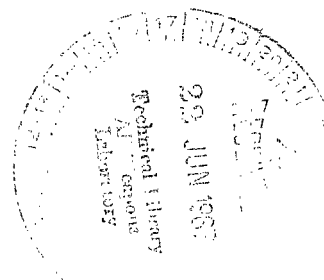**NASA TECHNICAL NOTE**

NASA TN D-3146
c.1

NASA TN D-3146

LOW CYCLE FATIGUE OF NOTCHED SPECIMENS BY CONSIDERATION OF CRACK INITIATION AND PROPAGATION

by S. S. Manson and M. H. Hirschberg

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JUNE 1967

LOW CYCLE FATIGUE OF NOTCHED SPECIMENS BY CONSIDERATION

OF CRACK INITIATION AND PROPAGATION

By S. S. Manson and M. H. Hirschberg

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

LOW CYCLE FATIGUE OF NOTCHED SPECIMENS BY CONSIDERATION

OF CRACK INITIATION AND PROPAGATION*

by S. S. Manson and M. H. Hirschberg

Lewis Research Center

SUMMARY

An approximate analysis has been developed whereby the number of cycles required to start an "engineering size" crack and the number of cycles required to propagate this crack to failure could be estimated for a notched specimen from a knowledge of the fatigue behavior of unnotched specimens. Cracks are assumed to occur at the root of a notch in a number of cycles which depend only on the localized surface strain. Propagation of the crack, once initiated, is assumed to proceed in a manner similar to that in an unnotched specimen. Reasonably good predictions of crack initiation, propagation, and total life were obtained for annealed 4340 steel and 7075 T6 aluminum notched specimens with elastic stress concentration factors of 2 and 3.

INTRODUCTION

The fatigue life of any structure or specimen having a discontinuity or notch may be considered to be composed of three separate stages. The stages are (1) the initiation of a crack at the root of the notch, which is dependent on the local stress and strain at the notch root, (2) the propagation of the crack through the material, and finally (3) the fracture of the structure or specimen that occurs when the remaining cross-sectional area together with the localized stress concentration at the tip of the fatigue crack cannot sustain the applied load. Unless these three stages are separated and individually evaluated, the influence of the various testing variables becomes vague.

The stage of the fatigue process that has received the least attention from an engineering viewpoint is that of crack initiation. Although many fundamental studies have

---

*An expanded version of the paper presented at the International Conference on Fracture, Tohoku University, Sendai, Japan, September 12-17, 1965.

been made by several investigators such as Forsythe (ref. 1), Wood (ref. 2), and numerous others (ref. 3), the engineering significance of microcracks observed under high magnification early in the fatigue life of specimens by these investigators is not clear. It has been suggested (e.g., refs. 4 and 5) that the fatigue process be divided into two stages, crack initiation and crack propagation. No definite criterion, however, has been suggested to establish when a fatigue crack is of sufficient size to demarcate separation between crack initiation and propagation from an engineering point of view. One of the present authors in a recent report (ref. 6) has suggested a simple formula for estimating the number of cycles required to initiate a crack in an unnotched 1/4-inch diameter specimen. While this proposal has not yet received adequate engineering evaluation and further studies are obviously required before it can be accepted, it does provide a definite formulation for the number of cycles required to produce a crack of "engineering size." For this reason it can be useful in the fatigue notch problem investigated in this report.

In order to predict when fatigue crack initiation will occur, the local strain at the notch root must be determined first. The strain analysis at the root of a notch is extremely complicated and even in the elastic range has been determined only approximately; however, within the limits of experimental accuracy, the stresses and strains in the elastic range can be determined by classical methods (refs. 7 and 8). Relatively little work has been done to establish the stress and strain distribution in the plastic range. Two interesting proposals have been made, one by Stowell as modified by Hardrath and Ohman (ref. 9) and the other by Neuber (ref. 10). These methods provide approximations whereby the stresses and strains at the root of a notch may be determined from the knowledge of the elastic stress-concentration factor and the stress-strain curve of the material. While their validity has not been extensively evaluated experimentally for either monotonic or cyclic loading, both of these methods provide a simple means whereby this aspect of the fatigue problem can be made tractable.

Crack propagation has been studied rather extensively in recent years. Most of these studies have dealt with cracks in thin sheets under loads pulsating from zero to a maximum value. Many fatigue specimens, however, are not of this configuration. For example, the most extensively investigated configuration for low cycle fatigue is an hourglass-shaped specimen of circular cross section having a minimum diameter of approximately 1/4 inch. Boettner, Laird, and McEvily (ref. 11) have made a limited study of crack propagation in this type of specimen. Their results, and the relation of these results to the present study, are presented in reference 12.

This report presents the results of an exploratory study developed to permit a simple analysis of the fatigue process in hourglass specimens with simple machined notches subjected to uniaxial reversed loading. The study is limited to and presents preliminary results for the estimation of crack initiation and propagation to failure for two materials, annealed 4340 steel and 7075 T6 aluminum. The investigation was conducted to deter-

mine the degree of validity that could be expected when highly simplified assumptions were made with regard to three major facets of the fatigue problem of notch specimens: (a) the determination of the strain at the root of the notch, (b) the number of cycles required to develop a crack of engineering size, and (c) the number of cycles required to propagate the crack of engineering size to complete fracture.

ANALYTICAL METHOD

The method used in analyzing the notched specimens of this investigation is shown schematically in figure 1. The total longitudinal strain range $\Delta\epsilon$ is plotted against the number of cycles to produce either initial cracking for the two different crack depths indicated $N_{0,B}$ and $N_{0,C}$ or final failure N_f. The failure curve GH represents the fatigue characteristics of the material based on uniaxial push-pull loading of smooth 1/4-inch-diameter hourglass-shaped specimens, determined either experimentally or by analytical prediction according to the method of reference 6 (modified in ref. 12). The curves EF and E'F' represent the cycles to initiate cracks of 0.003-inch depth and 0.013-inch depth, respectively. The number of cycles to propagate these cracks to failure at a constant strain range would then be represented by the horizontal distance between the initiation curves and the failure curve. Both initiation and propagation are assumed to be related to failure by the following equations (appendix A).

For a 0.003-inch-deep crack

$$\Delta N_B = \frac{0.443\, N_f}{\sqrt[5]{1 + \left(\dfrac{N_f}{730}\right)^2}}, \qquad (1a)$$

$$N_{0,B} = N_f - \Delta N_B \qquad (1b)$$

For a 0.013-inch-deep crack

$$\Delta N_C = \frac{0.286\, N_f}{\sqrt[5]{1 + \left(\dfrac{N_f}{730}\right)^2}}, \qquad (1c)$$

$$N_{0,C} = N_f - \Delta N_C \qquad (1d)$$

Figure 1. - Total fatigue life as sum of initiation and propagation components for specimen with a notch 0.010 inch deep.

Suppose a specimen with a ma-

chined notch of 0.010-inch depth is strain cycled so that the nominal longitudinal strain range is $\Delta\epsilon_1$. If the strain concentration factor is K_ϵ, the localized strain range at the root of the notch is then $K_\epsilon \Delta\epsilon_1 = \Delta\epsilon_2$. It will be assumed that the crack will be initiated in a number of cycles dependent only on the localized strain range at the root of the notch. Thus, for a strain range $\Delta\epsilon_2$, the number of cycles to initiate the crack of 0.003-inch depth will be AB, designated $N_{0,B}$.

Once the crack has penetrated to a depth of 0.003 inch below the surface of the root of the notch, it will be assumed to propagate in a manner similar to that which would have occurred if the crack of corresponding size were propagating in an initially smooth specimen (absence of a notch). Thus, once the crack has begun and grown in a notch to a sufficient depth such that the strain field at the crack tip is no longer significantly affected by the initial notch geometry, the number of cycles to failure will be approximately the same as that to propagate a crack to failure in a smooth specimen at the strain range $\Delta\epsilon_1$. However, if a smooth specimen had been brought to the point of crack propagation as in the notched specimen with a crack depth of 0.003 inch below the root of the notch, the total crack length of the smooth specimen would be 0.013 inch, which is equivalent to adding the 0.010-inch notch depth to the 0.003-inch crack. Thus, in changing over to a consideration of the further crack propagation of the 0.003-inch crack, the segment CD is referred to in figure 1, where the strain range has been reduced to the nominal strain range $\Delta\epsilon_1$, but where the point C is taken at the point on the curve for the smooth specimen at which the crack has achieved a 0.013-inch depth. The number of cycles represented by CD is designated ΔN_C. The total life of the notched specimen considered, therefore, is the sum AB and CD. The total life of a smooth specimen subjected to the same nominal strain range $\Delta\epsilon_1$ would have been A'D; the notch therefore has the effect of reducing the life of the smooth specimen by the number of cycles associated with the line B'C.

