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POWER AND CROSS-POWER SPECTRUM ANALYSIS BY HYBRID COMPUTERS

by Raymond S. Lim and William D. Cameron

Ames Research Center
Moffett Field, Calif.

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BY HYBRID COMPUTERS

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SUMMARY

This paper describes a method for analyzing power and cross-power spectra from 0.1 Hz to 20 kHz with general purpose hybrid computers. The method is valid for analyzing continuous and discrete spectra of periodic, aperiodic, complex, and stationary random signals. A spectrum is analyzed entirely on the analog computer by a pseudo band-pass filter method. The bandwidth and the averaging time are varied to suit the computation requirement of each spectrum. The digital computer is used for automatic control of the analog computer, for data acquisition, and for compensation of magnetic tape static skew. The definition of power and cross-power spectra, and the validity of various methods of analysis are comprehensively reviewed.

INTRODUCTION

Power and cross-power spectra are analyzed by filtering, phase shifting, squaring, and averaging. During the past 20 years, many methods have been devised for investigating problems in subjects ranging from aeronautics to zoology and many special analyzers have been built. Reference 1 presents a survey of spectrum analyzers. The flexibility of such analyzers is limited in frequency range, filter bandwidth, signal skew error compensation, averaging time, and the number of channels that can be analyzed simultaneously. Recently, digital computers have been used for spectrum analysis (in a very limited range).

The expanding technology of aeronautical and biomedical engineering requires a more sophisticated method for spectrum analysis. This requirement prompted the development of hybrid computers for spectrum analysis. The purpose of this paper is (1) to present a hybrid-computer technique for analyzing continuous and discrete spectra of periodic, aperiodic, complex, and stationary random signals from 0.1 Hz to 20 kHz, (2) to clarify well-established theories that describe random data, and (3) to clarify mechanization methods for power and cross-power spectrum analyses.

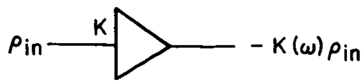
Existing hybrid computers, with a few additional special instruments, can be programmed to form a high quality spectrum analyzer. Hybrid computers are useful for spectrum analysis because they have the speed and flexibility of an analog computer, the dynamic range and accuracy of a digital computer, and the automatic output of results for documentation and plotting.

At Ames Research Center, a hybrid computing system is used to compute power and cross-power spectra of all types of data. The data, recorded on analog magnetic tape, have a frequency range from 0.01 Hz to 20 kHz, and may or may not be stationary throughout the entire data record. The length of the data record ranges from 50 msec to 60 sec with a dynamic range to 100 dB. Three channels of data can be analyzed simultaneously by the analog computer. The digital computer automatically controls the analog computer, performs the data acquisition, and compensates for the static skew error of the magnetic tape. It should be noted that any random data recorded on magnetic tape can be treated as stationary random data if the entire record length is analyzed.

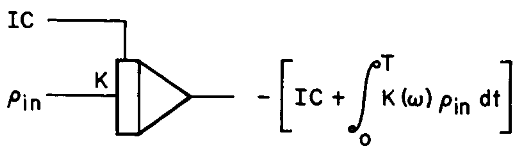
NOTATION

$C_{12}(\omega)$	cospectrum
$Q_{12}(\omega)$	quadspectrum
$R_{11}(\tau)$	autocorrelation function
$R_{12}(\tau)$	cross-correlation function
T	averaging time
$X(\omega)$	Fourier transform of $x(t)$
$x_{\Delta\omega}(t)$	$x(t)$ passed through a band-pass filter with bandwidth $\Delta\omega$
$x^{o}_{\Delta\omega}(t)$	$x_{\Delta\omega}(t)$ delayed by 90°
$\overline{x(t)}$	time average of $x(t)$
$\widetilde{x(t)}$	ensemble average of $x(t)$
$\Delta\omega$	bandwidth
$\Phi_{11}(\omega)$	power spectral density of $x_1(t)$
$\Phi_{22}(\omega)$	power spectral density of $x_2(t)$
$\Phi_{xx}(\omega)$	power spectral density of $x(t)$
ω_c	cutoff frequency of low-pass filter
ω_o	local oscillator frequency

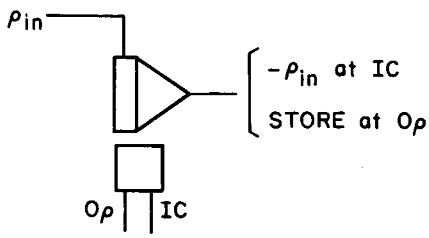
ANALOG SYMBOLS



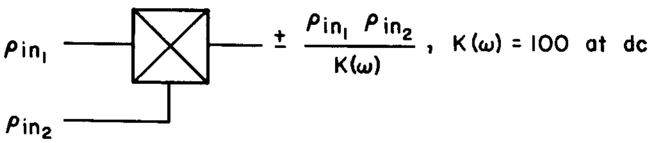
amplifier



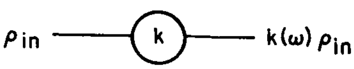
integrator



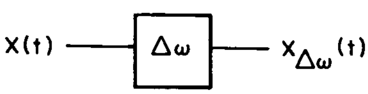
track/store amplifier



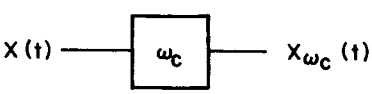
multiplier



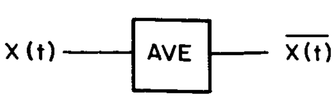
potentiometer



band-pass filter



low-pass filter



average circuit

MATHEMATICAL THEORY

Definition of Power Spectrum

There are at least two ways to calculate the power spectrum of stationary random data: (1) by direct calculation and (2) by transforming the autocorrelation function. By these methods, three basic mathematical definitions for the power spectrum are generally derived, only two of which are valid for random data. Let $\Phi_{11}(\omega)$ be defined as the power spectral density (psd); $x(t)$, a stationary random signal; and T, the interval of the data to be analyzed. The three definitions of psd are then:

1. From direct calculation,

$$\Phi_{11}(\omega) = \lim_{T\to\infty} \frac{1}{2T} \left| \int_{-T}^{T} x(t)e^{-j\omega t}\, dt \right|^2 \tag{1}$$

2. From direct calculation,

$$\Phi_{11}(\omega) = \lim_{T\to\infty} \frac{1}{2T} \overline{\left| \int_{-T}^{T} x(t)e^{-j\omega t}\, dt \right|^2} \tag{2}$$

3. From transformation of the autocorrelation function,

$$\Phi_{11}(\omega) = \int_{-\infty}^{\infty} R_{11}(\tau)e^{-j\omega\tau}\, d\tau \tag{3a}$$

$$= 2\int_{0}^{\infty} R_{11}(\tau)\cos \omega\tau \, d\tau \tag{3b}$$

$$= \lim_{\substack{\Delta\omega\to 0 \\ T\to\infty}} \frac{1}{2\Delta\omega T} \int_{0}^{T} x_{\Delta\omega}^{2}(t)\, dt \tag{3c}$$

where

$$R_{11}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T} x(t)x(t - \tau)\, d\tau \tag{4}$$

Equations (3) are mathematically correct and are excellent definitions for mechanizing on either digital or analog computers. For digital computation, equation (3b) is generally used, while for analog computation, equation (3c) is generally preferred. Equation (3a) is mathematically related to the correlation analysis of random data. The proof for equation (3a) is sometimes called Wiener's theorem for autocorrelation, and it can be found in reference 2. The derivation of equation (3c) from (3a) is relatively straightforward, but since this derivation is not often found in published literature, it is developed in appendix A. Equation (3c) can be implemented to obtain the density spectrum as follows:

1. $x(t)$ is passed through a band-pass filter with bandwidth $\Delta\omega$ to obtain $x_{\Delta\omega}(t)$,

2. $x_{\Delta\omega}(t)$ is squared through a multiplier to obtain $x^2_{\Delta\omega}(t)$,

3. finally, $x^2_{\Delta\omega}(t)$ is averaged to obtain $\overline{x^2_{\Delta\omega}(t)}$.

