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TROPOSPHERIC AND IONOSPHERIC EFFECTS
UPON RADIO FREQUENCY (VHF-SHF) COMMUNICATION

BY
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ABSTRACT

The primary purpose of this paper is to provide a realistic estimate of the tropospheric and ionospheric influence upon the range and range-rate measurements.

In connection with the LOCAST (Location and Communication with Aircraft by Satellite Transponder) Project it was necessary to provide a basis for selecting a suitable operating frequency where the range and range-rate errors introduced by the atmosphere will allow precise determination of the location of high-speed aircraft by means of a synchronous altitude satellite.

TROPOSPHERIC AND IONOSPHERIC EFFECTS
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INTRODUCTION

The regions of the atmosphere which affect the propagation of electro-magnetic waves are the troposphere and the ionosphere.

Signal deterioration results from the fact that there exists in the atmosphere spatial inhomogeneties which are continuously varying as a function with time.

A radio wave traveling from some object in space and incident upon the earth's surface has of necessity traversed the earth's atmosphere; a region in which the refractive index may deviate significantly from unity. Particularly at frequencies below 1000 MHz the atmosphere may affect any or even all of the five parameters defining the radio wave (i.e., its amplitude, direction of propagation, phase, frequency, and polarization).

The time and space variations of these five parameters may be affected by atmospheric irregularities of various sizes and, for a moving source, even by a smooth, nonvarying ionosphere.

A study of the effects of the atmosphere on radio wave propagation necessitates a knowledge of the height-variations of the dielectric constant, or refractive index, in the tropospheric and ionospheric regions. Since the magnitude of the dielectric constant is a function of such parameters as the geographic location on the earth, weather, time of day and season of the year, it becomes an overwhelming task to completely analyze atmospheric propagational effects under all parametric conditions.

To simplify the analytical problem, atmospheric models [1,2,3] which are representative of average conditions are employed to estimate the influence of the earth's atmosphere (tropospheric and ionospheric regions) upon measurements of range and range-rate at various frequencies.

The effects of the troposphere and ionosphere upon radiowave propagation can best be predicted if an accurate refractive profile of the atmosphere is available. To a certain extent the systematic or biasing effects of the atmosphere are predictable and can be corrected for if sufficient data regarding the atmospheric state are available.

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TROPOSPHERE

Figure 1 indicates the physical relationship between the troposphere and ionosphere above the earth's surface. The index of refraction, n, in the troposphere is given by

$$n = \frac{c}{v_p} = \sqrt{\epsilon_r \mu_r},$$ (1)

where

c = speed of light,

v_p = velocity of propagation in medium of index n,

ϵ_r = relative dielectric constant of medium, and

μ_r = relative permeability of medium.

For all practical purposes the relative permeability of the troposphere is that of free space (i.e., μ_r = 1.0) and, therefore, the dielectric constant at any point within the troposphere is a measure of the index of refraction at that point. Since the index of refraction, n, associated with the atmosphere is always near unity, it is general practice to describe this index in terms of the "refractivity," N, where

$$N = 10^6 (n - 1).$$ (2)

The above expression for the refractivity of air is independent of frequency (100 MHz to 10,000 MHz).

It can be computed by means of the equation[4]

$$N = \frac{77.6}{T} \left[P + \frac{4810e}{T} \right]$$ (3)

where T is temperature in degrees Kelvin, P is total atmospheric pressure, and e is the partial pressure of water vapor, both in millibars.

Bean and Thayer [3,5] describe the tropospheric refractivity profile by the following equation:

$$N = N_s \exp \left[-k(h - h_s) \right]$$ (4)

where

N_s = refractivity at earth's surface

k = decay constant (per km)

h = height above surface corresponding to N (km)

h_s = height of reference point above mean sea level (km)

The decay constant is generally calculated from a refractivity measurement at a height of 1 km. That is,

$$k = \ell n \frac{N_s}{N_s + \triangle N} \qquad (5)$$

$N_s + \triangle N$ is the refractivity at height of 1 km above surface [3] .

Figure 2 shows a typical measured tropospheric index of refraction profile compared to exponential model. The variations in surface refractivity are directly related to overall tropospheric profile variations, and hence to the error introduced in ranging measurements, by means of the exponential profile ray tracings of B. R. Bean and G. D. Thayer [3] .

The National Bureau of Standards has compiled a book of surface refractivity charts and data based upon eight years of surface weather data from 60 weather stations in the United States and five years from 306 worldwide stations [6] .

TROPOSPHERIC RANGE ERROR

The tropospheric range measurement error is primarily due to the electrical path length difference between a line-of-sight path measured in terms of the velocity of light in vacuum and the line-of-sight electrical path length due to the actual velocity of propagation associated with the troposphere.

When electromagnetic waves are propagated through a medium whose dielectric constant or index of refraction is a varying function of the path, the wave undergoes a change in direction, or refractive bending, and a retardation in the velocity of propagation.

The path length between groundstation and target is given by

$$R_o = \int_1^2 ds = \text{line-of-sight range (or true range)} \tag{6}$$

The "apparent range" due to a velocity of propagation less than that of light in vacuum is given by

$$R = \int_1^2 n\,ds = \text{apparent range}. \tag{7}$$

The range error $\triangle R$ is the difference between apparent range R and the true range R_0.

$$\triangle R = \int_1^2 (n - 1)\,ds \tag{8}$$

Using (2) and $ds = dh / \sin \psi$

where

ψ = elevation angle,

h_1 = height of surface above sea level,

h_2 = target height,

$$\triangle R \doteq \frac{10^{-6}}{\sin \psi} \int_{h_1}^{h_2} N\,dh \tag{9}$$

It can be shown (reference [8]) that equation (8) can, for elevation angles $\psi > 10$ degrees, be approximated by (9), which for $\psi > 10$ degrees accounts for 97% of the range error.

