ON THE PULSATION HYPOTHESIS FOR MASSIVE RED SUPERGIANTS

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ABSTRACT

The fundamental mode of radial pulsation has been calculated for convective envelopes in adiabatic equilibrium including radiation pressure. The theoretical Q values are compared with the observational Q values of variable M supergiants in galactic associations. The variability of these massive red supergiants can be explained on the pulsation hypothesis if a suitable readjustment of some of the observed periods is made. Pulsation seems to be suggested by (1) the cyclical variations in magnitude, spectrum, and radius in individual stars, and (2) a period-luminosity relation and (3) a period-spectrum relation for the group of stars.

I. INTRODUCTION

It has long been assumed that the variability of bright red supergiants associated with young populations is due to radial pulsation. Some photometric and spectroscopic studies have strongly supported the hypothesis. In the interest of bringing further evidence to bear on the point, this paper presents a theoretical and observational determination of the period-root-mean-density quantity

$$Q = \mathrm{Per}\ (\overline{\rho}/\overline{\rho}_{\odot})^{\frac{1}{2}}$$

for massive stars. The theoretical Section II contains a brief, but complete, discussion of the fundamental pulsational properties of convective envelopes in adiabatic equilibrium, while Section III compares these results with the observations of variable M supergiants in galactic associations, whose distances are known.

II. THEORETICAL MODEL ENVELOPES

The structure and pulsational characteristics of convective stellar envelopes are very difficult to calculate in detail. However, the superadiabatic region near the surface is, in general, rather small and the solutions converge rapidly toward an adiabatic solution in the interior. The particular adiabat which is attained depends on the properties of the superadiabatic region and may be characterized by a parameter K (Hayashi, Hōshi, and Sugimoto 1962).

In the following, we make two simplifying approximations. First, we assume adiabaticity throughout the envelope and leave K a parameter to be specified. Second, we consider the equation of state to be represented simply by a mixture of perfect gas and radiation. The latter approximation is certainly valid in the interior. Previous work on the pulsational characteristics of completely adiabatic envelopes neglected radiation pressure. It is necessary to include it here explicitly because of its importance at the high masses which are typical of red supergiants. Since the few equations required here are scattered in five different publications, it is useful and convenient to assemble and discuss them briefly together. Defining $1-\beta$ as the ratio of radiation pressure to total pressure, we may describe the state of the gas by the two adiabatic exponents (Chandrasekhar 1939)

$$\Gamma_1 = \frac{32-24\beta-3\beta^2}{24-21\beta}, \qquad \Gamma_2 = \frac{32-24\beta-3\beta^2}{24-18\beta-3\beta^2}. \tag{1}$$

The basic differential equations describing the structure of a convective envelope in adiabatic equilibrium may be expressed in non-dimensional form,

$$\frac{x}{q}\frac{dq}{dx} = \frac{\beta p x^3}{qt} = U, \qquad -\frac{x}{p}\frac{dp}{dx} = \frac{\beta q}{xt} = V,$$
$$1-\beta = A\,\frac{t^4}{p}, \qquad \frac{t}{p}\frac{dp}{dt} = \frac{\Gamma_2}{\Gamma_2-1} = n+1, \tag{2}$$

where we have adopted the usual Schwarzschild variables and have included radiation pressure (e.g., Hayashi et al. 1962). We shall assume zero boundary conditions at the surface, $p = t = 0$ at $q = x = 1$. Substituting the expression for the second adiabatic exponent, we integrate the last equation to obtain

$$p = Et^{5/2}\left(1 + \frac{1-\beta}{\beta}\right) e^{4(1-\beta)/\beta}, \tag{3}$$

where E is the constant of integration. Near the surface we have approximately

$$q = 1, \quad t = \frac{\beta}{n+1}\left(\frac{1}{x}-1\right), \quad V = \frac{n+1}{1-x}. \tag{4}$$

A convective envelope is specified by the parameters E and A, where

$$E = 4\pi G^{3/2}(\mu H/k)^{5/2} M^{\frac{1}{2}} R^{3/2} K, \qquad A = (4\pi/3) G^{3} (\mu H/k)^{4} M^{2}. \tag{5}$$

A solution loops in the U-V plane toward the point $U = 0$, $V = (n+1)_a$, where $(n+1)_a = 2.5$ if $A = 0$ and $(n+1)_a = 4$ otherwise. In the latter case, the relative radiation pressure $1-\beta$ increases monotonically from zero at the surface to unity at the asymptotic limit of mass fraction, q_a. For this reason, we define a mean value, $\overline{1-\beta} = \int(1-\beta)\,dx$, which indicates the relative importance of radiation pressure. As E increases with A held fixed, the mass of the envelope increases. Ultimately, a maximum value of E is attained, beyond which no solutions exist.

