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THE MAIN SECTION OF AXISYMMETRICAL JETS OF AN INCOMPRESSIBLE FLUID FLOWING FROM AN APERTURE OF FINITE SIZE INTO A CO-AXIAL UNIFORM FLOW OF THE SAME FLUID

by

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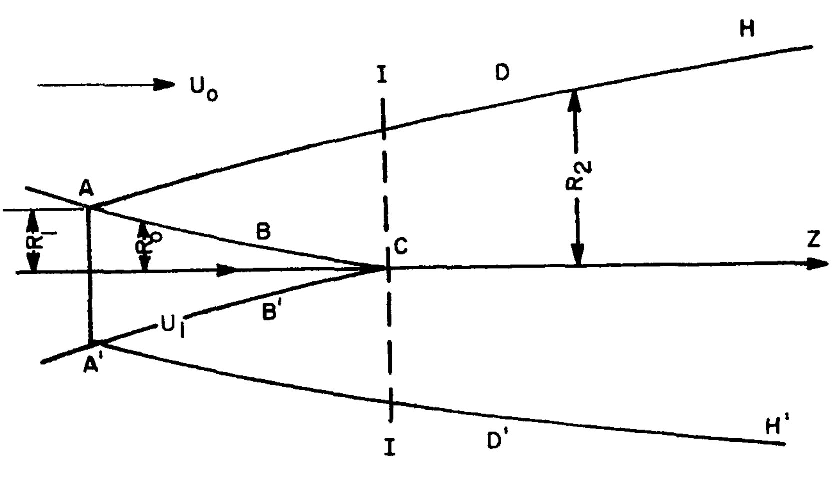
The existing approximate solutions of this problem are based on the use of predetermined velocity profiles in transverse sections of the flow[1,2]. The present article proves that a solution can also be obtained by strict integration of equations dealing with the boundary layer.

1. Integral Relations and Posing of the Boundary Value Problem

We shall use a cylindrical system of coordinates z, R, and φ, and shall consider the jet emerging in the direction of the Oz axis--the axis of symmetry--from a round aperture with a radius R_1 into a co-axial flow of the same fluid, uniform and one-dimensional at infinity, and possessing a velocity U_0, parallel to the Oz axis. Both in the exit cross section of the jet and in the cross sections of its potential core, we shall consider the velocity as being the same--parallel to the Oz axis and equal to U_1. The region of turbulent mixing is separated from the jet core by surface A A' B B' C, and from the external potential-flow by surface A A' D D' H H' (see figure, below). The radii of these surfaces of revolution are designated, respectively, R_0 and R_2. The region of turbulent mixing, as usual, shall be divided into an initial sector, which extends from section A A' to the end of the core, and the main section, which lies beyond the end point C of the flow core.



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[1] H. B. Squire and I. Trouncer, ROUND JETS IN A GENERAL STREAM, Reports and Memoranda, A.R.C., 1944, No. 1974.

[2] G. N. Abramovich, THEORY OF TURBULENT FLOWS (Teoriya turbulentnykh struy), Fizmatgiz, 1960.

If we overlook changes in the jet pressure and consider the mean flow as established, we can obtain relationship[3,4]

$$2\pi\int_0^{R_2} \rho V_z^2 R dR - 2\pi U_0 \int_0^{R_2} \rho V_z R dR = \rho U_1 (U_1 - U_0) \pi R_1^2, \qquad (1.1)$$

which reflects the law of the conservation of linear momentum (where ρ is the mass density of the fluid and V_z is the mean value of the longitudinal component of velocity at point z, R). However, instead of using the mean components of total velocity, V_z and V_R, it is more convenient to use the components U and V, defined as

$$U \equiv V_z - U_0, \qquad V = V_R . \qquad (1.2)$$

By substituting (1.2) in (1.1), we get the corresponding equation for the conservation of linear momentum expressed in terms of (1.2):

$$2\pi\int_0^{R_2} \rho U^2 R dR + 2\pi U_0 \int_0^{R_2} \rho U R dR = \rho U_1 (U_1 - U_0) \pi R_1^2 . \qquad (1.3)$$

We shall assume the following boundary conditions: a smooth transition of longitudinal velocity at the jet boundary to a velocity of a uniform, unidirectional flow, and the continuity of the flow along its axis (M-const)--

$$U\Big|_{R=0} < M, \quad \frac{\partial U}{\partial R}\Big|_{R=0} = 0, \quad V\Big|_{R=0} = 0, \quad U\Big|_{R=R_2} = 0, \quad \frac{\partial U}{\partial R}\Big|_{R=R_2} = 0 . \qquad (1.4)$$

2. Dimensionless Equation of Motion

We select radius R_1 of the exit cross section as the scale of length and the corresponding velocity U_1 at the exit point as the scale of speed. Passing to dimensionless variables, we get:

$$x \doteq \frac{z}{R_1}, \qquad r = \frac{R}{R_1}, \qquad r_2 = \frac{R_2}{R_1}, \text{ etc;} \qquad (2.1)$$

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[3] Squire and Trouncer, loc. cit.
[4] Abramovich, loc. cit.

$$\upsilon_z = \frac{V_z}{U_1}, \quad u = \frac{U}{U_1}, \quad \upsilon = \frac{V}{U_1}, \text{ etc.} \tag{2.2}$$

The quantities J_i and Q_i, having dimensions of momentum (flux) and mass (flux), are nondimensionalized using J_0 and Q_0, the momentum (flux) and mass (flux) at the exit cross section.

