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AN ORBITAL ANALYSIS TECHNIQUE FOR SHARPLY VARYING GEOPHYSICAL PHENOMENA

by

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ABSTRACT

A technique has been developed for increasing the time resolution of high accuracy satellite orbital parameters. The technique may be used to allow the association of orbital parameters with the precise time of experiment data sampling rather than with the uniformly spaced times normally available. Thus, geophysical phenomena having high spatial gradients may be resolved more clearly with respect to latitude, longitude, and altitude. Using this method an elliptical orbit is adjusted to one point on the non-elliptical, perturbed orbit allowing the extrapolation of orbital parameters to times near the initial point. The equations for this extrapolation follow directly from a consideration of the sub-satellite path. By virtue of its intended application, the method may be interpreted as an interpolation, with no sacrifice in accuracy, between successive orbital data samples. An accuracy of .01 degree is obtained for latitude and longitude in regions where these parameters vary most rapidly and an accuracy of 1 kilometer is obtained for altitude (all after an extrapolation time of 3 minutes).

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INTRODUCTION

The number of direct measurement studies of atmospheric parameters is increasing at a pace which requires the development of new analytical tools. The increasing number of satellites carrying direct measurement devices, and the interexperiment comparisons possible on the "omnibus" satellites allow for new sophistication and precision in the discussion of ionospheric dynamics. To utilize more fully the potential of multiexperiment satellites having high sampling rates, a study has been made to allow the association of orbital parameters with the precise time of experiment data handling.

Measurements of the latitudinal distribution of atmospheric ion density have indicated the existence of sharp, high-latitude variations in density and their possible correlation with certain diurnally varying geophysical phenomena. Taylor et al. (1966) have observed, for example, that changes in ion density of more than one order of magnitude occur at high latitudes in an interval of no more than one degree. Similar indications of the desirability of high orbital resolution have been shown in the work of Thomas et al. (1966) on Alouette I where electron density variations of a factor of three or four have been noted, again in less than one degree of latitude. Satellites having a polar orbit, such as the Orbiting Geophysical Observatory II (OGO-II), change longitude (and hence local time) very rapidly in these interesting high latitude regions which further complicates the already rapid traversal of latitude. Referring satellite orbital parameters to the precise time at which an ionospheric sample was taken allows the delineation of these narrow variations and the full utilization of the inherently high satellite-experiment resolution.

To produce orbital parameters at times other than the uniform intervals normally supplied, an extrapolation technique has been devised which requires the use of certain orbital data typically available and builds upon it. Orbital information is generally obtained, in conjunction with experiment data on magnetic tapes, at each whole minute of satellite life. This whole minute data is computed with all the perturbing influences (radiation pressure, drag, lunar and solar gravitational effects) taken into account and is therefore considered to be the most accurate information obtainable. The technique described in this paper involves essentially the adjustment of that sub-satellite path resulting from a Keplerian orbit to the actual path. Other orbital parameters also follow logically from this analysis, all with a precision depending upon the adequacy of the Keplerian approximation to a given orbit. Although the technique, as described, is actually an extrapolation from one point on an orbit, it is used to increase the time resolution of information available periodically and hence can be viewed as an interpolation between successive orbital samples. However, the method possesses certain advantages over both linear and higher order interpolation which will be discussed later in the report. With this interpretation the function of the method is to fill in orbital data, with no sacrifice in accuracy, between the available times and not to act as a long-term orbital predictor.

With regard to producing orbital parameters at irregular intervals, it is of course true that the sophisticated analysis that works at each whole minute could just as well be applied at the sample time. This method makes a more efficient utilization of the complex orbital analysis already completed by using an orbital model which minimizes the additional computational labor needed to increase the orbit data time resolution. Using the IBM 7094, the time required to compute one hour of orbital information at an interval of one second is reduced by a factor of five to slightly over one minute. This time savings has led to the adoption of this specific approach.