Equations (3) are applicable to stationary random signals as well as periodic and nonperiodic signals. Equation (3c) is an estimate of equation (3a).

Reference 3 (Aseltine) shows that equation (2) is correct for $x(t)$. (The wavy bar denotes the ensemble average.) This equation has little practical use in spectrum analysis.

The first definition, equation (1), has been shown to be mathematically invalid for all classes of random signals (ref. 3, 4, 5, or 6).

Definition of Cross-Power Spectrum

Cross-power spectrum analysis obtains the amplitude and phase-spectrum information between two independent random signals, $x_1(t)$ and $x_2(t)$. From the electrical engineering point of view, a cross-power spectrum analyzer is a phase meter for measuring the phase and amplitude relationship between two complex or random signals. The cross-power spectrum is a vector quantity, or a complex function. The real part is called the cospectrum and the imaginary part, the quadspectrum.

The cross-power spectrum function $\Phi_{12}(\omega)$ can be defined as the Fourier transformation of the cross-correlation function in a manner similar to defining the power spectrum as

$$\Phi_{12}(\omega) = \int_{-\infty}^{\infty} R_{12}(\tau)e^{-j\omega\tau}\, d\tau \tag{5}$$

Mathematically, this is the only known correct definition for random signals. The definition of the cross-power spectrum function from direct calculation as

$$\Phi_{12}(\omega) = \lim_{T\to\infty} \frac{1}{T}\int_0^T x_1(t)e^{j\omega t}\, dt \int_0^T x_2(t)e^{-j\omega t}\, dt \tag{6}$$

is incorrect when $x_1(t)$ and $x_2(t)$ are random signals (for the reason given in the previous section for power spectrum). However, if $x_1(t)$ and $x_2(t)$ are periodic functions, Papoulis (ref. 7) shows that equation (6) is correct.

Equation (5) can be defined in terms of the cospectrum and the quadspectrum as

$$\Phi_{12}(\omega) = C_{12}(\omega) + jQ_{12}(\omega) \tag{7}$$

where

$$C_{12}(\omega) = \int_{-\infty}^{\infty} R_{12}(\tau)\cos \omega\tau \, d\tau \tag{8a}$$

$$= \lim_{\substack{\Delta\omega\to 0 \\ T\to\infty}} \frac{1}{2\Delta\omega T} \int_0^T x_{1_{\Delta\omega}}(t) x_{2_{\Delta\omega}}(t) dt \tag{8b}$$

and

$$Q_{12}(\omega) = \int_{-\infty}^{\infty} R_{12}(\tau)\sin \omega\tau \tag{9a}$$

$$= \lim_{\substack{\Delta\omega\to 0 \\ T\to\infty}} \frac{1}{2\Delta\omega T} \int_0^T x_{1_{\Delta\omega}}(t) x^{o}_{2_{\Delta\omega}}(t) dt \tag{9b}$$

In the above equations, $C_{12}(\omega)$ is the cospectrum, $Q_{12}(\omega)$ is the quadspectrum, $R_{12}(\tau)$ is the cross-correlation function, $x_{2_{\Delta\omega}}(t)$ is $x_2(t)$ passed through a band-pass filter with bandwidth $\Delta\omega$, and $x^{o}_{2_{\Delta\omega}}(t)$ is $x_{2_{\Delta\omega}}(t)$ shifted 90°. (Equations (8b) and (9b) are derived in appendix B.)

Basic Method of Power Spectrum Analysis

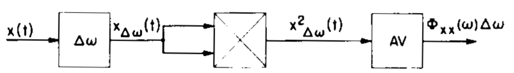


Figure 1.- Analog method of power spectrum analysis.

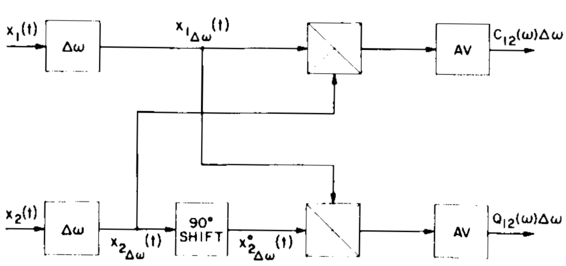


Figure 2.- Analog method of cross-power spectrum analysis.

The analog method of computing power spectra is well-known. As indicated by equations (3), (8b), and (9b), the basic method of computing the power and cross-power spectra is by band-pass filtering, phase shifting, squaring, and averaging the data (figs. 1 and 2, respectively).

Actually, the methods in figures 1 and 2 cannot be reasonably mechanized because it is difficult to design high-frequency narrow band-pass filters ($\Delta\omega$ of 1 Hz at 10 kHz), band-pass filters that can automatically select the bandwidth and the center frequencies independently, and two band-pass filters with an accurately matched phase for cross-spectrum analysis. Consequently, the basic analog method is not generally used. In its place, a pseudo band-pass filter (sometimes called tracking filter) is mechanized by the heterodyne principle. But because of this filtering technique, the method is valid only for analyzing data with a sufficiently long record. It is inaccurate for analyzing short records of a transient nature (such as impact research or heartbeat data), unless the data are made periodic. (This will be clarified in subsequent discussions.) In the present technology, analog multipliers (for multiplying and squaring) and 90° phase-shift networks with the required bandwidth are readily available.

TYPES OF RANDOM SIGNAL TO BE ANALYZED

The types of random data to be analyzed, by the techniques described in this report, can be classified into three groups:

1. Aerodynamic data on nonsteady phenomena. The frequency range of interest is from 10 Hz to 20 kHz with a dynamic range of 70 dB and a data record 60 sec long. The data are similar to white noise mixed with a few periodic signals, and may or may not be stationary within the 60-sec length. The spectrum is continuous.

2. Impact research data. The frequency range of interest is from 1 to 2500 Hz with a dynamic range of 80 dB and a data record 50 to 100 msec long. The data are similar to a decaying oscillation mixed with a small amplitude random signal. The spectrum is not continuous. These types of data must be converted to periodic data before the analysis so that they may be averaged.

3. Physiological and biological data. The frequency range is from 0.1 to 1000 Hz with a dynamic range of 60 dB and a data record between 0.1 to 2 sec long. Most physiological data analyzed are heartbeats and brain waves, both human and animal. The spectrum is not continuous. These data must also be converted to periodic data before the analysis.