Thus: $$J_0 = \rho U^2{}_1 \pi R_1{}^2, \quad Q_0 = \rho U_1 \pi R_1{}^2, \tag{2.3}$$

and the corresponding dimensionless quantities are

$$i_i = \frac{J_i}{\rho U_1{}^2 \pi R_1{}^2}, \qquad q_i = \frac{Q_i}{\rho U_1 \pi R_1{}^2} \ . \tag{2.4}$$

Dividing both parts of equation (1.3) by $\rho U_1{}^2 \pi R_1{}^2$ and using (2.1) and (2.2), we obtain the following statement of conservation of momentum:

$$2\int_0^{r_2} u^2 r dr + 2\lambda \int_0^{r_2} u r dr = 1 - \lambda \qquad \left(\lambda = \frac{U_0}{U_1}\right) . \tag{2.5}$$

For an established mean flow of an axisymmetrical jet, ignoring changes in pressure and using Prandtl's formula for turbulence friction, we obtain the following equations[5]:

$$V_z \frac{\partial V_z}{\partial z} + V_R \frac{\partial V_z}{\partial R} = -\frac{1}{R}\frac{\partial}{\partial R}\left[R l^2 \left(\frac{\partial V_z}{\partial R}\right)^2\right],$$

$$\frac{\partial V_z}{\partial z} + \frac{\partial V_R}{\partial R} + \frac{V_R}{R} = 0, \tag{2.6}$$

which may be restated in terms of (1.2) as

$$U_0 \frac{\partial U}{\partial z} + U \frac{\partial U}{\partial z} + V \frac{\partial U}{\partial R} = -\frac{1}{R}\frac{\partial}{\partial R}\left[R l^2 \left(\frac{\partial U}{\partial R}\right)^2\right],$$

$$\frac{\partial U}{\partial z} + \frac{\partial V}{\partial R} + \frac{V}{R} = 0, \tag{2.7}$$

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[5] L. G. Loytsyanskiy, FLUID AND GAS MECHANICS (Mekhanika zhidkosti i gaza), Fizmatgiz, 1959.

where l is the mixing length in the main section of the jet, regarded as being proportional to R_2, stated[6] as

$$l = cR_2 \qquad \text{(c-const)}. \tag{2.8}$$

Substituting (2.8) into (2.7) and using (2.1), (2.2), and (2.5) as well as taking into account that R_2 is a function of z only, we get the following equations for the main section of the jet:

$$\lambda \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial x} + \upsilon \frac{\partial u}{\partial r} = - c^2 r_2^2 \left[\frac{1}{r} \left(\frac{\partial u}{\partial r}\right)^2 + 2 \frac{\partial u}{\partial r} \frac{\partial^2 u}{\partial r^2} \right], \tag{2.9}$$

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial r} + \frac{\upsilon}{r} = 0. \tag{2.10}$$

3. Criterion of Similarity

The general character of the mean flow at large distances from the exit section of the jet ought to be determined by the quantities J, U_0, and ρ from which, however, it is impossible to construct a dimensionless parameter (J, the total momentum of the jet). From (1.1) we have $J = \rho U_1 (U_1 - U_0) \pi R_1^2$. From this and (2.3) and (2.5) it follows that

$$J = J_0 (1 - \lambda). \tag{3.1}$$

Therefore, the assignment of a value to J is reduced to assigning values to J_0 and to λ, and--as stated previously--the determining factors here are λ, J_0, U_0, and ρ, from which it is impossible to deduce any dimensionless parameter other than λ itself.

For this reason, the criterion of similarity of mean flows in two turbulent jets, 1 and 2, is the existence of the following condition:

$$\lambda^{(1)} = \lambda^{(2)}. \tag{3.2}$$

One of the conclusions which can be drawn from (3.2) is the well-known fact that all turbulent jets emerging into a stationary fluid are dynamically similar (for all such jets, $\lambda = 0$). As is known, the laws governing the motion of the fluid (in the mean) in turbulent jets

[6] Squire and Trouncer, loc.cit.

emerging into a homogenous collinear flow [7, 8], depend primarily upon the value of λ.

4. Form of the Bounding Surface of the Turbulence Area

Let us consider that the equation of boundary surface of a turbulence area can be given in the form of a series

$$r_2 = \alpha x^k + \ldots . \tag{4.1}$$

(On the right side of this expression is entered only the leading term of the expansion of r_2 in powers of x, where α is a dimensionless coefficient.)