ANALYSIS

Outline

Referring to Figure 1, which gives a schematic depiction of the extrapolation process, the approach is as follows:

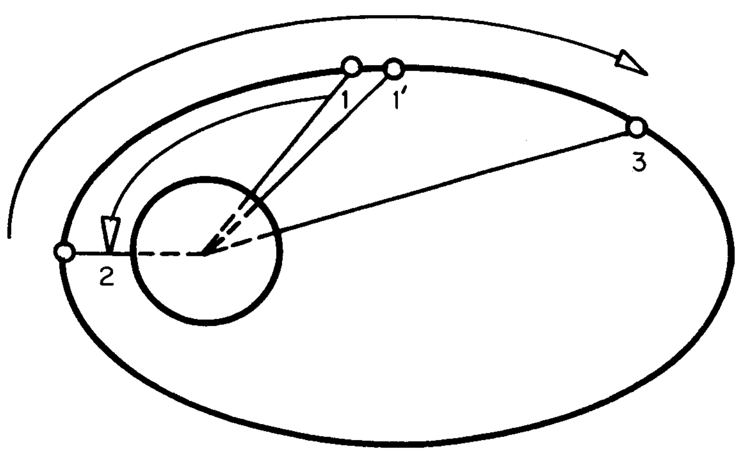

Figure 1–Schematic illustration of extrapolation technique.

Accurate orbital information is available associated with point 1, some arbitrary point on the orbit. By means of this information, the orbital parameters associated with point 2, satellite perigee, are calculated. These numbers are used as initial values in the Kepler equations allowing the generation of an entire orbit. Then the orbital parameters at any other point, say point 3, may be computed using the model of the full orbit. Point 3 is the position of the experiment sample of interest.

The orbital information needed for point 1 consists of the following: sub-satellite geodetic latitude, sub-satellite longitude, altitude above the earth spheroid, true anomaly, and time. In addition, the period, p, orbital inclination, θ, and eccentricity, ϵ, of the satellite must also be known, but these quantities are given for an entire orbit rather than a single point.

