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DEVELOPMENT OF SONIC-BOOM SIGNATURES IN A STRATIFIED ATMOSPHERE

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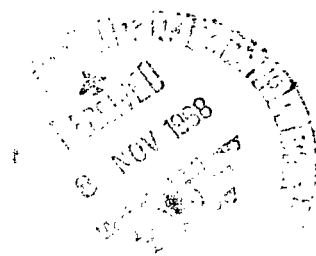
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SUMMARY

Equations describing the development of the pressure signature of a supersonic projectile or equivalent body of an airplane in straight steady level flight in an atmosphere having vertical gradients of temperature, pressure, and wind have been derived. The analytical method utilizes results of ray-tube calculations for a stratified atmosphere in the context of an analysis by Whitham.

INTRODUCTION

The sonic-boom literature contains many treatments, which use the acoustic weak shock approximation, of the effects of nonuniform atmosphere on the boom intensity. (See, for example, ref. 1.) However, this acoustic approximation treats the wave as a single pulse and does not account for any details of the nature or development of the pressure signature. Even in the uniform atmosphere limit this approximation predicts an incorrect variation of amplitude with radial distance from the flight path. An attempt to apply this linear acoustic theory to the problem of calculating the pressure signature of a supersonic body of revolution in an atmosphere with a temperature gradient has been described in reference 2. However, this linear theory fails to predict the formation of shock waves and other nonlinear features of the signature development.

On the other hand, these nonlinear phenomena are predicted by the theory (refs. 3 and 4) that Whitham has developed and applied to the calculation of signatures in a uniform atmosphere. The present analysis represents an extension or application of the nonlinear theory of Whitham used with a ray-tube calculation procedure similar to that used in reference 5 to describe the development of the signature in a stratified atmosphere with horizontal winds but without vertical winds.

SYMBOLS

The symbols a, p, u, v, λ, μ, ν, and ρ without a subscript or tilde apply for conditions undisturbed by the wave, z units below the flight level.

a	sound speed
$\tilde{a}$	local sound speed
A	function proportional to $\tilde{A}$
$\tilde{A}$	ray-tube cross-sectional area

$$c = \gamma \left(\frac{a_0 p_0 M^5}{2\beta^2} \right)^{1/2} F(\tau)$$

$$C = \left(\frac{\gamma + 1}{2\gamma} \right) \frac{c}{F(\tau)}$$

$$C_1 = a_0 M$$

$$C_2 = \frac{a_0 M \csc \omega}{\beta}$$

f	defined by equation (14)

$$F(\tau) = \frac{1}{2\pi} \int_0^{V\tau} \frac{S''(\xi) d\xi}{\sqrt{V\tau - \xi}}$$

$$K = \frac{A}{\tilde{A}}$$

M	Mach number
$\vec{n}$	unit vector normal to element of wave front associated with ray under consideration
$\tilde{p}$	local pressure
s	distance along ray path
$S(\xi)$	source distribution due to supersonic body
t	time at which ray under consideration intersects horizontal plane z units below flight altitude

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T	time variable used in equation (4)
u,v	wind components relative to X,Y,Z coordinate system
U	particle velocity in wave of finite amplitude
$\vec{V}$	flight velocity vector relative to air at flight altitude
$V = \left\|\vec{V}\right\|$	
$\vec{w}$	wind velocity vector relative to air at flight altitude
$w_n = \vec{w} \cdot \vec{n}$	
X,Y,Z	right-hand rectangular coordinate system moving with atmosphere at flight altitude, with Z-axis directed vertically downward and X-axis directed opposite to direction of flight; origin located at nose of generating body at time $t = \tau = 0$
x,y,z	coordinates referred to X-, Y-, and Z-axes, respectively
$\vec{w}^*$	wind velocity vector at flight altitude relative to ground
u^*, v^*	components of $\vec{w}^*$ referred to Ξ, H system
$\beta = \sqrt{M^2 - 1}$	
γ	ratio of specific heats
θ	angle of inclination of ray with horizontal
λ, μ, ν	components of $\vec{n}$
ψ	angle of rotation of Ξ, H coordinate system relative to X,Y system
ρ	density
τ	time at which element of generating body, initially at position x,0,0 crosses origin of X,Y,Z system

ω angle between vertical plane through flight path and plane determined by flight direction and initial ray direction

