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WIND VECTOR CALCULATION USING CROSSED-BEAM DATA AND DETECTOR ARRANGEMENT FOR MEASURING HORIZONTAL WINDS

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ABSTRACT

In atmospheric experimentation three ground-based detectors suffice for determining wind vectors that could be present in a certain space volume. While this is true in general, the geometry simplifies a great deal, when these vectors can be taken as horizontal. Since to do so will often be warranted, the pertinent method for a proper arrangement of detector positions and attitudes has been developed in detail. With a suitable layout winds reasonably constant near a selected height, if blowing into and out of a quarter of the rose, can be calculated with sufficient accuracy and confidence. The error analysis assumes that the observational time uncertainty is at most ± 0.1 sec.

To observe wind vectors in the remaining directions, or at several heights simultaneously, the use of more detectors (or of the available multiple detectors) is called for. This is subject matter for future investigation, as is the more intricate problem posed by non-horizontal winds.

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AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

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SUMMARY

The report's first part (sections I through V) contains the geometric foundation and subsequent development of a general method for measuring atmospheric winds by exploiting the ideas of crossed-beam experimentation. It shows that (and how) with three single-beam detectors at ground level one should be able to monitor a variety of unknown winds that could be blowing within a limited space volume, provided any such wind, once present, remains sufficiently stable during observation.

A second part (sections VI through VIII) applies -- and further develops -- the method in the practically meaningful and geometrically least complex case where the winds can be taken as horizontal. A criterion for this to be justifiable is given. Flow conditions, wind identification, error analysis, and the desire to keep the detector arrangement compact impose a number of restraints that are reflected in the experimental layout finally suggested. These restraints are more acutely felt the higher up one proposes to measure. Equations for actually computing the two velocity components are supplied.

A concluding section delineates problems yet to be solved.

I. INTRODUCTION

As it is well known, wind determination by crossed beams presents a number of problems. By way of examples we may list: record evaluation complications through long observation times and non-stationary statistics, interacting intensity fluctuations through the scattering of light from different sources (sky and sun), the disturbing influence of cumuli and other clouds, unwanted features in absorption physics. Most of them are or have been under vigorous attack elsewhere.

The report on hand is concerned with the specific difficulties encountered when the net results of the experimentation proper, i.e., the characteristic delay times, are to be used for calculating the winds they are associated with.

To achieve this end, resources of spatial geometry are assembled in a preparatory section. Still there are quite a few problems left that must be overcome. In the first place, covariance peaks at characteristic times will be registered whenever eddy trains travel on paths that happen to connect beams; one must sort out those that belong to one and the same wind. In the second, one should be able to say in what space region the latter was blowing. Thirdly, one cannot be satisfied to detect just this one wind on the chance it might be present, but wishes to monitor an array, as large as possible, of unknown wind vectors near a predetermined location. Fourth, observational error margins must be prevented from getting blown up to unacceptable dimensions. For this reason travel paths too short cannot be admitted. Too long ones are prohibited lest eddies lose identity. Finally, one would not like, on practical grounds, to have to place the detectors too far apart.

These warnings and requirements were heeded in working out the body of the paper. A rather definite set of detector specifications for observing horizontal winds is the outcome that leaves to free choice but two parameters out of ten.

II. GEOMETRIC PRELIMINARIES

Geometric analysis provides a number of results useful for the argument to follow and for the realization in general of spatial relationships associated with the crossed-beam method.

The two straight lines, a and b, established by the points P_1, P_2 and the direction vectors*,

$$\underline{\alpha} = \alpha_1\underline{i} + \alpha_2\underline{j} + \alpha_3\underline{k}$$

$$\underline{\beta} = \beta_1\underline{i} + \beta_2\underline{j} + \beta_3\underline{k},$$

*Vectors are designated by underlined symbols; e.g., $\underline{i}$, $\underline{j}$, $\underline{k}$ are the unit vectors in axis directions.

have the equations

$$\frac{x - x_1}{\alpha_1} = \frac{y - y_1}{\alpha_2} = \frac{z - z_1}{\alpha_3}$$

$$\frac{x - x_2}{\beta_1} = \frac{y - y_2}{\beta_2} = \frac{z - z_2}{\beta_3} .$$

With a view to later application it is assumed that they do not intersect, i.e., that

$$\Delta_{ab} = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{vmatrix} \neq 0. \qquad (1)$$

This excludes parallelism as well. Nevertheless, the lines do define two parallel planes in which they are contained. The direction of that pair's common normal is given by the unit vector (called the "binormal" direction here*)

$$\underline{n} = \frac{1}{\sin \omega_{ab}} [\underline{\alpha} \times \underline{\beta}], \qquad (2)$$

ω_{ab} being the angle made by the vectors $\underline{\alpha}$ and $\underline{\beta}$ when crossed:

$$\cos \omega_{ab} = \alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3.$$

Introduce now a further direction

$$\underline{m} = m_1\underline{i} + m_2\underline{j} + m_3\underline{k} \qquad (|\underline{m}| = 1)$$

---

*This agrees with the usage of the word in the theory of spatial curves, provided that the two planes are interpreted as the lines' tangential planes.

not parallel to those planes:

$$(m\alpha\beta) \equiv \begin{vmatrix} m_1 & m_2 & m_3 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{vmatrix} \neq 0.$$

The special case where $\underline{m} = \underline{n}$ evidently is not excluded.

Then there exist two -- and only two -- points, P_1^* and P_2^*, on the lines a and b such that $\overrightarrow{P_1^* P_2^*}$ is parallel to $\underline{m}$. Clearly, among the directions $\underline{m}$ only those are admitted as are capable of connecting a with b in that order (excluding all the reverse orientations). The position vectors, $\underline{r_1^*}$ and $\underline{r_2^*}$, of P_1^* and P_2^* are given* by

$$\left.\begin{aligned} \underline{r_1^*} &= \underline{r_1} + \frac{\underline{\alpha}}{(m\alpha\beta)} \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ \beta_1 & \beta_2 & \beta_3 \\ m_1 & m_2 & m_3 \end{vmatrix} \\ \underline{r_2^*} &= \underline{r_2} + \frac{\underline{\beta}}{(m\alpha\beta)} \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ m_1 & m_2 & m_3 \end{vmatrix} \end{aligned}\right\}, \qquad (3)$$

where $\underline{r_1}$ and $\underline{r_2}$ define P_1 and P_2. It is clear and can be shown mathematically* that the choice of these two points on a and b cannot affect the vectors $\underline{r_1^*}$ and $\underline{r_2^*}$. If the length $\overline{P_1^* P_2^*}$ is denoted by R_{ab}^* we may write

$$\overrightarrow{P_1^* P_2^*} \equiv \underline{r_2^*} - \underline{r_1^*} = R_{ab}^* \, \underline{m}.$$

Figure 1 illustrates this formula.

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*See: Heybey, W. H., "On the Wind Component Measurable by Crossed-Beam Arrangement," IN-AERO-7-67, October 12, 1967.

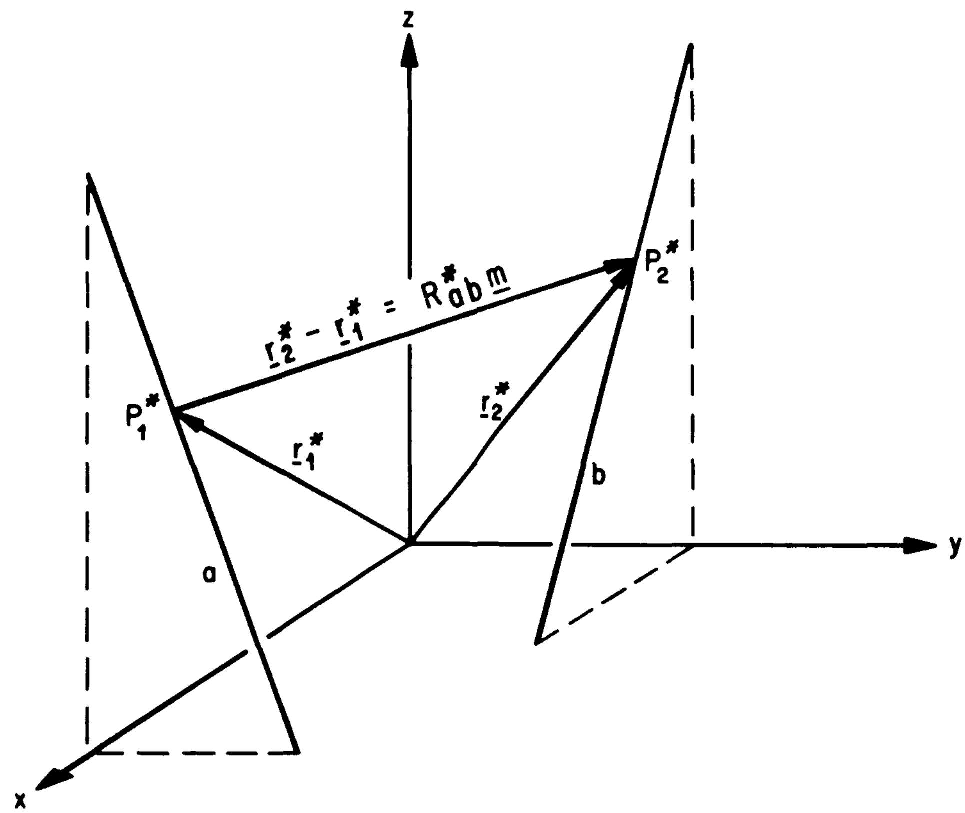

Figure 1. The vectors $\underline{r}_1^*$, $\underline{r}_2^*$, and $\underline{m}$

The projection onto the binormal of any straight segment connecting a and b is equal to the binormal distance, δ, itself. This is readily recognized by figure 2, which depicts the simplest geometric arrangement possible without loss of generality. Of the two parallel planes established by the lines a and b one is taken as the (z,x)-plane and the other as parallel to it. The y-axis is defined by the binormal. The projecting lines through the points P_1 or P_1^* on the one hand, and through P_2 or P_2^* on the other are the lines a and b themselves, as they are normal to the binormal.

Since thus the projections of $\overline{P_1^* P_2^*}$ and $\overline{P_1 P_2}$ are equal:

$$(\underline{r}_2^* - \underline{r}_1^*) \cdot \underline{n} = (\underline{r}_2 - \underline{r}_1) \cdot \underline{n},$$

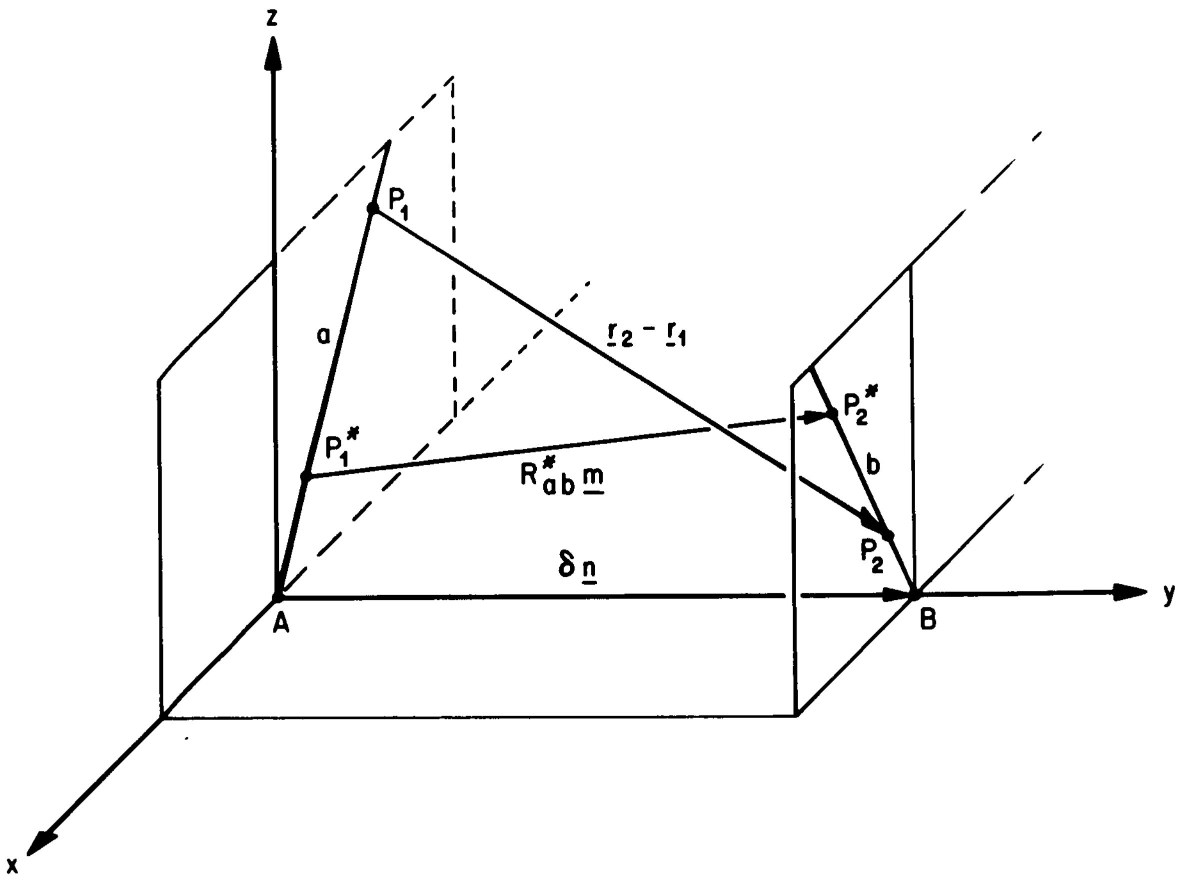

Figure 2. General two-beam geometry.