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By using (4), equation (9) can be written as,

$$\Delta R \doteq \frac{370.10^{-6}}{0.161 \sin \psi} \left[1 - \exp(-0.161\, h_2) \right] \text{ when } h_s \doteq h_1 \doteq o.$$

For $h \gqq 30.5$ km $\exp(-0.161 \times 30.5) = \exp(-4.91) \doteq 0$

$$\Delta R = \frac{370.10^{-6}}{0.161 \sin \psi}$$

Calculated Values:

Elevation angle ψ (degrees)	1	5	10	15	20	25	30
Range error ΔR (meters)	132	26.3	13	8.8	6.7	5.4	4.6

Equation (9) indicates that the range error is a function of the refractivity profile and elevation angle. Figures 3a and 3b show tropospheric range errors represented by the exponential model [3] and by the standard atmospheric models [7].

Tropospheric range errors decrease rapidly with increasing elevation angle and they are independent of frequency. The tropospheric error indicated is appropriate for lunar distances as well as earth orbits.

IONOSPHERE

In the troposphere the index of refraction is independent of frequency. For the ionosphere, however, the refractive index of the medium is a function of the transmitted frequency.

The effects of the ionosphere can be described to a sufficient approximation for $f \geq 100$ MHz by ascribing to the ionosphere an equivalent index of refraction given by

$$n = \frac{c}{v_p} = \left[1 - \frac{N_e\, e^2}{\epsilon_0\, m\, \omega^2} \right]^{1/2} \qquad \text{(10) (Reference 9)}$$

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where

v_p = phase velocity,

c = speed of light in vacuum,

N_e = electron density = electron/meter^3,

e = electron charge = 1.602×10^{-19} Coulombs,

m = electron mass = 9.11×10^{-31} kilograms

ϵ_0 = free space dielectric constant = 8.855×10^{-12} farad/meter.

IONOSPHERIC RANGE ERROR

The ionospheric range error due to time delay can be calculated, when

$$R_0 = \int_1^2 ds = \text{line of sight range (or true range)}$$

and the "apparent range" due to a phase velocity greater than that of light in vacuum, is given by

$$R = \int_1^2 \frac{ds}{n} = \text{measured range,}$$

where n = ionosphere index of refraction ($n < 1$).
Equation of R describes the apparent increase in range due to a group velocity less than the speed of light. The ionospheric range error ΔR is given by

$$\Delta R = R - R_0 = \int_1^2 \left(\frac{1}{n} - 1 \right) ds \qquad (11)$$

By using (2) and (10), equation (11) can be developed as,

$$n = \left[1 - \frac{N_e e^2}{\epsilon_0 m \omega^2} \right]^{1/2} \doteq 1 - \frac{N_e e^2}{2\epsilon_0 m \omega^2} \ldots ; \qquad N = (n-1) \cdot 10^6$$

$$\frac{1}{n} \doteq 1 + \frac{N_e e^2}{2\epsilon_0 m \omega^2}$$

$$\Delta R = \int_1^2 \frac{N_e e^2}{2\epsilon_0 m \omega^2} \, ds \qquad\qquad \omega = 2\pi f$$

$$\Delta R = \frac{1.6 \times 10^3}{(2\pi f)^2} \int_1^2 N_e \, ds \tag{12}$$

Reference (10) shows that if ψ is the angle between the horizontal and the direction of propagation, then

$$\int_1^2 N_e \, ds = (1 - 0.928 \cos^2 \psi)^{-1/2} \int N_e \, dh \tag{13}$$

The electron density, N_e, is a function of time of day, the magnitude of recent sun spot activity, the effect of local ionospheric storms, and, the local latitude, therefore, the integrated electron density will vary in a non-deterministic manner as a function of time. Several investigators [10] have observed a strong diurnal variation in the total electron content of the ionosphere. They agree that the daytime peak (early afternoon) is six to eight times greater than the nighttime minimum.

The highest values of $\int N_e \, dh$ occur, as expected, in the early afternoon with a range of $5.5 - 8.5 \times 10^{17}$ electrons per square meter (satellite height between 1227 - 1350 km).

A mean value of $\int N_e \, dh = 1.1 \times 10^{17}$ electrons per square meter for midnight [11,12]. Reference [13] gives the integrated electron density as a function of time of day measured on Sputnik III. An average value of $\int N_e \, dh$ is shown to

be about 2.4×10^{17} electrons per square meter; with a maximum value of approximately 8×10^{17} electrons per square meter, depending on the time of day.

Equation (12) can be written for elevation angle $\psi = 0$,

$$\Delta R = \frac{152}{f^2} \int N_e \, dh$$

where f in c/s,

ΔR in meters,

$$\int N_e \, dh \quad \text{in electrons per square meter.}$$

Ionospheric range error for elevation angle $\psi = 25°$ can be computed from Table 3 by using the following relation,

$$\Delta R_{(\psi = 25°)} = 0.55 \, \Delta R_{(\psi = 0°)}$$

Figure 4 shows the ionosphere range error for zero degree elevation angle and various integrated electron densities versus frequencies.

Range biasing effects due to the atmosphere decreases with increasing elevation angle ψ. The tropospheric range error is frequency independent while ionospheric range error is inversely proportional to the square of the operating frequency.

It is evident that the ionospheric range error is a maximum during the daytime. The major portion of the time delay occurs below an altitude of 500 km. At 200 MHz, for a one-way transmission path, the maximum error (curve A) ΔR is on the order of 910 meters. This value is approximately eight times larger than the maximum error introduced by the troposphere. At a frequency of about 575 MHz the tropospheric and ionospheric contributions to the propagational time delays are of the same order of magnitude. Above 575 MHz the tropospheric error is the more predominant one.

