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## ORBITAL CALCULATIONS AND TRAPPED

 RADIATION MAPPING$\qquad$
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George C. Marshall Space Flight Center, Huntsville, Alabama
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# ORBITAL CALCULATIONS AND TRAPPED RADIATION MAPPING 

## By

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#### Abstract

The Burrell Orbital Flux and Energy Spectra Code calculates the position of an earth satellite in geocentric and B-L coordinates. Utilizing the B-L coordinates, the code calculates the radiation flux and energy spectra for protons or electrons at any point in a circular or elliptical orbit.


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# ORBITA L CALCULATIONS AND TRAPPED RADIA TION MAPPING 

SUMMARY

Given the orbital parameters of an orbiting satellite, the BOFES code calculates the position of the satellite in geocentric and $B-L$ coordinates. Satellite positional data have been generated by the code duplicating the ephemeris data for the Pegasus satellites for a period of two weeks.

The code allows the complete mapping of the geomagnetic field traversed by the satellite on any feasible elliptical orbit out as far as the lunar orbit.

The code calculates the radiation intensity and energy spectral of trapped radiation at each orbital step and obtains a time flux weighted average at the end of each orbit. Later, a method will be added to the proton and also to the electron code to calculate the dose rates behind several shield thicknesses at each step in the orbit.

## INTRODUCTION

The Burrell Orbital Flux and Energy Spectra (BOFES) code gives an accurate calculation of the orbital position of earth satellites and simultaneously computes the geomagnetic $\mathrm{B}-\mathrm{L}$ coordinate of the satellite. The $\mathrm{B}-\mathrm{L}$ coordinates are calculated using the 48 -term representation of the geomagnetic field by Jensen and Cain or a simple dipole representation. The BOFES code utilizes the $B-L$ coordinates to determine the radiation fluxes and energy spectra at any point of the orbit. The code allows the complete mapping of the trapped radiation field traversed by the spacecraft on any feasible elliptical or circular orbit around the earth and out as far as the lunar radius. The number of orbits and also the number of steps per orbit are chosen at the convenience of the users.

## COORDINATE TRANSFORMA TION

To determine the position of a satellite, one must be given a set of six orbital elements that specify the orbit's orientation in space, its size and shape,
and the position occupied by the object at a specified time or the time at which it is at a specified point. For geocentric Orbits, such a set consists of three orientation elements and three dimensional elements.

The orbital elements are
(1) $\Omega$, the longitude of the node, measured in the plane of the equator from the direction of the vernal equinox eastward to the direction of the ascending node, or intersection of the orbit with the equator.
(2) $\omega$, the argument of perigee, the angle measured in the plane of the orbit from the direction of the ascending node to the direction of perigee.
(3) $\phi$, the inclination, the angle between the equatorial plane and the plane of the orbit.

The three remaining "dimensional" elements are
(1) a, the semimajor axis
(2) $\epsilon$, the eccentricity, the ratio of the distance from the center of the orbit to the focus (center of earth) to the semimajor axis.
(3) $T_{p}$, the time of perigee passage.

The orientation elements, $\Omega, \omega, \phi$, are typical Euler angles. Using these angles a coordinate transformation can be made from the orbital system to the earth coordinate system, to obtain the latitude and longitude. Using the Euler angles, the elements $\mathrm{a}_{\mathrm{ij}}$ of the coordinate transformation matrix are given by

$$
\begin{align*}
& a_{11}=\cos \omega \cos \Omega-\sin \omega \sin \Omega \cos \phi \\
& a_{12}=-\sin \omega \cos \Omega-\cos \omega \sin \Omega \cos \phi \\
& a_{13}=+\sin \phi \sin \Omega \\
& a_{21}=\cos \omega \sin \Omega+\sin \omega \cos \Omega \cos \phi \\
& a_{22}=-\sin \omega \sin \Omega+\cos \omega \cos \Omega \cos \phi \tag{1}
\end{align*}
$$

$$
\begin{align*}
& \mathrm{a}_{23}=-\sin \phi \cos \Omega \\
& \mathrm{a}_{31}=\sin \omega \sin \phi \\
& \mathrm{a}_{32}=\cos \omega \sin \phi \\
& \mathrm{a}_{33}=\cos \phi \tag{1}
\end{align*}
$$