we find that

$$R^*_{ab}(\underline{m} \cdot \underline{n}) = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{vmatrix} \frac{1}{\sin \omega_{ab}},$$

$$R^*_{ab}(\underline{m} \cdot \underline{n}) \sin \omega_{ab} = \Delta_{ab},$$

or that

$$R^*_{ab} = \frac{\Delta_{ab}}{(m\alpha\beta)}. \tag{4}$$

This is the shortest expression for the distance $P_1^* P_2^*$. That the quotient at right is always positive can be verified in a convenient manner if (as one is permitted to do) the defining points P_1, P_2 are replaced by the binormal terminals A, B with coordinates

$$x_1 = y_1 = z_1 = 0$$

$$x_2 = 0, \ y_2 = \delta > 0, \ z_2 = 0.$$

Then, since $\alpha_2 = \beta_2 = 0$,

$$\Delta_{ab} = \begin{vmatrix} 0 & \delta & 0 \\ \alpha_1 & 0 & \alpha_3 \\ \beta_1 & 0 & \beta_3 \end{vmatrix} = \delta \begin{vmatrix} \alpha_3 & \alpha_1 \\ \beta_3 & \beta_1 \end{vmatrix},$$

and

$$(m\alpha\beta) \equiv \begin{vmatrix} m_1 & m_2 & m_3 \\ \alpha_1 & 0 & \alpha_3 \\ \beta_1 & 0 & \beta_3 \end{vmatrix} = m_2 \begin{vmatrix} \alpha_3 & \alpha_1 \\ \beta_3 & \beta_1 \end{vmatrix}.$$

Because the vector $\underline{m}$ points from P_1^* to P_2^*, its y-component is positive: $m_2 > 0$. It follows that

$$\frac{\Delta_{ab}}{(m\alpha\beta)} = \frac{\delta}{m_2} > 0.$$

III. TWO-BEAM EXPERIMENTATION

Broadly speaking, wind determination by crossed beams assumes that the bulk speed of eddy propagation can be taken as a sufficiently close approximation to the main wind speed. In the simplest atmospheric experimentation two detectors receive natural light out of a conical volume extending from their objectives to great distances. For the present purpose these "beams" are replaced by the axes of the cones.

Likewise, turbulent eddies will be idealized to have infinitesimal size. The time, τ^*_{ab}, an eddy needs to cover the distance between two points P^*_1 and P^*_2 is determined experimentally. It is assumed as equal to that particular delay time that produces maximum covariance between the intensity curves obtained from the two beams, and is positive or negative depending on whether, in order to locate the maximum, the b-record has to be delayed relative to the a-record or vice versa. If it is negative, the velocity

$$\underline{V} = \frac{R^*_{ab}}{\tau^*_{ab}} \underline{m} \qquad (5)$$

goes from P^*_2 to P^*_1, i.e, it is in the reverse direction of $\underline{m}$.

Wind variations pending observation are to be reckoned with. Fluctuations in strength tend to blur the covariance maximum, but would not blot it out completely except perhaps when excessive or unidirectional. Variation of direction appears more serious. The eddies leaving P^*_1 (or P^*_2) over a period of time may often miss the second beam*; those that do arrive at it may not be numerous enough for a clear definition of the maximum. Although the beams a and b are immersed in a three-dimensional wind field, the directions of the motion could be inappropriate at every point of the a- (or b-) beam, meaning that no single eddy path will cross the second beam; compare Figure 3 (where however two connecting paths are indicated). At any rate, the number of covariance maxima will be

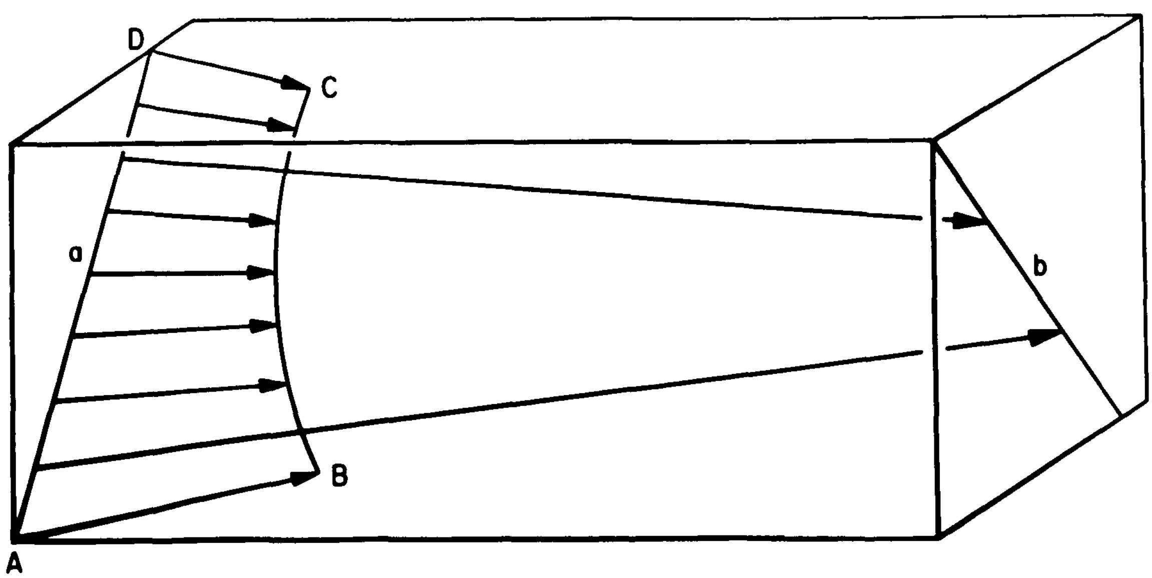

Figure 3. Curved eddy "sheet" ABCD downstream from beam a.

*In physical reality this danger abates somewhat because of the finite beam width and finite eddy size.

limited and may even be zero, for instance, if the wind is blowing in a direction normal to the beams' binormal. This observation is a first indication that more than just two detectors will be needed for measuring winds.

Counting on the finite beam width and eddy size, we infer from Figure 3 that it will be advantageous to have the beams pass each other at a relatively close distance* to insure eddy interception and an approximate wind vector finding. However, direct intersection would cause maximum correlation at the common point, irrespective of wind direction (which thus remains unknown) and leaving us with an indeterminate expression for the wind strength, V, the distance zero being covered in zero time.** A finite binormal distance is required which moreover ought not to be chosen too small in order to escape short travels with consequent large effects of observational inaccuracies. On the other hand, it must not be allowed to be too large either, lest eddy change or decay on the longer paths impair or destroy the needed correlation.

In an ideal atmosphere (considered first) the wind is constant within sight, its direction being given by the unit vector

$$\underline{v} = v_1 \underline{i} + v_2 \underline{j} + v_3 \underline{k}. \tag{6}$$

If we knew it, we should put

$$\underline{m} = \pm \underline{v} \tag{7}$$

and, by the system (3) could uniquely determine the position vectors $\underline{r}_1^*$ and $\underline{r}_2^*$ (the double sign cancels). The covariance curve would exhibit a single maximum at the time, τ_{ab}^*, needed for an eddy to move from P_1^* to P_2^* (or conversely). The strength of the wind would emerge as

$$V = \left| \frac{\underline{r}_2^* - \underline{r}_1^*}{\tau_{ab}^*} \right| = \frac{R_{ab}^*}{|\tau_{ab}^*|} .$$

---

* A later reasoning will throw support to beam nearness from quite a different angle.

**If, in a rare instance, the wind vector happened to be parallel to the plane of the intersecting beams, the situation would be quite as confused.

This result applies in wind tunnel experimentation when the convective motion can be taken as parallel to the tunnel axis. In the atmosphere a reasonable surmise regarding $\underline{v}$ will not in general be at hand. However, the component of the wind vector

$$\underline{V} = \underline{i}V_1 + \underline{j}V_2 + \underline{k}V_3 \tag{8}$$

in direction of the binormal is still obtainable. With the aid of expressions (5), (7), (4), and (2)

$$V_n = \pm \frac{R^*_{ab}}{\tau^*_{ab}} (\underline{v} \cdot \underline{n}) = \pm \frac{\triangle_{ab}}{\tau^*_{ab}} \frac{\underline{v} \cdot \underline{n}}{(\pm\, v\alpha\beta)} = \frac{\triangle_{ab}}{\tau^*_{ab}} \frac{(v\alpha\beta)}{(v\alpha\beta)} \frac{1}{\sin \omega_{ab}} ,$$

or

$$V_n = \frac{\triangle_{ab}}{\tau^*_{ab}} \frac{1}{\sin \omega_{ab}} . \tag{9}$$

Every term at right is known here.

On the other hand, writing $V_n = \underline{V} \cdot \underline{n}$ directly in terms of the expressions (8) and (2), we arrive at the important relation

$$\frac{\triangle_{ab}}{\tau^*_{ab}} = \begin{vmatrix} \alpha_2 & \alpha_3 \\ \beta_2 & \beta_3 \end{vmatrix} V_1 + \begin{vmatrix} \alpha_3 & \alpha_1 \\ \beta_3 & \beta_1 \end{vmatrix} V_2 + \begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} V_3, \tag{10}$$

which is basic for what follows.

IV. PRINCIPLES OF WIND VECTOR DETERMINATION - THREE-BEAM GEOMETRY

It will again be assumed in this section that the wind vector is constant throughout the observed atmosphere.

Let us introduce a third (c-) beam identified by a point P_3 and the unit direction vector

$$\underline{\gamma} = \gamma_1 \underline{i} + \gamma_2 \underline{j} + \gamma_3 \underline{k}.$$

It must not intersect with the first two beams, so that the conditions apply

$$\left.\begin{aligned} \Delta_{bc} &= \begin{vmatrix} x_3 - x_2 & y_3 - y_2 & z_3 - z_2 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix} \neq 0 \\ \Delta_{ca} &= \begin{vmatrix} x_1 - x_3 & y_1 - y_3 & z_1 - z_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \\ \alpha_1 & \alpha_2 & \alpha_3 \end{vmatrix} \neq 0 \end{aligned}\right\} . \tag{11}$$

Correlation of the b- and c-records and of the c- and a-records will result in two covariance maxima at $\tau = \tau^*_{bc}$ and $\tau = \tau^*_{ca}$, respectively.

