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A TECHNIQUE FOR LONG ARC FITTING AS APPLIED TO THE IMP 3 ORBIT

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ABSTRACT

An investigation of the IMP 3 (Explorer 28) orbit has been performed in an attempt to find a long arc consistent with the short arcs. An arc spanning 120 orbits has been obtained which does not differ from the short arcs in predicting the time of perigee passage by more than one minute in time (out of an orbital period of 140 hours.) It is concluded that highly eccentric orbits provide an opportunity for accurate long term studies of orbit determination and that interesting insights may be obtained from such studies.

LONG ARC FITTING OF HIGHLY ECCENTRIC ORBITS

INTRODUCTION

Highly eccentric orbits offer unusual opportunities for fitting long arcs with high precision. One major advantage is that a high eccentricity orbit is seldom, or never, affected by the earth's atmosphere. Since the atmospheric drag force is unpredictable, the uncertainties in close earth circular orbits have to be assumed to arise from this source, and indeed, close earth orbits are often used to derive the density of the atmosphere. In the case of high eccentricity orbits, however, precise determinations can be obtained and extrapolated forward with good accuracy.

A second major advantage is provided by the good resolution of the perigee position of the orbit. Because the raw data will yield a good definition of the position of perigee, the time of perigee passage becomes an important parameter in the orbit improvement. This parameter has the additional benefit of being readily derived from both the reduced observational data and the numerical computation, thus facilitating comparisons between the two. Further, the investigation of the long term behavior of this parameter yields some insights which may not be clearly visible in short arc fitting.

IMP 3 (Explorer 28) satellite is the best example of a high eccentricity orbit currently available. Radio tracking has been obtained for over 120 orbits, that is, 2 years of the 3 year lifetime. This paper discusses a technique of improving the orbit by finding a long arc which is consistent with the available short arc data.

DISCUSSION

The approach taken in this paper is to perform orbit improvement, rather than orbit determination. That is, a sequence of short arc orbit determinations was available and the objective was to try to obtain a good fit to the short arcs, rather than to attempt to match the original raw data as obtained from the range and range-rate tracking. The short arcs are in the form of World Maps, which express the longitude and latitude of the subsatellite point and the height above the earth's surface at intervals of one minute. These short arcs are determined over a period of about one month or 5 or 6 satellite revolutions.

For the purpose of obtaining a fit to the two years of data, two parameters were selected from the World Maps; the time of perigee passage and the perigee height. The World Map values, listed at intervals of one minute, were interpolated to obtain more precise values of the time of perigee passage. The initial orbital elements, obtained from the injection conditions as determined from the early orbits following launch, were used to initiate a numerical integration of the orbits. The values of the time of perigee passage and the perigee height for every orbit were obtained from the computation and subtracted with the "observed" values. When these differences, or residuals, of the first 5 or 10 orbits were examined, a linear growth would appear due to the accumulation of error when the orbital period assumed in the computation differs slightly from the actual period. The residuals between the observed and computed time of perigee passage on the ith orbit after launch

$$\Delta t_{p_i} = t_{p_i \, obs} - t_{p_i \, comp}$$

were summed by least squares to yield a change in orbital period

$$P_o' = \left[1 + \frac{\Sigma t_{p_{i\,comp}} \Delta t_{p_i}}{\Sigma t^2_{p_{i\,comp}}} \right] P_o$$

$$\Delta p = P_o' - P_o$$

which was converted into an improved value of the semimajor axis, a_o', by use of Kepler's law

$$a_o' = a_o + \Delta a = a_o + 2/3 \, \Delta p \, \frac{P_o}{a_o}$$

The numerical computation was performed again, utilizing the new initial value of the semimajor axis, a_o'. This process is repeated until the residuals no longer exhibit a linear trend; then the computation may be extended over more orbital passages, and, if necessary, the linear trend may again be removed. Only a few iterations are necessary. It was found that by using 25 orbits, all linear accumulations were removed and further data would yield no improvements. The residuals, shown in Fig. 1, are the result of performing orbital corrections on the first 25 orbits and then extrapolating the computation through the remainder of the 120 orbits. The extrapolation shows no accumulations in the residuals, but does exhibit an unexplained curve which has the appearance of a low frequency sinusoid. The development of this systematic, though non-secular, trend in the residuals in time of perigee passage, may be due to a small error in one of the other orbital elements, or may be due to a modeling error of undetermined origin.