The coordinate transformation to the earth system ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) is obtained by the following matrix multiplication:

$$
\left(\begin{array}{c}
x  \tag{2}\\
y \\
z
\end{array}\right)=\left(\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{array}\right)\left(\begin{array}{c}
x^{*} \\
y^{*} \\
0
\end{array}\right)
$$

where $\mathrm{x}^{*}, \mathrm{y}^{*}, \mathrm{z}^{*}$ is the coordinate system in the plane of the orbit with $\mathrm{x}^{*}$ drawn from the pericenter of the ellipse to the perigee and $z^{*}$ is the axis normal to the plane of the ellipse. The $z *$ element in the matrix of equation (2) is defined as zero since the point on the orbit is in the plane of the ellipse.

From equation (2) the geocentric coordinate system is given by

$$
\begin{align*}
& x=a_{11} x^{*}+a_{12} y^{*} \\
& y=a_{21} x^{*}+a_{22} y^{*}  \tag{3}\\
& z=a_{31} x^{*}+a_{32} y^{*}
\end{align*}
$$

Using the geometry of Figure 1 and spherical coordinates, the point $P\left(x^{*}, y *\right)$ is placed in the ( $x, y, z$ ) system by the following:

$$
\begin{align*}
& x=r \sin \lambda^{\prime} \cos \psi^{\prime} \\
& y=r \sin \lambda^{\prime} \sin \psi^{\prime}  \tag{4}\\
& z=r \cos \lambda^{\prime}
\end{align*}
$$

Substituting the above into equation (3) we obtain:

$$
\begin{equation*}
\cos \lambda^{\prime}=\frac{\mathrm{z}}{\mathrm{r}}=\frac{\mathrm{a}_{31} \mathrm{x}^{*}+\mathrm{a}_{32} \mathrm{y}^{*}}{\mathrm{r}} \tag{5}
\end{equation*}
$$



## VERNAL EQUINOX

FIGURE 1. COORDINATE TRANSFORMATION
Since $\lambda$ is the latitude of point $P\left(x^{*}, y^{*}\right)$ measured from the equatorial plane, then

$$
\lambda^{\prime}=90-\lambda
$$

and

$$
\cos \lambda^{\prime}=\cos (90-\lambda)=\sin \lambda .
$$

Thus equation (5) becomes

$$
\begin{equation*}
\sin \lambda=\frac{a_{31} x^{*}+a_{32} y^{*}}{r} . \tag{6}
\end{equation*}
$$

From Figure 1 and equation (3)

$$
\begin{equation*}
\tan \psi^{\prime}=\frac{y}{x}=\frac{a_{21} x^{*}+a_{22} y^{*}}{a_{11} x^{*}+a_{12} y^{*}} \tag{7}
\end{equation*}
$$

where $\psi^{\prime}$ is a displaced longitude (in the equatorial plane) of the point P ( $\mathrm{x} *, \mathrm{y} *$ ) measured eastward from the direction of the vernal equinox to the line of intersection of the projection of the Vector $\bar{r}$ onto the equatorial plane. Thus equations (6) and (7) give the latitude and a displaced longitude in the equatorial plane measured from the vernal equinox. To find the true longitude, one must know the angle " $\alpha$ " between the Greenwich Meridian and the vernal equinox at a specific time.

Thus the true longitude $\psi$ is given by

$$
\begin{equation*}
\psi=\psi^{\prime}-\alpha . \tag{7a}
\end{equation*}
$$

The method of determining $\alpha$ is discussed in the next section on methods.

METHOD

The BOFES code utilizes a constant angle step during an orbit. At each step the position relative to the earth coordinate system is computed. Simultaneously, the B-L coordinate of each point is also calculated. The B-L coordinate is then used to calculate the radiation flux and energy spectra from input data. The details of the methods used follow.

