1716.4827*
         1TMB(I-1)))                                            0502      ,75
          GØ TØ 60                                              0503      ,76
      131 T=TMB(I-1)+GLMB(I-1)*(H-HB(I-1))                      0504      ,77
          RHØ=RHØB(I-1)*EXP (-(1.+32.1740485/(1716.4827*GLMB
         1(I-1)))*ALØG  (T/TMB(I-1)))                                     ,78
          GØ TØ 60                                              0507      ,79
       50 RHØ=RHØB(N)*EXP (-(1.+32.1740485/(1716.4827*GLMB(N)))
         1*ALØG  (T/TMB(N)))                                              ,80
       60 V=SQRT (1.4*(1716.4827*T))                                      ,81
          P=RHØ*(1716.4827*T)                                   0511      ,82
          IF(C)1000,1000,260                                    0512      ,83
     1000 RETURN                                                0513      ,84
      260 CN=Z/R                                                0514      ,85
      202 IF(HNM-76.)1000,1000,250                              0517      ,86
      250 CL=X/R                                                0518      ,87
          CM=Y/R                                                0519      ,88
          CLCLS=CL*CLS                                          0520      ,89
          CMCMS=CM*CMS                                          0521      ,90
          CNCNS=CN*CNS                                          0522      ,91
          CØPSIP=(CLCLS+CMCMS)*CØS (.55)+(CM*CLS-CL*CMS)
```

```
        KILGØ  PØEAT                                                       06/18/65
            EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT    -   INTERNAL FØRMULA NUMBER(S)

      1*SIN (.55)+CN*CNS                                                             092
       F10=1.5+.8*CØS (PI*D/2010.)                                                   093
       FB=.85*F10                                                         0526       094
       IF(HNM-108.)300,350,301                                            0527       095
   301 IF (HNM-378.)350,400,400                                           0528       096
   138 RHØ=RHØ*(1.-0.3*CN**3*(1.-CØS (2.*PI*(HNM-16.)/34.))
      1*CØS (2.*PI*(D+9.)/365.))                                                     097
       P=RHØ*(1716.4827*T)                                                0531       098
       V=SQRT (1.4*(1716.4827*T))                                                    099
       RETURN                                                             0533       0100
   300 RHØ=5.606E-12*(76./HNM)**7.18*((108.-HNM)/32.+FB*                  0534
      1((HNM-76.)/32.)**(4.0/3.0))*(1.+(HNM-76.)/153.*((1.+CØPSIP)/2.)    0535
      2**3)                                                               0536       0101
       P=RHØ*(1716.4827*T)                                                0537       0102
       V=SQRT (1.4*(1716.4827*T))                                                    0103
       RETURN                                                             0539       0104
   350 RHØ=FB*EXP (TENBE*(-15.738-.00368*HNM+6.363*EXP (-.0048
      1*HNM)))*(1.+.19*(EXP (0.0102*HNM)-1.9)*((1.+CØPSIP)/2.)**3)                   0105
       P=RHØ*(1716.4827*T)                                                0542       0106
       V=SQRT (1.4*(1716.4827*T))                                                    0107
       RETURN                                                             0544       0108
   400 RHØ=0.00504*F10/(HNM)**5*(((1.+CØPSIP)/2.)**3*(1.-6.E+6            0545
      1/HNM**3)+6.E+6/HNM**3)                                             0546       0109
       P=RHØ*(1716.4827*T)                                                0547       0110
       V=SQRT (1.4*(1716.4827*T))                                                    0111
       RETURN                                                             0549       0112
       END                                                                0550       0113
```

```
     KILGØ   SMATMS                                              06/18/65
         EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT   -   INTERNAL FØRMULA NUMBER(S)