Although equations (1) are essentially the same as given in reference 12, the form has been changed. For this reason, a review of the assumptions and derivations is included in appendix A of this report.

Strictly speaking, a transfer period between the crack initiation phase AB and the crack propagation phase CD must be considered. This compromise is accomplished in the present analysis by defining $N_{0,B}$ not as the number of cycles to form a microcrack, but rather as the number of cycles to form an engineering-size crack of 0.003-inch depth according to equation (1b). While it is recognized that the simplification assumed here may be highly questionable, the results associated with the assumption will be compared with the experiments to ascertain its validity.

Examination of figure 1 reveals at once a limitation to the concept outlined for a material that displays a distinct knee in the fatigue curve. If such a material is cycled at a nominal strain range below the knee, the crack propagation period CD becomes indeter-

minate because it lies below the range where the initiation and propagation curves are defined. It is well known, however, that once a crack is started at the root of a stress concentration, this crack can propagate at a nominal applied strain range below the fatigue limit of a smooth specimen. A modification of the procedure is required for treatment of this case, but this modification is considered beyond the scope of the present report.

The data (ref. 12) used to arrive at equations (1) were obtained from materials that had reductions in area greater than 30 percent and that were sufficiently notch-ductile to permit appreciable crack-growth stages. For very low ductility or highly notch-sensitive material, such that the critical crack depth for fracture hardly exceeds the initial notch depth, equations (1) would not be valid since crack growth could not take place, and therefore, crack initiation and failure would occur at approximately the same number of cycles. Modifications to the proposed method to permit the prediction of the notch fatigue life of notch brittle materials are discussed in reference 12. The method outlined in this report should therefore be limited to the prediction of the notch fatigue life of materials sufficiently notch-insensitive to exhibit reasonable crack-growth stages.

EXPERIMENTAL PROCEDURE

The procedure and equipment used for this investigation were essentially the same as those used previously by the authors in evaluating the fatigue behavior of smooth specimens and are described in detail in reference 13. Additional equipment was required to observe the initiation of cracks at roots of notches during testing. A modification was also made to the method of strain cycling since the dilatometers described in reference 13 would have interfered with the visual observation of the notches.

Method of Strain Cycling

The procedure used to strain cycle smooth specimens was to maintain a constant diametral strain range by altering the applied stress range as necessary during the test. Materials that strain harden under cyclic loading require a progressive increase in stress range while cyclic strain softening materials require a progressive decrease in stress range. For each smooth (unnotched) specimen tested a record was kept of the stress range necessary to maintain a constant diametral strain range as a function of the number of cycles. The diametral strain range is used only in running the tests, while the computed longitudinal strain range is used in all the analyses. A more complete description of this procedure and sample curves of stress range against cycles for various materials are reported in reference 13.

In order to strain cycle notched specimens without the aid of dilatometers, use was
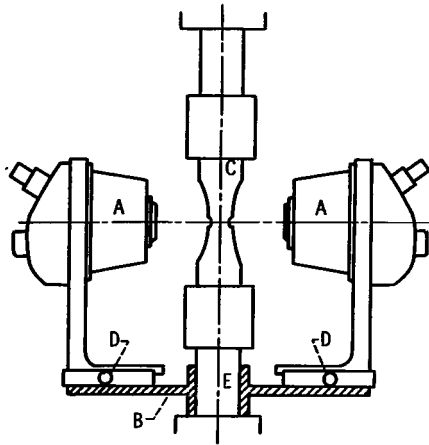
Figure 2. – Test setup for measurement of cracks at root of notched fatigue specimens.

made of the data of stress range against cycles previously obtained from smooth specimens. The stress range against cycle history that had been obtained by strain cycling the smooth specimen was duplicated in the notched specimen. The nominal strain range in the notched specimen was then assumed equal to that of the smooth specimen that produced the loading history being followed.

Method of Detecting Crack Initiation

The optical system used for the detection of cracks is shown in figure 2. Two microscopes (A) with continuously variable (zoom) magnification ranging from 7 to 30 were mounted on opposite ends of extension arms (B) in such a way that the centerlines of the microscopes and the two notches on the specimen (C) coincided. The microscopes were independently focused from knobs (D), and the notch roots could be examined from any angle by simply rotating the whole assembly about the lower loading rod (E). The notch configurations used will be described in the next section.

Crack initiation was defined as the number of cycles required to produce a crack of engineering size, which was the crack length associated with the previously discussed compromise. Once a detectable crack was developed at the root of a notch, its surface length increased to approximately 0.01 inch within a negligible percentage of the total crack propagation stage. Therefore, the point of initiation was taken at a surface crack length ranging from 0.006 to 0.010 inch, which was found experimentally to be a crack approximately 0.003 inch deep.

DETERMINATION OF LOCAL STRAIN AT NOTCH ROOT

In order to obtain the number of cycles for crack initiation from equations (1a) and (b), it is first necessary to estimate the strain concentration at the notch root. As already indicated, the process is complicated by plasticity, and no general solution has as yet been found. Several simplified approaches are available, however, that for the present will be assumed adequate for the treatment of this problem. It will be seen in the section COMPUTATION OF TOTAL LIFE that a high degree of accuracy in this phase of the analysis is not essential because only the crack initiation period is influenced by the results of the computation, and in many cases involving notched specimens, this initial period is relatively short compared with the total life of the part.

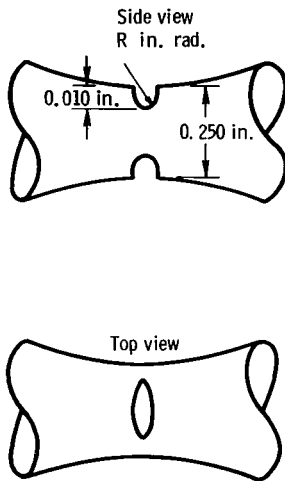| Material	Radius, R, in.	Calculated elastic-stress concentration factor, K_e	Measured fatigue notch factor, K_f
7075 T6 aluminum	0.008	3	2.9
7075 T6 aluminum	.025	2	1.7
4340 steel (annealed)	.008	3	2.9
4340 steel (annealed)	.025	2	2.0

Figure 3. - Test section of notched fatigue specimen.

Figure 3 shows the notch configurations that were used for experimental purposes in this investigation. The specimen used had a conventional circular hourglass shape with a minimum diameter of 1/4 inch. Slot notches were machined into the specimens as shown in the figure. Two geometries were used having radii of 0.008 and 0.025 inch for which the elastic stress-concentration factors K_e were 3 and 2, respectively. The computations to determine K_e were made by two methods. In the first, the assumption was made that the notch was machined entirely around the specimen, thus producing a configuration as treated by Peterson (ref. 8). In the second method of computation, the assumption was made that the specimen was square and that the notches were machined across only two of the opposite faces having the same notch dimensions as those given in figure 3. This configuration is also treated by Peterson in reference 8. Both methods of determining K_e agreed sufficiently to conclude that the theoretical stress-concentration factors were approximately 3 and 2 as shown in the table in figure 3. These approximate values of K_e were used in designing the specimens. The analysis to follow uses measured values of fatigue notch factors in place of approximate stress-concentration factors.

The fatigue notch factors K_f were determined experimentally by dividing the stress which resulted in failure of a smooth specimen in 10^6 cycles by that which resulted in the same life in the notched specimen. Approximately the same fatigue notch factors would have been obtained by using stress ratios at other values of high lives greater than 10^5 cycles. The measured values based on a life of 10^6 cycles are tabulated in figure 3. For the elastic stress-concentration factor K_e of 3 both 7075 T6 aluminum and annealed 4340 steel produced fatigue notch factors of 2.9. For the elastic stress-concentration factor of 2, the annealed 4340 steel produced a notch fatigue factor of 2, while the 7075 T6 aluminum gave a notch fatigue factor of 1.7.