Introducing the quantities

$$\langle u \rangle = \frac{2}{r_2^2} \int_0^{r_2} ur\,dr, \qquad \langle u^2 \rangle = \frac{2}{r_2^2} \int_0^{r_2} u^2 r\,dr = \xi^2 \langle u \rangle^2 \tag{4.2}$$

we may write (2.5) in this form[9]:

$$\langle u \rangle^2 + \frac{\lambda}{\xi^2} \langle u \rangle - \frac{1 - \lambda}{\xi^2 r_2^2} = 0 \tag{4.3}$$

where $\langle u \rangle$ and $\langle u^2 \rangle$ are clearly mean quantities associated with flux of the fluid (mass) and flux of momentum. [In (4.3) the coefficient ξ, which, according to (4.2), takes into account the difference between $\langle u \rangle$ and $\langle u^2 \rangle^{\frac{1}{2}}$, is close to 1.] From the quadratic equation (4.3) it follows that

$$\langle u \rangle = - \frac{\lambda}{2\xi^2} + \left(\frac{\lambda^2}{4\xi^4} + \frac{1 - \lambda}{\xi^2 r_2^2} \right)^{\frac{1}{2}} \tag{4.4}$$

Since the turbulence area spreads as the distance from the exit section increases, it follows that when $\lambda \gg 0$ and when the distance from the exit point is also sufficiently great, the second summand under the radical in (4.4) becomes much smaller than the first one.

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[7] Squire and Trouncer, loc. cit.
[8] Abramovich, loc. cit.
[9] Here and in all that follows, $\langle \ \rangle$ designates means.

Expanding the radical in a series of powers of r_2^{-2}, we get:

$$\langle u \rangle = \frac{a_1}{r_2^2} + \frac{a_2}{r_2^4} + \dots, \qquad a_2 = a_2(\lambda). \tag{4.5}$$

From (4.5) there follows an estimate of the order of magnitude of u when x is large:

$$\langle u \rangle \sim r_2^{-2}. \tag{4.6}$$

In estimating the mean value of υ with the aid of (2.10), we find

$$\langle \upsilon \rangle \sim \langle u \rangle r_2 / x. \tag{4.7}$$

In estimating orders of magnitude for the various terms, we take[10]

$$u \sim \langle u \rangle, \quad \upsilon \sim \langle \upsilon \rangle, \quad \frac{\partial}{\partial x} \sim \frac{1}{x}, \quad \frac{\partial}{\partial r} \sim \frac{1}{r_2}, \quad \frac{1}{r} \sim \frac{1}{r_2}.$$

And in estimating the order of magnitude of various individual summands of the left-hand side of the equation of motion (2.9) and using (4.7), we get

$$\lambda \frac{\partial u}{\partial x} \sim \frac{\langle u \rangle}{x}, \qquad u \frac{\partial u}{\partial x} \sim \frac{\langle u \rangle^2}{x}, \qquad \upsilon \frac{\partial u}{\partial r} \sim \frac{\langle u \rangle^2}{x}. \tag{4.8}$$

From (4.6) and from the fact that r_2 increases with x, it follows that when x is sufficiently large, $\langle u \rangle$ is small, hence,

$$\langle u \rangle^2 < \langle u \rangle. \tag{4.9}$$

The expressions (4.8) and (4.9) lead to the estimate

$$u \frac{\partial u}{\partial x} + \upsilon \frac{\partial u}{\partial r} < \lambda \frac{\partial u}{\partial x},$$

which shows--along with (4.8)--that the left-hand side of the equation (2.9) is on the order of $x^{-1} \langle u \rangle$. Both summands of the right-hand side of equation (2.9) are of the same order, $r_2^{-1} \langle u \rangle^2$. Comparing the orders of magnitude of both sides, we get

$$\langle u \rangle \sim r_2 x^{-1}. \tag{4.10}$$

[10] L. D. Landau and E. M. Livshits, MECHANICS OF CONTINUOUS MEDIA (Mekhanika sploshnykh sred), Gostekhizdat, 1954.

And from (4.10)--as well as (4.6) and (4.1)--it follows that

$$r_2 = \alpha x^{1/3} + \dots . \tag{4.11}$$

Substituting (4.11) into (4.5), we find

$$\langle u \rangle = b_1 x^{-2/3} + b_2 x^{-4/3} + \dots, \qquad b_i = b_i(\lambda) . \tag{4.12}$$

From this it can be assumed that all dimensionless functions defining jet flow are representable in series of powers of $x^{-2/3}$. (This is, in fact, later confirmed.) Hence,

$$r_2 = \alpha(\lambda) \left[x^{1/3} + \beta(\lambda) x^{-1/3} + \gamma(\lambda) x^{-1} + \dots\right]. \tag{4.13}$$

The initial series (4.5) is valid when λ is sufficiently greater than 0 and when the values of x are sufficiently large. For this reason, an adequate correlation of the resulting series can be expected only within the range of the values of λ and x as indicated above. When λ is close to zero, it is difficult to estimate the order of magnitude of the summands under the radical in (4.4) or to determine the type of series for $\langle u \rangle$. As a result, one cannot successfully solve this problem when λ is close to zero. For the case of an axisymmetrical jet emerging into a stationary fluid (i.e., for the case when $\lambda = 0$), there exists the Tolmin solution[11]. Index k in equation (4.1), defining the contour of the boundary surface, in this case equals 1. It is obvious that in the range of small values of λ, the power index k in equation (4.1) varies from $k = 1$ when $\lambda = 0$ to $k = 1/3$, for levels of λ for which the solution given here already applies.