The values of the perigee height residuals were used to adjust the initial eccentricity. It was found that a change in the initial value of the eccentricity, or one of the angular elements, that is, inclination, argument of perigee, or the right ascension of the ascending node, necessitated a change in the initial value of the semimajor axis although the value of the semimajor axis prior to the change in another orbital element had yielded satisfactory residuals in the time of perigee passage. This coupling of the semimajor axis with the other orbital elements is apparently due to the oblateness of the earth which strongly influences the energy of the orbit. (The oblateness causes a difference between the anomalistic and the osculating value of the period at perigee of about 4 hours on the IMP 3 orbital period of 140 hours.) However, satisfactory values of the semiajor axis can be obtained equally well after small changes in the orbital elements, providing that the remaining elements are modified first, and then the iterations performed with only an input change to the semimajor axis until the procedure converges.

Due to this coupling of the corrections to the semimajor axis with corrections to other orbital elements, the initial conditions which provide satisfactory residuals in the time of perigee passage are not unique. For example, little change in the residuals produced, if either set of orbital elements

	Set 1	Set 2
a	21.726218	21.726138
ω	135.73696	135.72696
e	.95251083	Same
i	33.828445	Same
Ω	-138.45267	Same

is used to initiate the numerical computation. It might be argued that this occurs because the time of perigee passage is only one observational parameter for testing the orbit and that consideration of two or more variables might provide a better fit of the orbit; however, this "observed" quantity is particularly sensitive and will in fact partially determine the choice of other quantities once a semimajor axis has been chosen. Further, not only corrections to the orbital elements but also model errors such as the value of the earth's harmonics, may be eliminated by an appropriate choice of the semimajor axis. It should be stressed that the semimajor axis in Set 2 could not be substituted into Set 1 and produce satisfactory residuals. A change of .5 km in a in Set 1 would produce a substantial linear term in the residuals.

The numerical computation was performed by ITEM (Ref. 1), an Encke method program for computing earth orbits and solar-system trajectories. The applicable forces it computes on the IMP 3 orbit are the earth's oblateness and lower harmonics, the gravitation of the moon and sun, and the solar radiation pressure. A comparison of two different integration intervals yielded differences in the time of perigee passage as large as the residuals in the data.

CONCLUSIONS

1. Using the technique described to find a long arc consistent with the short arcs which were obtained by the usual orbit determination method, an ephemeris of high precision has been obtained for IMP 3. This ephemeris was sufficiently precise to predict the acquisition of the satellite by the Smithsonian Astrophysical Observatory's Baker-Nunn camera network after a lapse of one year of observation. Furthermore, the ephemeris allowed an accurate prediction of the time and location of the IMP 3.

The maximum residual in the time of perigee passage was one minute during the two years of radio tracking data.

2. Long arc determinations may be useful in evaluating the methods and the results of short arc determinations. In particular, it may be that ambiguities in the energy determination similarly affect short arc determinations so that equally good fitting initial conditions could be obtained with low errors ascribed to each but with larger differences between the two sets than the errors. Also, model errors may be compensated for by changes in the initial conditions producing an illusion of a good fit. It may be worthwhile to determine model errors by identifying observable quantities which are particularly sensitive to a model quantity--such as the application of resonant orbits to the earth's harmonics.

3. The time of perigee passage is an exceptionally sensitive parameter when it is available for large number of orbits and is therefore to be recommended for utilization in long arc studies. It insures that the instantaneous position of the satellite corresponds closely to the predicted position. Further, it appears that this quantity is determined very accurately when raw data has been reduced to produce a short arc of 5 or 6 orbits. The accuracy of the data is inferred because there is no strong discontinuity in the residuals from short arc between one short arc and the next.

REFERENCE

1. "ITEM Program Manual Fortran IV Version", F. Whitlock, et. al., X-643-67-364, August 1967.