The elastic stress-concentration factors shown in figure 3 are valid when the strain at the root of the notch is very near or below the elastic limit. When larger strains are imposed, resulting in plasticity at the root of the notch, the stress and strain concentration factors cannot be determined according to the elastic analysis of Peterson (ref. 8). Two alternative methods, one based on an equation derived by Stowell (ref. 9) and the other on an equation derived by Neuber (ref. 10), were used in the present investigation

to determine the strain-concentration factor K_ϵ in terms of the stress-concentration factor K_σ and either the elastic stress-concentration factor K_e or the fatigue notch factor K_f. In this investigation, since the fatigue notch factor was experimentally determined, it was used in the analysis. When this factor is not available, the analysis described in the following paragraphs may be conducted with estimated values of K_f or with K_e replacing K_f.

The first method is based on a proposal of Stowell modified to a more general form by Hardrath and Ohman (ref. 9):

$$K_\sigma = 1 + (K_f - 1)\frac{E_2}{E_1} \tag{2}$$

Equation (2) gives the stress-concentration factor K_σ in terms of the fatigue notch factor K_f and two quantities E_1 and E_2. These quantities are secant moduli on the stress-strain curve of the material (fig. 4). The term E_1 refers to the secant modulus at point 1 on the curve, where point 1 represents the nominal condition in the cross section of the notched specimen. For purposes of illustration, E_1 was chosen to be on the elastic portion of the stress-strain curve, but it could also be in the region of plasticity. The secant modulus at point 2, E_2, represents the true condition at the root of the notch when the actual stress and strain concentrations that develop as a result of notch geometry are taken into account. The location of point 1 is known; hence, E_1 is known. The value of the fatigue notch factor K_f is also assumed to be known. Equation (2) can be solved iteratively for K_σ and E_2 from a knowledge of the stress-strain curve (refs. 14 and 15); however, it is possible to derive a graphical procedure that eliminates the need for a trial-and-error solution. Equation (2) may be rewritten by using the definitions of E_1 and E_2 (fig. 4) to yield
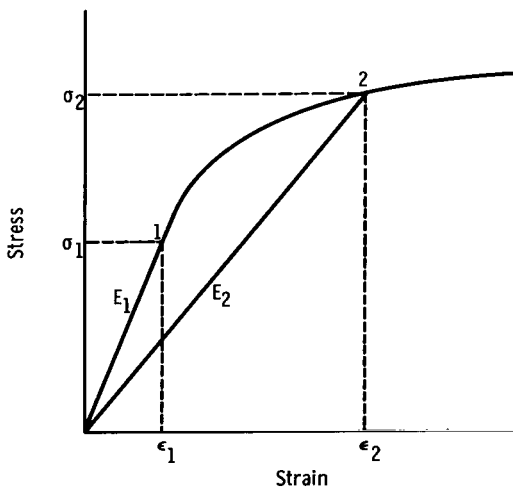

Figure 4. - Schematic stress-strain curve to illustrate terminology in equation (2).

$$E_1 = \frac{\sigma_1}{\epsilon_1} \tag{3}$$

and

$$E_2 = \frac{\sigma_2}{\epsilon_2} \tag{4}$$

Then, by substitution of equations (3) and (4) into equation (2),

$$K_\sigma = 1 + (K_f - 1)\frac{\sigma_2/\sigma_1}{\epsilon_2/\epsilon_1} \tag{5}$$

or

$$K_\sigma = 1 + (K_f - 1)\frac{K_\sigma}{K_\epsilon} \qquad (6)$$

Finally,

$$K_\sigma = \frac{K_\epsilon}{K_\epsilon - K_f + 1} \qquad (7)$$

Equation (7) relates the stress-concentration factor at the root of the notch to the strain-concentration factor and to the fatigue notch factor. An auxiliary diagram (fig. 5) can be used in the solution of this equation. The solid lines in figure 5 are representations of K_σ plotted against K_ϵ for selected values of K_f according to equation (7). The solid diagonal line is the elastic line where $K_\sigma = K_\epsilon = K_f$. Since $K_\epsilon = (\epsilon_2/\epsilon_1)$ and $K_\sigma = (\sigma_2/\sigma_1)$, a curve can also be plotted in figure 5, which is derived from the stress-strain curve of the material. The dot-dash line in figure 5 is obtained by dividing the values of stress and strain at a number of selected values of point 2 $(\sigma_2$ and $\epsilon_2)$ by the known nominal values of stress and strain at point 1 $(\sigma_1$ and $\epsilon_1)$ and by plotting the resulting stress ratio against the strain ratio. The intersection of this normalized stress-strain curve with the curves for the appropriate fatigue notch factor constitutes the solution of equation (7). The intersections for $K_f = 2$ and $K_f = 3$ are shown as points A and B, respectively, in figure 5.
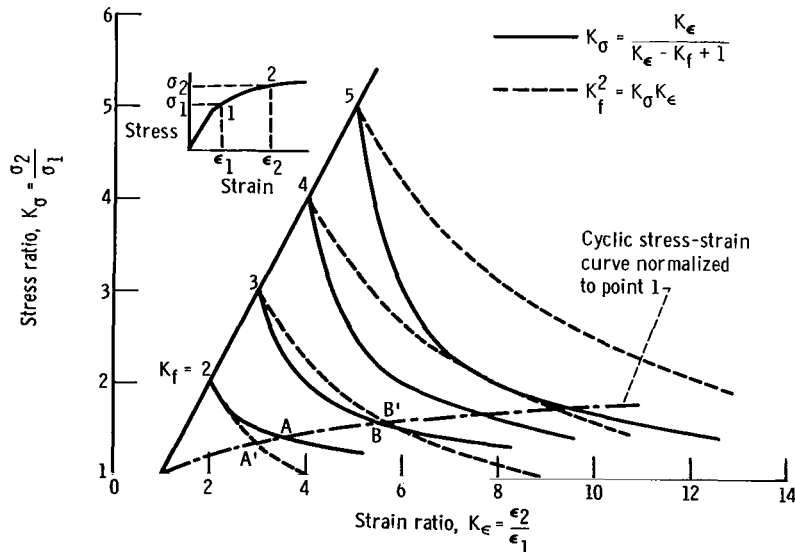
Figure 5. - Graphical determination of stress and strain concentration factors.

An alternative approach is based on a suggestion of Neuber (ref. 10). In his investigation, Neuber considered the case of a specimen subjected to shear and concluded that, as the stress-concentration factor is reduced by plasticity at the root of a discontinuity, the strain-concentration factor is increased, so that the product of stress and strain concentration factors remains approximately constant. Since initially the condition is completely elastic, both the stress and strain concentration factors are equal to K_f; therefore, the product must remain K_f^2. Thus,

$$K_\sigma K_\epsilon = K_f^2 \tag{8}$$

The solution of equation (8) in association with the actual stress-strain curve of the material can also be facilitated by the plot shown in figure 5. Here the dotted curves represent the hyperbolas required by plots of K_σ against K_ϵ according to equation (8). Again, the dot-dash curve is the relation between K_σ and K_ϵ for the test material normalized to the nominal conditions in the test section. The intersection of the dotted curves and the dot-dash curve constitutes the solution of equation (8). For illustrative purposes they are shown in figure 5 as points A' and B' for K_f values of 2 and 3, respectively.

While neither Neuber (ref. 10) nor Stowell-Hardrath-Ohman (ref. 9) proposed the use of their respective relations for computations under cyclic loading, it seems reasonable to suppose that their methods could be applied by using the cyclic stress-strain curve (ref. 6) in the analysis rather than the stress-strain curve obtained under monotonic loading. The cyclic stress-strain curves for the two materials tested and analyzed in the present investigation are shown in figure 6.
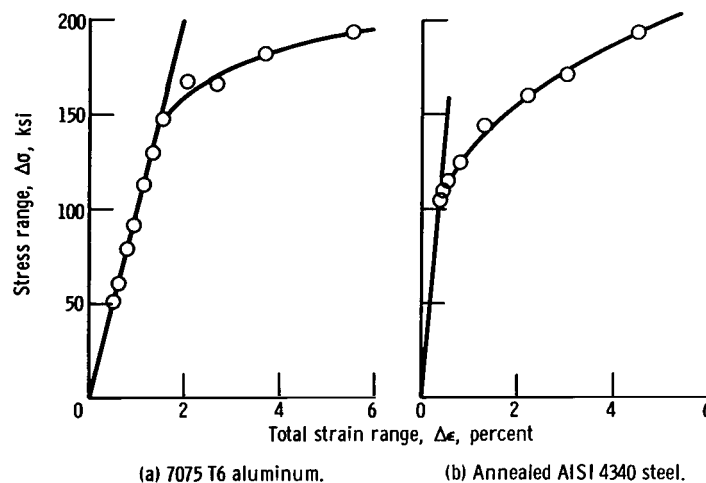

(a) 7075 T6 aluminum. (b) Annealed AISI 4340 steel.