      SUBRØUTINE SMATMS(X,Y,Z,SS,T,RHØ,HKM)
      DIMENSIØN RHØS (53)
      S=SS                                                          ,1
      IF(IFLAG-1)600,300,600                                        ,2
  600 CØNTINUE                                                      ,3
      PI=3.1415927                                                  ,4
      PI2=2.*PI                                                     ,5
      PI4=4.*PI                                                     ,6
      CØNV=920.                                                     ,7
      RHØS=0.                                                       ,8
      RHØS(2)=0.                                                    ,9
      RHØS(3)=0.0                                                   ,10
      RHØS(4)=1.225E-3                                              ,11
      RHØS(5)=0.                                                    ,12
      RHØS(6)=1.225E-3                                              ,13
      DØ90 I=7,49,2                                                 ,14
   90 RHØS(I)=RHØS(I-2)+5.                                          ,15   ,16
      RHØS(8)=7.3643E-4                                             ,17
      RHØS(10)=4.1351E-4                                            ,18
      RHØS(12)=1.9475E-4                                            ,19
      RHØS(14)=8.8910E-5                                            ,20
      RHØS(16)=4.0084E-5                                            ,21
      RHØS(18)=1.8410E-5                                            ,22
      RHØS(20)=8.4634E-6                                            ,23
      RHØS(22)=3.9957E-6                                            ,24
      RHØS( 24)=1.9663E-6                                           ,25
      RHØS( 26)=1.0269E-6                                           ,26
      RHØS( 28)=5.6075E-7                                           ,27
      RHØS( 30)=3.0592E-7                                           ,28
      RHØS( 32)=1.6665E-7                                           ,29
      RHØS( 34)=8.7535E-8                                           ,30
      RHØS( 36)=4.335E-8                                            ,31
      RHØS( 38)=1.999E-8                                            ,32
      RHØS( 40)=7.955E-9                                            ,33
      RHØS( 42)=3.170E-9                                            ,34
      RHØS( 44)=1.265E-9                                            ,35
      RHØS( 46)=5.070E-10                                           ,36
      RHØS(48)=2.070E-10                                            ,37
      RHØS(50)=8.750E-11                                            ,38
      RHØS(51)=116.0                                                ,39
      RHØS(52)=3.253E-11                                            ,40
      RHØS(53)=1.E22                                                ,41
      IFLAG=1                                                       ,42
  300 R=SQRT (X**2+Y**2+Z**2)                                       ,43
      CL=X/R                                                        ,44
      CM=Y/R                                                        ,45
      CN=Z/R                                                        ,46
      IF (HKM) 200,200,201                                          ,47
  200 RHØ =2.3765E-3                                                ,48
      GØ TØ 1000                                                    ,49
  201 IF(HKM-116.)311,312,312                                       ,50
  311 CALL TBL(HKM,RHØS)                                            ,51
      IF (HKM-30.) 110,110,111                                      ,52
  111 IF (HKM-90.) 112,112,110                                      ,53
  112 C=1.-0.3*CN**3*(1.-CØS (PI2*((HKM-30.)/60.)))*CØS (PI2*((T-36194.)
```

```
         KILGØ   SMATMS                                                    06/18/65
              EXTERNAL FØRMULA NUMBER    -    SØURCE STATEMENT    -    INTERNAL FØRMULA NUMBER(S)

          1/365.25))                                                                    ,54
           RHØ=RHØS*C                                                                   ,55
           GØ TØ 1000                                                                   ,56
      110  RHØ=RHØS                                                                     ,57
           GØ TØ 1000                                                                   ,58
      312  CØNTINUE                                                                     ,59
           HKM=HKM+4.C                                                                  ,60
           IF (S) 100,10C,101                                                           ,61
      100  FBAR=135.+75.*CØS (PI2*(T-36340.)/4C90.)+15.*CØS (PI4*(T-3603C.)
          1/4090.)                                                                      ,62
           TCØN=(T-38C47.)/365.25                                                       ,63
           GT=.025*CØS (PI2*TCØN)-.06*CØS (PI4*TCØN)                                    ,64
           S=(50.+.8*FBAR)*EXP (GT)                                                     ,65
      101  CALL HPC(CL,CM,CN,HKM,S,T,RHØHC)                                             ,66
           RHØ=RHØHC*CØNV                                                               ,67
     1000  RETURN                                                                       ,68
           END                                                                          ,69
```

```
         KILGØ   TBL                                                       06/18/65
              EXTERNAL FØRMULA NUMBER    -    SØURCE STATEMENT    -    INTERNAL FØRMULA NUMBER(S)

