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STATISTICAL ESTIMATION OF TROPOSPHERIC TEMPERATURES AND RELATIVE HUMIDITIES FROM REMOTE RADIOMETRIC MEASUREMENTS

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ABSTRACT

A formulation is given for making statistical estimations of tropospheric temperatures, relative humidities, and total water vapor content from remote spectral measurements of the infrared radiation emitted by the earth and atmosphere. Simultaneous measurements in the 6.3μ water vapor band, the atmospheric "window" near 11μ, and in the 15μ carbon dioxide band are required. The formulation can serve as a framework for analyzing the expected information content of measurements of the type to be obtained from forthcoming satellite experiments such as the infrared interferometer spectrometer (IRIS). Examples are given of applications to five ensembles of atmospheric soundings chosen to be representative rather than exhaustive. The fractions of the variance of temperature and relative humidity accounted for by the radiation measurements are found to vary considerably from one type of ensemble to another, and in all cases are strong functions of instrumental noise. For realistic instrumental noise levels, the standard errors of estimate for temperature are for the most part less than 2°K at levels below 200 mb, and the fraction of the variance in total water vapor content accounted for ranges from .65 to .83, depending on the ensemble.

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INTRODUCTION

Measurements of thermally emitted infrared radiation from the earth and atmosphere obtained with satellite-borne instruments contain information on the vertical temperature profile and the vertical distribution of optically active gases in the lower atmospher . Since this was first pointed out (King, 1958; Kaplan, 1959), considerable effort has been expended in the development of computational techniques for extracting the desired information from the data. Much of the early work was concerned with directly solving the integral equation relating the spectral intensity of the upwelling radiation to the temperature profile (Wark, 1961; Yamamoto, 1961). The principal difficulty encountered was the extreme sensitivity of the desired solutions to noise in the measurements, and methods of obtaining smoothed solutions were developed by King (1964) and by Twomey (1963; 1965) who employed a technique suggested by Phillips (1962).

Quite generally, the temperature profile at a given location can be regarded as consisting of a mean over some specified time interval plus a fluctuating component. Now since the mean can be specified from data covering the past history of the location when such data exist, the "new" information to be extracted from the satellite measurements consists of the departure from the mean at any given time, and it is the effect of this fluctuation on the upwelling radiation that constitutes the "signal" to be detected. In addition to defining mean values, the a priori data permit statistical parameters for the fluctuating component to be estimated (e.g., covariance matricies for the temperatures at various levels). An early effort to incorporate this a priori information into the formulation of the problem by Wark and Fleming (1966) involved the expansion of the indicial function of the integral equation in terms of empirical orthogonal functions (also called characteristic patterns) constructed using in situ measurements (Holmström, 1963; Alishouse, et al, 1967; Lorenz, 1956; Obukhov, 1960).

Recently, formulations using linear statistical estimation techniques have shown considerable promise for determining atmospheric temperature profiles (Rodgers, 1966; Strand and Westwater, 1968a; 1968b; Westwater and Strand, 1968). It is the purpose of the present paper to attempt to utilize estimation theory in the formulation of a technique for inferring relative humidities as well as temperatures at selected pressure levels, and to use the formulation in an analysis of the information content of medium resolution infrared spectral measurements of the type obtainable from instrumentation to be flown in forthcoming meteorological satellites such as the infrared interferometer spectrometer (IRIS) experiment (Hanel and Chaney, 1966). The consideration of tropospheric relative humidities is motivated by earlier studies of the distribution of tropospheric relative humidity on a quasiglobal basis, using measurements in the 6 to 6.5 micron water vapor absorption band and the 8 to 12 micron "window" obtained with satellite-borne radiometers (Möller, 1961; 1962; Möller and Raschke, 1964; Raschke and Bandeen, 1967). In these studies, an equivalent constant tropospheric relative humidity consistant with the observed radiances was derived. With the prospect of obtaining more refined measurements with spectral resolution elements small compared to the total width of an absorption band, the possibility of obtaining information on the vertical distribution of relative humidity can be considered. In a previous study (Conrath, 1969), the problem was examined using a direct estimation with a parametric representation of the relative humidity profile. In addition, a statistical estimation formulation was given, but only the variance in the relative humidities was considered. In the present paper, an attempt is made to consider the variance of both temperature and relative humidity simultaneously.

In the following sections, the application of statistical estimation theory to the remote radiometric inference of atmospheric parameters is considered, and a formulation is given for estimating relative humidities and temperatures at selected pressure levels in the troposphere along with total water vapor content in an atmospheric column. Criteria are introduced for defining the information content of measurements in a set of spectral intervals for a given ensemble of atmospheres. These techniques are then applied to selected ensembles of atmospheric soundings in an effort to gain some feeling for the information which might be extracted from forthcoming satellite measurements.

FORMULATION OF THE PROBLEM

For a non-scattering atmosphere in local thermodynamic equilibrium, the spectral intensity of upwelling thermal radiation at the top of the atmosphere can be written in the form

$$I(\nu) = B\left[\nu, T(x_s)\right] \tau\left(\nu, x_s\right) - \int_{x_\infty}^{x_s} B\left[\nu, T(x)\right] \frac{\partial \tau(\nu, x)}{\partial x} dx \qquad (1)$$

where the independent variable x is an arbitrary function of pressure, x_∞ and x_s refer to the top of the atmosphere and the surface respectively, $B(\nu, T)$ is the Planck intensity at wave number ν and temperature T, and $\tau(\nu, x)$ is the atmospheric transmittance at wave number ν from level x to the top of the atmosphere. The intensity $I(\nu)$ depends on the temperature profile $T(x)$ through the Planck intensity and on the amount and vertical distribution of the optically active gases through the transmissivity. Thus, if the distribution of gases which are optically active in the spectral region is known, measurements of $I(\nu)$ in that region will provide information on the temperature profile. If, however, the temperature profile is known, we can use the measurements to deduce information on the distribution and total amount of the optically active gases; this problem has been treated formally by King (1963). Recently Smith (1967; 1968a) has suggested the use of a multi-channel radiometer to infer water vapor mixing ratios as well as temperatures, and Prabhakara (1969) has examined the problem of obtaining information on the vertical ozone distribution from measurements in the 9.6 micron band.

The present study is concerned with the information on tropospheric relative humidities and temperatures obtainable from measurements in the 6.3 micron water vapor absorption band, the atmospheric "window" near 11 microns, and the 15 micron carbon dioxide band. The uniform mixing of carbon dioxide permits information on the atmospheric temperatures to be obtained from the 15 micron band; information on the relative humidity can be obtained from the 6.3 micron band and the "window" region provides information on the surface temperature. Since it is necessary to have information on the temperature to estimate the relative humidity and there is absorption due to water vapor in the "window" region and also in the 15 micron region, measurements from all three spectral regions will be employed in estimating both relative humidity and temperature.

We shall consider the information content of measurements in these spectral regions within the framework of a linear statistical estimation formulation. Let us define a vector of parameters to be estimated $b^T = [T_1 \cdots T_n, r_1 \cdots r_n]$ where $T_1 \cdots T_n$ and $r_1 \cdots r_n$ are temperatures and humidities respectively at selected levels in the troposphere. The superscript T denotes matrix transposition. Let us also define a vector $\theta^T = [\theta_1 \cdots \theta_m]$ where $\theta_1 \cdots \theta_m$ are the brightness temperatures associated with the intensities $I(\nu)$ in selected spectral intervals covering the three general spectral regions described above. The brightness temperature θ_i for the ith spectral interval is defined by

$$B(\nu_i, \theta_i) = I(\nu_i) . \tag{2}$$

Treating the components of $\mathbf{b}$ and $\boldsymbol{\theta}$ as random variables, a linear unbiased estimate for $\mathbf{b}$ which we shall call $\hat{\mathbf{b}}$ can be written

$$\hat{\mathbf{b}} = \langle \mathbf{b} \rangle + H\left(\boldsymbol{\theta} - \langle \boldsymbol{\theta} \rangle\right) \tag{3}$$

where H is a $2n \times m$ matrix to be determined, and the angular brackets denote an ensemble mean; i.e., for an arbitrary vector $\mathbf{v}$

$$\langle \mathbf{v} \rangle \equiv E\{\mathbf{v}\} \tag{4}$$

where E{ } is the expectation value operator. The matrix H can be determined by requiring that the quadratic risk function

$$R = E\left\{ (\hat{\mathbf{b}} - \mathbf{b})^T (\hat{\mathbf{b}} - \mathbf{b}) \right\} \tag{5}$$

be a minimum. It can be shown (Deutsch, 1965; p. 66) that the minimum value of R is obtained when

$$H = \Psi_{b\theta} \Psi_{\theta\theta}^{-1} . \tag{6}$$

In the notation used here, the covariance matrix of two arbitrary vectors $\mathbf{u}$ and $\mathbf{v}$ is written as

$$\Psi_{uv} \equiv E\left\{\left(\mathbf{u} - \langle\mathbf{u}\rangle\right) \left(\mathbf{v} - \langle\mathbf{v}\rangle\right)^T\right\} . \quad (7)$$

If we have available a joint ensemble of measurements of corresponding values of $\mathbf{b}$ and $\boldsymbol{\theta}$, we can estimate the covariances required in (6). This possibility, which has been considered by Rodgers (1966) and by Smith (1968b), is attractive because it avoids the use of the physics of the radiative transfer process, and a knowledge of the atmospheric transmissivities is not required. However, since adequate sets of such data do not presently exist, we shall use ensembles of radiosonde data along with the theoretical relation for the intensities (1) to investigate the potential usefulness of forthcoming satellite data for estimating tropospheric temperatures and relative humidities.