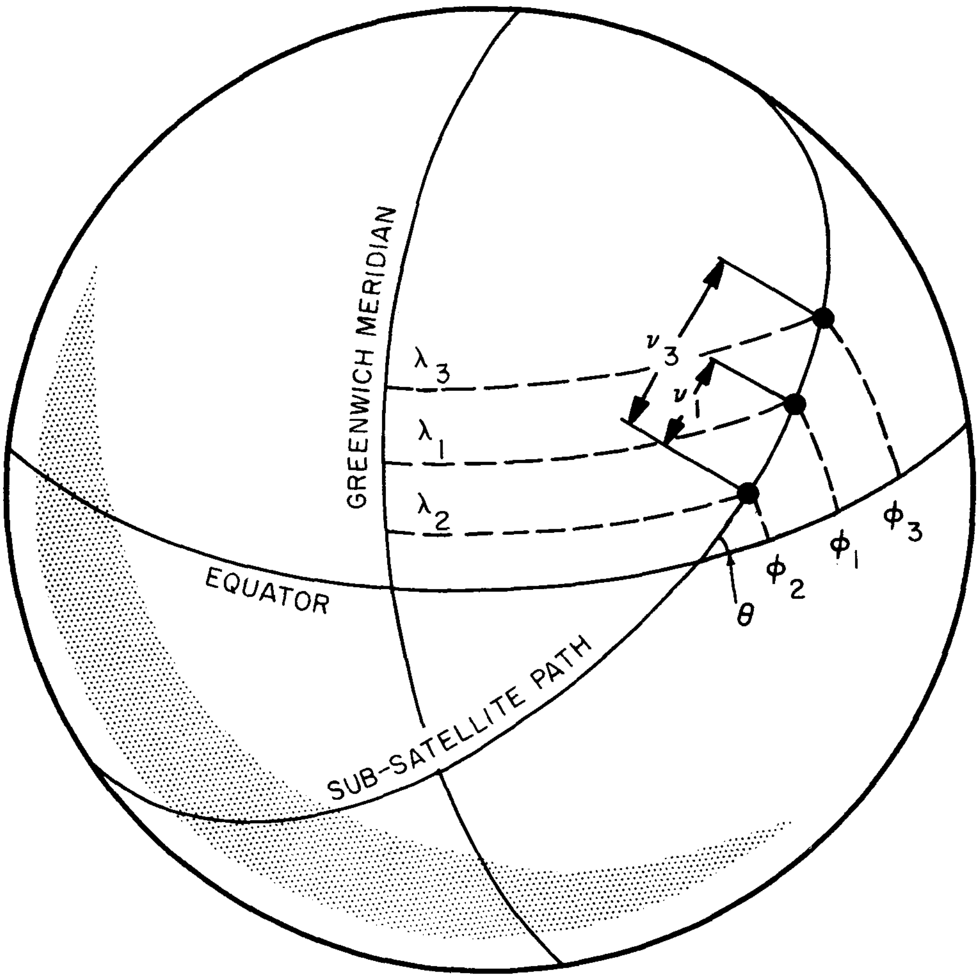

Figure 2–Angular variables used in orbital extrapolation.

Analysis of Sub-Satellite Path

Symbols used in the following analysis are defined in Table I. Figure 2 shows the sub-satellite path with the three sets of angular parameters subscripted in an order corresponding to their labels in Figure 1. Note that ν_2, the true anomaly of perigee, is zero and is not shown.

Table I
Symbols Used in Orbital Analysis

Symbol	Meaning
P	Satellite orbital period
ϵ	Orbital eccentricity
θ	Orbital plane inclination
λ_1	Sub-satellite latitude at given point
φ_1	Sub-satellite longitude at given point
r_1	Altitude above spheroid at given point
ν_1	True anomaly of given point
E_1	Eccentric anomaly of given point
t_1	Time of given point
λ_2	Sub-satellite latitude at perigee
φ_2	Sub-satellite longitude at perigee
r_2	Altitude above sphere at perigee
t_2	Time of perigee
λ_3	Sub-satellite latitude at experiment sample time
φ_3	Sub-satellite longitude at experiment sample time
r_3	Altitude above spheroid at experiment sample time
ν_3	True anomaly at experiment sample time
E_3	Eccentric anomaly at experiment sample time
t_3	Time of experiment sample
λ_{det}	Latitude in a geodetic coordinate system
λ_{cent}	Latitude in a geocentric coordinate system
r_{cent}	Geocentric distance
h	Altitude above the earth spheroid
a	Equatorial radius of earth
b	Polar radius of earth

Given the parameters of one point on the orbit and a direction of motion, enough information is now available to determine the sub-satellite location, altitude, and time of perigee, that is, the position of point 2, in Figure 1. By examining Figure 2, one may obtain the relationship

$$\nu_1 = \sin^{-1} (\sin \lambda_1 / \sin \theta) - \sin^{-1} (\sin \lambda_2 / \sin \theta) \tag{1}$$

The true anomaly of the given point, ν_1, the latitude of the given point, λ_1, and the orbital inclination, θ, are all known and hence the latitude of perigee, λ_2, may be found. Also from Figure 2, one may obtain the analogous relationship for longitude

$$\varphi_1 - \varphi_2 = \sin^{-1} (\cot \theta \tan \lambda_1) - \sin^{-1} (\cot \theta \tan \lambda_2) \tag{2}$$

Knowing the longitude of the given point, φ_1, using λ_1 and θ from above and now knowing λ_2, one solves for φ_2, the longitude of perigee.

To determine the time of perigee, the relationship between eccentric and true anomaly is used to compute the eccentric anomaly at the given point

$$E_1 = 2 \tan^{-1} \left[\left(\frac{1 - \epsilon}{1 + \epsilon} \right)^{1/2} \tan \frac{\nu_1}{2} \right] \tag{3}$$

Kepler's equation furnishes the necessary time dependence by relating eccentric anomaly with time. Solving it for the time of perigee, one finds

$$t_2 = t_1 - \frac{P}{2\pi} (E_1 - \epsilon \sin E_1) \tag{4}$$

where P is the period, t_1 the time of the given orbit point, and t_2 the time of perigee.

The equation for the radius, from the focus, of an elliptical orbit, gives the following relationship for perigee geocentric distance

$$r_2 = r_1 \frac{1 + \epsilon \cos \nu_1}{1 + \epsilon} \tag{5}$$

Here r_1 is the geocentric distance of the given point and r_2 the geocentric distance at perigee.