Figure 6. - Cyclic stress-strain behavior for reversed axial strain cycling.

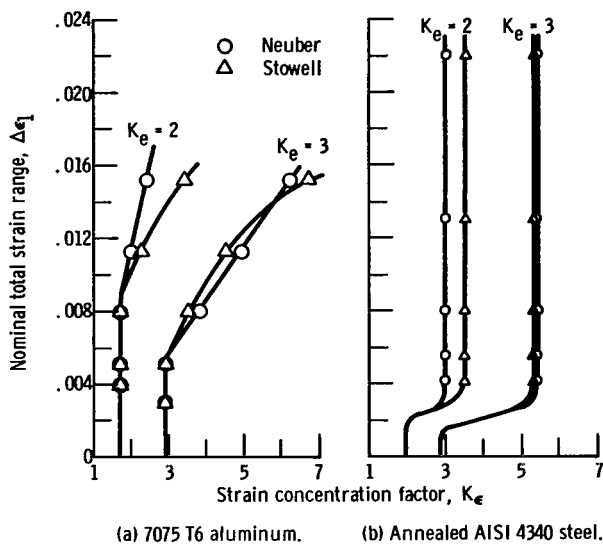(a) 7075 T6 aluminum.　　(b) Annealed AISI 4340 steel.

Figure 7. - Variation of strain concentration factor at root of notch as function of nominal applied strain range.

Figure 7 shows the two nominal stress-concentration factors used and the results of computations by equations (7) and (8) to determine the strain-concentration factor K_ϵ at the root of the notch for each of the two materials. Figure 7(a) shows the results for 7075 T6 aluminum; figure 7(b), the results for annealed 4340 steel. For nominal strains below the elastic limit both methods obviously produce the same results equal to the fatigue notch factors. As the nominal strain across the section is increased, the strain-concentration factor becomes greater than the elastic stress concentration factor. The two methods do not completely agree regarding the determination of the strain-concentration factor; however, as already noted, the results of the life computations were not significantly altered by these differences. An example of the method of determining the strain-concentration factor is given in appendix B.

COMPUTATION OF CRACK INITIATION

As discussed in reference 6 and modified in reference 12, the number of cycles at which a crack of engineering size (approx. 0.003 in. deep) develops in the notched specimens of this investigation is given approximately by equation (1b). For the two materials that were tested in the present investigation, no direct observations were made on the smooth specimens to determine the number of cycles required to initiate an engineering size crack. Therefore, for purposes of analysis, the number of cycles to produce and propagate a crack was computed from equations (1b) and (c) by using the observed number of cycles to failure N_f. Figures 8 and 9 show the results of the computations for the two materials. In each plot, the solid line represents the experimentally determined life of unnotched specimens as a function of strain range. The dotted curve represents the calculated number of cycles to initiate a crack of 0.003-inch depth. The dot-dashed curve represents the calculated number of cycles required to propagate a crack from a 0.013-inch depth to failure.

The number of cycles required to initiate a crack in the presence of the stress concentration can now be determined by using the strain-concentration factors from figure 7
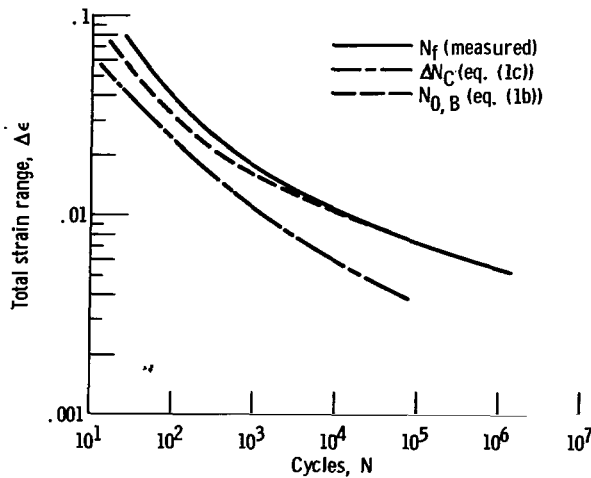
11

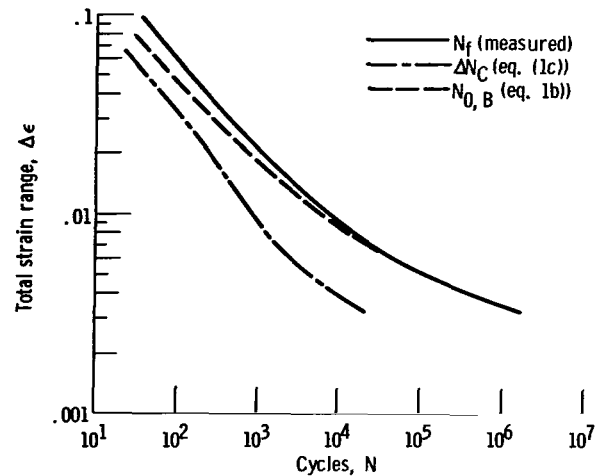Figure 8. - Initiation, propagation, and failure curves for 7075 T6 aluminum.

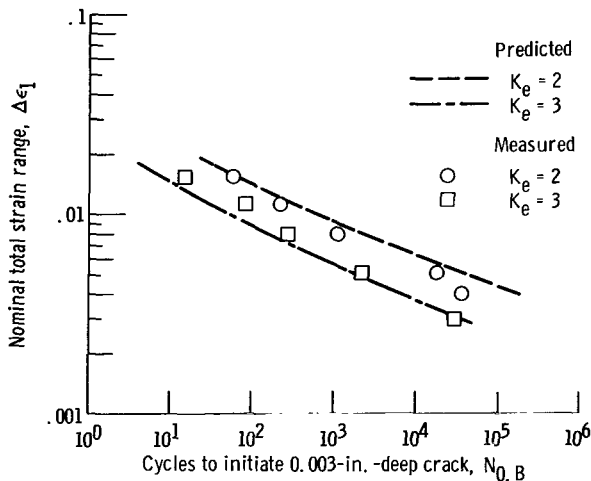Figure 9. - Initiation, propagation, and failure curves for annealed AISI 4340 steel.

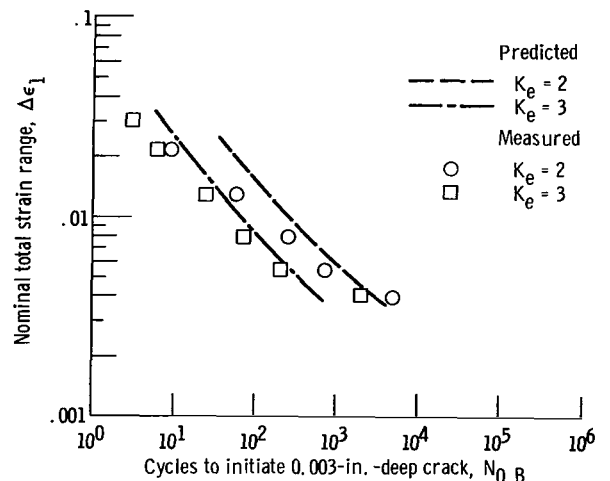Figure 10. - Crack initiation of notched 7075 T6 aluminum.

Figure 11. - Crack initiation of annealed AISI 4340 steel.

and the value of the applied nominal strain range. Multiplying the applied nominal strain range by the strain-concentration factor gives the local strain range at the root of the notch. Using this strain range in conjunction with figures 8 or 9 establishes the number of cycles to initiate the crack in the presence of the strain concentration.