In general, a measurement of $\boldsymbol{\theta}$ will consist of a "true" value $\boldsymbol{\theta}_0$ and a noise component $\boldsymbol{\epsilon}$, i.e.,

$$\boldsymbol{\theta} = \boldsymbol{\theta}_0 + \boldsymbol{\epsilon} . \quad (8)$$

Under the assumptions that $\boldsymbol{\epsilon}$ is uncorrelated with $\boldsymbol{\theta}_0$ and has a zero mean value, (6) becomes

$$H = \Psi_{b\theta_0}\left(\Psi_{\theta_0\theta_0} + \Psi_{\epsilon\epsilon}\right)^{-1} . \quad (9)$$

In previous work on the temperature inversion problem (Rodgers, 1966; Strand and Westwater, 1968a; 1968b), the integral on the right hand side of a linearized version of (1) was expressed in terms of numerical quadrature with the parameters to be estimated taken as the values of the indicial function at the quadrature points. In this way, it is possible to express the covariance matricies occurring in (9) directly in terms of the covariance matrix of the estimated parameter. The resulting formulation is similar to one given by Foster (1961). (See also Deutsch, 1965, p. 88.) In an earlier formulation of the statistical estimation of relative humidity, a linearization was used to express H in terms of the relative humidity covariance matrix (Conrath, 1969). However, in the present work we shall estimate the covariance matricies occurring in (9) directly by calculating the intensities, using (1), for each member of the ensemble of radiosonde data employed. Instead of estimating the unknown parameters at the integration quadrature points, we shall use the following levels: 100 mb, 150 mb, 200 mb, 300 mb, 400 mb, 500 mb, 700 mb; 850 mb, and surface.

Several authors have considered the problem of defining the information content of measurements used in remote sensing problems. Twomey (1966) has given a non-statistical formulation which considers the behavior of the kernel of the integral equation, and Mateer (1965) and Rodgers (1966) have defined the information content in terms of "number of independent pieces of information" based on a factor analysis of the measurements using characteristic patterns. Westwater and Strand (1968) have analyzed information content in terms of the new information provided by the measurements beyond that already known from a priori statistics alone. They suggest the use of three possible parameters for judging the information content: the reduction in total variance, the fractional reduction in total variance, and the standard deviation of the estimate error per atmospheric level. Similar concepts will be used here, only we shall be interested in displaying the measures of information at each level explicitly rather than using single parameter sums over all levels.

The covariance matrix of the estimate error $V \equiv E\{(\hat{b} - b)(\hat{b} - b)^T\}$ corresponding to the use of (9) for H can be written

$$V = \Psi_{bb} - \Psi_{b\theta_0}\left(\Psi_{\theta_0\theta_0} + \Psi_{\epsilon\epsilon}\right)^{-1} \Psi^T_{b\theta_0} \tag{10}$$

The standard deviation of the estimate error at a given atmospheric level is $S_i = \sqrt{V_{ii}}$ where V_{ii} is the ith diagonal element of V. In analogy with regression analysis we shall refer to this parameter as the standard error of estimate (Ezekiel and Fox, 1959). The fraction of the variance of an atmospheric parameter at a given level which can be accounted for by the radiation measurements can be written

$$c_i = 1 - \left(S_i/\sigma_{b_i}\right)^2 \tag{11}$$

where σ_{b_i} is the standard deviation of the atmospheric parameter at the level considered. Again in analogy with regression analysis, we shall refer to this parameter as the coefficient of multiple correlation (Ezekiel and Fox, 1959). It provides a convenient measure of the information at each level, being zero in the case where no information is provided by the radiation measurements and unity in the case of perfect information retrieval.

APPLICATIONS TO SELECTED ENSEMBLES

The techniques discussed in the preceding section will now be applied in analyses of the information obtainable from radiation measurements from selected ensembles of atmospheres. The study is not exhaustive, but is confined to a few representative ensembles in an effort to obtain a rough idea of the information derivable on tropospheric water vapor and temperatures from forthcoming satellite-borne experiments.

The spectral intervals used in this study are listed in Table 1, and are all within the 5-20 micron range of the first of the IRIS experiments to be flown in the near future. The spectral intervals were chosen as representative and are not necessarily optimum in the sense of maximizing any of the measures of information content. The required spectral intensities were calculated from model atmospheres using (1). In the 6.3 micron water vapor band, a random band atmospheric transmissivity model was used (Goody, 1952), incorporating the parameters given by Williamson and Houghton (1965). The water vapor transmissivity for the "window" interval near 11 microns was taken from Möller and Raschke (1964) as were the water vapor transmissivities for the region overlapping the 15 micron CO_2 absorption. The 15 micron carbon dioxide transmissivities were based on a direct line-by-line integration technique and were provided by V. Kunde (personal communication). In the overlapping region, it was assumed that the total transmissivity is given by the product of the separate water vapor and carbon dioxide transmissivities.

The model atmospheres incorporated in the various ensembles are based on collections of radiosonde data. These do not contain usable data on the relative humidity profiles in the uppermost tropospheric layers and stratosphere. Since the radiation measurements, especially in the more opaque parts of the 6.3 micron band, are sensitive to water vapor in the upper troposphere, the extrapolation of the relative humidity soundings into this region is problematic. We have elected to extrapolate the relative humidity up to 100 mb at a constant value equal to the value at the highest level for which a measurement exists. This should be borne in mind when considering the results presented below, especially with regard to the parameters calculated for the relative humidity in the upper troposphere.

The effects of instrumental errors enter into the calculations through the error covariance matrix $\Psi_{\epsilon\epsilon}$. It can be seen that the estimation is stable against instrumental errors in the sense that S_i approaches σ_{b_i} as an upper bound as $\Psi_{\epsilon\epsilon} \rightarrow \infty$ in which case $C_i \rightarrow 0$ indicating a complete lack of information recovery from the radiation measurements. In the present study we assume no correlation in the errors for the various spectral intervals so $\Psi_{\epsilon\epsilon}$ is diagonal. For the instrumentation being considered, the rms error σ_I in the spectral intensities is very nearly the same in all the spectral intervals so the diagonal elements of the brightness temperature noise covariance matrix can be approximated by

$$\left(\Psi_{\epsilon\epsilon}\right)_{ii} \cong \left[\frac{\partial B\left(\nu_i, \theta_i\right)}{\partial \theta_i}\right]^{-2} \sigma_I^2 \tag{12}$$

where the factors $\partial B/\partial \theta_i$ in each case are evaluated at the ensemble mean values of θ_i. Calculations have been made in each case for a range of instrumental errors σ_I; 0, 0.2, 0.5, 1.0, and 2.0 erg cm^{-2} sec^{-1} ster^{-1} cm. The value expected in the first of the IRIS satellite experiments is about 0.5 erg cm^{-2} sec^{-1} ster^{-1} cm.

The ensembles employed are based on radiosonde ascents and were constructed as follows:

Low Latitude Ensemble—based on 200 ascents at Ascension Island sampled over the 1962-1966 interval with a distribution essentially uniform over all seasons. The mean total water vapor content is 2.67 g cm^{-2}.

Mid Latitude Summer Ensemble—based on 117 ascents at Wallops Island, Virginia, sampled between April 15 and October 14 for the years 1962-1966. The mean total water vapor content is 2.86 g cm^{-2}.

Mid Latitude Winter Ensemble—based on 124 ascents at Wallops Island, Virginia, between October 15 and April 14 for the years 1962-1967. The mean total water vapor content is 1.26 g cm^{-2}.

High Latitude Summer Ensemble—based on 101 ascents at Fort Churchill, Canada, between April 15 and October 14 for the years 1964-1967. The mean total water vapor content is 1.22 g cm^{-2}.

High Latitude Winter Ensemble—based on 104 ascents at Fort Churchill between October 15 and April 14 for the years 1964-1967. The mean total water vapor content is .30 g cm^{-2}.

Mean temperatures and humidities along with their standard deviations for the various ensembles are listed in Tables 2-6.

As examples of the results obtained, coefficients of multiple correlation and standard errors of estimate for each of the five ensembles are given in Figures 1 and 2. Only the values calculated using an instrumental noise of 0.5 erg cm^{-2} sec^{-1} $ster^{-1}$ cm are given. To illustrate the reaction of the statistical parameters to instrumental noise, the coefficients of multiple correlation and standard errors of estimate for the Mid Latitude Winter Ensemble are shown in Figures 3 and 4 for the entire set of instrumental noise values employed in the calculations. The results for the remaining ensembles are qualitatively similar.

A statistical estimation method was formulated for total water vapor similar to that employed for relative humidities and temperatures, using the same spectral intervals. The resulting standard errors of estimate S_u and coefficients of multiple correlation C_u are shown as functions of instrumental noise in Figure 5.

Up to this point, a perfect knowledge of the covariance matricies has been assumed in the analysis. However, in practical applications to satellite data it will generally be necessary to estimate these quantities from samples of in situ measurements from previous times and in many instances from different geographic locations. Thus, it is important to get some feeling for the effects of applying an estimation relation derived for one ensemble to data from a different ensemble. The H matrix of (9) along with the mean values $\langle \mathbf{b} \rangle$ and $\langle \theta \rangle$ were derived using an ensemble based on 77 ascents taken during the summer months of 1965 at Columbia, Missouri, and were applied to the Mid Latitude Summer ensemble considered above. The resulting rms errors can be expressed in terms of the statistical parameters of both ensembles. Using the relation derived in the Appendix, the rms errors for temperatures, relative humidities, and total water vapor content were calculated for the case of an instrumental noise of 0.5 erg cm^{-2} sec^{-1} $ster^{-1}$ cm. Results for the temperatures and relative humidities are shown in Figure 6 along with the standard errors of estimate incurred using the exact estimation relations for the ensemble (from Figure 2). The rms error in the total water vapor content using the Columbia parameters is 0.51 g cm^{-2} or 18% of the ensemble mean while the standard error of estimate using the exact estimation relations is .44 g cm^{-2} or 15% of the ensemble mean.

