23. OPTICAL CROSSED-BEAM INVESTIGATION OF LOCAL SOUND GENERATION IN JETS

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SUMMARY

Optical cross-correlation techniques are being developed to provide many of the experimental inputs which semiempirical models need for prediction of noise generation at the source.

The location of sound source intensities in jet shear layers requires area integrals of correlated density fluctuations. Two narrow light beams are set to detect sound sources in the flow by triangulation. A digital cross correlation of the optical signals from the two beams is then used to approximate the desired area integration of correlated disturbances. This optical integration over correlation areas is confirmed experimentally by verifying the radial resolution and spectra that are expected for point-area-product mean values. The determination of noise sources in jet shear layers will be started as soon as a new infrared crossed-beam system, which should give density-related local beam modulations, is finished.

The measurement of streamwise velocity variations in supersonic jets is required to apply present models of noise generation in turbulence—shock-wave interaction zones. Mean velocity profiles have already been measured in supersonic flows by using a statistical cross-correlation between streamwise separated and delayed beams. The measurement of root-mean-square velocity fluctuations will be started as soon as the appropriate sample reduction and digital filtering techniques are completed.

INTRODUCTION

The prediction of jet noise intensities and spectra requires, in many instances, that the source of the noise be found. Present theoretical studies indicate that the shear layer and the turbulence in the jet are significant generators of noise. The problem is thus to formulate present mathematical noise prediction models in terms of experimentally accessible shear layer and turbulence parameters and to develop probes which could measure these parameters in the jet. This problem was recognized early while trying to predict and to suppress the extreme jet noise levels that are produced during the firing of large rocket motors. One possible set of experimentally accessible shear layer and turbulence parameters is provided by the point area correlation of density fluctuations and by the statistical distribution of streamwise velocity variations. Both of these parameters can conceivably be measured in remote or inaccessible flow regions through the digital correlation of the photometer outputs from two narrow light beams. The progress in developing this crossed-beam concept and the experimental technique is now reported.

This paper includes many unpublished analytical and experimental results. The details of the mathematical derivations will be covered in a future NASA publication. The experiments with ultraviolet beams in supersonic jets were performed by M. J. Fisher and R. J. Damkevala. (See ref. 1.)

SYMBOLS AND NOTATIONS

a	speed of sound
b	shear-layer width
D	nozzle exit diameter
f	frequency
G	normalized two-beam-product mean value, defined by equation (12)
G_ρ	point-area-product mean value, defined by equation (5)
I	detected radiative power (watts)
i	beam modulation, $I - \langle I \rangle$
K'	fluctuation of extinction coefficient, $K - \langle K \rangle$
M	Mach number
P	probability distribution
p	pressure

S	turbulence spectrum
T	integration (averaging) time
t	time
$\vec{U}$	velocity components in earth-fixed frame, (U,V,W)
U_c	group velocity (convection speed of turbulence)
x	longitudinal coordinate along jet axis
Y	lateral coordinate in microphone plane
z	lateral coordinate normal to microphone plane
ξ	minimum beam separation distance (space lag)
η	length coordinate along beam B
ζ	length coordinate along beam **A**
$\vec{\xi}$	point vector in beam-oriented frame, (ξ,η,ζ)
P	density
X	optical wavelength
AX	optical band pass
τ	time lag
< >	time average

Superscripts:

$\rightarrow$	vector
'	fluctuation

*	drifting or vortex-fixed system
–	mean value of point property

Subscripts:

A	beam A
B	beam B
crit	critical
f	far field
max	maximum
o	nozzle exit

EXPERIMENTAL INPUTS FOR MATHEMATICAL NOISE PREDICTION MODELS

The noise generation of rocket engines is often so intense that individual sound waves become visible on shadowgraphs. Figure 1 illustrates such sound emanation from a cold model air jet. Three distinct sets of sound waves are clearly recognizable outside the jet and have been enhanced by white lines for illustration purposes. The origin of these sound waves may be found by tracing the wave normals back to the jet. The sound sources are apparently located at the nozzle lip, in the jet shear layer, and in the turbulence-shock interaction zone. It is therefore hoped that the mathematical jet noise prediction models which have been developed for shear layers and turbulence in aircraft engine jets may also be applied to supersonic rocket exhausts. Experimental input parameters which could be used in both subsonic and supersonic jets are sought. Such parameters would allow the noise investigations in rocket engines to be based on the large body of experience that has been accumulated for present aircraft engines. Using such parameters to design experiments on sound generation at the source would result in instrumentation and data reduction systems that could be applied to both aircraft and rocket engines, as well as to the supersonic transports.

The main results of the literature search for suitable experimental inputs are illustrated in figure 2. Sound generation in subsonic and supersonic shear layers is predicted as a consequence of vortices that are not alined in the flow direction. The main experimental input that could be used to study the sound generation by such vortices is a point-area correlation of density fluctuations. In supersonic jets additional noise is predicted for streamwise velocity variations upstream of shocks. Such velocity variations produce a shock-wave oscillation which will, in turn, generate sound even if no vortices are present. The main experimental input that could be used to study sound generation by turbulence-shock interaction is thus the probability $P(U)$ of streamwise velocity fluctuations.