The equation describing small, radial adiabatic pulsations is

$$\frac{d^2\xi}{dx^2}+\frac{d\xi}{dx}\left[\frac{2}{x}-\frac{V}{x}(1+b)\right] + \xi\left[\omega^2 \quad \frac{Vx}{\Gamma_1 q}-\frac{2}{x^2}-\frac{2V}{x^2}(1+b-\frac{2}{\Gamma_1})\right] = 0, \qquad (6)$$

where

$$b = \frac{(1-\beta)(7\Gamma_1-8-2\beta)}{\Gamma_1(8-7\beta)}\left(\frac{n-3}{n+1}\right), \qquad \omega^2 = \left(\frac{2\pi}{Per}\right)^2 \frac{R^3}{GM}, \qquad (7)$$

and ξ is the radial displacement (Stothers 1965). The period-root-mean-density relation is simply

$$Per \quad \bar{\rho}^{\frac{1}{2}} = (3\pi G/\omega^2)^{\frac{1}{2}}. \qquad (8)$$

At the surface $V = \infty$, so we require the following regularity condition, after normalizing the displacement,

$$x = 1, \quad \xi = 1, \quad d\xi/dx = (\omega^2+4)/\Gamma_1-2. \qquad (9)$$

An eigenvalue ω^2 is determined whenever the solution converges to $\xi = 0$ at the center (or inner boundary). Rabinowitz (1957) has discussed methods of solution and the rapidity of convergence.

Results of integrations of the structure and pulsation equations are presented in Table 1 over the permissible ranges of E and A. Pulsational eigenfrequencies are given for the fundamental mode of pulsation only. The structures with no radiation pressure ($A = 0$) have already been treated by several authors (Osterbrock 1953; Härm and Schwarzschild 1955). In a few cases their pulsational properties have also been calculated (Cowling 1934; Rabinowitz 1957). It may be noted that the case $E = 45.5$ is simply the polytrope of index 3/2 with $\Gamma = 5/3$. The case $E = 0$ corresponds to an atmosphere of no mass, in which equations (4) are exact everywhere; however, the pulsation equation (6) may still be solved in this case.

A few structures including radiation pressure have been calculated by other authors. The structure with infinite mass ($A = \infty$) is simply the polytrope of index 3 with $\Gamma = 4/3$. The case $E = 0$ corresponds to an atmosphere of no mass with $\Gamma = 4/3$. A few non-degenerate structures have been calculated by Hayashi et al. (1962). However, most of Table 1 is new. The maximum value of ω^2 anywhere occurs in the case $A = 0$, $E = 0$.

The effect of radiation pressure is to deepen the convective envelope and to reduce the pulsational eigenfrequency. Therefore, for the same envelope depth in mass fraction, more massive stars tend to have pulsational eigenfrequencies quite comparable with those of less massive stars. Massive stars, however, can possess very deep convective envelopes during advanced stages of evolution (cf. references in Stothers 1968). The physically realizable portion of Table 1 for massive stars lies between $A \sim 10$ and $A \sim 3000$. These limits correspond, respectively, to stars of $10M_\odot$ with an age-zero chemical composition and $60M_\odot$ with a heavily metal-enriched chemical composition. The possible values of Q are therefore confined to the range

$$0.05 \leq Q \leq 0.10 \quad \text{day.}$$

It is somewhat surprising that the derived upper limit for Q is not considerably larger, in view of the large radiation pressure at high masses.

III. VARIABLE RED SUPERGIANTS

The known variable M supergiants in galactic associations are listed in Table 2. The spectral types and magnitudes are those adopted in the accompanying paper (Stothers 1968, Table 1). The magnitudes selected as applicable are median magnitudes. The effective temperatures and bolometric corrections (for the various spectral subdivisions) have been taken from Johnson (1966). The radii can then be derived in the usual way from the effective temperatures and bolometric magnitudes. The periods are the ones listed in the <u>General Catalogue of Variable Stars</u> (Kukarkin, Parenago, Efremov, and Kholopov 1958). The mass of an individual red supergiant has been assumed equal to the average (or, in some cases, the most frequent) mass of blue supergiants in the same association (Stothers 1968). For each variable we have formed an observational Q value.