Ξ,H coordinate system fixed relative to earth with origin directly below nose of generating body at $t = \tau = 0$, and with Ξ-axis directed opposite to direction of flight relative to ground

ξ,η coordinates referred to Ξ,H system

ζ dummy integration variable

Subscript:

0   undisturbed conditions at flight level

ANALYSIS

General Considerations

In deriving the equations for calculating the pressure signature, the coordinate system used is assumed to have its origin at flight altitude and to be moving with the atmosphere at this level so that in this system there is no wind in the plane $Z = 0$. The X-axis is directed opposite to the direction of flight, and the origin is located at the nose of the body at time $t = 0$. Subsequently, this system will be related to a system fixed relative to the ground.

Throughout the analysis the atmospheric variation is assumed to be slight over a distance of the order of the length of the signature. If this condition is not applied, certain assumptions of geometrical acoustics fail. However, when the atmosphere is such that the condition does not hold, turbulence is likely to occur and give rise to a different type of sonic-boom distortion which is not described by the present analysis.

Basic Equations

The equation for the rays associated with the acoustic wave fronts can be determined by a procedure analogous to that used in reference 3 for propagation in a uniform atmosphere. The basic concept is to start with the equation for a sound ray as determined by the assumption of ordinary acoustic propagation, and then modify that equation by replacing the expression for the undisturbed sound speed by the actual speed of propagation as influenced by the finite overpressure in the wave. For any pressure signature there is at least one point of zero overpressure, to which corresponds a ray associated with a wavelet moving at the undisturbed sound speed. Then this ray is subject to the ordinary acoustic refraction laws, as described, for example, in reference 6. The following approximation

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procedure is used: The ray associated with a point of zero overpressure is adopted as a "typical" ray for the signature for the purpose of determining the wave-front normal direction and the ray-tube cross-sectional area. Then the nonlinear effect of the motion of the other points of the signature relative to the point associated with this ray is determined by adjusting the speed of those points to account for the finite overpressure associated with them.

A fundamental expression required in this procedure is the time at which the wavelet associated with the typical ray arrives at a level z units below the plane $z = 0$

$$t = \tau + \int_0^z \frac{1}{a\nu} \, d\zeta \tag{1}$$

(See eq. (47) of ref. 6.)

In order to account for the finite overpressure associated with some other ray, equation (1) is again used, but with the undisturbed sound speed a replaced by the actual speed of the wavelet

$$\tilde{a} + U = a\left(1 + \frac{\tilde{a} - a}{a} + \frac{U}{a}\right) \equiv a\left[1 + \left(\frac{\gamma + 1}{2\gamma}\right)\left(\frac{\tilde{p} - p}{p}\right)\right]$$

(See ref. 3.)

The variation due to refraction of the overpressure $\Delta p = \tilde{p} - p$ along a ray tube has been determined in reference 7 (eq. (28) in section III of ref. 7). With the use of substitutions from equations (5) and (13) in section III of reference 7, Δp can be expressed in the form

$$\Delta p \propto \left[\frac{\rho a^3}{\left(a + w_n\right)\left|\vec{w} + a\vec{n}\right|A}\right]^{1/2}$$

or, with $\rho a^2 = \gamma p$

$$\Delta p = c\left[\frac{pa}{\left(a + w_n\right)\left|\vec{w} + a\vec{n}\right|A}\right]^{1/2}$$

The corresponding uniform atmosphere expression is

$$\Delta p = p_0 \gamma \, F(\tau)\left[\frac{M^5}{2A(M^2 - 1)}\right]^{1/2}$$

where

$$F(\tau) = \frac{1}{2\pi} \int_0^{V\tau} \frac{S''(\xi)d\xi}{\sqrt{V\tau - \xi}}$$

(ref. 3, eqs. (7) and (54), where in eq. (54) γ should be in the numerator).