For the unknown components V_1, V_2, V_3, three linear equation can than be set up according to pattern (10):

$$\left.\begin{aligned} L_1 \equiv \frac{\Delta_{ab}}{\tau^*_{ab}} &= \begin{vmatrix} \alpha_2 & \alpha_3 \\ \beta_2 & \beta_3 \end{vmatrix} V_1 + \begin{vmatrix} \alpha_3 & \alpha_1 \\ \beta_3 & \beta_1 \end{vmatrix} V_2 + \begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} V_3 \\ L_2 \equiv \frac{\Delta_{bc}}{\tau^*_{bc}} &= \begin{vmatrix} \beta_2 & \beta_3 \\ \gamma_2 & \gamma_3 \end{vmatrix} V_1 + \begin{vmatrix} \beta_3 & \beta_1 \\ \gamma_3 & \gamma_1 \end{vmatrix} V_2 + \begin{vmatrix} \beta_1 & \beta_2 \\ \gamma_1 & \gamma_2 \end{vmatrix} V_3 \\ L_3 \equiv \frac{\Delta_{ca}}{\tau^*_{ca}} &= \begin{vmatrix} \gamma_2 & \gamma_3 \\ \alpha_2 & \alpha_3 \end{vmatrix} V_1 + \begin{vmatrix} \gamma_3 & \gamma_1 \\ \alpha_3 & \alpha_1 \end{vmatrix} V_2 + \begin{vmatrix} \gamma_1 & \gamma_2 \\ \alpha_1 & \alpha_2 \end{vmatrix} V_3 \end{aligned}\right\} , \tag{12}$$

which are solved to give

$$\left.\begin{aligned} (\alpha\beta\gamma)\ V_1 &= \gamma_1 L_1 + \alpha_1 L_2 + \beta_1 L_3 \\ (\alpha\beta\gamma)\ V_2 &= \gamma_2 L_1 + \alpha_2 L_2 + \beta_2 L_3 \\ (\alpha\beta\gamma)\ V_3 &= \gamma_3 L_1 + \alpha_3 L_2 + \beta_3 L_3 \end{aligned}\right\} . \tag{13}$$

The determinant

$$(\alpha\beta\gamma) = \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix} \tag{14}$$

must not be zero. None of the beam directions $\underline{\alpha}$, $\underline{\beta}$, $\underline{\gamma}$ is allowed to be parallel to the plane of the other two if crossed. In particular, parallelism of any beam pair (e.g., $\alpha_i = \beta_i$, i = 1,2,3) is excluded; it would cause the determinant to vanish.

By the system (13) three beams suffice to measure winds; however, not all conceivable winds. If, e.g., $\underline{V}$ happened to be parallel to the pair of parallel planes established by two beams, one of the right sides in equations (12) would be identically zero and thus remove the equation from the system. If $\underline{V}$ were parallel to one single beam, two equations would become meaningless for the same reason. Such velocities therefore cannot be observed by the lineal beams as used. Although one can show that more than just the above few directions are not accessible through a set of three (single beam) detectors alone, such a set is being drawn on throughout the report as paradigmatic and basic.

After solving for V_1, V_2, V_3 the points P_1^*, P_2^* and the corresponding pairs P_3^*, P_4^* (on b and c), P_5^*, P_6^* (on c and a) can be determined (Figure 4). In doing so, one is led to use the unit vector

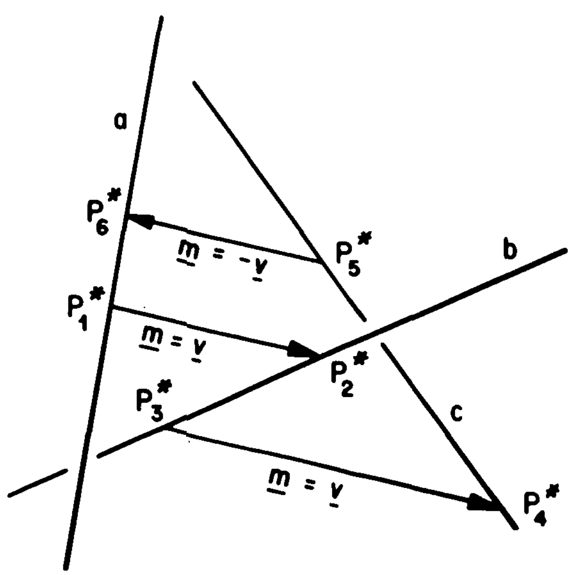

Figure 4. The six points $P_1^* \ldots P_6^*$.

$$\underline{v} = \frac{V_1\underline{i} + V_2\underline{j} + V_3\underline{k}}{\sqrt{V_1^2 + V_2^2 + V_3^2}}$$

and finds that

$$\underline{r_1^*} = \underline{r_1} + \frac{\underline{\alpha}}{(v\alpha\beta)} \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ \beta_1 & \beta_2 & \beta_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$\underline{r_2^*} = \underline{r_2} + \frac{\underline{\beta}}{(v\alpha\beta)} \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$\underline{r_3^*} = \underline{r_2} + \frac{\underline{\beta}}{(v\beta\gamma)} \begin{vmatrix} x_3 - x_2 & y_3 - y_2 & z_3 - z_2 \\ \gamma_1 & \gamma_2 & \gamma_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$\underline{r_4^*} = \underline{r_3} + \frac{\underline{\gamma}}{(v\beta\gamma)} \begin{vmatrix} x_3 - x_2 & y_3 - y_2 & z_3 - z_2 \\ \beta_1 & \beta_2 & \beta_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \qquad (15)$$

$$\underline{r_5^*} = \underline{r_3} + \frac{\underline{\gamma}}{(v\gamma\alpha)} \begin{vmatrix} x_1 - x_3 & y_1 - y_3 & z_1 - z_3 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$\underline{r_6^*} = \underline{r_1} + \frac{\underline{\alpha}}{(v\gamma\alpha)} \begin{vmatrix} x_1 - x_3 & y_1 - y_3 & z_1 - z_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Note that one may write V_i for v_i, since the common factor $\sqrt{V_1^2 + V_2^2 + V_3^2}$ cancels.

Like these position vectors the lengths of the travel paths are needed for later application; they can be written down according to expression (4) where, since $\underline{m} = \pm\, \underline{v}$, absolute marks will be added:

$$\overline{P_1^* \, P_2^*} \equiv R^*_{ab} = \left| \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{vmatrix}}{\begin{vmatrix} v_1 & v_2 & v_3 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{vmatrix}} \right|$$

$$\overline{P_3^* \, P_4^*} \equiv R^*_{bc} = \left| \frac{\begin{vmatrix} x_3 - x_2 & y_3 - y_2 & z_3 - z_2 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix}}{\begin{vmatrix} v_1 & v_2 & v_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix}} \right| \qquad (16)$$

$$\overline{P_5^* \, P_6^*} \equiv R^*_{ca} = \left| \frac{\begin{vmatrix} x_1 - x_3 & y_1 - y_3 & z_1 - z_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \\ \alpha_1 & \alpha_2 & \alpha_3 \end{vmatrix}}{\begin{vmatrix} v_1 & v_2 & v_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \\ \alpha_1 & \alpha_2 & \alpha_3 \end{vmatrix}} \right|$$

It is of some interest to also consider the point pairs P_1^*, P_6^* on beam a, P_3^*, P_2^* on b, P_5^*, P_4^* on c (Figure 4). Their distances $\underline{r}_6^* - \underline{r}_1^*$ etc. appear in a mathematically compact form if the velocity components in expressions (15) are replaced by the solutions (13). One then arrives at

$$\underline{r}_6^* - \underline{r}_1^* = \frac{L_2\,\underline{\alpha}}{(\alpha\beta\gamma)}\,(\tau_{ab}^* + \tau_{bc}^* + \tau_{ca}^*) \equiv \frac{L_2\,\underline{\alpha}}{(\alpha\beta\gamma)}\,\sigma$$

$$\underline{r}_2^* - \underline{r}_3^* = \frac{L_3\,\underline{\beta}}{(\alpha\beta\gamma)}\,\sigma$$

$$\underline{r}_4^* - \underline{r}_5^* = \frac{L_1\,\underline{\gamma}}{(\alpha\beta\gamma)}\,\sigma.$$

By conditions (1) and (11), none of the L_i defined in the system (12) are ever zero. If, however, a measurement shows that $\sigma = 0$, all three distances must have been zero, meaning that the two points on each beam coincide (Figure 4). An easy reasoning leads to the conclusion that in such a case a straight line parallel to wind direction exists containing the three double points in one row and thus connecting all three of the beam tracts. Obviously, two of the τ^* will have the same sign. Evaluation of the records must have indicated, say, maxima at positive delay times for the (a,b)- and (b,c)-covariances, and a maximum at a (fitting) negative delay time for the (c,a)-covariance.

V. LOCALLY VARYING WINDS

We now turn to an atmosphere in which the wind vector is allowed to vary with location but not with time, or only insignificantly so. (Remarks to the latter point have been made in section III.) In other words, for the duration of the observation the wind motion is supposed to be reasonably steady in the aerodynamic sense of the word.

With varying winds a peculiar difficulty arises in that the observed covariance peaks may have been caused by different winds blowing in different regions of the atmosphere. The system (12) must not be solved in these circumstances, since the times τ_{ab}^*, τ_{bc}^*, τ_{ca}^* might be associated with three different eddy motions, $\underline{V}'$ from a to b, $\underline{V}''$ from b to c, $\underline{V}'''$ from c to a. The system would break up into mutually independent equations.

To escape deception as far as possible one should keep the beams running close together in a narrow space region one wishes to explore. The wind can then be taken as constant there, and the formulism of the preceding section applies. Furthermore, the beams should be guided so as to enable a large number of unknown eddy trains to connect *all three* within that region. This furnishes us with an equally large number of meaningful travel time triads. If one of them is actually observed, the actual presence of the pertaining wind in the region is strongly indicated. To be sure, there might be connecting paths outside it, associated with eddy courses in more or less distant regions. But there will be few of them (Figure 3), and there is a slim chance only that the corresponding triads will fall within the compass of those established as meaningful, leading to the acceptance of a spurious wind. For added confidence, one may arrange for a fourth beam to be sent through the same narrow space volume; if the solutions of the four systems (12) that emerge with the possible three-beam combinations yield, by and large, four identical wind vectors, the likelihood of having observed a true wind can be said to approach certainty.

It is seen that the problem comes up of finding serviceable beam configuration which can respond to a relatively large variety of constant winds that may exist in a given small space volume. It will be dealt with here under the simplifying assumption that the natural wind can be taken as horizontal. For such winds, direction more than strength turns out to be the quantity limiting the scope of trustworthy measurement. A number of trials has led to the conclusion that the azimuthal range should not be taken larger than 90°, or, positively speaking, that with three detectors horizontal winds blowing into or out of a quarter of the rose can be observed. This range could be slightly extended at the expense of mathematical simplicity (and perhaps physical accuracy). To cover the entire rose is not feasible. One cannot connect three lines by horizontal parallels which, whatever direction they may have, are always at about the same height.