The results of such computations are shown in figures 10 and 11 for the two materials evaluated. Only the computations making use of the Neuber prediction for strain concentration factor are shown here; the computations based on the Stowell-Hardrath-Ohman analysis gave results that differed inappreciably. The dotted curves in figures 10 and 11 represent the predictions for an elastic stress concentration factor of 2.0, and the dot-dash curves represent the predictions for an elastic stress concentration factor of 3.0 for both materials. The circles and squares show the experimental results. Good

agreement was obtained for both materials and for each of the notches evaluated. An example of the method of computation is given in appendix B.

COMPUTATION OF CRACK PROPAGATION

As already shown in figure 1 (p. 3), the crack propagation period for a given strain range depends to a first approximation on the applied strain range only and not on the strain-concentration factor imposed; the crack propagates under a strain-concentration factor essentially unrelated to the strain concentration that caused it to form. This strain concentration depends largely on the crack itself and not on the geometric conditions existing prior to the development of the crack. Thus, the prediction for the period of crack propagation is the same for both notch configurations investigated. These predictions are shown by the solid lines in figures 12 and 13. The experimental data agree well with the predictions for the 7075 T6 aluminum (fig. 12), and it will be seen that the measured period of crack propagation is almost identical for each of the two notch configurations investigated. For the annealed 4340 steel (fig. 13) the agreement is good at crack propagation lives above approximately 1000 cycles. Below this value, however, the experimental results are lower than the predictions. The measured values of ΔN_C are, again, almost the same for each of the two notch configurations investigated. An example of the method used to compute ΔN_C is also given in appendix B.

One possible reason why the predictions of ΔN_C are greater than those measured for the annealed 4340 steel in the low cyclic life region can be seen in figure 14, where a plot is shown of the strain softening characteristic of this material for a strain range of 0.022 on a smooth specimen. As in most strain cycling tests, it is necessary to change the
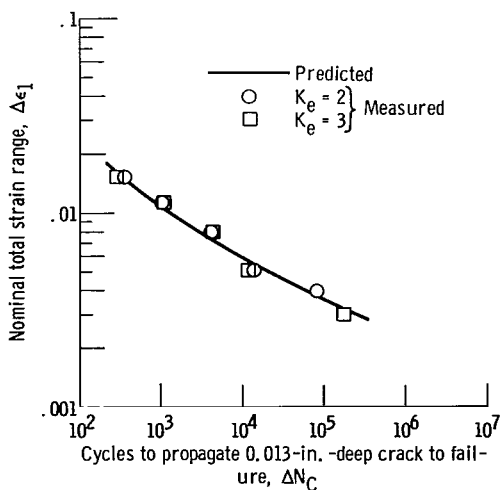

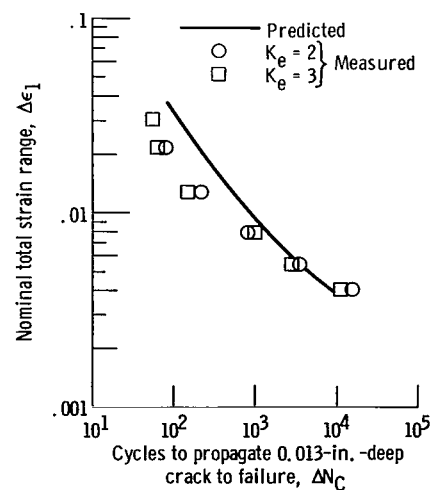Figure 12. - Crack propagation of notched 7075 T6 aluminum.

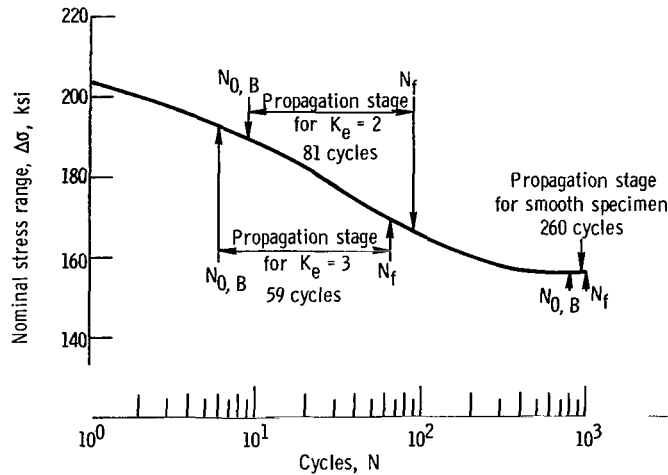Figure 13. - Crack propagation of annealed AISI 4340 steel.

$$N_{0, B}$$ Propagation stage for $K_e = 2$ 81 cycles $$N_f$$

Propagation stage for smooth specimen 260 cycles

Propagation stage for $K_e = 3$ 59 cycles $$N_{0, B} \quad N_f$$

$$N_{0, B} \quad N_f$$

Nominal stress range, $\Delta\sigma$, ksi

220
200
180
160
140

10^0 10^1 10^2 10^3

Cycles, N

Figure 14. – Variation of stress range with cycles for annealed AISI 4340 steel. $\Delta\epsilon = 0.022$.

stress range during the progressive cycling of the specimen in order to maintain constancy of strain range. The 4340 steel is a strain softening material in which the stress to maintain a constant strain range is progressively decreased as cycles are accumulated. For the smooth specimen, the estimated number of cycles required to cause a crack to initiate is shown in figure 14 at about 800 cycles for a life to failure of approximately 1050 cycles. During the crack initiation stage, numerous cycles are therefore imposed for which the applied stress range is relatively low. Once the crack is formed, it will grow under the influence of this lower stress range; however, when the nominal strain range of 0.022 is imposed in the presence of a notch, the number of cycles to crack initiation is considerably lower, and the bulk of the specimen has not yet softened to the same stress level that a smooth specimen reaches at its time of crack initiation. Because of the strain softening characteristic of the material, the number of cycles to propagate a crack at high stress range might well be expected to be less than that predicted on the assumption that the stress range remains constant for a given strain range (ref. 6). The analysis for 7075 T6 aluminum that does not strain soften but rather strain hardens moderately does not show this characteristic discrepancy (fig. 12).

In making an analysis of the crack propagation period for the notched specimens that have already been investigated by studying smooth specimens of the same size, it is not necessary to consider the crack growth process in detail. The number of cycles to crack initiation and the number of cycles for propagation can be computed from equations (1) from the measured number of cycles to failure for the smooth specimen. In more complicated cases, however, or when greater generality is sought, it is necessary to make a detailed examination of the equations governing crack growth and final fracture. Such analysis is beyond the scope of the present report. It is touched on, however, in reference 12.

COMPUTATION OF TOTAL LIFE

The total fatigue life of a notched specimen is the sum of the crack initiation and the crack propagation periods (fig. 1). Thus, by the addition of the proper curves of figures 10 to 13 the total life can be predicted for each of the two notch configurations
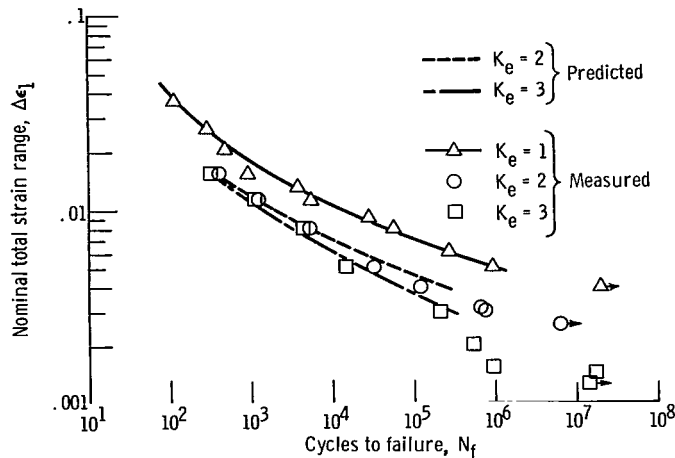
14

Figure 15. – Axial fatigue of notched 7075 T6 aluminum. (Arrows on data points indicate no failure.)
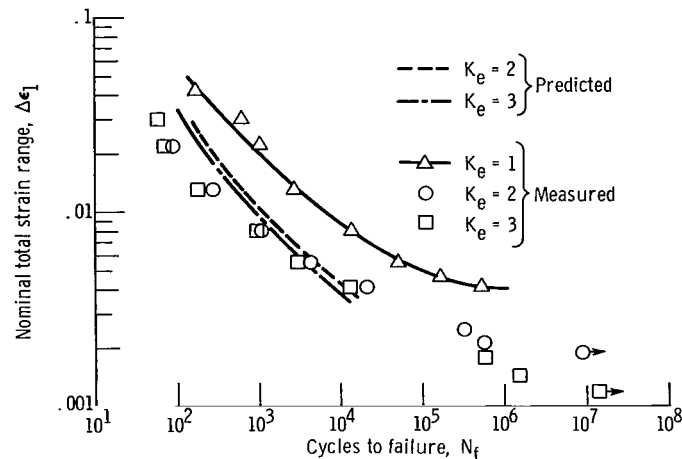

Figure 16. – Axial fatigue of annealed AISI 4340 steel. (Arrows on data points indicate no failure.)