By comparing the expression for Δp with the previous Blokhintzev expression in the limit of a uniform atmosphere, the parameter c is determined to be

$$c = \gamma \left[\frac{p_0 a_0 M^5}{2(M^2 - 1)}\right]^{1/2} F(\tau)$$

Therefore, the expression for Δp becomes

$$\Delta p = \gamma \left[\frac{p_0 a_0 M^5}{2(M^2 - 1)}\right]^{1/2} F(\tau) \left[\frac{pa}{(a + w_n)|\vec{w} + a\vec{n}|A}\right]^{1/2} \tag{2}$$

Here, the factor $F(\tau)$ accounts for the variation due to the finite amplitude of the wave, while the final factor accounts for the effects of refraction. Replacing a in equation (1) with $\tilde{a} + U$ and using equation (2) yields approximately

$$t \cong \tau + \int_0^z \frac{1}{\nu a}\left(1 - \frac{\gamma + 1}{2\gamma}\frac{\Delta p}{p}\right)d\zeta$$

that is

$$t = \tau + \int_0^z \frac{d\zeta}{\nu a} - \frac{\gamma + 1}{2}\left[\frac{a_0 p_0 M^5}{2(M^2 - 1)}\right]^{1/2} F(\tau) \int_0^z \frac{d\zeta}{\nu \left[pa(a + w_n)|\vec{w} + a\vec{n}|A\right]^{1/2}} \tag{3}$$

The new time variable T defined by

$$T = t - \int_0^z \frac{d\zeta}{\nu a}$$

is displaced from the original time variable by an amount that is constant for fixed z; that is, T is independent of τ. In terms of T, equation (3) becomes

$$T = \tau - CF(\tau) \int_0^z \frac{d\zeta}{\nu \left[pa(a + w_n)|\vec{w} + a\vec{n}|A\right]^{1/2}} \tag{4}$$

Derivation of Ray-Tube Area

Equations (2) and (4) are the fundamental equations for calculating the signature, but before they can be used, A and ν must be expressed as functions of z. The following procedure for determining the ray-tube cross-sectional area $\tilde{A}$ (which is proportional to A) is based on the method used in reference 5.

The rays considered are those associated with the propagation of the Mach wavelets. These wavelets propagate as acoustic waves and near the X-axis have the shape of a cone, which forms an angle with the axis equal to the complement of the Mach angle. In order to specify a particular ray, it is convenient to consider the initial incremental section of the

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ray cone (fig. 1). Then any given ray can be specified by the angle ω that the projection of the ray onto the cross section of the cone makes with the vertical.

The ray tube to be considered is defined by four bounding rays: the ray emitted at $x = 0$ and specified by the angle ω, the ray emitted at $x = 0$ and specified by the angle $\omega + \Delta\omega$, and the two corresponding rays emitted at $\Delta x = V \Delta\tau$. (See fig. 2.) For a ray associated with the angle ω, the constant "trace" velocities C_1 and C_2 in the stratified atmosphere are defined by (ref. 6, eqs. (41) and (42), where $u_0 = v_0 = 0$)

$$C_1 = \frac{a_0}{\lambda_0} = a_0 M \tag{5}$$

since $\lambda_0 = \frac{1}{M}$.

$$C_2 = \frac{a_0}{\mu_0} = \frac{a_0}{\sqrt{1 - \lambda_0{}^2 - \nu_0{}^2}}$$

or, since

$$\nu_0 = \frac{\beta \cos \omega}{M}$$

(ref. 5, eq. (14)) then

$$C_2 = \frac{a_0 M \csc \omega}{\beta} \tag{6}$$

Consequently

$$\frac{C_1}{C_2} = \beta \sin \omega \tag{7}$$

Since the vertical component of the wind velocity is assumed to be negligible, the equations of refraction are (ref. 6, eqs. (41) and (42))

$$a + \lambda u + \mu v = \lambda C_1 \tag{8}$$

$$a + \lambda u + \mu v = \mu C_2 \tag{9}$$

From equations (8) and (9)

$$\mu = \frac{C_1}{C_2} \lambda \tag{10}$$

Equations (8) to (10) can be solved for λ and μ, with the results

$$\lambda = \frac{a}{C_1 - \frac{C_1}{C_2} v - u} \tag{11}$$

and

$$\mu = \frac{a}{C_2 - v - \dfrac{C_2}{C_1}u} \qquad (12)$$

Then ν is determined by

$$\nu = \sqrt{1 - \lambda^2 - \mu^2}$$

or with a substitution from equation (10)

$$\nu = \sqrt{1 - \left(1 + \frac{C_1^2}{C_2^2}\right)\lambda^2} \qquad (13)$$

Define the function $f(z,\omega)$ by

$$f\left[z, C_1(\omega), C_2(\omega)\right] = \frac{v(z) + a(z)\mu(C_1, C_2, z)}{a(z)\nu(C_1, C_2, z)} \qquad (14)$$

