

Oxygen and Carbon Monoxide Cooling  
in Planetary Thermospheres

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## ABSTRACT

A study of the effectiveness of atomic oxygen and carbon monoxide as cooling agents in a planetary thermosphere is carried out. Analytic expressions for the total flux radiated by these constituents are derived based on the assumption of a sharply defined radiation zone in which the scale height variation is linear. It is found that the total flux radiated by atomic oxygen can exceed that radiated by carbon monoxide if the temperature in the radiating region is less than  $300^{\circ}\text{K}$ .

The time-dependent thermal conduction equation is considered briefly. It is shown that the cooling time for carbon monoxide is of the order of a few hours, in contrast to the several-day cooling time of atomic oxygen. A substantial overnight cooling is thus to be expected in planetary thermospheres which contain carbon monoxide.

## 1. Introduction

Atomic oxygen and carbon monoxide are present in varying degree in the upper atmospheres of the Earth, Mars and Venus. The Earth's thermosphere is predominately atomic oxygen and does not contain much carbon monoxide. The atmospheres of Mars and Venus, on the other hand, contain substantial  $\text{CO}_2$  and must therefore contain an abundance of both of the dissociation products, CO and O.

Each of these constituents can cool a planetary thermosphere by infrared emission of some of the thermal energy deposited by solar extreme ultraviolet (EUV) radiation. The possible importance of the magnetic dipole transition  $\text{O}(^3\text{P}_1) \rightarrow \text{O}(^3\text{P}_2)$  within the ground state triplet of oxygen in cooling the Earth's thermosphere was pointed out by Bates (1951). McElroy et al. (1965) have studied the effectiveness of CO rotational transitions in cooling the Martian thermosphere.

The power radiated by atomic oxygen is only weakly temperature dependent, whereas the power radiated by carbon monoxide is proportional to the square of the temperature. However, direct comparison of the power

radiated by each constituent can be misleading. The physically significant quantity is the total energy loss per  $\text{cm}^2$  column of atmosphere by each constituent. The calculations presented below show that the total energy loss by O and CO is more strongly temperature dependent than is the power radiated locally. The importance of the  $62_{\mu}$  radiation of atomic oxygen relative to rotational loss by CO is significantly greater than could be inferred solely from comparison of the power radiated by each constituent. If the temperature at the altitudes where radiative loss occurs is less than  $300^{\circ}\text{K}$ , atomic oxygen can be the dominant radiator. A precise determination depends on the scale height gradients in the radiation zone, the temperatures at which radiative loss commences (assuming it commences at a well-defined optical depth in each constituent), and whether or not the constituents are in diffusive equilibrium. Even for temperatures in the radiation zone of  $\sim 500^{\circ}\text{K}$  the ratio of O/CO energy loss may be  $\sim .5$ .

The analytic expressions developed in this paper for the flux radiated by O and CO may serve as a useful check on the results of detailed integrations of the thermal

conduction equation.

The time-dependent form of the thermal conduction equation is considered with a view to obtaining a qualitative idea of the amplitude of the temperature drop expected during the night. When the terms representing conductive loss and radiative loss due to atomic oxygen are dropped, a simple expression for the temperature decay may be obtained. A cooling time for CO radiation is defined as the time in which the temperature would fall to half its initial value due to CO radiative loss alone. This cooling time is found to vary from about 7 hours for an initial temperature of  $1000^{\circ}\text{K}$  to 12 hours for an initial temperature of  $600^{\circ}\text{K}$ . In the absence of a nocturnal heat source, CO cooling alone will cause the exospheric temperature to fall to  $\leq 300^{\circ}\text{K}$  in a 12 hour period for initial temperatures  $\leq 1000^{\circ}\text{K}$ .