According to the pulsation theory of Section II, these variables should have Q values in the range $0.05 \leq Q \leq 0.07$ day if they are undergoing radial pulsation in the fundamental mode. This range corresponds to the portion of Table 1 covered by the relevant A values for 14 to 25 $M_{\odot}$. Since the convective envelopes in published models of red supergiants (cf. references in Stothers 1968) are of intermediate depth, we can safely assume a uniform Q value,

$$Q = 0.06 \text{ day.}$$

Five of the ten variables with listed periods in Table 2 have Q values in reasonable accord with the pulsation hypothesis. Only one of the ten, RS Per, has a Q value which is significantly smaller (corresponding approximately to a fifth overtone!). The other four have incompatibly large Q values. If it is assumed that the light variation in all cases is due to pulsation, then we must seek sources of possible error in our evaluation of Q for these stars.

In terms of quantities directly used to determine Q, we have

$$Q = \left(\frac{L}{L_\odot}\right)^{-3/4} \left(\frac{M}{M_\odot}\right)^{\frac{1}{2}} \left(\frac{T_e}{5800}\right)^{3} \text{Per.}$$

The uncertainty associated with each of these quantities will be discussed in turn.

Luminosity. The errors in the derived bolometric magnitudes are due to (1) observational errors and (2) association back-to-front differences and possible association non-membership. The observational error (Stothers 1968) is ±0.3 to ±0.7 mag., which corresponds to an error in Q of only 20 to 50 per cent. This is unimportant. In the outer I Per aggregate, T Per and S Per are very likely to be members of the background Perseus arm, lying at $(m-M)_o = 12.7$ (Schild 1967). In the I Ori association, α Ori is probably a foreground member with $M_v \sim -5.7$ to -6.1 (Wilson 1959; Jenkins 1963). For T Per, S Per, and α Ori, we have used the adjusted luminosities to obtain the Q values listed in Table 2, rather than the luminosities in Stothers (1968). The other stars questioned by Stothers — TV Gem, α Sco, and RS Per — are probably normally associated with their respective aggregates. Although the relatively late spectral type (M4.5) and therefore possibly low luminosity of RS Per suggest that it may be a foreground star (larger Q value), the star lies in the center of χ Per against the sky and is almost surely a cluster member.

Mass. In order to bring the observed values of Q for α Sco, α Ori, T Per, and S Per into line with the pulsation hypothesis, the masses of these stars would have to be reduced to values 3 to 100 times smaller than those listed in Table 2. Such a large amount of implied mass loss is inadmissible. Most of the stars would have radiative envelopes and appear blue in color, not red, apart from the unlikelihood that mass loss would affect certain stars of a given spectral type so much more than other stars of the same spectral type. Moreover, the observed mass-loss rate for α Ori (Weymann 1962) is quite incapable of causing such a drastic amount of mass loss. In the anomalous case of RS Per, its present mass would have to be 25 times greater than its adopted mass to explain the discrepancy. This is also inadmissible.

Effective temperature. The scale of effective temperature as a function of spectral type is not known very well for M supergiants. However, by intercomparing the various determinations of effective temperature for α Sco, α Ori, and α^1 Her (Johnson 1966), we find that the error in the adopted value is probably less than $\pm 300^{\circ}$K. This causes a possible error in Q of only 30 per cent. In any case, the pulsationally discrepant stars are evenly distributed among the same spectral subdivisions as the concordant stars.

Period. It is likely that virtually all the discrepancies in Q in Table 2 are due to errors in the listed periods. The long cycles and relatively small amplitudes (typically 1 mag. photographic) of variable M supergiants are very difficult to determine and are usually semiregular in nature. Multiperiodicity (or simply irregularity) is not unknown; for example, Palmér (1939) lists periods of 11, 250, and 2070 days for α Ori and 840, 1120, and 3360 days for S Per.

With our adopted periods, no period-luminosity nor period-spectrum relation emerges from the data. Considering, however, only the five stars with Q values consistent with the pulsation hypothesis, we find both such relationships emerging (if YZ Per is assumed to be slightly in the foreground of the outer I Per aggregate). Since the Q value is determined principally by the period and luminosity, the discovery of a period-luminosity relation for stars selected by Q value is not surprising. However, since our Q values are also dependent on spectral subdivision (through the effective temperature), the discovery of a period-spectrum relation could be further evidence in favor of the pulsation hypothesis.