           SUBRØUTINE TBL (HKM,RHØS)
           DIMENSIØN RHØS(1)
           DØ 10 I=5,51,2                                                               ,1
           IF(HKM-RHØS(I))20,3C,10                                                      ,2
      10   CØNTINUE                                                                     ,3      ,4
      20   RHØS=RHØS(I-1)+(RHØS(I+1)-RHØS(I-1))*(HKM
          1-RHØS(I-2))/(RHØS(I)-RHØS(I-2))                                              ,5
      25   RETURN                                                                       ,6
      30   RHØS=RHØS(I+1)                                                               ,7
           GØ TØ 25                                                                     ,8
           END                                                                          ,9
```

```
      SUBRØUTINE HPC(CL,CM,CN,HKM,S,T,RHØA)
      DIMENSIØN S1(35),S2(35),S3(35),S4(35),S5(35),SS1(35),SS2(35),
     1SS3(35),SS4(35),SS5(35),Q(3,3),P(3),H(35),R(3,3),PP(3),C(5)
      DATA (S1(I),I=1,35)/
     1-24.065,-25.887,-27.310,-28.493,-29.530,-31.299,-32.772,
     1-34.068,-35.267,-36.395,-37.447,-38.396,-39.194,-39.817,
     1-40.289,-40.660,-40.973,-41.257,-41.524,-41.782,-42.033,
     1-42.280,-42.523,-42.762,-42.996,-43.566,-44.109,-44.622,
     1-45.101,-45.541,-45.938,-46.293,-46.604,-46.875,-47.109/
      DATA (S2(I),I=1,35)/
     1-24.065,-25.904,-27.228,-28.284,-29.197,-30.758,-32.072,
     1-33.218,-34.260,-35.235,-36.161,-37.044,-37.871,-38.620,
     1-39.268,-39.804,-40.232,-40.577,-40.864,-41.114,-41.344,
     1-41.560,-41.768,-41.970,-42.167,-42.648,-43.111,-43.557,
     1-43.985,-44.395,-44.784,-45.152,-45.496,-45.816,-46.111/
      DATA (S3(I),I=1,35)/
     1-24.065,-25.930,-27.137,-28.046,-28.814,-30.120,-31.232,
     1-32.213,-33.094,-33.903,-34.670,-35.400,-36.102,-36.777,
     1-37.424,-38.036,-38.607,-39.125,-39.584,-39.981,-40.317,
     1-40.601,-40.844,-41.056,-41.246,-41.664,-42.041,-42.397,
     1-42.738,-43.069,-43.389,-43.698,-43.997,-44.286,-44.564/
      DATA (S4(I),I=1,35)/
     1-24.065,-25.954,-27.077,-27.889,-28.558,-29.681,-30.643,
     1-31.496,-32.270,-32.983,-33.646,-34.272,-34.869,-35.443,
     1-35.998,-36.536,-37.056,-37.558,-38.039,-38.496,-38.924,
     1-39.320,-39.680,-40.002,-40.287,-40.856,-41.282,-41.625,
     1-41.927,-42.207,-42.473,-42.729,-42.977,-43.217,-43.451/
      DATA (S5(I),I=1,35)/
     1-24.065,-25.976,-27.036,-27.783,-28.385,-29.377,-30.221,
     1-30.980,-31.669,-32.304,-32.896,-33.454,-33.984,-34.490,
     1-34.976,-35.446,-35.903,-36.347,-36.778,-37.197,-37.604,
     1-37.997,-38.375,-38.736,-39.077,-39.837,-40.443,-40.912,
     1-41.278,-41.577,-41.835,-42.069,-42.287,-42.495,-42.696/
      DATA (SS1(I),I=1,35)/
     1-24.065,-25.852,-27.238,-28.339,-29.227,-30.677,-31.866,
     1-32.888,-33.805,-34.657,-35.464,-36.233,-36.963,-37.658,
     1-38.307,-38.899,-39.421,-39.869,-40.242,-40.553,-40.816,
     1-41.043,-41.247,-41.435,-41.612,-42.027,-42.420,-42.797,
     1-43.162,-43.514,-43.854,-44.182,-44.497,-44.799,-45.087/
      DATA (SS2(I),I=1,35)/
     1-24.065,-25.867,-27.155,-28.153,-28.950,-30.214,-31.250,
     1-32.149,-32.953,-33.685,-34.370,-35.022,-35.648,-36.252,
     1-36.836,-37.398,-37.935,-38.443,-38.915,-39.348,-39.735,
     1-40.075,-40.371,-40.626,-40.848,-41.300,-41.668,-41.996,
     1-42.301,-42.594,-42.877,-43.150,-43.416,-43.673,-43.922/
      DATA (SS3(I),I=1,35)/
     1-24.065,-25.893,-27.067,-27.948,-28.653,-29.731,-30.599,
     1-31.352,-32.027,-32.646,-33.222,-33.765,-34.279,-34.773,
     1-35.248,-35.708,-36.153,-36.586,-37.007,-37.415,-37.811,
     1-38.193,-38.558,-38.907,-39.235,-39.959,-40.536,-40.983,
     1-41.334,-41.623,-41.874,-42.103,-42.317,-42.522,-42.719/
      DATA (SS4(I),I=1,35)/
     1-24.065,-25.921,-27.012,-27.813,-28.451,-29.424,-30.184,
     1-30.841,-31.431,-31.971,-32.474,-32.945,-33.392,-33.817,
     1-34.223,-34.614,-34.992,-35.358,-35.715,-36.062,-36.402,
```