## 2. Power Radiated Locally by O and CO

Radiative loss via transitions within the ground state triplet of atomic oxygen were considered by Bates (1951) in conjunction with his discussion of heating of the Earth's upper atmosphere. The ground state of atomic oxygen is an inverted triplet with the  $^3\text{P}_0$  and  $^3\text{P}_1$

levels lying .028 and .020 e.v. respectively above the  $^3P_2$  level. The Einstein coefficients for these transitions are  $A_{02} = 1.7 \times 10^{-5} \text{ sec}^{-1}$  and  $A_{12} = 8.9 \times 10^{-5} \text{ sec}^{-1}$ . Due to the greater excitation energy of the  $^3P_0$  level, the small statistical weight,  $g_0 = 1$ , and the relative magnitudes of the Einstein coefficients, emission from  $^3P_0$  will be much smaller than that from  $^3P_1$  and may be neglected. The question of thermodynamic equilibrium within the  $O(^3P_{0,1,2})$  triplet was considered by Bates, who concluded that collisional excitation of the excited levels would probably occur much faster than photon emission if the total particle density were  $\geq 10^9 \text{ cm}^{-3}$ . It will be shown below that infrared emission in the 62 $\mu$  oxygen line is almost certain to be effective at much higher densities and that the assumption of equilibrium within the oxygen ground state is therefore justified. With this assumption the power radiated in the 62 $\mu$  line is

$$R_s = \epsilon_1 A_{12} n(O | ^3P_1) = \epsilon_1 A_{12} n(O | ^3P_{0,1,2}) \frac{g_1 e^{-e_1/kT}}{g_2 + g_1 e^{-e_1/kT} + g_0 e^{-e_0/kT}} \quad (1)$$

where  $\epsilon_1$  and  $\epsilon_0$  are the energies of the  $^3P_1$  and  $^3P_0$  levels,  $g_0$ ,  $g_1$ , and  $g_2$  are the statistical weights (the subscript refers to the corresponding J value) and  $n$  refers to the oxygen number densities. Evaluating the constants results in

$$R_s = 2.85 \times 10^{-18} n_{OX}(z) f(T) \text{ ergs/cm}^3 \text{ sec} \quad (2)$$

where  $n_{OX}(z)$  is the total oxygen density and

$$f(T) = \frac{3e^{-228/T}}{5 + 3e^{-228/T} + e^{-325/T}} \quad (3)$$

The temperature dependence of the power radiated locally by atomic oxygen is not very great. The factor  $f(T)$  varies from 0 to 1/3 as  $T$  varies from 0 to  $\infty$ .

The expression for the power radiated locally by CO was also given by Bates. The power radiated is proportional to the square of the temperature in this case and is given by

$$R_r = 2.58 \times 10^{-23} n_{CO} T^2 \text{ ergs/cm}^3 \text{ sec} \quad (4)$$

Rotational states are readily populated by collisions (Zener, 1931) and it is highly probable that carbon monoxide is in rotational equilibrium well above the region of radiative loss.

### 3. Equation of Radiative Transfer for a Doppler-Broadened Line

An accurate evaluation of the total energy lost by a radiating constituent such as O or CO requires a determination of the altitude at which unit optical depth occurs. We will assume that radiative loss is fully effective above unit optical depth and ineffective at lower altitudes. An expression for the optical depth in Doppler-broadened lines may be obtained from the equation of radiative transfer. We consider a two-level system with the upper energy state denoted by subscript 2 and the lower state by subscript 1, and a radiation field of density  $\rho_\nu$ . The rate of decrease in the population of the upper energy state is

$$-\frac{dn_2}{dt} = n_2 A_{21} + n_2 B_{21} \rho_\nu - n_1 B_{12} \rho_\nu \quad (5)$$

where  $A_{21}$ ,  $B_{21}$ , and  $B_{12}$  are the Einstein coefficients for



spontaneous and induced emission and for absorption.