Then the y coordinate of the intersection of a ray specified by the angle ω with a horizontal plane z units below the source of emission is (ref. 6, eq. (46))

$$y = \int_0^z f\left[\zeta, C_1(\omega), C_2(\omega)\right] d\zeta \qquad (15)$$

The corresponding intersection point of a ray specified by the angle $\omega + \Delta\omega$ is displaced in the y-direction by the increment

$$\Delta y = \frac{dy}{d\omega} \Delta\omega \qquad (16)$$

where $\dfrac{dy}{d\omega}$ is determined by an application of Leibniz's rule

$$\frac{dy}{d\omega} = \int_0^z \frac{\partial f}{\partial \omega} d\zeta \qquad (17)$$

In order to evaluate this quantity, $\dfrac{\partial f}{\partial \omega}$ must be expressed as a function of z. The first step is to expand the derivative by the chain rule

$$\frac{\partial f}{\partial \omega} = \frac{\partial f}{\partial \mu} \frac{\partial \mu}{\partial \omega} + \frac{\partial f}{\partial \nu} \frac{\partial \nu}{\partial \omega}$$

or

$$\frac{\partial f}{\partial \omega} = \frac{\partial f}{\partial \mu}\left(\frac{\partial \mu}{\partial C_1} \frac{dC_1}{d\omega} + \frac{\partial \mu}{\partial C_2} \frac{dC_2}{d\omega}\right) + \frac{\partial f}{\partial \nu}\left(\frac{\partial \nu}{\partial C_1} \frac{dC_1}{d\omega} + \frac{\partial \nu}{\partial C_2} \frac{dC_2}{d\omega}\right) \qquad (18)$$

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The quantities on the right-hand side of equation (18) can be obtained from previous relations. From equation (14)

$$\frac{\partial f}{\partial \mu} = \frac{1}{\nu} \tag{19}$$

and

$$\frac{\partial f}{\partial \nu} = - \frac{v + a\mu}{a\nu^2} \tag{20}$$

From equations (5) and (6)

$$\frac{dC_1}{d\omega} = 0 \tag{21}$$

and

$$\frac{dC_2}{d\omega} = - \frac{a_0 M}{\beta} \csc \omega \cot \omega \tag{22}$$

In view of equation (21), only $\frac{\partial \mu}{\partial C_2}$ and $\frac{\partial \nu}{\partial C_2}$ are required to complete the evaluation of $\frac{\partial f}{\partial \omega}$ in equation (18). These derivatives are obtained from equations (11) to (13). The resulting expressions are

$$\frac{\partial \mu}{\partial C_2} = -\left(\frac{1 - \frac{u}{C_1}}{a}\right)\mu^2 \tag{23}$$

$$\frac{\partial \nu}{\partial C_2} = -\left(\frac{\lambda}{\sqrt{1 - \lambda^2 - \mu^2}} \frac{\partial \lambda}{\partial C_2} + \frac{\mu}{\sqrt{1 - \lambda^2 - \mu^2}} \frac{\partial \mu}{\partial C_2}\right)$$

$$= -\left[\frac{\lambda}{\sqrt{1 - \lambda^2 - \mu^2}} \frac{\partial \lambda}{\partial C_2} + \frac{\mu}{\sqrt{1 - \lambda^2 - \mu^2}}\left(\frac{C_1}{C_2} \frac{\partial \lambda}{\partial C_2} - \frac{C_1}{C_2^2} \lambda\right)\right]$$

or

$$\frac{\partial \nu}{\partial C_2} = \frac{1}{\sqrt{1 - \lambda^2 - \mu^2}}\left[\left(1 + \frac{C_1^2}{C_2^2}\right)\frac{v\lambda\mu^2}{C_1 a} + \frac{\mu^2}{C_2}\right] \tag{24}$$

These results may be summarized as follows: When a, u, and v are specified as functions of z, then λ and μ are determined as functions of z by equations (11) and (12). In turn, these expressions substituted into equations (19) to (24) determine the quantities needed to express $\frac{\partial f}{\partial \omega}$ in equation (18) as a function of z. Then the integral of equation (17) can be evaluated so that the increment Δy in equation (16) is obtained as a function of z. Now consider the ray specified by the angle ω, as was the original ray, but emitted at a time $\Delta\tau$ units later, so that it is displaced by the distance $\Delta x = V \Delta\tau$ in the x-direction. Inasmuch as the atmosphere is assumed to be stratified,