```
     KILGØ  HPC                                                                06/18/65
         EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT   -   INTERNAL FØRMULA NUMBER(S)

   1-36.733,-37.057,-37.373,-37.681,-38.414,-39.088,-39.689,
   1-40.208,-40.645,-41.004,-41.300,-41.548,-41.762,-41.952/
    DATA (SS5(I),I=1,35)/
   1-24.065,-25.947,-26.978,-27.721,-28.309,-29.211,-29.898,
   1-30.488,-31.016,-31.500,-31.950,-32.372,-32.772,-33.151,
   1-33.513,-33.859,-34.192,-34.514,-34.826,-35.128,-35.423,
   1-35.710,-35.990,-36.265,-36.534,-37.182,-37.798,-38.379,
   1-38.923,-39.425,-39.880,-40.285,-40.639,-40.945,-41.206/
    PARF(X,Y,Z)=X+DT*(Y*(DT2-DT)/DT1+Z*(DT1+DT)/DT2)/(DT1+DT2)
    IF(IFLAG-1)601,600,601                                                  ✓1
 601 CØNTINUE                                                                ✓2
    H(1)=120.                                                                ✓3
    DØ 1400 I=1,34                                                           ✓4
    IF(I-4)1030,1030,1031                                                    ✓5
1030 H(I+1)=H(I)+20.                                                         ✓6
    GØ TØ 1400                                                               ✓7
1031 IF(I-24)1032,1032,1033                                                  ✓8
1032 H(I+1)=H(I)+40.                                                         ✓9
    GØ TØ 1400                                                               ✓10
1033 H(I+1)=H(I)+100.                                                        ✓11
1400 CØNTINUE                                                                ✓12   ,13
    PI=3.1415927                                                             ✓14
    IFLAG=1                                                                  ✓15
 600 CØNTINUE                                                                ✓16
    CAPM=.017203*(T-36203.)                                                  ✓17
    SL=CAPM+.0335*SIN (CAPM)-1.41                                            ✓18
    CLS=CØS (SL)                                                             ✓19
    SINSL=SIN (SL)                                                           ✓20
    CMS=.9175*SINSL                                                          ✓21
    CNS=.3977*SINSL                                                          ✓22
 400 DØ 20 I=1,35                                                            ✓23
    DT=H(I)-HKM                                                              ✓24
    IF(DT)20,21,21                                                           ✓25
  20 CØNTINUE                                                                ✓26   ,27
    GØ TØ 9001                                                               ✓28
  21 LPT=I                                                                   ✓29
    IF(I-35)1002,9001,9001                                                   ✓30
9001 RHØA=0.0                                                                ✓31
    GØ TØ 3000                                                               ✓32
1002 CØNTINUE                                                                ✓33
    DT1=H(LPT)-H(LPT-1)                                                      ✓34
    DT2=H(LPT+1)-H(LPT)                                                      ✓35
    DT=-DT                                                                   ✓36
    IF(S-150.)30,31,31                                                       ✓37
  30 DØ 40 I=1,3                                                             ✓38
    LP=LPT-2+I                                                               ✓39
    Q(1,I)=S1(LP)                                                            ✓40
    Q(2,I)=S2(LP)                                                            ✓41
  40 Q(3,I)=S3(LP)                                                           ✓42   ,43
    IGØ=1                                                                    ✓44
  42 DØ 41 I=1,3                                                             ✓45
    Q1=Q(I,2)                                                                ✓46
    Q2=Q(I,2)-Q(I,1)                                                         ✓47
    Q3=Q(I,3)-Q(I,2)                                                         ✓48
  41 P(I)=PARF(Q1,Q2,Q3)                                                     ✓49   ,50
    GØ TØ (5000,5001),IGØ                                                    ✓51
```