5. Expansion in Series of the Equations of Motion and of Those Defining Boundary Conditions

Equations (2.10), (4.12), and (4.13) show that dimensionless flow function ψ, which is connected with the components of velocity u and υ by the equations

$$u = \frac{1}{r}\frac{\partial \psi}{\partial r}, \qquad \upsilon = -\frac{1}{r}\frac{\partial \psi}{\partial x} \tag{5.1}$$

can be found in the form of an expansion of

$$\psi = \psi_1 + \psi_2 x^{-2/3} + \psi_3 x^{-4/3} + \dots . \tag{5.2}$$

[11]Abramovich, loc. cit.

Here the functions ψ_1 depend only on the variable η, which is determined by the equation

$$\eta = \frac{r}{r_2} = \frac{r}{a\left|x^{1/3} + \beta x^{-1/3} + \gamma x^{-1} + \dots\right|}$$

$$= \frac{r}{a}\left|x^{-1/3} - \beta x^{-1} + (\beta^2 - \gamma)\, x^{-5/3} + \dots\right| . \qquad (5.3)$$

With the aid of equations

$$\psi_1^* = \psi_1, \qquad \psi_2^* = \psi_2 c^{4/3}, \qquad \psi_3^* = \psi_3 c^{8/3} \dots, \qquad (5.4)$$

$$a^* = a c^{-2/3}, \qquad \beta^* = \beta c^{4/3}, \qquad \gamma^* = \gamma c^{8/3} \dots, \qquad (5.5)$$

$$x^* = x c^2, \qquad \partial(\cdot)/\partial x = c^2\, \partial(\dots)/\partial x^* \qquad (5.6)$$

it is possible to convert to new dimensionless functions ψ_1^* as well as to coefficients a^*, β^*, γ^* ... and the longitudinal coordinate x^*, thus excluding the structural constant c from all equations.

By substituting (5.4), (5.5), and (5.6) in (4.13), (5.2), and (5.3), we find[12]:

$$r_2 = a\left|x^{1/3} + \beta x^{-1/3} + \gamma x^{*-1} + \dots\right|, \qquad (5.7)$$

$$\psi = \psi_1 + \psi_2 x^{-2/3} + \psi_3 x^{-4/3} + \dots, \qquad (5.8)$$

$$\eta = a^{-1}\, r\left|x^{-1/3} - \beta x^{-1} + (\beta^2 - \gamma)\, x^{-5/3} + \dots\right| . \qquad (5.9)$$

We now use (5.6) and (5.9) to obtain the following for our operations of differentiation:

$$\frac{\partial}{\partial r} = \frac{1}{a}\left|x^{-1/3} - \beta x^{-1} + (\beta^2 - \gamma)\, x^{-5/3} + \dots\right| \frac{d}{d\eta} , \qquad (5.10)$$

$$\frac{\partial}{\partial x} = c^2\left\{\left(\frac{\partial}{\partial x}\right) + \left[-\frac{1}{3}x^{-1} + \frac{2}{3}\beta x^{-5/3} - \left(\frac{2}{3}\beta^2 - \frac{4}{3}\gamma\right)x^{-7/3} + \dots\right]\eta\frac{d}{d\eta}\right\} . \qquad (5.11)$$

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[12]For the sake of simplicity, from now on we shall omit the asterisk from ψ_1, ψ_2, ψ_3, a, β, γ, and x. These values will be presumed to be those defined by equations (5.4), (5.5), and (5.6).

With the aid of (5.2) and (5.8) through (5.11), we obtain the necessary series

$$u = \frac{1}{a^2}\left\{\Phi_1 x^{-2/3} + [\Phi_2 - 2\beta\Phi_1] x^{-4/3} + [\Phi_3 - 2\beta\Phi_2\right.$$

$$\left. + (3\beta^2 - 2\gamma)\Phi_1] x^{-2} + \ldots\right\}, \tag{5.12}$$

$$\frac{\partial u}{\partial r} = \frac{1}{a^3}\left\{\Phi_1' x^{-1} + [\Phi_2' - 3\beta\Phi_1'] x^{-5/3} + [\Phi_3' - 3\beta\Phi_2'\right.$$

$$\left. + (6\beta^2 - 3\gamma)\Phi_1'] x^{-7/3} + \ldots\right\}, \tag{5.13}$$

$$\frac{\partial^2 u}{\partial^2 r} = \frac{1}{a^4}\left\{\Phi_1'' x^{-4/3} + [\Phi_2'' - 4\beta\Phi_1''] x^{-2} + [\Phi_3'' - 4\beta\Phi_2''\right.$$

$$\left. + (10\beta^2 - 4\gamma)\Phi_1''] x^{-9/3} + \ldots\right\}, \tag{5.14}$$

$$\frac{\partial u}{\partial x} = \frac{c^2}{3a^2}\left\{[-\Phi_1'\eta - 2\Phi_1] x^{-5/3} + [-\Phi_2'\eta - 4\Phi_2 + 5\beta\Phi_1'\eta + 8\beta\Phi_1] x^{-7/3}\right.$$

$$+[-\Phi_3'\eta - 6\Phi_3 + 5\beta\Phi_2'\eta + 12\beta\Phi_2 - (14\beta^2 - 7\gamma)\Phi_1'\eta$$

