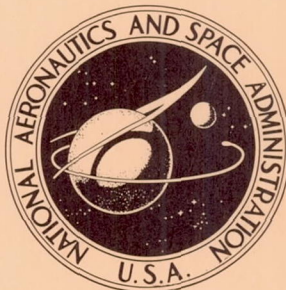


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FORTRAN PROGRAM FOR MACHINE  
COMPUTATION OF GROUP TABLES  
OF FINITE GROUPS

*by Gabriel Allen, David D. Evans, and Paul Swigert*

*Lewis Research Center  
Cleveland, Ohio*

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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## ABSTRACT

A FORTRAN program has been written for machine computation of group tables for finite groups. The listing presented is for use on the NASA Lewis Research Center IBM 360-67 and is machine dependent. The use of the method depends on the fact that every finite group of order  $n$  is isomorphic to some subgroup of the symmetric group  $S_n$ . The procedure for using the program is as follows: After the proper isomorphism has been established, the appropriate elements of  $S_n$  are entered into the program as input data. The program computes and prints out the group table for these elements of  $S_n$ . Then the translation is made from the  $S_n$  elements back to the original group elements. Two examples are shown: One is the entire group  $S_4$ , and the other is  $A_5$ , the even permutation subgroup of  $S_5$ .

# FORTRAN PROGRAM FOR MACHINE COMPUTATION OF GROUP TABLES OF FINITE GROUPS

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## SUMMARY

A FORTRAN program has been written for machine computation of group tables for finite groups. The listing presented is for use on the NASA Lewis Research Center IBM 360-67 and is machine dependent. The use of the method depends on the fact that every finite group of order  $n$  is isomorphic to some subgroup of the symmetric group  $S_n$ . The procedure for using the program is as follows: After the proper isomorphism has been established, the appropriate elements of  $S_n$  are entered into the program as input data. The program computes and prints out the group table for these elements of  $S_n$ . Then the translation is made from the  $S_n$  elements back to the original group elements. Two examples are shown: One is the entire group  $S_4$ , and the other is  $A_5$ , the even permutation subgroup of  $S_5$ .

## INTRODUCTION

Because of the sheer computational labor involved in constructing group tables for large finite groups, the possibility of programming such tasks for machine computation is very appealing. However, group multiplication cannot be carried out on a machine by simply inserting an \* between two elements of a group. A machine program for computing group tables is actually more closely related to some of the programs recently devised for proving mathematical theorems and algorithms than to ordinary arithmetic programming (refs. 1 to 3).

All of these programs, however, require the group table to be fed in as input, after which the group can be manipulated in any of several ways to extract the desired information about its structure and properties (ref. 3). It appears that the published literature still does not contain a specific program that begins only with group elements as input and then constructs actual group tables (see ref. 4 for an up-to-date survey of com-

puters in group theory). Since such a program would be very useful to anyone working with large finite groups, a program has been devised for constructing a group table or any subsection of such a table for the symmetric group  $S_n$ . This machine-dependent program, which is written in H level FORTRAN for the NASA Lewis Research Center IBM 360-67, uses as input only the group elements in the form of cycles. As is well known, every finite group of order  $n$  is isomorphic to some subgroup of  $S_n$  (ref. 5). Therefore, a table for any finite group  $G$  can be obtained from the table of the appropriate symmetric group once the isomorphism between the elements of  $G$  and those of the subgroup of  $S_n$  has been given.

Core storage limits the size of the multiplication table which can be handled by the program. Therefore, a subroutine which stores four 8-bit words in a single 32-bit storage location is used to gain more storage. This procedure is referred to as "packing" the storage locations.

Examples of output for  $S_4$  and  $A_5$  (the alternating group on five letters) are presented.

## DEFINITIONS AND CONVENTIONS

The entire report makes use of properties of the permutation group  $S_n$ . An element of the permutation group represents a rearrangement of a number of labeled objects among a number of labeled locations. To begin with, each object is assumed to be in the location which bears the same label as the object. The term standard configuration (SC) is used to describe that arrangement in which the  $i^{\text{th}}$  object is in the  $i^{\text{th}}$  location for  $i = 1$  to  $N$ , where  $N$  is the number of objects and/or locations. When  $N = 3$ , for example, SC means

|          |   |   |   |
|----------|---|---|---|
| Location | 1 | 2 | 3 |
| Object   | 1 | 2 | 3 |

Every group element in  $S_n$  can be written as a product of cycles. A cycle is a shorthand notation which describes the number and nature of changes effected by a given permutation. For example, the cycle (1, 3, 2) indicates that objects 1, 2, and 3 are to be permuted in some manner and that all other objects are to remain fixed. The notation is so economical that it cannot describe the nature of the permutation without some additional convention. Even though these conventions are well known, they are described here for convenience.

If the individual numbers or letters in each cycle are called units, then a unit may signify either a location or an object. Secondly, the convention may be adopted of going from left to right or from right to left within a cycle, so that there are four choices altogether. There are only two independent choices, however, since moving objects to the left must result in the same distribution of objects among locations as moving locations to the right and vice versa. The convention used here is that in which the units denote objects and each unit is moved to the location currently occupied by the unit to its left. An example should make this clear. Consider the effect of operation (1, 3, 2). When it operates on the SC, the result is

$$(1, 3, 2) \times \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 1 & 2 & 3 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 3 & 1 & 2 \\ \hline \end{array} \begin{array}{l} \text{Location label} \\ \text{Object label} \end{array}$$

When (1, 3, 2) operates on the configuration on the right side, the result is

$$(1, 3, 2) \times \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 3 & 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 2 & 3 & 1 \\ \hline \end{array}$$

An interchange of two units is called a transposition. A property of permutations which is very useful in programming group operations is that any permutation can be obtained by successive transpositions. For example, (1, 2, 3) = (1, 2)(2, 3). Therefore, a product of two group elements can be expressed as a single chain of transpositions. An examination of the result of this chain on the SC can then show the single group operation which would yield the same rearrangement. This group operation is then the product of the two original operations. The preceding few sentences are the heart of the method used herein.

## USE OF CLASS ALGEBRA TABLES

It is clear that, in the case of large finite groups, even the group table itself is difficult to use because of the sheer physical size of the table and the inherent difficulty of accurately locating the proper entry in a given row and column. It so happens that for most groups there is a useful way of breaking up the group table into smaller blocks which provides important information about the gross structure of the group. This method is simply to break up the group into what are called class product blocks. Let  $K_i$  denote the set of group elements in the  $i^{\text{th}}$  class of group  $G$ . Then  $K_I * K_J$  is

called the  $(IJ)^{\text{th}}$  class product block. Such a block is the table of group multiplication of all the elements of the  $J^{\text{th}}$  class used as column headings by all the elements in the  $I^{\text{th}}$  class used as row headings. The resulting gross structure of the group can be schematically summarized in the class algebra table (ref. 6) for the group.

A very simple example of such a breakup can be shown here by considering  $S_3$ . It has two classes in addition to the trivial class  $K_0$ , which consists entirely of the identity element  $E$ . Class  $K_2$  is defined here as the collection of cycles of length three. Thus,  $K_2$  consists of the two group elements  $(1, 2, 3)$  and  $(3, 2, 1)$ . The collection of two-unit cycles (or transpositions) makes up the third class. Thus,  $K_3$  consists of the elements  $(1, 2)$ ,  $(1, 3)$ , and  $(2, 3)$ . Therefore, the gross structure of the 36 entry group table for  $S_3$  can be shown by the nine-element class algebra table:

|       | $K_0$ | $K_2$        | $K_3$         |
|-------|-------|--------------|---------------|
| $K_0$ | $K_0$ | $K_2$        | $K_3$         |
| $K_2$ | $K_2$ | $2K_0 + K_2$ | $2K_3$        |
| $K_3$ | $K_3$ | $2K_3$       | $3K_0 + 3K_2$ |

In examining this table, it should be noted that an entry such as  $3K_0 + 3K_2$  in the class product block (33) means that, in this block, every element in class  $K_2$  and in class  $K_0$  appears exactly three times.

The five class product blocks  $K_0 * K_0$ ,  $K_0 * K_2$ ,  $K_0 * K_3$ ,  $K_2 * K_0$ , and  $K_3 * K_0$  consist merely of the three classes themselves. Since, for any two classes  $K_i$  and  $K_j$ ,  $K_i * K_j = K_j * K_i$ , all the additional information about the gross structure of  $S_3$  can be obtained from the three class product blocks  $K_2 * K_2$ ,  $K_3 * K_3$ , and  $K_2 * K_3$ .

### Examples

Before examining the output of some actual cases, it should be noted that the form in which these tables appears is slightly more efficient than the conventional group table. A conventional group table for a group of order  $n$  containing the group elements  $E, g_2, \dots, g_n$  has the form

|       |       |       |         |       |
|-------|-------|-------|---------|-------|
|       | $E$   | $g_2$ | $\dots$ | $g_n$ |
| $E$   | $E$   | $g_2$ | $\dots$ | $g_n$ |
| $g_2$ | $g_2$ | $X$   | $\dots$ | $X$   |
| .     | .     | .     | $\dots$ | .     |
| .     | .     | .     |         | .     |
| .     | .     | .     |         | .     |
| $g_n$ | $g_n$ | $X$   |         | $X$   |

The first row and first column merely repeat the row and column headings and are, in a sense, redundant. The form of the tables in this program omits the first row and first column of the group table. Of course, the entries still "appear" on the page, but only as row or column headings.