Sound Generation Studies in Jet Shear Layers

The theory of sound generation in subsonic jet shear layers was formulated by M. J. Lighthill in 1952 and 1954. (See refs. 2 and 3.) His quadrupole sources are physically identical with the deformation of vortex rings (ref. 4). Any vortex which is not alined to the local flow direction will exert a force on the surrounding fluid particles and thereby slightly change the local pressure. This pressure change will, in turn, slightly change the local gas density in an almost isentropic fashion. Consider many different realizations of a vortex that is traversing the nozzle-fixed point $\vec{x}$ during the moment t of the realization. The deformation of this vortex could be recognized by the temporal variations of the density $\partial\rho/\partial t$ in a vortex cross section. The size of the deformed cross section will then depend on the area within which the density changes are correlated. The expected jet noise contribution per cross section of a moving vortex may thus be expressed by the following average E over all realizations:

$$F_{\text{supersonic}}(\vec{x}, \ldots) = \int_{\substack{\text{Vortex-fixed}\\ \text{frame}}} E\left[\frac{\partial}{\partial t}\rho\left(\vec{\xi}^{*} = 0, t\right) \frac{\partial}{\partial t}\left(\vec{\xi}^{*}, t\right)\right] d\eta^{*}\, d\zeta^{*} \tag{1}$$

The integrand represents a correlation between the traveling vortex center $\vec{\xi}^{*} = 0$ and the vortex cross section $\xi^{*} = \text{Constant}$. It can be related to a similar correlation in a nozzle-fixed frame through the following translation of the separation coordinates:

$$\left.\begin{aligned} \xi &= \xi^{*} + U_c\tau_1 \\ \eta &= \eta^{*} \\ \zeta &= \zeta^{*} \end{aligned}\right\} \tag{2}$$

Lighthill (ref. 2) further assumes that the density time derivative in the nozzle-fixed spot $\vec{x} + \vec{\xi}$ may be represented by a statistically stationary time series. In this event, the ensemble average over realizations E may be replaced with a time average $\langle\rangle$ and the correlation of the time derivative may be replaced .with the time-lag derivative of the temporal fluctuations:

$$p' = p - \langle p \rangle \tag{3}$$

As a result of the translation and time-averaging procedures, the noise contributions from a deforming-vortex cross section might be expressed by an experimentally accessible point-area correlation of density variations:

$$F(\vec{x},\xi) = \lim_{\tau_1 \to 0} \frac{\partial^2}{\partial \tau_1^2} \int_{\substack{\text{Moving-vortex} \\ \text{cross section}}} \langle \rho'(0;t)\, \rho'(\vec{\xi}^*;t+\tau_1) \rangle \, d\eta^* \, d\zeta^*$$

$$= \lim_{\tau_1 \to 0} \frac{\partial^2}{\partial \tau_1} \int_{\substack{\text{Nozzle-fixed} \\ \text{frame}}} \langle \rho'(x;y;z;t)\, \rho'(x+\xi+U_c\tau_1;\ y+\eta;\ z+\zeta;\ t+\tau_1) \rangle \, d\eta \, d\zeta \tag{4}$$

The integrand represents one special case of the following point-area-product mean value:

$$G_\rho(\vec{x},\xi,\tau) = \int_{\substack{\text{Nozzle-fixed flow} \\ \text{cross section}}} \langle \rho'(\vec{x};t)\, \rho'(\vec{x}+\vec{\xi};\ t+\tau) \rangle \, d\eta \, d\zeta \tag{5}$$

The desired relation between such correlation measurements in the flow and the associated mean-square pressure fluctuations in the far field follows by substituting equations (1), **(4),** and (5) into the mathematical model of quadrupole sound. The result is a fourth-order integral that accounts for all jet locations $\vec{x}$ and all cross sections ξ.

$$\langle p'^2(x_f) \rangle_{\text{Subsonic shear layer}}$$

$$= \frac{1}{16\pi^2} \int_{\text{Jet}} \frac{(x_f - x)^2 (y_f - y)^2}{\left[|\vec{x}_f - \vec{x}| - \frac{U_c(\vec{x}_f - \vec{x})}{a_o} \right]^6} - \left[\frac{\partial}{\partial y} \left(\frac{a^2}{a_o^2} U \right) \right]^2 \times$$

$$\left(\int_{\substack{\text{Eddy} \\ \text{length}}} \frac{\partial^2}{\partial \tau_1^2} G_\rho\left(\vec{x};\ \xi(\tau_1 \to 0);\ \tau = \tau_1 \to 0\right) d\xi \right) dx \, dy \, dz \tag{6}$$

Vortices generate very intense and directional sound waves outside the jet as soon as their speed exceeds the speed of sound at the microphone $(U_c/a_o > 1)$. Such vortex speeds will occur both in hot subsonic jets and in cold or hot supersonic jets, and the associated "Mach wave sound emission" may be of primary importance for rocket engines and the supersonic transport. Ffowcs Williams and Maidanik (ref. 5) predicted this type of sound radiation from a theoretical study of the singularity that arises in equation (6) along the particular direction

$$\theta = \cos^{-1}\frac{a_o}{\overline{U}_c} \tag{7}$$

His mathematical model can also be represented in terms of point-area correlation of density fluctuations (ref. 6), except that the area deformation must be applied to a nozzle-fixed flow cross section instead of a drifting cross section. To a nozzle-fixed observer, the cross section of a passing vortex will change because of the variation of the vortex shape. Sound is thus generated even if the vortex is not deformed at all. The sound waves which emanate from the traveling vortex are very similar to the Mach waves which a bullet traveling along the same streamline would generate.

The desired relation between density correlation measurements in heated subsonic or any supersonic shear layer and the associated mean-square pressure fluctuations at the microphone follows by substituting equations (4) and (5) into the mathematical model of Mach wave sound emission. This substitution gives

$$\langle p'^2(\vec{x}_f)\rangle_{\text{Supersonic or heated shear layer}}$$

$$= \frac{1}{16\pi^2}\int_{\text{Jet}} \frac{(x_f - x)^2 (y_f - y)^2}{\left|\vec{x}_f - \vec{x}\right|^6} \left(\frac{\partial \overline{U}}{\partial y}\right)^2 \left(\int_{\substack{\text{Nozzle-fixed}\\ \text{frame}}} \frac{\partial^2}{\partial \tau^2} G_\rho(x,\xi;0)\, d\xi\right) dx\, dy\, dz \tag{8}$$

The integrand of equation (8) closely resembles the integrand of equation (6). The main difference between the associated density correlation measurements in subsonic and supersonic shear layers is in the interpretation of the temporal correlation function. Equation (8) requires the curvature $\partial^2 G/\partial\tau^2$ of the temporal correlation between a point and an area at a fixed separation ξ. Equation (6) requires the curvature $\partial^2 G/\partial\tau_1^2$ of the envelope to all time correlation curves $G(\tau)_\xi$ that have been obtained for different point-area separations.