$$\left. - (18\beta^2 - 12\gamma)\Phi_1] x^{-3} + \ldots\right\}, \tag{5.15}$$

$$\upsilon = \frac{c^2}{3a}\left\{\psi_1' x^{-4/3} + \left[\psi_2' - 3\beta\psi_1' + \frac{2\psi_2}{\eta}\right] x^{-2}\right.$$

$$\left. + \left[\psi_3' - \beta\psi_2' + (5\beta^2 - 5\gamma)\psi_1' + \frac{4\psi_3}{\eta} - \frac{2\beta\psi_2}{\eta}\right] x^{-8/3} + \ldots\right\} \tag{5.16}$$

wherein, for purposes of brevity, are introduced--alongside the functions ψ_1--also the functions

$$\Phi_1 = \psi_i'/\eta . \tag{5.17}$$

The prime mark (') used in (5.13) through (5.17) and everywhere hereafter indicates the derivatives for η.

By substituting (5.7), (5.9), and (5.12) through (5.16) into the equation of motion (2.9) and then adjusting the coefficients for equal powers of x in both the left-hand and the right-hand parts of this equation, we obtain a group of ordinary differential equations for functions ψ_1--as well as for functions Φ_1, which are connected with the former by (5.17).

Thus, in adjusting the coefficients for $x^{-5/3}$, we get the following equation for the function Φ_1:

$$\tfrac{1}{3}\lambda\alpha^3\,[\Phi_1\eta^2]' = [\Phi_1'^2\eta]' \,. \tag{5.18}$$

After adjusting the coefficients for $x^{-7/3}$ and using equation (5.18), we can obtain a differential equation for function ψ_2,

$$\frac{2}{\alpha^3}\,[\Phi_1'\Phi_2'\eta]' - \frac{\lambda}{3}\,[\psi_2'\eta + 2\psi_2]' = \frac{2}{3\alpha^2}\,\frac{\psi_1'^2}{\eta} + \frac{\lambda\beta}{3}\,[\psi_1'\eta]' \,; \tag{5.19}$$

while adjusting the coefficients for x^{-3} and using (5.18) and (5.19) leads to a differential equation for function ψ_3,

$$\begin{aligned}[2\Phi_1'\Phi_3'\eta]' - \frac{\lambda\alpha^3}{3}\,[\psi_3'\eta + 4\psi_3]' &= \frac{\lambda\alpha^3\beta}{3}\,[\psi_2'\eta]' + 2\lambda\alpha^3\beta\psi_2' \\ &- [\Phi_2'^2\eta]' - \frac{2\alpha}{3}\,[\psi_2\Phi_1]' - \frac{\lambda\alpha^3}{3}\,(\beta^2 + \gamma)\,[\psi_1'\eta]' \\ &+ \frac{8\alpha}{3}\,\psi_2'\Phi_1 - \frac{2\alpha\beta}{3}\,\frac{\psi_1'^2}{\eta} \,. \end{aligned} \tag{5.20}$$

From (5.3) it follows that the flow axis on boundary η has the following values:

$$\eta\big|_{r=0} = 0, \qquad \eta\big|_{r=r_2} = 1 \,. \tag{5.21}$$

Converting the foregoing to dimensionless values with the aid of (2.1) and (2.2), for conditions of (1.4), and using (5.21), we get the following boundary conditions:

$$\begin{aligned} &u\big|_{\eta=0} < M, \quad \frac{\partial u}{\partial r}\bigg|_{\eta=0} = 0, \quad \upsilon\big|_{\eta=0} = 0, \quad u\big|_{\eta=1} = 0, \\ &\frac{\partial u}{\partial r}\bigg|_{\eta=1} = 0 \quad (M = \text{const}) \,. \end{aligned} \tag{5.22}$$

We now substitute (5.12) into (2.5) and with the use of (5.9), (5.17), and (5.21) find that the left-hand part of (2.5) contains terms with x^0, $x^{-2/3}$, $x^{-4/3}\ldots$, while the right-hand side contains one term with x^0. Adjusting the coefficients for equal powers of x on both sides of the equation, we get

$$2\lambda\int_0^1 \psi_1'\,d\eta = 1 - \lambda, \quad \int_0^1 \left[\frac{\psi_1'^2}{a^2\eta} + \lambda\psi_2'\right] d\eta = 0$$

$$\int_0^1 \left[\frac{2\psi_2'\psi_1' - 2\beta\psi_1'^2}{a^2\eta} + \lambda\psi_3'\right] d\eta = 0 \quad . \tag{5.23}$$