Example 1 -  $S_4$ . - This group is small enough so that the entire group table can be printed out in a reasonable space. Therefore, the entire set of 24 group operations was entered as data and the group table constructed. The output for this group follows.





|             |            |            |            |            |            |            |            |
|-------------|------------|------------|------------|------------|------------|------------|------------|
| (1,2,4):    | (1,3,2,4): | (4,3,2,1): | (1,2,3,4): | (3,4):     | (1,2):     | (1,3):     | (2,4):     |
| {4,2,1):    | {3,4,2,1)  | {4,3)      | {4,2,3,1)  | {2,4,3,1)  | {4,1)      | {3,2,4,1)  | {2,1)      |
| {2,3,4):    | {3,1)      | {2,4,3,1)  | {3,2)      | {4,3,2,1)  | {4,2)      | {3,4,2,1)  | {4,1)      |
| {4,3,2):    | {2,3,4,1)  | {3,4,2,1)  | {3,2,4,1)  | {3,2)      | {3,4,2,1)  | {4,2,3,1)  | {4,3)      |
| {1,4,3):    | {3,2)      | {3,2,4,1)  | {2,1)      | {4,2)      | {4,3,2,1)  | {2,4,3,1)  | {3,2)      |
| {3,4,1):    | {4,3,2,1)  | {3,2)      | {2,4,3,1)  | {4,1)      | {2,4,3,1)  | {4,3)      | {4,2,3,1)  |
| {3,2,1):    | {2,4,3,1)  | {4,2,3,1)  | {4,3)      | {3,1)      | {3,2)      | {2,1)      | {3,4,2,1)  |
| {1,2,3):    | {4,2)      | {4,1)      | {3,4,2,1)  | {2,3,4,1)  | {3,1)      | {2,1)      | {3,2,4,1)  |
| {1,4)(2,3): | {2,1)      | {4,2)      | {4,2)      | {4,2,3,1)  | {3,2,4,1)  | {2,3,4,1)  | {4,3,2,1)  |
| {1,2)(3,4): | {4,2,3,1)  | {3,1)      | {4,2)      | {2,1)      | {4,3)      | {4,3,2,1)  | {2,3,4,1)  |
| {1,3)(2,4): | {4,3)      | {2,3,4,1)  | {4,3,2,1)  | {3,2,4,1)  | {4,2,3,1)  | {4,2)      | {3,1)      |
| {1,3,4,2):  | {4,3,1)    | {2,3,1)    | {4,3,2)    | {3,2,1)    | {3,4,2)    | {4,2,1)    | {3,4,1)    |
| {2,4,3,1):  | {3,4,2)    | {3,4,1)    | {4,2,1)    | {2,4,1)    | {4,3,1)    | {4,3,2)    | {2,3,1)    |
| {4,2,3,1):  | F          | {2,4,1)    | {3,2,1)    | {4,1)(3,2) | {3,1)(4,2) | {3,4,2)    | {4,3,1)    |
| {1,3,2,4):  | {2,1)(4,3) | {3,4,2)    | {4,3,1)    | {3,1)(4,2) | {4,1)(3,2) | {2,4,1)    | {3,2,1)    |
| {4,3,2,1):  | {2,3,1)    | {3,1)(4,2) | E          | {4,2,1)    | {4,3,2)    | {2,1)(4,3) | {4,1)(3,2) |
| {1,2,3,4):  | {4,2,1)    | E          | {3,1)(4,2) | {2,3,1)    | {3,4,1)    | {4,1)(3,2) | {2,1)(4,3) |
| {3,4):      | {4,1)(3,2) | {3,2,1)    | {2,4,1)    | E          | {2,1)(4,3) | {4,3,1)    | {3,4,2)    |
| {1,2):      | {3,1)(4,2) | {4,3,1)    | {3,4,2)    | {2,1)(4,3) | {3,4,1)    | {3,2,1)    | {2,4,1)    |
| {1,3):      | {4,3,2)    | {4,1)(3,2) | {2,1)(4,3) | {4,3,2)    | {4,2,1)    | {3,1)(4,2) | E          |
| {2,4):      | {3,4,1)    | {2,1)(4,3) | {4,1)(3,2) | {4,3,1)    | {2,4,1)    | {3,4,1)    | {4,2,1)    |
| {1,4):      | {3,2,1)    | {4,3,2)    | {2,3,1)    | {4,3,1)    | {3,2,1)    | {2,3,1)    | {4,3,2)    |
| {2,3):      | {2,4,1)    | {4,2,1)    | {3,4,1)    | {3,4,2)    |            |            |            |

|             |            |            |
|-------------|------------|------------|
| (1,2,4):    | (1,4):     | (2,3):     |
| {4,2,1):    | {4,2)      | {2,3,4,1)  |
| {2,3,4):    | {2,1)      | {4,2,3,1)  |
| {4,3,2):    | {2,3,4,1)  | {4,2)      |
| {1,4,3):    | {3,2,4,1)  | {4,3)      |
| {3,4,1):    | {3,1)      | {4,3,2,1)  |
| {3,2,1):    | {4,3)      | {3,2,4,1)  |
| {1,2,3):    | {4,3,2,1)  | {3,1)      |
| {1,4)(2,3): | {4,2,3,1)  | {2,1)      |
| {1,2)(3,4): | {3,2)      | {4,1)      |
| {1,3)(2,4): | {3,4,2,1)  | {2,4,3,1)  |
| {1,3,4,2):  | {2,4,3,1)  | {3,4,2,1)  |
| {2,4,3,1):  | {2,1)(4,3) | {3,1)(4,2) |
| {4,2,3,1):  | {3,1)(4,2) | {2,1)(4,3) |
| {1,3,2,4):  | {2,3,1)    | {4,2,1)    |
| {4,3,2,1):  | {4,3,2)    | {3,4,1)    |
| {1,2,3,4):  | {3,2,1)    | {4,3,1)    |
| {3,4):      | {3,4,2)    | {2,4,1)    |
| {1,2):      | {3,4,1)    | {4,3,2)    |
| {1,3):      | {4,2,1)    | {2,3,1)    |
| {2,4):      | {4,3,1)    | {3,2,1)    |
| {1,4):      | {2,4,1)    | {3,4,2)    |
| {2,3):      | E          | {4,1)(3,2) |
|             | {4,1)(3,2) | E          |

Example 2 -  $A_5$ . - The group  $A_5$  is the group of even permutations of  $S_5$ . It is, therefore, a proper subgroup of  $S_5$ . There are five classes of  $A_5$  including  $K_0$ . The elements of  $S_5$  which are expressible as five-unit cycles fall into two distinct classes of 12 group elements each. These will be called  $K_2$  and  $K_3$ . The 20 elements expressible as three unit cycles form a class called  $K_4$ , and the 15 elements which may be expressed as products of two independent transpositions (e. g. , (12) (35)) are in the class  $K_5$ . One should not be confused by the fact that the 24 five unit cycles of  $S_5$  (which fall into a single class of  $S_5$ ) fall into two distinct classes in  $A_5$ . It requires conjugation by an odd permutation of  $S_5$  to take an element of  $K_2$  into  $K_3$  and vice versa. The only group elements available for conjugation in  $A_5$  are the even permutations of  $S_5$ , and no conjugation of any element in  $K_2$  by such an even permutation can take this element into  $K_3$ .

Since  $A_5$  is of order 60, it is a good choice for breaking up the group table into class product blocks. The program can readily handle such blocks. However, class products do not have the closure property and elements can arise which are not included in the input for either class in the class product block. Therefore, this example serves to illustrate the warning that IDENT (see program description) has to be made long enough to accommodate the longest chain of changes in locations which any element of the full group can cause. Note, for instance, that whereas the class  $K_4$  consists of cycles three units long, the class product block for  $K_4 * K_4$  contains cycles five units long. The following class algebra table is not readily available in the literature and was obtained from the class product blocks computed by this program:

| $K_0$ | $K_2$                       | $K_3$                       | $K_4$                               | $K_5$                               |
|-------|-----------------------------|-----------------------------|-------------------------------------|-------------------------------------|
| $K_2$ | $12K_0 + 5K_2 + K_3 + 3K_4$ | $K_2 + K_3 + 3K_4 + 4K_5$   | $5K_2 + 5K_3 + 3K_4 + 4K_5$         | $5K_3 + 3K_4 + 4K_5$                |
| $K_3$ | $K_2 + K_3 + 3K_4 + 4K_5$   | $12K_0 + K_2 + 5K_3 + 3K_4$ | $5K_2 + 5K_3 + 3K_4 + 4K_5$         | $5K_2 + 3K_4 + 4K_5$                |
| $K_4$ | $5K_2 + 5K_3 + 3K_4 + 4K_5$ | $5K_2 + 5K_3 + 3K_4 + 4K_5$ | $20K_0 + 5K_2 + 5K_3 + 7K_4 + 8K_5$ | $5K_2 + 5K_3 + 6K_4 + 4K_5$         |
| $K_5$ | $5K_3 + 3K_4 + 4K_5$        | $5K_2 + 3K_4 + 4K_5$        | $5K_2 + 5K_3 + 6K_4 + 4K_5$         | $15K_0 + 5K_2 + 5K_3 + 3K_4 + 2K_5$ |

Of course, the class algebra table only gives gross information about the structure of the group. For example, although  $K_2 * K_4 = K_4 * K_2$ , both class product blocks are needed since one cannot predict the details of block  $K_2 * K_4$  from  $K_4 * K_2$  or vice versa. Therefore, all 16 nontrivial class product blocks were obtained and are shown in the following pages.