VI. MEASURING HORIZONTAL WINDS

With detectors P_1, P_2, P_3 at ground level and beams pointing upward, $z_1 = z_2 = z_3 = 0$, and $\alpha_3 > 0$, $\beta_3 > 0$, $\gamma_3 > 0$. For ease of writing and greater perspicuity we introduce the abbreviations

$$\frac{\alpha_k}{\alpha_3} = a_k, \quad \frac{\beta_k}{\beta_3} = b_k, \quad \frac{\gamma_k}{\gamma_3} = c_k, \quad k = 1,2. \tag{17}$$

The determinants (1) and (11) can then be written as

$$\left.\begin{aligned}
\Delta_{ab} &= \alpha_3\beta_3 \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & 0 \\ a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \end{vmatrix} \equiv \alpha_3\beta_3\Delta_1 \\
\Delta_{bc} &= \beta_3\gamma_3 \begin{vmatrix} x_3 - x_2 & y_3 - y_2 & 0 \\ b_1 & b_2 & 1 \\ c_1 & c_2 & 1 \end{vmatrix} \equiv \beta_3\gamma_3\Delta_2 \\
\Delta_{ca} &= \gamma_3\alpha_3 \begin{vmatrix} x_1 - x_3 & y_1 - y_3 & 0 \\ c_1 & c_2 & 1 \\ a_1 & a_2 & 1 \end{vmatrix} \equiv \gamma_3\alpha_3\Delta_3
\end{aligned}\right\} . \tag{18}$$

The three determinants Δ_i newly defined here appear in the system (13) for the V_i which goes into

$$\left.\begin{aligned}
\frac{(\alpha\beta\gamma)}{\alpha_3\beta_3\gamma_3} V_1 &= c_1 \frac{\Delta_1}{\tau^*_{ab}} + a_1 \frac{\Delta_2}{\tau^*_{bc}} + b_1 \frac{\Delta_3}{\tau^*_{ca}} \\
\frac{(\alpha\beta\gamma)}{\alpha_3\beta_3\gamma_3} V_2 &= c_2 \frac{\Delta_1}{\tau^*_{ab}} + a_2 \frac{\Delta_2}{\tau^*_{bc}} + b_2 \frac{\Delta_3}{\tau^*_{ca}} \\
\frac{(\alpha\beta\gamma)}{\alpha_3\beta_3\gamma_3} V_3 &= \frac{\Delta_1}{\tau^*_{ab}} + \frac{\Delta_2}{\tau^*_{bc}} + \frac{\Delta_3}{\tau^*_{ca}}
\end{aligned}\right\} . \tag{19}$$

If, with a given beam configuration (yet to be developed) experimentation yields three values τ^*_{ab}, τ^*_{bc}, τ^*_{ca} such that the ratio

$$v_3 = \frac{V_3}{\sqrt{V_1^2 + V_2^2 + V_3^2}}$$

turns out to be very small, the simplification $v_3 = 0$ seems warranted; we are dealing with an essentially horizontal wind.

On condition that $V_3 = 0$ the third line of the system (19) can be used to eliminate, e.g., the ratio Δ_3/τ^*_{ab}. V_1 and V_2 are then given by

$$\left.\begin{aligned} \frac{(\alpha\beta\gamma)}{\alpha_3\beta_3\gamma_3} V_1 &= (c_1 - b_1)\frac{\Delta_1}{\tau^*_{ab}} + (a_1 - b_1)\frac{\Delta_2}{\tau^*_{bc}} \\ \frac{(\alpha\beta\gamma)}{\alpha_3\beta_3\gamma_3} V_2 &= (c_2 - b_2)\frac{\Delta_1}{\tau^*_{ab}} + (a_2 - b_2)\frac{\Delta_2}{\tau^*_{bc}} \end{aligned}\right\} . \tag{20}$$

It will be noted here that the determinant

$$\begin{vmatrix} \Delta_1(c_1 - b_1) & \Delta_2(a_1 - b_1) \\ \Delta_1(c_2 - b_2) & \Delta_2(a_2 - b_2) \end{vmatrix} = \Delta_1\Delta_2 \frac{(\alpha\beta\gamma)}{\alpha_3\beta_3\gamma_3}$$

is different from zero by earlier hypotheses which exclude beam intersection and parallelism. Equations (20), for the moment considered as a linear system for $1/\tau^*_{ab}$ and $1/\tau^*_{bc}$, can therefore be solved; the ratio of the travel times emerges as

$$Q \equiv \frac{\tau^*_{bc}}{\tau^*_{ab}} = \frac{\Delta_2}{\Delta_1}\frac{(a_2 - b_2) - q(a_1 - b_1)}{q(c_1 - b_1) - (c_2 - b_2)} \tag{21}$$

where the quantity

$$q = \frac{V_2}{V_1} = \tan\varphi \tag{22}$$

defines the azimuthal angle, φ, as counted from the positive x-axis. The derivative of Q with respect to q shows that the function Q(q) is monotonic* for all q, that is, forever increasing or decreasing, with

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*It represents a hyperbola with a horizontal and a vertical asymptote.

an infinite discontinuity at $q' = \frac{c_2 - b_2}{c_1 - b_1}$. This particular value of q is not permissible (zero or infinite travel times are not admitted); neither is the value $q'' = \frac{a_2 - b_2}{a_1 - a_1}$, at which the numerator of Q(q) is zero. From physical necessity alone then certain q-ranges around q' and q'' are inaccessible; horizontal winds in certain φ-ranges cannot be observed with a three-detector setup.

One may arrange beam directions such that both q' and q'' are negative:

$$\frac{c_2 - b_2}{c_1 - b_1} < 0, \quad \frac{a_2 - b_2}{a_1 - b_1} < 0. \tag{23}$$

By definitions (17), these conditions impose restrictions on the direction cosines α_i, β_i, γ_i.

Is it then possible to "catch" at least all winds with positive values of $q (0 \leqq q \leqq \infty)$, i.e., with azimuths in $0 \leqq \varphi \leqq 90°$, $180° \leqq \varphi \leqq 270°$? In these ranges the ratio of travel times will monotonically vary between the values

$$Q(0) \equiv Q_0 = -\frac{\Delta_2}{\Delta_1}\frac{a_2 - b_2}{c_2 - b_2}$$

and

$$Q(\infty) \equiv Q_\infty = -\frac{\Delta_2}{\Delta_1}\frac{a_1 - b_1}{c_1 - b_1}. \tag{24}$$

These are uniquely determined by the detector positions and beam directions. In the latter course of the investigation the quantities Q_0 and Q_∞, conversely, will be prescribed and then aid in shaping the experimental configuration. They constitute parameter combinations and have been introduced to lessen the number of significant parameters.

To answer the above question it is at first necessary to decide on the location at which the measurement is to be made, especially on the height interval circumscribed by the positions of the points P_i^*. The heights of P_2^*, P_4^*, P_6^* can be disregarded as equal to those of P_1^*, P_3^*, P_5^*, respectively. Since the z_i are zero, the system (15) yields relatively simple expressions for the z-components:

$$\left.\begin{aligned} z_1^*(q) &= \frac{(y_2 - y_1) - (x_2 - x_1)q}{(a_2 - b_2) - (a_1 - b_1)q} \\ z_3^*(q) &= \frac{(y_3 - y_2) - (x_3 - x_2)q}{(b_2 - c_2) - (b_1 - c_1)q} \\ z_5^*(q) &= \frac{(y_1 - y_3) - (x_1 - x_3)q}{(c_2 - a_2) - (c_1 - a_1)q} \end{aligned}\right\} . \qquad (25)$$

In a rectangular (q,z^*)-system the curves $z_i^*(q)$ present themselves as hyperbolas with asymptotes parallel to the system axes (provided that the factors of q in the denominators are not zero). They favor our purpose in that they offer wide q-ranges in which the z^*-values are nearly constant. Winds blowing at (practically) the same level from a rather large compass of directions have thus a chance to be detected. This decisive advantage is lost with the straight lines that would appear if the q-coefficients in the denominators were set zero; this would also entail that $(\alpha\beta\gamma) = 0$.* Let us agree then to impose the further conditions

$$(a_1 - b_1) \neq 0, \quad (b_1 - c_1) \neq 0, \quad (c_1 - a_1) \neq 0$$

on the beams' direction cosines.

While it is true that

$$z_1^*(q'') = \pm\infty, \; z_3^*(q') = \pm\infty,$$

these discontinuities, corresponding to the vertical asymptotic lines $q = q''$ and $q = q'$, are placed outside the interval to be investigated $(0 \leq q \leq \infty)$, both q'' and q' being negative. To ensure the same feature regarding the third hyperbola, $z_5^*(q)$, we will require that

$$\frac{c_2 - a_2}{c_1 - a_1} < 0.$$

*To avoid this predicament, only one of the coefficients in fact could be taken as zero. One can show that all three vanish if two do so.

The three equilateral hyperbolas will then, in $0 \leqq q \leqq \infty$ run more or less parallel to their horizontal asymptotes.

For a more detailed description, the coordinates of the three detectors must be taken into account, as appearing in the numerators of the functions $z_i^*(q)$. Let us place P_1 at the origin, P_2 and P_3 in the upper right and the lower left quadrants, respectively (Figure 5). This arrangement has been found advantageous for measuring horizontal

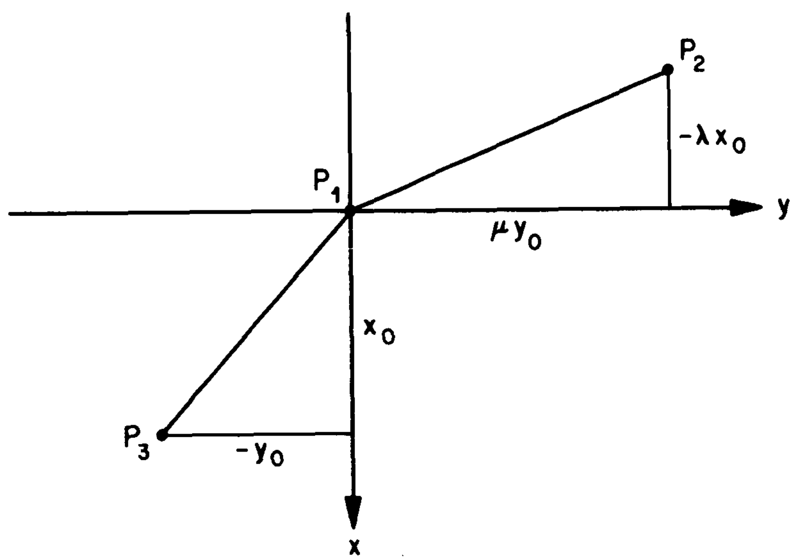

Figure 5. Detector arrangement and coordinates.

winds with azimuthal angles in the remaining two quadrants. The four positive position parameters x_0, y_0, λ, μ are introduced for practical reasons and are left indeterminate at present. With their aid the set (24) can be rearranged to give

$$\left.\begin{aligned} z_1^*(q) &= \frac{\mu y_o + \lambda x_o q}{(a_2 - b_2) - (a_1 - b_1)q} \\ z_3^*(q) &= -\frac{(\mu + 1)y_o + (\lambda + 1)x_o q}{(b_2 - c_2) - (b_1 - c_1)q} \\ z_5^*(q) &= \frac{y_o + x_o q}{(c_2 - a_2) - (c_1 - a_1)q} \end{aligned}\right\} . \qquad (25)$$

These heights all ought to be near a preselected height, $z = h > 0$, near which the constant wind vector is being sought. This is especially true at the proposed ends, $q = 0$ and $q = \infty$, of the q-interval. With positive numbers D_i, E_i not far from unity we may therefore put

$$\left.\begin{array}{llll} \dfrac{\mu y_o}{a_2 - b_2} = hD_1, & -\dfrac{(\mu + 1)y_o}{b_2 - c_2} = hD_3, & \dfrac{y_o}{c_2 - a_2} = hD_5 & (q = 0) \\ -\dfrac{\lambda x_o}{a_1 - b_1} = hE_1, & \dfrac{(\lambda + 1)x_o}{b_1 - c_1} = hE_3, & -\dfrac{x_o}{c_1 - a_1} = hE_5 & (q = \infty) \end{array}\right\}. \tag{26}$$

As an immediate consequence

$$\left.\begin{array}{lll} (a_2 - b_2) > 0, & (b_2 - c_2) < 0, & (c_2 - a_2) > 0 \\ (a_1 - b_1) < 0, & (b_1 - c_1) > 0, & (c_1 - a_1) < 0 \end{array}\right\}. \tag{27}$$

These conditions summarize and specify all restrictions placed so far on the direction cosines. They still leave much room for free choice. But, if they are satisfied, the values of z_1^*, z_3^*, z_5^*, for all wind directions $0 \leqq q \leqq \infty$, vary monotonically* between hD_1 and hE_1, hD_3 and hE_3, hD_5 and hE_5, thus insuring that one is measuring effectively "at" the height h. This, however, is not enough.