Substituting series (5.12), (5.13), and (5.16) into (5.22), using (5.17) and adjusting the coefficients for similar powers of x in the left-hand and right-hand sides of the resulting expressions, we find the boundary conditions for functions Φ_1 and ψ_1 to be as follows:

$$\Phi_1\big|_{\eta=0} < M = \text{const}, \quad \Phi_1'\big|_{\eta=0} = 0, \quad \psi_1'\big|_{\eta=0} = 0, \quad \psi_1'\big|_{\eta=1} = 0,$$

$$\psi_1''\big|_{\eta=1} = 0; \quad \dot{\Phi}_2\big|_{\eta=0} < M, \quad \Phi_2\big|_{\eta=0} = 0, \quad \psi_2^1\big|_{\eta=0} = 0,$$

$$\eta^{-1}\psi_2\big|_{\eta=0} = 0, \quad \psi_2'\big|_{\eta=1} = 0, \quad \psi_2''\big|_{\eta=1} = 0; \quad \Phi_3\big|_{\eta=0} < M, \tag{5.24}$$

$$\Phi_3'\big|_{\eta=0} = 0, \quad \psi_3'\big|_{\eta=0} = 0, \quad \eta^{-1}\psi_3\big|_{\eta=0} = 0, \quad \psi_3'\big|_{\eta=1} = 0,$$

$$\psi_3''\big|_{\eta=1} = 0.$$

6. First Approximation

Integrating (5.18), we get

$$\tfrac{1}{3}\lambda a^3\Phi_1\eta^2 = \Phi_1'^2\eta + C_1 \; . \tag{6.1}$$

By assuming, in (6.1) that $\eta = 0$ and using (5.24), we find that the integration constant $C_1 = 0$, whereupon (6.1) takes this form:

$$\Phi_1'^2 = \tfrac{1}{3}\lambda a^3\Phi_1\eta \; . \tag{6.2}$$

In the cross sections of the flow $\partial u/\partial r \leqslant 0$ (assuming λ <) and $\alpha > 0$. Therefore, it follows from (5.13) that $\Phi_1' \leqslant 0$. Taking this into account, and extracting the root from both parts of (6.2), we get:

$$\Phi_1' = -\sqrt{{}^1\!/_3\lambda\alpha^3\,\eta\,\Phi_1} \quad . \tag{6.3}$$

Integrating (6.3) for η, we find $\Phi_1{}^{1/2} = C_2 - \sqrt{{}^1\!/_{27}\lambda\alpha^3\eta^3}$. From this it follows, by using (5.17) that

$$\psi_1' = \Phi_1\eta = \eta\,\left|C_2 - \sqrt{{}^1\!/_{27}\lambda\alpha^3\cdot\eta^3}\right|^2 \quad \text{or } \psi_1' = {}^1\!/_{27}\lambda\alpha^3\eta\,\left|1 - \eta^{3/2}\right|^2. \tag{6.4}$$

Integration constant C_2 was determined under condition $\psi_1'\big|_{\eta=1}\big| = 0$.

Constant α is determined from the integration condition (5.23) into which we substitute (6.4) and on computing the integral find

$$\alpha = [105(1 - \lambda) \;/\; \lambda^2]^{1/3} \quad . \tag{6.5}$$

With the aid of (6.4) it is easy to see that all remaining conditions of (5.24) are thereupon automatically satisfied.

7. Second Approximation

With the aid of (6.4), we get

$$\int\frac{\psi_1'^2}{\eta}\,d\eta = \frac{\lambda^2\alpha^6}{2\cdot 729}\,\eta^2\left[1 - \frac{16}{7}\,\eta^{3/2} + \frac{12}{5}\,\eta^3 - \frac{16}{13}\,\eta^{9/2} + \frac{1}{4}\,\eta^6\right] + \text{const.} \tag{7.1}$$

By integrating (5.19) and utilizing (7.1), we get the following equation:

$$\frac{2}{\alpha^3}\,[\Phi_2'\Phi_1'\eta] - \frac{\lambda}{3}\,[\psi_2'\eta + 2\psi_2] = \frac{\lambda\beta}{3}\,\psi_1'\eta + \frac{\lambda^2\alpha^4}{2187}\,\eta^2\,\Bigg[1 - \frac{16}{7}\,\eta^{3/2}$$
$$+ \frac{12}{5}\,\eta^3 - \frac{16}{13}\,\eta^{9/2} + \frac{1}{4}\,\eta^6\Bigg] + D_1 \quad . \tag{7.2}$$

If we assume in (7.2) that $\eta = 0$ and use (5.24), we see that the integration constant $D_1 = 0$. Taking this into account and using (5.17) and (6.4), we obtain from (7.2)

$$\left(1 - \eta^{3/2}\right)\left(\psi_2'' \eta - \psi_2'\right) + {}^3/_2\, \eta^{1/2}\left(\psi_2'\eta + 2\psi_2\right) = -\frac{1}{18}\lambda a^3\beta\eta^{5/2}\left(1 - \eta^{3/2}\right)^2$$

$$- \frac{1}{2\cdot 243}\lambda a^4\eta^{5/2}\left(1 - \frac{16}{7}\eta^{3/2} + \frac{12}{5}\eta^3 - \frac{16}{13}\eta^{9/2} + \frac{1}{4}\eta^6\right) . \qquad (7.3)$$

The coefficients of the linear differential equation (7.3) and its right-hand side are polynomials, the subsequent terms of which contain η in a power which is greater by $^3/_2$ than the preceding term. The structure of the polynomials is such that a general solution of equation (7.3) can be sought in the form of a product of $\eta^{1/2}$ and an infinite series in powers of $^3/_2$. It is easy to see that the infinite series in the present case becomes a polynomial, and that the solution looks as follows:

$$\psi_2 = A_1\eta^2 + A_2\eta^{7/2} + A_3\eta^5 + A_4\eta^{13/2} + A_5\eta^8 . \qquad (7.4)$$