```
         KILGØ  HPC                                                                    06/18/65
              EXTERNAL FØRMULA NUMBER   -    SØURCE STATEMENT   -   INTERNAL FØRMULA NUMBER(S)

5000 DØ 240 I=1,3                                                                      ,52
     LP=LPT-2+I                                                                        ,53
     R(1,I)=SS1(LP)                                                                    ,54
     R(2,I)=SS2(LP)                                                                    ,55
 240 R(3,I)=SS3(LP)                                                                    ,56    ,57
5001 DØ 241 I=1,3                                                                      ,58
     R1=R(I,2)                                                                         ,59
     R2=R(I,2)-R(I,1)                                                                  ,60
     R3=R(I,3)-R(I,2)                                                                  ,61
 241 PP(I)=PARF(R1,R2,R3)                                                              ,62    ,63
     GØ TØ (43,44),IGØ                                                                 ,64
  43 DT=S-100.                                                                         ,65
     DT1=30.                                                                           ,66
     DT2=50.                                                                           ,67
  45 RHØN=EXP (PARF(P(2),P(2)-P(1),P(3)-P(2)))                                         ,68
     RHØX=            EXP (PARF(PP(2),PP(2)-PP(1),PP(3)-PP(2)))                        ,69
     GØ TØ 410                                                                         ,70
  31 DØ 140 I=1,3                                                                      ,71
     LP=LPT-2+I                                                                        ,72
     Q(1,I)=S3(LP)                                                                     ,73
     Q(2,I)=S4(LP)                                                                     ,74
 140 Q(3,I)=S5(LP)                                                                     ,75    ,76
     IGØ=2                                                                             ,77
     DØ 340 I=1,3                                                                      ,78
     LP=LPT-2+I                                                                        ,79
     R(1,I)=SS3(LP)                                                                    ,80
     R(2,I)=SS4(LP)                                                                    ,81
 340 R(3,I)=SS5(LP)                                                                    ,82    ,83
     GØ TØ 42                                                                          ,84
  44 DT=S-200.                                                                         ,85
     DT1=50.                                                                           ,86
     DT2=50.                                                                           ,87
     GØ TØ 45                                                                          ,88
 410 SIG=(S-160.)/90.                                                                  ,89
     SIG2=SIG**2                                                                       ,90
     PHIN=591.+32.5*SIG-8.5*SIG2                                                       ,91
     PHIM=96.+20.*SIG+10.*SIG2                                                         ,92
     HN=1115.+507.5*SIG+52.5*SIG2                                                      ,93
     DELH=590.+355.*SIG+35.*SIG2                                                       ,94
     HS=325.+27.*SIG-5.*SIG2                                                           ,95
     ZETA=((HKM-HN)/DELH)**2                                                           ,96
     FZ=-.06+.03*ZETA+1.06*EXP (-3.7*ZETA)                                             ,97
     PHI=PI*(PHIN-PHIM*FZ-4.47+.01174*HKM+EXP (-.04*(HKM-HS)))/1200.                   ,98
     IF(PHI-2.)420,421,421                                                             ,99
 421 PHI=2.                                                                            ,100
     GØ TØ 430                                                                         ,101
 420 IF(PHI-1.)425,430,430                                                             ,102
 425 PHI=1.                                                                            ,103
 430 EPSI=ALØG  (1.+SQRT (RHØX/RHØN))/ALØG  (2./(1.+CØS (PHI)))                        ,104
     GAMMA=PI*(18.5+30.*EXP (-.00567*(HKM-200.)+EXP (-.01455*(HKM-200.
    1)))+(18.5+21.5*EXP (-.0315*(HKM-200.)))*SIG+4.*(1.-SIG2))/180.                    ,105
     IF(GAMMA-5.)320,320,2003                                                          ,106
2003 GAMMA=5.                                                                          ,107
 320 CPSIPU=(CL*CLS+CM*CMS)*CØS (GAMMA)+(CM*CLS-CL*CMS)*SIN (GAMMA)+CN*
    1CNS                                                                               ,108
1000 RHØA=RHØN+          (RHØX-RHØN)*((1.+CPSIPL)/2.)**EPSI                            ,109
```