TROPOSPHERIC AND IONOSPHERIC RANGE-RATE ERROR

1. TROPOSPHERIC RANGE-RATE ERROR

The one way range error ΔR due to the troposphere was

$$\Delta R = \int_1^2 (n - 1) \, ds \tag{8}$$

The tropospheric refraction contribution to the Doppler shift of the signal received from a passing satellite for a transmitter frequency f, is

$$\Delta f_{tro} = -\frac{f}{c} \left[\frac{\partial(\Delta R)}{\partial t} \right] \tag{14}$$

where

$$\Delta f_{tro} = \text{tropospheric Doppler frequency error,}$$

$$\partial(\Delta R)/\partial t = \Delta(\partial R/\partial t) = \Delta \dot{R} = \text{range-rate error,}$$

$$\partial R/\partial t = \dot{R} = \text{range-rate.}$$

As previously mentioned, the refractivity N is independent of frequency, therefore, the Doppler frequency error due to the troposphere is directly proportional to frequency.

The computed refraction contributions to the Doppler shift produced by the troposphere as shown in Table 4 has been derived from the observed data by Hopfield[14]. Table 4 indicates that the tropospheric Doppler frequency error decreases very rapidly with increasing elevation angle.

2. IONOSPHERIC RANGE-RATE ERROR

The ionospheric range error ΔR is given by

$$\Delta R = R - R_0 = \int_1^2 \left(\frac{1}{n} - 1 \right) ds \tag{11}$$

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or,
$$\Delta R = \frac{1.6 \times 10^3}{(2\pi f)^2} \int_1^2 N_e ds .$$
(12)

Similar to Δf_{tro}, by using (12) we can write for the ionospheric Doppler error Δf_{ion},

$$\Delta f_{ion} = -\frac{f}{c} \left[\frac{\partial(\Delta R)}{\partial t} \right] \text{ where } \frac{\partial(\Delta R)}{\partial t} = \Delta \left(\frac{\partial R}{\partial t} \right) = \Delta \dot{R} \text{ range-rate error}$$

$$\Delta f_{ion} = -\frac{f}{c} \frac{1.6 \times 10^3}{(2\pi f)^2} \left[\frac{\partial}{\partial t} \int N_e ds \right]$$

$$\Delta f_{ion} = -\frac{1.6 \times 10^3}{(2\pi)^2 f c [1 - 0.928 \cos^2 \psi]^{1/2}} \left[\frac{\partial}{\partial t} \int N_e dh \right]$$
(15)

Δf_{ion}, the ionospheric Doppler frequency error is at maximum during the daytime and at low elevation angles. The ionospheric Doppler frequency error is inversely proportional to frequency.

A study and an experimental investigation of the behavior of ionospheric contributions to the Doppler shift was done at the University of Texas [15]. The purpose of this study was to present experimental data which illustrate the separation of frequency dependent refraction errors. The experimental data gave evidence of the presence of ionospheric refraction errors of unusually high magnitude.

The following data in Table 5 are derived from this study by assuming the maximum observed values of total refraction error, first order refraction error, and third order refraction error, 27, 18.5, and 10 c/s respectively, referenced to 54 MHz.

The values of Table 5 exceed previous estimates by a substantial amount [16, 17]. During periods where the ionosphere is not seriously disturbed, the magnitude of the various ionospheric contributions to the Doppler shift are most likely a factor of 3 — 10 lower.

The maximum range-rate error due to the ionosphere for 0°, and 25° and 70° elevation angle versus frequency can be seen in Figure 5.

Figure 6 shows a comparison of tropospheric and ionospheric range-rate errors versus elevation angle at 136 MHz and 1,500 MHz. It can be seen at a frequency of 1,500 MHz or higher the tropospheric range-rate error will predominate by a factor approximately four or more for elevation angle of zero, but with increasing elevation angle the tropospheric range-rate error falls off quite rapidly. As seen in Figure 6 the tropospheric range-rate errors at 136 MHz and 1,500 MHz are equal because they are frequency independent.

Errors in range and range-rate measurement caused by propagation effects through the atmosphere are to some extent computable errors and, therefore, reducible errors. The extent to which it is reducible depends on how accurately one can know tropo- and ionospheric characteristics over the region of interest. It is well known that seasonal day-to-day and diurnal variations in the integrated electron density occur, so that a continuous monitoring would be required. (The maximum standard deviation of the diurnal changes is about 30 percent.)

At the present only the troposphere refractivity profile is known with sufficient accuracy to permit adjustment of error data.

Models based on average daytime and nighttime ionosphere electron density profiles, however, are useful in estimating average biasing effects which can be attributed to the ionosphere. However, correction procedures based on use of ionospheric profiles have not proved very accurate and may not lead to any significant reduction in error[18]. The inability to predict the error encountered at a given time indicates that accurate systems must operate at frequencies high enough to reduce the initial values of ionospheric error to a tolerable value.

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Table 1

Tropospheric range errors for a standard atmosphere with 100% relative humidity, one way transmission path at the altitude $\geq$ 30.5 km

ψ (degrees)	0	1	2	3	5	10	20
ΔR (meters)	116	73	51.3	38.2	26	12.8	7.6

Table 2

Tropospheric range error for a standard atmosphere with 0% relative humidity, one way transmission path at the altitude $\geq$ 30.5 km.