Columbia and Wallops Island are at similar latitudes, but the two ensembles are somewhat different with the Columbia ensemble having a mean total water vapor content of 3.15 g cm^{-2} and a greater variance in both relative humidities and temperatures at most levels. However, the rms errors are only slightly larger than the standard errors of estimate for both relative humidities and temperatures.

SUMMARY AND DISCUSSION

A linear statistical estimation formulation has been developed for obtaining information on tropospheric temperatures, relative humidities, and total water vapor content from remote radiometric measurements. The information derivable from spectral measurements of the type obtainable from forthcoming satellite experiments was analyzed within the framework of the formulation. The construction of the ensembles was quite arbitrary and does not necessarily represent in any sense an optimum sampling for the calculation of parameters required in the statistical estimation relations. Rather, the ensembles were chosen as representative in order to provide some idea of the results which might be expected from the satellite measurements.

It has been found that the coefficients of multiple correlation vary considerably among the ensembles, being largest for those ensembles possessing the greatest variance. In the case of the Low Latitude Ensemble, the coefficients of multiple correlation (Figure 1) are considerably lower than for the other cases considered, but the temperature and humidity standard deviations reveal the ensemble is quite homogeneous (Table 2). In all of the ensembles, the coefficients of multiple correlation are strong functions of the instrumental noise variance. The resulting standard errors of estimate for temperature for the typical instrumental noise standard deviation of 0.5 erg sec^{-1} cm^{-2} $ster^{-1}$ cm are for the most part less than 2°K at the levels below 200 mb. The information obtainable on the surface temperature is found to be higher in all cases than that for any other single parameter. This reflects the fact that intensities in spectral intervals both in the "window" region and the wings of the absorption bands are strong functions of the surface temperature through the boundary term in (1).

For all ensembles, with the exception of that for high latitude winter, the coefficient of multiple correlation for relative humidity is greatest at the 400 mb level. This reflects the fact that for all of the spectral intervals used in the 6.3μ band the intensity of the upwelling radiation contains at least some contribution from atmospheric layers near the 400 mb level; therefore, measurements in all of these spectral intervals provide some information on the relative humidity near 400 mb. The other levels considered tend to be weighted less heavily. The high latitude winter ensemble shows low coefficients of multiple correlation for relative humidity and strong sensitivity to instrumental noise, which can be traced to the small temperature lapse rates for this ensemble. The fraction of the variance in total water vapor content which can be accounted for by the radiation measurements for the cases considered here varies from .65 to .83 for an instrumental noise of .5 erg cm^2 sec^{-1} $ster^{-1}$ cm.

Application of statistical estimation relations derived for one ensemble to a second ensemble (Figure 6) results in rms errors in temperature a maximum of 0.8°K greater than the standard errors of estimate obtained using the estimation relations derived from the second ensemble itself. At several levels the rms errors exceed the standard errors of estimate by only 0.3°K or less. Similarly, the rms errors in fractional relative humidity are less than .03 larger than the standard errors of estimate at all levels. While for similar latitudes and seasons, the two ensembles differ somewhat in detailed behavior.

The analyses have assumed cloud-free conditions in all cases. Unless one is content to utilize only those data from cloud-free situations, it will be necessary to cope with cases in which the instrumental field-of-view is partially cloud filled. Suggestions have been made for treating the problem by using additional information in the form of higher resolution spatial scans within the principal field-of-view (Smith, 1967; 1968a) or by using part of the spectral information (Smith, 1968b). Information on the clouds could also be incorporated directly into the statistical estimation formulation, at least in principle. In most cases, the satellite experiments considered here will be accompanied by other instrumentation such as television cameras and infrared radiometers which will provide cloud data. Any complete estimation formulation should utilize all of the available information.

The formulation considered provides one possible approach to the problem of estimating tropospheric temperature and moisture and serves as a framework for analyzing the information on these parameters contained in remote infrared measurements. Applications to representative ensembles, such as those given here, should provide a basis for the planning of research projects and applications using the satellite data.

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APPENDIX

Let us consider the application of a linear estimation relation to radiation data for a given ensemble. Instead of the statistical parameters H, $\langle \mathbf{b} \rangle$, and $\langle \theta \rangle$ of the ensemble, assume we employ another set of parameters K, $\mathbf{c}$ and $\boldsymbol{\phi}$ in our estimation formula which have been derived for a different ensemble. The estimation relation is then given by

$$\hat{\mathbf{b}} = \mathbf{c} + K(\boldsymbol{\theta} - \boldsymbol{\phi}) \tag{A1}$$

We would like to express the resulting rms error in terms of mean values and covariance matricies for the ensemble.

The rms error for each atmospheric parameter resulting from the estimation is given by the square root of the diagonal elements of the matrix

$$U = E\left\{(\hat{\mathbf{b}} - \mathbf{b})\,(\hat{\mathbf{b}} - \mathbf{b})^T\right\} \tag{A2}$$

Substitution of (A1) into (A2) gives

$$U = E\left\{\mathbf{b}\mathbf{b}^T\right\} + E\left\{\left[\mathbf{c} + K(\boldsymbol{\theta} - \boldsymbol{\phi})\right]\left[\mathbf{c} + K(\boldsymbol{\theta} - \boldsymbol{\phi})\right]^T\right\}$$
$$- E\left\{\mathbf{b}\mathbf{c}^T + \mathbf{b}(\boldsymbol{\theta} - \boldsymbol{\phi})^T K^T\right\} - E\left\{\mathbf{c}\mathbf{b}^T + K(\boldsymbol{\theta} - \boldsymbol{\phi})\,\mathbf{b}^T\right\} \tag{A3}$$

Rearranging and introducing the covariance matricies for the ensemble, we obtain

$$U = \Psi_{bb} + \left(\langle \mathbf{b}\rangle - \mathbf{c}\right)\left(\langle \mathbf{b}\rangle - \mathbf{c}\right)^T + K\left[\Psi_{\theta\theta} + \left(\langle\theta\rangle - \boldsymbol{\phi}\right)\left(\langle\theta\rangle - \boldsymbol{\phi}\right)^T\right] K^T$$

$$- K\left[\Psi_{b\theta}^T + \left(\langle\theta\rangle - \boldsymbol{\phi}\right)\left(\langle \mathbf{b}\rangle - \mathbf{c}\right)^T\right] - \left[\Psi_{b\theta} + \left(\langle \mathbf{b}\rangle - \mathbf{c}\right)\left(\langle\theta\rangle - \boldsymbol{\phi}\right)^T\right] K^T \quad . \qquad \text{(A4)}$$

Defining the matricies $\Gamma \equiv \left(\langle \mathbf{b}\rangle - \mathbf{c}\right)\left(\langle \mathbf{b}\rangle - \mathbf{c}\right)^T$, $\Lambda \equiv \left(\langle\theta\rangle - \boldsymbol{\phi}\right)\left(\langle\theta\rangle - \boldsymbol{\phi}\right)^T$, and $\Delta \equiv \left(\langle \mathbf{b}\rangle - \mathbf{c}\right)\left(\langle\theta\rangle - \boldsymbol{\phi}\right)^T$, (A4) becomes

$$U = \Psi_{bb} + \Gamma + K\left(\Psi_{\theta\theta} + \Lambda\right) - K\left(\Psi_{b\theta} + \Delta\right) - \left(\Psi_{b\theta} + \Delta\right)^T K^T \quad . \qquad \text{(A5)}$$

The required rms errors are given by $\sqrt{U_{ii}}$. The matricies Γ, Λ, and Δ appear because the estimate will not in general be unbiased $\left(\text{i.e.,} \langle \hat{\mathbf{b}}\rangle \neq \langle \mathbf{b}\rangle\right)$. Note that in the special case when $K = H$, $\mathbf{c} = \langle \mathbf{b}\rangle$, and $\boldsymbol{\phi} = \langle\theta\rangle$, U reduces to V (Equation (10), but in all other cases $\sqrt{U_{ii}}$ will be larger than the standard error of estimate $\sqrt{V_{ii}}$.

Table 1

Spectral Intervals Employed in the Analysis

Absorption Bands	Central Frequency (cm^{-1})	Bandwidth (cm^{-1})
15μ CO_2 and Rotational H_2O	690	5
	700	5
	723	5
	747	5
"Window," H_2O	920	40
6.3μ H_2O	1225	50
	1275	50
	1325	50
	1375	50
	1425	50
	1475	50

Table 2

Mean Temperatures $\langle T \rangle$, Mean Relative Humidities $\langle r \rangle$, Temperature Standard Deviations σ_T, and Relative Humidity Standard Deviations σ_r for Low Latitude Ensemble

Pressure (mb)	$\langle T \rangle$ (°K)	σ_T (°K)	$\langle r \rangle$	σ_r
100	196.0	2.42	.22	.06
150	205.8	1.36	.22	.06
200	218.1	1.09	.22	.06
300	238.7	1.90	.19	.06
400	253.6	1.75	.18	.06
500	266.4	2.03	.17	.09
700	281.6	1.50	.30	.13
850	289.9	1.33	.47	.08
Surface	298.5	1.54	.67	.06

Table 3

Mean Temperatures $\langle T \rangle$, Mean Relative Humidities $\langle r \rangle$, Temperature Standard Deviations σ_T, and Relative Humidity Standard Deviations σ_r for Mid Latitude Summer Ensemble

