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# OPTICAL AND MICROWAVE COMMUNICATIONS-A COMPARISON

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BY

F. KALIL

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OPTICAL AND MICROWAVE COMMUNICATIONS-  
A COMPARISON

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ABSTRACT

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Some preliminary comparisons were made of microwave, millimeter, and optical communication systems for space communication from a spacecraft at Mars distances. An attempt was made to be realistic with regard to technology. Some discussion of thermal, quantum, and sky noise is included, as well as some discussion and analyses of microwave, millimeter wave, and optical technology; acquisition and tracking; and some mission analysis. Based on the considerations herein, it appears likely that in the radio spectrum the S-Band is the better place to operate. However, it appears likely that optical communication systems have the greater potential for higher data rates—up to about  $10^8$  bps at Mars distances (see Table 8 which summarizes the results).

# OPTICAL AND MICROWAVE COMMUNICATIONS-

## A COMPARISON

### I. INTRODUCTION

A comparison of Lasers versus microwaves (and millimeter waves) in space communications has been reported in the literature by S. Gubin, R. B. Marster, and D. Silverman, (ref. 1). However, in that study (ref. 1) a laser, diffraction limited beamwidth of 20 arc seconds was assumed. Primarily because of this large beamwidth, the laser communication systems did not compare favorably with the microwave system. Furthermore, the best available laser source was not considered in Reference 1, namely, the 130 watt, CW, 10.6  $\mu$  wavelength, CO<sub>2</sub> laser with an excellent efficiency of ~13%, developed by Bell Telephone Laboratories, (ref. 2, Hughes NAS5-9637). The largest CW laser source they considered was a 2W ionized argon gas laser at 0.4880  $\mu$  wavelength, and in the microwave region they only considered S-Band, 10 Gc, and 25 Gc systems.

In this paper similar comparisons will be made using more up-to-date values for CW laser output powers of 4W and 130W (ref. 2), and laser diffraction limited beamwidths of 1 arc second. This is a factor of 10 worse than the present goal of 0.1 arc sec diffraction limited beamwidth (ref. 3, R. Chase optical tech. conference), being emphasized in the research programs sponsored by the Office of Advanced Research and Technology, NASA Headquarters and by the NASA Electronics Research Center. (ref. 4, NBS lecture series).

### II. ANALYSIS

In the following analysis, the simple range equation will be used to evaluate the maximum amount of data (Bits/sec) which can be transmitted to the earth from a spacecraft at Mars distances with various communication system. The distance chosen for this analysis was  $1.852 \times 10^8$  km or  $10^8$  n. mi.

#### Range Equation

The simple range equation is based on the following considerations. Assume that transmitting and receiving antennae are separated by a distance  $R \gg \lambda$  (wavelength), such that the received wavefront may be considered planar over the receiving antenna cross section. If the transmitter antenna were omnidirectional (isotropic radiator), then the inverse square law would be applicable, i.e.,

the transmitted power  $P'_T$  would be uniformly spread out over a sphere, so that the power per unit area at the receiver would be  $P'_T/4\pi R^2$ . However, if the transmitting antenna is directional with a directive gain of  $G_T$ , then the power per unit area at the receiver would be  $G_T P'_T/4\pi R^2$ , because by definition "the directive gain of an antenna is the ratio of the power received or radiated in a given direction to that which would be received or radiated if the antenna were nondirectional," and is given by

$$G = \frac{4\pi A_{eff}}{\lambda^2} \quad (1)$$

where  $A_{eff}$  is the effective capture cross section of the antenna and for circular apertures is taken to be about  $0.54 A_R$ . Thus, the received power is

$$P'_R = \frac{P'_T G_T A_{R,eff}}{4\pi R^2} \quad (2)$$

If one wishes to define  $P_R$  as the received power at the receiver input terminals, and  $P_T$  as the power output at the transmitter power amplifier terminals, then one must take account of the received and transmitter line losses  $L_R$  and  $L_T$ , in which case

$$P_R = \frac{P_T G_T A_{R,eff}}{4\pi R^2 L_T L_R} \quad (3)$$

### Noise

The received carrier signal to noise ratio is

$$\frac{C}{N} = \frac{P_T G_T A_{R,eff}}{4\pi R^2 L_T L_R N} \quad (4)$$

where  $N$  is the total noise at the receiver input terminals and is given by

$$N = \psi B \quad (5)$$



where  $\psi$  is the noise power density and B is the effective detection bandwidth and has been extensively treated (refs. 5 - 9). B. M. Oliver (ref. 5) has shown that the total noise power density of an ideal amplifier is given by (see also ref. 1)

$$\psi_a = \frac{hf}{e^{hf/kT} - 1} + hf \quad (6)$$

where T is the effective noise temperature in degrees Kelvin at the input,  $h = 6.624 \times 10^{-34}$  watt-sec<sup>2</sup> is Planck's constant,  $k = 1.38047 \times 10^{-23}$  watt-sec-deg<sup>-1</sup> is Boltzmann's constant, and f is the frequency in cycles per sec. Similarly, the total noise power density of an ideal linear amplitude detector or phase detector is

$$\psi_a = \frac{hf}{e^{hf/kT} - 1} + \frac{hf}{2} \quad (7)$$

An ideal amplifier or detector is "noiseless," i.e. the signal-to-noise ratio is the same at its output terminals as at its input terminals. In terms of noise factor (or noise figure as it is sometimes called), a "noiseless" amplifier, detector, or network in general, has a noise figure of unity. A more thorough and detailed discussion of noise figure and noise temperature is given in the appendix. Furthermore, in subsequent sections of this report it will become clear as to how a nonideal amplifier or detector could be analyzed with the aid of the foregoing considerations.

It should be noted that in the case of a non-coherent power detector such as a photo detector which is not used as a mixer, then the right hand term of equation 7 becomes 2hf (refs. 5, 10).

The first term on the right hand side of Equations 6 and 7 is the "thermal noise" caused by thermal agitation of the molecules or electric charge in the "equivalent resistive element" of the circuit (See also refs. 8, 9). The second term on the right hand side of these equations is the "quantum noise" which comes about because of the well established and experimentally verified principles of quantum mechanics, namely:

1. The intensity of a radiation field, i.e. the product of its amplitude vector by the complex conjugate of its amplitude vector, specifies the probability of intercepting a photon. Therefore, even if the received radiation is a coherent monochromatic wave of constant power P, photons will be received in a random fashion.

2. The Heizenberg's Uncertainty Principle which states that two canonically conjugate variables cannot be determined precisely simultaneously; in particular

$$\Delta E \Delta t \geq \frac{h}{4\pi} \quad (8)$$

where  $E = hf$  is the photon energy, and hence  $\Delta E$  is the uncertainty in photon energy and  $\Delta t$  is the uncertainty in the time of arrival of the photon. The physical essence of Equation 8 is that the more precisely the photon's energy is known, the less precise its time of arrival can be determined. Basically, it is this uncertainty in time of arrival of the photons which causes this shot noise being referred to here.

To facilitate plotting (Equations 6 and 7) we divide through by  $kT$  and plot  $\psi/kT$  as a function of  $hf/kT$  as shown in Figure 1. This figure shows how the thermal, quantum, and total noise vary with  $hf/kT$  and how they compare with each other. It can be seen that at

$$\alpha = \frac{hf}{kT} \ll 1,$$

or  $hf \ll kT$ , then the thermal noise predominates and the detector is said to be thermal noise limited; while at

$$\alpha = \frac{hf}{kT} \gg 1$$

or  $hf \gg kT$  the quantum noise predominates, and the detector is said to be quantum noise limited. Thus there are two regions in the noise spectrum: 1) when  $hf < kT$  in which case  $\psi \approx kT$  which is the familiar expression for noise density; and 2) when  $hf > kT$  in which case  $\psi_a \cong hf$  for an ideal amplifier and coherent heterodyne detection, while  $\psi_d \approx hf/2$  for an ideal linear amplitude or phase detector or coherent homodyne detection, and in the case of non-coherent power detection  $\psi_d \approx 2hf$ . Shown in Figure 2 is the noise power density as a function of frequency  $f$  for various temperatures  $T$ . It can be seen from this figure that the thermal noise drops off sharply with increased frequency after  $hf$  begins to become significantly larger than  $kT$ .

It can be seen from these Figures 1 and 2 that if thermal noise were the only noise present, then the received signal power required for a given communication rate or a given signal to noise ratio would also decrease rapidly. Thus,

by choosing the carrier frequency high enough, one might falsely conclude that it would be possible to receive the entire contents of a book with a total received energy equivalent to one photon or less. The fact that this is not possible provides further evidence of the validity of the quantum mechanics principles and the existence of quantum noise.

### Microwave and Millimeter Systems

In the case of microwave and millimeter systems wherein  $kT \gg hf$ , the noise power density  $\psi$  becomes

$$\psi \approx kT \quad (9)$$

where  $T$  is the effective noise temperature in degrees Kelvin the input. Various noise sources can be included in the temperature  $T$  by equating them to an equivalent temperature and adding the effective temperatures contributed by each to get a total equivalent system noise temperature (see the Appendix for further details). In addition to the noise generated within the receiver, other noise sources include atmospheric attenuation, atmospheric noise, galactic noise, side lobe noise, back lobe noise (in the case of a mesh type antenna), and "spill-over" noise (noise which enters the antenna input by optical paths other than via the lobes).

The resultant tropospheric contribution to the noise temperature of a narrow beam antenna whose radiation pattern admits no side or back lobes is given by

$$T_{\text{troposphere}} = \int_0^{\infty} \alpha T \left[ \exp \int_0^r \alpha dr \right] dr \quad (10)$$

where  $\alpha$  and  $T$  are the absorption coefficient (reciprocal length units) and temperature respectively, at any point in the atmosphere at a distance  $r$  from the antenna. Shown in Figure 12 - 18 of ref. 10 are the calculated values of  $T_{\text{troposphere}}$  versus frequency at various antenna beam elevation angles. These computed curves are in essential agreement with experimental measurements. Shown in Figure 8 (ref. 5), are some typical effective antenna temperatures which include the effects of cosmic noise for both quiet and noisy sky, atmospheric absorption for good and bad weather, and ground radiation scattered by rain or snow.

Using the foregoing information, the system capabilities for an S-Band system were computed and are tabulated in Table 1. The assumptions are all in the table, but some are repeated here. For example, the transmitter powers used

Table 1  
 Analysis of Microwave Transmission from Mars Space Vehicle to  
 Earth-Based Station for  $P_T = 20 \text{ W}$  and  $100 \text{ W}$

	$P_T = 20\text{W}$	$P_T = 100\text{W}$
Frequency	2.3 Mc	2.3 Mc
Wavelength	13 cm (0.428 ft)	13 cm (0.428 ft)
Range	$1.852 \times 10^8 \text{ km}$ ( $10^8 \text{ n.mi.}$ )	$1.852 \times 10^8 \text{ km}$ ( $10^8 \text{ n.mi.}$ )
Transmitter Power, $P_T$	20W      13.0dbW	100W      20.0dbW
S/V Antenna Gain, $G_T$	4.88m (16 ft) 38.8db	4.88m (16 ft) 38.8db
S/V Transmission Loss	- 0.5db	- 1.0db
Free Space Path Loss, $\lambda^2 / (4\pi R)^2$	-265.1db	-265.1db
Ground Antenna Gain, $G_R^{(1)}$	64m (210 ft) 60.0db	64m (210 ft) 60.0db
Receiver Power, $P_R$	-153.8dbW	-147.3dbW
$\text{CNR} = P_R / \text{KTB}$	10.0db	10.0db
Allowable Noise Power	-163.8dbW	-157.3dbW
System Noise Temperature <sup>(2)</sup>	50°K $-211.7 \frac{\text{dbW}}{\text{cps}}$	50°K $-211.7 \frac{\text{dbW}}{\text{cps}}$
Noise Bandwidth	61.5kc $47.9 \frac{\text{db}}{\text{cps}}$	275kc $54.4 \frac{\text{db}}{\text{cps}}$
Maximum Transmission Rate <sup>(3)</sup>	$6.15 \times 10^4 \text{ bits/sec}$	$2.75 \times 10^5 \text{ bits/sec}$

NOTES:

- (1) Including ground line loss (ref. 1).
- (2) Includes  $10^\circ\text{K}$  receiver noise,  $30^\circ\text{K} \pm 10^\circ\text{K}$  sky noise (i.e., overall background noise from sky, spillover, sidelobes and backlobes).
- (3) No margins included.

were 20 watts (typical of the Apollo S/C system) and 100 watts (projected future capability). The value of 100 watts for projected future capability may be conservative, because an 8 Kw-CW, C-Band TWT (traveling wave tube) is now an off-the-shelf item (ref. 11). Although this TWT operates in the C-Band, it is at least indicative of what might be in the offing at S-Band for future space borne applications. At the same time, however, it must be kept in mind that the necessary power supply may not be conducive for spaceborne use because of size and weight. It was for such reasons as this that the 100 watt spaceborne transmitter power was used in Table 1 as a future projected capability for a Mars mission.

Because of the atmospheric "windows" at about 16 Gc, 34 Gc, and 94 Gc (see Figure 3a in this report and Figure 12-17 of ref. 10), and the availability of S/C transmitter sources at these frequencies (some off-the-shelf and some still in the laboratory stage, see refs. 2, 12), systems capabilities were also computed at these frequencies and are tabulated in Tables 2 and 3. The value used for the receiver noise temperatures were for the best available parametric amplifiers at 16 Gc and 34 Gc, while at 94 Gc the computations for the system capability was done for each of the best available crystal mixer, parametric amplifier and maser (ref. 2).

### Optical Systems

In the case of optical systems where  $hf \gg kT$ , the noise power density  $\psi$  for ideal detectors is given by (ref. 5).

$$\psi \approx \begin{cases} hf/2 \text{ homodyne detection} \\ hf \text{ heterodyne detection} \\ 2hf \text{ non-coherent detection} \end{cases}$$

In practice however, the detectors are not ideal.

For carrier signals, in the visible and near infrared spectrum (i.e. about  $0.4 \mu$  to  $1.1 \mu$ ), photo emissive devices, such as photomultiplier tubes are suitable detectors. The response times of ordinary photomultipliers are between 1 and 3 nanoseconds enabling them to detect modulation frequency in the order of 300 mc. Although the performance of commercial photomultipliers begins to be degraded between 50 mc and 150 mc modulation frequency, beat notes of up to 300 mc in modulation frequency have been detected with an ordinary 7102 multiplier phototube (ref. 13). However, such devices emit a current even in

Table 2  
 Analysis of Millimeter (16 Gc, 34 Gc) Transmission from Mars  
 Space Vehicle to Earth-Based Station.

Frequency		16 Gc <sup>(4)</sup>		34 Gc <sup>(4)</sup>	
Wavelength		18.75mm (0.06152 ft)		8.82mm (0.02893 ft)	
Range		1.852 × 10 <sup>8</sup> km (10 <sup>8</sup> n.mi.)		1.852 × 10 <sup>8</sup> km (10 <sup>8</sup> n.mi.)	
Transmitter Power, P <sub>T</sub>		200W <sup>(1)</sup>	23.0dbW	200W <sup>(1)</sup>	23.0dbW
S/V Antenna Gain, G <sub>T</sub>		4.85m (15 ft) 55.0db		4.85m (15 ft) 61.7db	
Transmitter Line Loss, 1/L <sub>T</sub>		~ - 1.5db		~ - 3.0db	
Free Space Path Loss, $\lambda^2/(4\pi R)^2$		-281.8db		-288.4db	
Receiver Line Loss, <sup>(5)</sup> 1/L <sub>R</sub>		~ - 0.5db		~ - 1.0db	
Ground Antenna Gain, G <sub>R</sub>		9.16m (30 ft) 61.2db		9.16m (30 ft) 67.8db	
Receiver Power, P <sub>R</sub>		-144.6dbW		-139.9dbW	
CNR = P <sub>R</sub> /KTB		10.0db		10.0db	
Allowable Noise Power		-154.6dbW		-149.9dbW	
Effective System Noise Temperature	Receiver Noise	438°K <sup>(2)</sup>	-201.8 $\frac{\text{dbw}}{\text{cps}}$	290°K <sup>(2)</sup>	-202.2 $\frac{\text{dbw}}{\text{cps}}$
	Sky Noise	14°K <sup>(3)</sup>		51°K <sup>(3)</sup>	
	Line Noise	32°K		43°K	
Noise Bandwidth		52.5kcs	47.2 $\frac{\text{db}}{\text{cps}}$	170kcs	52.3 $\frac{\text{db}}{\text{cps}}$
Maximum Transmission Rate		5.25 × 10 <sup>4</sup> bits/sec.		1.70 × 10 <sup>5</sup> bits/sec.	

- NOTES: (1) Assumed value based on 200W, CW, TWT at 94 Gc developed by Hughes Aircraft Co. (See ref. 2).  
 (2) Receiver noise for best available Paramp (See ref. 2).  
 (3) Sky noise, includes atmospheric absorption, good weather, quiet sky (ref. 9, H. H. Grimm).  
 (4) Best available parametric amplifiers (See ref. 2).  
 (5) Assumes receiver first stage is mounted near antenna input to minimize receiver line loss.