Given the semimajor axis a the period of the orbit is determined by

$$
\begin{equation*}
\mathrm{T}=2 \pi \sqrt{\frac{\mathrm{a}^{3}}{\mu}} ; \quad \mu=\mathrm{GM}, \tag{8}
\end{equation*}
$$

where $G$ is the gravitational constant and $M$ is the mass of the earth.
The radial distance of the satellite (Fig. 2) and the flight time ( $\Delta \mathrm{t}$ ) for each constant step in angle $(\Delta \theta)$ are given by, $\theta_{k+1}=\theta_{k}+\Delta \theta$, where $\theta_{0}=0^{\circ}$ at perigee

$$
\begin{align*}
& r_{k+1}=\frac{a\left(1-\epsilon^{2}\right)}{1+\epsilon \cos \theta_{k+1}}  \tag{9}\\
& \gamma=\frac{a-r_{k+1}}{a \epsilon}  \tag{10}\\
& t_{k+1}=\sqrt{\frac{a^{3}}{\mu}}\left[\cos ^{-1}(\gamma)-\epsilon \sqrt{1-\gamma^{2}}\right]  \tag{11}\\
& \Delta t=t_{k+1}-t_{k}
\end{align*}
$$

where $r_{k+1}$ is the radial distance measured from pericenter to the position of the spacecraft, $\theta_{k+1}$ is the angle measured from perigee to the spacecraft position in the orbit plane, $\epsilon$ is the eccentricity of the orbit, $\gamma$ is equivalent to the cosine of the eccentric anomaly and $t_{k+1}$ is the flight time of the spacecraft corresponding to the angle $\Theta_{k+1}$. Using equation (6), the radius of the earth for the latitude, $\lambda$, is computed by

$$
\begin{equation*}
R=\frac{0.996633 \mathrm{Re}}{\sqrt{1-0.0067227 \cos ^{2} \lambda}} \tag{12}
\end{equation*}
$$



FIGURE 2. ORBIT GEOMETRY OF SATELLITE
where Re is the equatorial radius [1]. The altitude of the spacecraft above the earth's surface is given by

$$
\begin{equation*}
\mathrm{H}=\mathrm{r}-\mathrm{R}, \tag{13}
\end{equation*}
$$

where $r$ is obtained from equation (9); units of the altitude are the same as Re and the semimajor axis a.

The geodetic latitude is computed [1], using equation (6), as

$$
\begin{equation*}
\lambda_{d}=\tan ^{-1}\left[\frac{\tan \lambda}{0.9932773}\right] \tag{14}
\end{equation*}
$$

The longitude $\psi$, measured eastward from the Greenwich Meridian (Fig. 3), is given by

$$
\begin{equation*}
\psi=\psi^{\top}-\alpha \quad(0 \leq \psi<2 \pi) \tag{15}
\end{equation*}
$$



FIGURE 3. DEFINITION OF $\psi$ AND $\psi_{0}$

The angle $\alpha$ (in radians) is given by

$$
\begin{equation*}
\alpha=\psi_{\mathbf{o}}+6.300388 \mathrm{t} \tag{16}
\end{equation*}
$$

where the constant is the rate of change of the mean sidereal time in radians/day; $t$ is the time of flight (in days) measured from beginning of the calculations; and $\psi_{o}$ is angular distance (in radians) between the vernal equinox and the Greenwich Meridian at the beginning time of the calculation. The $\psi_{0}$ is given by

$$
\begin{equation*}
\psi_{\mathrm{o}}=.98564995[\mathrm{MJD}-35838.0]+99.39235, \tag{17}
\end{equation*}
$$

where MJD is the mean Julian date of the beginning of the calculation; the constant 99.39235 is $\psi_{0}$ at MJD 35838.0 ; and 0.98564995 is the rate of change of the mean sidereal time in degrees/day. (The times of epoch are given in Julian days, and for convenience 2400000.5 has been subtracted to give a "modified" Julian day.) From equations (13), (14), and (15), the position of the spacecraft is determined at each point in time.

In the BOFES codes, all computations are begun at the perigee point in the orbit. When given the orbital parameters for an epoch $T_{0}$, which may or may not be the time of perigee passage, the time of perigee passage must be calculated and the argument of perigee $\omega$ and the longitude of the ascending node $\Omega$ must be corrected. The time of the previous perigee passage corresponding to the epoch $\mathrm{T}_{0}$ is given by

$$
\mathrm{T}_{\mathrm{p}}=\mathrm{T}_{0}-\frac{\mathrm{T} \mathrm{M}_{0}}{2 \pi}
$$

where $\mathrm{T}_{0}$ is the epoch time; $\mathrm{M}_{0}$ is the mean anomaly corresponding to $\mathrm{T}_{0}$; and T is the period of the orbit. When $\mathrm{T}_{\mathrm{p}}$ is computed, the orbital parameters $\omega$ and $\Omega$ are computed for the time of perigee passage $\mathbf{T}_{\mathbf{p}}$. These computations are performed by hand. The time of perigee passage $T_{p}$ is input data designated as the sum of DATE and TIME, where DATE is the Julian date (days), midnight Greenwhich time, at which computations are to be made and TIME is the fraction of a day (day units) after midnight when computations are performed.