Pressure (mb)	$\langle T \rangle$ (°K)	σ_T (°K)	$\langle r \rangle$	σ_r
100	210.1	3.58	.30	.13
150	212.0	3.60	.30	.13
200	218.0	3.21	.30	.13
300	237.3	2.60	.30	.13
400	251.7	2.89	.31	.1[illegible]
500	263.7	2.59	.32	.1[illegible]
700	278.6	3.10	.45	.25
850	286.6	3.22	.60	.16
Surface	294.2	3.93	.81	.15

Table 4

Mean Temperatures $\langle T \rangle$, Mean Relative Humidities $\langle r \rangle$, Temperature Standard Deviations σ_T, and Relative Humidity Standard Deviations σ_r for Mid Latitude Winter Ensemble

Pressure (mb)	$\langle T \rangle$ (°K)	σ_T (°K)	$\langle r \rangle$	σ_r
100	211.6	6.18	.32	.13
150	214.8	6.55	.32	.13
200	217.2	7.14	.32	.13
300	228.9	5.63	.32	.13
400	242.8	5.68	.34	.15
500	254.3	5.64	.34	.20
700	269.2	5.80	.38	.22
850	275.4	6.64	.50	.22
Surface	281.2	7.03	.70	.19

Table 5

Mean Temperatures $\langle T \rangle$, Mean Relative Humidities $\langle r \rangle$, Temperature Standard Deviations σ_T, and Relative Humidity Standard Deviations σ_r for High Latitude Summer Ensemble

Pressure (mb)	$\langle T \rangle$ (°K)	σ_T (°K)	$\langle r \rangle$	σ_r
100	223.8	4.86	.28	.12
150	223.9	4.47	.28	.12
200	223.7	4.75	.28	.11
300	226.2	3.87	.33	.12
400	238.4	5.19	.38	.14
500	250.0	6.69	.40	.19
700	265.2	7.10	.52	.24
850	271.4	7.80	.63	.15
Surface	278.0	9.14	.75	.13

Table 6

Mean Temperatures $\langle T \rangle$, Mean Relative Humidities $\langle r \rangle$, Temperature Standard Deviations σ_T, and Relative Humidity Standard Deviations σ_r for High Latitude Winter Ensemble

Pressure (mb)	$\langle T \rangle$ (°K)	σ_T (°K)	$\langle r \rangle$	σ_r
100	218.6	5.11	.28	.12
150	219.3	4.35	.28	.12
200	219.2	4.12	.30	.12
300	219.8	3.54	.37	.12
400	228.6	5.04	.41	.14
500	237.6	6.01	.45	.19
700	250.6	6.88	.52	.25
850	251.0	7.83	.59	.15
Surface	251.3	9.87	.67	.12

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Figure 6. Standard errors of estimate S_T and S_r resulting from an application of self-consistant estimation relations to the Mid Latitude Summer Ensemble (Wallops Island) compared with rms errors Σ_T and Σ_r resulting from an application of estimation relations derived from a Columbia, Mo. ensemble. The results for both (a) temperature and (b) relative humidity are shown along with the a priori standard deviations σ_T and σ_r. An instrumental error of 0.5 erg cm^{-2} sec^{-1} $ster^{-1}$ cm was employed.

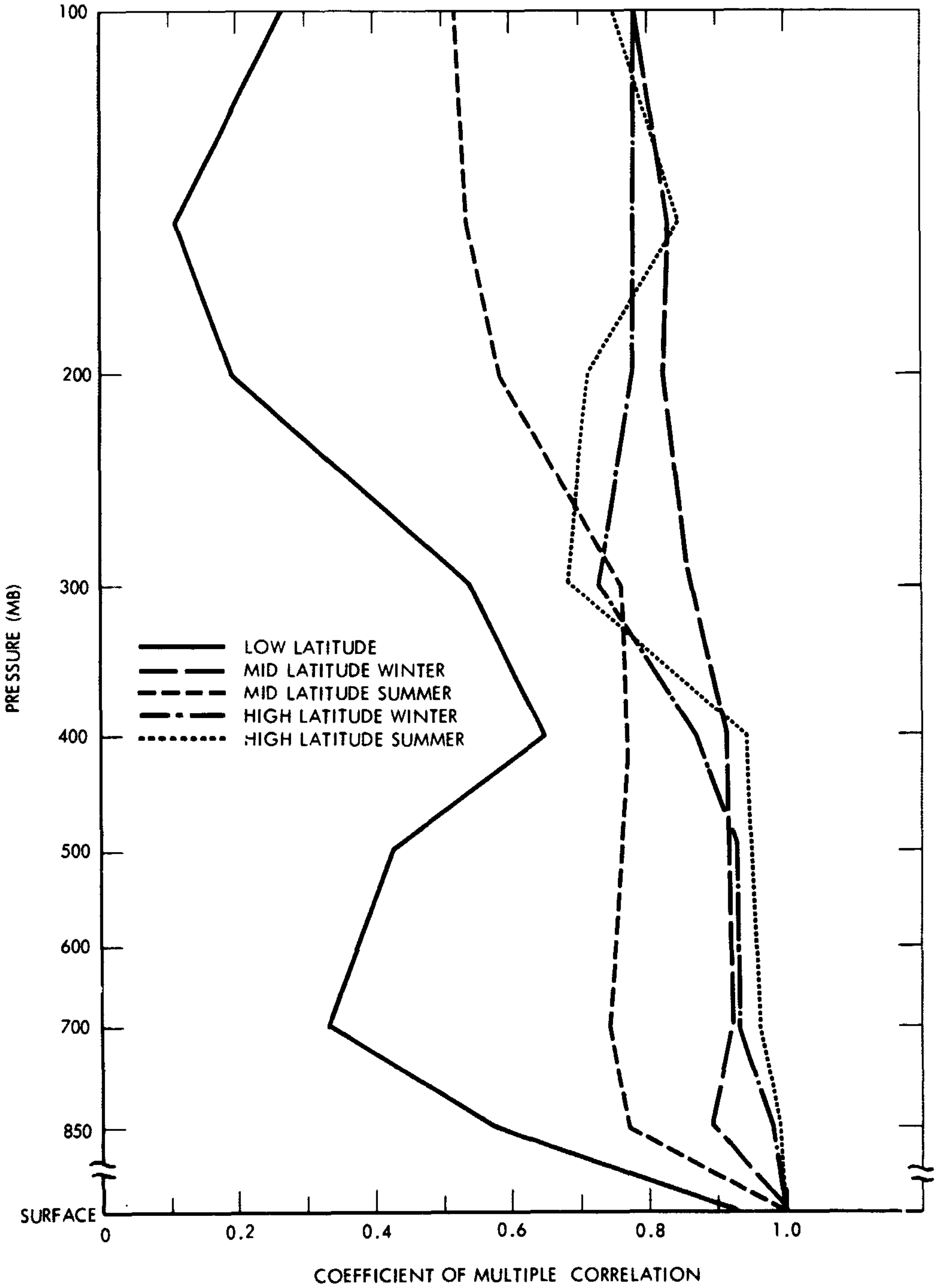

Figure 1a. Coefficients of multiple correlation as functions of atmospheric pressure level for temperature. The results were obtained from an application of the statistical estimation formulation to selected ensembles of atmospheric soundings, assuming an instrumental noise level of 0.5 erg cm^{-2} sec^{-1} $ster^{-1}$ cm.