ψ (degrees)	0	1	2	3	5	10	20
ΔR (meters)	88.5	60	44	33.8	22	11.6	4.6

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Table 3

Ionospheric Range Error at 0° Elevation Angle, Various
Integrated Electron Densities, and Frequencies

| Frequency f (MHz) | A. Average Value $\int N_e\,dh = 2.4 \times 10^{17}$ el/m² ΔR (meters) | DAYTIME | | MIDNIGHT |
		B. Mean Value $\int N_e\,dh = 6.9 \times 10^{17}$ el/m² ΔR (meters)	C. Maximum Value $\int N_e\,dh = 8.5 \times 10^{17}$ el/m² ΔR (meters)	D. Mean Value $\int N_e\,dh = 1.1 \times 10^{17}$ el/m² ΔR (meters)
100	3,640	10,200	12,700	1,670
136	1,960	5,650	6,860	902
200	910	2,620	3,182	418
400	228	657	798	105
500	154	447	539	71
800	56.8	164	199	26
1,000	36.4	102	127	16.7
1,500	16.2	46.7	56.7	7.5
2,000	9.1	26.2	31.8	4.2

Table 4

Maximum Ionospheric and Tropospheric Doppler Errors ($\triangle f$) and Range-Rate Errors ($\triangle \dot{R}$):

IONOSPHERE

Frequency 136 MHz	ELEVATION ANGLE (degrees)					
	0	5	15	25	35	70
Doppler Error $\triangle f_{ion}$ (c/s)	28	26.3	20.7	15.4	12.30	7.98
Range-Rate Error $\triangle \dot{R}$ (meters/sec)	62	58	45.7	34	27.10	17.60
Frequency 1,500 MHz	ELEVATION ANGLE (degrees)					
	0	5	15	25	35	70
Doppler Error $\triangle f_{ion}$ (c/s)	2.34	2.2	1.74	1.3	1.03	0.67
Range-Rate Error $\triangle \dot{R}$ (meters/sec)	0.47	0.43	0.35	0.26	0.21	0.13

TROPOSPHERE

Frequency 136 MHz	ELEVATION ANGLE (degrees)					
	0	2	5	8	15	
Doppler Error $\triangle f_{tro}$ (c/s)	0.88	0.38	0.13	0.06	0.025	
Range-Rate Error $\triangle \dot{R}$ (meters/sec)	1.96	0.83	0.28	0.14	0.05	
Frequency 1,500 MHz	ELEVATION ANGLE (degrees)					
	0	2	5	8	15	
Doppler Error $\triangle f_{tro}$ (c/s)	9.8	4	1.4	0.7	0.27	
Range-Rate Error $\triangle \dot{R}$ (meters/sec)	1.96	0.8	0.28	0.14	0.05	

Table 5

Maximum Ionospheric Doppler Errors (Range-Rate Errors)

Elevation Angle $\psi = 0°$	FREQUENCY (MHz)								
	54	100	136	150	324	400	1,000	1,500	2,000
Doppler Error Δf_{ion} (c/s)	100	40.5	28	24.8	11	8.8	3.5	2.34	1.75
Range-Rate Error $\Delta \dot{R}$ (meters/sec)	552	121.5	62	49.7	10.18	6.67	1.15	0.47	0.26

Elevation Angle $\psi = 70°$	FREQUENCY (MHz)								
	54	100	136	150	324	400	1,000	1,500	2,000
Doppler Error Δf_{ion} (c/s)	28.5	11.57	7.98	7.1	3.13	2.52	1.01	0.67	0.5
Range-Rate Error $\Delta \dot{R}$ (meters/sec)	158	34.70	17.60	14.20	2.90	1.90	0.30	0.13	0.07

APPARENT
PATH

RAY
PATH

DIRECT PATH
(OR LINE OF
SIGHT)