Table 3  
 Analysis of Millimeter Wave (94 Gc) Transmission From a  
 Mars Space Vehicle to Earth-Based Station

	Crystal Mixer <sup>(1)</sup>	Paramp <sup>(1)</sup>	Maser <sup>(1)</sup>
Frequency	94 Gc	94 Gc	94 Gc
Wavelength	3.2mm (0.01048 ft)	3.2mm (0.1048 ft)	3.2mm (0.01048 ft)
Range	1.852 × 10 <sup>8</sup> (10 <sup>8</sup> n.mi.)	1.852 × 10 <sup>8</sup> (10 <sup>8</sup> n.mi.)	1.852 × 10 <sup>8</sup> (10 <sup>8</sup> n.mi.)
Transmitter Power, P <sub>T</sub>	200W	200W	200W
S/V Antenna Gain, G <sub>T</sub>	4.58m (15 ft) 70.0db	4.58m (15 ft) 70.0db	4.58m (15 ft) 70.0db
Transmitter Line Loss, 1/L <sub>T</sub>	- 4.0db	- 4.0db	- 4.0db
Free Space Path Loss, $\lambda^2 / (4\pi R)^2$	-297.2db	-297.2db	-297.2db
Receiver Line Loss, <sup>(4)</sup> 1/L <sub>R</sub>	- 1.5db	- 1.5db	- 1.5db
Ground Antenna Gain, G <sub>R</sub>	4.58m (15 ft) 70.0db	4.58m (15 ft) 70.0db	4.58m (15 ft) 70.0db
Received Power, P <sub>R</sub>	-139.7db	-139.7db	-139.7db
CNR = P <sub>R</sub> /KTB	10.0db	10.0db	10.0db
Allowable Noise Power	-149.7dbW	-149.7dbW	-149.7dbW
Effective Receiver Noise <sup>(1)</sup>	5500°K <sup>(1)</sup>	870°K <sup>(1)</sup>	200°K <sup>(1)</sup>
System Noise <sup>(2), (3)</sup>	212°K <sup>(2)</sup>	212°K <sup>(2)</sup>	212°K <sup>(2)</sup>
Temperature	85°K <sup>(3)</sup>	85°K <sup>(3)</sup>	85°K <sup>(3)</sup>
Noise Bandwidth	13.5 kcs	67.5 kcs	15.5 kcs
Maximum Transmission Rate	1.35 × 10 <sup>4</sup> bits/sec	6.75 × 10 <sup>4</sup> bits/sec	1.55 × 10 <sup>5</sup> bits/sec

NOTES: (1) Best available (see ref.2).

(2) Grimm, H.H., ref. 9.

(3) Blake, L.V., ref.27.

(4) Assumes receiver first stage mounted near antenna input to minimize receiver line loss.

the absence of illumination. This current is called a dark current and is a source of noise. The resulting noise equivalent power (NEP) in a one cycle bandwidth for some typical photo emissive type tubes in the red visible region varies between  $2 \times 10^{-12}$  watt-sec<sup>-1/2</sup> and  $10^{-17}$  watts-sec<sup>-1/2</sup> (see refs. 10, 13), where the NEP is equal to the input signal which produces the same output voltage as is present in one cycle bandwidth due to noise alone.

For carrier signals of wavelength greater than about  $1.1 \mu$ , where photo emissive devices are no longer operative, the p-n or p-I-n junction devices used in the photovoltaic mode might be used. They have response times of about 1 microsecond beyond  $\sim 1.0 \mu$ , and nanosecond response times have been reported (see ref. 13), particularly in the visible and near infrared spectrum. Their disadvantages are their capacity, which restricts the bandwidth over which they can be operated and their sensitive area which must be kept small to keep the capacitance small and response times fast. The most important source of noise in a photovoltaic detector is shot noise caused by the particle nature of the current. There is also thermal noise in the various resistive elements in the diode circuit. However, the shot noise due to the quantum effects, namely the quantum nature of electric charge and photons, can be made to predominate over the thermal noise by cooling the detector.

In the case of photo-emissive type detectors, it is possible to achieve a condition where shot noise is dominant by using heterodyne operation (refs. 5, 13). It has been shown (ref. 5) that both the shot noise power and signal power increase in the same proportion as the local oscillator power making it possible for the shot noise power to overshadow the dark current noise without degrading the signal-to-noise ratio.

On the other hand, heterodyne operation has some disadvantages. First, there is the requirement for close alignment of the received signal beam with the local oscillator beam, because constructive interference between the two beams can occur only if they are aligned within an angle  $\Delta\theta \leq \lambda/d$ , where  $\lambda$  is the wavelength of the received beam and  $d$  is the diameter of the collecting optics (ref. 13). Other disadvantages include the problems of coping with the local oscillator instabilities and relatively large doppler shifts due to the relative motion of transmitter and receiver. For instance the one-way Doppler shift is

$$\Delta f_{\text{Doppler}} \approx \frac{\dot{r}}{\lambda} \quad (11)$$



where  $\dot{r}$  is the relative speed of the source and observer, or, in terms of a ground tracking station terminology,  $\dot{r}$  is the range rate. In the case of a 200 day trajectory to Mars, launched December 26, 1971, after twelve hours out, the  $\dot{r}$  varies from about 3 km/sec (~10,000 fps) to about 16.4 km/sec (~54,000 fps). From this, the resulting Doppler shift is tabulated below for two of the more promising laser sources.

Type of laser source	Wavelength $\lambda$ , in microns	Doppler, cps	
		12 hrs. after injection	200 days after injection
Argon II	0.5	$6 \times 10^9$	$32.8 \times 10^9$
Co <sub>2</sub>	10.6	$2.8 \times 10^8$	$15.5 \times 10^8$

In addition to the shot noise, dark current noise and thermal noise, there is also the background noise (i.e., all other noise entering the detector with the signal including the noise due to signal fluctuations).

The problems of optical background noise have been discussed, documented, and summarized in ref. 10 which in turn utilizes a large number of references. The optical background noise includes: "cosmic" background (see Figure 4), solar radiation background (see Figure 5), lunar and planetary radiation (see Figure 6), and (in the case of a spacecraft "looking" at the earth) reflected solar and total earth radiation (see Figure 7). This latter figure does not include the fine spectra which could be an important factor in the selection of the optimum frequency for a ground beacon for acquisition and tracking of the earth terminal by the spacecraft.

Shown in Figures 8 and 9 are the atmosphere's transmission at sea level for various elevation angles (or varying optical air masses) over the wavelength regions  $0.3\mu$  to  $1.3\mu$  and  $1.2\mu$  to  $5.0\mu$ , respectively.

Faraday rotation. An electromagnetic wave propagating through an ionized medium in the presence of a magnetic field undergoes a rotation of its plane of polarization. This is called Faraday rotation. In the propagation path between an earth terminal and a space vehicle, the ionized medium is the earth's ionosphere and the magnetic field is that of the earth. Because of the inverse-square relationship between frequency and Faraday rotation, the rotation could be a fraction of a radian at L-Band (ref. 22), and about 1 arc minute at light frequencies (ref. 18).

In recent years, the sun's magnetic field has been inferred from the polarization of sun spots. The polarization does not change with slanting look angles (ref. 18). In addition, measured polarization in the light from Crab Nebula and other nebulae tends to confirm physical theory which says that the atmosphere can have no more than small effects on the plane and degree of polarization of light. According to theory (ref. 22, p. 605), the one-way rotation of the plane of polarization may be written as

$$\Omega(\text{rad}) = \frac{2.35 \times 10^4}{f^2} \int_{h_1}^{h_2} NH \cos \theta \sec \chi \, dh \quad (12)$$

f = frequency of electromagnetic radiation, cps

N = number of electrons per cubic centimeter

H = strength of geomagnetic field, emu (gauss)

$\theta$  = angle between the direction of propagation and magnetic field

$\chi$  = angle between direction of propagation and zenith at point where electromagnetic ray passes through the ionosphere

dh = element of height (cm) along line of sight between transmitter and receiver antennae.

The factors N, H, and  $\theta$  are included under the integral sign because of their altitude dependence.

After a careful consideration of the foregoing factors and information capacity, it may be concluded (See also ref. 13) that:

1. The choice of a laser source in the atmosphere windows of the I-R (infrared) spectrum, namely at  $\sim 3.5 \mu$  and  $\sim 10.6 \mu$ , seem highly desirable particularly if suitable detectors can be found, because the information capacity increases with wavelength (See Figure 9, ref. 13).

2. Existing lasers limit practical consideration to wavelengths  $\leq 10.6 \mu$ , the wavelength of the CO<sub>2</sub> laser.

3. Antenna sizes and weights favor higher frequency operation to the point where the beam becomes so narrow that the problems of pointing and tracking and atmosphere image motion limit the advantages to be gained.

4. Using a ground based terminal instead of a spaceborne relay, favors non-coherent operation because of atmospheric effects and the larger apertures may be used. However, heterodyne operation, if possible, would permit narrow band (IF) filtering which would be valuable for decreasing the background noise.

5. Incoherent analogue modulation techniques do not compete with the more efficient time quantized forms of modulation.

6. PCM and PPM are the most efficient of the time-quantized forms of modulation.

7. For a high background noise environment, in the absence of "signal noise," PCM is superior to PPM.

8. The three main forms of incoherent PCM modulation (namely PCM/AM, PCM/FSK, and PCM/PL) exhibit nearly the same communication system efficiency.

9. PCM/FSK and PCM/PL have the advantage over PCM/AM. Because with a laser of peak power limitation, PCM/AM will be restricted to an operation at one-half the average power transmission of PCM/FSK and PCM/PL.

10. The Faraday effect causes a rotation of the plane of polarization of about 1 arc min for light (ref. 18) which is not believed to be overly detrimental to the PCM/PL mode of operation.

It is interesting to note that a PCM/PL (pulse code, polarization modulation) high data rate (~30 M bps), Argon II laser communication system with about 2 watts to 4 watts CW power output is presently under development for the NASA Manned Spacecraft Center, Houston, Texas (Contract NAS9-4266). The receiver utilizes the noncoherent detection mode. In essence, depending on the polarization, the received signal will be separated by a prism (such as a nichol prism, for example, see ref. 12, p. 499) and passed onto one of two photocathode type detectors for non-coherent detection; i.e., they act essentially as photon counters.

For the ground terminal receiver, it has been proposed (see ref. 21) that 30 meter spherical antenna be built as an optical analogue of the 1000 ft Arecibo radio astronomy antenna. Although it is not probable that such an antenna will be ready for the missions to take place in the 1970's, it is included in the analysis. (Note: It was also included in the analyses of ref. 1).

Shown in Table 4 are some characteristics for several gas lasers. Based on: 1) the transmissivity of the atmosphere; and 2) the "best" gas laser sources

Table 4  
CW Laser Oscillators (See ref. 10)

Active Material	Wavelength ( $\mu$ )	Output Power	Dimensions of Active Material	Comments	References in ref. 10
1. He-Ne	0.6118 0.6328 1.084 1.152	5 mW 50 mW 5 mW 20 mW	6 mm x 1.8 m	Single mode, commercially available	1
2. He-Ne	0.6328	900 mW			
3. He-Ne	0.6328	100 mW	10 mm x 5.5 m	research devices	2
4. Xe	3.5 9.0	0.1 mW 0.5 mW	5 mm x 1.2 m		
5. Ar <sup>+</sup>	0.4579(0.05) 0.4765(0.1) 0.4880(25) 0.4965 (0.1) 0.5107 (0.1) 0.5145 (0.4)	10 W	6 mm x 60 cm	research devices, 0.1 - 0.2% efficiency	4
7. Ar <sup>+</sup>	0.4880	1 W	3 mm x 45 cm	airborne development device	6
8. Co <sub>2</sub>	10.57 (0.75) 10.59 (0.25)	16 W	25 mm x 2.0 m	4.0% efficiency, single mode for each line 15% efficiency	7
		135 W			8
9. Cr <sup>+3</sup>	0.6943	70 mW	2 mm x 2.54 cm	water cooled	9
10. Nd <sup>+3</sup> (CaWO <sub>4</sub> )	1.06	1 W	3 mm x 3.5 cm	methyl alcohol cooling (approximately 300°K)	10
11. Nd <sup>+++</sup> (YAG)	1.06	1.5 W	2.5 mm x 3.0 cm		11
12. Nd <sup>+++</sup> (YAG)	1.06	0.5 W	---	water cooled, commercially available, portable	12
13. Dy <sup>+2</sup> (CaF <sub>2</sub> )	2.36	0.75 W	4.8 mm x 2.54 cm	liquid Neon (27°K) bath	13
14. GaAs	0.84	12 W	0.5 mm x 0.4 cm (diode dimensions)	liquid He (4°K) bath, 23% efficiency	12

(Table 4); some laser system communication capabilities were computed and are tabulated in Table 5 for the case of communications from a spacecraft at Mars distances to an earth based terminal utilizing the non-coherent detection mode.

Table 5  
Mars Vehicle to Earth-Based Station Laser Transmission Analysis

Type	Ionized Argon Gas		CO <sub>2</sub>	
Wavelength	0.5 $\mu$		10.6 $\mu$	
Range, R	1.852 $\times 10^8$ km (10 <sup>8</sup> n. mi.)		1.852 $\times 10^8$ km (10 <sup>8</sup> n. mi.)	
Transmitter Power	4.0W	6.0 dbW	130.0W	21.1 dbW
DSV Antenna Gain <sup>(1)</sup>	12.7 cm	115.3 db	1 m	106.7 db
DSV Transmission Loss <sup>(2)</sup>	$\tau = 0.85$	-0.7 db	$\tau \approx 0.7$	-1.4 db
Spreading Loss	$\frac{1}{4\pi R^2}$	-236.7 $\frac{\text{db}}{\text{ft}^2}$	$\frac{1}{4\pi R^2}$	236.7 $\frac{\text{db}}{\text{m}^2}$
Minimum Atmosphere Loss <sup>(3)</sup>	0.40	-4.0 db	0.40	-4.0 db
Receiver Aperture Area <sup>(4)</sup>	78.5 m <sup>2</sup>	18.9 dbm <sup>2</sup>	78.5 m <sup>2</sup>	29.3 dbm <sup>2</sup>
Receiver Loss <sup>(5)</sup>	0.27	-5.7 db	0.27	-5.7 db
Received Power	-106.4 dbW		-100.6 dbW	
CNR <sup>(6)</sup>	10.0 db		10.0 db	
Detector Quantum Efficiency, $\eta$	0.20	-7.0 db	0.20	-7.0 db
Allowable Noise Power	-123.4 dbW		-117.6 dbW	
hf	3.97 $\times 10^{-19}$	-184.1 $\frac{\text{dbW}}{\text{cps}}$	1.88 $\times 10^{-20}$	-197.3 $\frac{\text{dbW}}{\text{cps}}$
Noise Bandwidth (2 B <sub>0</sub> )	1.15 Mc	60.7 dbcps	93 Mc	79.7 dbcps
Maximum Transmission Rate	1.15 $\times 10^6$ bits/sec		9.3 $\times 10^7$ bits/sec	

NOTES:

- (1) For diffraction limited beamwidth of 1 arc second at  $\lambda = 0.5\mu$  and 2.67 arc sec. at  $\lambda = 10.6\mu$
- (2) Beam deflector  $\tau_1 = 0.85$ , Modulator  $\tau_2 = 0.85$ .
- (3) Rain loss 30db, Fog and Snow loss 80 db. (Laser Letter July 1964, p. 3; See Ref. 1 also.)
- (4) Assuming 30 meter spherical antenna, effective diameter is 10 meters.
- (5) 10 A° Filter,  $\tau_1 = 0.35$ ; Antenna,  $\tau_2 = 0.90$ ; Beam Deflector,  $\tau_3 = 0.85$ .
- (6)  $\text{CNR} = \eta P_s / 2hf B_0$ , Quantum Noise Limited.  $\text{CNR} = 10$  db for  $P_{e.s.} \approx 2.3 \times 10^{-5}$
- (7) Rain Margins Not Included.

In these computations, it was assumed that the minimum atmosphere loss is 0.4 or -4 db (See ref. 1) which seems to be a reasonable value when one notes the differences reported (ref. 13, Fig. 11 which is repeated here as Fig. 10) in the bandwidth capability for free space transmission versus ground based terminal in daytime and nighttime operation. The two lasers used were the 4W Argon II laser and the 130W CO<sub>2</sub> laser given in Table 4. The HeXe laser at 3.5 μ wavelength, which is at one of the atmosphere windows, was not treated because of its low power output and its relatively low efficiency. Table 6 lists some semiconductor materials which might be considered for the detector in a 10.6 μ Laser system. Table 6 also gives their characteristic cut-off wavelength and maximum operating temperature. Their response times are not known, because there apparently had been no need to measure it heretofore. Table 7 gives some Baker Nunn sites showing percent of time lost due to cloud cover. More will be said about this in the next section. Table 8 summarizes the results of Tables 1 to 5.

Table 6  
 Characteristics of semiconductor materials  
 for 10.6 μ detector (see ref. 10)

Detector Material	$\lambda_{1/2}, \mu^{(a)}$	$T_{max}, ^\circ K^{(b)}$
Ge: Au	~ 9	70
Ge: Hg	14	40
Ge: Cd	22	25
Ge: Cu	28	18
Hg <sub>1-x</sub> Cd <sub>x</sub> Te	12	77

(a) Wavelength at which detectivity decreases to 1/2 its peak value.

(b) Temperature at which detectivity decreases to 1/√2 of its maximum value.