Using the position elements of the satellite, the $\mathrm{B}-\mathrm{L}$ coordinates can be determined simultaneously by utilizing the 48 -term representation of the geomagnetic field by Jensen and Cain or a simple dipole representation [2] may be obtained by using equations (18) and (19). Hence,

$$
\begin{equation*}
L=\frac{r}{\cos ^{2} \lambda} \tag{18}
\end{equation*}
$$

and

$$
\begin{equation*}
B=\frac{M}{r^{3}} \quad\left(4-\frac{3 r}{L}\right)^{1 / 2} \tag{19}
\end{equation*}
$$

where $\lambda$ is the geocentric latitude given in equation (6); $r$ is the radial distance (in earth radii) obtained by equation (9) ; and $M$ is the dipole moment (. 311653 gauss $\mathrm{Re}^{3}$ ).

Since the radiation (proton and electron) fluxes are functions of the $B-L$ coordinates, the BOFES code utilizes the B-L coordinates to determine the radiation intensity at any point in an orbit and as far out as the lunar radius. The code is also used to determine the artificial belt radiation level at any point on the orbit.

## PROTON CODE

The proton BOFES code computes the proton flux $\Phi[B(t), L(t)]$ at each orbital step. Also a time average flux is determined continuously and printed at the end of each orbit. The time average flux is given by

$$
\begin{equation*}
\bar{\Phi}=\frac{\sum_{i=1}^{N} \Phi_{i}[B(t), L(t)] \Delta t_{i}}{\sum_{i=1}^{N} \Delta t_{i}} \tag{20}
\end{equation*}
$$

where $\sum_{i=1}^{N} \Delta t_{i}$ is the accumulative time at the end of the $N^{\text {th }}$ step in the orbit.
The B-L coordinate is also used to determine the corresponding exponential energy parameter $\mathrm{E}_{0}$ which is employed to find the differential and integral proton energy spectrum at each point in the orbit calculation.

After the proton flux, $\Phi_{i}$, has been determined, the time-weighted proton differential and integral energy spectra are found for certain specified input proton energies, $E(n)$. The code computes these energy spectra for each point in the orbit and keeps running totals for each proton energy, $E(n)$. The timeweighted average of the integral and differential energy spectra for the $\mathrm{N}^{\text {th }}$ energy group are

$$
\begin{equation*}
I\left(>E_{n}\right)=\frac{\sum_{i=1}^{N} \Phi_{i}(B, L) e \frac{E_{M}-E(n)}{E_{o i}(B, L)} \Delta t_{i}}{\sum_{i} \Delta t_{i}}, \tag{21}
\end{equation*}
$$

$$
\Phi\left(E_{n}\right)=\frac{\sum_{i=1}^{N} \frac{\Phi_{i}(B, L)}{E_{o i}(B, L)} e \frac{E_{M}-E(n)}{E_{o i}(B, L)} \Delta t_{i}}{\sum_{i} \Delta t_{i}}
$$

The above are printed out after each orbit for each proton energy $E(n) . E_{M}$ is the minimum energy of the proton data, $\mathrm{E}_{0}$ is the exponential energy parameter corresponding to each point in the orbit, $\Delta t_{i}$ is the time step along the orbit and the summation is the accumulation of the total flight time of the spacecraft.

## ELECTRON CODE

The BOFES code for electrons computes the instantaneous electron flux at any point in the orbit and also carries a time average of the electron flux given by equation (20).

The electron's energy spectra above 0.5 MeV are given in eight energy bands and are normalized so that

$$
\begin{equation*}
\sum_{j=1}^{8} f_{j}(E)=1 \tag{22}
\end{equation*}
$$

The electron energy spectra are a function only of the L coordinate. Therefore, for any point in the orbit, the energy bands corresponding to the $L$ coordinate are obtained, and an accumulative time-flux-weighted average is printed out at the end of each orbit and is given by

$$
\begin{equation*}
F_{j}(E)=\frac{\sum_{i} \Phi_{i}(B, L) f_{j}[E(t), L(t)] \Delta t_{i}}{\sum_{i} \Phi_{i}(B, L) \Delta t_{i}} \tag{23}
\end{equation*}
$$

where

$$
j=1,2, \ldots 8
$$

The energy groups represented above are the same as those in Reference 3, page 48 (i.e., $0.5-1,1-2,2-3 \mathrm{MeV}, \ldots$ ).