Sound Generation Studies in Turbulence-Shock Interaction Zones

Streamwise velocity fluctuations upstream of a shock create a nonuniform pressure jump across the shock. The shock starts oscillations which, in turn, propagate pressure waves into the downstream flow. In 1967, H. S. Ribner (ref. 7) established a mathematical relation between axial acoustical flux I_{ac} downstream of the shock ($x \to \infty$) and the mean-square velocity variation $\langle U'^2 \rangle$ upstream of the shock ($x \to -0$).

$$I_{ac}(x \to \infty) = \frac{3}{2} \frac{\bar{p}^2}{\bar{\rho} \bar{a}} (\vec{x} \to m, \ldots) \frac{\langle U'^2(x \to -0) \rangle}{a^{*2}} \times$$

$$\int_{\theta=\theta_{crit}(M;\gamma)}^{\theta=\pi/2} |P(\theta)|^2 \cos^3 \theta \left[1 + M \sin \theta_2 (\theta;M;\gamma)\right] \left(1 + \sin \theta_2\right) d\theta \qquad (9)$$

In this relation, M is the Mach number upstream of the shock, γ denotes the ratio of specific heats, and a^* is the critical speed of sound. The functions a^* and θ_{crit} can be obtained from isentropic flow and oblique-shock tables. The functions θ_2 and $P(\theta)$ were tabulated by Ribner in 1955. (See ref. 8.)

The flux of acoustic energy is steady across the jet boundaries. Just outside the jet, the acoustic flux should be the same as that calculated inside the jet. This invariance allows the estimation of the mean-square pressure fluctuation, just outside the jet, which turbulence-shock interaction will generate far downstream of the shock:

$$\langle p'^2(\vec{x}_f) \rangle_{\substack{\text{Shock-turbulence} \\ \text{interaction}}} = \rho_o a_o I_{ac} \qquad (10)$$

In equation (10), the acoustic flux may be calculated numerically for any given Mach number from equation (9). In fact, a similar integral applicable to oblique shocks can be obtained. Thus, there is a direct relationship between the mean-square pressure fluctuation at the downstream microphone and the longitudinal root-mean-square velocity. This model is still idealized since the finite extent of the potential core and the interaction between the shear layer and the shock root are not yet included. However, Ribner' s formulation could be extended to these more realistic flow fields. Within this model, or any of its future extensions, the measurement of the streamwise velocity variations upstream of shocks seems to be the most important input that would be required in future investigations of jet noise at the source.

CROSSED-BEAM MEASUREMENT OF POINT-AREA CORRELATIONS

Two narrow light beams are set to select by triangulation a point in the jet, as shown in figure 3. Each of these beams is then collected by photodetectors A and B. Consider now the passage of vortices, which are slightly more opaque or transparent than the surrounding gas. The passage of such a vortex will slightly change the received radiative power. The fluctuations of the photodetector output signals therefore contain information on the vortices that traverse the beams. These fluctuations i_A and i_B are recorded by using ac-coupled amplifiers. Each recorded signal will account for any vortex that is traversing the line of sight, whereas information is sought on only those vortices that pass the beam intersection. These "common" signal components can now be retrieved by a digital cross-correlation computation. During such a calculation, all radiative power modulations which occur only in one beam without affecting the other beam are canceled.

The cancellation of "noise" components depends on the number of oscillations in the record and will be very complete, if the digital cross-correlation computations are applied to a very long record. Noise components may thus be suppressed to the level of the temporal variations of the boundary conditions by integrating over a long record. All the crossed-beam data have been reduced with a special digital computer program which provides for a time integration of any desired length, by accumulating the time integral in a recursion formula. In this recursion, only one short data piece at a time is used on the computer memory. The associated "piecewise" correlation computations have successfully retrieved common signal components, which have been an order of magnitude smaller than the root-mean-square values of the recorded signals i_A and i_B.

Practical limitations, such as dynamic interchannel time displacement errors of the data acquisition system and temporal trends in the experimental boundary conditions, provide a lower limit for the smallest signal that can be retrieved, The continued development of piecewise correlation techniques fortunately shows promise for the partial elimination of the variations of boundary conditions in uncontrolled environments. (See ref. 9.) Environmental variations have been partially eliminated through piecewise modification of the signal time histories. (See ref. 10.)

Optical Approximation of Point-Area Correlations

The common signals which remain after the digital cross-correlation computations are produced only by those vortices which passed the beam intersection. All beam modulations by other vortices and by light source and detector noise should cancel each other. (See ref. 11.) The physical interpretation of the common signals thus needs to consider only those vortices which passed through the beam intersection point. Analytical studies of the optical integration along those beam elements which intersect the "common" vortices now indicate that this optical integration may be arranged to provide the desired point-area correlation. (See ref. 6.) The main results of these and more recent analytical studies are summarized in figure 4 and in the following equation:

$$\frac{<i_A(t)\ i_B(t)>}{<I_A><I_B>} - \int_{\substack{\text{Eddy cross}\\ \text{section}}} <K'(x;y;z;t)\ K'(x;y+\eta;z+\zeta;t)>\,d\eta\,d\zeta \tag{11}$$

The left-hand side of equation **(11)** is an experimentally accessible quantity and is denoted by the term "two-beam-product mean value." The numerator is calculated by adding the instantaneous product of the ac-coupled photometer outputs i_A and i_B. The denominator is given by the mean value of the dc-coupled photometer records I_A and I_B. The time average $< >$ has been obtained by integrating over a record length T which was sufficient to reduce all remaining variations of the two-beam-product mean value below a desired error margin $(\text{rms noise}/I \leqq 10^{-4})$.