### Mission Analysis

Since the communication downlink at Mars distances is being considered in this report, let us now examine a Mars mission, at least in a preliminary fashion. Consider the case where a manned or unmanned spacecraft is on its way to Mars and is ~1.8 × 10<sup>8</sup> km away from the earth (See Fig. 11). It is being tracked by the earth tracking network, and it is communicating with an earth ground station via a narrow laser beam. Ground stations strategically located to alleviate the cloud cover problem may be feasible (See ref. 15). For example, for a Baker-Nunn site located in New Mexico (253° 27'E, 32° 25'N) the observation time lost due to

Table 7  
Baker Nunn sites showing percent of time lost due to clouds (see Ref. 10)

Station Coordinates (Long.)	Lat.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
New Mexico 253° 27'(E)	+32° 25'	36	25	27	15	5	12	49	32	48	19	26	22
	1963	34	27	21	29	30	13	48	57	28	16	27	17
South Africa 028° 15'(E)	-25° 58'	42	29	31	26	4	6	1	5	8	29	53	37
	1963	50	23	27	28	15	21	20	2	7	30	55	49
Australia 136° 46'(E)	-31° 06'	32	21	21	9	34	12	22	22	27	38	15	18
	1963	16	22	22	22	51	35	47	21	18	17	18	22
Spain 353° 48'(E)	+36° 28'	52	32	81	43	33	15	21	15	34	47	40	43
	1963	72	49	39	39	35	32	4	19	22	19	60	45
Peru 288° 30'(E)	-16° 28'	81	71	56	38	10	2	5	6	35	22	42	55
	1963	88	95	63	56	23	2	6	15	21	26	13	52
Iran 052° 31'(E)	+29° 38'	28	44	33	54	10	1	18	16	03	nil	14	38
	1963	17	40	30	47	30	7	15	23	1	12	35	22
Curacao 291° 10'(E)	+12° 05'	52	43	56	53	74	66	46	38	60	39	45	50
	1963	70	59	51	68	70	40	64	55	63	50	65	62
Florida 279° 53'(E)	+27° 01'	55	40	62	55	36	61	41	55	52	45	55	43
	1963	57	58	47	28	50	42	44	29	49	32	49	43
Argentina 294° 54'(E)	-31° 57'	53	52	38	72	51	27	39	32	22	29	35	43
	1963	*	35	40	27	39	33	36	32	57	46	46	30
Hawaii 203° 45'(E)	+20° 43'	23	41	56	39	33	5	38	18	20	16	26	33
	1963	59	17	70	78	52	28	29	23	39	39	32	30

\*No photography attempted for a 3 week period when the mirror was removed for realuminizing.

Table 8  
Comparison of Some Plausible Microwave, Millimeter Wave, and  
Optical Communication Systems at Mars Distances

$R = 1.852 \times 10^8$ km ( $10^8$ n. mi), $C/N = 10.0$ db						
	Radio				Optical	
	S-Band	16 Gc	34 Gc	94 Gc	Argon II	CO <sub>2</sub>
f	2.3 Mc	16 Gc	34 Gc	94 Gc		
$\lambda$	13.05 cm (0.428 ft)	1.875 cm (0.06152 ft)	8.82 mm (0.02893 ft)	3.19 mm (0.01048 ft)	$0.5\mu$	$10.6\mu$
$P_T$	100 W	200 W	200 W	200 W	4 W	130 W
$d_T$	4.88 m (16 ft)	4.58 m (15 ft)	4.58 m (15 ft)	4.58 m (15 ft)	12.7 cm	1 m
$G_T$	38.8 db	55.0 db	61.7 db	70.0 db	115.3 db	106.7 db
$\frac{1}{L_T}$	~-1.0 db	~-1.5 db	~-3.0 db	~-4.0 db		
$\frac{1}{L_R}$		~-0.5 db	~-1.0 db	~-1.5 db		
$d_R$	64 M (210 ft)	9.16 m (30 ft)	9.16 m (30 ft)	4.58 m (15 ft)	10 meter, eff.	10 meter, eff.
$G_R$	60.0 db	61.2 db	67.8 db	70.0 db		
$T^*$	50°K	484°K	384°K	497°K		
$\tau_R$					0.27	0.27
$\tau_T$					0.85	0.7
$\tau_a$					0.4	0.4
$\eta$					0.2	0.2
$B_{max}^{**}$ bps	$2.8 \times 10^5$	$5.3 \times 10^4$	$1.7 \times 10^5$	$1.6 \times 10^5$	$1.2 \times 10^6$	$9.3 \times 10^7$

NOTES:

\*Effective system noise temperature and includes receiver noise, sky noise and atmospheric attenuation.

\*\*No margins included.



cloud cover varies between 5% and 49% depending on the time of year (see Table 13-2, ref. 10, given here as Table 7). The average time lost is approximately 25%. If one were to strategically locate a number of such sites, then the probability that at least one of these sites will not have cloud cover is

$$P_{\text{no cloud}} = 1 - (P_{\text{cloud}})^n \quad (13)$$

where

$n$  = no. of sites

$P_{\text{cloud}}$  = probability of cloud cover at a site

$P_{\text{no cloud}}$  = probability of no cloud cover at at least one of the sites.

Hence if  $n = 4$  and  $P_{\text{cloud}} = 0.25$  or 25%, then  $P_{\text{no cloud}} = 0.996$ .

It should be noted that in equation 13 it is assumed that the probabilities of cloud cover at the various sites chosen are not correlated. Hence, in this sense the results obtained with equation 13 may be optimistic. Furthermore, the results are pertinent only to occultations due to cloud cover. They do not consider occultations due to the earth's rotation. For example, in the case shown in Figure 11, where an occultation is about to occur due to the earth's rotation, the spacecraft must have the capability of switching over from one station to another whenever an occultation occurs whether it be due to cloud cover or planetary rotation, etc. The problem of acquisition and tracking associated with use of very narrow laser beams has been studied to a limited extent (refs. 16, 17), and the results of these studies will be utilized in a further analysis of this problem by Hughes Aircraft Company on Contract NAS5-9637 (see refs. 2, 10).

One possible acquisition and tracking mode which bears consideration is the use of the technique used by Perkin-Elmer (ref. 18) on Stratoscope II. Experience with the balloon-borne Stratoscope II astronomical telescope, which utilizes stellar guidance techniques, indicates that the 3 ton gimballed structure was stabilized at 1 or 2 arc seconds rms while its optical line of sight is directable by transfer lens action towards distant stars with pointing errors well within the 0.15 arc-second diffraction limit of the instrument. Measurements indicate that line of sight errors in the order of 1/50 arc second or better may be expected with 9th magnitude or brighter stars for the 36 inch aperture instrument with an optical efficiency of about 30% from the aperture to the detectors (ref. 18). Consider the case wherein an earth laser beacon is utilized on the ground and the Stratoscope II technique is used to acquire and "lock onto" (i.e. track) the ground

beacon. Because of the propagation time delay (it takes a photon about 10 min. to traverse  $1.8 \times 10^8$  km in space) and the velocity aberration effects, the point ahead angle could be 300 times larger than the beam spread, assuming a very narrow laser beam of No. 1 arc sec for the downlink. Hence, the point ahead angle in this case must be controlled to within one part in 300 (See Appendix B for details). The vehicle may utilize the same telescope as both receiver and transmitter antenna and would acquire and track a light beacon on the earth. A course acquisition of the earth, which could appear as bright as about a -4th magnitude star at 1 A.U. (ref. 18), might be performed by the astronaut and the acquisition and tracking system might then perform the more vernier pointing by "locking onto" the earth beacon which should be operating at a different laser frequency from the down-link. However, the earth is not always this bright, in which case direct detection of the earth beacon without resort to earth shine detection appears to be a requirement.

The "magnitude" of a star is its apparent brightness. The ancient Greeks devised the system still in use today, whereby the dimmest of stars ordinarily visible to the naked eye is +6, ranging upward to +1, 0, and -1 for the very bright stars, -12 for the full moon as seen from the earth, and -27 for the sun. Each successive step on the scale represents a 2.5 multiplication of brightness.

Letting the vehicle share the same telescope for transmitter and receiver antennae has the advantages of smaller size and weight. However, it will be necessary to operate the earth beacon at a different laser frequency than the vehicle transmitting laser so that the vehicle can transmit and receive simultaneously without interference from scattered light or other detrimental effects.

Assume that the ground beacon uses an Argon II, CW, gas laser at  $0.5\mu$  wavelength and the vehicle uses the  $\text{CO}_2$ , CW, gas laser for the down-link. The problem is, what is the amount of transmitted power required of the ground beacon? We will now address ourselves to this problem at least in a preliminary way. The position of the space vehicle may be determined to within a few hundred km based on Mariner IV success (see also ref. 19, 26). We will use an error of 400 km. The beamwidth of the ground beacon must be large enough to insure that the vehicle lies within the beamwidth, in which case

$$\theta_{\text{Beam Gnd Beacon}} = 2(3) \sqrt{\left(\frac{\sigma_{\text{S/C, pos}}}{R}\right)^2 + (\sigma_{\text{Beam point error}})^2} \quad (14)$$

where

$\sigma_{S/C, pos}$  = standard deviation of spacecraft position error

$\sigma_{\text{Beam point error}}$  = standard deviation of pointing the earth beacon

R = slant range from ground beacon to spacecraft

The factor of 2 is used because the errors can be plus or minus, while the factor of 3 is used because the errors are one sigma and to ensure a high probability (99.7%) that the spacecraft is in the beamwidth. Using  $R = 1.852 \times 10^8$  km  $\sigma_{S/C, pos} \cong 400$  km, and  $\sigma$  (beam point error) = 2 arc seconds, which is within the capabilities of a high quality ground telescope pointing system, then  $\theta_T = \theta$  (beam, gnd. beacon) = 12 arc sec.  $\doteq 58 \mu$  rad. We will use  $\theta_T = 20$  arc sec to be conservative. It is assumed here that the atmosphere may be considered as part of the ground beacon "optics" so that  $\theta_T$  is the width of the beam after it leaves the atmosphere. It is not known at this time just how much of the beam divergence is due to the atmosphere.

Figure 16 gives a model block diagram which illustrates the operation of an optical, direct detection receiver (i.e. a non-coherent detection receiver). For such a receiver, it can be shown (ref. 10) that the signal-to-noise ratio at the detector output is given by

$$\frac{S}{N} = \frac{\eta P_s^2}{2B_0 hf (P_s + P_b)} \quad (15)$$

where it is assumed that: (1) the photodetector is a photomultiplier with a gain of the order of  $10^{-6}$  so that the shot noise and background noise are much larger than the thermal noise; (2) the detector is cooled so that its dark current may be neglected; and (3) the shot noise caused by the background is much larger than the background noise itself, which is usually the case (i.e. background shot noise  $\gg$  direct background noise). When looking at an earth beacon, which is the case being considered here, then the received background power  $P_b$  due to the earth shine may be expected to be larger than the received signal power in which case the signal-to-noise ratio at the detector output is

$$\frac{S}{N} = \frac{\eta P_s^2}{2B_0 hf P_b} \text{ for background limited operation} \quad (16)$$

where  $B_0$  is the effective bandwidth of the electrical filter following the detector,  $h$  is Planck's constant ( $h = 6.624 \times 10^{-34}$  watt-sec<sup>2</sup>),  $f$  is the laser transmitter frequency,  $\eta$  is the quantum efficiency of the detector (i.e.  $\eta$  is the number of photo-electrons emitted by the photo-cathode per incident photon),  $P_b$  is the received background noise power which will be discussed subsequently, and  $P_s$  is the received signal power. We will use a value of 20 cps for  $B_0$  (Mariner IV used a bandwidth of 1 bps for commands). It will be noted that when  $P_s$  is greater than the background power,  $P_b$ , then

$$\frac{S}{N} = \frac{\eta P_s}{2B_0 hf} \quad (17)$$

for signal noise limited operation, which is the expression used earlier in computing the maximum information bandwidth in bits per second using a laser on the down-link from the spacecraft to the earth.

Let us now direct our attention to the uplink using an Argon II laser as the ground beacon on the earth. The signal power received by the photodetector on the spacecraft is

$$P_s = \frac{P_T A_R \tau_a \tau_T \tau_R}{\frac{\pi}{4} R^2 \theta_T^2} \quad (18)$$

where

$P_T$  = power transmitted by the laser ground beacon, chosen to be an Argon II laser for this example.

$A_R$  = effective receiver aperture area

$\tau_a$  = atmosphere transmissivity

$\tau_T$  = transmitter transmissivity

$\tau_R$  = receiver transmissivity

$R$  = distance between transmitter and receiver

$\theta_T$  = whole beamwidth of the transmitted beam at the half power points

Assuming  $\tau_a = 0.4$ ;  $\tau_R = 0.27$  (filter  $\tau_1 = 0.35$ , antenna  $\tau_2 = 0.90$ ; beam deflector  $\tau_3 = 0.85$ );  $\tau_T = 0.7$  (beam deflector  $\tau_1 = 0.85$ , modulator  $\tau_2 = 0.85$ ) (see also ref. 1);  $A_R = 0.785 \text{ m}^2$ ;  $R = 1.852 \times 10^8 \text{ km}$ ;  $\theta_T = 10^{-4} \text{ rad}$  as discussed above; then for the uplink

$$P_s = 1.74 \times 10^{-16} P_T \quad (19)$$

As will subsequently be seen for the case of the uplink, where the earth shine is a source for relatively large background noise, the  $P_b > P_s$  so that background limited operation results and equation (16) should be used in computing the received signal-to-noise ratio.

The background noise power received by the photodetector may be computed as follows. The radiant emittance of a Lambertian radiating source (the earth in this case) in watts per unit solid angle is given by

$$J = \frac{A'_{\text{earth}}}{\pi} \int_{\lambda_1}^{\lambda_2} W_\lambda d\lambda \quad (20)$$

where  $A'_{\text{earth}}$  is the area of the earth within the field of view of the spacecraft receiver optics, and where  $W_\lambda$  is the radiant emittance of the earth in watts per unit area per unit wavelength. Typical values of  $W_\lambda$  for the earth over the spectrum of interest are shown in Figure 7. The  $\lambda_2 - \lambda_1$  is the bandwidth of the optical filter in the receiver, which for the uplink case being considered is on the spacecraft. Practical values for  $\Delta\lambda = \lambda_2 - \lambda_1$  are from 1 to 10  $\text{\AA}$ , or  $10^{-4}$  to  $10^{-3} \mu$ . Hence the background noise power,  $P_b$ , received at the photodetector is given by

$$P_b = J \tau_R d\Omega \quad (21)$$

where  $d\Omega$  is the solid angle subtended by the receiver aperture and is

$$d\Omega = \frac{A_R}{R^2} \quad (22)$$

Substituting into equation (21) for  $J$  and  $d\Omega$  from equations (20) and (22), then

$$P_b = \frac{A'_{\text{earth}} W_{\lambda_1 - \lambda_2} \Delta \lambda A_R \tau_R}{\pi R^2} \quad (23)$$

Let us now examine if the optical bandwidth  $\Delta \lambda = 1 \mu$  is wide enough to handle the change in wavelength due to Doppler shifts,  $\Delta \lambda_{\text{Doppler}}$ , which is given by

$$\frac{\Delta \lambda_{\text{Doppler}}}{\lambda} \doteq \frac{\Delta f_{\text{Doppler}}}{f} \doteq \frac{v}{c} \quad (24)$$

from which it is found that

$$\Delta \lambda_{\text{Doppler}} \doteq 0.5 \times 10^{-5} \mu \text{ to } 2.7 \times 10^{-5} \mu$$

Hence, it appears likely that an optical filter as narrow as  $10^{-4} \mu$  (or  $1 \text{ \AA}$ ) might be practical. In any event, a  $10^{-4} \mu$  optical filter will be assumed for this example. From Figure 7, at  $\lambda = 0.5 \mu$ , the  $W_{\lambda} = 1.3 \times 10^2 \text{ watts/m}^2\text{-}\mu$ . The received background power falling on the photo detector is, for the case considered

$$P_b = 2.56 \times 10^{-26} A'_{\text{earth}}, \text{ watts} \quad (25)$$

when  $A'_{\text{earth}}$  is in units of  $\text{m}^2$ . In the following paragraph, the  $A'_{\text{earth}}$  will be examined from an overall system's point of view.

As pointed out earlier, the optical line of sight of the spacecraft receiver optics is directable towards distant stars to well within the 0.15 arc sec. diffraction limit of a 1 meter aperture instrument with an optical efficiency of about 30% from the aperture to the detectors (ref. 18). Hence, assume that the spacecraft receiver optics is "looking" at the bright earth and its field of view is such that it only "sees" one-hundredth of the earth's surface, i.e.

$$A'_{\text{earth}} = \frac{A_{\text{earth}}}{100} = 1.277 \times 10^{12} \text{ m}^2 \quad (26)$$

Then the angular field of view  $\theta'_R$  of the spacecraft receiver optics is obtained from

$$A'_{\text{earth}} = \frac{\pi}{4} (R \theta'_R)^2 \quad (27)$$

$$\therefore \theta'_R = 6.9 \times 10^{-6} \text{ rad} = 1.42 \text{ arc sec},$$

which is an order of magnitude larger than the capability of directing the receiver telescope to well within 0.15 arc sec. It should be noted that the angular field of view is not the diffraction limited beam angle. The field of view of the spacecraft receiver optics depends on the optical focal length,  $f$ , and the diameter of the field stop,  $d_{\text{field stop}}$ , of the receiver optics, i.e.

$$\theta'_R = \frac{d_{\text{field stop}}}{\text{focal length}} \quad (28)$$

while the diffraction limited beam angle,  $\theta_R$ , is related to the wavelength,  $\lambda$ , of the beam and the diameter,  $d_R$ , of the receiver aperture, i.e.

$$\theta_R = \frac{\lambda}{d_R}. \quad (29)$$

The difference between the diffraction limited beam angle and the field of view is diagrammatically depicted in Fig. 17.