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and since both rays have the same initial inclination angle, they are subject to exactly the same refractive effects. Consequently, these rays are parallel, and when they intersect the plane z units below the flight level, they are still displaced the same distance Δx in the x-direction. Consequently, the area of the intersection of the ray tube with this horizontal plane is

$$\Delta x \, \Delta y = V \frac{\partial y}{\partial \omega} \Delta \omega \, \Delta \tau \tag{25}$$

The inclination θ of a ray z units below the flight plane is determined by

$$\sin \theta = \frac{1}{ds/dz}$$

or

$$\sin \theta = \frac{a\nu}{\left| \vec{w} + a\vec{n} \right|} \tag{26}$$

Then the actual ray-tube cross-sectional area is determined by

$$\tilde{A} = \Delta x \, \Delta y \sin \theta$$

with $\Delta x \, \Delta y$ given by equation (25) and $\sin \theta$ by equation (26).

Since for steady motion in a uniform atmosphere, the function $A(z)$ as used by Whitham (ref. 3, eq. (52), where $\lambda = \infty$) is defined by

$$A = s = z \frac{M}{\beta} \sec \omega$$

the coefficient K relating A and $\tilde{A}$

$$A = K\tilde{A}$$

is given by

$$K = \frac{M^2}{\beta^2 \Delta x \, \Delta \omega}$$

Consequently

$$A = \frac{M^2}{\beta^2} \frac{dy}{d\omega} \sin \theta \tag{27}$$

Equations (17), (26), and (27) define the function $A(z)$ to be used in equations (2) and (4) for the signature calculation. This calculation for the pressure as a function of time can be performed in a manner similar to that described in reference 8 for the pressure as a function of distance. The characteristic parameter designated in reference 8 as a distance measured along the longitudinal axis from the body nose is replaced with τ, and the quantity $x - \beta r$ of reference 8 is replaced with T.

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Transformation to Ground-Fixed Coordinates

The y coordinate of the intersection of the ray specified by the angle ω with a horizontal plane can be determined by equations (14) and (15). The x coordinate of the intersection point is given by

$$x = \int_0^Z \frac{u + a\lambda}{a\nu} d\zeta \qquad (28)$$

and the time of intersection by

$$t = \int_0^Z \frac{d\zeta}{a\nu} \qquad (29)$$

Equations (28) and (29) are derived as equations (45) and (47) of reference 6. In order to determine the location of the ray-ground intersection with respect to a coordinate system fixed relative to the earth (the Ξ,H system), the relationship of this system to the original system at the time of intersection must be determined. The Ξ- and H-axes are chosen so that the flight velocity vector relative to the ground is directed along the negative Ξ-direction.

Then the Ξ,H origin is located directly below the X,Y origin at $t = 0$. It is assumed that the velocity vector $\vec{V}$, which is directed along the negative X-axis and has the magnitude of the airplane airspeed, is determined from the airplane's instruments. The wind vector at flight altitude relative to the ground $\vec{w}^*$ is determined from meteorological sounding data. Then the relation between the X,Y and Ξ,H systems at $t = 0$ is as shown in figure 3. At $t = 0$, the Ξ,H system is rotated relative to the X,Y system by an angle ψ such that

$$\xi = x \cos \psi - y \sin \psi$$

and

$$\eta = x \sin \psi + y \cos \psi$$

At the time of intersection, a translation of axes has also occurred because of the motion of the atmosphere at flight altitude. Consequently, at time t

$$\xi - u^* t = x \cos \psi - y \sin \psi$$

and

$$\eta - v^* t = x \sin \psi - y \cos \psi$$

where u^* and v^* are the components of $\vec{w}^*$ referred to the Ξ,H system.

CONCLUDING REMARK

Equations for calculating the pressure signature of an airplane in straight uniform flight in a stratified atmosphere have been derived.