The right-hand side is identical with the desired point-area integration. The point is selected by the beam intersection. The area is generated by moving one beam parallel to the other. The integral may thus be described as a "point-area-product mean value." The correlation refers, however, to the local changes of radiative power, which are caused by the changes of opacity or transparency between passing vortices and the surrounding gas. The scalar K' is thus given by the local variation of light extinction or light emission coefficients. Two-beam-product mean values (eq. **(11)**) would therefore give the desired point-area integration of density variations (eq. (5)) only if the local variations in radiative power K' can be related to density variations ρ'.

The optical approximation of point-area approximation can be extended to points and areas which are separated along a streamline. Let the beams A and B intersect the common streamline at the positions x and $x + \xi$. The optical point-area correlation would then be given by

$$G(T;x;y;z;\xi;\ldots) = \frac{<i_A(t)\ i_B(t)>}{<I_A><I_B>}$$

$$= \int_{\substack{\text{Vortex cross}\\ \text{section}}} <K'(x;y;z;t)\ K'(x+\xi;y+\eta;z+\zeta;t)>\,d\eta\,d\zeta \tag{12}$$

The claim of point-area correlation (eq. (12)) cannot be verified with point probes, since such a test would require a large number of phase-matched probes which possess a linear and time-invariant frequency response and which do not interfere with the flow. However, the point-area correlation does lead to two important conclusions on spatial resolution and spectral characteristics, which can be verified rather easily. The first conclusion is obtained by considering the isolation of individual streamlines through the location of the outside point. Thus individual streamlines which are separated by no more than a beam diameter could be distinguished. In flows with a dominant direction, the spatial resolution of streamlines should thus be possible inside a typical vortex. The spatial resolution of flow traverses should therefore approach the resolution of point probes.

The second conclusion is reached by considering cross-flow components. Any directional fluctuation will displace a vortex only inside the beam front. The area integral across the vortex is not changed by such a displacement, and the associated two-beam product mean value will therefore not depend on cross-flow components. This discrimination against cross-flow components is the essential difference between one- and three-directional wave number spectra. Crossed-beam spectra should approximate three-dimensional spectrum function and not the one-dimensional spectrum that is given by point probes.

Experimental Verifications

The first crossed-beam tests were designed to check the optical approximation of point-area correlations by verifying these conclusions. All tests were conducted in the initial portion of two supersonic jet shear layers, which were generated by expanding highly dried and unheated air through two nozzles. These nozzles produce shock-free jets of the same thrust at Mach numbers of 2.42 (**D** = 4.57 cm) and 3.34 (D = 5.47 cm).

The shadowgraph of figure 1 illustrated the jet from the M = 3.34 nozzle at off-design conditions. Typical sound pressure levels exceed 150 dB in the area where most of the crossed-beam instrumentation was located. All measurements were thus performed under extremely adverse noise and vibration environments.

Most crossed-beam measurements used focused ultraviolet beams generated by projecting the image of a hydrogen discharge (ref. 12) on the desired points of minimum beam separation. The beams were mounted on a lathe bed in such a manner that the points of minimum beam separation would always be parallel to the jet axis, that is, to the dominant direction of the streamlines. This point pair could traverse the jet axially and radially, and the separation could be varied in an axial direction. The light was extinguished in the flow by using the ultraviolet absorption of oxygen at an optical wavelength of 1850 Å inside a spectroscopic bandpass of 50 Å. Additional light extinction occurred by scattering from natural tracers, generated by condensation of entrained wet ambient air, and from air liquefaction at high chamber pressures. Crossed-beam tests with collimated helium-neon laser beams of approximately 2 mm in diameter were performed more recently to obtain additional information on scattering without simultaneous absorption.

The modulations of the ultraviolet beam were recorded with two photomultiplier tubes, EMR Model N 541A-05-14. The output signals from the two phototubes were amplified by specially built amplifiers and stored on an FM tape recorder, CEC, Model VR-3600, at a speed of 120 ips. Special multiplex equipment was integrated into the FM tape recorder so that any dynamic interchannel time displacement errors could be avoided by using two center frequencies (450000 cps and 825000 cps) on the same tape track. This multiplexing technique led to a flat frequency response from 500 cps to 500000 cps and relative phase distortions of less than 3^{o}. The analog tapes were then converted to digital tapes by running them through the analog-digital conversion systems of the Marshall computation laboratory at the reduced speed of 15 ips. Because of this stretching of the time scale, the effective sampling rate was 160 000 samples per second. The digital tapes were then processed on an IBM 7094 computer by using the piecewise correlation program.

The cancellation of cross-flow components by optical area integration may be judged from the spectral measurements summarized in figure 5. The ordinate displays crossed-beam spectra which have been calculated by including a time delay in the cross-correlation calculations:

$$S(f) = \frac{1}{2\pi} \int_{\tau=-0.003 \text{ sec}}^{\tau=0.003 \text{ sec}} \frac{\langle i_A(t)\, i_B(t+\tau) \rangle}{\langle i_A{}^2 \rangle^{1/2} \langle i_B{}^2 \rangle^{1/2}} \cos(2\pi f \tau)\, d\tau \tag{13}$$

These spectral estimates were then normalized with the maximum value; thus, the peak of the spectrum is always equal to one. The abscissa gives a normalized frequency which was based on the shear-layer width b and the turbulent convection speed U_c (given subsequently in fig. 8). This normalization has been successful in collapsing hot-wire spectra in subsonic jet shear layers on a universal curve.