Here, A_i represents the dimensionless constant coefficients (All A_i are alike in that they are equal to zero, when $i > 5$). Solution of (7.4) satisfies all boundary conditions of (5.24) when $\eta = 0$. By substituting (7.4) in (7.3) and adjusting the coefficients for the highest power of $\eta_1^{7/2}$ in both parts of the equation, we find

$$A_5 = \frac{1}{8\cdot 729\cdot 11}\lambda a^4 \qquad (7.5)$$

Adjusting the coefficients for η^7, $\eta^{11/2}$, η^4, and $\eta^{5/2}$ and utilizing, consecutively, (7.5) . . . , (7.6), we then get:

$$A_4 = -\frac{4\cdot 31}{729\cdot 121\cdot 13}\lambda a^4, \qquad A_3 = \frac{58}{243\cdot 5\cdot 121}\lambda a^4 + \frac{1}{81}\lambda a^3\beta$$

$$(7.6)$$

$$A_2 = -\frac{250}{729\cdot 7\cdot 121}\lambda a^4 - \frac{2}{81}\lambda a^3\beta*, \qquad A_1 = \frac{1}{729\cdot 121}\lambda a^4 + \frac{1}{81}\lambda a^3\beta$$

To determine constant β, we shall now turn to the integral expression (5.23), which on the basis of $\psi_2 |_{\eta = 0}$ and of (7.1), assumes this form:

$$\psi_2 |_{\eta=1} = -\frac{1}{\lambda a^2}\int_0^1 \frac{\psi_1'^2}{\eta}\, d\eta = -\frac{\lambda a^4}{120\cdot 7\cdot 13} . \qquad (7.7)$$

From (7.4) and (7.7) it follows:

$$A_1 + A_2 + \ldots + A_5 = -\frac{\lambda a^4}{120\cdot 7\cdot 13} . \qquad (7.8)$$

Substituting expressions (7.5) and (7.6) into (7.8), we get

$$\beta = -\,{}^3/_{121}\; a = -0.024793\; a . \qquad (7.9)$$

And by using (7.5), (7.6), and (7.9), we find

$$A_1 = -\,2.9475\cdot 10^{-4}\lambda a^4, \quad A_2 = 2.0730\cdot 10^{-4}\lambda a^4, \quad A_3 = 8.8426\cdot 10^{-5}\lambda a^4$$

$$A_4 = -\,1.0813\cdot 10^{-4}\lambda a^4, \quad A_5 = 1.5588\cdot 10^{-5}\lambda a^4 . \qquad (7.10)$$

(7.4) and (7.10) will prove to us that the remaining boundary conditions (5.24) are satisfied automatically.

8. Third Approximation

Equation (5.20) is integrated in the same way as equation (5.19) for the second approximation. The solution appears as a polynomial, thus

$$\psi_3 = B_1\eta^2 + B_2\eta^{7/2} + B_3\eta^5 + B_4\eta^{13/2} + B_5\eta^8 + B_6\eta^{19/2} + B_7\eta^{11} \qquad (8.1)$$

wherein the coefficients have the following values:

$$B_1 = -\,4.8905\cdot 10^{-6}\lambda a^5, \quad B_2 = 5.8683\cdot 10^{-6}\lambda a^5,$$

$$B_3 = -\,1.9689\cdot 10^{-6}\lambda a^5, \quad B_4 = -\,5.9013\cdot 10^{-7}\lambda a^5,$$

$$B_5 = 5.0840\cdot 10^{-7}\lambda a^5, \quad B_6 = -\,1.3244\cdot 10^{-7}\lambda a^5, \qquad (8.2)$$

$$B_7 = 1.0291\cdot 10^{-8}\lambda a^5.$$

From the integral expression (5.23), we determine the constant γ:

$$\gamma = -\,2.2010\cdot 10^{-4} a^2 \qquad (8.3)$$

We also determine another value which will be needed later:

$$\psi_3\big|_{\eta=1} = B_1 + B_2 + \ldots + B_7 = -\,1.1950\cdot 10^{-6}\lambda a^5. \qquad (8.4)$$

9. Basic Characteristics of the Flow

The excess expenditure of fluid Q through the cross section of the flow is determined by the equation:

$$Q = 2\pi\rho\int_0^{R_2} U R \, dR \, . \tag{9.1}$$

Dividing (9.1) by $\rho U_1 \pi R_1^2$ and using (2.1), (2.2), (2.4), and (5.1), we obtain an expression defining this dimensionless excess expenditure as follows:

$$q = 2\int_0^{R_2} ur \, dr = 2\int_0^{R_2} \frac{\partial\psi}{\partial r} dr = 2\,[\psi\,(x,\ r_2) - \psi\,(x,\ 0)] \, . \tag{9.2}$$