GROUP (1,2,3,4,5)  
K2\*K2

EXAMPLE 2 - A5

|              |              |              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| (1,2,3,4,5): | (1,2,3,4,5): | (1,2,5,3,4): | (1,4,5,2,3): | (1,4,2,3,5): | (1,3,5,4,2): | (1,3,4,2,5): | (5,4,3,2,1): |
| (1,2,5,3,4): | (3,5,2,4,1)  | (3,5,4,2,1)  | (5,3,2,4,1)  | (5,2,4,3,1)  | (4,3,1)      | (4,3,5,2,1)  | E            |
| (1,4,5,2,3): | (5,2,4,3,1)  | (5,4,2,3,1)  | (4,3,2)      | (4,5,2)      | (4,5,1)      | (4,5,2,3,1)  | (3,5,1)      |
| (1,4,5,2,3): | (3,5,4,2,1)  | (3,5,1)      | (5,3,4,2,1)  | (5,4,3,2,1)  | (4,3,2)      | (5,4,3)      | (2,4,1)      |
| (1,4,5,3,5): | (3,2,5,4,1)  | (3,2,1)      | (2,5,4,3,1)  | (2,5,4,3,1)  | (5,2,4,3,1)  | (5,4,3,2,1)  | (4,5,2)      |
| (1,3,5,4,2): | (5,3,2)      | (4,3,2)      | (2,5,1)      | (2,5,3,4,1)  | (5,2,3,4,1)  | (5,3,2,4,1)  | (4,5,2,3,1)  |
| (1,3,5,4,2): | (5,3,2,4,1)  | (5,4,3,2,1)  | (2,4,1)      | (2,4,5,3,1)  | (4,5,2,3,1)  | (4,5,3,2,1)  | (3,5,2)      |
| (5,4,3,2,1): | E            | (4,5,2)      | (3,5,1)      | (3,4,1)      | (2,5,3,4,1)  | (2,4,1)      | (4,2,5,3,1)  |
| (5,2,1,4,3): | (5,4,2)      | E            | (3,4,2,5,1)  | (3,2,5,4,1)  | (5,3,2,4,1)  | (5,4,1)      | (2,4,5,3,1)  |
| (5,4,1,3,2): | (5,3,1)      | (5,2,4,3,1)  | E            | (4,5,3)      | (2,3,4,5,1)  | (2,4,5,3,1)  | (4,2,3,5,1)  |
| (5,3,2,4,1): | (4,3,1)      | (4,5,2,3,1)  | (5,4,3)      | E            | (2,5,1)      | (2,3,1)      | (3,4,2,5,1)  |
| (5,3,1,2,4): | (4,3,5,2,1)  | (4,2,3,5,1)  | (5,4,3,2,1)  | (5,2,1)      | E            | (3,5,2)      | (3,4,1)      |
| (5,2,4,3,1): | (4,2,1)      | (4,5,1)      | (3,5,4,2,1)  | (3,2,1)      | (5,3,2)      | E            | (2,5,3,4,1)  |

|              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|
| (1,2,3,4,5): | (5,2,1,4,3): | (5,4,1,3,2): | (5,3,2,4,1): | (5,3,1,2,4): | (5,2,4,3,1): |
| (1,2,5,3,4): | (5,3,1)      | (4,2,1)      | (5,4,2)      | (3,2,5,4,1)  | (5,3,2)      |
| (1,4,5,2,3): | E            | (4,2,3,5,1)  | (3,5,4,2,1)  | (5,4,3,2,1)  | (3,2,1)      |
| (1,4,5,2,3): | (5,3,2,4,1)  | E            | (2,5,1)      | (3,4,2,5,1)  | (2,5,3,4,1)  |
| (1,4,2,3,5): | (2,4,5,3,1)  | (5,2,1)      | E            | (3,4,1)      | (4,5,3)      |
| (1,3,5,4,2): | (2,3,4,5,1)  | (5,2,4,3,1)  | (4,3,1)      | E            | (4,5,1)      |
| (1,3,4,2,5): | (2,3,1)      | (4,3,5,2,1)  | (5,4,3)      | (5,4,1)      | E            |
| (5,4,3,2,1): | (3,4,2,5,1)  | (2,4,5,3,1)  | (4,5,2,3,1)  | (3,5,2)      | (4,2,3,5,1)  |
| (5,2,1,4,3): | (3,2,4,5,1)  | (5,3,1)      | (2,3,1)      | (3,4,2)      | (2,3,4,5,1)  |
| (5,4,1,3,2): | (3,4,2)      | (2,4,3,5,1)  | (4,3,5,2,1)  | (5,2,1)      | (4,2,1)      |
| (5,3,2,4,1): | (5,4,2)      | (2,3,4,5,1)  | (3,4,5,2,1)  | (4,3,5,2,1)  | (3,5,4,2,1)  |
| (5,3,1,2,4): | (5,4,1)      | (3,4,2)      | (3,4,2,5,1)  | (4,3,2,5,1)  | (3,2,5,4,1)  |
| (5,2,4,3,1): | (2,2,5,4,1)  | (4,5,3)      | (2,3,4,5,1)  | (4,2,3,5,1)  | (2,3,5,4,1)  |

GROUP (1,2,3,4,5)  
K3\*K3

|              |              |              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| (1,3,5,2,4): | (1,3,5,2,4): | (1,5,4,2,3): | (1,5,3,4,2): | (1,2,5,4,3): | (1,5,2,3,4): | (1,4,5,3,2): | (5,3,1,4,2): |
| (1,5,4,2,3): | (5,4,3,2,1)  | (2,5,1)      | (2,3,1)      | (4,5,1)      | (2,5,4,3,1)  | (3,4,2)      | E            |
| (1,5,3,4,2): | (4,5,3)      | (4,3,5,2,1)  | (4,3,2,5,1)  | (3,5,2,4,1)  | (4,5,3,2,1)  | (2,5,1)      | (2,4,3,5,1)  |
| (1,2,5,4,3): | (3,4,2)      | (3,5,2,4,1)  | (3,2,5,4,1)  | (3,5,2)      | (3,2,4,5,1)  | (2,5,4,3,1)  | (2,3,5,4,1)  |
| (1,5,2,3,4): | (4,5,3,2,1)  | (2,4,3,5,1)  | (4,5,1)      | (5,3,2,4,1)  | (4,2,1)      | (3,5,1)      | (3,2,4,5,1)  |
| (1,4,5,3,2): | (2,5,1)      | (3,4,1)      | (3,5,2,4,1)  | (3,5,1)      | (2,4,5,3,1)  | (5,4,2)      | (5,4,3)      |
| (5,3,1,4,2): | E            | (3,4,5,2,1)  | (3,5,2,4,1)  | (3,4,2)      | (3,5,1)      | (5,2,4,3,1)  | (5,2,3,4,1)  |
| (5,4,1,3,2): | (2,1)(4,3)   | (4,5,1)      | (3,2,4,5,1)  | (5,2,3,4,1)  | (3,2,1)      | (2,4,3,5,1)  | (2,3,4,5,1)  |
| (5,1,2,4,3): | (5,4,2,3,1)  | (5,3,2)      | E            | (4,5,3,2,1)  | (4,3,1)      | (3,5,2)      | (4,5,2)      |
| (5,2,1,3,4): | (4,3,2,5,1)  | (2,4,1)      | (2,3,5,4,1)  | E            | (5,4,2)      | (3,4,1)      | (3,2,1)      |
| (5,1,4,3,2): | (2,3,1)      | (4,5,3)      | (4,5,2)      | (5,3,4,2,1)  | E            | (5,4,2,3,1)  | (5,4,1)      |
| (5,4,1,2,3): | (5,3,4,2,1)  | (4,3,2,5,1)  | (4,3,1)      | (3,2,4,5,1)  | (4,2,5,3,1)  | (3,5,2,4,1)  | (3,4,5,2,1)  |