One must also see to it that, near height h, the three beams are close to each other; i.e., that the travel paths R_{ab}^*, R_{bc}^*, R_{ca}^* are reasonably short line segments. Evaluation of expressions (16) in terms of horizontal winds and the quantities introduced in the meantime yields

$$\left.\begin{array}{l} R_{ab}^* = \sqrt{1 + q^2}\; \dfrac{\mu y_o}{\dfrac{\mu\nu}{\lambda}\dfrac{E_1}{D_1} + q} \left|\dfrac{D_1 - E_1}{D_1}\right| \\ R_{bc}^* = \sqrt{1 + q^2}\; \dfrac{(\mu + 1)y_o}{\dfrac{\mu + 1}{\lambda + 1}\,\nu\,\dfrac{E_3}{D_3} + q} \left|\dfrac{D_3 - E_3}{D_3}\right| \\ R_{ca}^* = \sqrt{1 + q^2}\; \dfrac{y_o}{\nu\dfrac{E_5}{D_5} + q} \left|\dfrac{D_5 - E_5}{D_5}\right| \end{array}\right\} \tag{28}$$

*This can be seen by consulting the differential quotients dz_i/dq. It is also evident from the course of a hyperbola near its horizontal asymptote.

where ν stands for y_o/x_o:

$$\nu = \frac{y_o}{x_o} . \tag{29}$$

These results indicate that, even though the D and E are all to be chosen close to unity, one must select them such that

$$\left.\begin{aligned}
\frac{D_1}{E_1} &= -\frac{\mu\nu}{\lambda}\,\frac{a_1 - b_1}{a_2 - b_2} \neq 1 \\
\frac{D_3}{E_3} &= -\frac{\mu + 1}{\nu + 1}\,\nu\,\frac{b_1 - c_1}{b_2 - c_2} \neq 1 \\
\frac{D_5}{E_5} &= -\nu\,\frac{c_1 - a_1}{c_2 - a_2} \neq 1
\end{aligned}\right\} . \tag{30}$$

More conditions for the relationship of direction cosines and detector locations will appear in section VII.

At this station it will merely be pointed out that the distances (28) attain minima for certain values $q = q_{min}$:

$$\left.\begin{aligned}
R^*_{ab}\Big|_{min} &= x_o \frac{\lambda}{\sqrt{1 + q^2_{min}}} \left|\frac{D_1 - E_1}{E_1}\right| \quad \text{with} \quad q_{min} = \frac{\lambda}{\mu\nu}\,\frac{D_1}{E_1} \\
R^*_{bc}\Big|_{min} &= x_o \frac{\lambda + 1}{\sqrt{1 + q^2_{min}}} \left|\frac{D_3 - E_3}{E_3}\right| \quad \text{with} \quad q_{min} = \frac{\lambda + 1}{\mu + 1}\,\frac{1}{\nu}\,\frac{D_3}{E_3} \\
R^*_{ca}\Big|_{min} &= x_o \frac{1}{\sqrt{1 + q^2_{min}}} \left|\frac{D_5 - E_5}{E_5}\right| \quad \text{with} \quad q_{min} = \frac{1}{\nu}\,\frac{D_5}{E_5}
\end{aligned}\right\} . \tag{31}$$

While at $q = 0$ and $q = \infty$ the points P^* must not be too far apart to avoid physically prohibited long travel times, the minimum distances must not be too short to prevent experimental inaccuracies from falsifying the outcome.*

The core of the space volume traversed by closely bundled beams is a triangular prism with (non-parallel) bases, $P_1^* P_3^* P_6^*$ and $P_2^* P_4^* P_5^*$ on figure 4. Their shape is not critical; no further restrictions on direction cosines issue from position requirements for the corners. The latter's heights (z-coordinates) are approximately equal. Computation (with $v_3 = 0$) of the x- and y-coordinates as appearing in the formulas (15) reveals that they all can be written in terms of z_1^*, z_3^*, z_5^*:

$$\left.\begin{aligned}
x_1^* &= a_1 z_1^*(q) & y_1^* &= a_2 z_1^*(q) \\
x_2^* &= -\lambda x_o + b_1 z_1^*(q) & y_2^* &= \mu y_o + b_2 z_1^*(q) \\
x_3^* &= -\lambda x_o + b_1 z_3^*(q) & y_3^* &= \mu y_o + b_2 z_3^*(q) \\
x_4^* &= x_o + c_1 z_3^*(q) & y_4^* &= -y_o + c_2 z_3^*(q) \\
x_5^* &= x_o + c_1 z_5^*(q) & y_5^* &= -y_o + c_2 z_5^*(q) \\
x_6^* &= a_1 z_5^*(q) & y_6^* &= a_2 z_5^*(q)
\end{aligned}\right\} . \qquad (32)$$

Although these coordinates depend on q (i.e., on wind direction), they ordinarily would not vary very much, as the $z_i^*(q)$ remain close to the fixed height h. An exception occurs if one wishes to examine a small space volume at a large lateral distance from the origin. The beams then must be sharply inclined towards the ground causing some of the $|a_k|$, $|b_k|$, $|c_k|$ to be large, perhaps much larger than unity. In such cases the lateral location of the space volume cannot be adequately known beforehand, because it will markedly shift with the still undetermined wind direction prevailing at height h. By contrast, if, e.g., the a-beam is sent up vertically ($a_1 = a_2 = 0$), one is sure to measure more or less overhead, the corners P_1^* and P_6^* of the prism being located on the z-axis. The other four corners remain dependent on q, however.

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*If from an independent source the wind is known as horizontal, these requirements could be relaxed regarding R_{ca}^*, since τ_{ca}^* is not needed for the determination of V_1 and V_2.

In concluding this section the system (20) will be put into a form more suitable for the argumentation in the next one. By repeatedly employing expressions (24) and (26) one finally arrives at the representation

$$\left.\begin{aligned} FV_1 &= \frac{1}{\tau^*_{ab}} - \frac{Q_\infty}{\tau^*_{bc}} \\ FV_2 &= C\left(-\frac{1}{\tau^*_{ab}} + \frac{Q_o}{\tau^*_{bc}}\right) \end{aligned}\right\}, \tag{33}$$

where

$$\left.\begin{aligned} F &= \frac{1}{c_1 - b_1} \frac{(a_1 - b_1)(c_2 - b_2) - (a_2 - b_2)(c_1 - b_1)}{\lambda x_o (a_2 - b_2) + \mu y_o (a_1 - b_1)} = \frac{1}{\lambda x_o} \frac{1 - \frac{Q_\infty}{Q_o}}{\frac{D_1}{E_1} - 1} \\ C &= \frac{c_2 - b_2}{b_1 - c_1} = \frac{\mu + 1}{\lambda + 1} \nu \frac{E_3}{D_3} > 0 \end{aligned}\right\}. \tag{34}$$

The ten parameters x_o, λ, μ, ν, a_k, b_k, c_k are here reduced to four: F, C, Q_∞, Q_o.

Looking at F it is clearly necessary that

$$\frac{Q_\infty}{Q_o} = \frac{a_1 - b_1}{c_1 - b_1} \frac{c_2 - b_2}{a_2 - b_2} \neq 1. \tag{35}$$

By requirements (27) this ratio is positive, so that Q_∞ and Q_o must have the same sign in addition. That D_1/E_1 should be different from unity has already been deduced from the physical necessity to have the travel paths R^*_{ab} as different from zero; see set (30). The lines a and b must not intersect.

Truly important are the last three parameters, as will be seen shortly. If they can be assigned values so as to keep error transmission, height variation, and detector distances in bounds, the problem will have been solved. Pertinent analyses are offered in the next two sections. They will show the requirements as partly incompatible; a trade-off is unavoidable.

The knowledge of all four parameters is necessary for computing V_1 and V_2 from the set (33).

VII. ERROR PROPAGATION IN DETERMINING HORIZONTAL WINDS

Freedom in the choice of the beam arrangement is further narrowed down by the desire to minimize the effect of observational errors on the wind vector results. For the investigation to follow let us agree on a pivotal travel time of 1 second subject to a 10 percent error in ascertaining it. The ensuing uncertainty of ± 0.1 second will be taken as typical for all travel times. This excludes τ's materially smaller than 1 second as suspect of too great inaccuracy. The observational error should remain small relative to the peak times.*

If winds up to k knots, i.e., up to (approximately) 1/2 k m/sec, are to be measured with some confidence, none of the minimum path lengths (31) will be allowed to be appreciably shorter than 1/2 k m, so that not less than 1 second is needed for the eddies to cover the travel distance in between beams. This places a restriction on detector locations and beam directions. Another one stems from the necessity of limiting the lengths of the longest paths (at q = 0 and q = ∞) in order not to obliterate the correlation. It is bound up with the behavior of the turbulent flow under observation, and therefore less definite unless pertinent flow characteristics can reasonably well be guessed at.

By the system (33) the magnitude of the wind vector becomes

$$V = \frac{1}{|F|} \sqrt{\left(\frac{1}{\tau^*_{ab}} - \frac{Q_\infty}{\tau^*_{bc}}\right)^2 + C^2 \left(-\frac{1}{\tau^*_{ab}} + \frac{Q_o}{\tau^*_{bc}}\right)^2} . \qquad (36)$$

*Other assumptions on it would of course alter the analysis of the present section.

The direction of the wind is given by

$$\frac{V_2}{V_1} = \tan \varphi \equiv q = \frac{1}{C} \frac{Q - Q_\infty}{Q_o - Q}. \qquad (37)$$

No decision can be made here on the (equal) signs of Q_∞ and Q_o. Whether they are both positive or both negative, the condition

$$0 \leqq q \leqq \infty$$

is always met, the constant C being positive.

There are practical advantages in using a beam configuration where τ^*_{ab} and τ^*_{bc} are of equal sign ($Q > 0$); the mathematical notation is simpler, and the records handle more easily, since the travel time τ^*_{ca} will as a rule have the opposite sign. To show this, suppose that the beams are running such that, if at height h a horizontal wind is blowing with an azimuth in $0 \leqq \varphi \leqq 90°$, the registering eddies proceed from a to b ($\tau^*_{ab} > 0$) and from b to c ($\tau^*_{bc} > 0$) (although at slightly different levels close to h). At a third level near h they should move from a to c in a majority of cases; but, since in Figure 4 the "positive" direction $\underline{m}$ was taken from c to a, $\underline{m} = -\underline{v}$, causing τ^*_{ca} to be negative (compare expression (5) for $\underline{V}$, as it would be written for the transition $c \rightarrow a$).

Let us then simply require that

$$0 < Q_\infty \leqq Q \leqq Q_o. \qquad (38)$$

Is there a way of assigning favorable values to the parameters Q_∞, Q_o, C such that variations of $\pm$ 0.1 second in the τ's have as little effect as possible on V and φ? Those values could not be expected to minimize the errors in all admissible wind vectors; in fact, they can achieve that for some directions and strengths only. However, one may endeavor to keep the error within certain bounds, say 10° in direction and 10% in strength.

As regards direction errors it follows from relation (37) that

$$d\varphi = \frac{\cos^2\varphi}{C} \frac{Q_o - Q_\infty}{(Q_o - Q)^2} dQ.$$

Writing, for short,

$$\tau^*_{ab} = \tau_1, \qquad \tau^*_{bc} = \tau_2, \tag{39}$$

$$dQ = \frac{\tau_1\, d\tau_2 - \tau_2\, d\tau_1}{\tau_1^2} = \frac{d\tau_2 - Q d\tau_1}{\tau_1} .$$

Thus, in a first approximation, the angle error engendered by the errors $\Delta\tau_1$ and $\Delta\tau_2$ becomes

$$\Delta\varphi = \frac{C(Q_o - Q_\infty)}{C^2(Q_o - Q)^2 + (Q - Q_\infty)^2} \frac{\Delta\tau_2 - Q\Delta\tau_1}{\tau_1} . \tag{40}$$

Since by definition (21)

$$\frac{1}{\tau_1} = \frac{Q}{\tau_2} , \tag{41}$$

it is seen that, at a fixed value of Q, the error will increase when $|\tau_1|$ or $|\tau_2|$ decrease. This is further reason for not admitting travel times below 1 second.