HYBRID COMPUTER METHOD OF SPECTRUM ANALYSIS

Method of Power and Cross-Power Spectrum Analysis

Power and cross-power spectra are analyzed by a hybrid computer in a manner similar to the method discussed under Basic Method of Power Spectrum Analysis. There are many combinations of analog and digital computers that are called hybrid computers. The hybrid computer used to implement the spectrum analysis discussed in this report is the EAI HYDAC 2000 system, which consists of a 231R-V analog computer and a DOS-350 logic computer. The method

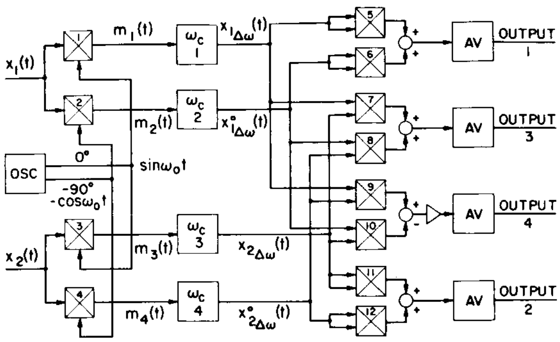


Figure 3.- Method of power and cross-power spectrum analysis.

of analysis is shown in figure 3, and is thoroughly analyzed in appendix C. As shown in appendix C, the spectra of $x_1(t)$ and $x_2(t)$ are

$$\Phi_{11}(\omega) = \frac{2}{\Delta\omega} \text{ (Output 1)} \qquad (10a)$$

$$\Phi_{22}(\omega) = \frac{2}{\Delta\omega} \text{ (Output 2)} \qquad (10b)$$

Equations (10) are not ambiguous and are correct forms for random signals as well as complex or periodic signals. The $\Delta\omega$ division in equations (10) is for bandwidth normalization, and should be used for a continuous spectrum only. For cross-power spectrum, the cospectrum and quadspectrum are

$$C_{12}(\omega) = \frac{2}{\Delta\omega} \text{ (Output 3)} \qquad (11a)$$

$$Q_{12}(\omega) = -\frac{2}{\Delta\omega} \text{ (Output 4)} \qquad (11b)$$

In particular, for inputs of

$$x_1(t) = A_1 \sin \omega_1 t$$

$$x_2(t) = A_1 \sin(\omega_1 t - \theta_2)$$

θ_2 = variable, from 0° to 360°, $C_{12}(\omega)$ and $Q_{12}(\omega)$ are

$$C_{12}(\omega) = \frac{A_1{}^2 B^2}{4} \cos \theta_2 \left(\frac{2}{\Delta\omega}\right) \qquad (12a)$$

$$Q_{12}(\omega) = \frac{A_1{}^2 B^2}{4} \sin \theta_2 \left(\frac{2}{\Delta\omega}\right) \qquad (12b)$$

regardless of $\omega_1 \geq \omega_o$ or $\omega_1 \leq \omega_o$. This means that there is no ambiguity on the output sign of the quadspectrum.

The implementation of the power and cross-power spectrum analysis between computer elements of the method shown in figure 3 is

1. Analog computer; all spectrum computations.

2. Digital computer;

(a) Logic computer - for automatic control of the analog computer and analyzer output digitizing and storage.

(b) IBM 7040/7094 - off-line spectrum scaling, coherence function computation, cross-power phase-angle computation, magnetic tape recorder static-skew error compensation, and plotting.

In addition to the standard HYDAC 2000 system, a programmable two-phase oscillator system and a low-pass filter system are also required.

Perhaps the best way to explain how the hybrid computer is used to mechanize the spectrum analysis is to divide the system in figure 3 into six major parts, and then describe each part in detail; these six parts are: (1) a programmable two-phase oscillator, (2) a pseudo band-pass filter, (3) squaring and averaging, (4) output equations, (5) automatic program control and analog data digitizing, and (6) scaling and static skew-error correction.

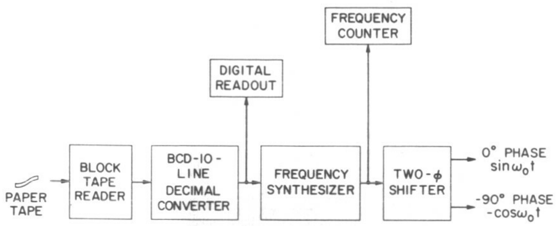


Figure 4.- Programmable two-phase oscillator.

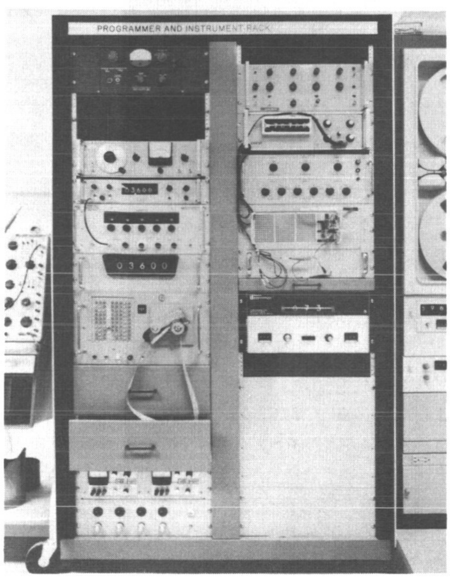


Figure 5.- Auxiliary equipment required for spectrum analysis.

Programmable two-phase oscillator.- A block diagram of the programmable two-phase oscillator circuit is shown in figure 4. The hardware is shown in figure 5. The frequency of the spectrum to be analyzed is punched onto the paper tape to be read by the block reader. The frequency synthesizer is a new instrument and is simply a digital programmable oscillator. The two-phase shifter is an all-pass active network with two outputs. One output is the reference phase; the other is the -90° phase. The two output phases are always 90 ±5° apart for the frequency within the band of 0.1 to 5000 Hz. Before 1966, a two-phase synthesizer was not available. The amplitude response of the best single-phase frequency synthesizer is ±12 percent, and this larger amplitude variation must be compensated in the two-phase shifter to within ±1 percent or better. At the present, it is reasonable to expect that a two-phase synthesizer with an amplitude response of ±2 percent or better and a phase linearity of ±2° throughout the frequency range will soon be available.

Pseudo band-pass filter.- The mechanization of the pseudo band-pass filter is shown in figure 6. Consider the pseudo band-pass filter 1, which consists of multiplier 1 and low-pass filter 1. The input signal $x(t)$ is multiplied, or heterodyned, with the 0° phase of the oscillator. The resultant output is a sum and difference frequency pair for each frequency ω_i of $x(t)$. At the output of low-pass filter 1, only those frequencies within ω_c are passed. Thus the multiplier low-pass filter combination is, in effect, a pseudo band-pass filter. The center frequency is determined by the oscillator, and the bandwidth $\Delta\omega$ is equal to $2\omega_c$. In effect, $x_{\Delta\omega}(t)$ is $x(t)$ passed through a band-pass filter with bandwidth $\Delta\omega$.