## SPECIAL FEATURES OF CODE

The option MT determines the position of the orbit angle step relative to the time step. For MT = 0, 1 the angle step along an orbit is halfway between the time step, or the angle step does not coincide with the time step. For MT $=-1$, the angle and time step coincide, i. e. , $\mathrm{t}_{\mathrm{k}+1}$ is the flight time for the orbit angle $\theta_{\mathrm{k}+1}$.

The option JOB distinguishes the type of calculations to be performed and the output format to be used. When $\mathrm{JOB}=1,2$, or 3 the output is at each orbital step; for $\mathrm{JOB}=4$, the output is at the end of each orbit only. When $\mathrm{JOB}=1$, the code computes and outputs the position (degrees) of the spacecraft in orbit $(\Theta)$, the altitude ( H ), latitude ( 2 d ), longitude $(\psi)$, the time step ( $\Delta \mathrm{t}$ ) and the total flight time (TT), respectively. When JOB=2, the output is the same as for $\mathrm{JOB}=1$ with the B and L coordinates included. For $\mathrm{JOB}=3$, the output is the same as for $\mathrm{JOB}=2$, but includes the radiation flux (particles per $\mathrm{cm}^{2}-\mathrm{sec}$ ), a running sum of the product of flux and the time step (no. of particles crossing unit area during the time step), also the reciprocal of the exponential energy parameter $\mathrm{E}_{0}$ (proton code). The reciprocal is printed as zero if the energy spectra option is $1(k=1)$. For JOB $=4$, the output includes the total flight time from beginning of calculations (TT), the product of the flux and the time step (SFLUX) and the time averaged particle flux (SFLUX/TT) given by equation (20).

An option $A L$ is also included which determines the maximum value of the L coordinate to be calculated using the 48 -term representation of the geomagnetic field. All $L$ values (in earth radii units) equal to or greater than the value of AL will be computed using a dipole representation. For altitudes below 800 km and longitudes between 90 and 240 degrees east, a 50 percent reduction [4] in run time can be realized if one bypasses the calculation of the geomagnetic field using the 48 -term expansion by Jensen and Cain. If the position of the spacecraft falls within the above region, a test is provided to bypass the multipole expansion of the geomagnetic field and computes the dipole field and sets the radiation flux equal to 1 particle per $\mathrm{cm}^{2}-\mathrm{sec}$. An option to bypass this geographic test is provided because, if one is interested in the geomagnetic field distribution, the B-L coordinates must be computed at each position traversed by the spacecraft.

The energy spectra option $k$ provides for the calculation of energy spectra (protons and electrons) at each orbital position. When $\mathrm{k}=2$, the electron energy bands corresponding to the $L$ coordinate are determined and printed after
each orbital step and a time-flux-weighted average is printed at the end of each orbit. For protons ( $k=2$ ), the time-flux-weighted average of the differential and integral energy spectra (Eq. 21) corresponding to the proton energies $E(n)$ are computed and printed after each orbit. When $k=1$, the energy spectra for protons and the eight bands (electrons) are not given as output.

## RESULTS AND CONCLUSIONS

Figures 4 through 7 show the different types of computations possible using the BOFES (proton and electron) codes.

Figure 4 is a plot of the proton flux at an altitude of 1000 km computed along a circular polar orbit. Curve $B$ shows the total proton flux per orbit while curve A is a running average of the total flux per orbit, which tends to stabilize after several orbits. Figure 5 shows the proton flux on a circular orbit, inclined $32^{\circ} .5$, at an altitude of 286 kilometers. Again, curve B is the total flux per orbit, which shows a greater fluctuation than Figure 4. Curve A is the running average of the proton flux. Similar electron data can be obtained using the electron flux data AE1.

Figure 6 shows the electron flux plotted as a function of altitude for four different inclinations using the electron data (AE1).

Figure 7 shows the proton flux as a function of altitude using the proton data (Ap1).

Given the orbital parameters of an orbiting satellite, the BOFES code can compute the satellite positional data duplicating the satellite ephemerous data from more elaborate codes with only small errors in the position of the satellite for a period of two weeks.