Each transition resulting in a decrease in  $n_2$  corresponds to the addition of a photon to the radiation field. Since this emission occurs over a frequency range the decrease in  $n_2$  corresponds to an increase in the photon intensity (in units  $\text{cm}^{-2} \text{steradian}^{-1} \text{sec}^{-1} (\text{cps})^{-1}$ ) integrated over frequency and direction, i.e.,

$$-\frac{dn_2}{dt} = \int d\Omega \left[ \frac{d}{ds} \right]_0^\infty d\nu I_\nu \quad (6)$$

where  $d\Omega$  is the element of solid angle,  $I_\nu$  the radiation intensity, and  $ds$  an element of path length in the medium. If the spectral line due to transitions between states 2 and 1 is Doppler-broadened we may write

$$\int_0^\infty I_\nu d\nu = \int_0^\infty I_{\nu_0} e^{-[(\nu-\nu_0)/\alpha_D]^2} d(\nu-\nu_0) = \frac{\sqrt{\pi}}{2} \alpha_D I_{\nu_0} \quad (7)$$

where  $\alpha_D$  is the Doppler width and  $I_{\nu_0}$  the intensity at the line center. We now make use of the relationship between the radiation density and intensity,

$$\rho_\nu = \frac{1}{c} \int I_\nu d\Omega = \frac{4\pi}{c} \bar{I}_\nu \quad (8)$$

between the Einstein coefficients

$$\frac{B_{21}}{A_{21}} = \frac{c}{8\pi} \left[ \frac{c}{\nu} \right]^2 = \delta_{\nu} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = \frac{g_2}{g_1} \quad (9)$$

where  $g_2$  and  $g_1$  are the statistical weights of levels 2 and 1, and substitute the combined results of (6) and (7) in (5) to obtain the radiative transfer equation

$$\frac{1}{4} \frac{dF_{\nu}}{dz} = \frac{1}{2\pi^{3/2}} \frac{A_{21}}{\alpha_D} n_2 \left[ 1 + \frac{4\pi\delta_{\nu}}{c} \left( 1 - \frac{g_2}{g_1} \frac{n_1}{n_2} \right) \bar{I}_{\nu} \right] \quad (10)$$

where  $F_{\nu} = \frac{1}{\pi} \int \mu I_{\nu} d\Omega$  is the photon flux. The subscript 0 on  $\nu$  has been dropped. The relation between the directions specified by  $z$  and  $s$  is  $dz = ds \cos \theta$  where  $\theta$  is the angle between directions  $z$  and  $s$ . If we now assume that the levels are in thermal equilibrium we have

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} e^{\epsilon_{21}/kT} \quad \text{and} \quad n_2 = \frac{g_2}{Z(T)} e^{-\epsilon_2/kT} n \quad (11)$$

where  $\epsilon_{21} = \epsilon_2 - \epsilon_1$  is the energy difference between states 2 and 1,  $n$  is the total number density and  $Z(T)$  is the partition function for the system. With the use of (11) the radiative transfer equation is

$$\frac{1}{4} \frac{dF_{\nu}}{dz} = -\sigma_D n \bar{I}_{\nu} + \epsilon_D n \quad (12)$$

where

$$\sigma_D = \frac{1}{4\pi} \frac{g_2 A_{21}}{3/2} \frac{1}{Z(T)} \left[ \frac{m}{2kT} \right]^{1/2} \left[ \frac{c}{v} \right]^3 e^{-\epsilon_1/kT} \left[ 1 - e^{-\epsilon_{21}/kT} \right] \quad (13)$$

is the absorption cross section in the line and

$$\epsilon_D = \frac{1}{2\pi} \frac{g_2 A_{21}}{3/2} \frac{1}{Z(T)} \left[ \frac{m}{2kT} \right]^{1/2} \left[ \frac{c}{v} \right] e^{-\epsilon_2/kT} \quad (14)$$

is the emission coefficient. The subscript D refers to the fact that these expressions hold for Doppler-broadened lines. The ratio  $\epsilon_D/\sigma_D$  is just the Plank distribution (for photons) as it must be since thermal equilibrium has been assumed.