The synthesized band-pass filter in figure 6 is not a true band-pass filter. It measures only that component of the signal inphase with the reference oscillator, as shown in appendix C. Therefore, the application of one pseudo band-pass filter for spectrum analysis is not sufficient. Two pseudo band-pass filters must be used to measure both the inphase and out-of-phase components of the signal, since, in general, there is a random phase angle between the input data and the oscillator. If this phase angle is represented by θ_i, the output of the first pseudo band-pass filter is $A_i \cos(\omega t + \theta_i)$ and the second, $A_i \sin(\omega t + \theta_i)$. To obtain a true measure of the signal, it is necessary to add these two components vectorially in the manner of

$$A_i^2 = [A_i \cos(\omega t + \theta_i)]^2 + [A_i \sin(\omega t + \theta_i)]^2$$

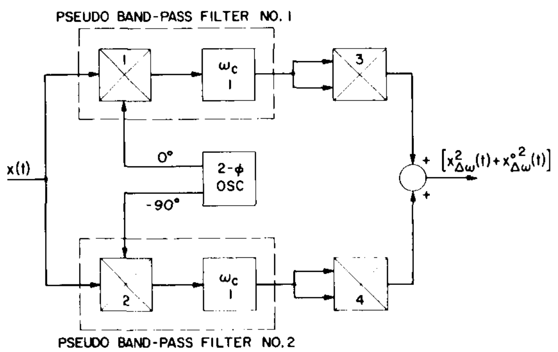


Figure 6.- Pseudo band-pass filter and ripple cancellation.

The pseudo band-pass filtering circuit is shown in figure 7. Only the $x_1(t)$ channel is shown; the $x_2(t)$ channel is the same. The circuit shown in figure 7 should be self-explanatory. The outputs of low-pass filters 1 and 2 are $x_{1_{\Delta\omega}}(t)$ and $x^o_{1_{\Delta\omega}}(t)$, respectively. The phase and the amplitude responses of the $x_1(t)$ channel and the $x_2(t)$ channel must be matched in order for the cross-power spectrum outputs to be meaningful. Any mismatch will be system error. The system error in using the HYDAC 2000 system and the auxiliary equipment is approximately $\pm 4^\circ$ of measurement with a full scale of 360°.

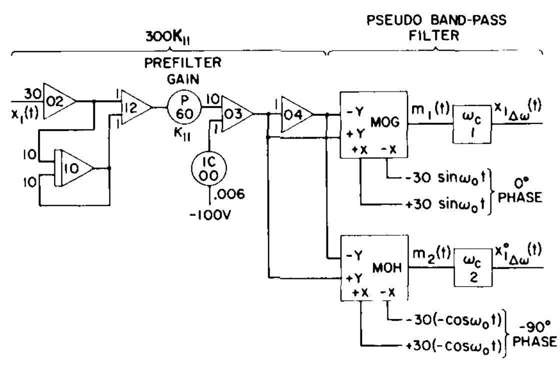


Figure 7.- Analog mechanization of pseudo band-pass filtering system.

Squaring and averaging.- The squaring and averaging circuit system is shown in figure 8 for $\Phi_{11}(\omega)$, and in figure 9 for $Q_{12}(\omega)$. The circuits for $\Phi_{22}(\omega)$ and $C_{12}(\omega)$ are assumed to be understood. True integration is used. The integration time is set and controlled by the DOS-350 logic computer in accordance with the mode-logic table of the integrator (A05). A05 is programmed to integrate for a fixed length of time T and then to hold its value at the end of time T. During this hold period, track/store amplifier A06 tracks A05 and stores its value. After A06 has stored the value of A05, A05 is commanded to reset. Operation of the circuit in figure 9 is similar to that in figure 8.

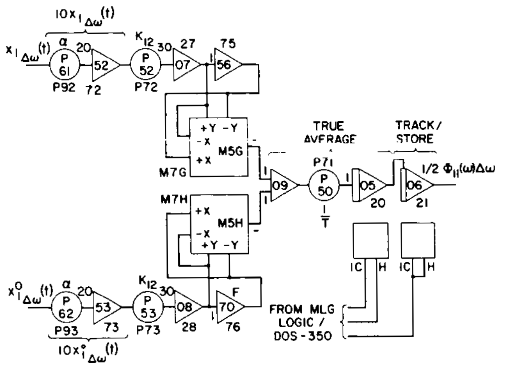


Figure 8.- Squaring and averaging system, $x_1(t)$ power spectrum channel.

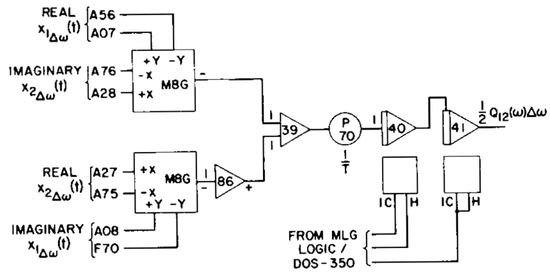


Figure 9.- Squaring and averaging system, $Q_{12}(w)$ channel.

Output equations.- There are four outputs from the analyzer when two channels of data are being analyzed: $\Phi_{11}(\omega)$, $\Phi_{22}(\omega)$, $C_{12}(\omega)$, and $Q_{12}(\omega)$. After the signal is traced through the analog circuits and all gain factors are accounted for, these four outputs become

$$\Phi_{11}(\omega) = \frac{2(\text{Output 1})}{7.29\times10^{6}K_{11}^{2}K_{12}^{2}} \frac{1}{\Delta\omega} \frac{S_A}{S_o} \tag{13}$$

$$\Phi_{22}(\omega) = \frac{2(\text{Output 2})}{7.29\times10^{6}K_{21}^{2}K_{22}^{2}} \frac{1}{\Delta\omega} \frac{S_A}{S_o} \tag{14}$$

$$C_{12}(\omega) = \frac{2(\text{Output 3})}{7.29\times10^{6}K_{11}K_{21}K_{12}K_{22}} \frac{1}{\Delta\omega} \frac{S_A}{S_o} \tag{15}$$

$$Q_{12}(\omega) = \frac{2(\text{Output 4})}{7.29\times10^{6}K_{11}K_{21}K_{12}K_{22}} \frac{1}{\Delta\omega} \frac{S_A}{S_o} \tag{16}$$

where the above equations are in mean-square power, and

S_A magnetic tape loop speed playback when analyzing data

S_o magnetic tape speed at which the data were recorded originally

The factor S_A/S_o is necessary because the multipliers in the 231R-V are accurate only to about 5 kHz. To analyze to 20 kHz, a speed reduction of 4 is required. The normalization of the output equations by the bandwidth is valid only for continuous spectra. (A continuous spectrum is one whose output is directly proportional to the analyzer bandwidth.)

Automatic program control and analog data digitizing.- The complete program for controlling the analog computer for automatic spectrum analysis is provided by the DOS-350 logic computer. The DOS-350 also digitizes the four outputs from the analyzer and punches these outputs on paper tape for subsequent spectrum scaling and static skew-error compensation on the IBM 7040/7094 system. Specifically, the DOS-350 performs the following functions during spectrum analysis:

1. Senses the control signal recorded on one channel of the tape loop and starts the analysis for that frequency point as programmed on the frequency synthesizer. The splice problem on the loop is eliminated by using a step-type control signal on the tape loop.

2. Generates a second set of control signals for spectrum analysis at higher frequencies if the data are stationary throughout the record.

3. Generates a control signal to advance the block paper-tape reader by one block (one frequency point) at the end of the analysis time T.