The diameter  $d'_{\text{earth}}$  of the earth's area  $A'_{\text{earth}}$  "seen" by the receiver optics, i.e. within the field of view, at Mars distances is

$$d'_{\text{earth}} = R \theta'_{\text{earth}} = 1280 \text{ km (690 n.mi.)} \quad (30)$$

which is one-tenth the diameter of the earth. Hence, in order to acquire the ground beacon, the spacecraft could scan a  $10 \times 10$  raster with a total scan time ( $T_{\text{scan}}$ ) of 100 seconds (1-2/3 min), which corresponds to a dwell time of 1 sec per "field of view."

During the scan mode the probability of detecting the ground beacon (assuming that it is in the search field), is a function of both the signal-to-noise ratio and the  $\log_{10} (T_{\text{fa}} / t_d)$ , see Figure 13-4 of reference 28, where

$$T_{fa} = \frac{T_{scan}}{\eta_{fa}} = \text{mean time between false alarms} \quad (31)$$

$\eta_{fa}$  = number of false alarms for each complete scan of the search field  
(in this case the earth)

$t_d$  = dwell time, i.e. the time that the instantaneous field of view rests on  
each point in the total field

Hence

$$t_d = \frac{T_{scan}}{[A_{earth}/A'_{earth}]} \quad (32)$$

= 1 sec in this case

For  $\eta_{fa} = 10^{-2}$ , i.e. 1 false alarm per 100 complete scans of the search  
field, then

$$\begin{aligned} \log_{10} (T_{fa}/t_d) &= \log_{10} \left( \frac{A_{earth}}{\eta_{fa} A'_{earth}} \right) \\ &= 2 - \log_{10} \eta_{fa} = 4 \end{aligned} \quad (33)$$

For a signal-to-noise power ratio of 10, which corresponds to a peak signal  
voltage to rms noise voltage ratio of 4.5, then from Figure 13-4 of ref. 28, the  
probability of detecting the ground beacon in one scan of the search field ( $P_D$ ) is

$$P_D = 0.6$$

The cumulative probability ( $P_C$ ) of detecting the ground beacon after  $j$  scans  
of the search field is

$$P_C = 1 - (1 - P_D)^j \quad (34)$$



assuming independent results are obtained on each scan (see ref. 14). Hence for  $j = 3$ ,

$$P_C = 0.936$$

That is, the cumulative probability of detecting the ground beacon in 3 complete scans (or 5 min. of scan time at 100 sec per scan) is 93.6% for a  $S/N = 10$ , which will subsequently be used to determine the required beacon transmitter power.

Using

$$A'_{\text{earth}} = \frac{A'_{\text{earth}}}{100} = 1.277 \times 10^{12} \text{ m}^2$$

the received background noise power at the detector from equation (24) is

$$P_b = 3.26 \times 10^{-14} \text{ watts} \quad (35)$$

Solving for the required ground beacon transmitter power,  $P_T$ , as given by equation (16)

$$P_T = 8.55 \sqrt{\frac{S}{N}}, \text{ watts} \quad (36)$$

Hence, for a signal-to-noise ratio of 10,  $P_T \doteq 27$  watts, which is beyond the present state-of-the-art technology for an Argon II laser, but it is not an overwhelming obstacle.

It is interesting to note that for this case of background noise limited operation, increasing the power transmitted by a factor of 2 increases the signal-to-noise ratio by a factor of 4 and would increase the probability of detection,  $P_D$ , discussed earlier from 0.6 to 0.9999 per scan of the search field.

It should be noted that in planning a Mars mission, the launch date and flight time should be scheduled so that the earth-sun-Mars angle at the time of intercept is small enough (preferably  $\simeq 90^\circ$ ) so that the background noise from the sun would be low; i.e., the sun would be far removed from the line of sight between the vehicle and earth. Shown in Figure 12 are: a) typical distances (R) of Mars from the earth at time of intercept and b) earth-sun-Mars angle at time of intercept (ref. 20). Shown in Figure 13 is a possible Mars trajectory wherein the

earth-sun-Mars angle of about  $90^\circ$  so that background noise from the sun would be small as discussed above. The launch date for this trajectory is November, 1966, and an intercept date of September, 1967. Furthermore, a patched conic Mars trajectory (ref. 23) was made to determine the time history of a flight trajectory to Mars during the year 1971. Table 9 gives the time history of this trajectory. The following are the symbol definitions used in Table 9, and Figure 14 pictorially defines these symbols. Figure 15 is a plot of this trajectory. From the viewpoint of solar background noise, this latter trajectory appears to be somewhat better than the former trajectory.

Symbols used in Table 9 and Fig. 14:

- RVS - Angle between the reference vector and the sun, read as Reference, Vehicle, Sun Angle
- RVE - Reference, Vehicle, Earth Angle
- RVT - Reference, Vehicle, Target Angle
- RFT - Radius from the Target body
- RFS - Radius from the Sun
- RFE - Radius from the Earth
- EVSA - Earth, Vehicle, Sun angle

### III. SUMMARY

Given in Table 7 is a summary comparison of some "plausible" microwave, millimeter wave and optical communication systems at Mars distances. It is believed that the systems considered are "plausible" in the sense that the values used for the various parameters are representative of state-of-the-art hardware, either off-the-shelf or now working in the laboratory, except in the cases of: (a) the optical ground receiver antenna (30 meter spherical) which has been proposed (see refs. 1, 21) as an optical analogue of the 1000 ft Aricebo radio astronomy antenna, and (b) the present lack of suitable detectors in the I-R spectrum. From Table 8, and Figure 3a, it may be concluded that for spacecraft to ground communications: (1) in the radio spectrum the S-Band appears to be the better place to operate; and (2) the optical communication systems show considerable promise for supplying high data rates (theoretically up to  $\sim 10^8$  bps) at Mars distances. However, considerable work remains to be done to make the optical systems operational, particularly the development of flight tested hardware, improving the lifetime expectancy of the laser tubes, solving the problem of acquisition and tracking associated with the very narrow laser

Table 9  
Earth-Mars Trajectory Data

INJECTION: JUNE 11, 1971  
MARS INTERCEPT: DEC. 28, 1971

JULIAN DATE: 2441114.2530768  
JULIAN DATE: 2441314.25307083

D	H	M	V, km/sec	$\gamma$ , deg.	$\dot{r}$ , km/sec	$\dot{r}$ , fps	RFS, km	RFE, km	RFT, km	RVE, °	RVS, °	RVT, °	ESVA, °
0	0	19.5	11.43	0	0	0	151,910,000	6,560	89,559,090	90.0°	174.98	57.24	.0000057
0	12	19.5	3.65	83.76	3.6	11,800	152,200,000	188,980	88,893,130	126.4°	174.88	57.39	0.055
1	0	19.5	3.386	86.26	3.3	10,850	152,120,000	339,360	88,272,720	123.56°	174.74	57.55	0.104
5	0	0	3.154	89.26	3.1	10,200	152,830,000	1,440,290	83,508,450	120.62°	172.34	58.68	0.437
10	0	0	3.157	89.47	3.1	10,200	153,880,000	2,802,310	77,769,030	120.51°	167.97	59.94	0.793
20	0	0	3.264	87.10	3.2	10,500	156,470,000	5,565,43	67,092,870	121.59°	158.46	61.88	1.281
30	0	0	3.514	84.51	3.5	11,500	159,630,000	8,472,660	57,583,510	123.29°	148.99	62.94	1.415
40	0	0	4.027	82.59	4.0	13,100	163,270,000	11,685,300	49,313,550	125.398°	139.85	63.04	1.141
50	0	0	4.803	83.47	4.8	15,800	167,260,000	15,456,000	42,301,130	127.38°	131.08	62.14	0.4666
60	0	0	5.751	86.03	5.7	18,700	171,490,000	19,983,000	36,504,510	128.69°	122.72	60.29	0.8095
70	0	0	6.899	88.76	6.8	22,400	175,870,000	25,429,000	31,818,830	129.15°	114.76	57.69	2.394
80	0	0	8.164	85.21	8.1	26,700	180,300,000	31,923,000	28,079,890	128.62°	107.19	54.61	4.429
90	0	0	9.492	80.72	9.3	30,600	184,710,000	39,483,000	25,070,110	127.12°	99.98	51.42	6.873
100	0	0	10.93	76.23	10.6	35,000	189,020,000	48,118,000	22,596,590	124.82°	93.10	48.44	9.707
110	0	0	12.39	71.72	11.8	38,800	193,190,000	57,797,000	20,421,790	121.81°	86.52	45.90	12.899
120	0	0	13.88	67.57	12.8	42,100	197,160,000	68,426,000	18,386,490	118.22°	80.22	43.90	16.42
130	0	0	15.42	63.54	13.6	44,700	200,898,000	79,940,000	16,369,220	114.18°	74.15	42.45	20.24
140	0	0	16.94	59.59	14.5	47,600	204,360,000	92,220,000	14,294,380	109.76°	68.31	41.49	24.33
150	0	0	18.46	55.90	15.3	50,300	207,530,000	105,150,000	12,124,180	105.02°	62.65	40.94	28.66
160	0	0	20.01	52.29	15.9	52,300	210,380,000	118,600,000	9,848,820	100.04°	57.15	40.68	33.20
170	0	0	21.52	48.73	16.0	52,500	212,899,000	132,440,000	7,477,420	94.85°	51.80	40.62	37.93
180	0	0	23.01	45.36	16.4	54,000	215,060,000	146,510,000	5,030,370	89.48°	46.57	40.63	42.80
190	0	0	24.50	45.03	16.3	53,600	216,870,000	160,680,000	2,533,420	83.96°	41.44	40.54	47.81
200	0	0	24.74	39.69	15.7	51,700	218,290,000	174,810,000	12,831	78.31°	36.40	56.06	52.90

beams, gaining a better understanding of the atmospheric effects on laser beams, and development of suitable detectors in the I-R spectrum.

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## VI. APPENDIX

### A. Noise Figure and Effective Noise Temperature

An ideal amplifier or detector is one which is "noiseless," that is it introduces no noise onto the noise already present at the input. A noise factor,  $F$ , (sometimes called a noise figure) is commonly used to describe the noisiness of a network.

The "noisiness" of a particular system or part thereof can be measured by comparing  $S/N$  at output and input. This measure of the noisiness of a system is called the noise figure,  $F$ , of the system and it is defined as

$$F = \frac{S_s/N_s}{S_0/N_0} \quad (\text{A-1})$$

with  $S_0/N_0$  the signal-to-noise ratio in power at the output and  $S_s/N_s$  the signal-to-noise ratio in power at the input (source). An ideal network is thus one whose noise figure is unity (that is, no additional noise introduced in the system). As  $F$  increases, the "noisiness" of the system increases (see also ref. 24).

Noise figures are frequently measured or given in decibels (since  $F$  is a ratio of power ratios, the conversion simply being  $10 \log_{10} F$ ).

The concept of noise figure is particularly useful in the radio wave spectrum such as the microwave and millimeter wave for instance. Radar receivers in the Gc range and using crystal converters frequently have noise figures ranging from 10 to 15 db, i.e.,  $F$  ranging from 10 to 40. Most (or much) of the noise is developed in the system. A decrease of only 3 db in the noise figure of a typical system would reduce the power requirements of the radar transmitter by a factor of two. Hence, the question of decreasing system noise figures is of great importance.

The maximum power available at the output of a system, under matched conditions, is frequently called the available power. Thus, for a source represented by in rms signal voltage  $e_s$  and output resistance  $R_s$  (sometimes referred to as the internal resistance of an equivalent signal source generator, see Figure A1) and under matched conditions (i.e., the load resistance is matched to  $R_s$ ), the signal-to-noise ratio at the source is (see ref. 24, p. 231)

$$\frac{S_s}{N_s} = \frac{\text{available signal power}}{\text{available noise power}} \quad (\text{A-2})$$

If  $G$  is the available power gain of a network and is defined to be the ratio of available output signal power to available input signal power, i.e.,  $S_0 \equiv GS_s$ , then the noise figure can be written as

$$F = \frac{N_0}{GN_s} \quad (\text{A-3})$$

The equation  $F = (N_0/GN_s)$  presents an alternative form for the equation of noise figure (aside from the definition in terms of S/N ratios) and is frequently given as the basic definition of  $F$ . Thus  $F$  may be defined as the ratio of actual noise power available from a network to that which would be available if the network were noiseless (ref. 24, p. 232).

Let us now examine how the noise figure can be related to an effective (or equivalent) system noise temperature,  $T$ .

In the case of microwave and millimeter systems where  $kT \gg hf$ , then the noise power density given by equation 6 becomes

$$\psi \cong kT \quad (\text{A-4})$$

and

$$N = kTB \quad (\text{A-5})$$

with  $B$  being the effective bandwidth of the system.

Consider first a single linear network, and then a system of cascaded networks. Referring to Figure A1, the available noise power at the network output is

$$N_0 = G_1 N_s + N_1 \quad (\text{A-6})$$

where  $N_1$  is the noise power contributed by the network at its output. Thence

$$F_1 \equiv \frac{N_0}{G_1 N_s} = 1 + \frac{N_1}{G_1 N_s} \quad (\text{A-7})$$



or

$$(F_1 - 1) G_1 N_s = N_1 \quad (\text{A-8})$$

But  $N_1$ , the noise contributed by the network, is equivalent to adding a noise  $N_e = N_1/G_1$  at the input, that is  $N_e$  is the equivalent network noise, or in other words,  $N_e$  is  $N_1$  referred to the input terminals of the network. Replacing  $N_1$  by  $G_1 N_e$ ; substituting  $kT_s B$  for  $N_s$  and  $kT_e B$  for  $N_e$ ; and solving for  $T_e$

$$T_e = (F_1 - 1) T_s \quad (\text{A-9})$$

= equivalent network noise temperature referred to the network input terminals.

The temperature  $T_s$ , which appears in this latter equation, is the temperature at the input to the network (see also ref. 22, p. 363). Present measurement standards requires that the noise figure (or noise factor as it is sometimes called) of receivers be measured with respect to an input termination at a reference temperature  $T_0 = 290^\circ\text{K}$ . Thus, we replace  $T_s$  by  $T_0$  to give

$$T_e = (F_1 - 1) T_0 \quad (\text{A-10})$$

so that in terms of the equipment (or effective) noise temperature of the network, the noise factor becomes

$$F_1 = 1 + \frac{T_e}{T_0} \quad (\text{A-11})$$

The concept of noise figure was originally formulated to describe the performance of relatively noisy receivers. The use of the noise figure with its standard temperature  $T_0 = 290^\circ\text{K}$  is not as convenient with low noise devices as is the effective (equivalent) system noise temperature. Although Figure A1 shows only one network, cascaded networks can be treated also. It can be shown (ref. 22, p. 364) that the noise figure  $F_0$  for  $n$  cascaded networks is

$$F_0 = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots + \frac{F_n - 1}{G_1 G_2 \dots G_{n-1}} \quad (\text{A-12})$$

Similarly, the effective noise temperature  $T_e$  of  $n$  networks in cascade is

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \dots + \frac{T_n}{G_1 G_2 \dots G_n} \quad (\text{A-13})$$

or

$$T_e = (F_0 - 1) T_0 \quad (\text{A-14})$$

where  $F_0$  is given by equation A-12.

The noise factor  $F_0$  and the corresponding  $T_e$  may be referred to any point in the passive r-f line system preceding the receiver. However, we have, throughout this report, been referring the noise temperature to the input terminals of the receiving system. Hence, we must also consider the effective noise temperature,  $T_{L,e}$ , due to the r-f line losses between the antenna and the receiver input terminals and the sky noise temperature,  $T_{sky}$ , due to background radiation (galactic noise, planetary noise, solar noise), side lobe noise, back lobe noise, spillover, and atmospheric attenuation. Since the sky noise is attenuated by the lossy r-f line between the antenna and the receiver input terminals, then the effective sky noise temperature  $T_{sky,e}$  referred to the receiver input terminals is

$$T_{sky,e} = T_{sky}/L_R \quad (\text{A-15})$$

where  $L_R$  is the receiver line loss factor. In terms of  $L_R$ , the  $T_{L,e}$  is given by (See ref. 25, p. 124)

$$T_{L,e} = T_L \left( 1 - \frac{1}{L_r} \right) \quad (\text{A-16})$$

where  $T_L$  is the actual line temperature. Therefore, the total effective system noise temperature referred to the input,  $T_i$ , becomes (See also ref. 25, pp. 124-5)

$$T_i = T_0 (F_0 - 1) + T_{sky,e} + T_{L,e} \quad (\text{A-17})$$

Thence, this  $T_1$  is the temperature  $T$  to use in computing the total available noise power ( $N = kTB$ ) referred to the input terminals of the receiver (See Figure A-2).

It should be noted that "the effective noise temperature and the noise figure both describe the same property of the receiver. Controversy has existed over which is better. There seem to be, however, areas of usefulness for both definitions, and it is likely that they will both continue to be applied. The effective noise temperature is preferred for describing low-noise devices, and the noise figure is generally preferred for conventional receivers" (ref. 22, p. 366). Furthermore, it should be noted that the effective noise temperature and noise figure are useful when dealing with systems which operate in the radio frequency spectrum, such as the microwave and millimeter wave systems, where  $kT \gg hf$  and thus the thermal noise predominates over the quantum noise. However, the effective system noise temperature is not useful in the optical spectrum, where  $hf \gg kT$  and thus the quantum noise predominates over the thermal noise. In the optical spectrum the background noise and noise contributed by the detector are handled somewhat differently as demonstrated earlier in the text in the section on Mission Analysis where a laser ground beacon is detected by the spacecraft receiver in the presence of earth and reflected solar radiation.

### B. The Point Ahead Angle

The point ahead angle (or lead angle) is the angle with which the optical beam must be pointed ahead of a target which is moving relative to the sources because of the finite amount of time it takes the signal to reach the target.