|              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|
| (1,3,5,2,4): | (5,4,1,3,2): | (5,1,2,4,3): | (5,2,1,3,4): | (5,1,4,3,2): | (5,4,1,2,3): |
| (1,5,4,2,3): | (5,1)(4,3)   | (4,5,3,2,1)  | (5,4,2,3,1)  | (4,5,3)      | (4,3,2,5,1)  |
| (1,5,3,4,2): | (4,5,2)      | (3,4,1)      | (5,3,2)      | (2,4,1)      | (3,4,5,2,1)  |
| (1,2,5,4,3): | (4,5,2,3,1)  | E            | (4,3,2,5,1)  | (2,3,1)      | (4,5,2)      |
| (1,2,5,4,3): | (4,2)(5,3)   | (5,2,3,4,1)  | E            | (3,5,2,4,1)  | (5,3,4,2,1)  |
| (1,5,2,3,4): | (4,5,1)      | (3,2,1)      | (4,2,5,3,1)  | E            | (3,2,4,5,1)  |
| (1,4,5,3,2): | (2,3,1)      | (5,4,2)      | (2,4,3,5,1)  | (5,4,2,3,1)  | E            |
| (5,3,1,4,2): | (3,5,2)      | (5,4,1)      | (4,3,2)      | (4,3,2)      | (2,3,5,4,1)  |
| (5,4,1,3,2): | (2,4,3,5,1)  | (5,3,4,2,1)  | (2,3,1)      | (4,2)(5,3)   | (5,1)(4,3)   |
| (5,1,2,4,3): | (5,3,4,2,1)  | (4,5,2,3,1)  | (5,4,1)      | (3,4,5,2,1)  | (4,2,5,3,1)  |
| (5,2,1,3,4): | (4,3,1)      | (5,3,2)      | (4,2,3,5,1)  | (5,3,1)      | (4,3,2)      |
| (5,1,4,3,2): | (2,1)(5,3)   | (5,4,2,3,1)  | (2,4,1)      | (3,5,4,2,1)  | (5,3,1)      |
| (5,4,1,2,3): | (5,1)(4,2)   | (3,4,5,2,1)  | (5,3,1)      | (4,5,2)      | (3,4,2,5,1)  |

GROUP (1,2,3,4,5)  
K4\*K4

|          |             |             |             |             |             |             |             |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| (1,2,3): | (1,2,3):    | (2,3,4):    | (1,3,4):    | (1,2,4):    | (1,2,5):    | (1,3,5):    | (2,3,5):    |
| (2,3,4): | (3,2,1)     | (2,1)(4,3)  | (3,4,2)     | (3,1)(4,2)  | (3,1)(5,2)  | (3,5,2)     | (2,1)(5,3)  |
| (1,3,4): | (3,1)(4,2)  | (4,3,2)     | (4,1)(3,2)  | (3,4,1)     | (3,4,2,5,1) | (4,2,3,5,1) | (4,2)(5,3)  |
| (1,2,4): | (2,4,1)     | (3,1)(4,2)  | (4,3,1)     | (2,1)(4,3)  | (2,5,3,4,1) | (4,1)(5,3)  | (3,5,2,4,1) |
| (1,2,5): | (4,1)(3,2)  | (2,3,1)     | (3,1)(4,2)  | (4,2,1)     | (5,1)(4,2)  | (4,1)(5,2)  | (2,3,5,4,1) |
| (1,3,5): | (5,1)(3,2)  | (2,3,4,5,1) | (3,4,2,5,1) | (5,1)(4,3)  | (2,4,3,5,1) | (3,1)(5,2)  | (2,3,1)     |
| (2,3,5): | (2,5,1)     | (3,4,2,5,1) | (5,1)(4,3)  | (3,5,2,4,1) | (2,1)(5,3)  | (5,3,1)     | (3,1)(5,2)  |
| (1,4,5): | (3,1)(5,2)  | (4,2,3,5,1) | (3,5,1)     | (3,5,2,4,1) | (3,5,1)     | (5,1)(3,2)  | (5,3,2)     |
| (2,4,5): | (2,3,4,5,1) | (3,5,2)     | (3,5,2,4,1) | (2,5,1)     | (2,1)(5,4)  | (3,1)(5,4)  | (4,5,2,3,1) |
| (3,4,5): | (4,5,2,3,1) | (4,2)(5,3)  | (4,1)(5,3)  | (4,1)(5,2)  | (4,5,1)     | (3,2,4,5,1) | (3,2)(5,4)  |
| (3,2,1): | (2,4,5,3,1) | (3,4,1)     | (2,1)(4,3)  | (2,5,3,4,1) | (2,3,4,5,1) | (4,5,1)     | (4,5,2)     |
| (4,3,2): | E           | E           | (2,4,1)     | (4,1)(3,2)  | (5,3,2)     | (2,1)(5,3)  | (3,5,1)     |
| (4,3,1): | (4,3,1)     | (4,2,1)     | E           | (2,3,1)     | (4,3,2,5,1) | (2,4,3,5,1) | (5,4,3)     |
| (4,2,1): | (2,1)(4,3)  | (4,1)(3,2)  | (3,2,1)     | (4,5,2)     | (2,5,4,3,1) | (5,4,3)     | (4,3,5,2,1) |
| (5,2,1): | (3,4,2)     | (5,2,3,4,1) | (4,5,3)     | (2,4,5,3,1) | E           | (3,5,4,2,1) | (5,1)(3,2)  |
| (5,3,1): | (3,5,2)     | (5,3,4,2,1) | (5,3,4,1)   | (5,3,2,4,1) | (2,3,1)     | (5,2,1)     | E           |
| (5,3,2): | (2,1)(5,3)  | (4,5,3)     | (2,5,3,4,1) | (2,1)(5,4)  | (2,4,1)     | (3,4,1)     | (5,2,3,4,1) |
| (5,4,1): | (5,3,1)     | (5,4,2,3,1) | (3,1)(5,4)  | (5,4,1)     | (5,1)(4,2)  | (3,4,2,5,1) | (3,4,2)     |
| (5,4,2): | (2,3,5,4,1) | (3,2)(5,4)  | (3,2,5,4,1) | (2,3,5,4,1) | (2,4,3,5,1) | (5,1)(4,3)  | (5,2)(4,3)  |
| (5,4,3): | (2,5,4,3,1) | (5,4,2)     | (5,4,1)     | (2,3,5,4,1) | (2,4,3,5,1) |             |             |
|          |             |             |             |             |             |             |             |
| (1,2,3): | (1,4,5):    | (2,4,5):    | (3,4,5):    | (3,2,1):    | (4,3,2):    | (4,3,1):    | (4,2,1):    |
| (2,3,4): | (4,5,2,3,1) | (2,4,5,3,1) | (2,3,4,5,1) | E           | (2,4,1)     | (4,1)(3,2)  | (4,3,1)     |
| (1,3,4): | (2,3,4,5,1) | (4,5,3)     | (3,2)(5,4)  | (4,2,1)     | E           | (2,3,1)     | (2,1)(4,3)  |
| (1,2,4): | (4,5,3)     | (3,4,5,2,1) | (3,1)(5,4)  | (4,1)(3,2)  | (3,2,1)     | E           | (3,4,2)     |
| (1,2,5): | (4,5,2)     | (2,1)(5,4)  | (2,4,5,3,1) | (3,4,1)     | (2,1)(4,3)  | (4,3,2)     | E           |
| (1,3,5): | (4,1)(5,2)  | (2,4,1)     | (2,5,3,4,1) | (3,5,1)     | (2,4,3,5,1) | (4,3,2,5,1) | (4,5,1)     |
| (2,3,5): | (4,1)(5,3)  | (3,5,2,4,1) | (3,4,1)     | (5,1)(3,2)  | (3,2,4,5,1) | (4,5,1)     | (4,2,3,5,1) |
| (1,4,5): | (4,2,3,5,1) | (4,2)(5,3)  | (3,4,2)     | (5,2,1)     | (4,5,2)     | (4,5,2,3,1) | (4,3,5,2,1) |
| (2,4,5): | (5,4,1)     | (4,1)(5,2)  | (4,1)(5,3)  | (3,2,4,5,1) | (4,3,2,5,1) | (5,1)(4,3)  | (5,1)(4,2)  |
| (3,4,5): | (5,1)(4,2)  | (5,4,2)     | (4,2)(5,3)  | (3,4,5,2,1) | (5,2)(4,3)  | (5,2,4,3,1) | (5,2,1)     |
| (3,2,1): | (5,1)(4,3)  | (5,2)(4,3)  | (5,4,1)     | (3,4,1)     | (5,3,2)     | (5,3,1)     | (5,3,4,2,1) |
| (4,3,2): | (4,5,3,2,1) | (3,2,4,5,1) | (3,4,2,5,1) | (2,3,1)     | (3,1)(4,2)  | (4,2,1)     | (4,1)(3,2)  |
| (4,3,1): | (3,2,4,5,1) | (3,2)(5,4)  | (4,5,2)     | (2,1)(4,3)  | (3,4,2)     | (3,1)(4,2)  | (3,2,1)     |
| (4,2,1): | (3,1)(5,4)  | (4,5,2,3,1) | (4,5,1)     | (4,3,2)     | (4,1)(3,2)  | (3,4,1)     | (3,1)(4,2)  |
| (5,2,1): | (2,1)(5,4)  | (4,5,1)     | (4,5,3,2,1) | (3,1)(4,2)  | (4,3,1)     | (2,1)(4,3)  | (2,4,1)     |
| (5,3,1): | (4,2,1)     | (5,1)(4,2)  | (5,3,4,2,1) | (3,1)(5,2)  | (5,2,4,3,1) | (4,3,5,2,1) | (4,1)(5,2)  |
| (5,3,2): | (4,3,1)     | (5,2,4,3,1) | (5,1)(4,3)  | (5,3,2)     | (5,3,2,4,1) | (4,1)(5,3)  | (4,2,5,3,1) |
| (5,4,1): | (4,3,2,5,1) | (4,3,2)     | (5,2)(4,3)  | (2,1)(5,3)  | (4,2)(5,3)  | (4,2,5,3,1) | (4,5,3,2,1) |
| (5,4,2): | F           | (5,2,1)     | (5,3,1)     | (3,2,5,4,1) | (5,4,3)     | (5,4,3)     | (5,4,2)     |
| (5,4,3): | (2,5,1)     | E           | (5,3,2)     | (3,5,4,2,1) | (5,4,3)     | (2,5,4,3,1) | (2,1)(5,4)  |
|          | (3,5,1)     | (3,5,2)     | E           | (5,4,3,2,1) | (3,2)(5,4)  | (3,1)(5,4)  | (3,5,4,2,1) |
|          |             |             |             |             |             |             |             |
| (1,2,3): | (5,2,1):    | (5,3,1):    | (5,3,2):    | (5,4,1):    | (5,4,2):    | (5,4,3):    |             |
| (2,3,4): | (5,3,1)     | (5,1)(3,2)  | (2,5,1)     | (5,4,2,3,1) | (2,5,4,3,1) | (2,3,5,4,1) |             |
| (1,3,4): | (5,3,4,2,1) | (5,4,2,3,1) | (5,4,2)     | (5,2,3,4,1) | (3,2)(4,3)  | (3,5,2)     |             |
| (1,2,4): | (5,2,3,4,1) | (5,4,1)     | (3,2,5,4,1) | (5,1)(4,3)  | (3,4,2,5,1) | (3,5,1)     |             |
| (1,2,5): | (5,4,1)     | (5,3,2,4,1) | (2,5,3,4,1) | (5,1)(4,2)  | (2,5,1)     | (2,4,3,5,1) |             |
| (1,3,5): | E           | (5,3,2)     | (2,1)(5,3)  | (5,4,2)     | (2,1)(5,4)  | (2,5,4,3,1) |             |
| (2,3,5): | (3,5,2)     | E           | (3,2,1)     | (5,4,3)     | (5,4,4,2,1) | (3,1)(5,4)  |             |
| (1,4,5): | (2,1)(5,3)  | (2,3,1)     | E           | (2,3,5,4,1) | (5,4,3)     | (3,2)(5,4)  |             |
| (1,4,5): | (4,5,2)     | (4,5,3)     | (4,5,3,2,1) | E           | (4,2,1)     | (4,3,1)     |             |
| (2,4,5): | (2,1)(5,4)  | (2,4,5,3,1) | (4,5,3)     | (2,4,1)     | E           | (4,3,1)     |             |
| (3,4,5): | (3,4,5,2,1) | (3,1)(5,4)  | (3,2)(5,4)  | (3,4,1)     | (3,4,2)     | (4,3,2)     |             |
| (3,2,1): | (5,1)(3,2)  | (5,2,1)     | (3,1)(5,2)  | (3,4,1)     | (3,4,2)     | E           |             |
| (4,3,2): | (5,4,3,2,1) | (5,2,4,3,1) | (5,2)(4,3)  | (5,4,3,2,1) | (3,2,5,4,1) | (3,5,4,2,1) |             |
| (4,3,1): | (5,2,4,3,1) | (5,1)(4,3)  | (4,3,2,5,1) | (5,3,1)     | (4,2,5,3,1) | (4,1)(5,3)  |             |
| (4,2,1): | (5,1)(4,2)  | (5,3,4,2,1) | (4,2,5,3,1) | (5,2,1)     | (4,1)(5,2)  | (4,3,5,2,1) |             |
| (5,2,1): | (2,5,1)     | (2,1)(5,3)  | (5,3,1)     | (2,1)(5,4)  | (5,4,1)     | (5,4,3,2,1) |             |
| (5,3,1): | (3,1)(5,2)  | (3,5,1)     | (5,1)(3,2)  | (3,1)(5,4)  | (5,4,2,3,1) | (5,4,1)     |             |
| (5,3,2): | (3,2,1)     | (3,1)(5,2)  | (3,5,2)     | (3,2,5,4,1) | (3,2)(5,4)  | (5,4,2)     |             |
| (5,4,1): | (4,1)(5,2)  | (4,1)(5,3)  | (5,3,2,4,1) | (4,5,1)     | (5,1)(4,2)  | (5,1)(4,3)  |             |
| (5,4,2): | (4,2,1)     | (4,2,5,3,1) | (4,2)(5,3)  | (4,1)(5,2)  | (4,5,2)     | (5,2)(4,3)  |             |
| (5,4,3): | (4,3,5,2,1) | (4,3,1)     | (4,3,2)     | (4,1)(5,3)  | (4,2)(5,3)  | (4,5,3)     |             |