Information relating to point 1 (referring to Figure 1) has been used to generate the orbital parameters of point 2, satellite perigee. By use of essentially the same equations, the data relating to point 1 will now be used to compute the desired orbital information for point 3, i.e., the time of the experiment sample of interest.

The time of perigee, τ_2, is now known and a numerical solution of Kepler's equation or E_3, the eccentric anomaly at the time of the experiment sample, may be found.

$$E_3 - \epsilon \sin E_3 = \frac{2\pi}{P} (t_3 - t_2). \tag{6}$$

E_3 may now be used to compute ν_3, the true anomaly at the time of the experiment sample, from

$$\nu_3 = 2 \tan^{-1} \left[\left(\frac{1 + \epsilon}{1 - \epsilon} \right)^{1/2} \tan \frac{E_3}{2} \right] \tag{7}$$

A relationship analogous to Equation (1) exists between points 2 and 3

$$\nu_3 = \sin^{-1} (\sin \lambda_3 / \sin \theta) - \sin^{-1} (\sin \lambda_2 / \sin \theta) \tag{8}$$

and may be used to find λ_3, satellite latitude at the time of the data sample. The same analogy holds for Equation (2), producing

$$\varphi_3 - \varphi_2 = \sin^{-1} (\cot \theta \tan \lambda_3) - \sin^{-1} (\cot \theta \tan \lambda_2) \tag{9}$$

to give φ_3, the desired longitude.

For the geocentric distance of the desired point, the analog of Equation (5) is

$$r_3 = \frac{r_2 (1 + \epsilon)}{1 + \epsilon \cos \nu_3} = \frac{r_1 (1 + \epsilon \cos \nu_1)}{1 + \epsilon \cos \nu_3} \tag{10}$$

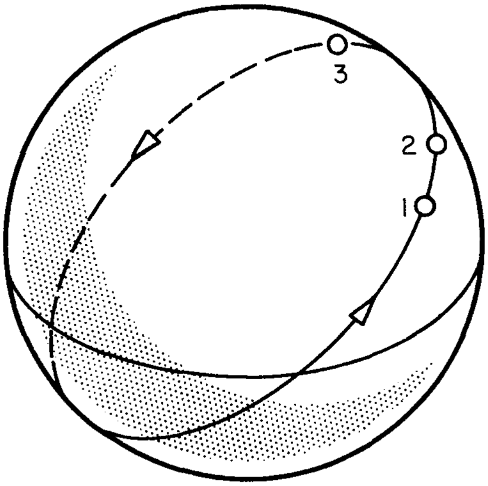

Figure 3–Potential ambiguity arising in the determination of the initial direction of Satellite motion.

Direction of Satellite Motion

It will be noticed that Figure 1 shows another point, 1', not mentioned in the schematic description of the process. The sub-satellite latitude of this point is needed to establish an initial direction of motion for the orbit. An ambiguity may occur here if the two points to be compared have latitudes on opposite "sides" of the maximum value latitude may reach, namely the orbital inclination. This situation is illustrated in Figure 3, where the northward direction associated with point 1 may be reversed if a latitude comparison is made with point 3 instead of point 2.

If the difference between successive samples of latitude is used as the criterion to determine direction, it is clear from Figure 3 that the test should be made with points on the same "side," (here points 1 and 2). Initially, two successive points from those supplied at uniform intervals must be used for this comparison. To insure that the directional ambiguity does not occur, these points must be ignored if they are in the $|\text{latitude}| \simeq$ inclination region, which will be referred to as the "inclination" region. Points sufficiently different in latitude from the value of inclination to insure that they are both on the same side must be used to generate an orbit. This approach of temporarily evading the direction problem and then coming back to the inclination region has been chosen for reasons of generality and simplicity in contrast to certain more complex difference criteria valid for only one set of orbital parameters or requiring more than two latitudes samples. Having available the parameters for any point on the orbit, it is then possible to fill in the orbital values for the experiment sample times in the inclination region by looking back in time to that area.