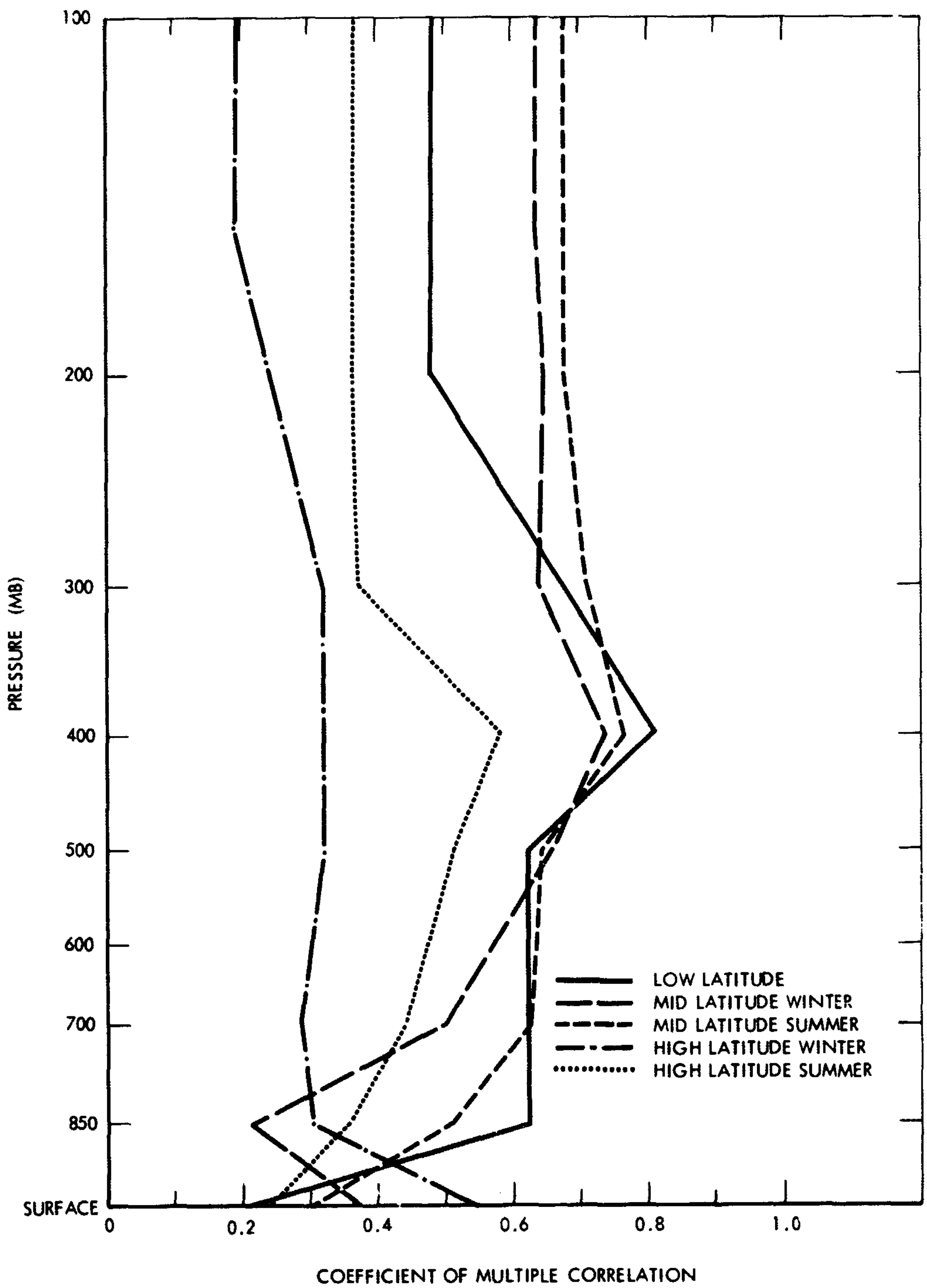

Figure 1b. Coefficients of multiple correlation as functions of atmospheric pressure level for relative humidity. The results were obtained from an application of the statistical estimation formulation to selected ensembles of atmospheric soundings, assuming an instrumental noise level of 0.5 erg cm^{-2} sec^{-1} $ster^{-1}$ cm.

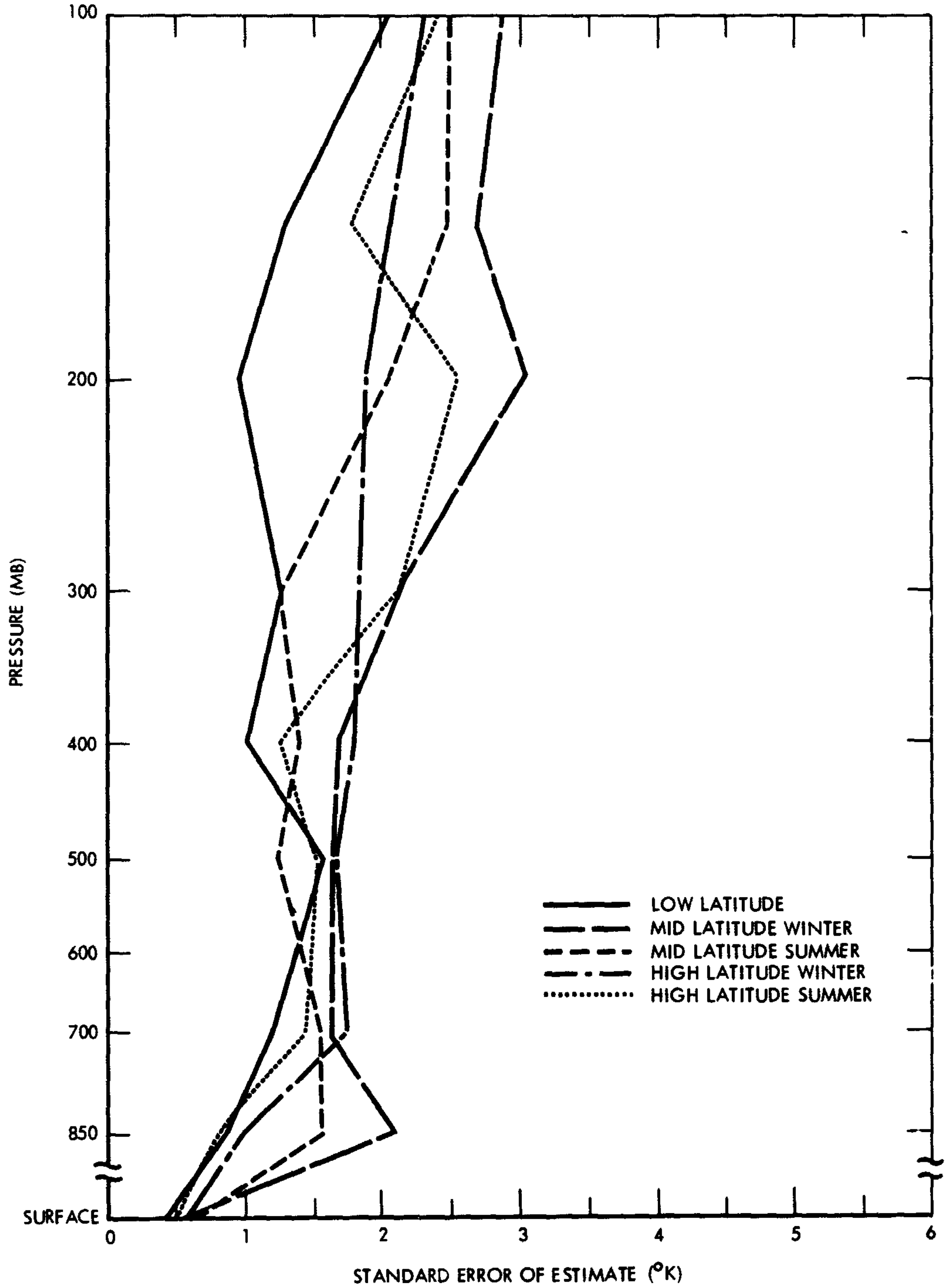

Figure 2a. Standard errors of estimate as functions of atmospheric pressure level for temperature. The results pertain to the same conditions as those of Figure 1.

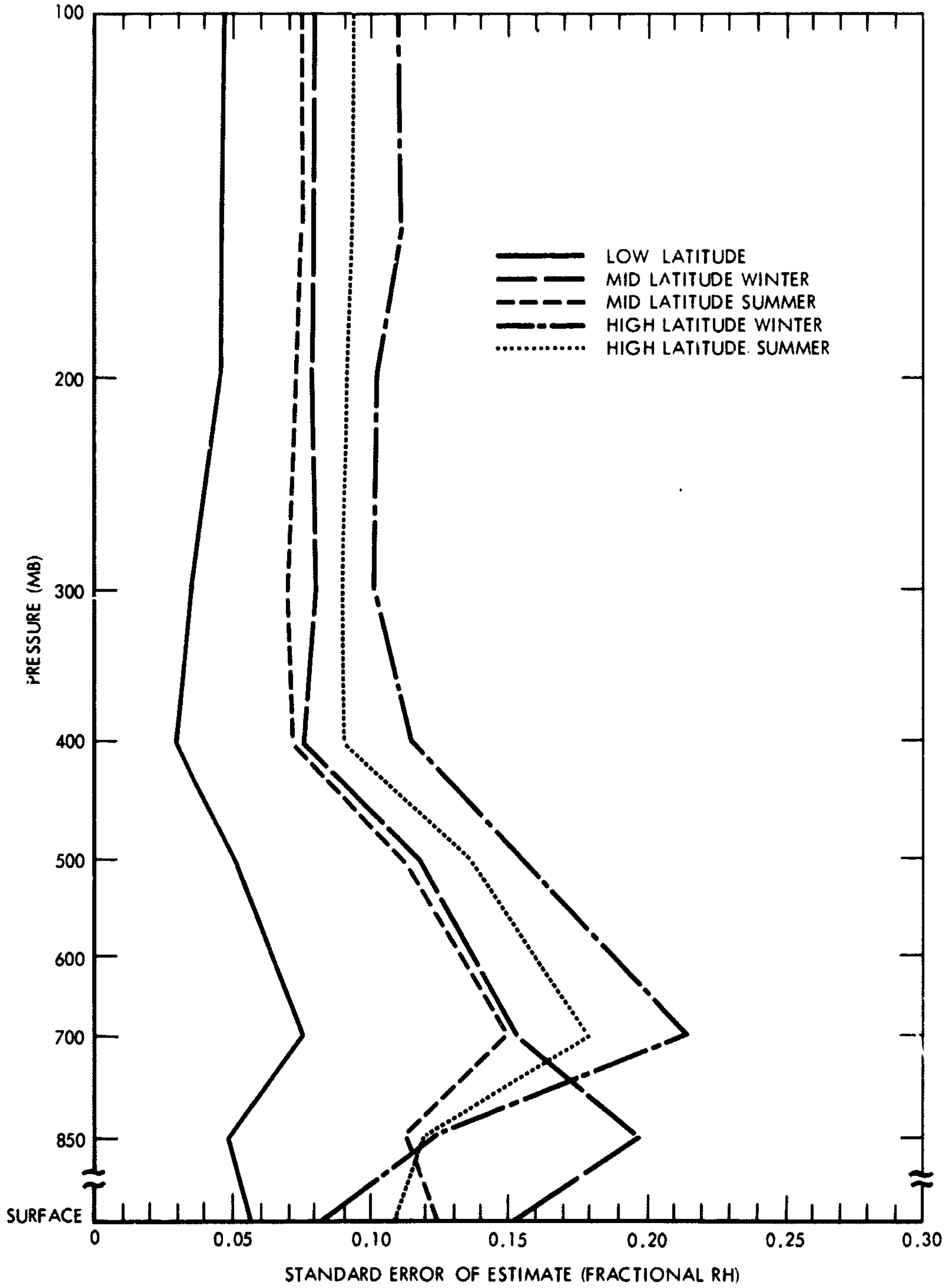

Figure 2b. Standard errors of estimate as functions of atmospheric pressure level for relative humidity. The results pertain to the same conditions as those of Figure 1.