The points in figure 5 represent crossed-beam spectra for six different locations in both the subsonic and the supersonic portions of the two jet shear layers. These points do approach a "universal" spectrum shape for the high frequencies. Thus, the local beam modulations in the supersonic flow seem to have the same length and velocity scales as the modulations in the subsonic flow.

The two solid lines in figure 5 represent the one-dimensional and three-dimensional spectra which the Heisenberg-Kolmogoroff theory would predict for locally isotropic turbulence. The position of these lines is immaterial; only the slope of the lines is important. One-dimensional spectra, which include cross-flow components, should follow a -5/3 power law and are thus indicated by a -5/3 slope. Three-dimensional spectra, which do not include cross-flow components, should follow a -11/3 power law. The measured spectra obviously approximate this -11/3 slope. This approximation of a three-dimensional spectrum function gives confidence in the optical approximation of point-area-product mean values.

The location of local sound sources may be judged from the crossed-beam intensity measurements summarized in figure 6. Intersecting light beams measured point-area correlations, and a pitot probe measured velocities at several traverses in the two jets. The results in figure 6 refer to one traverse of the $M = 3.34$ jet. The inner and outer edges of the shear layer are indicated by the velocity profile and are located at $y/D = 0.4$ and 0.8. The shear-layer thickness b is thus $(0.8 - 0.4)5.47 \times 2.2$ cm. A typical vortex cross section would extend over approximately 1/3 of this width.

Inspection of the two-beam-product mean values indicates that both the ultraviolet and the laser systems clearly resolve common beam modulations of less than 0.1 percent of the mean radiative power in the beam. These common modulations show a clear structure. The ultraviolet system produces, in the outer edge of the shear layer, a peak correlation which is probably associated with the condensation of the wet ambient air that was entrained by the jet. The laser system produces a peak correlation at the inner edge of the shear layer, which could probably be associated with the condensation of the natural CO_2 content of air.

The radial resolution of the two-beam-product mean values may be judged by comparing the width of the intensity peaks with the anticipated vortex diameter, $\approx b/3$. Both peaks have clearly been resolved inside a vortex. The beam diameter was approximately 2 mm; some of the experimental points show a clear and consistent variation of two-beam-product mean value along a 2-mm interval. The claim that the spatial resolution should approach the beam diameter seems therefore to be verified for this initial part of a supersonic shear layer.

There is thus every reason to believe that crossed beams do integrate over correlation areas as predicted by the analytical formulation in the section "Optical Approximation of Point-Area Correlations."

The successful approximation of point-area correlations would allow us to locate sound sources in jet shear layers if the local beam modulations are proportional to density variations. The laser beams show a peak intensity at the supersonic edge of the shear layer, and the ultraviolet beams show a peak intensity at the subsonic edge of the shear layer. However, the theory of Mach wave emission would predict the maximum sound source intensities in the center of the shear layer. It is therefore unlikely that the two systems have located sound sources; that is, the local beam modulations are not density related. Density-related beam modulations might be achieved by suppressing light scattering.

The local emission and absorption processes, which remain after scattering has been reduced, are uniquely determined by concentration, temperature, and density fluctuations. (See ref. **13.**) Furthermore, most jet noise theories assume that all thermodynamic variations in supersonic shear layers occur isentropically and do not change the composition of the gas. Small disturbances of densities and temperatures are then proportional to each other, and a relation to density is justified. However, chemical reactions in actual rocket exhausts will change the gas composition in an irreversible manner. Fortunately, the radiative power contributions due to temperature variations may then be suppressed by special settings of the wavelength A and the spectroscopic bandpass **AA.** (See ref. **14.**) It is therefore believed that local infrared beam modulations which are directly porportional to density fluctuations might be found in many jet noise investigations of practical interest.

Scattering will be reduced in future noise investigations by using infrared beams in heated flows. Heating should reduce the production of natural tracers by wet air entrainment or air liquefaction in the jet. The few remaining natural tracer particles should scatter the infrared light much less than the visible and ultraviolet beams now in use. Contamination by solid tracer particles might, however, present problems in some applications,

To study such problems, a special optical calibration cell which can be resonated like an organ pipe is being built. Wall pressure measurements in the cell should then provide a direct estimate of the local density fluctuations. The relation between beam modulation and density variation could thus be studied for various settings of the spectroscopic band pass and various gas samples.

VELOCITY MEASUREMENTS WITH STREAMWISE SEPARATED BEAMS

Streamwise separated beams may still contain a considerable amount of common signals, even if the separation exceeds several vortex diameters. Such commonality is a consequence of the flow, which transports the same emitting, absorbing, or scattering centers from the upstream to the downstream beam. A finite correlation might be expected if the signal of the downstream beam is delayed by the approximate transit time between the beams before the product mean value is calculated. Conversely, variable time delays might be introduced into the data reduction to analyze the motion along the streamline which has been selected by the beam triangulation. Without such time delays, widely separated beams could not contain common signals.

Conceptual Velocity Measurements

Consider the conceptual crossed-beam experiment illustrated in figure 7. Two mutually perpendicular beams have been separated along a streamline by the distance ξ. The ac-coupled signal from the downstream beam is delayed by the time interval τ, and the delayed and the undelayed signals $i_B(t - \tau)$ and $i_A(t)$ are then fed into a digital cross-correlation program. The output of this program will reach a maximum when the time delay is equal to the transit time which the average vortex needs to travel from the upstream to the downstream beam. The peak of the correlation curve may thus be used to identify the most probable velocity, as indicated in figure 7.