The comparison between given points is needed to establish an initial direction. The problem still remains, however, of associating a direction with those extrapolated points which may be near the region of inclination. For such points, computing the latitude for both the experiment data sample time and a time .1 second later allows changes in direction to be sensed.

Use of Spheroid Earth Model

The latitude, longitude, and altitude of the satellite at the time of the experiment sample have now been determined and are in a coordinate system appropriate to a spherical earth. Usually, the original orbit information from which these extrapolations are produced is in a system using a spheroidal earth model. The calculations above must be performed using "spherical" values, however, the best form for the final answers is again one referring to a spheroidal earth model. The equations for the "spherical-spheroidal" transformation, applicable for either direction, are from Cain et al. (1964).

$$\tan \lambda_{det} = \left(\frac{a}{b}\right)^2 \tan \lambda_{cent} \tag{11}$$

$$r^2_{cent} = h^2 + 2h\left(a^2 \cos^2 \lambda_{det} + b^2 \sin^2 \lambda_{det}\right)^{1/2} + \frac{a^4 \cos^2 \lambda_{det} + b^4 \sin^2 \lambda_{det}}{a^2 \cos^2 \lambda_{det} + b^2 \sin^2 \lambda_{det}} \tag{12}$$

where

λ_{det} = latitude in a geodetic coordinate system

λ_{cent} = latitude in a geocentric coordinate system

r_{cent} = geocentric distance

h = height above the earth spheroid

ACCURACY OF EXTRAPOLATION

Referring again to Figure 1, the net result of the entire process is to use point 1 (with the aid of 1') to gain information about point 3. Because of the numerous perturbing influences, the orbit is not truly a classical Keplerian ellipse and increasing the time separation of points 1 and 3 will lead to increasing error in the extrapolation process. Point 1 represents one value of a sequence of available, accurate orbital data. Normally, such points are available at uniform time intervals throughout the orbit and choosing for the extrapolation process the one closest to point 3, the experiment sample time, will minimize the error. A study has been made of the relationship between the time separation of points 1 and 3 and the error in the prediction of point 3 by predicting the satellite path for a period of several minutes. The satellite and region of orbit chosen for this study are such that the latitude and longitude change most rapidly in a small period of time. In the high latitude regions of the OGO II orbit, the traversal of latitude and longitude is at the rate of several degrees per minute. This is precisely the area in which very narrow geophysical phenomena, of less than one degree extent, have been detected. The variables thought to determine the location of these regions of rapid density variation behave in a fashion making convenient "cause and effect" tabulation very difficult. For example, neither the local time nor geomagnetic activity occurring during OGO II sampling periods varies in a manner allowing a clear-cut correlation between them and the possibly related ion density. The difficulty of obtaining an adequate set of density measurements as a function of these parameters makes accuracy in the location of these high-gradient areas a necessity if any casual explanation is to be confirmed. Orbital parameters available only once per minute will not allow the proper resolution of these sharp gradients.

Figure 4 shows the accuracy of the prediction of latitude, longitude, and altitude in these regions where they change most rapidly. Points on the orbit five minutes away from the time of the initially given point show an error of less than .01 degree in latitude, Figure 4a, and an error of less than one degree in longitude, Figure 4b. The longitude prediction improves greatly, to the same accuracy as the latitude prediction, in those non-polar regions where the change in longitude per unit time is smaller. An altitude error on the order of one kilometer is seen, also after 5 minutes. For both latitude and longitude the improvement to be gained by use of this technique as opposed to a linear interpolation between successive points is most noticeable in the high latitude regions. A linear approximation used here for the clearly non-linear time behavior of the orbit would obscure the locations of particle density gradients and the nature of their causative forces. If a linear interpolation were to be applied between consecutive minutes to the data shown in Figure 4a and 4b, an error of over one degree in latitude and over four degrees in longitude would result. The need for only one set of orbital parameters plus the latitude of a second point simplifies the computation and information handling problems associated with the process in comparison to a higher order interpolation technique. In addition, in contrast to a higher order interpolation, the method produces valid predictions in an orbit region well away from the area in which accurate information was available. Studies made with the very dissimilar orbits of OGO I and OGO II have shown the capability to predict latitude and longitude to better than one degree and altitude to 1 percent for times approaching one orbital period.