(K_e = 2 and 3) and for each material. The results are shown in figures 15 and 16 by the dotted and dot-dash curves, respectively along with the measured lives obtained from smooth specimens (K_e = 1). Again, the experimental results agree favorably with the predictions except for the very low cyclic lives in the case of the annealed 4340 steel, as already discussed.

Of special interest is the fact that below lives of 10^5 cycles, both the analytical predictions and the experimental observations indicate that notches of nominal stress-concentration factors of 3 do not have a much more detrimental effect on the cyclic life than do those having nominal stress concentration factors of 2. The reason for this can be seen by examination of figures 10 to 13. Even though the higher stress-concentration factors have a considerable effect in reducing the crack initiation periods (this effect differing appreciably for the two notch configurations investigated), the crack initiation period for notched specimens is generally a relatively small part of the total life. The crack propagation period is assumed to be independent of the stress-concentration factor of the notch and in most practical cases is the largest portion of the total life. Thus, only to a minor extent does the sum of the two components reflect the differences introduced by the higher nominal stress concentration.

This observation is in agreement with the general experimental finding that increasing the nominal stress-concentration factor of a notch does not produce a correspondingly large decrease in fatigue life in the low cycle range. In fact, as the nominal stress-concentration factor is increased, a value is reached beyond which further decrease in total life is negligible. In practice, fatigue notch factors greater than 7 are not encountered in conventional fatigue tests (ref. 16).

An example of the method used to calculate the total life of a notched fatigue specimen is given in appendix B.

DISCUSSION

The method discussed is, of course, only an approximation. The assumption that the fatigue process of a notched specimen (apart from the question of final fracture) can be divided into two distinct parts, the initiation period, which depends on the strain-concentration factor, and the propagation period, which is essentially independent of this factor, ignores the complex situation that truly exists during the transition period. A more detailed analysis would be desirable, but in view of the good agreement between analysis and experiment obtained through the simplifying assumptions, it may be questioned whether the complexity of a detailed analysis would be warranted by potentially small increases in accuracy.

The method illustrated is valid only for the prediction of the fatigue life of specimens of 1/4-inch diameter containing notches when corresponding tests have been conducted on similar specimens devoid of notches. For more complex shapes that have not previously been analyzed in the absence of notches, a more detailed analysis would be required to determine the details of the crack growth process. For crack initiation, it may be assumed that this period is more or less independent of the configuration and depends largely on the local strain at the surface of the notch root. If no observations have been made directly to relate strain range with the number of cycles to produce crack initiation, use can be made of the relation between strain range and the number of cycles to produce failure in a 1/4-inch-diameter specimen, equations (1) being applied to determine crack initiation.

It is interesting to make a comparison between the method described herein and that previously used by other investigators to predict the fatigue life of notched specimens. Peterson (ref. 14), for example, also used the con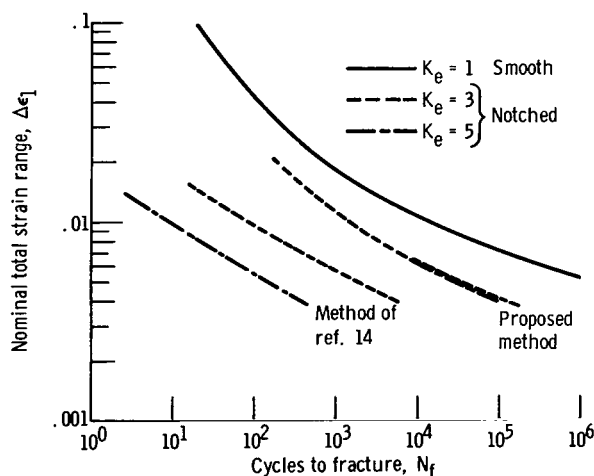cept of plastic strain concentration determined by the Stowell-Hardrath-Ohman approximation to obtain the strain range at the root of a notch. The strain range at the root of the notch was used in conjunction with the curve of strain range against life of the smooth specimen to predict life of the notched part. In essence, therefore, the strain concentration of the notch was applied to the entire life of the specimen, propagation as well as initiation. Applying the strain concentration to the propagation period obviously has the effect of producing more conservative calculations of life compared with the method discussed in this report. Figure 17 shows a

Figure 17. - Comparison of methods for predicting cyclic life of notched specimens for 7075 T6 aluminum.

16

comparison between calculations for 7075 T6 aluminum for elastic stress concentration factors of 3 and 5. The method of Peterson predicts large progressive decreases in life as the nominal stress-concentration factor is increased. The method described in this report does not. As already shown in figures 15 and 16, changing from a stress concentration of 2 to 3 did not significantly reduce the life. Preliminary tests with a very sharp notch having a nominal stress-concentration factor of approximately 4 also showed a relatively small reduction in life, as would be expected by noting figure 17. The approach used by Peterson would have predicted a much larger decrease than that observed.

Crews and Hardrath (ref. 15) have made the same assumption as Peterson regarding the use of the strain-concentration factor although they did recognize the fact that they were not representing the crack growth portion of the life adequately. In their study, they actually measured the stress and strain at the root of a notch and predicted notched specimen life from experiments in which stress across the unnotched specimen was equal to that measured at the root of the notch. Their predictions of life were generally lower than those observed experimentally (ref. 15). Again, the reason for this conservative result is probably that no separation was made between the initiation and the crack propagation phases. The strain-concentration factors were essentially applied to both phases. Both types of calculations, one giving full weight to the strain concentration at the root of the notch and the other entirely ignoring its effect during the propagation stage, represent extremes of approximation. It would appear, based on the results of this investigation, that the approximation involved in ignoring the effects of the notch during the propagation phase is the better compromise. It should be emphasized, however, that all of the preceding discussion relates only to materials that have sufficient ductility and are notch insensitive enough to exhibit reasonable crack growth stages.

CONCLUDING REMARKS

An approximate analysis has been developed whereby the number of cycles required to start an engineering size crack and the number of cycles required to propagate this crack to failure could be estimated for a notched specimen from a knowledge of the fatigue behavior of unnotched specimens. Reasonably good agreement with experimental results were obtained for the two materials and the two notch configurations tested. In the form presented, the method is presently limited to estimates of the fatigue life of notched specimens strain cycled above the fatigue limit of smooth specimens and to materials exhibiting a reasonable crack growth stage. Further evaluation with more mate-

rials and a wider range of notch geometries is desirable. The effects of cyclic strain hardening or softening on the crack propagation stage also require further evaluation.

APPENDIX A

EQUATIONS USED IN DESCRIBING CRACK INITIATION
AND CRACK PROPAGATION

The crack growth relation used in the analysis of this investigation is shown in figure 18 and described at greater length in reference 12. Briefly, the justification for the use of this relation is based on the following four independent assumptions:

(1) The crack growth rate is proportional to the crack depth.

(2) The maximum crack depth is a constant for all ductile materials.

(3) The remaining cyclic life in a notched specimen, once a small crack has started at the root of the notch, is a known, experimentally determined, function of the total life of the specimens.

(4) The full crack growth period is also a known function of the total life of the specimen.