The observational errors have been limited to ± 0.1 second. Drawing on the worst case we have therefore to consider the error possibilities

$$\left.\begin{aligned} \Delta\tau_1 &= \Delta\tau_2 = \pm\, 0.1 \text{ sec} \\ \Delta\tau_1 &= -\Delta\tau_2 = \mp\, 0.1 \text{ sec} \end{aligned}\right\}, \tag{42}$$

so that

$$\Delta\varphi = \Delta\tau_2 \frac{C(Q_o - Q_\infty)}{C^2(Q_o - Q)^2 + (Q - Q_\infty)^2} \frac{1 \mp Q}{\tau_1}. \tag{43}$$

With the upper sign the errors $\Delta\varphi$ are relatively small. Concentrating on the second possibility we shall take either $\tau_1 = +1$, or $\tau_2 = +1$* (the lowest values admitted).

*Negative unit values for τ_1 or τ_2 result in the same errors with the opposite sign.

Dealing at first with $\tau_1 = +1$ and writing

$$\Delta\varphi_1 = \Delta\tau_2 \cdot C(Q_o - Q_\infty)F_1(Q),$$

where

$$F_1(Q) = \frac{1 + Q}{C^2(Q_o - Q)^2 + (Q - Q_\infty)^2},$$

one is restricted to the interval $1 \leqq Q \leqq Q_o$ corresponding to $\tau_2 \geqq 1$.

Taking secondly $\tau_2 = +1$, one has to write

$$\Delta\varphi_2 = \Delta\tau_2 \cdot C(Q_o - Q_\infty)F_2(Q)$$

with

$$F_2(Q) = \frac{Q(1 + Q)}{C^2(Q_o - Q)^2 + (Q - Q_\infty)^2}$$

and the restriction $Q_\infty \leqq Q \leqq 1$, as $\tau_1 \geqq 1$ here. Note that to set $\tau_2 = 1$ in expression (43), one must introduce in it the relation (41).

On differentiating the functions F_1 and F_2 with respect to their argument Q one finds that both can have maxima within the respective Q-intervals on condition that

$$C^2 \geqq K_1 = \frac{1 - Q_\infty}{Q_o - 1}\,\frac{Q_\infty + 3}{Q_o + 3} \quad \text{in the first case,}$$

$$C^2 \leqq K_2 = \frac{1 - Q_\infty}{Q_o - 1}\,\frac{3Q_\infty + 1}{3Q_o + 1} \quad \text{in the second case.} \tag{44}$$

Neither the values Q_1^* and Q_2^* at which these maxima reside, nor the corresponding maximal errors are given here, because, with the relationship of Q_∞, Q_o and C^2 adopted later, the above conditions are not satisfied. In these circumstances the largest absolute error occurs at an interval terminal; one can show that in both cases this is Q = 1. The errors $\Delta\varphi_1$ and $\Delta\varphi_2$ are then equal and amount to

$$|\Delta\varphi|_{\text{largest}} = \frac{1}{5}\,\frac{C(Q_o - Q_\infty)}{C^2(Q_o - 1)^2 + (1 - Q_\infty)^2}, \tag{45}$$

$\Delta\tau_2$ being taken as $\pm 1/10$. Notice that this is an estimate only for the actual error (differences have been treated as differentials).

Up to now nothing more definite can be said about the three parameters than that they should so be selected that the largest angle error won't exceed 10°. Further hints would be expected to issue from a discussion of the strength errors.

With the aid of relation (41) logarithmic differentiation of expression (36) for V yields

$$\frac{dV}{V} = \frac{1}{\tau_2}\,\frac{(Q - Q_\infty)(-Q^2 d\tau_1 + Q_\infty d\tau_2) + C^2(-Q + Q_o)(Q^2 d\tau_1 - Q_o d\tau_2)}{(Q - Q_\infty)^2 + C^2(-Q + Q_o)^2} .$$

If again differences are written for differentials, the relative strength error in terms of $\Delta\tau_1$ and $\Delta\tau_2$ is being estimated as

$$\frac{\Delta V}{V} = \frac{1}{\tau_2}\,\frac{Q^2\Delta\tau_1[C^2(Q_o - Q) - (Q - Q_\infty)] + \Delta\tau_2[Q_\infty(Q - Q_\infty) - Q_o\, C^2(Q_o - Q)]}{(Q - Q_\infty)^2 + C^2(Q_o - Q)^2} . \qquad (46)$$

Like the φ-error it changes sign with τ_2 (and τ_1) and absolutely increases when τ_1 or τ_2 (taken as positive) decrease.

We wish to determine the worst possible errors. With this end in mind let us consider values of Q_∞ and Q_o such that

$$Q_\infty < 1 < Q_o. \qquad (47)$$

Then if τ_1 has the smallest admissible (worst) value ($\tau_1 = 1$), the travel time τ_2 can vary in

$$1 \leqq \tau_2 \leqq \tau_1 Q_o = Q_o; \qquad (48a)$$

if τ_2 is left constant as unity, τ_1 can move in

$$1 \leqq \tau_1 \leqq \frac{\tau_2}{Q_\infty} = \frac{1}{Q_\infty} . \qquad (48b)$$

It is advisable to examine first the second of the possibilities (42):

$$(\frac{\Delta V}{V})_2 = \frac{\Delta\tau_2}{\tau_2} \frac{(Q - Q_\infty)(Q_\infty + Q^2) - C^2(Q_o - Q)(Q_o + Q^2)}{(Q - Q_\infty)^2 + C^2(Q_o - Q)^2} . \quad (49)$$

At $Q = Q_\infty$, τ_2 can be taken as unity; τ_1 can be taken so at $Q = Q_o$. Hence the errors at the Q-interval boundaries will be written as

$$\left. \begin{aligned} (\frac{\Delta V}{V})_2\Big|_{Q = Q_\infty} &= -\frac{\Delta\tau_2}{\tau_2} \frac{Q_o + Q_\infty^2}{Q_o - Q_\infty} \\ (\frac{\Delta V}{V})_2\Big|_{Q = Q_o} &= \frac{\Delta\tau_2}{\tau_1} \frac{Q_\infty + Q_o^2}{Q_o(Q_o - Q_\infty)} \end{aligned} \right\}. \quad (50)$$

In the last expression relation (41) has been put to use again.

Assuming $Q_o \gg Q_\infty$ and $|\Delta\tau_2| = 0.1$, the worst terminal errors (at $\tau_2 = 1$ and $\tau_1 = 1$, respectively) are of the order of 10 percent. Since they are of opposite sign, the error zero occurs at least once in $Q_\infty \leqq Q \leqq Q_o$ (the observation errors cancelling each other). The value $Q = 1$ with which both τ_1 and τ_2 can be unity belongs to both the intervals (48a) and (48b) (standing at the lower end). While nothing much can be done to reduce the error at the upper end, we can see to it that at $Q = 1$ it becomes zero. Expression (49) then yields the relation:

$$C^2 = \frac{1 - Q_\infty^2}{Q_o^2 - 1} . \quad (51)$$

It simplifies further if we require (as seems best to do) that the longest travel paths associated with $\tau_1 = 1$ and $\tau_2 = 1$ should be equal. These are covered in Q_o and $1/Q_\infty$ sec., respectively, as can be seen by the ranges (48). Thus,

$$Q_o = 1/Q_\infty \quad (52)$$

and

$$C = Q_\infty. \quad (53)$$

As a consequence of these stipulations the worst relative error (49) may now be written as

$$\left.\begin{aligned} (\frac{\Delta V}{V})_2 &= \frac{\Delta\tau_2}{\tau_2}\,\frac{Q_m(Q^3 - 1) - Q^2 + Q}{Q_m(Q^2 + 1) - 2Q} \\ \text{where} \\ Q_m &= \frac{Q_o + Q_\infty}{2} \\ \tau_2 &= 1 \quad \text{in } Q_\infty \leq Q \leq 1 \\ \tau_2 &= Q\tau_1 = Q \quad \text{in } 1 \leq Q \leq Q_o \end{aligned}\right\} . \tag{54}$$

Clearly, the relative error is 0 at $Q = 1$. No other zero error $(\Delta V/V)_2$ exists, since the equation

$$Q_m(Q^2 + Q + 1) - Q = 0$$

has no real roots.

Relation (37) now attains the form

$$q = \frac{Q - Q_\infty}{1 - QQ_\infty}$$

and shows that $q = 1$ when $Q = 1$. That is, we have decided that the strength error zero existing when $\Delta\tau_1 = -\Delta\tau_2$ should occur for a wind with azimuth 45° (or 225°).

In the event that $\Delta\tau_1 = \Delta\tau_2$ (first of the possibilities (42)) one finds that

$$(\frac{\Delta V}{V})_1 = \frac{\Delta\tau_2}{\tau_2}\,\frac{-Q_m(Q^3 + 1) + Q^2 + Q}{Q_m(Q^2 + 1) - 2Q} \tag{55}$$

$$\left.\begin{aligned} \left(\frac{\Delta V}{V}\right)_1\Bigg|_{Q=Q_\infty} &= -\frac{\Delta\tau_2}{\tau_2}\,\frac{Q_o - Q_\infty^2}{Q_o - Q_\infty} \\ \left(\frac{\Delta V}{V}\right)_1\Bigg|_{Q=Q_o} &= \frac{\Delta\tau_2}{\tau_1}\,\frac{Q_\infty - Q_o^2}{Q_o(Q_o - Q_\infty)} \end{aligned}\right\} . \tag{56}$$

In formula (55) the relationships (52) and (53) are incorporated. The worst terminal errors again are about 10 percent with $Q_o \gg Q_\infty$, both of the same sign this time. One can show that there is no zero of the function $(\Delta V/V)_1$ in $Q_\infty \leqq Q \leqq Q_o$; however, absolute errors less than those at the terminals will prevail in the interior of the Q-interval.

Turning to the angle error we notice that the stipulations (52) and (53) can be combined to

$$C = \frac{1 - Q_\infty}{Q_o - 1} ,$$

so that, with $Q_\infty < 1$,

$$\frac{K_1}{C^2} = \frac{Q_o - 1}{1 - Q_\infty}\,\frac{Q_\infty + 3}{Q_o + 3} = \frac{Q_\infty + 3}{1 + 3Q_\infty} > 1,$$

$$\frac{K_2}{C^2} = \frac{3Q_\infty + 1}{3 + Q_\infty} < 1.$$

Conditions (44) are violated; the error expression (45) applies and can now be written as

$$|\Delta\varphi|_{\text{largest}} = \frac{1}{10}\,\frac{1 + Q_\infty}{1 - Q_\infty} .$$

If the largest deviation permitted is $10° \sim 0.175$, one obtains an upper bound for Q_∞ as well as a corresponding lower bound for Q_o:

$$Q_\infty \leqq \frac{3}{11} , \quad Q_o \geqq \frac{11}{3} = 3.67.$$

For deciding on a definite value for Q_o we recall that the ratio of travel times Q for a (constant) wind is equal to the ratio of the eddy travel paths. Thus, with $\varphi = 0$,

$$Q_o = \left.\frac{\tau_2}{\tau_1}\right|_{q=0} = \left.\frac{R^*_{bc}}{R^*_{ab}}\right|_{q=0} .$$

Since

$$\left.R^*_{ab}\right|_{q=0}$$

is already longer than the minimum path, one would not care to select Q_o too large. Table I below (slide rule computation) is based on

$$Q_o = 5.$$

If a turbulence pattern moving parallel to x-axis needs 1 second to cover the distance between the a- and b-beams, it requires 5 seconds for the distance between the b- and c-beams and therfore is assumed here to essentially preserve its characteristics during any 5 seconds of of observation time in order to insure a well-developed covariance peak. With a speed of 20 m/sec this corresponds to a 100 m stretch.

It is important to keep in mind that Q_∞ and Q_o (corresponding to $\varphi = 90°$ and $\varphi = 0°$) are the limits of serviceable peak time ratios. Observed ratios

$$\frac{\tau^*_{bc}}{\tau^*_{ab}} < \frac{1}{5} \quad \text{and} \quad \frac{\tau^*_{bc}}{\tau^*_{ab}} > 5$$

ought to be discarded since there is suspicion that they might not have been produced by a single wind present near height h.