#### 4. Optical Depths in O and CO

We will obtain some initial estimates of the overlying number densities in oxygen and carbon monoxide at which these constituents may begin to cool a planetary thermosphere by radiative loss. The optical depth in a line is defined by

$$d\tau = -\sigma_D n dz \quad (15)$$

where the altitude variable is denoted by  $z$ . If the constituent  $n$  is distributed in diffusive equilibrium in an isothermal atmosphere, the optical depth is

$$\tau = \sigma_D n H \quad (16)$$

and this unit optical depth occurs at an altitude where the overlying number density is

$$n H = \frac{1}{\sigma_D} \quad (17)$$

For the 62 $\mu$  line we evaluate the constants in (13) and use the partition function appropriate to the  $O(^3P_{0,1,2})$  triplet to find

$$\sigma_D(0) = 8.9 \times 10^{-16} \frac{1 - e^{-228/T}}{5 + 3e^{-228/T} + e^{-325/T}} \frac{1}{T^{1/2}} \text{ cm}^2 \quad (18)$$

If we take  $T = 300^\circ\text{K}$  as an estimate of the isothermal temperature, then the overlying oxygen number density at unit optical depth is

$$n_{\text{OX}} H_{\text{OX}} \Big|_{\tau=1} = 2.4 \times 10^{17} \text{ cm}^{-2} \quad (19)$$

We recall that the initial assumption of thermal equilibrium in the ground state oxygen triplet required total number densities  $\geq 10^9$ . That radiative cooling commences below the altitude at which departures from equilibrium occur is obvious from (19) since atomic oxygen number densities of  $10^9$  would correspond to unit optical depth in the  $62\mu$  line only if the scale height were  $\approx 1000$  km.

In the case of carbon monoxide we have transitions between rotational states  $J \rightleftharpoons J-1$  for all values of the angular momentum quantum number  $J$ . We will assume, as has been done by McElroy et al. (1965), that the optical depth in the carbon monoxide rotational lines is given by the optical depth in the line in which most energy is transferred. The rotational state with the greatest occupation is specified by

$$J_{\text{MAX}} = \left[ \frac{T}{2\theta} \right]^{1/2} \quad (20)$$

The Einstein coefficient for spontaneous emission in this line may be evaluated from the general expression for the A coefficient (Schiff, 1955, Chapter 10) using the dipole

matrix element for adjacent states of a rigid rotator.

We have

$$A_{J,J-1} = \frac{4}{3\hbar} (4\pi B)^3 d^2 \frac{J^4}{2J+1} \quad (21)$$

where B is the rotational constant of the molecule, d the dipole moment of CO and  $\hbar$  Plank's constant. Evaluating this and all other J-dependent terms in (13) at  $J = J_{MAX}$  gives the absorption cross section in the strongest rotational line as

$$\sigma_D(CO) = \frac{4\pi^{3/2}}{3\sqrt{e}} \frac{d^2}{\hbar} \left[ \frac{m}{kT} \right]^{1/2} \left[ \frac{\theta_r}{T} \right]^{1/2} \left[ 1 - e^{-(2\theta_r/T)} \right]^{1/2} \quad (22)$$

where  $\theta_r$  is the characteristic rotational temperature  $hcB/k$ . Evaluating the constants, with  $d = .12 \times 10^{-18}$  (Fowler, 1935),  $B = 1.93 \text{ cm}^{-1}$  (Bates, 1951), and expanding the exponential to first order we find

$$\sigma_D(CO) = \frac{1.35 \times 10^{-13}}{T^{3/2}} \text{ cm}^2 \quad (23)$$

This expression was given by McElroy et al. (1965). At a temperature  $T = 300^\circ\text{K}$  unit optical depth in the CO

rotational lines thus occurs at an overlying CO density of

$$n_{\text{CO}} H_{\text{CO}} |_{\tau=1} = \frac{1}{\sigma_D(\text{CO})} = 3.8 \times 10^{16} \text{ cm}^{-2} \quad (24)$$