$n=1$

IONOSPHERE $(n<1)$

$n=1$

FREE SPACE $(n=1)$

TROPOSPHERE $(n>1)$

$\Delta\psi$

ψ

HORIZON

O

1000 km

85 km $(N \doteq 0.004)$

30.5 km $(N \doteq 4)$

20 km $(N \doteq 20)$

10 km $(N \doteq 90)$

GROUND $N_S \doteq 313$

T

$n =$ INDEX OF REFRACTION

$N = (n-1)\, 10^6$

$\psi =$ ELEVATION ANGLE

$\Delta\psi =$ ELEVATION ANGLE ERROR
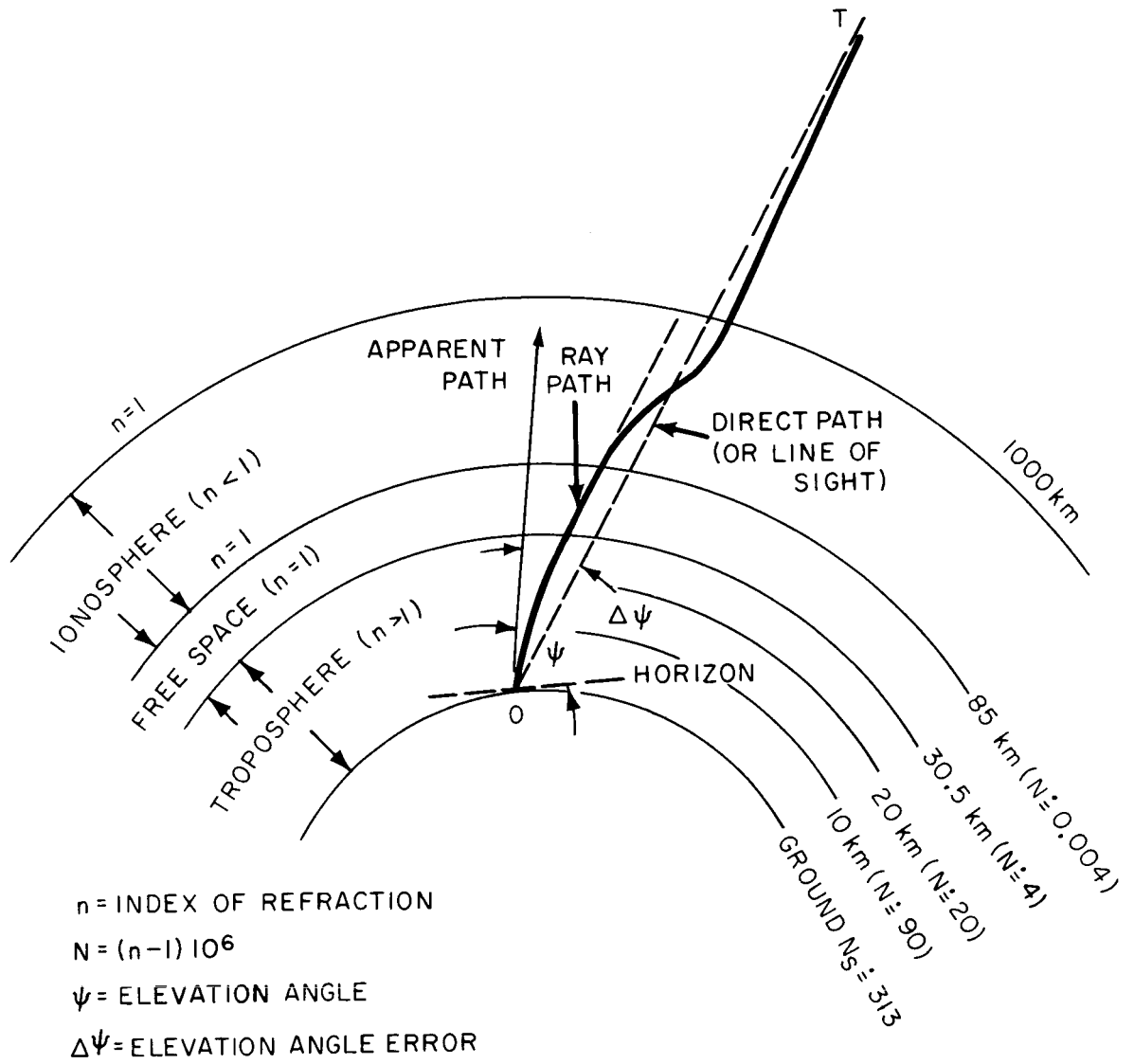
Figure 1—Typical Ray Path Trajectory Through the Atmosphere
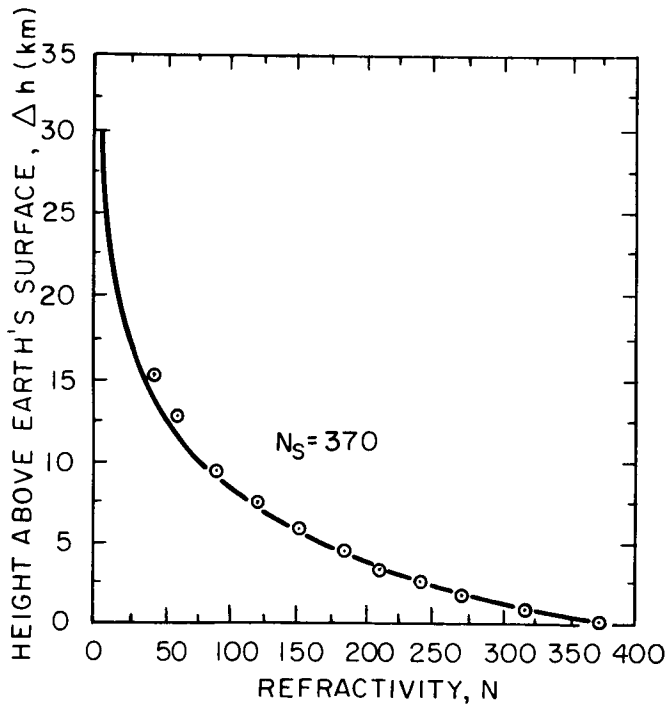
CALCULATED EXPONENTIAL PROFILE (NBS)

$N = N_S \exp(-k\Delta h) = 370 \exp(-0.161\Delta h)$

Δh = HEIGHT ABOVE STATION (km)

$K = \ln \dfrac{N_S}{N_S + \Delta N}$

N_S = 370 (SURFACE REFRACTIVITY)

ΔN = CHANGE IN N FOR Δh = 1 km.

$\Delta N = -55$

⊙ MEASURED PROFILE USING RADIOSONDE 8 AUGUST 1963, VALKARIA, FLORIDA. (REFERENCE 18, FIGURE 2.)

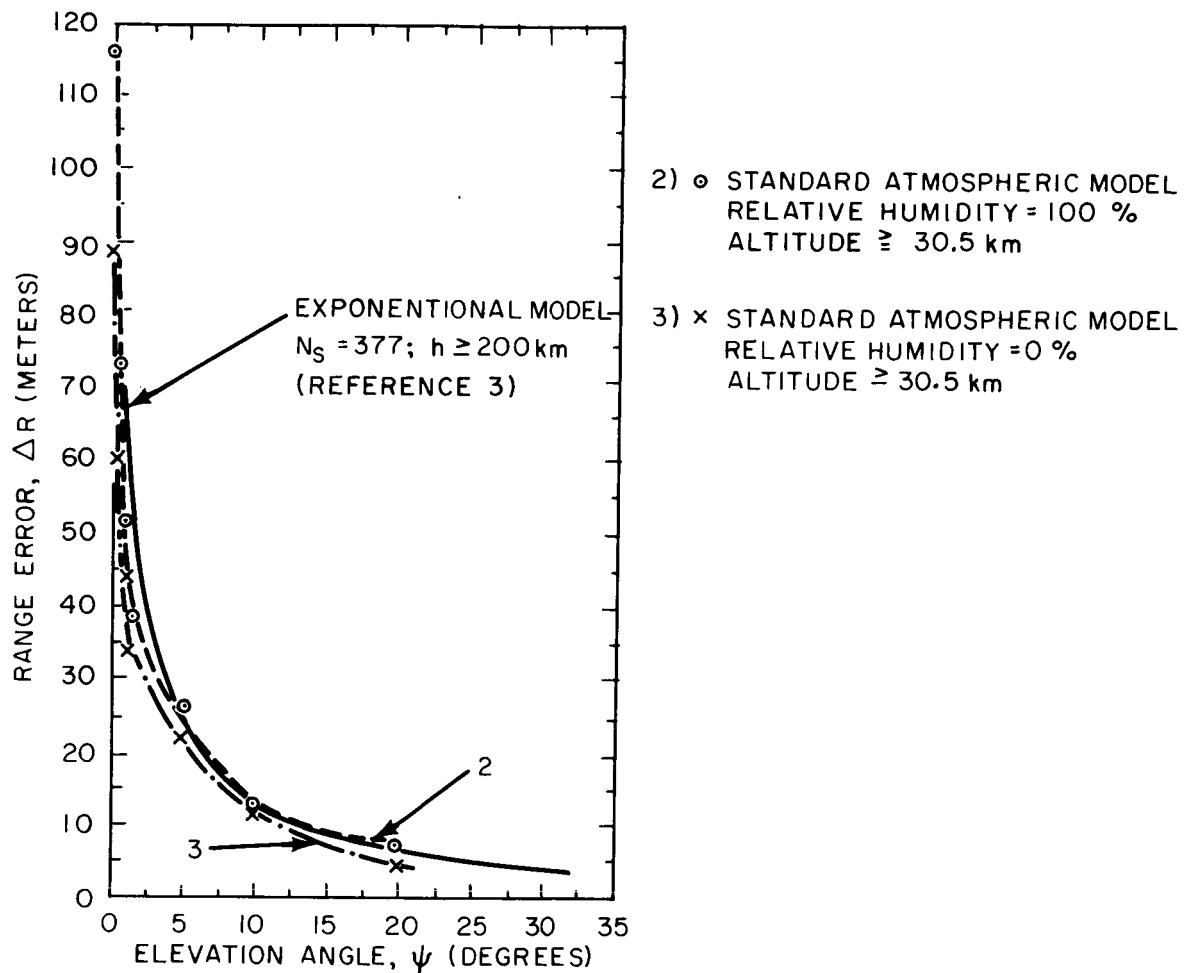Figure 2—Typical Measured Tropospheric Index of Refraction Profile Compared to Exponential Model.