The reasons for these assumptions and the justification for their use in deriving the crack growth relation of this analysis are as follows:

(1) The general form of the crack propagation law, which appears to represent the available limited data for the specimens, type, and conditions of tests performed in this investigation is $d\ell/dN = f(\Delta\epsilon_p)\ell$, where $f(\Delta\epsilon_p)$ is a general function of the plastic strain range and N is the number of cycles at crack length ℓ. Other forms of crack growth equation have been formulated in which $d\ell/dN$ is proportional to ℓ^γ where γ may be as high as high as 2 but that these equations were arrived at and are applicable to crack growth conditions where the nominal stresses are well below the fatigue limit for smooth specimens. The data and analysis of this investigation are limited to loading conditions above the fatigue limit where very limited data seem to indicate that the crack growth rate is proportional to ℓ directly (ref. 11). This assumption regarding the form of the propagation law results in a straight line relation between the log of the crack depth and the fraction of the crack propagation period used up. The procedure presented is, however, not limited to the value of γ of unity and corresponding equations could be formulated using other values of γ.
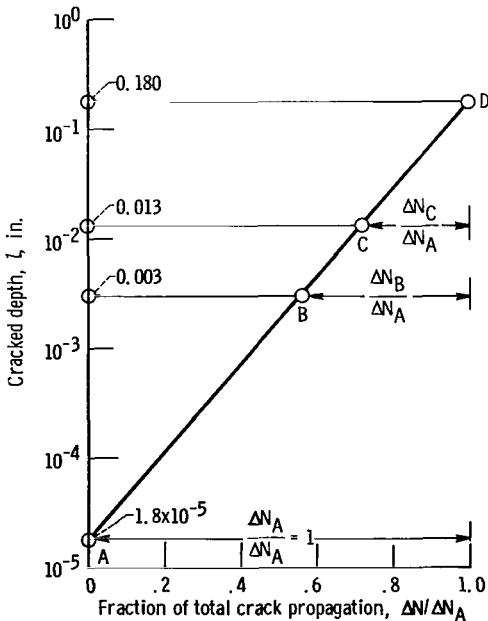

Figure 18. – Hypothesized crack growth relation.

19

(2) The depth of the fatigue crack, when tensile failure occurs at the end of the fatigue tests, is a constant and is taken as 0.18 inch for the materials used in this investigation. This assumption forces the straight line resulting from assumption (1) to pass through point D of figure 18. As pointed out in reference 12, the exact value of the crack depth assumed at failure has little effect on the life prediction as long as this value is an appreciable percentage of the specimen diameter, which is true for the ductile materials investigated. This insensitivity is due to the fact that the crack growth rate is very large once the crack is large, and therefore, any reasonable assumption regarding this value of total crack depth will result in approximately the same prediction of life.

(3) The number of cycles remaining in a specimen with a 0.010-inch-deep machined notch once a crack of 0.003-inch depth has been generated at the root of the notch can be given by $\Delta N_C = 4N_f^{0.6}$, as verified experimentally and presented in figure 10 of reference 12. This remaining life is further assumed to be equal to the remaining life of an initially smooth specimen with a fatigue crack depth of 0.013 inch. Assumptions (1), (2), and (3) lead directly to $\Delta N_B = 6.2\,N_f^{0.6}$ as the equation of the remaining number of cycles in an initially smooth specimen with a fatigue crack of 0.003-inch depth. This relation is easily arrived at from the similar triangles in figure 18 resulting in the following:

$$6.2\,N_f^{0.6} = 4N_f^{0.6} \times \frac{\log(0.18) - \log(0.003)}{\log(0.18) - \log(0.013)}$$

These equations alone do not define points B and C of figure 18; the total crack propagation period must also be established.

(4) The full crack growth period is assumed to be $14N_f^{0.6}$, as discussed in appendix B of reference 12, and is represented by ΔN_A in figure 18. This assumption results directly in point B being located at an abscissa value equal to $\left(1 - \dfrac{6.2}{14}\right) = 0.577$, point C being at $\left(1 - \dfrac{4}{14}\right) = 0.714$, and a crack depth associated with point A of 1.8×10^{-5} inch. As was noted in reference 12, $14N_f^{0.6}$ becomes greater than N_f for all values of N_f less than approximately 730 cycles. Since the full crack growth period can never be greater than the total life, it will be assumed that for all lives greater than 730 cycles, the full crack propagation period ΔN_A will be equal to $14N_f^{0.6}$; for all lives less than 730 cycles, a crack of depth 1.8×10^{-5} inch is assumed to appear on the first loading cycle, and therefore, $\Delta N_A = N_f$.

With assumptions (1) to (4), along with figure 18, the equations necessary to calculate the total crack propagation period ΔN_A, the number of cycles to initiate cracks of 1.8×10^{-5} inch $N_{0,A}$ and 0.003 inch $N_{0,B}$, and the number of cycles to propagate a crack from 0.013 inch to failure ΔN_C can be obtained. These equations are listed in table I. The equations for $N_{0,B}$ and ΔN_C are plotted as solid lines in figure 19.

The coefficient 14 used in the equation defining ΔN_A was obtained from the analysis

TABLE I. – SUMMARY OF CRACK INITIATION AND

CRACK PROPAGATION EQUATIONS

| Predicted number of cycles	Cycles to failure, N_f		Single relation for all values of N_f
	> 730	< 730	
Total crack propagation period, ΔN_A	$14\,N_f^{0.6}$	N_f	$\dfrac{N_f}{\sqrt[5]{1 + \left(\dfrac{N_f}{730}\right)^2}}$
Cycles to initiate 1.8×10^{-5}-in. cracks, $N_{0,A}$	$N_f - 14\,N_f^{0.6}$	0	$N_f - \dfrac{N_f}{\sqrt[5]{1 + \left(\dfrac{N_f}{730}\right)^2}}$
Cycles to initiate 0.003-in. cracks, $N_{0,B}$	$N_f - 6.2\,N_f^{0.6}$	$\dfrac{7.8}{14}\,N_f$	$N_f - \dfrac{\dfrac{6.2}{14}\,N_f}{\sqrt[5]{1 + \left(\dfrac{N_f}{730}\right)^2}}$
Cycles to propagate crack from 0.013 in. to failure, ΔN_C	$4N_f^{0.6}$	$\dfrac{4}{14}\,N_f$	$\dfrac{\dfrac{4}{14}\,N_f}{\sqrt[5]{1 + \left(\dfrac{N_f}{730}\right)^2}}$
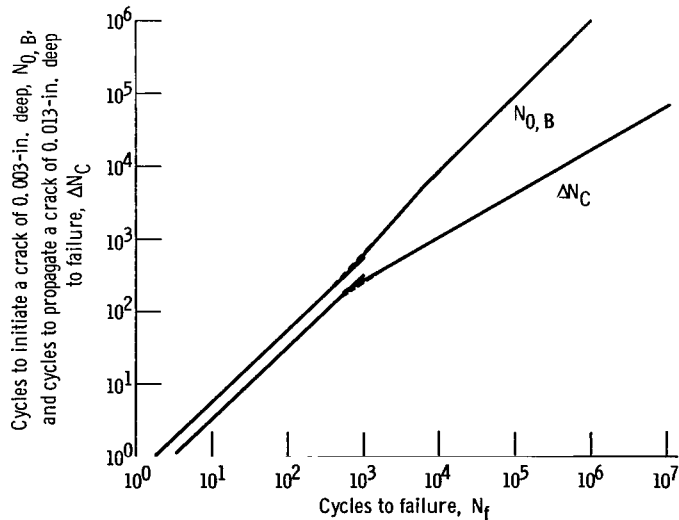
Figure 19. – Crack initiation and propagation as functions of cycles to failure.

of very limited cumulative fatigue data. Other extreme values for this coefficient were tried and found to have very little effect (at most a factor of 2) on life predictions below a few hundred cycles. The extreme numbers examined were 8 and 27. A coefficient of 8 would imply that, for lives below 180 cycles, a crack of 0.001-inch depth would be initiated on the first loading cycle, but such cracks were not detected experimentally; the assumption of the coefficient 27 would imply that a crack 1 angstrom deep (interatomic dimension) initiates on the first cycle for all lives below 4000 cycles. The number 14 is a compromise between these extremes but of course requires additional experimental verification. The use of the numbers 4, 6.2, 14, etc. are not to be considered arbitrary but do depend upon the form of the equations used to represent the data in figure 10 of reference 12, and are intended to be mutually consistant. Another form of the equation would have given a different set of constants but approximately the same predictions of life.