4. Generates integrator and track/store amplifier control signals to control the analog spectrum computation in a fully automatic mode.

5. Records the number of frequency points analyzed and shuts off the program at the end of the analysis.

6. Punches the program on paper tape in the proper format consistent with the IBM 7040/7094 system.

Scaling and static skew-error compensation.- Scaling is defined here as the conversion of the spectrum outputs to engineering units of the original experiment. For example, if the spectrum outputs should be in psf/Hz (pounds per square foot per hertz), then the original calibration of the transducer must be used to multiply the spectrum outputs at each frequency point. This type of scaling or multiplication is ideally suited to the digital computer.

There are two ways of compensating static skew error:

1. By an analog delay line at the output of the magnetic tape recorder, and

2. By computing $\theta \pm 2\pi f\tau$ at the digital computer after scaling, where τ is the time difference between recorder channels, and θ is the arctangent of $Q_{12}(\omega)$ over $C_{12}(\omega)$ in radians. If $x_1(t)$ leads $x_2(t)$ by τ, $\theta - 2\pi f\tau$ is used; otherwise, $\theta + 2\pi f\tau$ is used.

Static skew-error compensation is mandatory at high frequencies for the cross-power spectrum since present recorders can have static skew errors as large as 30 μsec, which represents 216° at 20 kHz.

Dynamic Range, Resolution, Bandwidth, Averaging Time, and Frequency Scan Rate

The dynamic range of the analyzer must be carefully considered before the type of analog computer is selected for spectrum analysis. Dynamic range here is defined as:

$$\text{dynamic range} = \frac{\text{maximum spectrum output obtainable}}{\text{minimum spectrum output observable}}$$

For a high quality ±100-V computer, the dynamic range is about 100 to 0.05 or 66 dB. To obtain the maximum 66-dB dynamic range at the output, two gain controls are used in each channel of the analyzer. In the $x_1(t)$ channel, K_{11} is the prefilter gain and K_{12} is the postfilter gain. Before each analysis, a search is required for the maximum spectrum output. After that particular frequency point is located, K_{11} and K_{12} are adjusted for a 100-V output. In a good ±100-V analog computer, the output is actually linear up to ±120 V or more.

Resolution and bandwidth are functions of the low-pass filter setting. The bandwidth should be chosen in accordance with the type of data to be analyzed. To simplify changes of bandwidth, standard low-pass filters are used.

The problem of averaging time is solved by using true averaging. The frequency scan rate has no meaning in this hybrid method of spectrum analysis, since a sweep-type oscillator is not used. After completing a frequency-point analysis, the next frequency point can be analyzed as soon as the system transient decays to zero.

Statistical Uncertainty

The predominant source of error or uncertainty in an analyzed spectrum is the random statistical variations of the sample record. Perhaps the most complete discussion, but not necessarily the easiest to understand, on the subject of statistical error is given by Chang (ref. 8). Some error discussions were also given by Bendat and Piersol (ref. 9). Confidence limits can be defined as relative for statistical error comparison. In reference 8, the confidence limits are defined as

$$\frac{1}{1 + a} < \frac{\Phi(\omega)}{\Phi(\omega)_a} < \frac{1}{1 - a} \tag{17}$$

where

$\Phi(\omega)$	true spectrum
$\Phi(\omega)_a$	analyzed spectrum
a	$k_p \epsilon \sqrt{\frac{2}{N}}$
ϵ	$\frac{1}{\sqrt{\Delta\omega T}}$
N	number of sample records
k_p	a constant depending on the specified probability, tabulated as follows:

P	k_p
0.5	0.477
.8	.906
.9	1.163
.95	1.386
.99	1.82
.999	2.32

Example: The signal to be analyzed has the following analyzer parameters:

$$N = 1 , \quad T = 10 \text{ sec} , \quad \Delta\omega = 10 \text{ Hz}$$

Now determine the 90-percent confidence limits.

Solution: The rms per unit error of each record is

$$\epsilon = \frac{1}{\sqrt{\Delta\omega T}} = 0.1$$

For P = 0.9 or 90 percent, $k_p = 1.163$. Therefore,

$$a = 1.163 \times 0.1 \times \sqrt{\frac{2}{1}} = 0.165$$

$$\frac{1}{1 - a} = 1.198 , \qquad \frac{1}{1 + a} = 0.858$$

Thus, there is a 90-percent probability that the true spectrum lies between 0.858 and 1.198 times the measured spectrum.

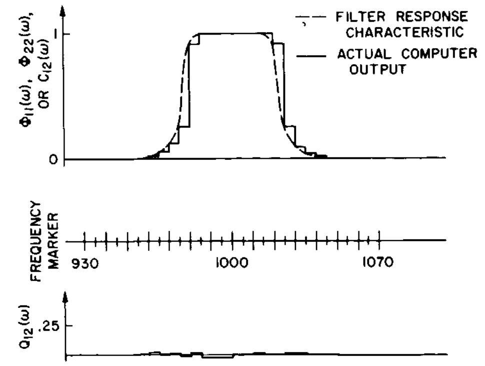


Figure 10.- Analysis of a sine wave.

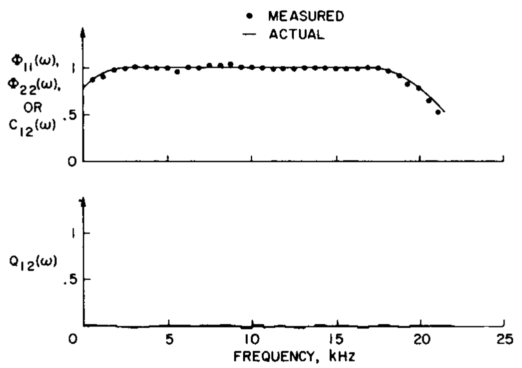


Figure 11.- Analysis of white noise from 10 Hz to 20 kHz.

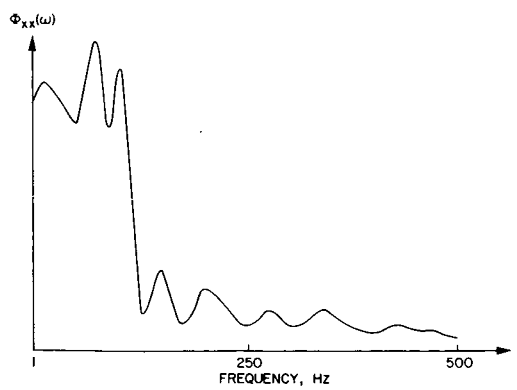


Figure 12.- Analysis of a heartbeat pulse.

Typical Results

Figure 10 shows the analysis of a single 0.707 Vrms sine wave at 1 kc as an input to both the $x_1(t)$ and $x_2(t)$ channels. The result shown in figure 10 is correct since the analysis of a single sine wave for a frequency band across the analyzer bandwidth is the amplitude response of the analyzer band-pass filter.

The analysis of a band-limited white noise from a General Radio GR-1390B generator is shown in figure 11. The noise is first passed through a low-pass filter with an effective bandwidth of 20 kHz. The amplitude of the noise is then adjusted to read 0.45 Vrms at the output of the filter (i.e., the input to the analyzer). This gives a power spectral density of 10^{-5} V^2/Hz. The scatter of the measured values about the actual values is within ±2 percent.