Equations (19) and (24) show clearly that conclusions as to the relative effectiveness of O and CO as cooling agents based on a direct comparison of the power radiated locally is misleading. The sum of the oscillator strengths for the CO rotational lines is greater than the oscillator strength of the  $62\mu$  O emission. However, this implies not only that CO is a more effective emitter in the infrared than is O, but is also a more effective absorber. Only a consideration of the balance between emission and loss can lead to valid conclusions regarding the relative effectiveness of these constituents as radiators.

##### 5. Comparison of Total Fluxes Radiated by O and CO

We will now assume that in the altitude region from which radiative loss occurs the scale height may be adequately described in terms of a linear variation i.e.,

$$H(z) = H_0 + \beta z \quad (25)$$

where  $H_0$  is the scale height at the bottom of the zone and  $\beta$  is the (constant) scale height gradient. The number density of a constituent is given in this case by

$$n(z) = \frac{n_0 T_0}{T(z)} \left[ \frac{H_0}{H} \right]^{1/\beta} \quad (26)$$

The subscript 0 on a quantity refers to the value of that quantity at the altitude of the bottom of the radiation zone, i.e., at unit optical depth. This altitude is different for different constituents, but this distinction need not be made yet. The optical depth in a given constituent is

$$\tau(z_0) = - \int_{z_0}^{z_1} n(z) \sigma_D(T) dz \quad (27)$$

where we assume that  $z_1$  denotes the altitude of the top of the radiating zone and that contributions to the optical depth from greater altitudes are negligible. The altitude  $z_1$  is not strictly defined, but should be chosen at least one scale height above  $z_0$ . Using (18)



and (23) for the absorption cross sections in O and CO we find the following expressions for the optical depths in these constituents

$$\tau(O) = \frac{\sigma_{D_0}(O)}{1 + \beta_{OX}/2} (n_{O,H_0})_{OX} \left[ 1 - \Lambda_{OX}^{(2+\beta_{OX})/2\beta_{OX}} \right] \quad (28)$$

$$\tau(CO) = \frac{\sigma_{D_0}(CO)}{1 + \frac{3}{2}\beta_{CO}} (n_{O,H_0})_{CO} \left[ 1 - \Lambda_{CO}^{(2+3\beta_{CO})/2\beta_{CO}} \right] \quad (29)$$

where  $\Lambda = T_0/T_1$  is the ratio of temperatures at the bottom and top of the radiation zone. Subscripts OX and CO have been appended to quantities which refer to atomic oxygen and carbon monoxide. In the derivation of (28) from (27) only the  $T^{-1/2}$  dependence of  $\tau_D(T)$  was explicitly included in the integration. The remaining temperature-dependent term in (18), which varies from 0 to 1/5 as  $T$  varies from  $\infty$  to 0, is assumed evaluated at  $T_0$ . This approximation will slightly overestimate the optical depth at 62 $\mu$  (for a given altitude) and correspondingly underestimate the effectiveness of O cooling. The total flux radiated by a constituent is given by

$$F_R = \int_{z_0}^{z_1} R(z, T) dz \quad (30)$$

where  $R(z, T)$  is the power radiated. Using (2) and (4) for the power radiated by O and CO we find

$$F_R(O) = 2.8 \times 10^{-18} (n_0 H_0)_{OX} f(T_0) \left[ 1 - \Delta_{OX}^{1/\beta_{OX}} \right] \quad (31)$$

$$F_R(CO) = \frac{2.58 \times 10^{-23} (n_0 T_0^2 H_0)_{CO}}{1 - 2\beta_{CO}} \left[ 1 - \Delta_{CO}^{(1-2\beta_{CO})/\beta_{CO}} \right] \quad (32)$$