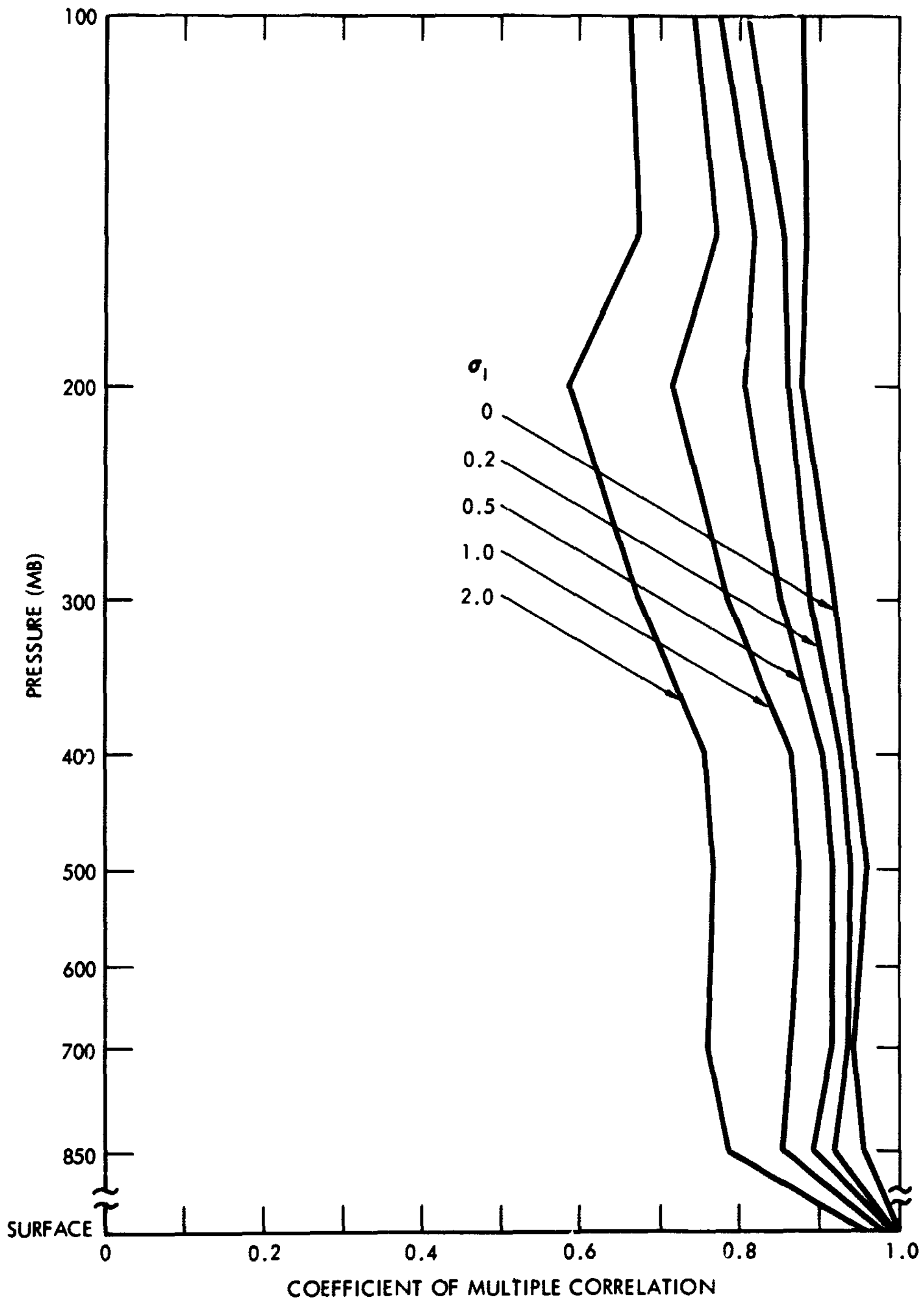

Figure 3a. Effects of instrumental noise on the coefficients of multiple correlation for temperature. A mid latitude winter ensemble of atmospheric soundings was employed. The values of instrumental noise σ_I are expressed in erg cm^{-2} sec^{-1} $ster^{-1}$ cm, and the curves labeled $\sigma_I = \infty$ are the ensemble standard deviations.

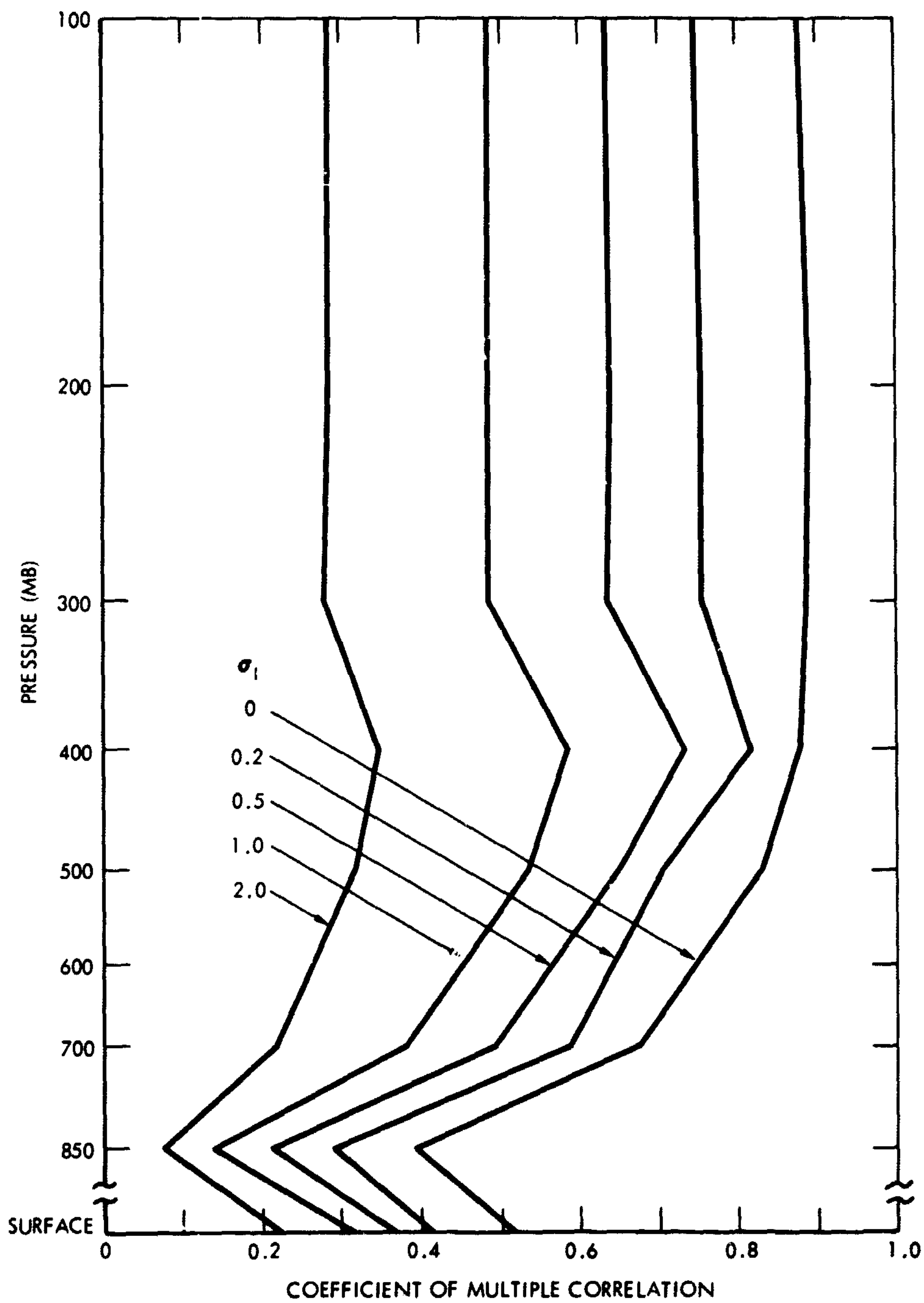

Figure 3b. Effects of instrumental noise on the coefficients of multiple correlation for relative humidity. A mid latitude winter ensemble of atmospheric soundings was employed. The values of instrumental noise σ_I are expressed in erg cm^{-2} sec^{-1} $ster^{-1}$ cm, and the curves labeled $\sigma_I = \infty$ are the ensemble standard deviations.