```
      KILGØ  HPC                                                        06/18/65
         EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT    -   INTERNAL FØRMULA NUMBER(S)

1001 BETA=TAN2PI(CL,CM)-TAN2PI(CLS,CMS)-180.*GAMMA/PI                               /110
2002 IF(BETA) 2000,2001,2001                                                         /111
2000 BETA=BETA+360.                                                                  /112
     GØ TØ 2002                                                                      /113
2001 CØNTINUE                                                                        /114
 700 AMP=HKM/4000.+(.91+.44*SIG+.38*SIG2)*EXP (-(2.-HKM/(405.+143.*SIG)
    1)**2)                                                                           /115
 710 AMPS=.245+.0425*SIG-.0625*SIG2                                                  /116
     U=AMP*(-.08*EXP (-((BETA-250.)/55.)**2)+AMPS*EXP (-((BETA-135.)/
    134.))**2))+AMP*4.E-6*BETA                                                       /117
     FACT=1.+(1.-CN**2)*U                                                            /118
     RHØA=RHØA*FACT                                                                  /119
3000 RETURN                                                                          /120
     END                                                                             /121
```

```
      KILGØ  TANZPI                                                     06/18/65
         EXTERNAL FØRMULA NUMBER   -   SØURCE STATEMENT    -   INTERNAL FØRMULA NUMBER(S)