The function  $f(T_0)$  appearing in (31) and given explicitly by (3) was not included in the integrand of (30) but is assumed evaluated at  $T_0$ . The variation of  $f(T)$  with  $T$  is relatively slow for  $T > 250^\circ K$ . The expression (32) was given by McElroy et al. (1965). If we now evaluate the product  $nH$  appearing in (31) and (32) at unit optical depth (using (28) and (29)) we obtain the total energy loss by O and CO in a planetary thermosphere (in ergs/cm<sup>2</sup> sec)

$$F_R(O) = 9.6 \times 10^{-3} \left( 1 + \frac{\beta_{OX}}{2} \right) E(T_0) T_0^{1/2} \left[ \frac{1 - \Delta_{OX}^{1/\beta_{OX}}}{(2+\beta_{OX})/2\beta_{OX}} \right] \quad (33)$$

$$F_R(CO) = 9.5 \times 10^{-11} \frac{2+3\beta_{CO}}{1-2\beta_{CO}} T_0^{7/2} \left[ \frac{1 - \Delta_{CO}^{(1-2\beta_{CO})/\beta_{CO}}}{(2+3\beta_{CO})/2\beta_{CO}} \right] \quad (34)$$

where

$$E(T_0) = \frac{1}{e^{228/T_0} - 1} \quad (35)$$

It must be borne in mind that  $T_0$  may refer to different temperatures in the expressions relating to O and CO.

For temperatures  $T_0 \gtrsim 300^\circ\text{K}$  the flux lost in the 62 $\mu$  line is roughly proportional to  $T_0^{3/2}$  since  $E(T_0) \approx T_0$ .

The presence of atomic oxygen in a planetary thermosphere therefore does, to some extent, act as a thermostat since the total energy loss increases more rapidly than the temperature. In an atmosphere containing both O and CO the thermostatic effect of CO will of course predominate since the energy loss by CO goes as  $T_0^{7/2}$ .

We will neglect the difference in temperatures at unit optical depths in O and CO and denote by  $T_R$  a mean temperature in the altitude region between which O and CO begin to radiate. The ratio of the total heat fluxes radiated by O and CO is then

$$R = \frac{F_R(O)}{F_R(CO)} = 5 \times 10^7 \frac{(2+\beta_{OX})(1-2\beta_{CO})}{(2+3\beta_{CO})} \frac{E(T_R)}{T_R^3} \quad (36)$$

$$\times \frac{\left[1 - \Delta_{OX}^{1/\beta_{OX}}\right] \left[1 - \Delta_{CO}^{(2+3\beta_{CO})/2\beta_{CO}}\right]}{\left[1 - \Delta_{OX}^{(2+\beta_{OX})/2\beta_{OX}}\right] \left[1 - \Delta_{CO}^{(1-2\beta_{CO})/\beta_{CO}}\right]}$$

Since  $E(T_R) \approx T_R$  for  $T_R \gtrsim 300^\circ\text{K}$  this ratio falls off roughly as  $T^{-2}$  at high temperatures. The ratio is plotted as a function of  $T_R$  in Fig. 1. Table 1 gives the values of the total flux radiated by O and CO from unit optical depth in each constituent for various values of temperature and scale height gradient in the radiation zone. The figures may serve as a useful check on the results of detailed numerical integration of the thermal conduction equation and as a basis for judging the relative effectiveness of these constituents as cooling agents. The temperature difference at unit optical depth in O and CO should be taken into account before quantitative comparisons are made.

## 6. Nocturnal Temperature Change

Bates compared the total energy content of the Earth's F region (per  $\text{cm}^2$  column) with the energy loss rate due to thermal conduction and  $62\mu$  O radiation. The character-

Table 1

Total flux radiated from a planetary thermosphere by O and CO. T refers to the temperature at unit optical depth in each constituent and  $\beta$  is the corresponding scale height gradient. The flux is in ergs/cm<sup>2</sup> sec.