The digital correlation computations may be so arranged that the shape of the correlation curve indicates streamwise velocity components other than the most probable value. The analytical studies indicate that the following correlation of the signal time derivatives should approximate the probability P that the streamwise velocities U had the value ξ/τ:

$$P\left(U = \frac{\xi}{\tau}\right) = \frac{\tau}{\xi} \frac{\left\langle \frac{\partial}{\partial t} i_A(t) \frac{\partial}{\partial t} i_B(t - \tau) \right\rangle}{\int_{-\infty}^{\infty} \frac{\left\langle \frac{\partial}{\partial t} i_A(t) \frac{\partial}{\partial t} i_B(t - \tau) \right\rangle}{\tau} d\tau} \tag{14}$$

Equation (14) is thus the key to the desired measurement of streamwise velocity variations. The root-mean-square value $\langle U'^2 \rangle^{1/2}$, which is required for sound generation studies in shock-turbulence interaction zones, could be read directly from the width of the correlation curves.

The approximation of the streamwise velocity distribution function P with crossed-beam measurements may be understood physically by identifying the time derivative of the common signal components with the slope of a passing vortex surface. A large oscillation is expected only if the head or the tail of a vortex traverses the beam. The correlation calculation of equation (14) then represents the correlation of traveling interfaces instead of the entire vortex. An interface will need only a very short time interval to travel its own length, and the correlation curve $P(\xi/\tau)$ should therefore resemble a very sharp peak if all interfaces would travel at the same transit time. Conversely, the spread of the correlation curve will give the distribution of interface transit time or speeds. The correlation of time derivatives is thus different from the usual correlation of the signal fluctuations, where the width of the curve is identified with the time a vortex needs to travel its own length and not with the spread of transit times.

Measurements of Convection Speed Profiles

Crossed-beam measurements of convection velocities have been made in several radial and axial positions in both of the supersonic jets using the ultraviolet and the laser systems. The main results are plotted in figure 8 in a nondimensional form'with the width of the shear layer as a length scale and the nozzle exit speed as a velocity scale. Pitot tube traverses of subsonic and supersonic jets suggest that this particular normalization should give a "universal" velocity profile that is valid for both subsonic and supersonic jets. (See ref. 15.) A similar result is predicted by identifying the turbulent velocity field with the random walk of a single fluid particle. The velocity measurements with pitot probes did collapse on the universal curve shown. The dashed curve represents subsonic convection speed measurements with a pair of hot wires that were also separated along streamlines. (See ref. 16.)

Inspection of the crossed-beam results leads to the following observations:

(a) In the subsonic portion of supersonic shear layers, crossed-beam measurements agree with scaled group velocity profiles from subsonic hot-wire measurements. The same agreement with hot-wire group velocities was found in our previous investigations of subsonic shear layers. (See refs. 17 and 18.)

(b) In the supersonic portion of the jet shear layers, laser crossed-beam measurements tend to follow the mass average speed indicated by pitot tubes. Convection velocities from ultraviolet measurements also tend to be higher than those predicted from hot-wire measurements in subsonic flows. Unfortunately, the scatter of the velocity points is still too large for any detailed analysis. This lack of resolution is probably due to the electronic and computational problems associated with retrieving very small signals out of noise. It is believed that the experience from these first crossed-beam tests in supersonic shear layers will allow us to improve data acquisition and reduction so that the spatial resolution of velocity profiles will be considerably better in future tests.

(c) Both laser and ultraviolet beams give approximately the same velocity profile although each beam responds to 'different light extinction processes.

Apparently, velocity measurements are independent of the type of local extinction phenomena to which the beams respond. This independence of the local extinction process suggests that velocity and length scale measurements can be conducted whether or not the beams respond to density variations. Scaling studies between subsonic and supersonic jets, between cold and hot jets, and between model jets and the prototype engine exhaust are thus within the capability of existing crossed-beam technology.

The measurement of streamwise velocity fluctuations and length scales requires the correlation of time derivatives. In the digital program, a time derivative is replaced with the moving central difference between three adjacent data samples. These differences are so small that the quantization errors are relatively large. Quantization errors can be reduced by sample reduction techniques whenever the sampling frequency exceeds the data frequency by one to two orders of magnitude. Adding many samples to obtain one data point reduces quantization errors since the average is more accurate than any of the original samples. Appropriate time-scale stretching, sample reduction, and digital filtering techniques for correlation of time derivatives are being developed. Preliminary checks of these techniques are encouraging, and it is believed that the measurement of streamwise root-mean-square velocity will be experimentally feasible.

CONCLUSIONS

Theoretical and experimental studies of crossed-beam test arrangements-indicate that the statistical correlation of optical signals has the potential to provide many of the measurements which are required for jet noise investigations at the source. Remote, or inaccessible, source areas are studied optically without the probe interference problems that plague present hot-wire investigations.

The intensity and spectra of jet noise may be related mathematically to the point-area correlation of density fluctuations in jet shear layers and to streamwise velocity fluctuations in shock-turbulence interaction zones.

The optical integration along the beams has been used to obtain point-area correlations that are not accessible to any point probe.

Sound sources may be located by varying the position of the beam intersection point in the jet. The two-beam-product mean value should provide direct estimates of sound source intensities and spectra, if an optical wavelength region can be found that provides density-related light extinction or emission processes.

The distribution of streamwise velocity variations could conceivably be measured in subsonic and supersonic jets with streamwise separated and delayed beams.

The velocity and length scale measurements in subsonic and supersonic jets show that present crossed-beam technology could be used for scaling studies between cold and hot jets and between model engines and the prototype.