GROUP (1,2,3,4,5)  
K2\*K3

|              |              |              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| (1,2,3,4,5): | (1,3,5,2,4): | (1,5,4,2,3): | (1,5,3,4,2): | (1,2,5,4,3): | (1,5,2,3,4): | (1,4,5,3,2): | (5,3,1,4,2): |
| (1,2,5,3,4): | (4,2,5,3,1): | (4,3,2)      | (5,4,3)      | (3,2,1)      | (4,2)(5,3)   | (5,4,1)      | (5,4,3,2,1)  |
| (1,4,5,2,3): | (5,3,2)      | (3,2,4,5,1)  | (3,1)(5,4)   | (5,1)(3,2)   | (3,1)(4,2)   | (5,4,3)      | (3,2)(5,4)   |
| (1,4,2,3,5): | (5,3,1)      | (2,1)(4,3)   | (2,4,3,5,1)  | (3,4,1)      | (2,1)(5,3)   | (5,1)(4,2)   | (5,1)(4,3)   |
| (1,3,5,4,2): | (5,1)(4,3)   | (5,2)(4,3)   | (4,3,2)      | (4,3,2)      | (5,3,2)      | (2,4,1)      | (2,1)(4,3)   |
| (1,3,4,2,5): | (4,3,1)      | (4,1)(5,2)   | (4,1)(3,2)   | (4,5,2)      | (4,3,2,5,1)  | (2,3,1)      | (2,3,1)      |
| (5,4,3,2,1): | (2,3,4,5,1)  | (4,1)(5,3)   | (3,2)(5,4)   | (5,2,1)      | (4,3,2)      | (2,3,5,4,1)  | (2,1)(5,4)   |
| (5,2,1,4,3): | (5,1)(3,2)   | (2,5,3,4,1)  | (4,1)(5,2)   | (4,2)(5,3)   | (4,5,1)      | (3,1)(5,2)   | (3,5,2,4,1)  |
| (5,4,1,3,2): | (2,1)(4,3)   | (4,5,1)      | (2,4,1)      | (4,5,3)      | (4,3,1)      | (3,1)(4,2)   | (3,4,1)      |
| (5,3,2,4,1): | (2,1)(5,4)   | (3,5,1)      | (3,1)(5,2)   | (4,2,3,5,1)  | (3,1)(5,4)   | (5,2)(4,3)   | (3,5,2)      |
| (5,3,1,2,4): | (5,4,2)      | (3,2,1)      | (3,5,1)      | (4,1)(3,2)   | (3,5,4,2,1)  | (5,1)(4,3)   | (5,1)(3,2)   |
| (5,2,4,3,1): | (3,2)(5,4)   | (2,1)(5,3)   | (2,5,1)      | (4,1)(5,3)   | (2,1)(5,4)   | (3,4,2,5,1)  | (3,5,1)      |
| (1,2,3,4,5): | (5,4,1,3,2): | (5,1,2,4,3): | (5,2,1,3,4): | (5,1,4,3,2): | (5,4,1,2,3): |              |              |
| (1,2,5,3,4): | (4,2,1)      | (3,1)(5,2)   | (4,1)(5,3)   | (5,2,1)      | (3,1)(4,2)   |              |              |
| (1,4,5,2,3): | (4,2,3,5,1)  | (5,2,1)      | (4,3,1)      | (3,5,2)      | (5,1)(4,2)   |              |              |
| (1,4,2,3,5): | (5,2,1)      | (3,2,5,4,1)  | (4,2)(5,3)   | (5,4,1)      | (3,2,1)      |              |              |
| (1,3,5,4,2): | (2,3,4,5,1)  | (3,1)(5,4)   | (5,3,2,4,1)  | (2,1)(5,4)   | (3,1)(5,2)   |              |              |
| (1,3,4,2,5): | (4,3,5,2,1)  | (4,5,3)      | (5,1)(3,2)   | (2,4,5,3,1)  | (5,2)(4,3)   |              |              |
| (5,4,3,2,1): | (2,4,5,3,1)  | (5,3,1)      | (4,1)(3,2)   | (2,1)(5,3)   | (5,2,4,3,1)  |              |              |
| (5,2,1,4,3): | (5,3,1)      | (3,4,2)      | (2,5,1)      | (3,1)(4,2)   | (4,5,3)      |              |              |
| (5,4,1,3,2): | (2,4,3,5,1)  | (3,2)(5,4)   | (5,1)(4,2)   | (3,1)(5,4)   | (5,3,2)      |              |              |
| (5,3,2,4,1): | (5,4,2)      | (3,2,4,2,1)  | (2,3,1)      | (4,2)(5,3)   | (5,1)(4,3)   |              |              |
| (5,3,1,2,4): | (2,3,4,5,1)  | (4,2,1)      | (2,5,4,3,1)  | (3,4,2)      | (4,5,1)      |              |              |
| (5,2,4,3,1): | (4,5,3)      | (4,1)(5,2)   | (5,4,3)      | (5,2,3,4,1)  | (4,2,1)      |              |              |
|              |              | (4,1)(3,2)   | (5,4,2)      | (3,4,1)      | (4,5,3,2,1)  |              |              |