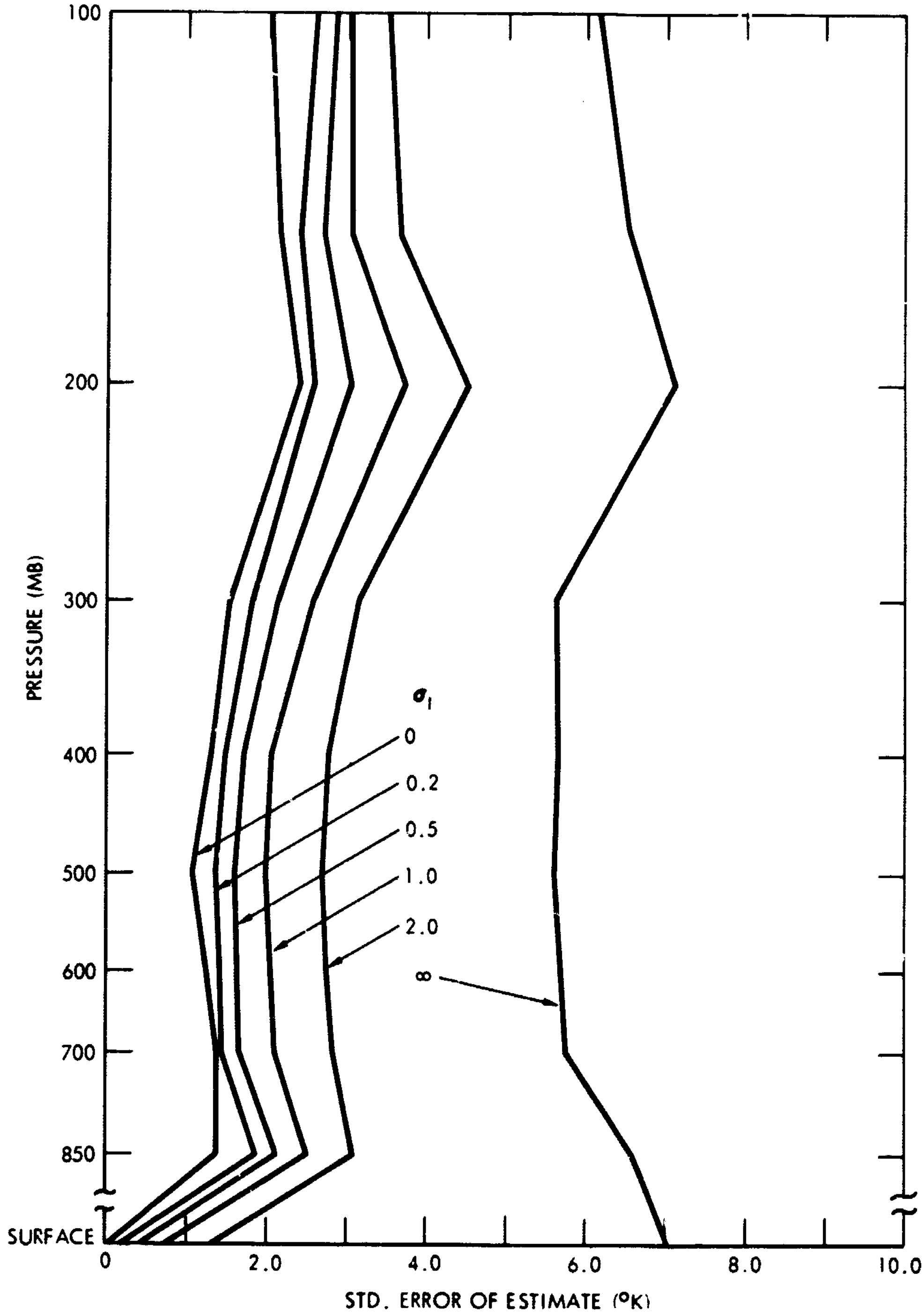

Figure 4a. Effects of the instrumental noise on the standard errors of estimate for temperature. The results pertain to the same conditions as those of Figure 3.

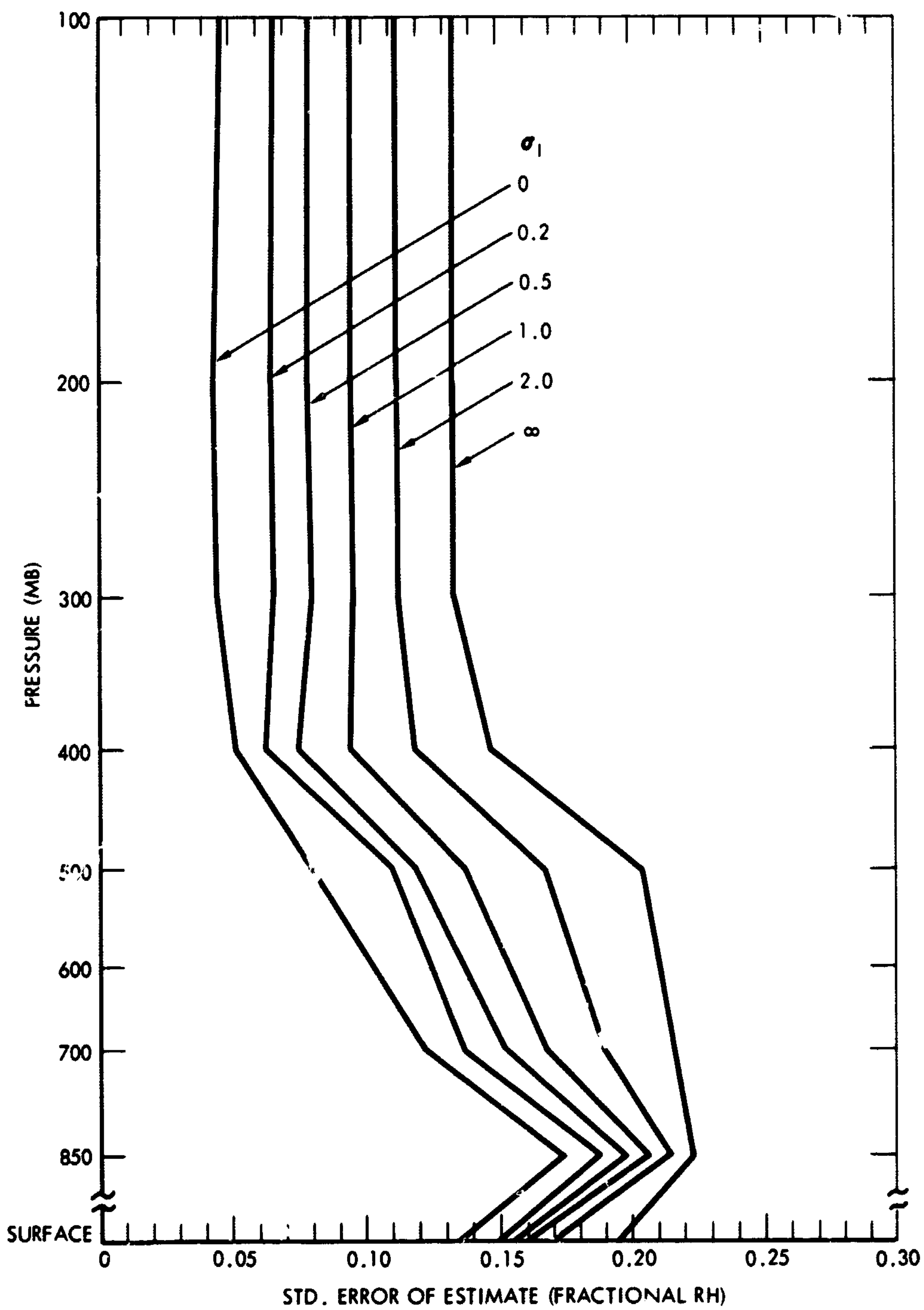

Figure 4b. Effects of the instrumental noise on the standard errors of estimate for relative humidity. The results pertain to the same conditions as those in Figure 3.

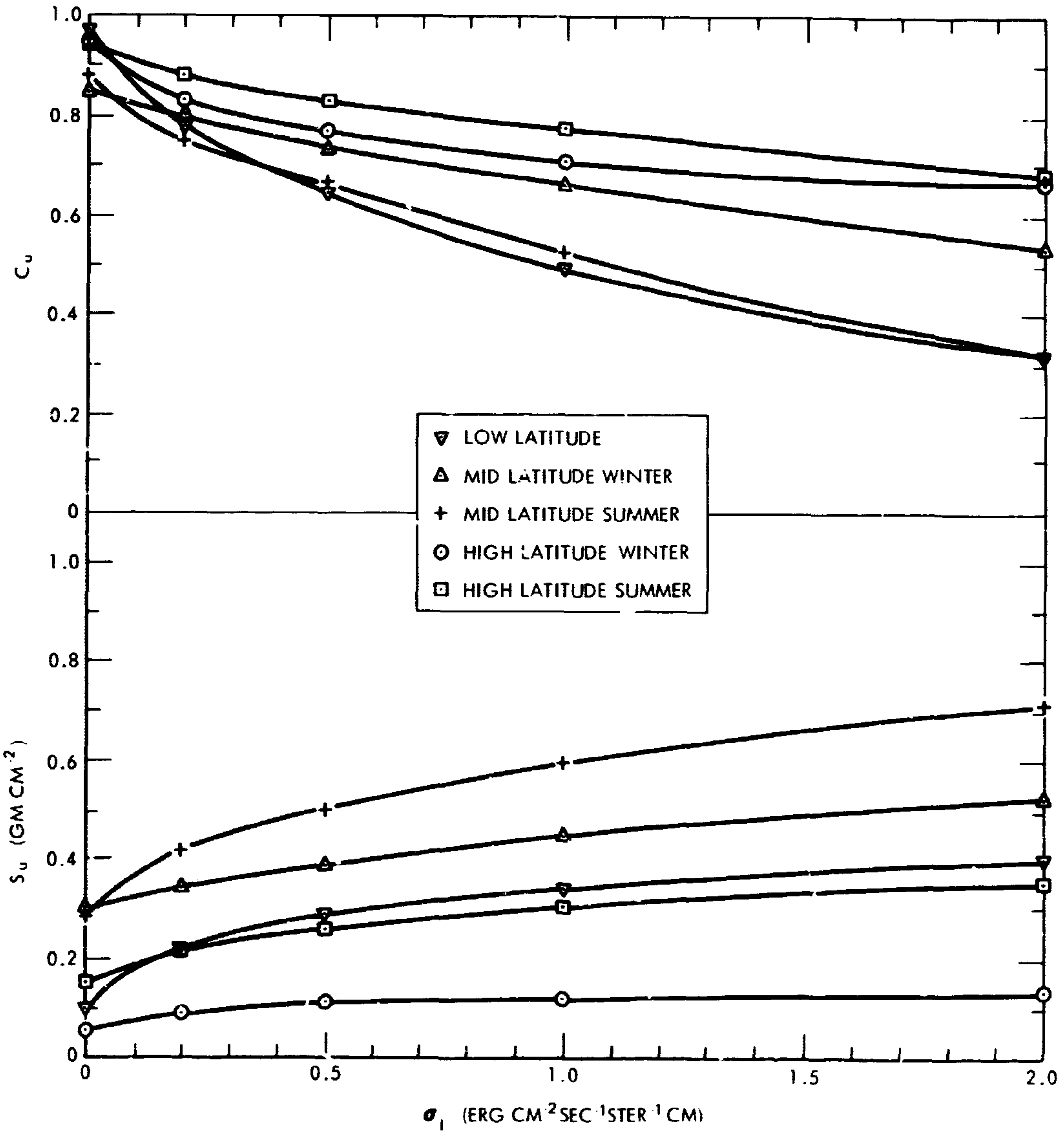

Figure 5. Statistical parameters as functions of instrumental noise σ_I obtained using the estimation formulation for total atmospheric water vapor content. The upper set of curves represents coefficients of multiple correlation C_u and the lower set represents standard errors of estimate S_u.