Subsequently, a method will be added to the proton, and possibly to the electron, code whereby the radiation dose intensity behind several shield thicknesses will be computed at each orbital step and a time and flux weighted average will also be computed for each orbit.

The code allows the complete mapping of the trapped radiation field traversed by the satellite on any feasible elliptical or circular orbit around the earth and out as far as the lunar radius.


FIGURE 4. PLOT OF THE PROTON FLUX (protons per $\mathrm{cm}^{2}-\mathrm{hr}$ ) CALCULA TED ON A CIRCULAR POLAR ORBIT AT AN ALTITUDE OF 1000 km


FIGURE 6. ORBITAL INTEGRATIONS WITH ELECTRON DATA AE1, E $>0.5$ MeV FOR CIRCULAR ORBITS (REF. 3, p. 97)


FIGURE 5. PROTON FLUX (protons per $\mathrm{cm}^{2}-\mathrm{hr}$ ) CALCULATED ON A CIRCULAR ORBIT AT AN ALTITUDE OF 286 km INCLINED $32: 5$ WITH THE EQUATOR.


FIGURE 7. ORBITAL INTEGRATIONS USING PROTON DATA AP1, E > 34 MeV (REF. 3, p. 159)

## Key to Proton Flux - Energy Spectra Data

The following is a description of the data computed by the BOFES proton code (Fig. A-1). The sample data is for printout option equal to 3 ( $\mathrm{JOB}=3$ ), which is inclusive of $\mathrm{JOB}=1,2$, and 4 . The energy spectra are not included in the output of $\mathrm{JOB}=1$ and 2 .

1. Orbital parameters described previously in the section Data Input, except LATO and LONGO which are the latitude and longitude of the initial starting position, respectively.
2. Minimum energy (in MeV units) of the proton data.
3. THETA - angle (degrees) measured from perigee of orbit to position of spacecraft.

ALT - Altitude of spacecraft (same units as semimajor axis).
LAT - Latitude (degrees) measured from the equator.
LONG - Longitude (degrees) measured eastward from Greenwich Meridian.

B - B coordinate (gauss units).
L - L coordinate (Earth radii units).
TT - Total orbital flight time of spacecraft measured from the beginning of computations (same units as time factor, TF).

FLUX - Proton Flux (particles $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ ) at spacecraft position.
SFLUX - Proton current (particles $\mathrm{cm}^{-2}$ ), essentially the product of the proton flux and time between each orbital step (DT) with a time factor correction.

1/EO - Reciprocal of energy parameter ( $\mathrm{MeV}^{-1}$ )
4. Total orbital flight time (TT), proton current (SFLUX) and the quotient of SFLUX and TT (equivalent to proton flux with time unit equal to time factor TF), respectively. This printout is taken from the last printout for each orbit and is the output from $\mathrm{JOB}=4$.
5. Time and flux-weighted average of the proton energy spectra data including proton energy ( E ) in MeV units, the differential proton flux at energy E and the integral proton flux for proton energies equal to and greater than energy $E$.

## Key to Electron Flux-Energy Spectra Data

The following is a description of symbols used in the output of the BOFES Electron code (Fig. A-2). The sample data is for output option 3 ( $\mathrm{JOB}=3$ ) which includes the output of $\mathrm{JOB}=1,2$, or 4 . The elctron energy spectra can be included only in the output from options 3 and 4.

1. Same as 1 for proton data.
2. Same as 3 for proton data.
3. Eight differential energy spectra groups (. $5-1 \mathrm{MeV}, 1-2,2-3, \ldots$ $6-7,7-\infty)$, respectively, satisfying the condition of equation (22). The differential energy spectra are a function of the coordinate only, therefore, the value of the energy spectra in each of the eight groups is determined by the L value printed on the line above the energy spectra.
4. Same as 4 for proton data.
5. Time and flux-weighted average of the differential energy spectra data for each point along the orbit.