     FUNCTIØN TAN2PI(X,Y)
C    TAN2PI=ARCTAN(Y/X)                                          0985
C    TAN2PI  EQUAL  ØR  LESS  THAN  2PI                          0986
C    TAN2PI  EQUAL  ØR  GREATER  THAN  ZERØ                      0987
     RADDEG=57.2957795                                           0988      /1
     IF(Y)1,2,3                                                  0989      /2
   2 IF(X)5,4,6                                                  0990      /3
   4 TAN2PI=1.0E+30                                              0991      /4
     GØ TØ 20                                                    0992      /5
   5 TAN2PI=180.0                                                0993      /6
     GØ TØ 20                                                    0994      /7
   6 TAN2PI=0.0                                                  0995      /8
     GØ TØ 20                                                    0996      /9
   1 IF(X)7,8,9                                                  0997      /10
   7 TAN2PI=180.+RADDEG*ATAN(Y/X)                                          /11
     GØ TØ 20                                                    0999      /12
   8 TAN2PI=270.0                                                09991     /13
     GØ TØ 20                                                    0999e     /14
   9 TAN2PI=360.+RADDEG*ATAN(Y/X)                                          /15
     GØ TØ 20                                                    0999C     /16
   3 IF(X)7,10,11                                                0999E     /17
  10 TAN2PI=90.0                                                 0999F     /18
     GØ TØ 20                                                    0999I     /19
  11 TAN2PI=RADDEG*ATAN(Y/X)                                               /20
  20 RETURN                                                      0999H     /21
     END                                                         0999J     /22
```

REFERENCES

1. Kilgo, H. F.: MSFC/LMSC Satellite Orbit Decay and Orbital Lifetime Program. Lockheed Missiles and Space Company Technical Report HREC/0082-1, LMSC/HREC A710725, June 18, 1965.

2. Small, H. W.: Atmospheric Density Between 70 and 200 Nautical Miles from Satellite Observations. Lockheed Missiles and Space Company, Tracking Note No. 23, LMSC/A376332, Sunnyvale, California, July 1964.

3. Solar Geophysical Data, Part B. U. S. Department of Commerce, National Bureau of Standards, Central Radio Propagation Laboratory, Boulder, Colorado.

4. Jacchia, Luigi G.: A Variable Atmospheric - Density Model from Satellite Accelerations. Smithsonian Institution Astrophysical Observatory Special Report No. 39. Cambridge, Massachusetts, March 30, 1960.

5. Kurtz, H. F., Jr.; Naumcheff, M.; and McNair, Ann R.: Orbital Tracking and Decay Analysis of the Saturn I Flights. AIAA/NASA Flight Testing Conference Paper Number 65-215, February 1965.

6. Nicolet, M.: Solar Radio Flux and Temperature of the Upper Atmosphere. J. Geophys. Rev. Vol. 68, No. 22, November 15, 1963.

7. Poe, R. F.: A Review of the Geophysical Model. Lockheed Missiles and Space Company, Tracking Note No. 2, LMSC/4082333, Sunnyvale, California, September 25, 1961.

8. Ladner, James E.; and Ragsdale, George C.: Earth Orbital Satellite Lifetime. George C. Marshall Space Flight Center Report MTP-AERO-62-77, Huntsville, Alabama, October 14, 1962.

NASA TM X-53385 APPROVAL

EARTH ORBITAL LIFETIME PREDICTION MODEL AND PROGRAM

By

Ann R. McNair and Edward P. Boykin

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This report has also been reviewed and approved for technical accuracy.

E. D. GEISSLER
Director, Aero-Astrodynamics Laboratory

DISTRIBUTION

R-AERO-DIR
Dr. Geissler
Mr. Jean

R-AERO-P
Mr. McNair
Mr. Thionnet
Mr. Teague
Mr. Ledford
Mr. McAnnally
Mr. Tomlin

R-AERO-A
Mr. Dahm
Mr. Ballance

R-AERO-D
Mr. Horn

R-AERO-DA
Mr. Winch
Mr. Cremin
Mr. Gillis
Mrs. Chandler
Mr. Lester

R-AERO-G
Mr. Baker
Mr. Schwaniger

R-AERO
Mr. Thomas
Mr. Scott

R-AERO-Y
Mr. Vaughn
Mr. P. Smith
Mr. Scoggins

R-AERO-F
Mr. Lindberg

R-AERO-FM
Mr. Stone
Mr. Hardage

R-AERO-F
Mr. Fulmer

R-AERO-FO
Mr. Hill (15)
Mr. Benson
Mr. Fleishman

R-ASTR-DIR
Dr. Haeussermann

R-ASTR-NG
Mr. B. Moore
Mr. S. Seltzer

R-ASTR-S
Mr. J. Mack

R-ASTR-A
Mr. F. Digesu

R-ASTR-I
Mr. L. Barr
Mr. H. Golden

R-P&VE-DIR
Mr. Cline

R-P&VE-A
Mr. E. Goerner

DISTRIBUTION (Concluded)

R-P&VE-VO
  Mr. W. Kistler

R-AS
  Mr. F. Williams
  Mr. H. Ruppe
  Mr. de Fries

I-MO
  Dr. F. Speer
  Mr. F. Kurtz
  Mr. M. Naumchiff
  Mrs. A. McNair (5)

I-I/IB-E
  Mr. F. Vreuls
  Mr. D. Germany
  Mr. R. Callaway

I-I/IB-F
  Mr. J. Fikes

I-V-E
  Mr. L. Bell

I-V-T
  Mr. H. Burns

EXTERNAL

MSC/FM
  Mr. J. Mayer
  Mr. J. McAnulty
  Mr. L. Jenkins
  Mr. L. Huss
  Mr. F. McCreary

MSC/PS3
  Mr. C. Perrins, Jr.
  Mr. R. Ward (5)

Headquarters
Bellcomm, 1124
  Mr. R. L. Wagner
  Mr. Kaskey
OMSF/MAT
  Mr. Smith
OMSF/MAO
  Mr. Abernathy

GSFC
  Dr. Von
  Dr. J. W. Serig/Code 554
  Mr. J. Donegan/Code 554
  Mr. P. Larson

North American Aviation, Inc.
Space and Information Systems Division
Building 6, Dept. 692-401
Downey, California
Attn: Mr. Earl Lucas

Lockheed Missles and Space Company
Huntsville Research & Engineering Center
4800 Bradford Drive
Huntsville, Alabama
Attn: Mr. G. Christopher (1)
  Dr. R. Beal (1)
  Mr. Cassara (1)
  Mr. T. Richards (5)
  Mr. H. Kilgo
  Mr. P. Boykin (5)

IBM
150 Sparkman Drive
Huntsville, Alabama
Attn: Mr. W. Bosarge

DISTRIBUTION (Concluded)

MS-IP

MS-IL (8)

MS-T
Jean Bond (6)

MS-H

CC-P

Scientific and Technical Information Facility (25)
Attn: NASA Rep. (S-AK/ RKT)
P. O. Box 33
College Park, Maryland