The reality of such a relation may be tested by adjusting the periods of all stars with "discrepant" Q values to fit an adopted Q = 0.06 day. The resulting "revised" periods are listed in the ninth column of Table 2. It is immediately clear that a period-spectrum relation is now supported by the data. This is basically a consequence of the luminosity-spectrum relation among the variables, the brighter being later in spectral type. It remains to be seen, of course, whether the observations permit such a readjustment of the periods to be made. In the case of α Ori, the revised period agrees well with one of the periods listed by Palmér (1939). In the other cases (except for T Per), the revised periods are nearly integral multiples or divisors of the original periods. Concerning the distances of α Ori, T Per, S Per, and YZ Per, our suspicions now seem to have been well founded since the revised distances conform well with the pulsation hypothesis. Finally, we mention the interferometric and radial-velocity studies of α Ori (Pease 1931; Sanford 1933) which suggest that this star undergoes a cyclical variation in radius.

The relation between luminosity, spectrum, and period for the semiregular red variables is shown in Figure 1. It should be remarked that the <u>visual</u> absolute magnitudes of these stars are very similar (because of the effect of the bolometric correction), averaging $\langle M_V \rangle = -5.4$. Since more than half the stars listed in Stothers (1968, Table 1) with spectral type M1 and later are known variables, it is possible, and even likely, that the remaining stars are also variables of small amplitude.

We conclude that the hypothesis of radial pulsation (in the fundamental mode) in the variable M supergiants tends to be supported by the available observational data.

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TABLE 1

Theoretical Convective Envelopes Including Radiation Pressure

	E	A						
		0	1	10	10^2	10^3	10^4	∞
q_a	0	1.000	1.000	1.000	1.000	1.000	1.000	0.000
	0.1	0.995	0.979	0.934	0.774	0.414	0.061	....
	1	0.951	0.912	0.816	0.570	0.213	....	....
	10	0.599	0.556	0.438	0.212	....	....	....
	E_{max}	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\overline{1-\beta}$	0	0.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.1	0.000	0.255	0.379	0.486	0.571	0.632	....
	1	0.000	0.137	0.253	0.375	0.479	....	....
	10	0.000	0.048	0.125	0.236	....	....	....
	E_{max}	0.000	0.004	0.031	0.145	0.364	0.601	1.000
ω^2	0	6.02	0.00	0.00	0.00	0.00	0.00	0.00
	0.1	6.01	4.99	4.98	4.97	4.42	1.69	....
	1	5.95	5.18	4.82	4.51	3.15	....	....
	10	5.29	4.96	4.24	3.10	....	....	....
	E_{max}	2.71	2.67	2.44	1.93	1.57	1.28	0.00
E_{max}		45.5	45.1	42.2	28.4	7.24	0.269	0.000