As pointed out earlier the point ahead angle must be controlled to within one part in up to about 300 (depending on the beamwidth and propagation time) because of earth and ground station motion relative to the spacecraft and the propagation times involved. More specifically, the earth's orbital speed is  $\sim 30$  km/sec. Since the spacecraft must first detect the ground beacon before pointing its own optical transmitter beam towards the ground, then the two-way propagation time should be used in this case to determine the point ahead or lead angle ( $\theta_{\text{Lead Angle}}$ ). In addition the Bradley effect (sometimes called angle of aberration,  $\alpha$ ) must be considered (see ref. 29, pg. 379).

For simplicity, and as shown in Figure B-1, let  $t_1$  be the time at which a "bundle of photons" are transmitted by the ground beacon;  $t_2$  be the time at which the spacecraft receives this "bundle of photons"; assuming a negligible turn around time so that  $t_2$  may be considered to also be the time at which the spacecraft transmits its "bundle of photons" towards the earth station; and  $t_3$  be the time at which the earth receives the downlink signal (i.e. the latter "bundle of

photons"). Figure B-1 also illustrates the Bradley effect. From Figure B-1, it may be seen that in the spacecraft's reference frame

$$\theta_{\text{Lead Angle}} \doteq \frac{S}{R} - \alpha \quad (\text{B-1})$$

where the negative sign is used before the  $\alpha$  because of the convention adopted here that velocity components are positive when directed along the positive direction of an axis and angles are positive when measured in the counterclockwise direction. In any event, it is clear that one must be careful to give the proper sign to the angle  $\alpha$ . From Figure B-1, and neglecting relativistic effects, the angle  $\alpha$  is

$$\alpha \doteq \frac{V_{\perp}}{c} \quad (\text{B-2})$$

$$V_{\perp} = V_{s/c, e} \cos \gamma \quad (\text{B-3})$$

$V_{s/c, e}$  = speed of spacecraft relative to the earth at time  $t_2$ .

The  $S$  is the distance that the ground station has traveled normal to the line of sight (or the slant range  $R$ ) during the two-way propagation time. Therefore

$$S = (\Omega_e R_e + v_e) 2 t_{\text{prop}} \quad (\text{B-4})$$

where  $\Omega_e \doteq 1/4$  deg/min, angular speed with which the earth rotates on its axis

$R_e = 6378.153$  km, earth's equatorial radius

$v_e \doteq 30$  km/sec, earth's orbital speed

$t_{\text{prop}} = R/c$ , one-way propagation time

$c = 3 \times 10^5$  km/sec, speed of light.

At  $R = 1.852 \times 10^8$  km,  $t_{\text{prop}}$  is 617 sec or about 10 min. and the two-way propagation time is about 20 min. Thus

$$S \doteq 36,300 \text{ km}$$

$$\frac{S}{R} \doteq 196 \mu \text{ rad}$$

$$\alpha \doteq \frac{V_{s/c,e} \cos \gamma}{c}$$

$$\doteq \frac{19 \text{ km/sec}}{3 \times 10^5 \frac{\text{km}}{\text{sec}}} \doteq 63 \mu \text{ rad} \quad (\text{B-5})$$

when the spacecraft is in the vicinity of Mars (see Table 9 for typical values of  $V_{s/c,e}$  and  $\gamma$ ), so that

$$\theta_{\text{Lead angle}} \doteq 133 \mu \text{ rad} \doteq 27.4 \text{ arc sec} .$$

However, the beamwidth of the downlink at the half power points is

$$\theta_T \doteq \frac{\lambda_{\text{co2}}}{d_T} \doteq \frac{10.6 \times 10^{-6} \text{ m}}{1 \text{ m}}$$

$$\doteq 10.6 \mu \text{ rad} \doteq 2.2 \text{ arc sec}$$

for the downlink beamwidth being considered, the point ahead angle is 12.5 times larger than the beamwidth and must be controlled to within one part in 12.5 ( $3\sigma$ ). Since this point ahead angle is relative to the apparent line of sight of the received beam from the ground beacon, it appears likely that it should be possible to control it to within one part in 12.5 or about 2 arc sec, because the point ahead angle will be of the order of 27.4 arc sec.

Since this point ahead angle is relative to the apparent line of sight of the beam received by the spacecraft from the ground beacon, then the "accuracy" with which it must be controlled is relative to this line of sight.

The beamwidth ( $\theta_{\text{Beam down}}$ ) of the spacecraft's Laser beam must be wide enough to assure that the ground station lies within this beamwidth at the time of arrival ( $t_3$ ) of the "bundle of photons" transmitted from the spacecraft. Assuming a normal distribution,

$$\theta_{\text{Beam down}} \doteq 2(3) \sqrt{\sigma_{\text{lead}}^2 + \sigma_{\text{point}}^2 + \sigma_{\text{LOS}}^2} \quad (\text{B-6})$$

where  $\sigma_{\text{lead}}$  = one sigma error in the predicted lead angle

$\sigma_{\text{point}}$  = one sigma error in controlling the point ahead angle relative to the apparent line of sight

$\sigma_{\text{LOS}}$  = one sigma error in the apparent line of sight of the ground beacon beam received by the spacecraft and could be significantly less than 0.15 arc sec (ref. 18).

The factor of 2 is used because the errors can be plus or minus, while the factor of 3 is used because the errors are one sigma and to assure a high probability (99.7%) that the ground station lies in the downlink beam.

Since  $v_e \gg \Omega_e R_e$ , then equation B-1 may be written as

$$\theta_{\text{lead}} = \frac{2}{c} (v_e - v_{\perp}) \quad (\text{B-7})$$

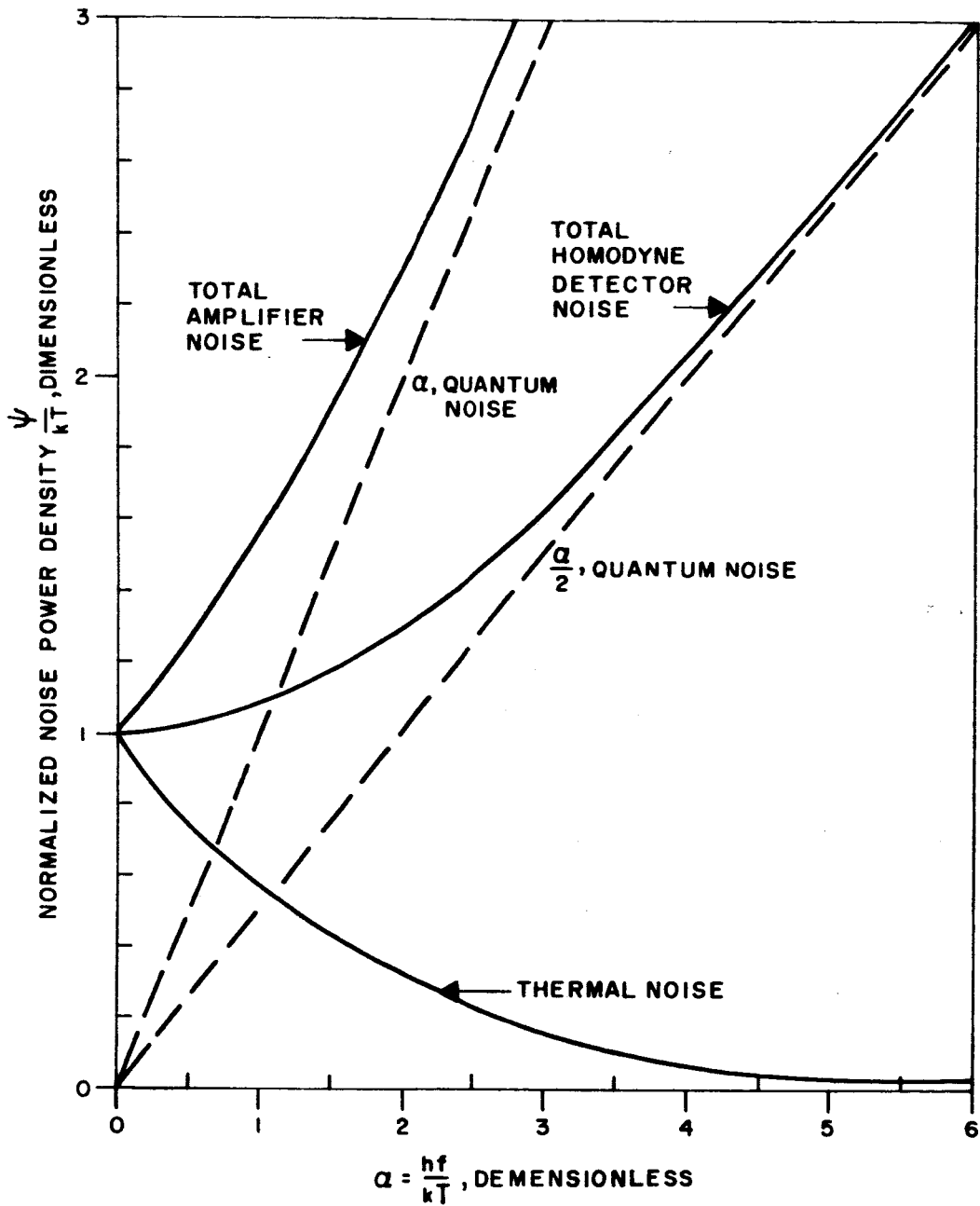
from which the uncertainty in the predicted lead angle is

$$\sigma_{\text{lead}} = \delta \theta_{\text{lead}} \approx \frac{\delta v_e}{c} \quad (\text{B-8})$$

Since the  $\delta v_{\perp}$  may be expected to be of the order of meters per second, then this error as well as  $\sigma_{\text{LOS}}$  may be considered to be negligibly small, and

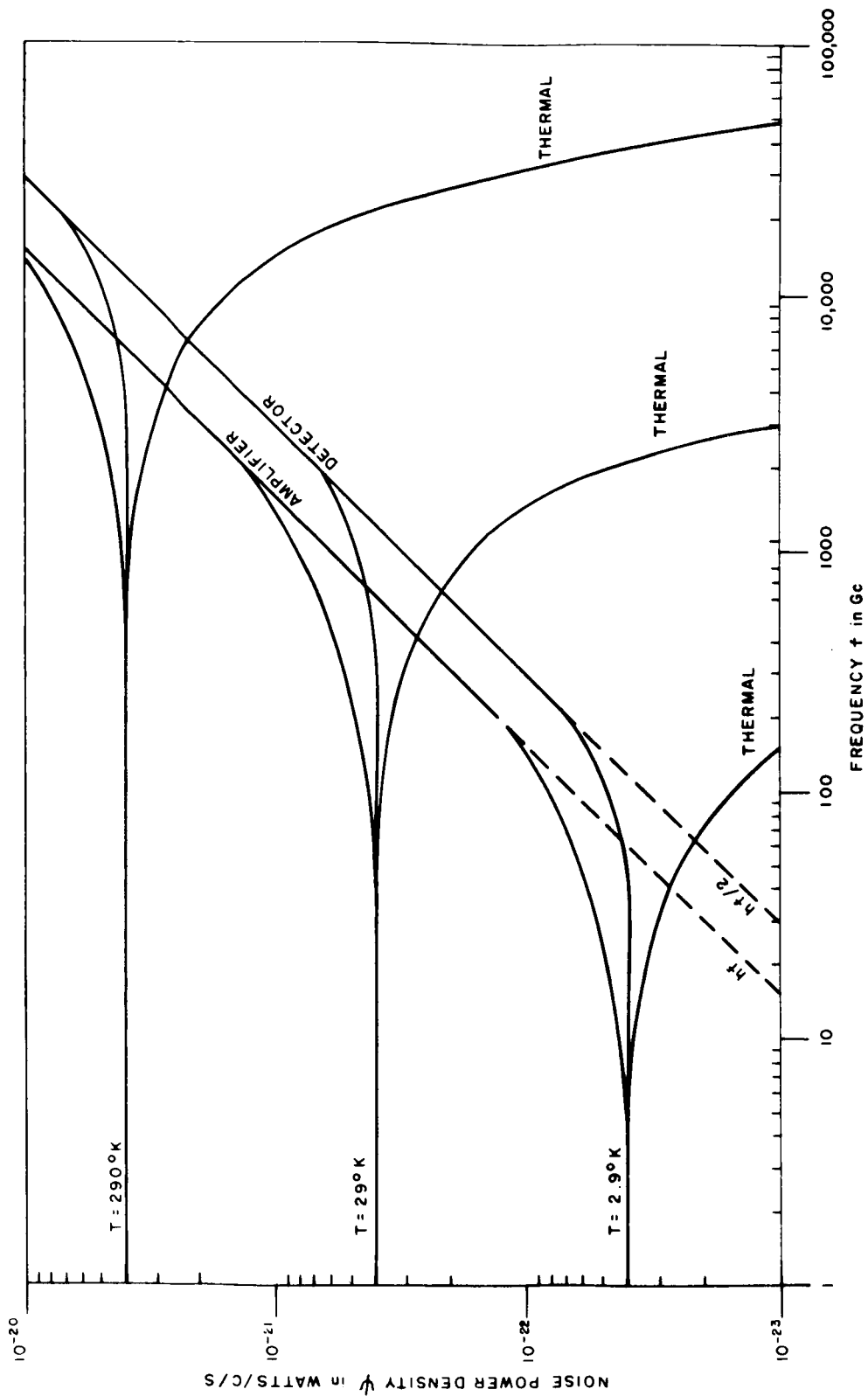
$$\theta_{\text{Beam down}} \cong 2(3\sigma_{\text{point}}) \quad (\text{B-9})$$

Hence, it may be concluded that the point ahead angle must be controlled to within half a beamwidth ( $3\sigma$ ). For the case being considered here, the point ahead angle is 12.5 times larger than the beamwidth of 2.2 arc sec, and hence must be controlled to within one part in 25 ( $3\sigma$ ), i.e. to within  $\pm 1.1$  arc sec. Since this point ahead angle is relative to the apparent line of sight of the beam received by the spacecraft from the ground beacon, it appears likely that it should be possible to control it to within the required 1.1 arc sec similar to what was done with a 3 ton gimbaled telescope on Stratoscope II (see ref. 18).



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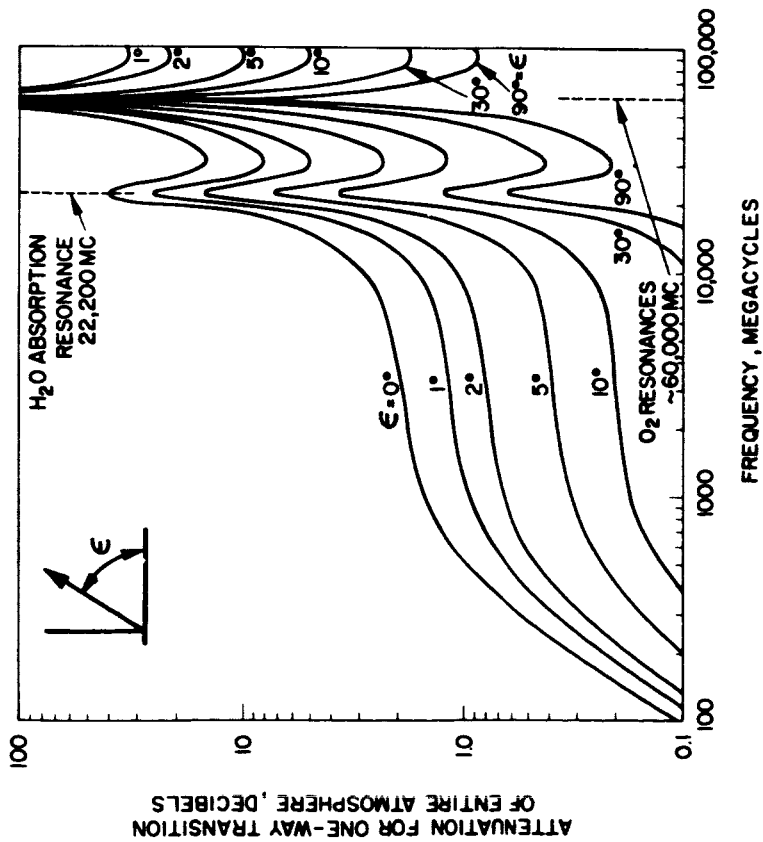
Figure 1-Comparison of Thermal, Quantum and Total Noise Power Density (B. M. Oliver, ref. 5)



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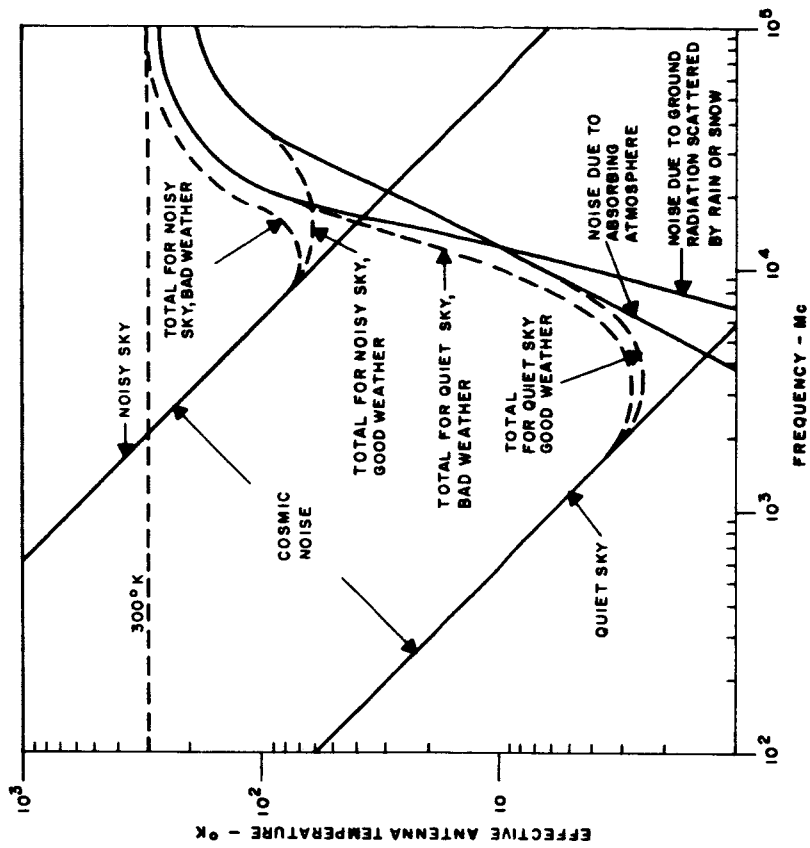
Figure 2—Thermal, Quantum, and Total Noise Power Density as a Function of Frequency at Various Temperatures (B. M. Oliver, ref. 5)