If we assume that $\psi_1|_{\eta=0} = 0$, which can be done because ψ_1 can be accurately determined up to the point of an arbitrary constant (see paragraph 6), and if we also use the conditions $\psi_2|_{\eta=0} = \psi_3|_{\eta=0} = 0$, we can get from (9.2), (5.8), and (5.21)

$$q = 2\,\left|\psi_1|_{\eta=1} + \psi_2|_{\eta=1}\, x^{-2/3} + \psi_3|_{\eta=1}\, x^{-4/3} + \ldots\right| \tag{9.3}$$

while it follows from (5.23) and the condition $\psi_1|_{\eta=0} = 0$ that

$$\psi_1|_{\eta=1} = \frac{1-\lambda}{2\lambda} \, . \tag{9.4}$$

We now introduce the value X with the aid of this equation,

$$X = x^* \alpha^{*-3/2} = x c^2 \alpha^{*-3/2} \quad , \tag{9.5}$$

and by using (9.4), (7.7), (8.4), (6.5), and (9.5), we obtain from (9.3)

$$q = \frac{1-\lambda}{\lambda}\left[1 - \frac{0.019231}{X^{2/3}} - \frac{0.00025095}{X^{4/3}} + \ldots\right] \, . \tag{9.6}$$

With the aid of (5.7), (6.5), (7.9), (8.3), and (9.5), we find:

$$r_2 = \frac{\sqrt{105(1-\lambda)}}{\lambda}\left[X^{1/3} - \frac{0.024793}{X^{1/3}} - \frac{0.00022010}{X} + \ldots\right]. \quad (9.7)$$

By assuming $\eta = 0$ in (5.12) and using (5.17), (6.4), (6.5), (7.4), (7.9), (7.10), (8.1), (8.2), (8.3), and (9.5), we find the excess dimensionless axial velocity of the flow as being

$$u_0 = \lambda\left[\frac{0.037037}{X^{2/3}} + \frac{0.0012470}{X^{4/3}} + \frac{0.000045592}{X^2} + \ldots\right] \quad (9.8)$$

while if we assume $\eta = 1$ in (5.16) and use (5.24), (6.5), (7.7), (7.9), (8.4), and (9.5), we obtain an expression for the dimensionless transverse component of velocity at the flow boundary υ°:

$$\upsilon^\circ = -\frac{c^2\lambda}{3}\left[\frac{0.00018315}{X^2} + \frac{0.0000093208}{X^{8/3}} + \ldots\right]. \quad (9.9)$$

Let us determine the profile of longitudinal velocities as defined by two first approximations. Retaining the first two terms in (5.12) and using (5.17), (6.4), (6.5), (7.4), (7.9), (7.10), (9.5), and (9.8), we get

$$\frac{u}{u_0} = \left[1 - \eta^{3/2}\right]^2 + \frac{-0.012258\eta^{3/2} + 0.027864\eta^3 - 0.018981\eta^{9/2} + 0.003375\eta}{X^{2/3}}. \quad (9.10)$$

Retaining in the right-hand side of (9.10) the one first summand, we obtain a velocity profile determined by the first approximation. Experimentally determined profiles are close to this theoretical profile[13, 14]. The second summand of the right-hand side of (9.10) turns out to be negative for the entire interval of $0 < \eta < 1$. This means that the velocity profiles defined in the second approximation are somewhat "sharper" than the velocity profile obtained in the first approximation. It must also be borne in mind that the second summand becomes appreciable only when the value of X is very small. Generally speaking, at a great distance from the exit cross section of the flow,

[13]Squire and Trouncer, loc. cit.
[14]Abramovich, loc. cit.

it is sufficient--in all the expansions obtained--to retain only the first terms, resulting in very simple expressions. Subsequent approximations need to be taken into account only to determine the flow profile at the beginning of the main section of the jet flow.

10. Starting Point for the Longitudinal Coordinate

In the solution as found above, one thing remained undetermined and that was the starting point of coordinate z. A rational choice of such a starting point and determination of its position with respect to the exit section of the flow can be made only after a solution for the initial section of the flow has been obtained and coupled with the solution for the main section.

The position of the starting point of the coordinate z with respect to transverse section 1, passing through the end C of the core of the flow (see figure), can be determined as follows.

Let us designate the coordinate of section 1 as z_0 (corresponding to the dimensionless coordinates x_0, x_0^*, X_0). The dimensionless excess axial velocity in section 1 equals $1 - \lambda$:

$$u_0^1 = 1 - \lambda . \qquad (10.1)$$

Substituting $X = X_0$ in (9.8) and using (10.1), we obtain the equation

$$\frac{0.037037}{X_0^{2/3}} + \frac{0.0012470}{\left(X^{2/3}\right)^2} + \frac{0.000045592}{\left(X_0^{2/3}\right)^3} + \ldots = \frac{1 - \lambda}{\lambda} , \qquad (10.2)$$

which serves for defining X_0. After determining X_0 and substituting $X = X_0$ in series (9.6) through (9.9), it is possible to find the basic flow characteristics in section 1. However, we refrain from doing so at the moment, because the flow parameters in the transition section 1 can be determined more accurately after the coupling of the solutions for the main and the initial sections of the jet flow.

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The main section of axisymmetrical jets of an incompressible fluid flowing from an aperture of finite size into a co-axial uniform flow of the same fluid is investigated. The existing approximate solutions are based on the use of predetermined velocity profiles in transverse sections of the flow. The present article proves that a solution can also be obtained by strict integration of equations dealing with the boundary layer.

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