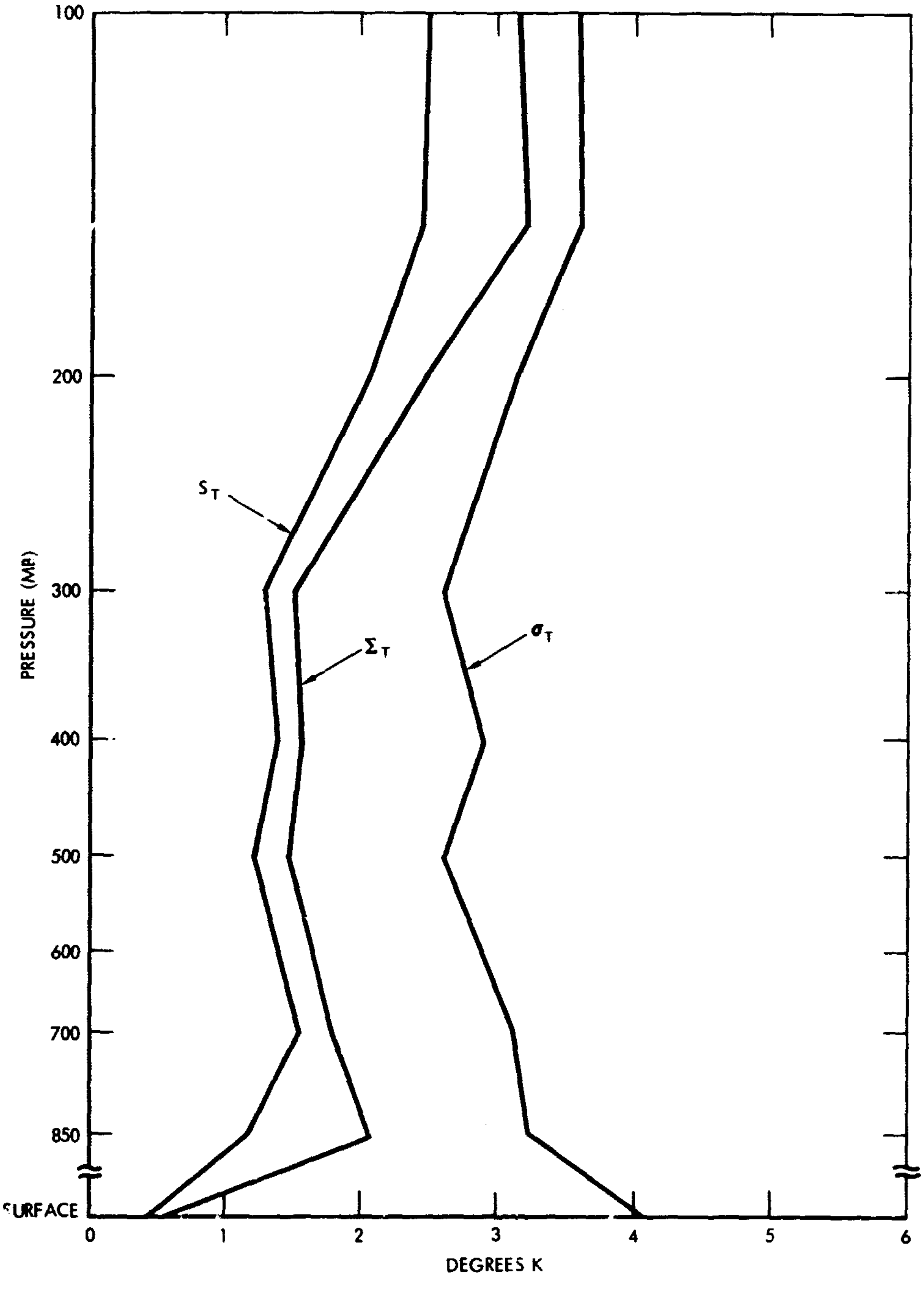

Figure 6a. Standard errors of estimate S_T and S_r resulting from an application of self-consistant estimation relations to the Mid Latitude Summer Ensemble (Wallops Island) compared with rms errors Σ_T and Σ_r resulting from an application of estimation relations derived from a Columbia, Mo. ensemble. The results for temperature are Shown along with the a priori standard deviations σ_T and σ_r. An instrumental error of 0.5 erg cm^{-2} sec^{-1} $ster^{-1}$ cm was employed.

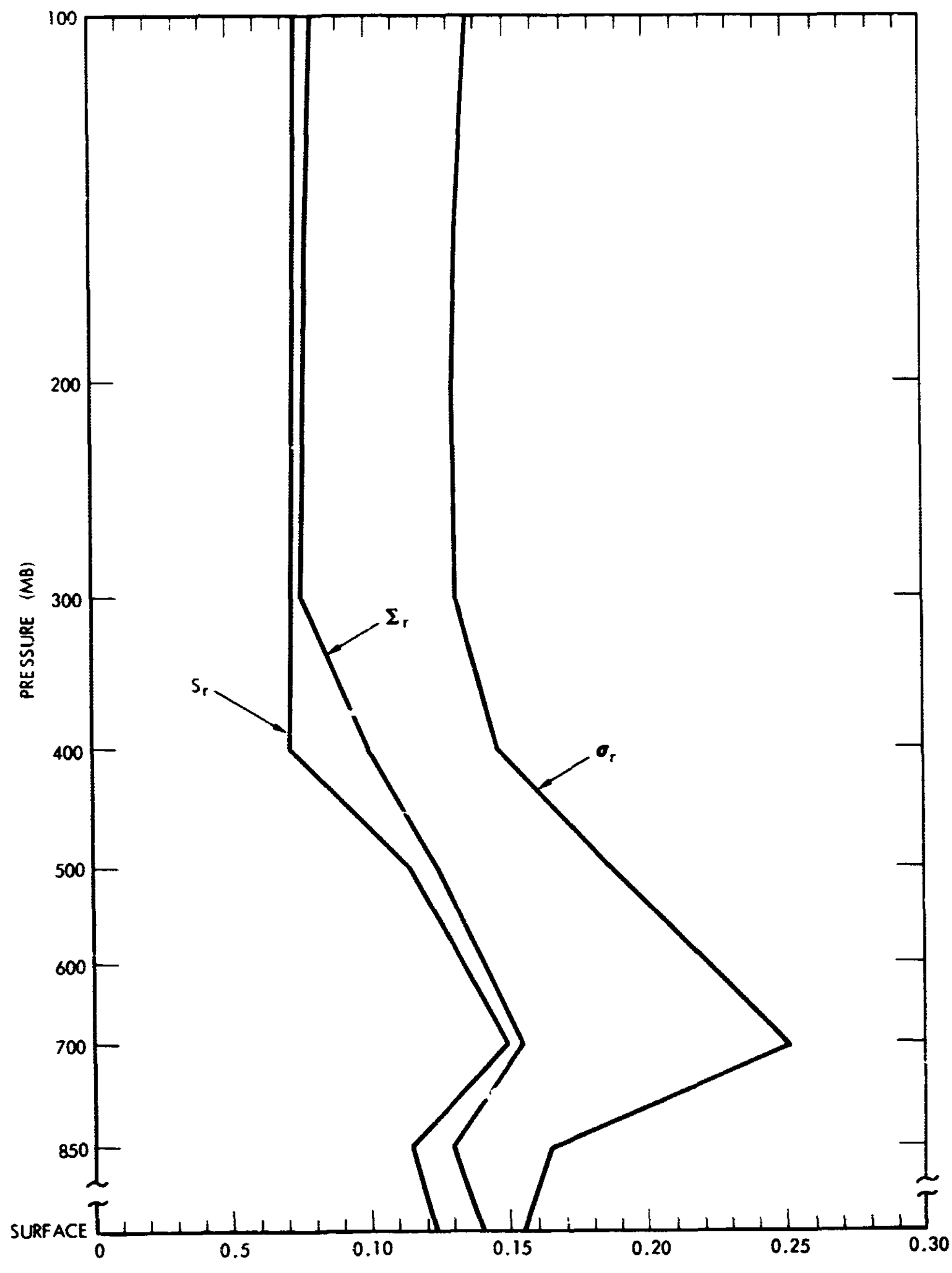

Figure 6b. Standard errors of estimate S_T and S_r resulting from an application of self-consistant estimation relations to the Mid Latitude Summer Ensemble (Wallops Island) compared with rms errors Σ_T and Σ_r resulting from an application of estimation relations derived from a Columbia, Mo. ensemble. The results for relative humidity are shown along with the a priori standard deviations σ_T and σ_r. An instrumental error of 0.5 erg cm^{-2} sec^{-1} $ster^{-1}$ cm was employed.