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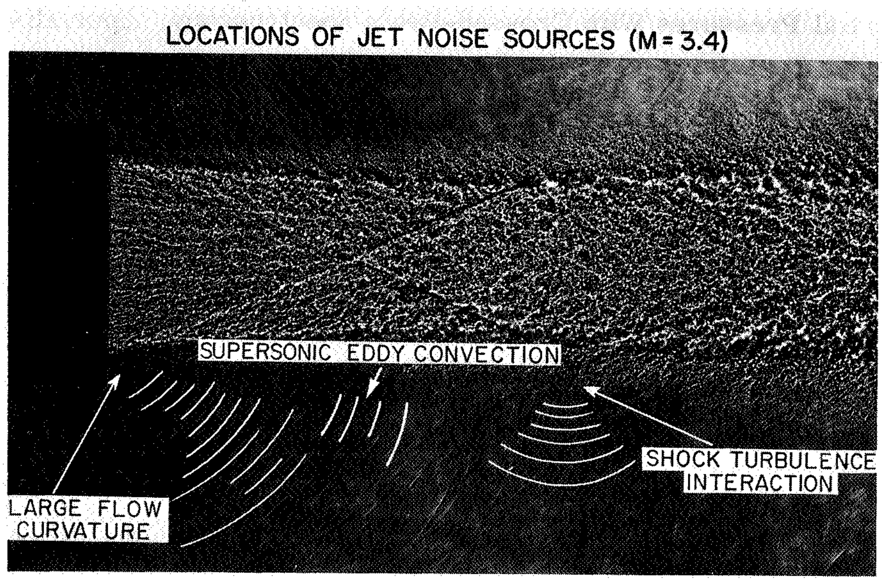