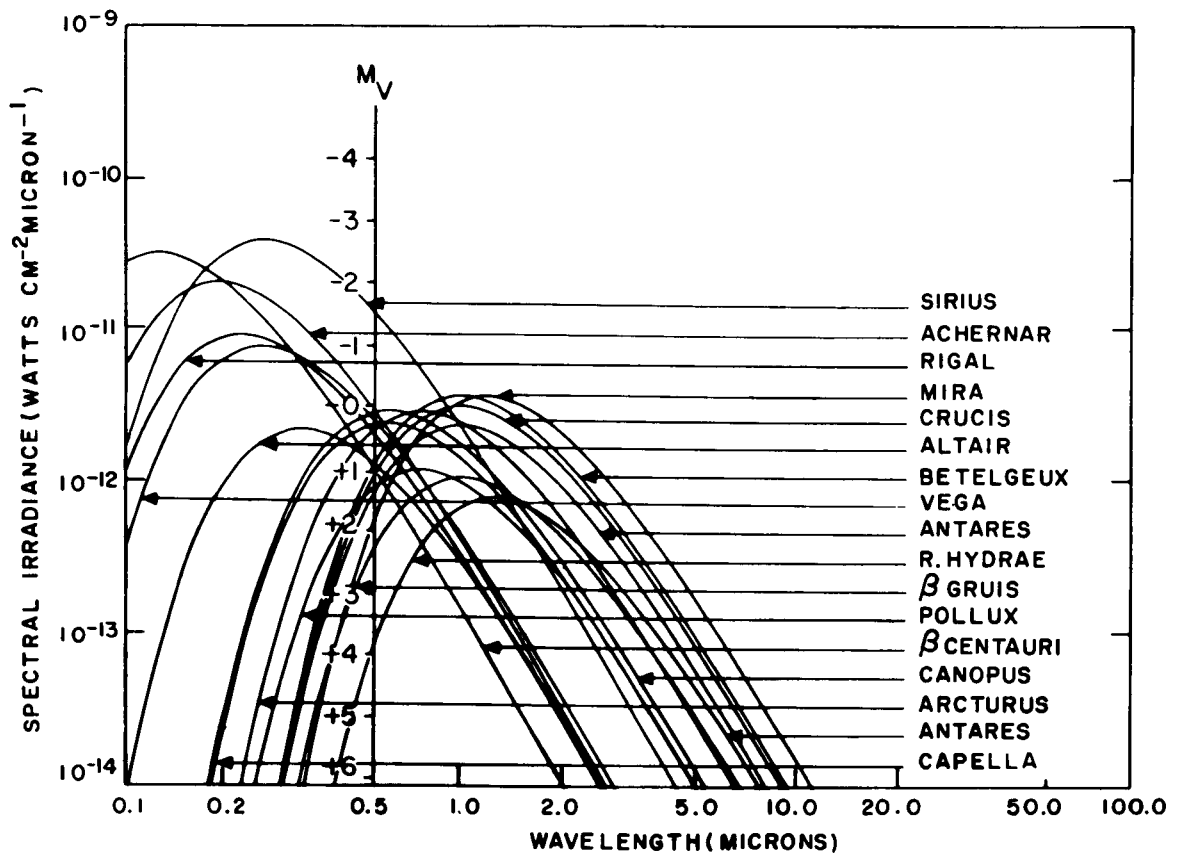
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Figure 3a—One Way Attenuation through the Standard Summer Atmosphere due to Oxygen and Water Vapor (Hughes Aircraft Co., ref. 10 and L. V. Blake, ref. 27)



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Figure 3b—Effective Antenna Temperature due to Galactic Noise and Atmospheric Absorption (H. G. Grimm, ref. 9)



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Figure 4—Spectral Irradiance of Brightest Stars outside the Terrestrial Atmosphere  
 (Hughes Aircraft Co., ref. 10)

THE SOLAR SPECTRUM

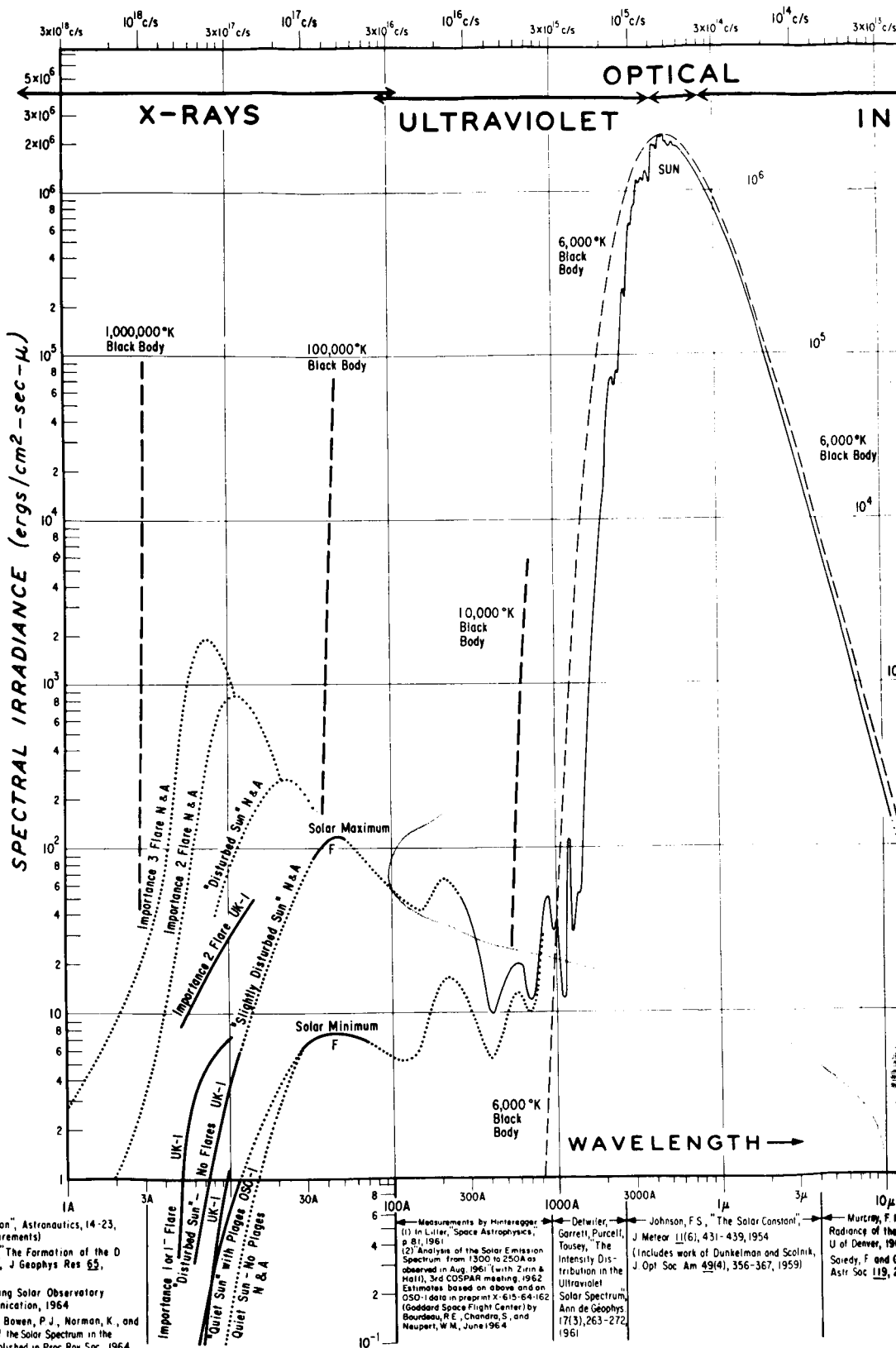
Displayed here is the entire radiation pattern of the sun, from X-rays through visible light to radio waves. The solar energy received at the top of the earth's atmosphere can be read from the vertical scales. Wavelengths to which these energies apply are indicated at the bottom, frequencies at the top.

The units of spectral irradiance read, in unabbreviated form, "ergs per square centimeter per second in a wavelength interval of one micron." Because of the enormous range of energies charted here, it was necessary to fold this logarithmic scale thrice in the infrared and radio regions. The folded sections are accompanied by the appropriate numerical labels.

Along the scale at the top, frequencies in cycles per second (c/s) increase toward the right. One gigacycle per second (Gc/s) is equivalent to  $10^9$  (one billion) cycles per second; therefore, 1,000 gigacycles equals  $10^{12}$  cycles. Farther to the right, one megacycle per second (Mc/s) is  $10^6$  (one million) cycles per second.

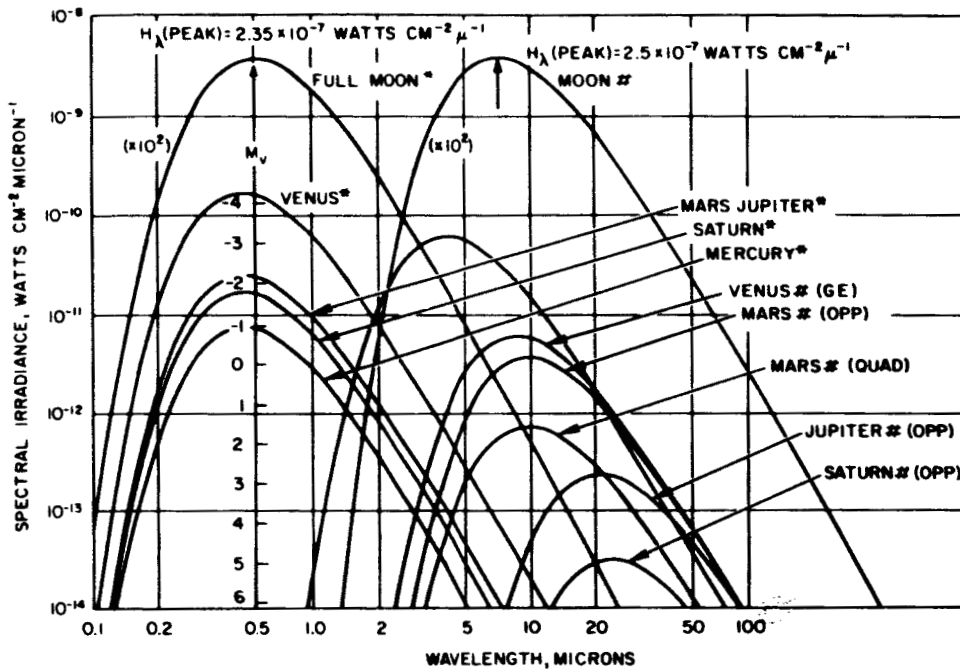
The wavelengths, given along the bottom scale, become longer toward the right; they are indicated successively in angstroms (A), microns ( $\mu$ ), millimeters (mm), centimeters (cm), and meters (m). One micron is 10,000 angstroms, and one millimeter is 1,000 microns.

- F: Friedman, H., "Solar Radiation", *Astronautics*, 14-23, August 1962 (rocket measurements)  
 N & A: Nicollet, M and Aikin, A C., "The Formation of the D Region in the Ionosphere", *J Geophys Res* 63, 1469-1483, 1960  
 OSO-1: Data from the first Orbiting Solar Observatory W A White, private communication, 1964  
 UK-1: Pounds, K A., Willmore, A P., Bowen, P J., Norman, K., and Sanford, P.W., "Measurements of the Solar Spectrum in the Wavelength Band 4-14A". Published in *Proc Roy Soc.*, 1964. Ariel 1 satellite data.



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\* - calculated irradiance from planets, at brightest, due to sun reflectance only.

# - calculated irradiance from planets, due to self-emission only.

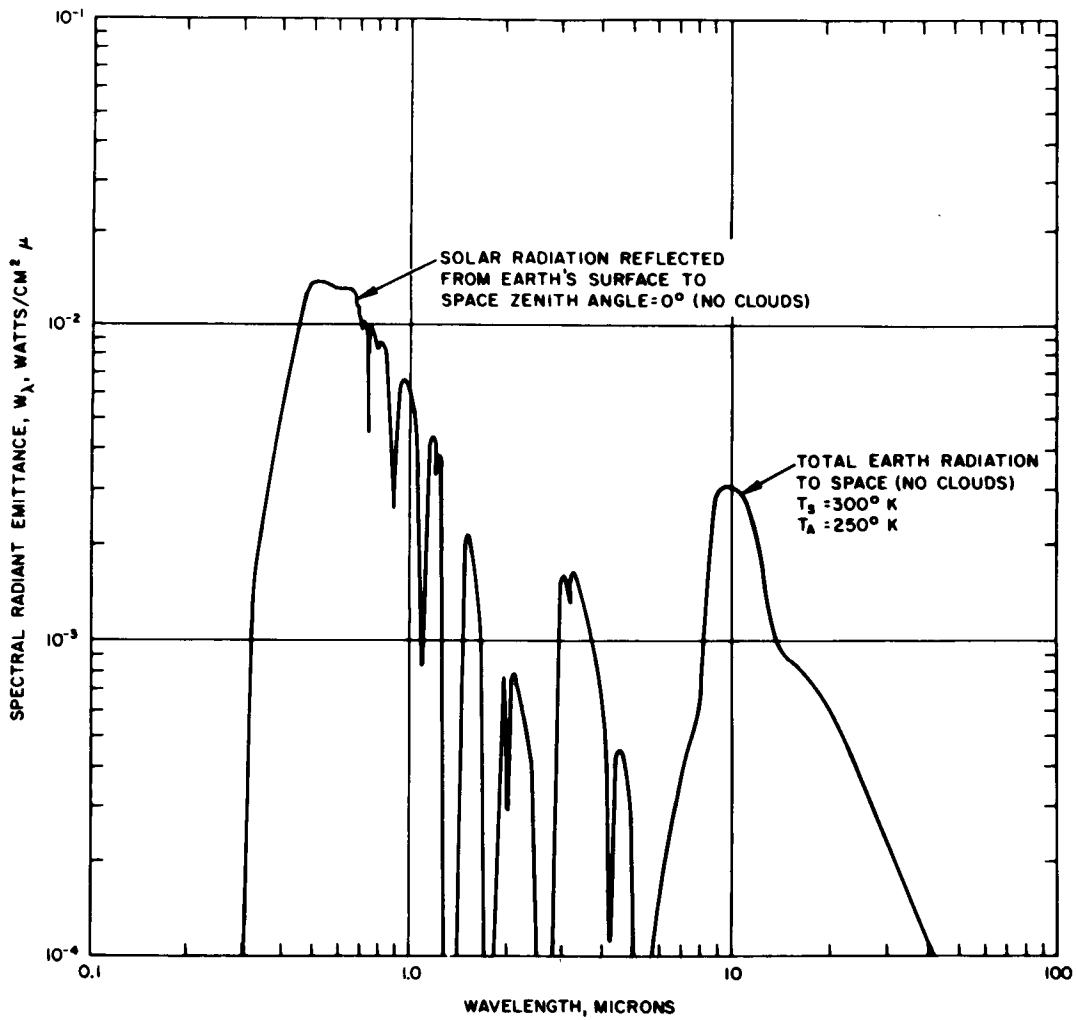
GE - inferior planet at greatest elongation.

OPP - superior planet at opposition.

QUAD - superior planet at quadrature.