Figure 3a—Tropospheric Range Error Vs. Elevation Angle.
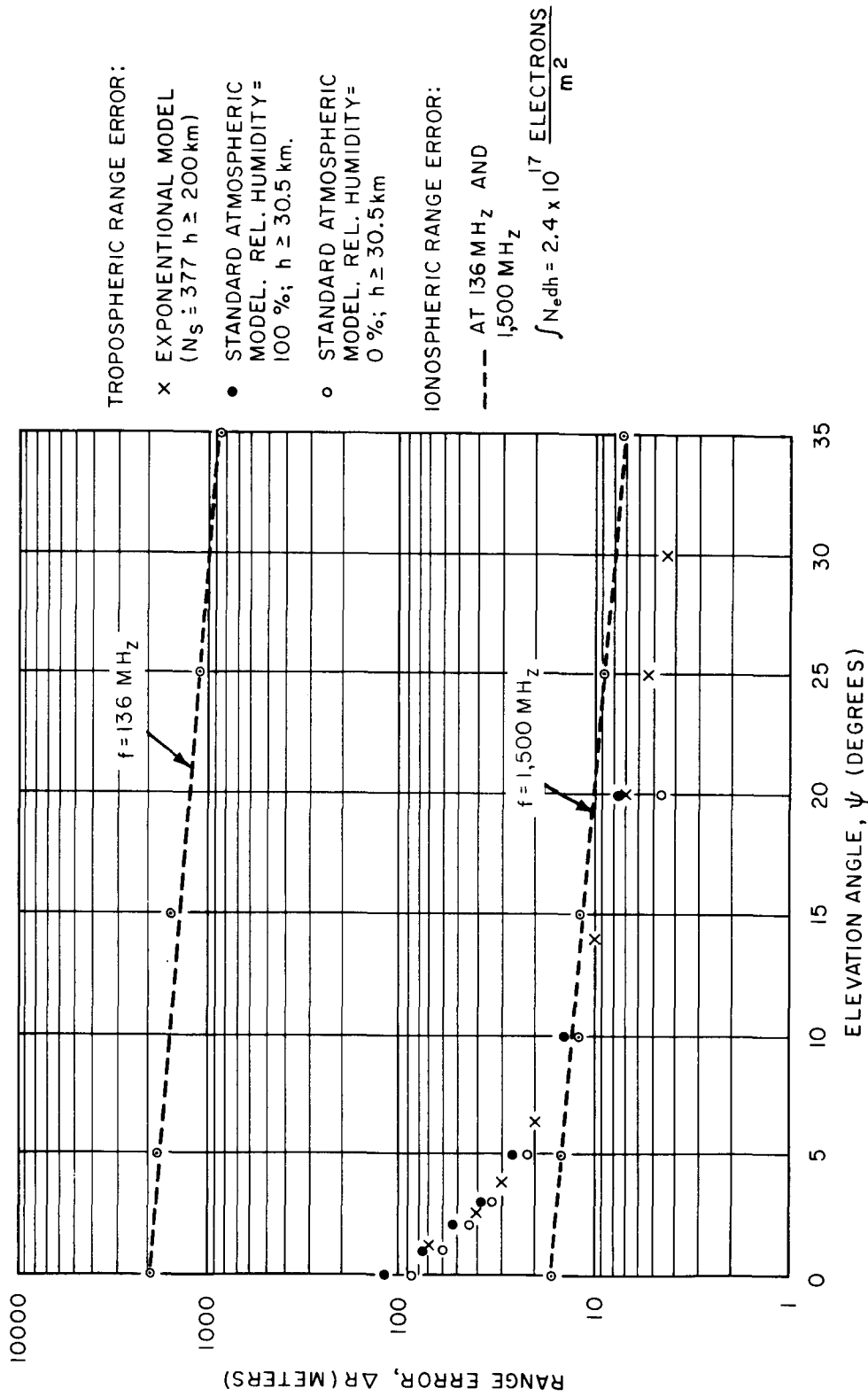
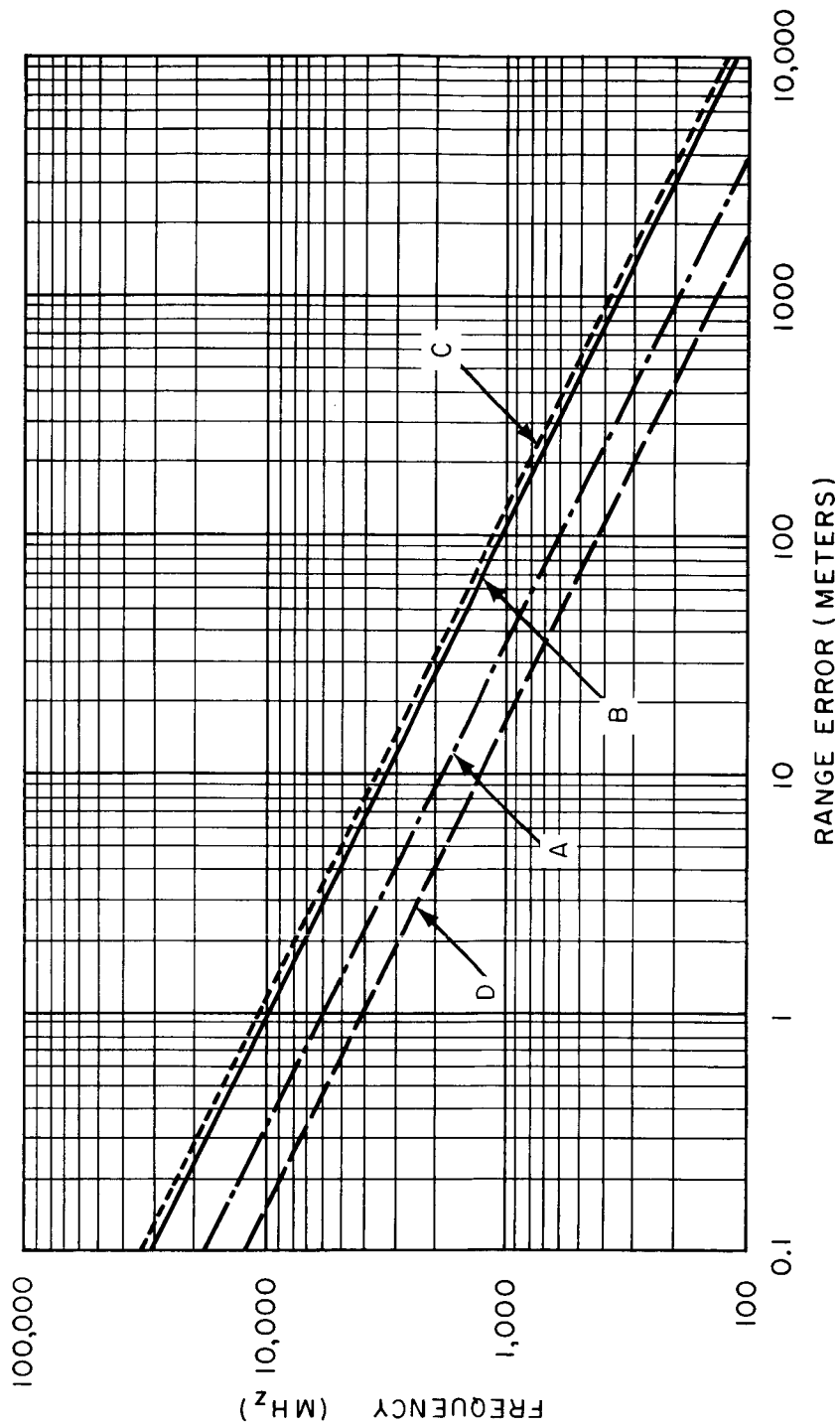Figure 3b—Tropospheric and Ionospheric Range Errors Vs. Elevation Angle.