Figure 12 shows an analysis of one heartbeat from medical research with a 0.2-Hz analyzer bandwidth from 0.5 to 125 Hz.

Figures 13 and 14 show the power and the cross-power spectra of two typical 20-kHz wind-tunnel data. These data are from transducers 1 inch apart, stations -18.19 and -17.19. These two figures are plotted by the digital computer side by side on a 31 × 31-inch sheet. The accuracy and confidence of these figures are confirmed by three CPRMS numbers. The CPRMS (wind tunnel) is the RMS value of the data measured at the wind tunnel during an experiment. The CPRMS (loop output) is the RMS value of the data after it is transferred onto an analog loop recorder for spectrum analysis. The CPRMS (psd area) is the RMS value of the data computed in the digital computer by taking the square root of the area underneath the power spectrum curve. Under ideal

TEST NUMBER	153	IDENTIFICATION	X	Y
RUN NUMBER	137	TRACK NUMBER	11	9
CONFIGURATION	27	PERIPHERAL LOCATION	0	0
MACH NUMBER	2	STATION LOCATION	−18.19	−17.19
DYNAMIC PRESSURE (Q)	760 psf	CPRMS (WIND TUNNEL)	.0297	.0292
VELOCITY (V)	1745 fps	CPRMS (LOOP OUTPUT)	.02994	.02914
CHARACTERISTIC LENGTH (L)	1 in.	CPRMS (PSD AREA)	.02942	.03009

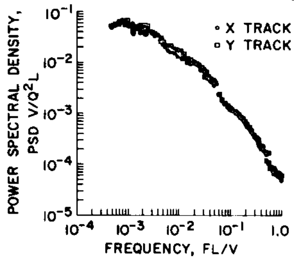


Figure 13.- Power spectral of two typical 20 kHz wind-tunnel data.

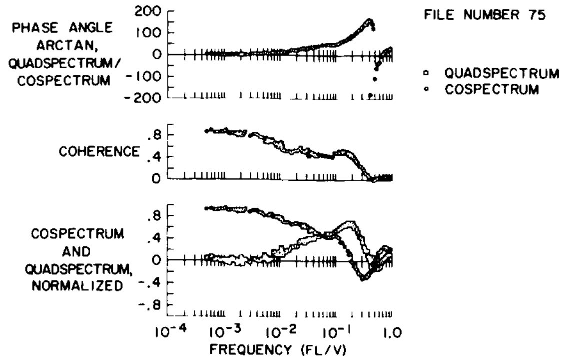


Figure 14.- Cross-power spectrum of two typical 20 kHz wind-tunnel data.

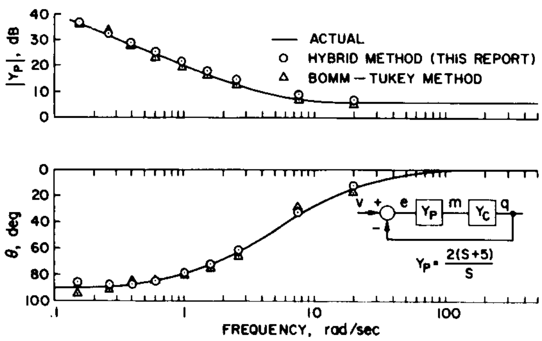


Figure 15.- Analysis of the transfer function.

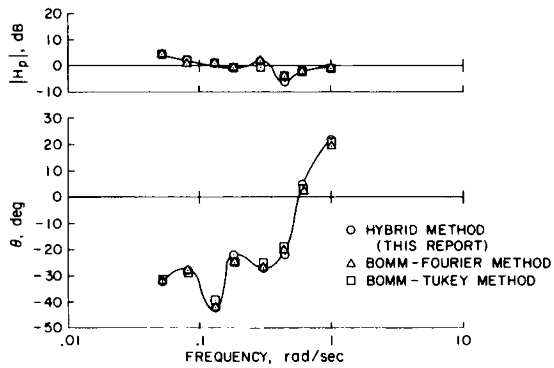


Figure 16.- Analyzed transfer function of a human pilot.

conditions when no error occurs, these three numbers should be equal. As indicated in figure 13, the error is about 3.15 percent.

Figure 15 shows the analysis of a known transfer function, Y_p. This same transfer function was analyzed independently in the digital computer by the BOMM-Tukey method (ref. 10). The results obtained are in close agreement with the actual frequency and phase response. Figure 16 shows the analysis of a human pilot. Note the close agreement of the results obtained by the three methods - the hybrid method, the BOMM-Fourier method, and the BOMM-Tukey method.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Sept. 27, 1966
124-11-04-06

APPENDIX A

DERIVATION OF $\Phi_{11}(\omega)$ FOR RANDOM SIGNALS FROM AUTOCORRECTION

The purpose of this appendix is to show that the power spectrum

$$\Phi_{11}(\omega) = \int_{-\infty}^{\infty} R_{11}(\tau)e^{-j\omega\tau}\, d\tau \qquad \text{(A1)}$$

can be estimated by

$$\Phi_{11}(\omega) \cong \lim_{\substack{\Delta\omega\to 0 \\ T\to\infty}} \frac{1}{2\Delta\omega T} \int_{0}^{T} x_{\Delta\omega}^{2}(t)\,dt \qquad \text{(A2)}$$

To proceed with the derivation, the inverse transform of equation (A1) is taken.

$$R_{11}(\tau) = \int_{-\infty}^{\infty} \Phi_{11}(\omega)e^{j\omega\tau}\, d\omega \qquad \text{(A3)}$$

Since

$$R_{11}(0) \geq |R_{11}(\tau)| \qquad \text{(A4)}$$

the maximum psd is obtained for $\tau = 0$. That is,

$$R_{11}(0) = \int_{-\infty}^{\infty} \Phi_{11}(\omega)\,d\omega \qquad \text{(A5a)}$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt \qquad \text{(A5b)}$$

In the derivation of a cross spectrum, τ cannot equal zero because $R_{12}(0)$ is not necessarily the maximum value. For this reason, the cross spectrum is generally complex. Equations (A5) should be clear intuitively since the total power of $x(t)$ is equal to the mean square value, and also to the total area underneath the power spectrum curve. Now, for small $d\omega$, $d\omega$ approaches $\Delta\omega$. In the limit, as $d\omega$ approaches zero,

$$\lim_{d\omega\to o} \int_{-\infty}^{\infty} \Phi_{11}(\omega)\,d\omega = \lim_{d\omega\to o} \sum_{n=-\infty}^{\infty} \Phi_{11_n}(\omega)\,d\omega \tag{A6a}$$

$$= \lim_{\Delta\omega\to o} 2 \sum_{n=o}^{\infty} \Phi_{11_{\Delta\omega_n}}(\omega)\Delta\omega \tag{A6b}$$

The interpretation of equations (A6) is that integrating $\Phi_{11}(\omega)$ is equivalent to obtaining the area underneath the spectrum curve. This area can also be obtained by dividing the area into n small areas with width $\Delta\omega$ and summing over n. In view of equations (A6), equation (A5a) becomes

$$R_{11}(0) = \lim_{\Delta\omega\to o} 2 \sum_{n=o}^{\infty} \Phi_{11_{\Delta\omega_n}}(\omega)\Delta\omega \tag{A7}$$

or

$$R_{11_{\Delta\omega}}(0) = \lim_{\Delta\omega\to o} 2\Phi_{11_{\Delta\omega}}(\omega)\Delta\omega \tag{A8}$$

from which

$$\Phi_{11_{\Delta\omega}}(\omega) = \lim_{\Delta\omega\to o} \frac{1}{2\Delta\omega} R_{11_{\Delta\omega}}(0) \tag{A9}$$

or, in general notation,

$$\Phi_{11}(\omega) = \lim_{\substack{\Delta\omega\to o \\ T\to\infty}} \frac{1}{2\Delta\omega T} \int_{o}^{T} x_{\Delta\omega}^{2}(t)\,dt \tag{A10}$$

where $x_{\Delta\omega}(t)$ is interpreted as $x(t)$ passed through a band-pass filter with bandwidth $\Delta\omega$. This completes the derivation.