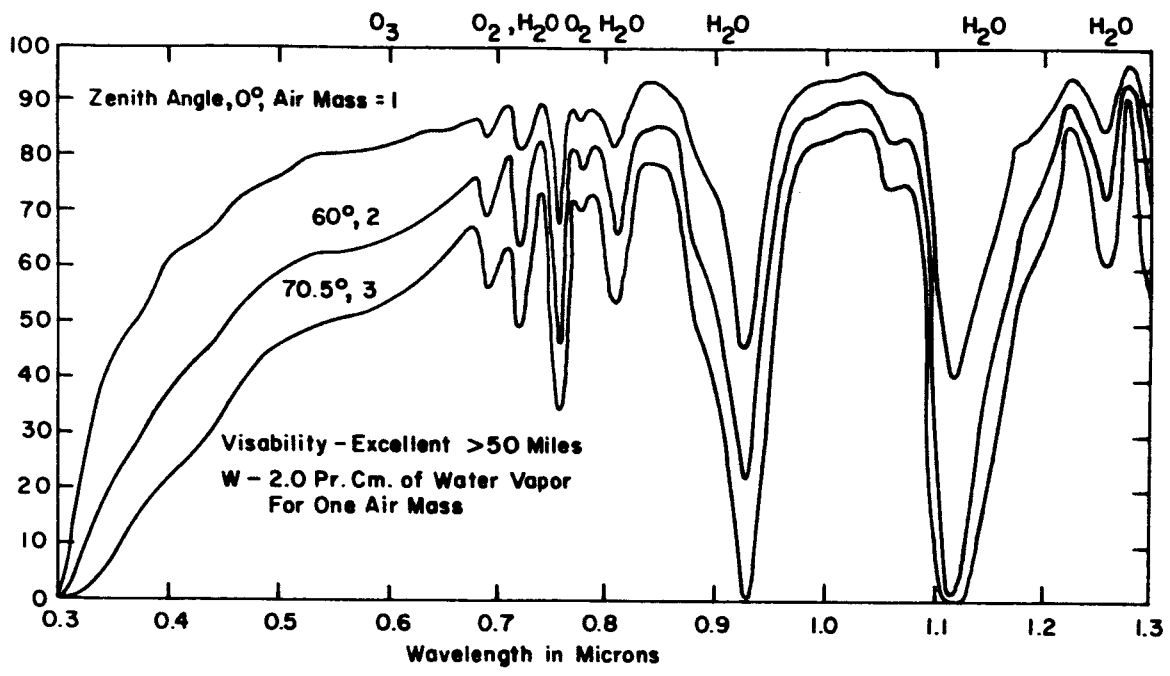
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Figure 6—Calculated Planetary and Lunar Spectral Irradiance outside the Terrestrial Atmosphere (Hughes Aircraft Co., ref. 10)



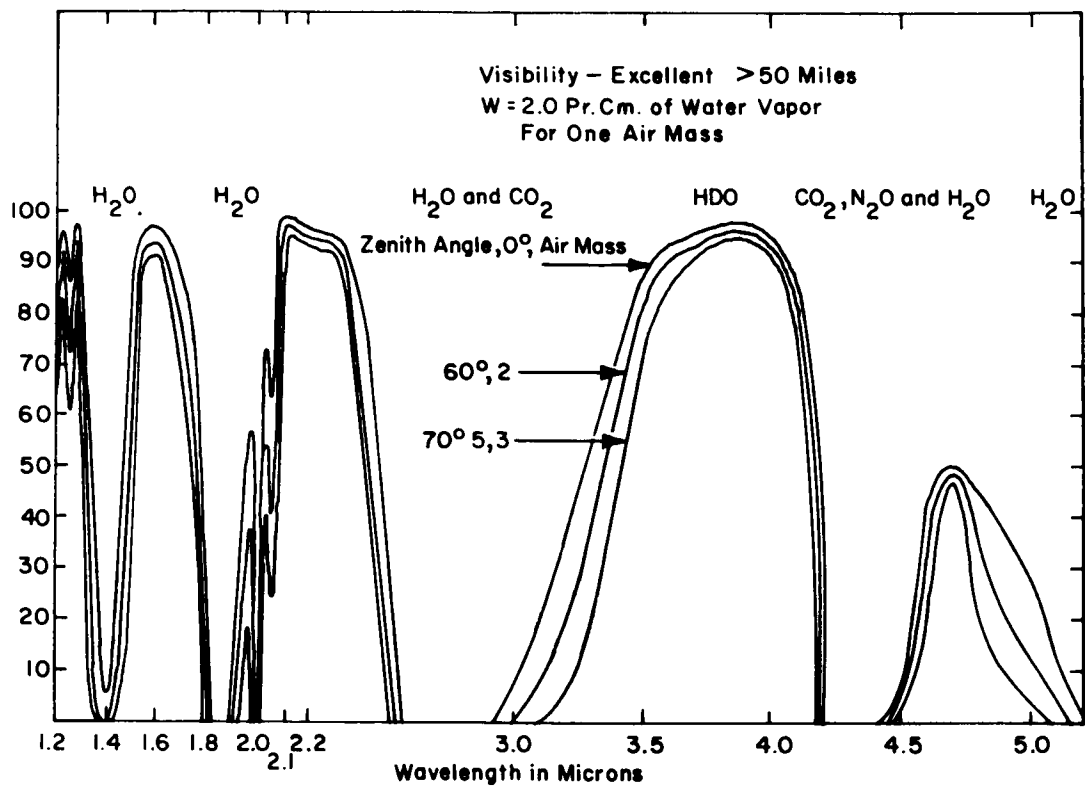
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Figure 7—Solar and Terrestrial Radiation. Reflected solar and total earth radiation to space values should be divided by  $\pi$  to obtain the radiance for each case.  $T_s$  is the surface Temperature and  $T_A$  is the effective radiating air temperature (Hughes Aircraft Co., ref. 10)



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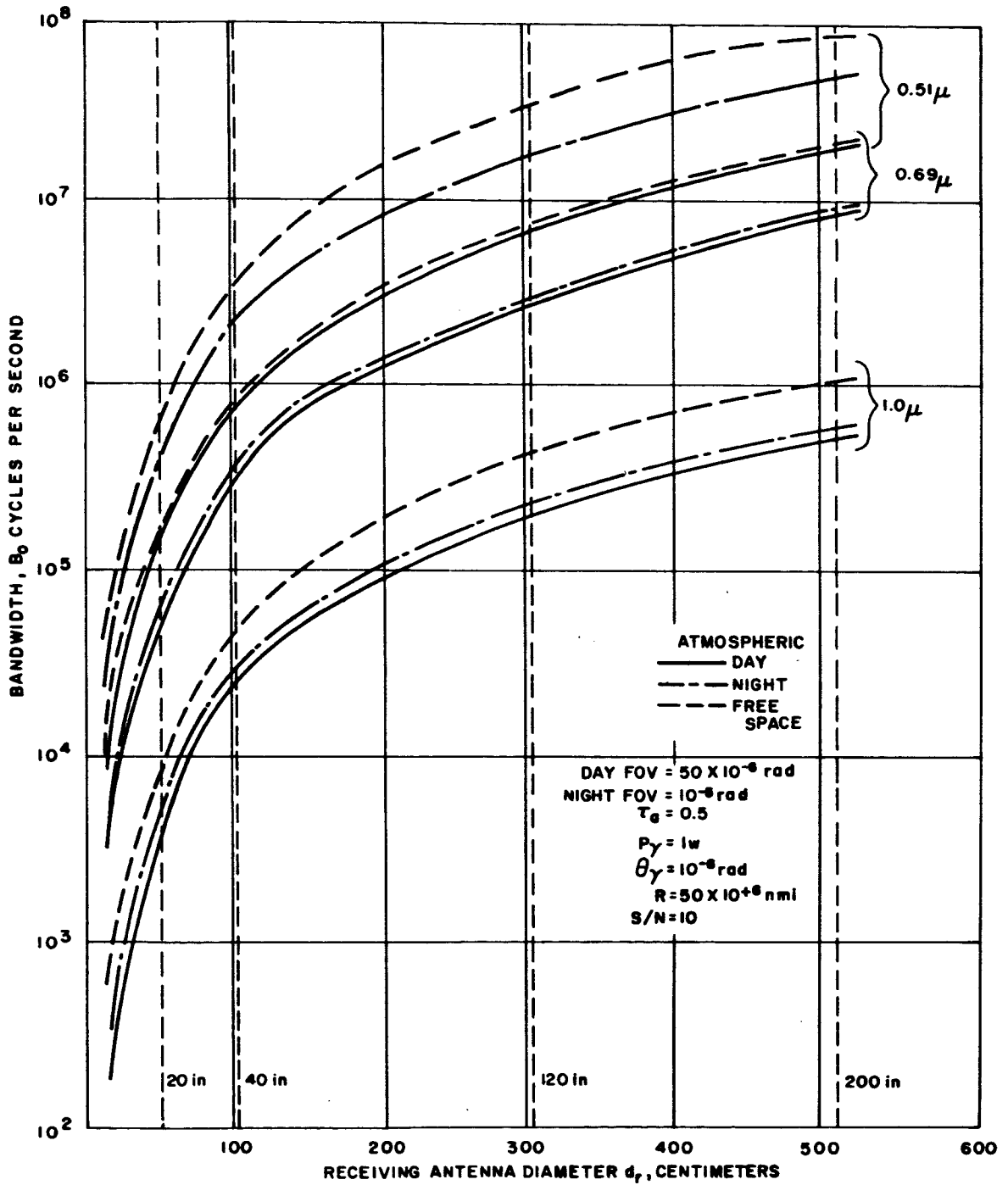
Figure 8—Transmission of the Atmosphere at Sea Level for Varying Optical Air Masses.  
 Atmospheric transmission, 0.3 to 1.3 microns (Hughes Aircraft Co., ref. 10)



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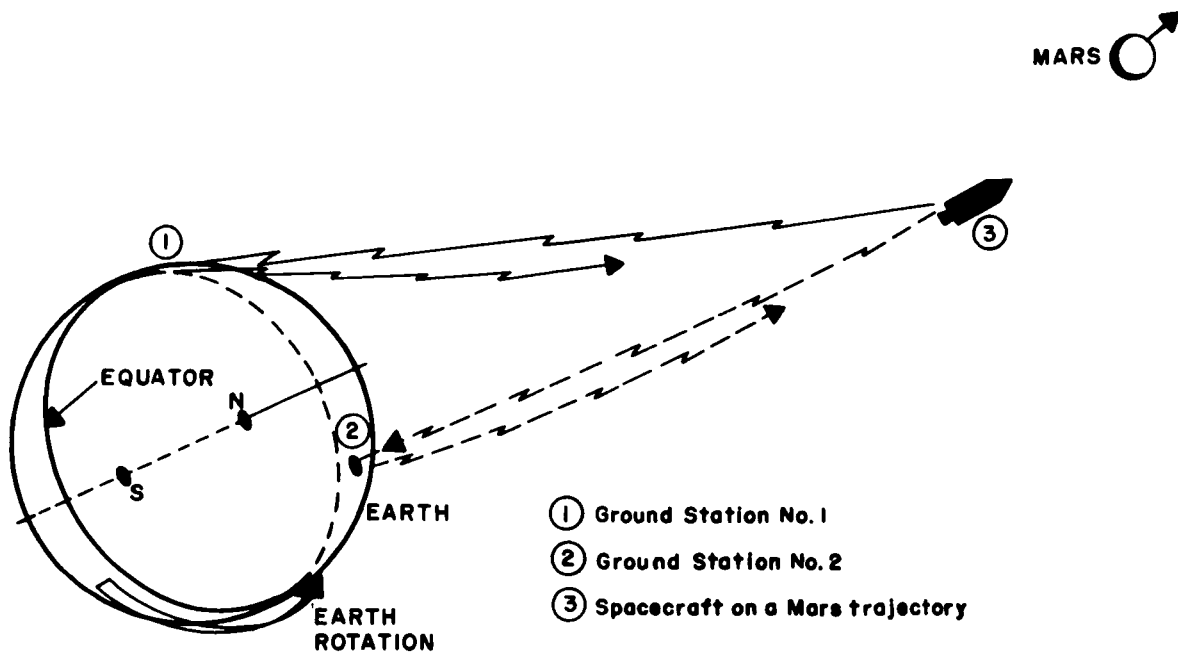
Figure 9—Transmission of the Atmosphere at Sea Level for Varying Optical Air Masses.  
 Atmospheric transmission, 1.2 to 5.0 microns (Hughes Aircraft Co., ref. 10)





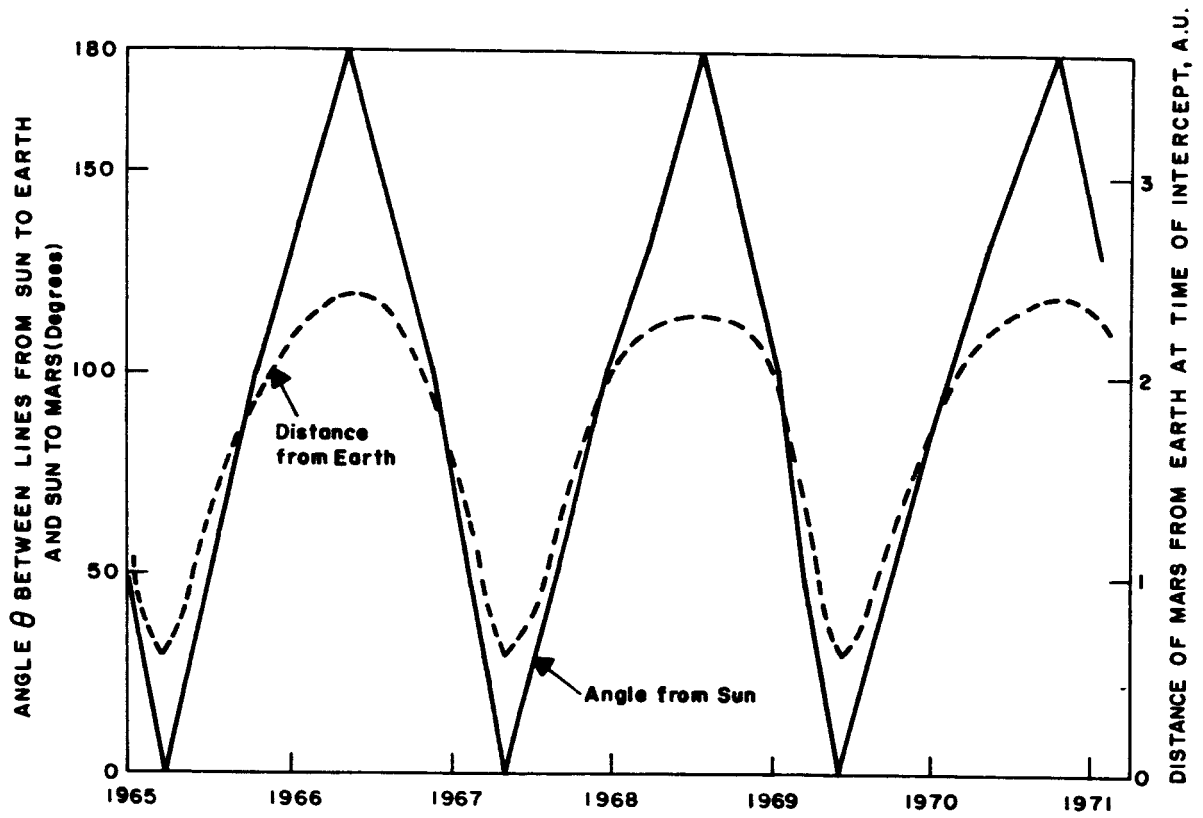
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Figure 10—Some Optical System Communication Capabilities at Daytime, Night, and in Free Space (H. L. Brinkman and W. K. Pratt, ref. 13)



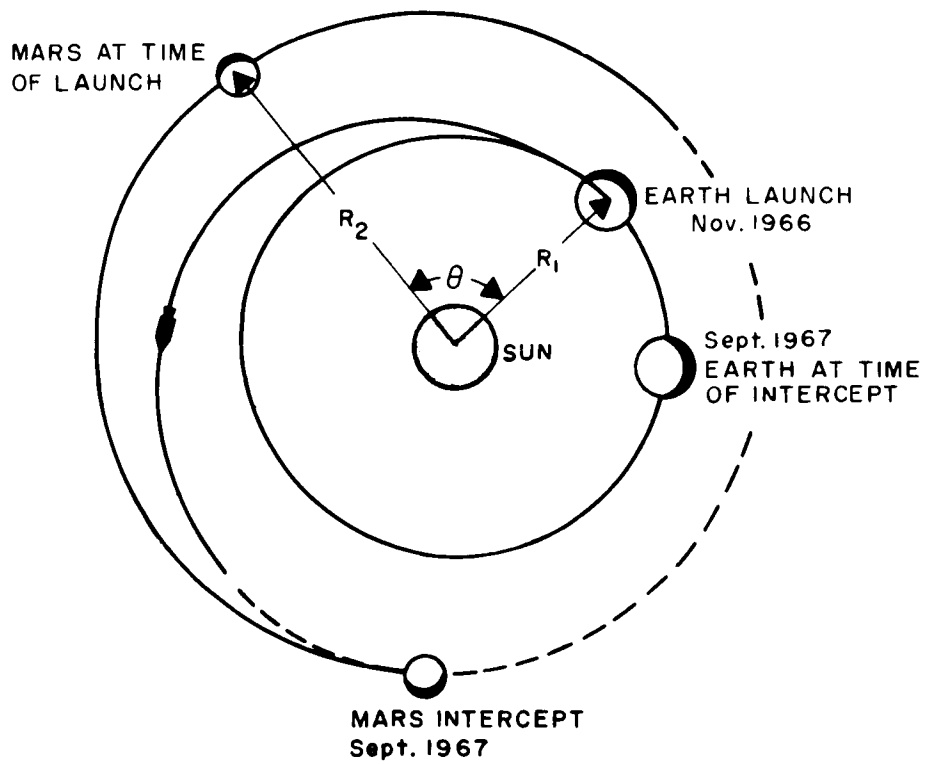
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Figure 11—Top View of the Ecliptic Plane with a Spacecraft on Its Way to Mars. An occultation is about to occur and the spacecraft must switch over communication from ground station 1 to ground station 2