T	$\beta = 0$		.1		.2		.3		.4	
	F <sub>O</sub>	F <sub>CO</sub>	F <sub>O</sub>	F <sub>CO</sub>	F <sub>O</sub>	F <sub>CO</sub>	F <sub>O</sub>	F <sub>CO</sub>	F <sub>O</sub>	F <sub>CO</sub>
100	.010	.002	.012	.002	.012	.003	.012	.004	.013	.005
200	.064	.022	.067	.026	.069	.032	.073	.041	.076	.052
300	.15	.089	.15	.11	.16	.13	.17	.17	.17	.21
400	.25	.24	.27	.29	.27	.37	.28	.45	.28	.55
500	.37	.53	.39	.65	.40	.49	.42	.96	.42	1.2
600	.51	1.0	.52	1.2	.55	1.5	.57	1.8	.59	2.2
700	.66	1.7	.69	2.1	.71	2.5	.73	3.0	.74	3.6
800	.83	2.8	.86	3.3	.88	4.0	.91	4.7	.93	5.5
900	.99	4.2	1.0	5.0	1.1	5.9	1.1	6.9	1.1	8.1
1000	1.2	6.0	1.3	7.1	1.3	8.4	1.3	9.8	1.3	11.

istic time which he obtained as the quotient of these quantities was of the order of several days, and he therefore concluded that nocturnal cooling would be slight. The time-dependent study by Harris and Priester (1962) gives a daily amplitude of exospheric temperature variation of roughly  $\pm 200^{\circ}\text{K}$  about a mean of  $1500^{\circ}\text{K}$ . The amplitude is thus relatively small ( $\pm 15\text{-}20\%$ ).

The situation is quite different if substantial CO is present. We will define the cooling time for CO as the time required for radiation from the rotational states to halve a given initial temperature. If we consider CO cooling alone, the temperature change is described by

$$\rho c_p \frac{dT}{dt} = -k_r n_{\text{CO}}(z) T^2 \quad (37)$$

where  $k_r = 2.58 \times 10^{-23} \text{ erg}/^{\circ}\text{K}^2 \text{ sec}$  is the coefficient of the term for the power radiated by CO (see equation (4)). The solution of (37) is

$$T(t) = \frac{T_0}{1 + t/\bar{\theta}} \quad (38)$$

where  $T_0$  is the initial temperature and

$$\theta = \frac{c_p \rho}{k_r n_{CO} T_0} \quad (39)$$

is the cooling time for CO. If we take the density  $\rho$  in (39) to be the sum of the O and CO densities,  $\theta$  may be expressed as

$$\theta = \frac{c_p m_{CO}}{k_r T_0} \left[ 1 + \frac{\sigma_D(CO)}{\sigma_D(O)} \left[ \frac{m_{OX}}{m_{CO}} \right]^2 \frac{\tau_{OX}}{\tau_{CO}} \right] \quad (40)$$

where  $m_{CO}$  is the mass of a CO molecule,  $m_{OX}$  the mass of atomic oxygen,  $\sigma_D(O)$  and  $\sigma_D(CO)$  are the Doppler absorption cross sections given in (18) and (23), and  $\tau_{OX}$  and  $\tau_{CO}$  are the isothermal optical depths in O and CO. The temperature dependence not taken into explicit account in passing from (37) to (38), as well as the altitude dependence of  $\theta$ , appears in the right hand term in the brackets of (40). The Doppler absorption cross sections are temperature-dependent, but the ratio  $\sigma_D(CO)/\sigma_D(O)$  does not depend strongly on temperature and is equal to about 5 over a wide temperature range. The first two factors in the right hand term in brackets of (40) are

thus of order unity. The ratio  $\rho_{\text{OX}}/\rho_{\text{CO}}$  is certainly less than one, but since the density,  $\rho$ , is likely to be slightly underestimated in (40) we will take the right hand term as equal to 1. The cooling time is thus

$$\theta \approx 2 \frac{c_p^m \text{CO}}{k_r T_0} \quad (41)$$

The approximation made in setting  $\rho_{\text{OX}}/\rho_{\text{CO}} \approx 1$  is thus roughly equivalent to the assumption that the carbon monoxide density ( $\text{gm/cm}^3$ ) in the radiation zone is half the total density. Equation (41) thus contains the implicit assumption that  $\text{CO}_2$  is a major atmospheric constituent at the altitude at which it dissociates.