For computing Table I the now particularly simple and symmetric equations (33) are available:

$$5FV_1 = \frac{5}{\tau_1} - \frac{1}{\tau_2}$$

$$5FV_2 = - \frac{1}{\tau_1} + \frac{5}{\tau_2}.$$

The indices 1 and 2 are interchangeable without altering the form of the equations. It suffices therefore to deal with the half-range $1 \leqq Q \leqq Q_o$ only; the same absolute errors prevail in $Q_\infty \leqq Q \leqq 1$. The value of 5F is inconsequential for our purpose which is concerned with ratios only:

$$\frac{V'}{V} = \sqrt{\frac{\left(\frac{5}{\tau_1'} - \frac{1}{\tau_2'}\right)^2 + \left(-\frac{1}{\tau_1'} + \frac{5}{\tau_2'}\right)^2}{\left(\frac{5}{\tau_1} - \frac{1}{\tau_2}\right)^2 + \left(-\frac{1}{\tau_1} + \frac{5}{\tau_2}\right)^2}},$$

$$\frac{V_2}{V_1} = \tan \varphi, \quad \frac{V_2'}{V_1'} = \tan \varphi' .$$

The primed quantities assume that the peak times as determined by the records deviate by as much as ± 0.1 second from the true times, τ_1 and τ_2. While these are already the largest deviations considered likely to occur, the error situation is still worsened by adopting the smallest values, $\tau_1 = 1$, $\tau_1' = 1.1$, as can be done in the above half-range. These are implicit in Table I which explicitly shows the τ_2- and τ_2'-ranges only.

TABLE I

Worst Expected Errors ($Q_o = 5$)

τ_2	τ_2'	$(\frac{V'}{V} - 1) \times 100\%$	$\varphi' - \varphi$	τ_2'	$(\frac{V'}{V} - 1) \times 100\%$	$\varphi' - \varphi$
5.0	4.9	-9.5	1°28'	5.1	-9.5	0°56'
4.5	4.4	-9.6	1°41'	4.6	-9.5	1°1'
4.0	3.9	-9.4	1°57'	4.1	-9.4	1°7'
3.5	3.4	-9.3	2°19'	3.6	-9.2	1°10'
3.0	2.9	-9.2	2°51'	3.1	-9.3	1°21'
2.5	2.4	-9.0	3°35'	2.6	-9.1	1°31'
2.0	1.9	-7.8	4°46'	2.1	-8.8	1°28'
1.5	1.4	-5.3	6°30'	1.6	-8.5	1°13'
1.1	1.0	0	8°10'	1.2	-8.6	0°13'

The above errors are found little affected by the choice of Q_o as long as $Q_o \geqq 4$. Attempts to reduce them by substituting for the conditions (51) and (52) other relations judged promising* have met with no decisive success; improvement in one place had as a rule to be paid for by worsening elsewhere.

The errors shrink when both travel times are larger than pivotal. This reflects the relative smallness of the constant observation error in such cases and could not be expected to occur if the latter, for example, were to be taken as increasing proportionately to observed peak time.

VIII. EXPERIMENTAL ARRANGEMENT

The (relatively large) value selected above for Q_o cannot be maintained. It must be relaxed in order

(1) to keep the D_i, E_i close to unity,

(2) to avoid too sprawling a detector configuration, and

(3) to secure reasonable travel path lengths.

The peak errors then will go up. We shall strive to hold them below 20° in direction and 15 percent in strength.

It will be seen shortly that conditions (51) and (52) need not be altered.

To inquire into these points we will have to make use of relationships that exist among the position parameters λ, μ, ν, the bounds Q_∞, Q_o, and the D_i, E_i.

*Such conditions were derived from the structure of the error approximations (43) and (46), and similar ones. Among others the case $1 < Q_\infty < Q_o$ was studied.

The first line in the set (26) may be written as

$$\left.\begin{aligned} a_2 - b_2 &= \frac{\mu y_o}{hD_1} \\ b_2 - c_2 &= -\frac{(\mu + 1)y_o}{hD_3} \\ c_2 - a_2 &= \frac{y_o}{hD_5} \end{aligned}\right\} . \tag{57}$$

Since the left sides add up to zero,

$$\mu = \frac{D_1}{D_5}\frac{D_5 - D_3}{D_3 - D_1}, \qquad \mu + 1 = \frac{D_3}{D_5}\frac{D_5 - D_1}{D_3 - D_1} . \tag{58}$$

In an analogous manner, the second line yields

$$\lambda = \frac{E_1}{E_5}\frac{E_5 - E_3}{E_3 - E_1}, \qquad \lambda + 1 = \frac{E_3}{E_5}\frac{E_5 - E_1}{E_3 - E_1} . \tag{59}$$

Because λ and μ are defined as positive, D_1, D_3, D_5 and E_1, E_3, E_5 must be in either ascending or descending sequence.

Again using the first line, we may write the first expression (24) as

$$Q_o = -\frac{\mu}{\mu + 1}\frac{D_3}{D_1}\frac{\Delta_2}{\Delta_1} .$$

Both the lines (26) help to transform the definitions (18) for Δ_2 and Δ_1 into

$$\Delta_2 = (\lambda + 1)(\mu + 1)\frac{x_o y_o}{h}\left(\frac{1}{E_3} - \frac{1}{D_3}\right)$$

$$\Delta_1 = \lambda\mu\frac{x_o y_o}{h}\left(\frac{1}{E_1} - \frac{1}{D_1}\right),$$

so that

$$Q_o = \frac{\lambda + 1}{\lambda} \frac{E_1}{E_1 - D_1} \frac{D_3 - E_3}{E_3} . \tag{60}$$

Analogous transformations yield

$$Q_\infty = - \frac{\lambda}{\lambda + 1} \frac{E_3}{E_1} \frac{\Delta_2}{\Delta_1} = \frac{\mu + 1}{\mu} \frac{D_3 - E_3}{D_3} \frac{D_1}{E_1 - D_1} . \tag{61}$$

Solving equations (60) and (61) for D_1/E_1 and D_3/E_3 one obtains

$$\left. \begin{aligned} \frac{D_1}{E_1} &= \frac{1 + \frac{\lambda + 1}{\lambda} \frac{1}{Q_o}}{1 + \frac{\mu + 1}{\mu} \frac{1}{Q_\infty}} \\ \frac{D_3}{E_3} &= \frac{1 + \frac{\lambda}{\lambda + 1} Q_o}{1 + \frac{\mu}{\mu + 1} Q_\infty} \end{aligned} \right\} . \tag{62}$$

By expressions (31), these quotients exercise the dominating influence on the minimum travel paths R^*_{ab} and R^*_{bc}. Which one of these is the shorter one cannot be seen at a glance. To clarify this question we note at first that by relations (34) and (62),

$$\left. \begin{aligned} \frac{D_3}{E_3} &= \frac{\mu + 1}{\lambda + 1} \frac{\nu}{C} \\ \frac{D_1}{E_1} &= \frac{\lambda + 1}{\lambda} \frac{\mu}{\mu + 1} \frac{Q_\infty}{Q_o} \frac{D_3}{E_3} = \frac{\mu}{\lambda} \frac{Q_\infty}{Q_o} \frac{\nu}{C} \end{aligned} \right\} . \tag{63}$$

With the aid of the results (62) and (63),

$$\frac{R^*_{bc}\Big|_{min}}{R^*_{ab}\Big|_{min}} = \sqrt{\frac{C^2 Q_o^2 + Q_\infty^2}{C^2 + 1}} .$$

This root can have positive values smaller than, larger than, or equal to unity. It would still have to be compared to (more involved) analogous ratios formed with

$$R^*_{ca}\Big|_{min.}$$

The virtually infinite number of possible minimum lengths is drastically reduced if one limits the competing cases by stipulating the paths

$$R^*_{bc}\Big|_{min} \quad \text{and} \quad R^*_{ab}\Big|_{min}$$

as equal, i.e., by requiring that

$$C^2 = \frac{1 - Q_\infty^2}{Q_o^2 - 1}.$$

Surprisingly, this condition* is the same as was found earlier in demanding that the zero strength error should occur at $Q = 1$. There is no reason why the second of the former conditions should not be adopted as well:

$$Q_oQ_\infty = 1, \tag{52}$$

so that the simple relation

$$C = Q_\infty \tag{53}$$

again applies.

To keep the minimum path lengths different from zero none of the ratios D_i/E_i must be equal to unity. On the other hand, the beams, as is required of them, will be running at approximately the same height in

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*It means that the equal minimum distances are found between the heights $z_1(q)$ and $z_3(q)$ defined by $q = Q_\infty$ and $q = Q_o$ (both of these values are in $0 \leqq q \leqq \infty$).

parts if the D_i and E_i are allowed to range between, say, 0.9 and 1.1 only. Their ratios then move between 0.818 and 1.222 with exclusion of 1.

For a more detailed investigation take

$$\frac{D_1}{E_1} > 1$$

to start with. The first of the expressions (62) coupled with condition (52) then yields

$$\frac{1}{\mu} < \frac{\lambda + 1}{\lambda} Q_\infty^2 - 1. \tag{64}$$

With $\mu > 0$ (as was agreed upon) the right side must be positive:

$$\frac{1}{\lambda} > Q_o^2 - 1. \tag{65}$$

From Figure 5,

$$\frac{1}{\lambda} = - \frac{x_3}{x_2} ,$$

so that $Q_o = 5$ would require an uncomfortably large value for this ratio of P_3, P_2-coordinates.

In the same way, when limiting D_1/E_1 to an upper bound of 1.2, we obtain the further condition

$$\frac{1}{\mu} > \frac{5}{6} Q_\infty \left(\frac{\lambda + 1}{\lambda} Q_\infty - \frac{1}{5}\right) - 1. \tag{66}$$

If we tentatively introduce the values $Q_o = 2$ ($Q_\infty = 1/2$) and $1/\lambda = 5$ compatible with the unequality (65), the quantity $\mu = -y_2/y_3$, according to inequalities (64) and (66), should be taken out of the range $2 < \mu < 6$. Let us select $\mu = 4$.

Before proceeding further, it is advisable to probe into the (approximate) error peaks to be expected when using the much lowered value of Q_o.

Regarding the angle error,*

$$|\Delta\varphi|_{\text{largest}} = \frac{1}{10}\,\frac{1 + Q_\infty}{1 - Q_\infty} = 0.3.$$

This corresponds to 17°11'.

It suffices to compute the worst strength errors at $Q = Q_o$. With $|\Delta\tau_2| = 0.1$ and $\tau_1 = 1$ expressions (50) and (56) give

$$\left(\frac{\Delta V}{V}\right)_2\Big|_{Q=Q_o} = -\frac{3}{20} \quad (\sim -15\%)^{**}$$

$$\left(\frac{\Delta V}{V}\right)_1\Big|_{Q=Q_o} = -\frac{7}{60} \quad (\sim -11.7\%).$$

This appears bearable. A table containing more accurate error results is supplied at the end of the section.

Based on $Q_o = 2$, $\lambda = 1/5$, $\mu = 4$,

$$\frac{D_1}{E_1} = \frac{8}{7} = 1.14$$

$$R^*_{ab}\Big|_{\min} = \frac{2x_o}{35\sqrt{5}} = 0.0256x_o = R^*_{bc}\Big|_{\min} .$$

---

* Conditions (44) are not satisfied, as they were not with $Q_o = 5$.

** To comply with the later Table II, we have taken here $\Delta\tau_1 = 0.1$, $\Delta\tau_2 = -0.1$.

The factor of x_o would increase (as one might desire), were the parameter μ to be taken as 5, for instance; but then the third distance,

$$R^*_{ca}\Big|_{min},$$

would become markedly shorter. For calculating it we have to use the quantity

$$\nu = \frac{y_o}{x_o} = C\,\frac{\lambda + 1}{\mu + 1}\,\frac{D_3}{E_3} = \frac{4}{35},$$

where $C = Q_\infty = 1/2$; D_3/E_3 is found from the second of the expressions (62). With the smallest admitted value, $9/11 = 0.818$, for D_5/E_5 one obtains

$$R^*_{ca}\Big|_{min} = 0.0252x_o,$$

which value increases to that of

$$R^*_{ab}\Big|_{min}$$

on relaxing D_5/E_5 to 0.816.