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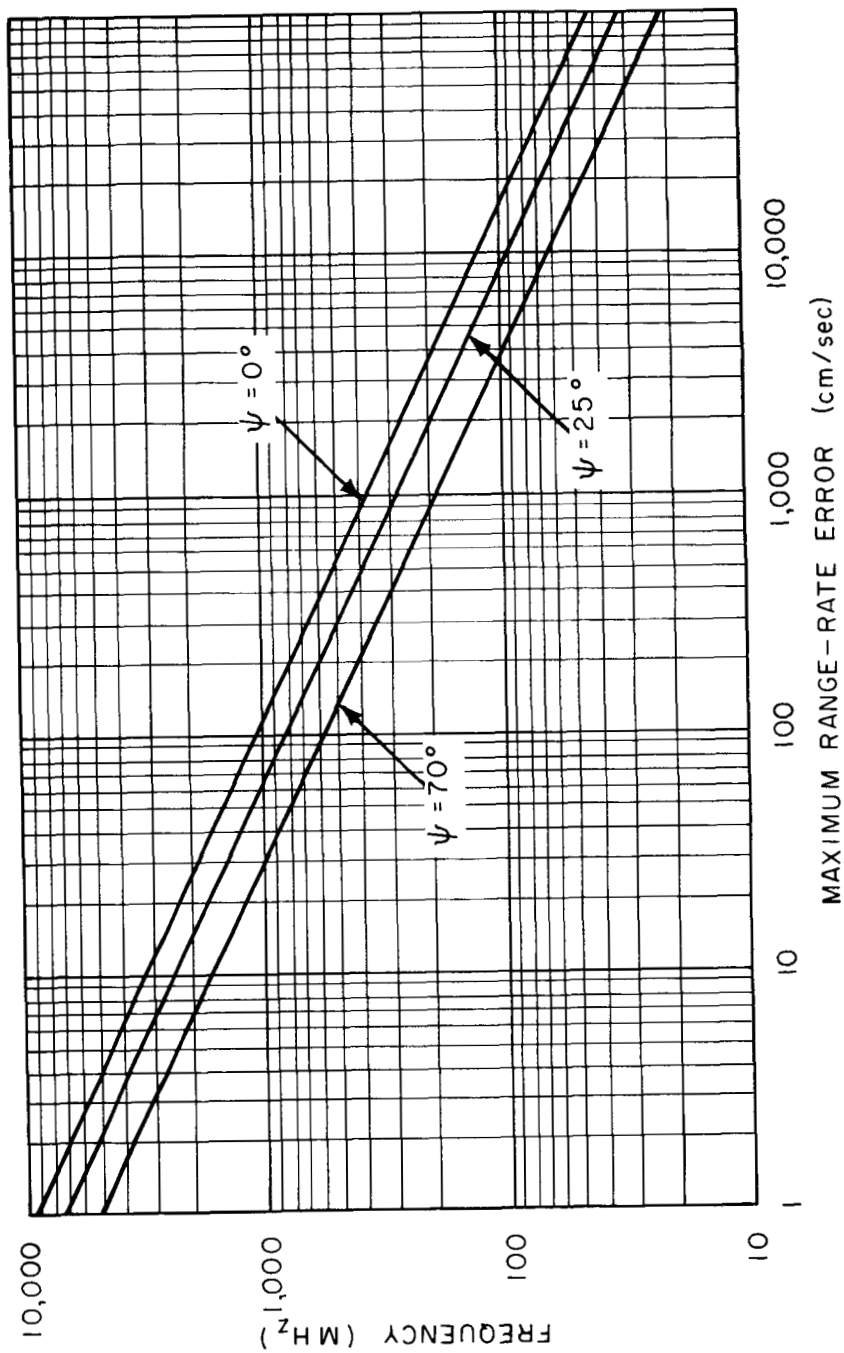Figure 4–Ionospheric Range Error for 0° Elevation and Various Integrated Electron Densities.