APPENDIX B

DERIVATION OF $\Phi_{12}(\omega)$ FOR RANDOM SIGNALS FROM CROSSCORRELATION

The purpose of this appendix is to show that the cross-power·spectrum

$$\Phi_{12}(\omega) = \int_{-\infty}^{\infty} R_{12}(\tau)e^{-j\omega\tau}\,d\tau \tag{B1}$$

can be estimated by

$$\Phi_{12}(\omega) \cong C_{12}(\omega) + jQ_{12}(\omega) \tag{B2}$$

where

$$C_{12}(\omega) = \lim_{\substack{\Delta\omega\to 0 \\ T\to\infty}} \frac{1}{2\Delta\omega T}\int_0^T x_{1_{\Delta\omega}}(t)x_{2_{\Delta\omega}}(t)\,dt \tag{B3}$$

$$Q_{12}(\omega) = \lim_{\substack{\Delta\omega\to 0 \\ T\to\infty}} \frac{1}{2\Delta\omega T}\int_0^T x_{1_{\Delta\omega}}(t)x^{o}_{2_{\Delta\omega}}(t)\,dt \tag{B4}$$

and $x^{o}_{2_{\Delta\omega}}(t)$ is $x_{2_{\Delta\omega}}(t)$ delayed 90°. To proceed with the derivation, the inverse transform of equation (B1) is taken. The variable delay τ cannot be set equal to zero because $R_{12}(0)$ is not equal to nor, in general, greater than $R_{12}(\tau)$. It is felt that the derivation that follows is heuristic, rather than rigorous. Now,

$$R_{12}(\tau) = \int_{-\infty}^{\infty} \Phi_{12}(\omega)e^{j\omega\tau}\,d\omega \tag{B4a}$$

$$= \int_{-\infty}^{\infty} [C_{12}(\omega) + jQ_{12}(\omega)]e^{j\omega\tau}\,d\omega \tag{B4b}$$

Since an arbitrary function can be decomposed into a sum of an even and an odd function (e.g., ref. 7), then

$$R_{12}(\tau) = R_{12}(\tau)_e + R_{12}(\tau)_o \tag{B5}$$

where $R_{12}(\tau)_e$ and $R_{12}(\tau)_o$ are the even and odd parts of $R_{12}(\tau)$, respectively. Now, the result of equating equations (B5) and (B4b) is

$$R_{12}(\tau)_e + R_{12}(\tau)_o = \int_{-\infty}^{\infty} C_{12}(\omega) e^{j\omega\tau} \, d\omega + (j) \int_{-\infty}^{\infty} Q_{12}(\omega) e^{j\omega\tau} \, d\omega \qquad \text{(B6)}$$

With some intuition, let us equate the even and odd parts of equation (B6). That is,

$$R_{12}(\tau)_e = \int_{-\infty}^{\infty} C_{12}(\omega) e^{j\omega\tau} \, d\omega \qquad \text{(B7a)}$$

$$R_{12}(\tau)_o = (j) \int_{-\infty}^{\infty} Q_{12}(\omega) e^{j\omega\tau} \, d\omega \qquad \text{(B7b)}$$

In a manner similar to the derivation of the power spectrum, the cospectrum can be derived as follows:

$$R_{12}(0)_e = \int_{-\infty}^{\infty} C_{12}(\omega) d\omega$$

$$= \lim_{d\omega \to o} \sum_{n=-\infty}^{\infty} C_{12_n}(\omega) d\omega$$

$$= \lim_{\Delta\omega \to o} 2 \sum_{n=o}^{\infty} C_{12_{\Delta\omega_n}}(\omega) \Delta\omega$$

$$R_{12_{\Delta\omega}}(0)_e = \lim_{\Delta\omega \to o} 2C_{12_{\Delta\omega}} \Delta\omega$$

$$C_{12_{\Delta\omega}} = \lim_{\Delta\omega \to o} \frac{1}{2\Delta\omega} R_{12_{\Delta\omega}}(0)_e \qquad \text{(B8)}$$

In general,

$$C_{12_{\Delta\omega}} = \lim_{\Delta\omega\to o} \frac{1}{2\Delta\omega} R_{12_{\Delta\omega}}(0)$$

$$= \lim_{\substack{\Delta\omega\to o \\ T\to\infty}} \frac{1}{2\Delta\omega T} \int_o^T x_{1_{\Delta\omega}}(t) x_{2_{\Delta\omega}}(t)\, dt \qquad \text{(B9)}$$

This completes the derivation for the cospectrum. Now, for the quadspectrum,

$$R_{12}(0)_o = j \int_{-\infty}^{\infty} Q_{12}(\omega)\, d\omega$$

$$-jR_{12}(0)_o = \lim_{d\omega\to o} \sum_{n=-\infty}^{\infty} Q_{12_n}(\omega)\, d\omega$$

$$= \lim_{\Delta\omega\to o} 2 \sum_{n=o}^{\infty} Q_{12_{\Delta\omega_n}}(\omega)\Delta\omega$$

$$-jR_{12_{\Delta\omega}}(0)_o = \lim_{\Delta\omega\to o} 2Q_{12_{\Delta\omega}}(\omega)\Delta\omega$$

$$Q_{12_{\Delta\omega}} = \lim_{\Delta\omega\to o} \frac{1}{2\Delta\omega} (-j) R_{12_{\Delta\omega}}(0)_o \qquad \text{(B10)}$$

In general,

$$Q_{12_{\Delta\omega}} = \lim_{\Delta\omega\to o} \frac{1}{2\Delta\omega} (-j) R_{12_{\Delta\omega}}(0)$$

$$= \lim_{\substack{\Delta\omega\to o \\ T\to\infty}} \frac{1}{2\Delta\omega T} (-j) \int_o^T x_{1_{\Delta\omega}}(t) x_{2_{\Delta\omega}}(t)\, dt$$

$$= \lim_{\substack{\Delta\omega\to o \\ T\to o}} \frac{1}{2\Delta\omega T} \int_o^T x_{1_{\Delta\omega}}(t) x^o_{2_{\Delta\omega}}(t)\, dt \qquad \text{(B11)}$$

This completes the derivation for the quadspectrum.