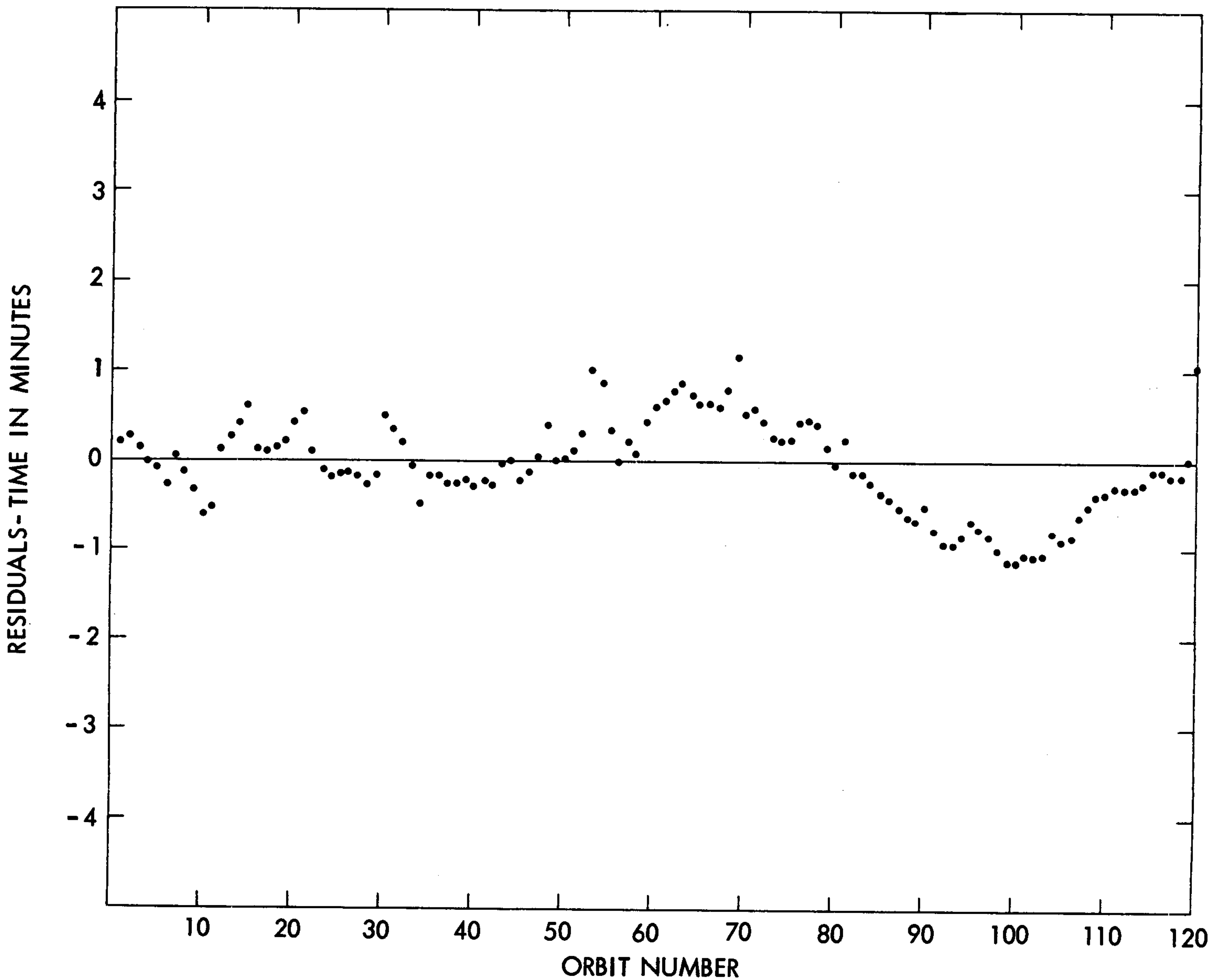
RESIDUALS- TIME IN MINUTES
4
3
2
1
0
-1
-2
-3
-4
10
20
30
40
50
60
70
80
90
100
110
120
ORBIT NUMBER

Figure 1