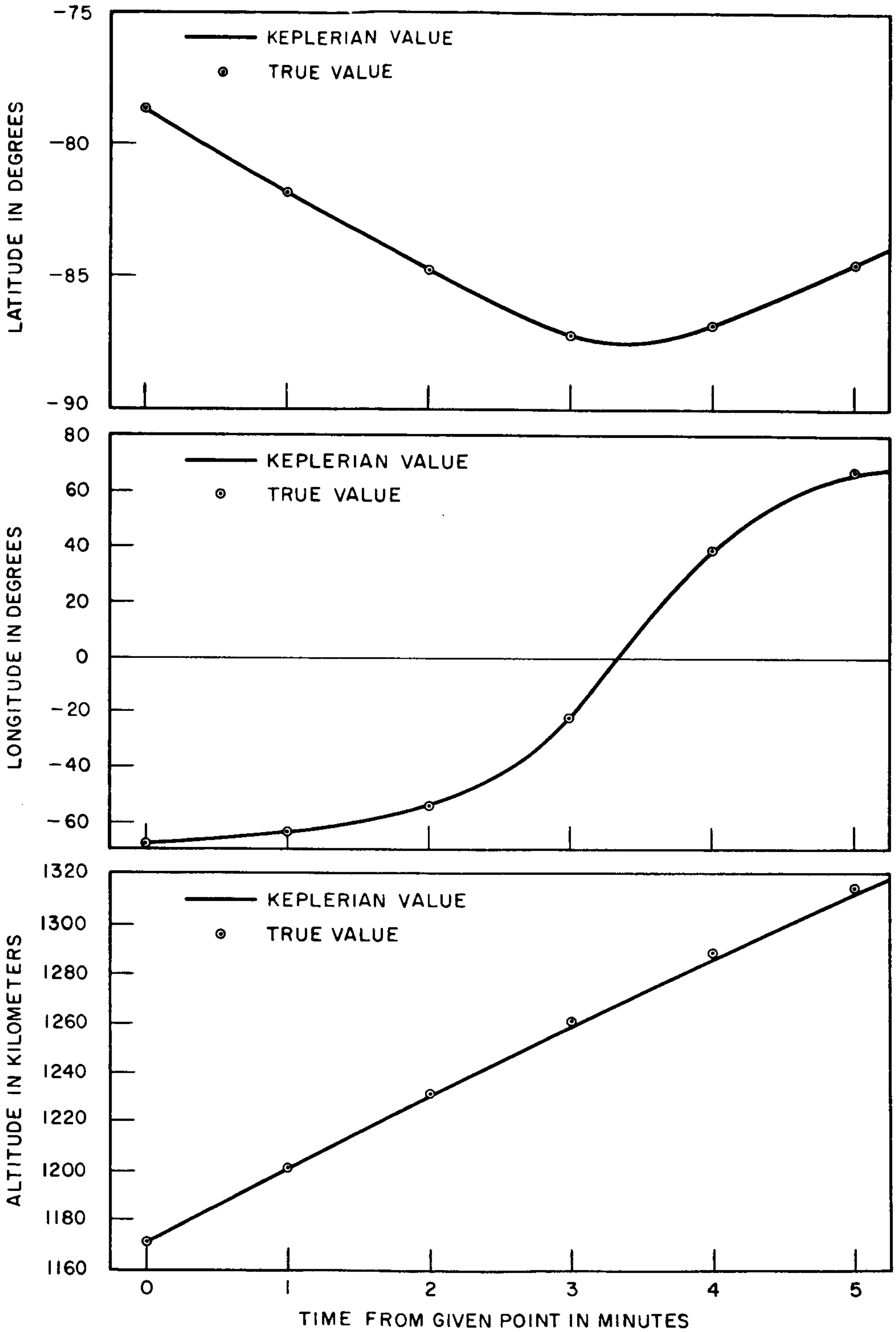

Figure 4–Comparison of extrapolated, Keplerian value of selected orbit parameters with true value for high latitude region of OGO II.