The temperature change as a function of time, evaluated from (38), over a 12-hour period is shown in Fig. 2 for various initial temperatures. The specific heat has been taken as  $c_p = 0.2 \text{ cal/}^\circ\text{K gm}$ . The cooling time given by (41) is about 7 hours at  $1000^\circ\text{K}$ .

The amplitude of the temperature decrease given in Fig. 2 should be representative of that to be expected in the Martian thermosphere since the rotational period of the planet is about 24 hours. Thus, if the peak



daytime temperature is  $\approx 1000^{\circ}\text{K}$  the temperature of the nighttime Martian exosphere must fall to  $\approx 300^{\circ}\text{K}$  unless there is a nocturnal heat source. The rotational period of Venus is relatively long, being about 250 days retrograde (Goldstein and Carpenter, 1963). The nocturnal temperature in the Venusian thermosphere could thus be quite low since equilibrium may depend either on energy transport from the day side, on infrared heating from the planetary spectrum, or on other low altitude heat sources. Thermal conduction from the day side could not maintain a very high temperature on the night side of the planet. Even with a day-night temperature difference of  $2000^{\circ}\text{K}$ , the mean horizontal temperature gradient would be  $\approx .05^{\circ}\text{K}/\text{km}$ , which would transport  $\sim .008 \text{ erg}/\text{cm}^2\text{sec}$ . From Table 1 we see that this flux could be substantially accommodated by O and CO at temperatures  $\approx 100^{\circ}\text{K}$ . A quantitative estimate of the temperature maintained by infrared heating would require a solution of the radiative transfer equation, but it would be less than the planetary effective temperature of  $\sim 230^{\circ}\text{K}$ .

## 7. Summary and Conclusions

The effectiveness of atomic oxygen and carbon monoxide

as radiative cooling agents in a planetary thermosphere has been investigated. Analytic expressions for the total flux radiated by each constituent were developed and it was shown that for temperatures  $\leq 300^{\circ}\text{K}$  in the radiation zone the total energy loss in the  $62\mu$  oxygen line may equal or exceed that in the CO rotational lines. The figures given in Table 1 for the total flux radiated by each constituent depend on the efficacy of a Bates' type of cooling approximation for their validity, but the conclusions drawn in this paper as to the relative cooling effectiveness of O and CO are not so dependent.

It has been shown that substantial overnight cooling will occur in planetary exospheres containing CO. The amplitude of the day-night temperature variation in the Martian exosphere is probably about 50% in contrast to a 15-20% diurnal variation in the Earth's exosphere. The nocturnal temperature minimum in the Martian exosphere must be  $\leq 300^{\circ}\text{K}$  if the peak daytime temperature is  $\leq 1000^{\circ}\text{K}$ , as it must be for the retention of atomic oxygen by the planet (Chamberlain, 1962). The Venusian exosphere must also be rapidly cooled by CO. Due to the length of the night on Venus the equilibrium exospheric temperature

must be maintained either by horizontal energy transport or by heating from lower altitudes. Horizontal conduction from the day side could not maintain a nighttime temperature in excess of  $\sim 100^{\circ}\text{K}$ .

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## LEGENDS

Fig. 1. Ratio of O to CO radiated flux. The curves are labeled according to the values of the scale height gradient  $\beta_{OX}$  for O.

Fig. 2. Decrease in temperature with time due to infrared radiation from CO.