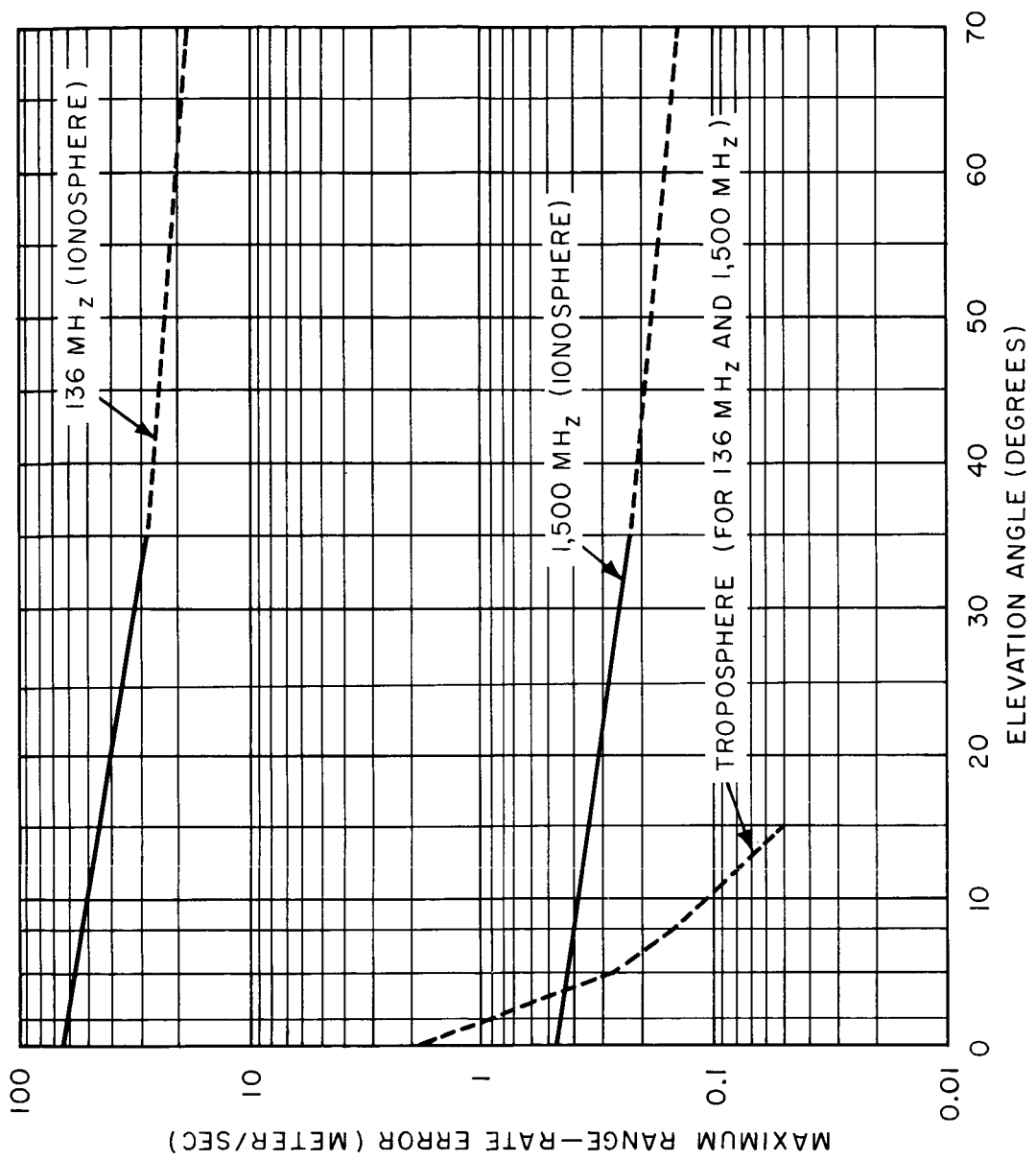Figure 5—Ionospheric Maximum Range-Rate Error for 0°, 25° & 70° Elevation Angle.

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Figure 6—Comparison of Tropospheric and Ionospheric Range-Rate Error
Vs. Elevation Angle (at 136 MHz and 1,500 MHz).