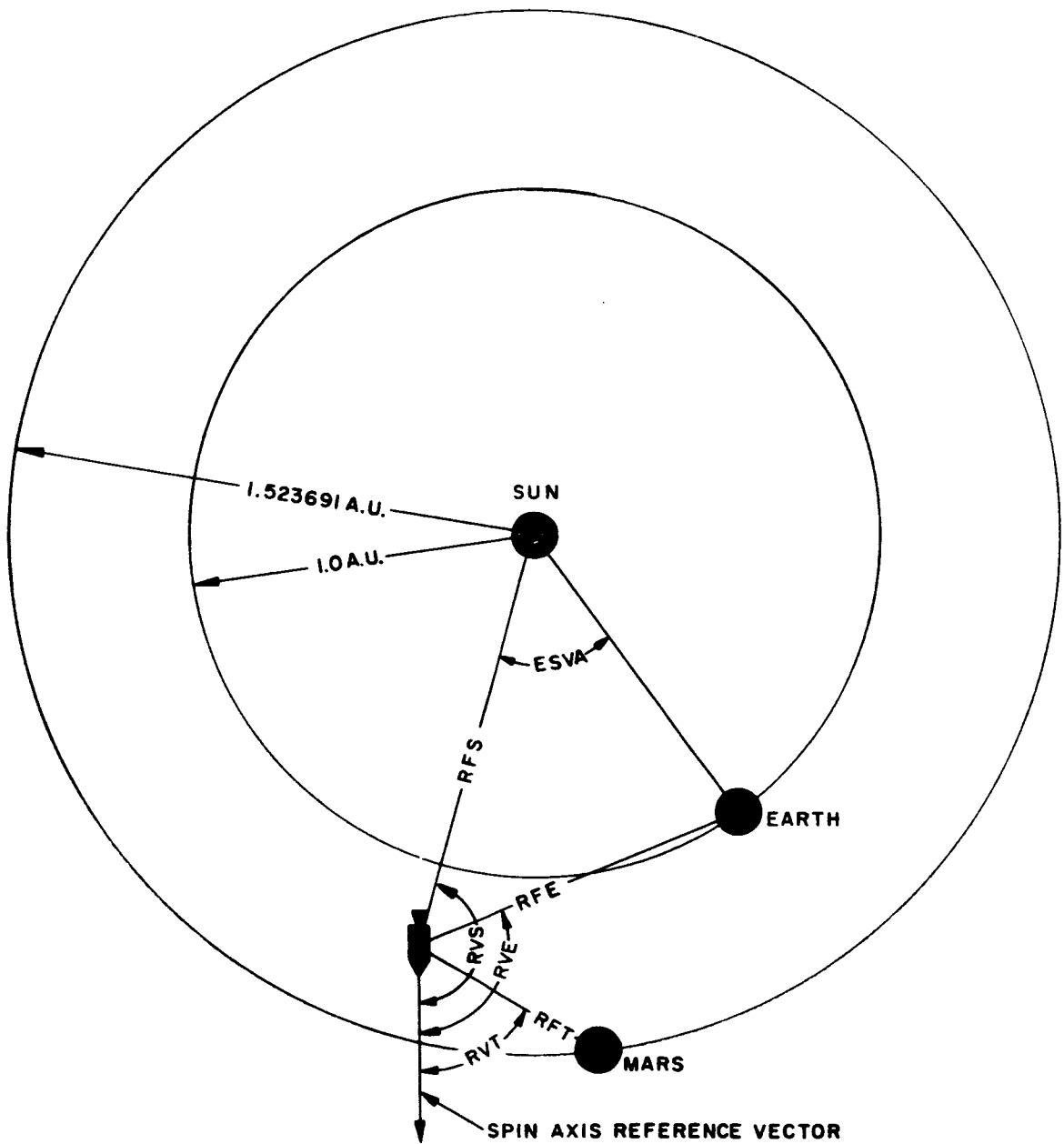
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Figure 12—Angle  $\theta$  and Distance from Earth to Mars at Time of Intercept (see also ref. 20)



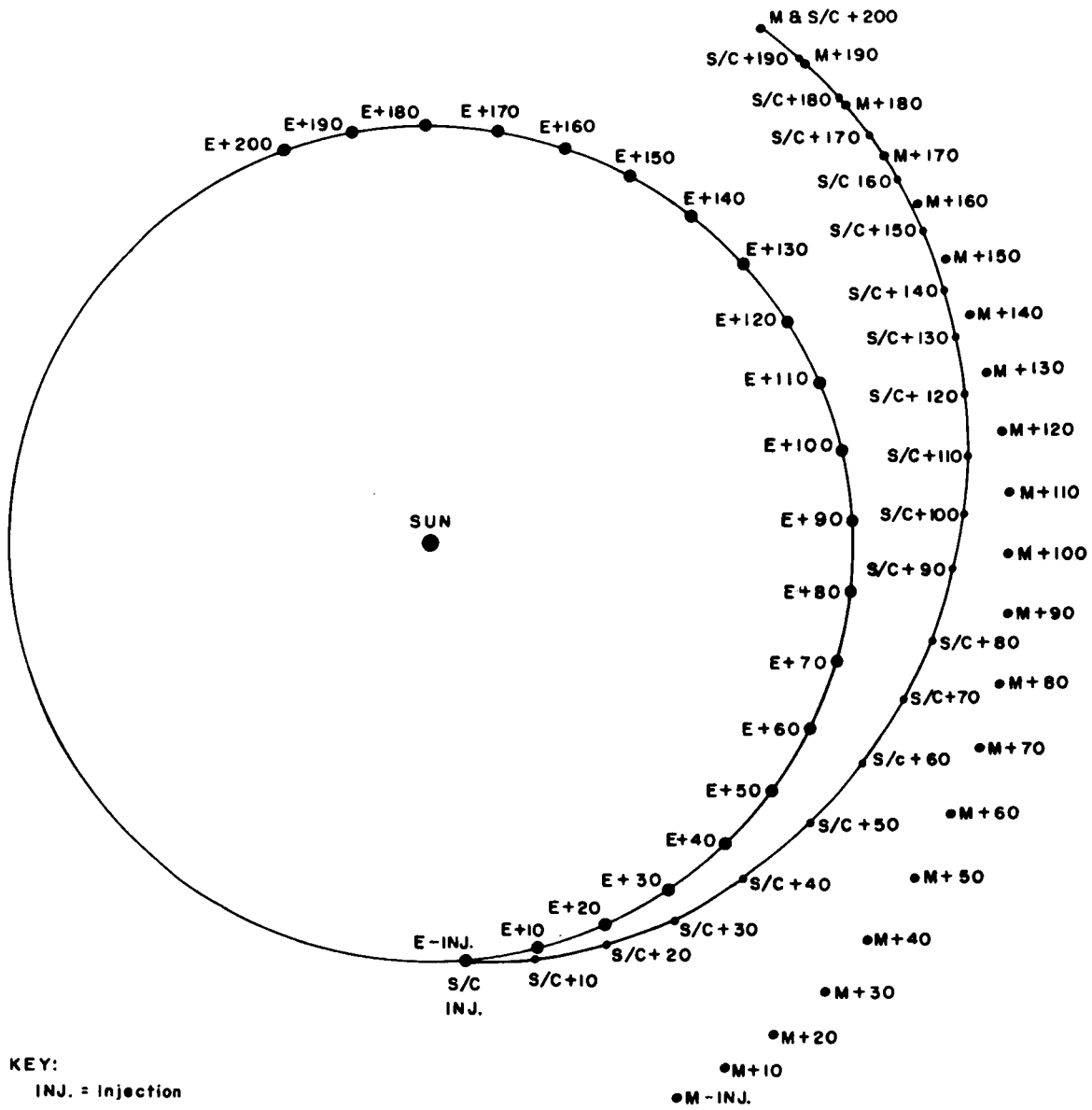
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Figure 13—Possible Mars Trajectory



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Figure 14—Earth-Mars Trajectory Geometry, Pictorially Defining Some Symbols used in Computer Printout



**KEY:**  
 INJ. = Injection  
 E +20 = Earth at 20 days after Injection  
 S/C +20 = Spacecraft at 20 days after Injection  
 M +20 = Mars at 20 days after Injection

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Figure 15—Earth-Mars Trajectory

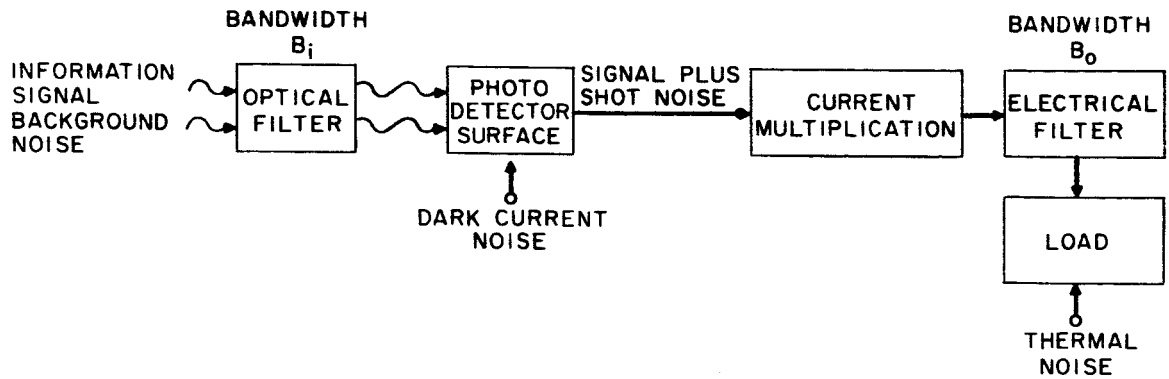


Figure 16—Direct or Noncoherent Detection Model (see also ref. 10)

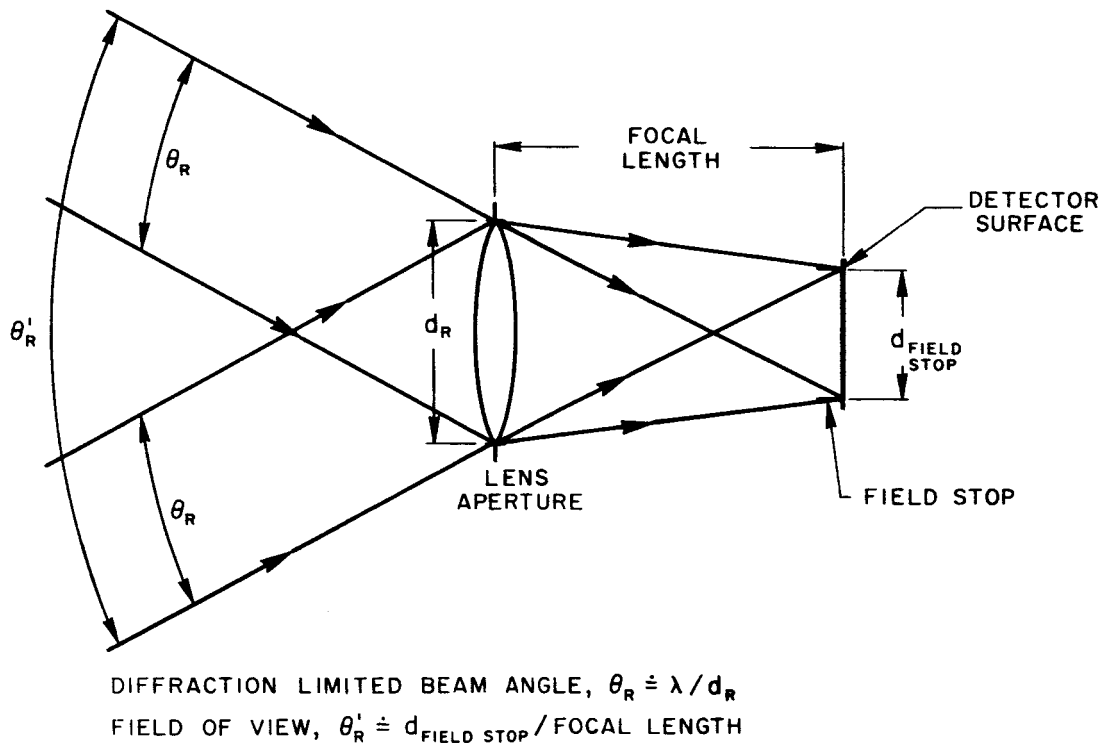


Figure 17—Illustration of Beam Divergence Angle ( $\theta_R$ ) and Angular Field of View ( $\theta'_R$ )

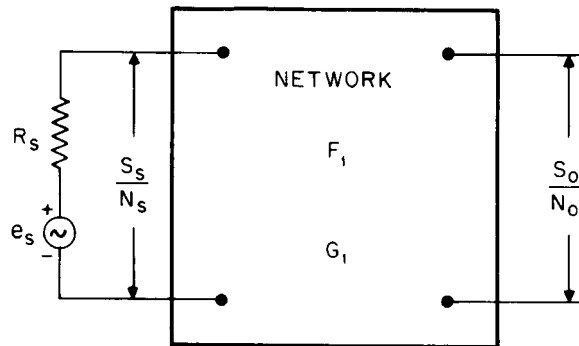
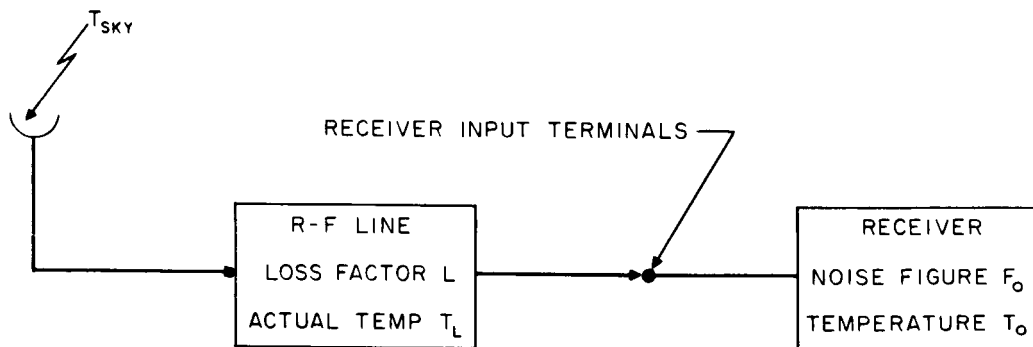


Figure A-1—Linear Network



$$T_i = T_0(F_0 - 1) + T_L(1 - 1/L) + T_{SKY}/L$$

= TOTAL EFFECTIVE NOISE TEMPERATURE  
AT RECEIVER INPUT TERMINALS

Figure A-2—Receiver System



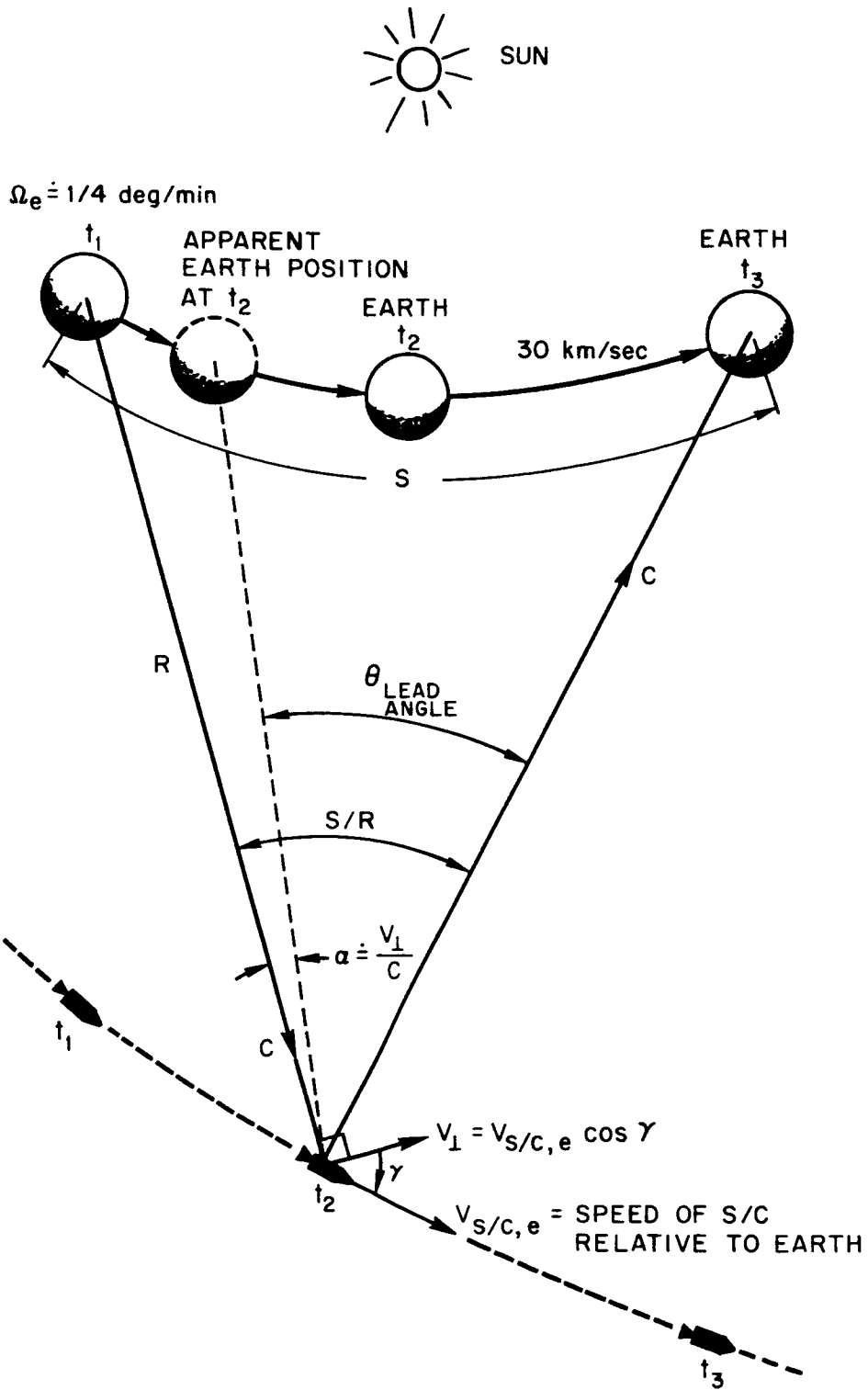


Figure B-1—Geometry for computing Lead Angle