ORBITAL PARAMETERS FOR BOFES ELECTRON CODE


## Data Input - Electron Code

The following table is a listing of the data input for the BOFES electron table.

| Vari | iable | Format | Description |
| :---: | :---: | :---: | :---: |
| (A) | MAX | I5 | Number of $L$ values for electron flux data. |
| (B) | KMAX (1), BL (1) | I5, F7. 0 | KMAX (1) is the number of B-flux pairs for $L$ value, BL (1). |
| (C) | $\mathrm{B}(\mathrm{J}, \mathrm{I}), \mathrm{FL}(\mathrm{J}, \mathrm{I})$ | $\begin{aligned} & \text { F6.0, E8. 0, } \\ & 3(\text { F7. 0, E8.0) } \end{aligned}$ | B-flux pairs for $L$ value, $B L(1)$ card (B) is repeated until KMAX (max) BL (max) is read. |
|  | LMAX | I5 | Number of $L$ values for energy spectra data. |
| (E) | BEL(1) | F5. 2 | L coordinate value. |
|  | $E(1,1)$ | 8E10.3 | Energy spectra, 8 energy bands.5-1 MeV, 1-2 MeV, ... 6-7 MeV, $7-\infty \mathrm{MeV}$. card ( $E$ ) is repeated. Until Bel(LMAX) is read. |
|  | E (8, 1) |  |  |
|  | NCASE | I5 | Number of different orbital cases to be run. |
|  | A MEG, A MEGA, PHI | 3E14. 8 | A MEG is argument of perigee (degrees) A MEGA, longitude of ascending node (deg), measured eastward from vernal equinox, PHI, inclination of orbit. (degrees). |

Variable
(I) DMEG, DMEGA, DATE, TIME
(J) A, ECC, TF, REE
(K) NC NS, ERR

## Format

4E14. 8

4E14. 8

2I5, F7. 0

DMEG, rate of change of A MEG (degrees/hr) DMEGA, rate of change of A MEGA (degrees/hr).

DATE, Julian date (days), midnight Greenwich time of day at which computations are to be made, TIME, fraction of day (day) after midnight when computations are performed.

A is the semimajor axis of orbit ECC is the eccentricity TF is a time factor, gives time unit used in computations. Ratio of (time unit used/hr)
$\mathrm{TF}=60$, time units in minutes
TF = 1, time units in hours $T F=1 / 24$, time unit in days, etc.
REE is equatorial radius of earth (A and REE must have same units).

NC, No. of orbits per case (having same orbital parameters). NS, no. of angle steps per orbit. ERR, accuracy of L computation eg. $E R R=.1$ accuracy $=1 \%$ $E R R=1$, accuracy $=$ $10 \%$ etc.

Variable
(L) MT, JOB, AL, K

2I5, F7.0, IS

Description

MT, position of orbital angle relative to time step. JOB, specifies type of orbital calculations to be made. AL, upper limit on $L$ computed by 48 -term representation of geomagnetic field. K, Energy spectra option, previously described.

A listing of the input data for the BOFES proton code is given in the following table.

| Variable | Format | Description |
| :---: | :---: | :---: |
| MAX | I5 | Same as (A) in electron code. |
| KMAX (1) , BL (1) | I5, F7. 0 | Same as (B) in electron code. |
| B(J,I), FL(J,I) | $\begin{aligned} & \text { F6. 0, E8.0 } \\ & \text { (F7.0, E8.0) } \end{aligned}$ | Same as (C) in electron code. |
| EMIN | F10.0 | Proton energy (MeV) of exponential parameter (EO) data. |
| LMAX | I5 | Same as (D) in electron code. |
| KEMAX (1), BEL(1) | I5, F7. 0 | KEMAX, no. of B-EO pairs for $L$ coordinate, BeL (1). |
| $\mathrm{BB}(1,1), \mathrm{EO}(1,1)$ | F6. 0, E8. 0 | B-exponential energy parameter (EO) pairs for $L$ coordinate, BeL (1). |
| BB(KEMAX, 1) , EO(KEMAX, 1) |  | Repeated until <br> KEMAX(LMAX) BeL(LMAX) is read. |
| NCASE | I5 | Same as (G) in electron code. |
| A MEG, A MEGA, PHI | 3E14. 8 | Same as (H) in electron code. |
| DMEG, DMEGA, DA TE, TIME | 4E14. 8 | Same as (I) in electron code. |
| A, ECC, TF, REE | 4E14.8 | Same as (J) in electron code. |
| NC, NS, ERR | 215, F7. 0 | Same as (K) in electron code. |
| MT, JOB, AL, K | 2I5, F7. 0, I5 | Same as (L) in electron code. |

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# ORBITAL CALCULATIONS AND TRAPPED RADIATION MAPPING 

By
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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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