APPENDIX C

DERIVATION OF OUTPUT EQUATIONS FOR ANALOG METHOD OF POWER AND CROSS-POWER SPECTRUM ANALYSIS

The schematic diagram for this method is shown in figure 3. Let the inputs be

$$x_1(t) = \sum_{i=1}^{\infty} A_i \sin(\omega_i t + \theta_i) \tag{C1}$$

$$x_2(t) = \sum_{i=1}^{\infty} C_i \sin(\omega_i t + \varphi_i) \tag{C2}$$

$$OSC = \begin{cases} B \sin \omega_o t & \text{for } 0^\circ \text{ phase} \\ -B \cos \omega_o t & \text{for } -90^\circ \text{ phase} \end{cases} \tag{C3}$$

where θ_i, in general, does not equal φ_i. The values θ_i and φ_i indicate that the inputs, in general, are random phase with respect to the oscillator. With the summation understood, the filtered outputs are

$$x_{1_{\Delta\omega}}(t) = \frac{B}{2} A_i \cos \delta_1 \tag{C4a}$$

$$x^o_{1_{\Delta\omega}}(t) = \frac{B}{2} A_i \sin \delta_1 \tag{C4b}$$

$$x_{2_{\Delta\omega}}(t) = \frac{B}{2} C_i \cos \delta_2 \tag{C5a}$$

$$x^o_{2_{\Delta\omega}}(t) = \frac{B}{2} C_i \sin \delta_2 \tag{C5b}$$

where

$$\delta_1 = (\omega_i t - \omega_o t + \theta_i)$$

$$\delta_2 = (\omega_i t - \omega_o t + \varphi_i)$$

The power spectrums of $x_1(t)$ and $x_2(t)$ in terms of mean square power are

$$\Phi_{11}(\omega) = \frac{2}{\Delta\omega}\frac{1}{T}\int_0^T \left[x^2_{1_{\Delta\omega}}(t) + x^{o2}_{1_{\Delta\omega}}(t)\right] dt$$

$$= \frac{2}{\Delta\omega}(K_1^2 + K_2^2 + \ldots + K_n^2) = \frac{2(\text{Output 1})}{\Delta\omega} \qquad \text{(C6a)}$$

$$\Phi_{22}(\omega) = \frac{2}{\Delta\omega}\frac{1}{T}\int_0^T \left[x^2_{2_{\Delta\omega}}(t) + x^{o2}_{2_{\Delta\omega}}(t)\right] dt$$

$$= \frac{2}{\Delta\omega}(l_1^2 + l_2^2 + \ldots \; l n^2) = \frac{2(\text{Output 2})}{\Delta\omega} \qquad \text{(C6b)}$$

where

$$K_i = \frac{A_i B}{2}, \qquad l_i = \frac{C_i B}{2}, \qquad i = 1, 2, 3, \ldots$$

$$|\omega_i + \omega_o| > \omega_c, \qquad |\omega_i - \omega_o| < \omega_c$$

Equations (C6) have no ambiguity and are correct forms for random signals as well as for complex or periodic signals. The division by $\Delta\omega$ in equations (C6) is for bandwidth normalization, and should be used for continuous spectra only. The cospectrum and quadspectrum in terms of mean squared power are

$$C_{12}(\omega) = \frac{2}{\Delta\omega}\frac{1}{T}\int_0^T \left[x_{1_{\Delta\omega}}(t)x_{2_{\Delta\omega}}(t) + x^o_{1_{\Delta\omega}}(t)x^o_{2_{\Delta\omega}}(t)\right] dt$$

$$= \frac{2}{\Delta\omega}\text{ Output 3} \qquad \text{(C7a)}$$

$$Q_{12}(\omega) = -\frac{2}{\Delta\omega}\frac{1}{T}\int_0^T \left[x_{1_{\Delta\omega}}(t)x^o_{2_{\Delta\omega}}(t) - x^o_{1_{\Delta\omega}}(t)x_{2_{\Delta\omega}}(t)\right] dt$$

$$= -\frac{2}{\Delta\omega}\text{ Output 4} \qquad \text{(C7b)}$$

This completes the derivation for the output equations. To show that the quadspectrum has no ambiguity in sign as a function of frequency,

$$\text{Output 4} = -\frac{1}{T}\int_0^T \frac{B^2}{4}\sum_{i=1}^{n}(A_i \cos \delta_1 C_i \sin \delta_2$$

$$- A_i \sin \delta_1 C_i \cos \delta_2)dt \qquad \text{(C8)}$$

To simplify equation (C8), let

$$\left.\begin{aligned} x_1(t) &= A_1 \sin \omega_1 t \\ x_2(t) &= A_1 \sin(\omega_1 t - \theta_2) \\ \theta_2 &= \text{a variable, from } 0^\circ \text{ to } 360^\circ \end{aligned}\right\} \tag{C9}$$

Then

$$\left.\begin{aligned} \delta_1 &= (\omega_1 - \omega_o)t \\ \delta_2 &= (\omega_1 t - \omega_o t - \theta_2) \end{aligned}\right\} \tag{C10}$$

and

$$\text{Output } 4 = -\frac{A_1{}^2B^2}{4}\frac{1}{T}\int_o^T [\cos(\omega_1 - \omega_o)t \sin(\omega_1 t - \omega_o t - \theta_2)$$

$$- \sin(\omega_1 - \omega_o)t \cos(\omega_1 t - \omega_o t - \theta_2)]dt \tag{C11}$$

<u>Case 1</u>. $\omega_1 > \omega_o$ by $\Delta\omega_o$; $\omega_1 = \omega_o + \Delta\omega_o$

$$\text{Output } 4 = -\frac{A_1{}^2B^2}{4}\frac{1}{T}\int_o^T [\cos(\Delta\omega_o)t \sin(\Delta\omega_o t - \theta_2)$$

$$- \sin(\Delta\omega_o)t \cos(\Delta\omega_o t - \theta_2)]dt$$

$$= -\frac{A_1{}^2B^2}{4}\frac{1}{T}\int_o^T [(-\cos^2 \Delta\omega_o t \sin \theta_2) - (\sin^2 \Delta\omega_o t \sin \theta_2)]dt$$

$$= \frac{A_1{}^2B^2}{4} \sin \theta_2 \tag{C12}$$

<u>Case 2</u>. $\omega_1 = \omega_o$

$$\text{Output } 4 = -\frac{A_1{}^2B^2}{4}\frac{1}{T}\int_o^T (-\sin \theta_2)dt$$

$$= \frac{A_1{}^2B^2}{4} \sin \theta_2 \tag{C13}$$

Case 3. $\omega_1 < \omega_o; \omega_1 = \omega_o - \Delta\omega_o$

$$\text{Output } 4 = -\frac{A_1{}^2B^2}{4}\frac{1}{T}\int_0^T [-\cos(\Delta\omega_o)t \sin(\Delta\omega_o t + \theta_2)$$

$$+ \sin(\Delta\omega_o)t \cos(\Delta\omega_o t + \theta_2)]dt$$

$$= -\frac{A_1{}^2B^2}{4}\frac{1}{T}\int_0^T [(-\cos^2 \Delta\omega_o t \sin \theta_2) + (-\sin^2 \Delta\omega_o t \sin \theta_2)]dt$$

$$= \frac{A_1{}^2B^2}{4} \sin \theta_2 \tag{C14}$$

Thus, for all three cases, output 4 (a quadspectrum output) is always a plus sine function, and there is no ambiguity.

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