GROUP (1,2,3,4,5)  
K2\*K4

|              |             |             |             |             |             |             |             |
|--------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| (1,2,3,4,5): | (1,2,3):    | (2,3,4):    | (1,3,4):    | (1,2,4):    | (1,2,5):    | (1,3,5):    | (2,3,5):    |
| (1,2,5,3,4): | (5,2,3,4,1) | (2,4,3,5,1) | (4,2,3,5,1) | (3,4,2,5,1) | (3,4,5,2,1) | (3,4,5,2,1) | (4,5,2,3,1) |
| (1,4,5,2,3): | (5,3,2,4,1) | (2,4,5,3,1) | (4,2,5,3,1) | (5,3,4,2,1) | (5,2,3,4,1) | (4,1)(5,2)  | (2,4,1)     |
| (1,4,2,3,5): | (3,4,5,2,1) | (3,4,5,2,1) | (3,5,2)     | (3,1)(5,2)  | (3,1)(5,4)  | (3,2)(5,4)  | (4,5,3,2,1) |
| (1,3,5,4,2): | (3,4,2,5,1) | (4,3,2,5,1) | (5,1)(3,2)  | (3,5,1)     | (3,5,4,2,1) | (5,4,2,3,1) | (4,2,5,3,1) |
| (1,3,4,2,5): | (5,4,2)     | (5,4,3,2,1) | (5,4,3)     | (5,4,3)     | (4,2)(5,3)  | (5,3,4,2,1) | (3,4,2,5,1) |
| (5,4,3,2,1): | (5,1)(4,2)  | (3,2,4,5,1) | (4,3,2,5,1) | (5,1)(4,3)  | (5,3,4,2,1) | (2,1)(4,3)  | (5,1)(4,3)  |
| (5,2,1,4,3): | (5,4,3)     | (5,4,1)     | (2,1)(5,4)  | (3,2)(5,4)  | (4,3,2)     | (2,1)(4,3)  | (3,4,1)     |
| (5,4,1,3,2): | (5,2)(4,3)  | (4,1)(5,2)  | (5,2,1)     | (3,5,2)     | (5,4,3)     | (5,4,3,2,1) | (4,3,2,5,1) |
| (5,3,2,4,1): | (5,4,1)     | (3,1)(5,4)  | (2,5,4,3,1) | (5,4,3,2,1) | (5,3,2,4,1) | (2,5,3,4,1) | (3,4,1)     |
| (5,3,1,2,4): | (4,1)(5,3)  | (5,3,1)     | (2,4,5,3,1) | (4,5,3,2,1) | (4,1)(3,2)  | (2,4,1)     | (5,4,1)     |
| (5,2,4,3,1): | (4,5,3,2,1) | (2,1)(5,3)  | (4,2)(5,3)  | (4,2,5,3,1) | (4,5,2,3,1) | (4,5,2)     | (2,1)(5,4)  |
|              | (4,3,5,2,1) | (5,2,1)     | (4,5,2)     | (4,5,2,3,1) | (4,3,1)     | (4,3,2)     | (5,4,3,2,1) |
| (1,2,3,4,5): | (1,4,5):    | (2,4,5):    | (3,4,5):    | (3,2,1):    | (4,3,2):    | (4,3,1):    | (4,2,1):    |
| (1,2,5,3,4): | (5,2,3,4,1) | (2,5,3,4,1) | (2,3,5,4,1) | (4,5,1)     | (2,5,1)     | (5,1)(3,2)  | (5,1)(4,3)  |
| (1,4,5,2,3): | (5,3,2,4,1) | (2,1)(4,3)  | (2,5,4,3,1) | (4,2)(5,3)  | (2,1)(5,3)  | (5,3,2)     | (4,5,3)     |
| (1,4,2,3,5): | (5,4,2,3,1) | (4,2,5,3,1) | (4,2,3,5,1) | (4,5,2)     | (4,1)(5,2)  | (5,2,3,4,1) | (5,2,4,3,1) |
| (1,3,5,4,2): | (2,3,5,4,1) | (4,1)(5,3)  | (4,1)(3,2)  | (5,1)(4,2)  | (4,5,1)     | (2,3,4,5,1) | (2,4,3,5,1) |
| (1,3,4,2,5): | (2,1)(5,3)  | (3,5,1)     | (3,2,1)     | (5,4,2,3,1) | (3,1)(5,4)  | (2,1)(5,4)  | (2,3,5,4,1) |
| (5,4,3,2,1): | (2,5,3,4,1) | (3,4,1)     | (3,2,5,4,1) | (4,2,3,5,1) | (3,5,1)     | (2,5,1)     | (2,3,4,5,1) |
| (5,2,1,4,3): | (3,2,1)     | (5,1)(3,2)  | (5,2,1)     | (2,5,4,3,1) | (5,4,2,3,1) | (3,5,4,2,1) | (3,2,5,4,1) |
| (5,2,1,4,3): | (3,2,4,2,1) | (4,2,3,5,1) | (4,2,1)     | (5,2,4,3,1) | (4,5,2,3,1) | (3,4,5,2,1) | (3,5,2,4,1) |
| (5,4,1,3,2): | (5,3,2)     | (3,2,1)     | (3,1)(5,2)  | (2,3,5,4,1) | (3,5,4,2,1) | (5,4,2)     | (3,2)(5,4)  |
| (5,3,2,4,1): | (4,3,2)     | (5,4,3,2,1) | (5,2,4,3,1) | (5,3,4,1)   | (5,3,4,2,1) | (4,2)(5,3)  | (5,3,2)     |
| (5,3,1,2,4): | (5,2,4,3,1) | (2,5,4,3,1) | (2,4,3,5,1) | (4,5,3)     | (2,5,3,4,1) | (5,3,2,4,1) | (5,3,1)     |
| (5,2,4,3,1): | (3,1)(4,2)  | (5,4,2,3,1) | (5,1)(4,2)  | (5,2)(4,3)  | (5,2,3,4,1) | (3,5,2,4,1) | (3,1)(5,2)  |
| (1,2,3,4,5): | (5,2,1):    | (5,3,1):    | (5,3,2):    | (5,4,1):    | (5,4,2):    | (5,4,3):    |             |
| (1,2,5,3,4): | (4,5,3)     | (3,2)(5,4)  | (2,1)(5,4)  | (3,4,2)     | (2,1)(4,3)  | (2,3,1)     |             |
| (1,4,5,2,3): | (3,4,1)     | (3,2,5,4,1) | (2,3,5,4,1) | (3,4,2,5,1) | (2,3,4,5,1) | (2,5,1)     |             |
| (1,4,2,3,5): | (2,4,5,3,1) | (2,3,4,5,1) | (4,5,1)     | (2,3,1)     | (4,3,1)     | (4,1)(3,2)  |             |
| (1,3,5,4,2): | (4,2)(5,3)  | (3,4,2)     | (4,2,1)     | (3,5,2)     | (4,3,5,2,1) | (4,5,2,3,1) |             |
| (1,3,4,2,5): | (4,2,3,5,1) | (5,4,2)     | (3,1)(4,2)  | (4,3,5,2,1) | (3,5,2,4,1) | (3,4,5,2,1) |             |
| (5,4,3,2,1): | (3,4,2)     | (4,3,5,2,1) | (3,5,4,2,1) | (5,2)(4,3)  | (3,4,5,2,1) | (3,1)(5,2)  |             |
| (5,2,1,4,3): | (4,2,3,5,1) | (4,3,5,2,1) | (5,2,4,3,1) | (4,5,3,2,1) | (5,3,2,4,1) | (5,3,4,2,1) |             |
| (5,4,1,3,2): | (2,1)(3,2)  | (2,1)(4,3)  | (4,3,1)     | (2,1)(5,3)  | (4,1)(5,3)  | (4,5,3,2,1) |             |
| (5,3,2,4,1): | (4,1)(3,2)  | (4,1)(5,2)  | (3,5,2,4,1) | (4,3,2,5,1) | (3,2,4,5,1) | (3,4,2,5,1) |             |
| (5,3,1,2,4): | (3,2,5,4,1) | (3,5,2,4,1) | (5,2,3,4,1) | (3,2,4,5,1) | (5,1)(3,2)  | (5,1)(4,2)  |             |
| (5,2,4,3,1): | (3,1)(5,4)  | (3,2,4,5,1) | (2,3,4,5,1) | (3,1)(4,2)  | (2,3,1)     | (2,4,1)     |             |
|              | (2,5,4,3,1) | (2,4,3,5,1) | (5,1)(4,3)  | (2,4,5,3,1) | (5,3,1)     | (5,3,2,4,1) |             |