Figure 1 L-68-8578

EXPERIMENTAL INPUTS TO JET NOISE PREDICTION MODELS

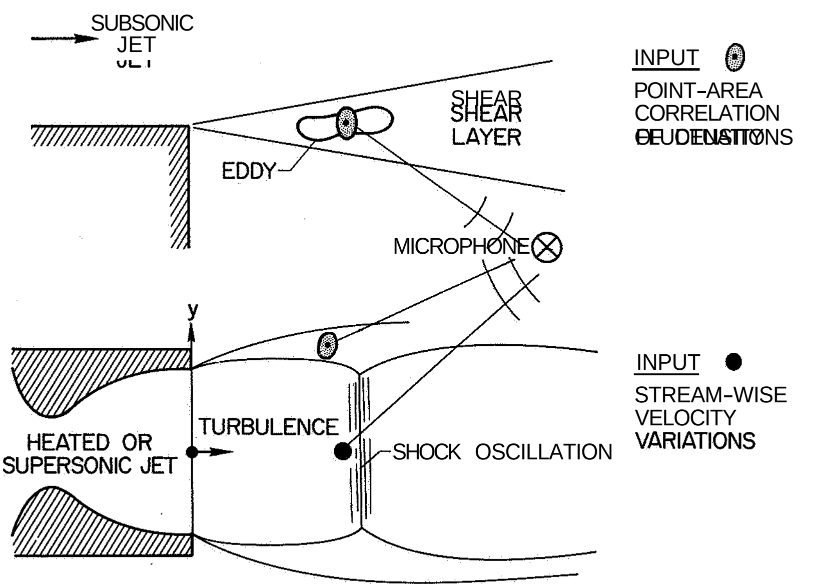

Figure 2

SOUND SOURCE IDENTIFICATION IN SUPERSONIC JETS

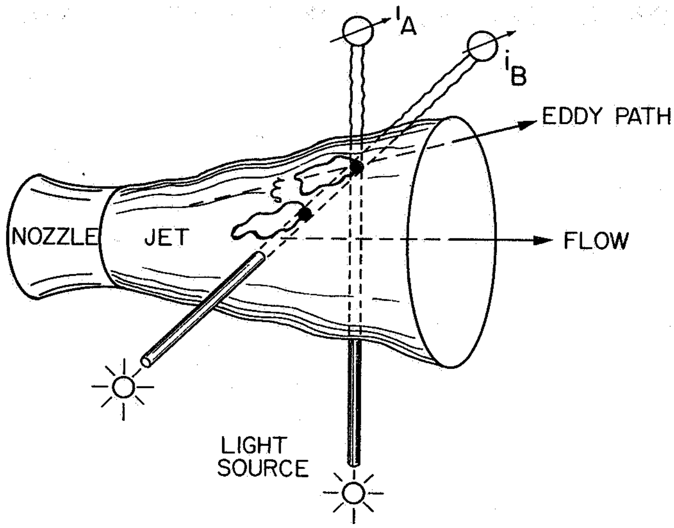

Figure 3

OPTICAL AREA INTEGRATION

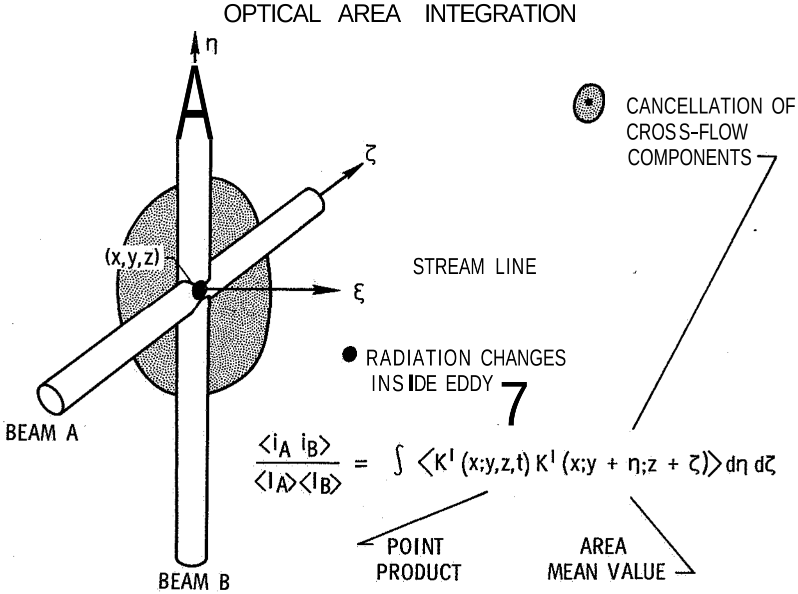

Figure 4

CROSS–BEAM SPECTRA IN JET–SHEAR LAYER

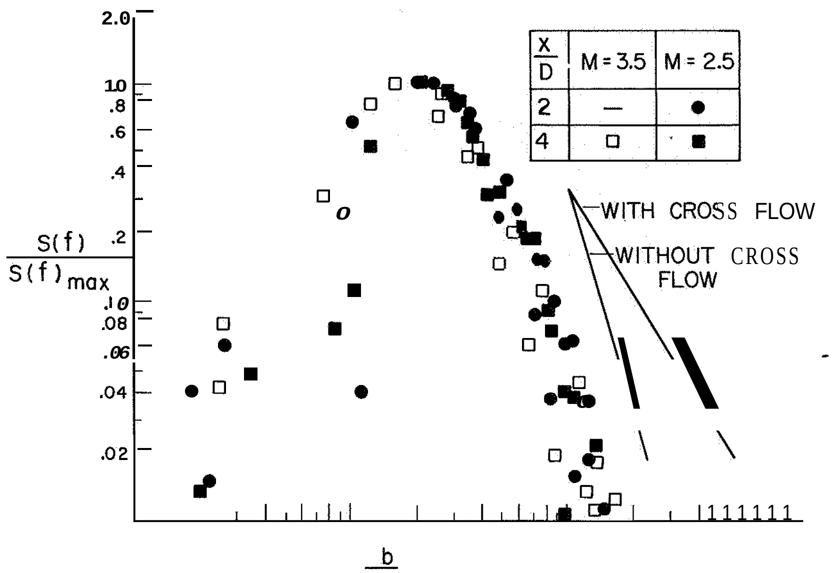

Figure *5*

CROSS-BEAM RESOLUTIONS OF POINT-AREA CORRELATIONS

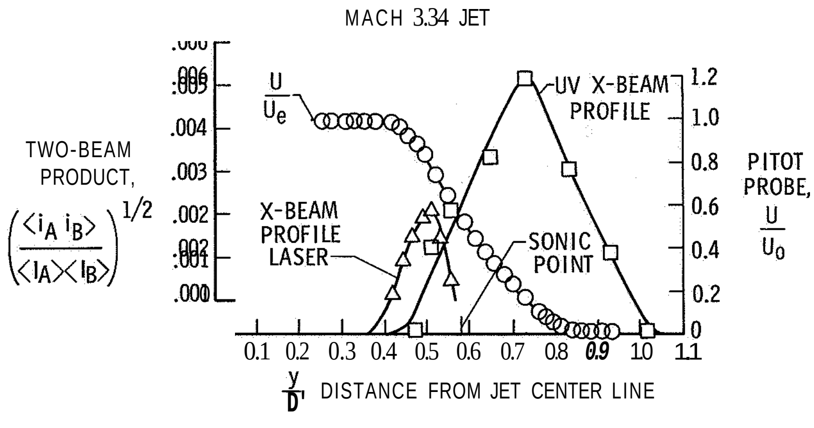

Figure *6*

MEASUREMENT OF STREAMWISE VELOCITY VARIATIONS

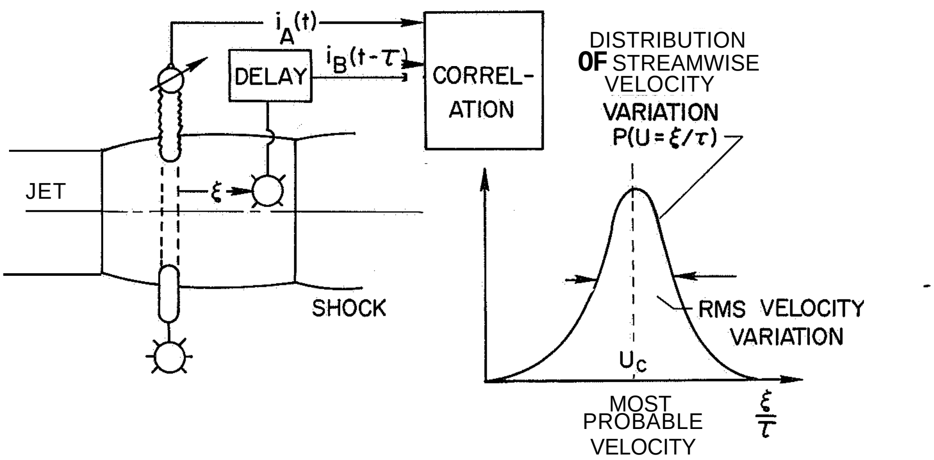

Figure 7

CONVECTION SPEED MEASUREMENTS
IN JET SHEAR LAYER

$\frac{x}{D}$	$U_0 = 624$ m/s	$U_0 = 556$ m/s	
2	o	●	U-V SYSTEM
4	□	□	U-V SYSTEM
4	+		LASER SYSTEM

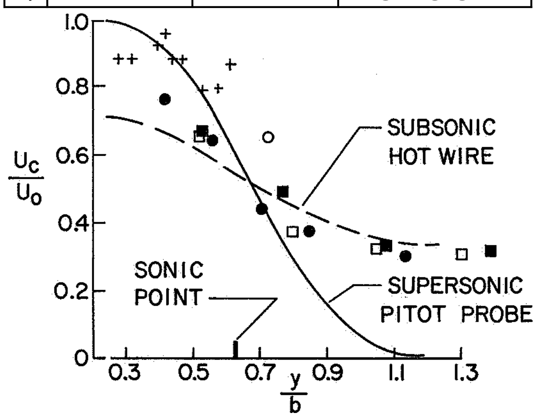

Figure 8