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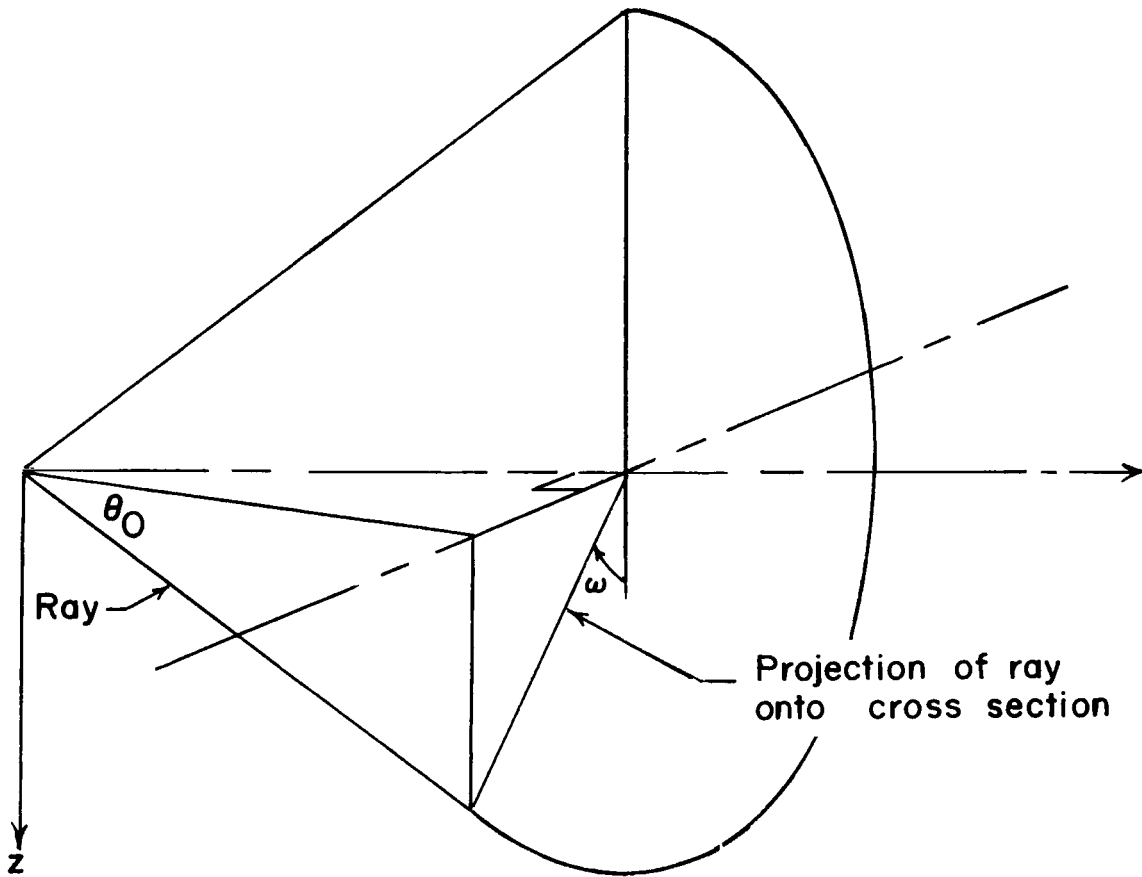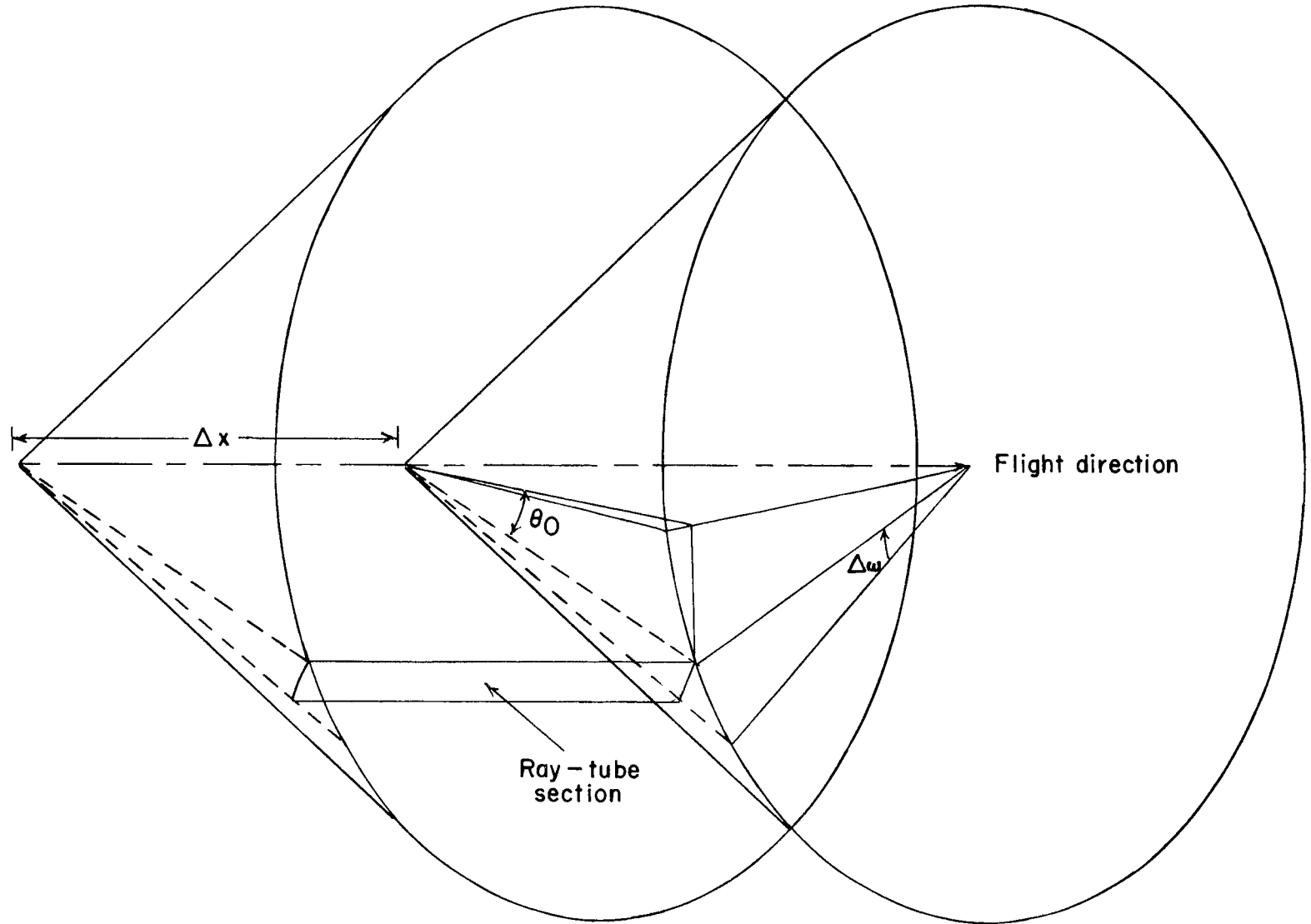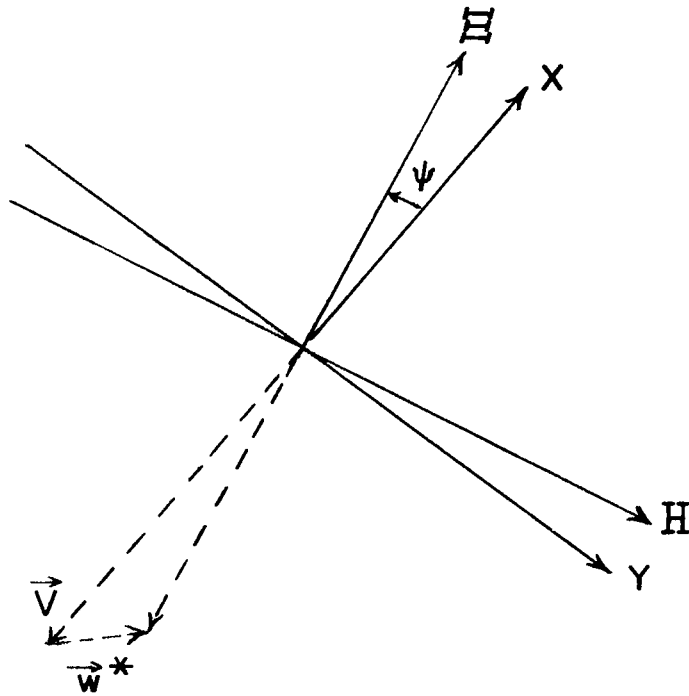
Figure 1.- Geometry for specifying a ray.

Δx

θ_O

$\Delta \omega$

Flight direction

Ray – tube
section

Figure 2.- Ray-tube geometry.

Figure 3.- Relation between coordinate system at t = 0.

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