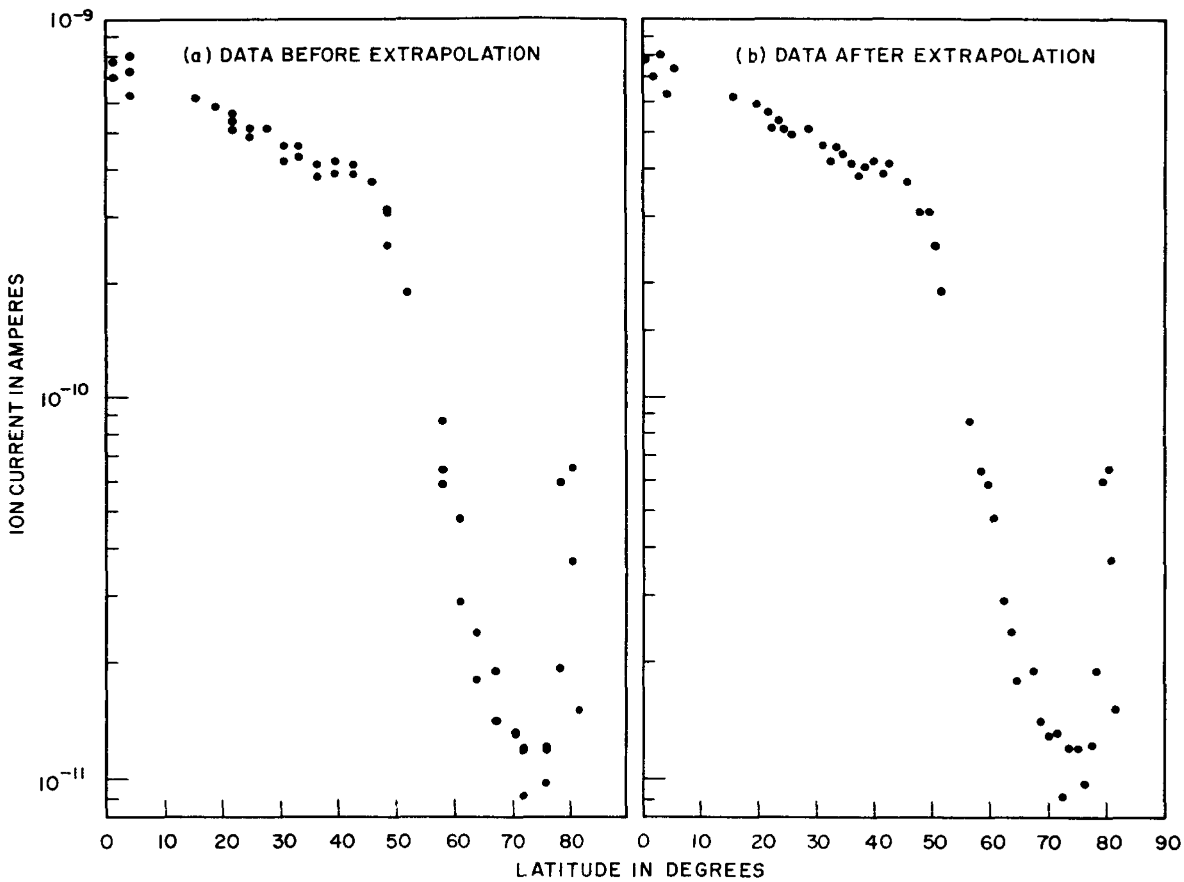

Figure 5—An example of ion current data before and after the association of latitude with the precise time of sampling.

CONCLUSION

Before this study was undertaken it was necessary to reference sampled data to the supplied orbit data having the nearest available time. If the sampling interval were less than the interval between the available orbit points, more than one experiment sample would be associated with one set of orbital parameters. The result is seen in Figure 5a where several ion density points seem to occur at the same latitude. Figure 5b shows the same data with the correct latitude computed for each experiment sample by means of the extrapolation process.

Therefore, a technique has been developed which allows the delineation of spatially narrow geophysical phenomena through the use of orbital information supplied at time intervals too large to show the sharp variations. The method is more efficient from a computational standpoint than the sole use of "sophisticated" orbit analysis or high order interpolation and more accurate than simple linear interpolation.

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