GROUP (1,2,3,4,5)  
K2\*K5

|              |             |             |             |             |             |             |             |
|--------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| (1,2,3,4,5): | (1,2)(3,4): | (1,3)(2,4): | (1,4)(2,3): | (1,2)(3,5): | (1,3)(2,5): | (1,5)(2,3): | (1,2)(4,5): |
| (1,2,5,3,4): | (3,5,1)     | (4,3,2,5,1) | (5,1)(4,2)  | (3,1)(5,4)  | (4,5,5,2,1) | (4,5,2)     | (3,4,1)     |
| (1,4,5,2,3): | (3,5,2,4,1) | (4,5,3,2,1) | (4,2)(5,3)  | (5,4,1)     | (4,1)(3,2)  | (3,5,2,4,1) | (5,1)(4,3)  |
| (1,4,2,3,5): | (3,2,4,5,1) | (5,2)(4,3)  | (5,2,1)     | (3,2,4,5,1) | (4,5,3)     | (2,1)(5,4)  | (3,1)(4,2)  |
| (1,3,4,2,5): | (3,2)(5,4)  | (5,1)(4,3)  | (2,5,1)     | (3,1)(4,2)  | (5,3,4,2,1) | (5,4,2)     | (3,5,2,4,1) |
| (1,3,4,2,5): | (5,1)(3,2)  | (5,4,1)     | (2,5,4,3,1) | (3,4,2)     | (5,1)(4,2)  | (4,2,5,3,1) | (3,5,2)     |
| (5,4,3,2,1): | (5,4,2)     | (4,5,1)     | (2,4,3,5,1) | (5,4,2,3,1) | (4,2,1)     | (4,2)(5,3)  | (5,2,3,4,1) |
| (5,2,1,4,3): | (4,5,2)     | (2,3,5,4,1) | (3,1)(5,2)  | (4,3,2)     | (2,4,3,5,1) | (4,3,1)     | (5,3,2)     |
| (5,4,1,3,2): | (5,4,2,3,1) | (2,1)(5,4)  | (5,4,3)     | (5,2)(4,3)  | (5,1)(4,3)  | (2,5,4,3,1) | (4,2)(5,3)  |
| (5,3,2,4,1): | (4,2,5,3,1) | (2,1)(5,3)  | (4,5,3)     | (4,1)(5,2)  | (2,4,1)     | (4,1)(5,3)  | (4,3,2,5,1) |
| (5,3,1,2,4): | (4,1)(5,3)  | (5,3,2)     | (5,3,2,1)   | (4,5,1)     | (3,2)(5,4)  | (3,4,5,2,1) | (4,3,1)     |
| (5,2,4,3,1): | (4,1)(5,2)  | (3,5,2)     | (3,4,5,2,1) | (4,3,2,5,1) | (5,4,3)     | (2,1)(4,3)  | (4,2,5,3,1) |

|              |             |             |             |             |             |             |             |
|--------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| (1,2,3,4,5): | (1,4)(2,5): | (1,5)(2,4): | (1,3)(4,5): | (1,4)(3,5): | (1,5)(3,4): | (2,3)(4,5): | (2,4)(3,5): |
| (1,2,5,3,4): | (5,3,4,2,1) | (5,2)(4,3)  | (4,1)(3,2)  | (5,4,2,3,1) | (3,5,2)     | (2,4,1)     | (2,5,4,3,1) |
| (1,4,5,2,3): | (3,4,2)     | (3,4,5,2,1) | (4,3,2,5,1) | (5,4,2)     | (3,1)(5,2)  | (2,4,3,5,1) | (2,1)(5,4)  |
| (1,4,2,3,5): | (5,3,1)     | (2,5,4,3,1) | (3,4,2)     | (5,1)(3,2)  | (2,3,5,4,1) | (4,2,1)     | (4,3,2,5,1) |
| (1,3,5,4,2): | (2,1)(5,3)  | (5,4,3)     | (5,2,3,4,1) | (2,3,1)     | (3,2)(5,4)  | (4,1)(5,2)  | (4,3,1)     |
| (1,3,5,4,2): | (2,4,3,5,1) | (4,1)(5,3)  | (5,2,1)     | (2,1)(4,3)  | (4,5,3,2,1) | (3,1)(5,2)  | (3,4,1)     |
| (1,3,4,2,5): | (2,1)(4,3)  | (4,5,3)     | (4,1)(5,2)  | (2,5,4,3,1) | (5,3,2)     | (3,5,2,4,1) | (3,1)(5,4)  |
| (5,4,3,2,1): | (3,2,4,5,1) | (4,1)(3,2)  | (2,1)(5,3)  | (3,4,5,2,1) | (4,2,1)     | (5,3,1)     | (5,2,3,4,1) |
| (5,2,1,4,3): | (3,5,1)     | (2,3,5,4,1) | (5,3,4,2,1) | (3,2,1)     | (2,1)(5,4)  | (4,2,5,3,1) | (4,1)(3,2)  |
| (5,4,1,3,2): | (4,3,2)     | (4,5,3,2,1) | (2,5,1)     | (5,2)(4,3)  | (4,2,5,3,1) | (3,5,1)     | (3,4,5,2,1) |
| (5,3,2,4,1): | (3,2)(5,4)  | (3,2,1)     | (2,4,3,5,1) | (4,5,2)     | (3,1)(4,2)  | (5,1)(4,3)  | (5,2,1)     |
| (5,3,1,2,4): | (5,4,2,3,1) | (3,1)(5,2)  | (4,3,2)     | (5,1)(4,2)  | (3,5,2,4,1) | (2,1)(4,3)  | (2,5,1)     |
| (5,2,4,3,1): | (3,1)(5,4)  | (2,3,1)     | (4,2)(5,3)  | (3,2,4,5,1) | (2,4,1)     | (5,3,4,2,1) | (5,1)(3,2)  |

|              |             |
|--------------|-------------|
| (1,2,3,4,5): | (2,5)(3,4): |
| (1,2,5,3,4): | (2,1)(5,3)  |
| (1,4,5,2,3): | (2,3,1)     |
| (1,4,2,3,5): | (4,1)(5,3)  |
| (1,3,5,4,2): | (4,5,3,2,1) |
| (1,3,4,2,5): | (3,2,4,5,1) |
| (1,3,4,2,5): | (3,2,1)     |
| (5,4,3,2,1): | (5,1)(4,2)  |
| (5,2,1,4,3): | (4,5,1)     |
| (5,4,1,3,2): | (3,1)(4,2)  |
| (5,3,2,4,1): | (5,4,2,3,1) |
| (5,3,1,2,4): | (2,3,5,4,1) |
| (5,2,4,3,1): | (5,4,1)     |