A wind of 20 m/sec ($\approx$ 40 knots) then covers a minimum distance in one second (smallest travel time permitted) if

$$x_o = 780 \text{ m}.$$

(Lesser winds require proportionately less.) Hence, the detector coordinates must be chosen as

$$P_1:\quad x_1 = 0, \qquad y_1 = 0,$$

$$P_2:\quad x_2 = -\lambda x_o = -156 \text{ m}, \qquad y_2 = \mu\nu x_o = 356.7 \text{ m},$$

$$P_3:\quad x_3 \equiv x_o = 780 \text{ m}, \qquad y_3 = -\nu x_o = -89.1 \text{ m}.$$

These figures represent the most compact detector configuration that can be obtained with the values chosen for Q_o, λ and μ, if winds up to 40 knots are to be measured reliably.

A check into the longest paths (at $q = 0$ and $q = \infty$) with the aid of expressions (28) reveals that all beam distances remain below 44.6 m, except the distance R^*_{ca} which drops from 143.5 m at $\varphi = 0$ ($q = 0$) to 47.2 m at $\varphi = 7°$ ($q = 0.123$) and continues down to the minimum. The relatively long c-a-paths may sometimes cause difficulties in covariance peak definition, but on the whole the beams appear running sufficiently close in the region proposed for measurement.

If the actual eddies are suspected to be very sizable, one might wish to increase the minimum distance to avoid spurious or untrustworthy correlation maxima. The detectors must then be located farther apart than above.

It will be noted that the observation height h does not directly enter into the determination of the detector coordinates.* However, according to set (26), it affects the calculation of the beam directions which also depends on the quantities D_i and E_i.

The small value assigned to the ratio D_5/E_5 suggests using the lowest possible figure for D_5. If we put

$$D_5 = 0.9,$$

$$E_5 = 1.102.^{**}$$

The rest of the D_i and E_i is found in a more cumbersome way. By combining expressions (58) through (61), one obtains the expressions

$$\frac{\mu}{\lambda} = \frac{D_1}{E_1}\frac{E_5}{D_5}\frac{\dfrac{E_3 - E_1}{E_5 - E_3}}{\dfrac{D_3 - D_1}{D_5 - D_3}}$$

$$\frac{Q_o}{Q_\infty} = \frac{1 + \dfrac{E_3 - E_1}{E_5 - E_3}}{1 + \dfrac{D_3 - D_1}{D_5 - D_3}} .$$

---

* An indirect influence will be recognized later.

** This figure, slightly larger than the admitted value 1.1, arises through adopting the ratio 0.816 instead of 0.818.

After applying the second of relations (63) and solving for

$$\delta \equiv \frac{D_3 - D_1}{D_5 - D_3} = \frac{1 - \dfrac{Q_\infty}{Q_o}}{\dfrac{C}{\nu}\dfrac{D_5}{E_5} - 1}$$

$$\epsilon \equiv \frac{E_3 - E_1}{E_5 - E_3} = \frac{Q_o}{Q_\infty}\frac{C}{\nu}\frac{D_5}{E_5}\frac{1 - \dfrac{Q_\infty}{Q_o}}{\dfrac{C}{\nu}\dfrac{D_5}{E_5} - 1},$$

one uses expressions (58) and (59) to obtain

$$D_1 = \mu D_5 \delta \qquad E_1 = \lambda E_5 \epsilon$$

$$D_3 = (\mu + 1)\ D_5 \frac{\delta}{\delta + 1} \qquad E_3 = (\lambda + 1)\ E_5 \frac{\epsilon}{\epsilon + 1}.$$

With the parameter values chosen so far,

$$\delta = \frac{3}{10.28}, \qquad \epsilon = \frac{10.71}{2.57}$$

$$D_1 = \frac{2.7}{2.57} = 1.051 \qquad E_1 = \frac{1.89}{2.056} = 0.92$$

$$D_3 = \frac{13.5}{13.28} = 1.017 \qquad E_3 = \frac{5.67}{5.312} = 1.068$$

$$D_5 = 0.9 \qquad E_5 = 1.102.$$

The D's and E's are in descending and ascending sequence, respectively.

These results imply that we have to require wind constancy from 0.9 h to 1.05 h at $\varphi = 0$, and from 0.92 h to 1.1 h at $\varphi = 90°$. In between the requirements are less. The three hyperbolas $z_i^*(q)$ approach each other as φ increases from $\varphi = 0$ on and actually intersect at one value* of $q = \tan \varphi$; for it, P_1^*, P_3^*, P_5^* lie on the same horizontal line, rendering the required layer thickness zero. Further increase of φ widens the traces up to the distance of 0.18 h at $\varphi = 90°$.

If one intends to measure at a large height, the vertical layer near the φ-terminals may be judged too extended for the winds to be constant in. In such a predicament, one would calculate the stretch, $q_1 \leqq q \leqq q_2$ of the z_i^*-curves in which they can be considered sufficiently close. This would exclude the measurement of winds blowing in a certain range of azimuths near $\varphi = 0$ and $\varphi = 90°$. Mathematically, new Q-terminals, $Q_1 > Q_\infty$, $Q_2 < Q_0$, obeying the relation

$$Q_{1,2} = \frac{Q_\infty + Cq_{1,2}\, Q_0}{1 + Cq_{1,2}} = \frac{Q_\infty + q_{1,2}}{1 + Q_\infty\, q_{1,2}}$$

would be introduced, causing a curtailing of the admissible τ-ratios and therefore of measurable wind directions. If this is undesirable, one would have to keep the D_i and E_i closer to unity than before with the consequence of a more sprawling detector layout, especially since, in addition, higher up one should have to allow for strong winds.

If we plan to use the detector locations as given above for measurement at h = 100 m, wind vector constancy is required at most from 8 m below to 10 m above h (when $\varphi = 90°$), and from 10 m below to 5 m above (when $\varphi = 0°$). Assuming this as warranted we can proceed to find the beam directions.

The system (57) for computing a_2, b_2, c_2 is underdetermined, since there is a linear relationship of the left sides. As a consequence, one equation must be left out. There is still freedom in choosing one of the unknowns, say $a_2 = \alpha_2/\alpha_3$. Thus

$$b_2 \equiv \frac{\beta_2}{\beta_3} = \frac{\alpha_2}{\alpha_3} - \frac{4 \times 89}{100 \times 1.05} = \frac{\alpha_2}{\alpha_3} - 3.39$$

$$c_2 \equiv \frac{\gamma_2}{\gamma_3} = \frac{\alpha_2}{\alpha_3} + \frac{89}{100 \times 0.9} = \frac{\alpha_2}{\alpha_3} + 0.99.$$

*This is $q = 0.275$, corresponding to $z_i^* = 1.035$ h.

In the same way, the second line (26) gives

$$b_1 \equiv \frac{\beta_1}{\beta_3} = a_1 + \frac{\lambda x_o}{hE_1} = \frac{\alpha_1}{\alpha_3} + 1.70$$

$$c_1 \equiv \frac{\gamma_1}{\gamma_3} = a_1 - \frac{x_o}{hE_5} = \frac{\alpha_1}{\alpha_3} - 7.08.$$

Together with the orthogonality conditions $\left(\sum \alpha_i^2 = 1, \text{ etc.}\right)$, these expressions determine $\underline{\beta}$ and $\underline{\gamma}$ once $\underline{\alpha}$ is chosen. Take, for example,

$$\alpha_3 = \frac{2}{7}\ (73.4°), \quad \alpha_2 = \frac{3}{7}\ (64.6°), \quad \alpha_1 = \frac{6}{7}\ (31°).$$

Then,

$$\beta_3 = \frac{1}{5.16}\ (78.8°), \quad \beta_2 = -\frac{1.89}{5.16}\ (111.5°), \quad \beta_1 = \frac{4.70}{5.16}\ (24.3°)$$

$$\gamma_3 = \frac{1}{4.88}\ (78.2°), \quad \gamma_2 = \frac{2.49}{4.88}\ (59.4°), \quad \gamma_1 = -\frac{4.08}{4.88}\ (146.7°).$$

The figures in parentheses give the angles made by the three beams with the positive coordinate axes. Those with the z-axis are rather large; the beams have relatively low inclinations towards the ground plane. When they approach each other in the region of interest, many of their horizontal connections can therefore run approximately at the same height (those with directions in $0 \leqq q \leqq \infty$).

Flatness of at least two beam courses is required at any height one wishes to explore. If it is large, the detectors will have to be placed widely apart, or else one must be content to monitor a smaller sector of the rose. (It will be remembered that covariance peaks could originate through unrelated winds and that, in order to minimize that danger, one has to force the beams into close neighborhood near h so that the atmospheric motion there can be considered uniform.)

Actual analysis errors encountered with $Q_o = 2$ are given in Table II which again is based on $\tau_1 = 1$, $\tau_1' = 1.1$ and is computed only for $1 \leqq Q \leqq Q_o = 2$. It is assumed as before that the worst observational errors in τ_2 are $\pm$ 0.1 second. Observed travel time ratios $\tau_2/\tau_1 > 2$ and $< 1/2$ ought to be discarded on suspicion they might not be related to a single wind blowing near the height h = 100 m.

TABLE II

Worst Expected Errors with $Q_o = 2$

τ_2	τ_2'	$(\frac{V'}{V} - 1) \times 100\%$	$\varphi' - \varphi$	τ_2'	$(\frac{V'}{V} - 1) \times 100\%$	$\varphi' - \varphi$
2	1.9	-13.3	6.4°	2.1	-10.4	1.9°
1.6	1.5	-12.1	9.9°	1.7	-10.0	2.0°
1.3	1.2	- 7.6	14.0°	1.4	- 9.0	1.6°
1.1	1.0	0	16.2°	1.2	- 8.9	0.7°

These errors reduce with slower winds, i.e., with larger values of the τ's. A wind half the strength of that in Table II would (with $\tau_1 = 2$, $\tau_2 = 4$) be subject to a largest error (incurred with $\Delta\tau_1 = 0.1$, $\Delta\tau_2 = -0.1$) of -7% in strength and 3° in direction, as compared to -13.3% and 6.4°, respectively. In like circumstances the large figure $\Delta\varphi = 16.2°$ on the last line would reduce to $\Delta\varphi = 8.2°$.

In general, if one decides not to use observed travel times below, say, 2 seconds (instead of 1 second), the requirements for the experimental arrangement will be less stringent under the assumption made in the present report of an observational error $|\Delta\tau| \leq 0.1$ second. The layout can be modified accordingly. The one described here is but one example; others can be constructed following the same guidelines with shifted accents or altered assumptions. A simple case in question is to select a different free beam direction.

IX. CONCLUDING REMARKS

Of the many parameters entering the problem only two remained arbitrary at the end, both direction cosines. Perhaps use could be made of the discretion thus given to us to forge a link with a fourth beam destined either to widen the range of azimuths measurable at one sitting or to explore winds at a different height simultaneously (four-detector arrangement). Similar aims could be pursued by utilizing existing multiple detectors that receive light from several directions (residing in one plane, however, with present versions).

If general (non-horizontal) winds are to be detected, the mathematics are considerably more involved. The number of height functions z_i^* will increase from three to six. They will depend on two variables ($q = v_2/v_1$ and $p = v_3/v_1$) and thus describe six surfaces instead of the three relatively simple hyperbolic lines. A rather large effort undoubtedly will have to be spent in solving the problem. It might be preferable to at first deal with winds, outside of storm clouds for example, that are predominantly vertical rather than horizontal.

Detectors mounted on a plane or satellite flying at constant velocity present different complications. Mathematically, two Galilei systems moving relative to each other are necessary for the description; physically, the limited choice of detector positions hampers the freedom in adjusting the parameters x_o, λ, μ, ν to meet given requirements. Some of them probably will have to be relaxed. Which to select and how far to yield will have to be discussed in the course of later work.

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WIND VECTOR CALCULATION USING CROSSED-BEAM DATA AND DETECTOR ARRANGEMENT FOR MEASURING HORIZONTAL WINDS

by W. H. Heybey

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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