Single equations representing crack initiation and crack propagation rather than separate ones for lives above and below 730 cycles are desirable. These separate equations can be closely approximated by a single equation in the following manner. Let F_1 represent the crack propagation relation in the high life range and F_2 the crack propagation relation in the low life range; then the single relation F that approximates F_1 and F_2 over the entire life range can be expressed as follows:

$$\left(\frac{1}{F}\right)^n = \left(\frac{1}{F_1}\right)^n + \left(\frac{1}{F_2}\right)^n \tag{9}$$

where n is some positive number. The greater the value of n, the closer F approaches F_1 and F_2 over the entire life range. For this investigation, $n = 5$ was arbitrarily chosen. Equation (9) can be rewritten as

$$F = \frac{F_2}{\left[1 + \left(\frac{F_2}{F_1}\right)^5\right]^{1/5}} \tag{10}$$

If the two equations used to represent ΔN_C are $4N_f^{0.6}$ for N_f greater than 730 cycles and $4/14\ N_f$ for N_f less than 730 cycles, then the one single equation for ΔN_C that can be used to replace these two equations is obtained from equation (10) as

$$\Delta N_C = -\frac{0.286\, N_f}{\left[1 + \left(\dfrac{0.286\, N_f}{4\, N_f^{0.6}}\right)^5\right]^{1/5}} \tag{11}$$

and by letting

$$\left(\frac{0.286\, N_f}{4\, N_f^{0.6}}\right)^5 = \left(\frac{N_f}{14\, N_f^{0.6}}\right)^5 = \left(\frac{N_f^{0.4}}{14}\right)^5 \approx \left(\frac{N_f}{730}\right)^2$$

$$\Delta N_C = \frac{0.286\, N_f}{\left[1 + \left(\dfrac{N_f}{730}\right)^2\right]^{1/5}} \tag{12}$$

In a similar manner, the equation for ΔN_B can be derived. Since $N_{0,B} = N_f - \Delta N_B$,

$$N_{0,B} = N_f - \frac{0.443\, N_f}{\left[1 + \left(\dfrac{N_f}{730}\right)^2\right]^{1/5}} \tag{13}$$

Equations (12) and (13) are the dashed curves plotted in figure 19, where it can be seen how well these single relations represent the sets of equations from which they were derived. Equations (12) and (13), which are also listed in table I, were used exclusively in this report to predict crack initiation, crack propagation, and failure for notched specimens.

APPENDIX B

SAMPLE CALCULATIONS

The following calculations are for an annealed 4340 steel notched fatigue specimen with an elastic stress-concentration factor of 2 subjected to an axial strain range of 0.008. For this material and notch configuration, the fatigue notch factor has been measured and was found to be equal to the elastic stress-concentration factor, namely 2. The following calculations are made by using this value of K_f.

Determination of Strain-Concentration Factor K_ϵ at Notch Root

For nominal applied strain range $\Delta\epsilon_1$ of 0.008, the nominal stress range is found from the cyclic stress-strain curve of figure 6(b) (p. 10) to be 123 ksi. It should be noted that, for this example, the nominal values are slightly into the plastic region of the cyclic stress-strain curve. The cyclic stress-strain curve is now normalized for these nominal values by choosing a number of points on this curve and dividing the stress ranges at these points by 123 ksi and the strain ranges by 0.008. This normalized cyclic stress-strain curve is the dot-dash curve plotted in figure 5 (p. 9).

The strain concentration factors K_ϵ determined from the methods of both Neuber and Stowell-Hardrath-Ohman are now read off the curves in figure 5 at the intersection of the K_f lines and the normalized cyclic stress-strain curve. The Neuber method results in a K_ϵ of 3.0, while the Stowell-Hardrath-Ohman method gives a K_ϵ of 3.5. These values are plotted in figure 7(b) (p. 11) for the nominal strain range of 0.008.

Determination of Number of Cycles to Initiate Crack $N_{0,B}$

The strain range at the root of the notch is equal to the nominal strain range multiplied by the strain-concentration factor. For this example the value of K_ϵ determined from the method of Neuber will be used; therefore, $\Delta\epsilon_2 = 0.008 \times 3.0 = 0.024$. The number of cycles to initiate a crack in this notched specimen is assumed to be the same number of cycles required to initiate a crack in an unnotched specimen subjected to a strain range equal to 0.024. The value of $N_{0,B}$ is therefore read off figure 9 (p. 12) for the strain range 0.024 and is equal to 460 cycles. This value is plotted in figure 11 (p. 12) at the nominal strain range 0.008.

Determination of Number of Cycles to Propagate Crack to Failure ΔN_C

The nominal strain range for the notched specimen is the same as the nominal strain range for the unnotched specimen and, since the notch is assumed to have no influence on the propagation stage, ΔN_C for the notched specimen is equal to ΔN_C for the unnotched specimen. This value is read off the dot-dash curve in figure 9 for the nominal strain range 0.008 and is equal to 1400 cycles. This value is again plotted in figure 13 at the nominal strain range.

Determination of Total Life N_f

The total life of the specimen used in this example is the sum of the number of cycles predicted to initiate the crack and the number of cycles predicted to propagate this crack to failure. For this case then, $N_f = 460 + 1400 = 1860$ cycles and is plotted in figure 16, once again at the nominal strain range.

REFERENCES

1. Forsyth, P. J. E.: A Two Stage Process of Fatigue Crack Growth. Proceedings of the Crack Propagation Symposium, Cranfield, England, vol. 1, 1962, pp. 76-94.

2. Wood, W. A.: Recent Observations on Fatigue Failure in Metals. Symposium on Basic Mechanisms of Fatigue, Boston, Mass., June 23, 1958, Spec. Tech. Publ. No. 237, ASTM, 1959, pp. 110-121.

3. Anon.: Proceedings of the International Conference on Mechanisms of Fatigue in Crystalline Solids, Orlando, Fla., Nov. 15-17, 1962. Acta Met., vol. 11, July 1963, pp. 639-828.

4. Grover, Horace: Size and Notch-Size Effects in Fatigue. Fatigue - An Interdisciplinary Approach. John J. Burke, Norman L. Reed and Volker Weiss, eds., Syracuse University Press, 1964, pp. 361-377.

5. Vagapov, R. D.: Methods for Determining the Fatigue Strength by Dividing the Process of Cyclic Loading into Two stages. Industrial Lab., vol. 30, Jan. 1965, pp. 921-927.

6. Manson, S. S. : Fatigue: A Complex Subject - Some Simple Approximations. Experimental Mech., vol. 5, no. 7, July 1965, pp. 193-226.

7. Neuber, Heinz: Theory of Notch Stresses, Principles for Exact Stress Calculation. J. W. Edwards Co., Ann Arbor, Mich., 1946.

8. Peterson, R. E.: Stress Concentration Design Factors; Charts and Relations Useful in Making Strength Calculations for Machine Parts and Structural Elements. John Wiley and Sons, Inc., 1953.

9. Hardrath, Herbert F.; and Ohman, Lachlan: A Study of Elastic and Plastic Stress Concentration Factors due to Notches and Fillets in Flat Plates. NACA TR-1117, 1953.

10. Neuber, H. : Theory of Stress Concentration for Shear-Strained Prismatical Bodies with Arbitrary Nonlinear Stress-Strain Law. J. Appl. Mech., vol. 83, no. 4, Dec. 1961, pp. 544-550.

11. Boettner, R. C.; Laird, C.; and McEvily, A. J.: Crack Nucleation and Growth in High Strain-Low Cycle Fatigue. AIME Trans., vol. 233, no. 2, Feb. 1965, pp. 379-387.

12. Manson, S. S.: Interfaces Between Fatigue, Creep, and Fracture. Int. J. Fracture Mech., vol. 2, no. 1, Mar. 1966, pp. 327-363.

13. Smith, Robert W.; Hirschberg, Marvin H.; and Manson, S. S.: Fatigue Behavior of Materials Under Strain Cycling in Low and Intermediate Life Range. NASA TN D-1574, 1963.

14. Peterson, R. E.: Fatigue of Metals. III - Engineering and Design Aspects. Materials Res. Standards, vol. 3, Feb. 1963, pp. 122-139.

15. Crews, J. H., Jr.; and Hardrath, H. F.: A Study of Cyclic Plastic Stresses at a Notch Root. Experimental Mech., vol. 6, no. 6, June 1966, pp. 313-320.

16. Frost, N. E.: Non-Propagating Cracks in Vee-Notched Specimens Subjected to Fatigue Loading. Aeron. Quart., vol. 8, pt. 1, Feb. 1957, pp. 1-20.

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546