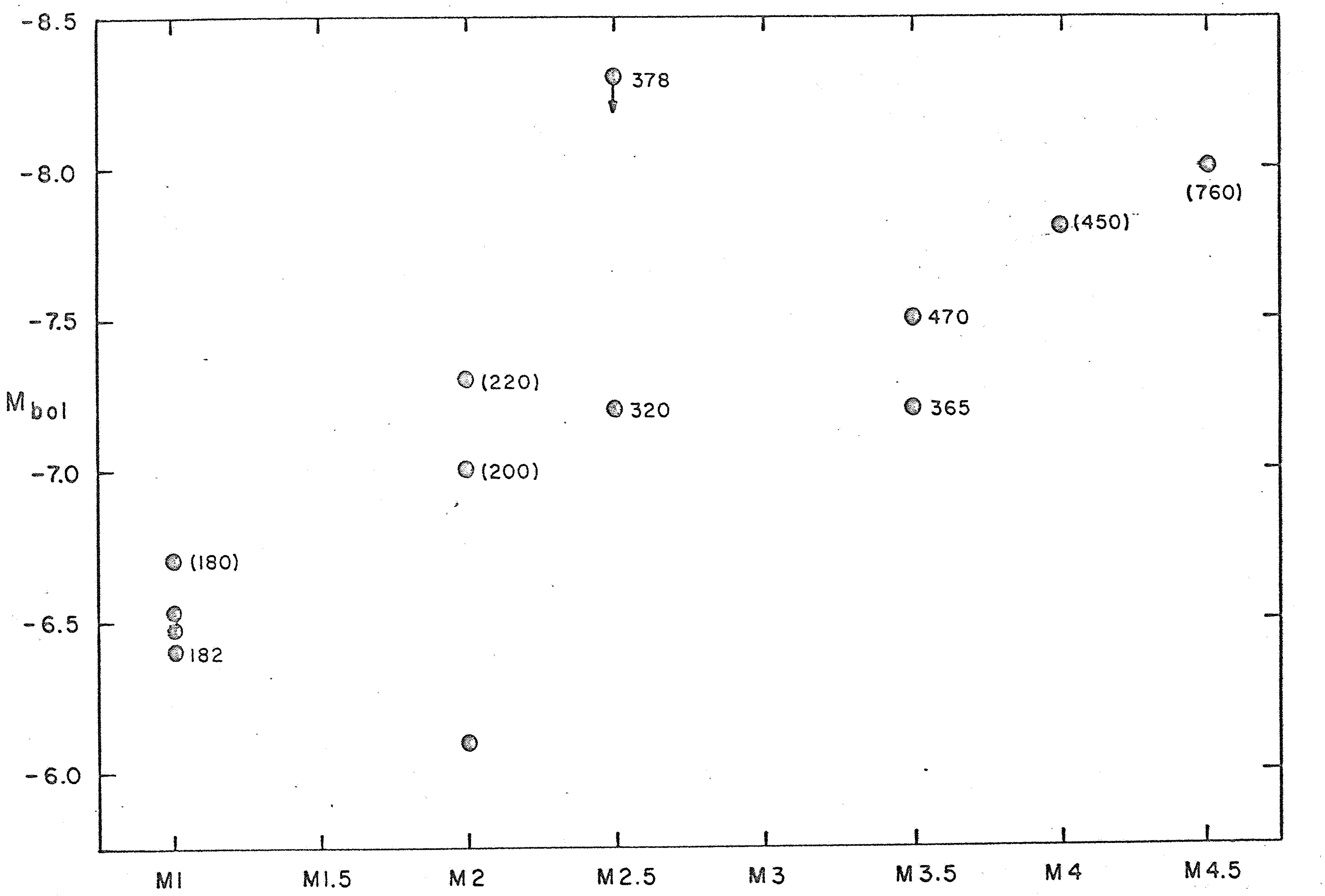
TABLE 2

Variable M Supergiants in Galactic Associations

Star	Sp	M_v	M_{bol}	Per(day)	T_e (°K)	$M/M_\odot$	Q(day)	"Revised" Per (day)	Association
TV Gem	M1	-5.1	-6.4	182	3600	14	0.077	182	I Gem
BU Gem	M1	-5.2	-6.5	...	3600	14	...	...	I Gem
FZ Per	M1	-5.2	-6.5	...	3600	14	...	...	I Per (inner group)
α Sco	M1	-5.4	-6.7	1733	3600	14	0.59	~180	II Sco
T Per	M2	-5.6	-7.0	326	3500	20	0.097	~200	I Per (outer group)
WY Gem	M2	-4.7	-6.1	...	3500	14	...	...	I Gem
α Ori	M2	-5.9	-7.3	2070	3500	25	0.56	~220	I Ori (foreground member)
AD Per	M2.5	-5.6	-7.2	320	3400	14	0.065	320	I Per (inner group)
YZ Per	M2.5	-6.7	-8.3	378	3400	20	0.043	378	I Per (outer group)
BU Per	M3.5	-5.1	-7.2	365±	3200	14	0.062	365±	I Per (inner group)
SU Per	M3.5	-5.4	-7.5	470	3200	14	0.065	470	I Per (inner group)
S Per	M4	-5.4	-7.8	840±	3100	20	0.11	~450	I Per (outer group)
RS Per	M4.5	-5.2	-8.0	152	3000	14	0.012	~760	χ Per

FIGURE CAPTION

Fig. 1. - Bolometric H-R diagram for the known variable M supergiants in galactic associations. The variables are labeled with their period in days, if known. Periods "revised" in accordance with the text are given in parentheses.



-8.5
-8.0
-7.5
-7.0
-6.5
-6.0
M_{bol}
378
(760)
(450)
470
(220)
320
365
(200)
(180)
182
M1
M1.5
M2
M2.5
M3
M3.5
M4
M4.5