GROUP (1,2,3,4,5)  
K3\*K2

|              |              |              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| (1,3,5,2,4): | (1,2,3,4,5): | (1,2,5,3,4): | (1,4,5,2,3): | (1,4,2,3,5): | (1,3,5,4,2): | (1,3,4,2,5): | (5,4,3,2,1): |
| (1,5,4,2,3): | (4,2,5,3,1)  | (4,3,1)      | (5,4,2)      | (5,3,2)      | (5,1)(3,2)   | (5,3,1)      | (2,3,4,5,1)  |
| (1,5,3,4,2): | (3,2,1)      | (3,2,4,5,1)  | (2,1)(5,3)   | (2,1)(4,3)   | (5,2)(4,3)   | (4,3,2)      | (4,1)(5,2)   |
| (1,2,5,4,3): | (4,3,2)      | (3,2)(5,4)   | (2,4,3,5,1)  | (2,4,1)      | (4,1)(5,2)   | (4,1)(3,2)   | (3,1)(5,2)   |
| (1,5,2,3,4): | (5,2,1)      | (5,1)(4,2)   | (3,2,1)      | (3,4,5,2,1)  | (4,5,3)      | (4,5,2)      | (4,1)(5,3)   |
| (1,5,2,3,4): | (3,1)(4,2)   | (3,1)(5,4)   | (4,3,2)      | (4,3,2)      | (4,3,2,5,1)  | (4,3,1)      | (2,5,1)      |
| (1,4,5,3,2): | (5,4,3)      | (3,5,2)      | (5,1)(4,3)   | (5,4,1)      | (2,4,1)      | (2,3,5,4,1)  | (3,1)(4,2)   |
| (1,5,2,3,4): | (5,4,3,2,1)  | (5,2)(3,2)   | (2,1)(4,3)   | (2,1)(5,4)   | (4,5,2)      | (3,2)(5,4)   | (3,5,2,4,1)  |
| (5,4,1,3,2): | (5,3,1)      | (5,2,4,3,1)  | E            | (4,5,3)      | (2,3,4,5,1)  | (2,4,5,3,1)  | (4,2,3,5,1)  |
| (5,1,2,4,3): | (4,1)(5,2)   | (4,2,1)      | (3,2,5,4,1)  | (3,1)(5,2)   | (5,3,1)      | (5,2,1)      | (4,5,3)      |
| (5,2,1,3,4): | (4,2)(5,3)   | (5,4,3)      | (5,1)(4,2)   | (5,3,2,4,1)  | (4,1)(3,2)   | (4,1)(5,3)   | (2,3,1)      |
| (5,1,4,3,2): | (5,4,1)      | (5,2,1)      | (3,4,1)      | (3,1)(5,4)   | (2,4,5,3,1)  | (2,1)(5,4)   | (4,2)(5,3)   |
| (5,4,1,2,3): | (3,1)(5,2)   | (3,1)(4,2)   | (5,3,2)      | (5,2)(4,3)   | (5,1)(4,3)   | (5,2,4,3,1)  | (4,5,1)      |

|              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|
| (1,3,5,2,4): | (5,2,1,4,3): | (5,4,1,3,2): | (5,3,2,4,1): | (5,3,1,2,4): | (5,2,4,3,1): |
| (1,5,4,2,3): | (3,2)(5,4)   | (5,1)(4,3)   | (2,1)(4,3)   | (4,2,1)      | (2,1)(5,4)   |
| (1,5,3,4,2): | (2,5,3,4,1)  | (4,5,2)      | (4,5,1)      | (3,5,1)      | (4,1)(5,3)   |
| (1,2,5,4,3): | (2,5,1)      | (4,5,2,3,1)  | (3,1)(5,4)   | (5,4,3)      | (3,5,1)      |
| (1,5,2,3,4): | (3,4,1)      | (4,2,3,5,1)  | (4,2,3,5,1)  | (5,1)(3,2)   | (4,1)(3,2)   |
| (1,4,5,3,2): | (5,3,2)      | (4,5,1)      | (2,1)(5,4)   | (3,5,4,2,1)  | (2,1)(5,3)   |
| (1,4,5,3,2): | (5,1)(4,2)   | (2,3,1)      | (5,1)(5,2)   | (5,2)(4,3)   | (3,4,2,5,1)  |
| (5,4,1,3,2): | (2,4,1)      | (3,5,2)      | (3,5,1)      | (5,1)(4,3)   | (3,4,1)      |
| (5,4,1,3,2): | (3,4,2)      | (2,4,3,5,1)  | (4,3,5,2,1)  | (5,2,1)      | (4,2,1)      |
| (5,1,2,4,3): | (3,1)(5,4)   | (5,3,4,2,1)  | (3,4,2)      | (4,1)(3,2)   | (3,2)(5,4)   |
| (5,2,1,3,4): | (5,1)(3,2)   | (5,4,3,1)    | (2,5,4,3,1)  | (5,4,2)      | (2,5,1)      |
| (5,1,4,3,2): | (3,1)(4,2)   | (2,1)(5,3)   | (3,5,2)      | (5,2,3,4,1)  | (3,4,2)      |
| (5,4,1,2,3): | (4,5,3)      | (5,1)(4,2)   | (4,2,1)      | (3,2,1)      | (4,5,3,2,1)  |





GROUP (1,2,3,4,5)  
K4\*K2

|          |              |              |              |              |              |               |              |
|----------|--------------|--------------|--------------|--------------|--------------|---------------|--------------|
| (1,2,3): | (1,2,3,4,5): | (1,2,5,3,4): | (1,4,5,2,3): | (1,4,2,3,5): | (1,3,5,4,2): | (1,3,4,2,5):  | (5,4,3,2,1): |
| (2,3,4): | (3,4,5,2,1): | (3,4,2,5,1): | (4,5,3,2,1): | (4,3,5,2,1): | (5,4,3):     | (5,2,1)(4,3): | (5,4,1):     |
| (1,3,4): | (3,2,4,5,1): | (3,2,5,4,1): | (2,4,5,3,1): | (2,4,3,5,1): | (4,3,5,2,1): | (4,3,2,5,1):  | (5,2,1):     |
| (1,2,4): | (2,4,5,3,1): | (2,5,4,3,1): | (4,5,2):     | (4,2)(5,3):  | (4,2,3,5,1): | (4,2,5,3,1):  | (5,1)(3,2):  |
| (1,2,5): | (4,5,2,3,1): | (4,2,5,3,1): | (3,2)(5,4):  | (3,5,2):     | (3,5,1):     | (3,1)(5,2):   | (5,1)(4,3):  |
| (1,3,5): | (5,2,3,4,1): | (5,3,4,2,1): | (4,1)(3,2):  | (4,5,2,3,1): | (3,1)(5,4):  | (3,4,5,2,1):  | (5,4,3):     |
| (2,3,5): | (2,5,3,4,1): | (2,1)(4,3):  | (4,1)(5,2):  | (4,2,5,3,1): | (5,4,2,3,1): | (5,3,4,2,1):  | (3,2)(5,4):  |
| (1,4,5): | (2,3,4,5,1): | 13,4,1,1     | 14,2,5,3,1,1 | 14,3,2,5,1,1 | 15,4,3,2,1,1 | 15,1,1(4,3)   | (2,1)(5,4)   |
| (2,4,5): | (2,3,5,4,1): | (2,1)(5,3):  | (5,2,3,4,1): | (5,4,2,3,1): | (5,4,2,3,1): | (3,1)(4,2):   | (4,3,2):     |
| (3,4,5): | (4,2,3,5,1): | (4,1)(5,3):  | (5,4,2,3,1): | (5,1)(3,2):  | (3,2,1):     | (3,5,1):      | (2,1)(4,3)   |
| (4,3,2): | (2,4,3,5,1): | (2,3,5,4,1): | (5,2,4,3,1): | (5,1)(4,2):  | (4,2,1):     | (4,2,3,5,1):  | (3,2,1):     |
| (4,3,1): | (4,5,3):     | (5,2)(4,3):  | (4,5,1):     | (4,1)(5,3):  | (2,3,5,4,1): | (2,5,3,4,1):  | (5,4,2,3,1)  |
| (4,3,2): | (4,5,1):     | (4,1)(5,2):  | (4,1)(5,4):  | (3,5,1):     | (2,1)(5,3):  | (2,5,1):      | (5,3,4,2,1)  |
| (4,3,1): | (2,1)(5,4):  | (2,5,1):     | (3,4,5,2,1): | (3,5,4,2,1): | (4,2)(5,3):  | (5,4,2):      | (5,3,2,4,1)  |
| (4,2,1): | (3,2)(5,4):  | (5,3,2):     | (2,3,4,5,1): | (2,3,5,4,1): | (3,5,2,4,1): | (3,2,5,4,1):  | (5,2,4,3,1)  |
| (5,2,1): | (3,4,2):     | (4,5,3):     | (4,2,3,5,1): | (4,1)(3,2):  | (3,2,5,4,1): | (3,4,1):      | (2,5,4,3,1)  |
| (5,3,1): | (2,1)(4,3):  | (2,3,4,5,1): | (4,3,5,2,1): | (4,2,1):     | (5,4,2):     | (5,4,2):      | (3,2,5,4,1)  |
| (5,3,2): | (5,1)(4,3):  | (5,2,3,4,1): | (4,3,1):     | (4,5,1):     | (2,1)(5,4):  | (2,3,4,5,1):  | (3,5,4,2,1)  |
| (5,4,1): | (2,3,1):     | (2,4,5,3,1): | (3,5,2):     | (3,4,2):     | (3,4,2,5,1): | (3,1)(4,2):   | (4,3,2,5,1)  |
| (5,4,2): | (5,1)(3,2):  | (5,3,2,4,1): | (2,3,1):     | (2,3,4,5,1): | (3,4,5,2,1): | (3,2,4,5,1):  | (4,3,5,2,1)  |
| (5,4,3): | (2,5,1):     | (2,4,1):     | (3,1)(5,2):  | (3,4,2,5,1): | (5,3,4,2,1): | (5,1)(4,2):   | (4,5,3,2,1)  |
| (1,2,3): | (5,2,1,4,3): | (5,4,1,3,2): | (5,3,2,4,1): | (5,3,1,2,4): | (5,2,4,3,1): | (5,3,2,4,1):  |              |
| (2,3,4): | (4,1)(5,3):  | (4,1)(5,2):  | (5,1)(4,2):  | (3,2,4,5,1): | (5,3,2,4,1): | (5,3,2,4,1):  |              |
| (1,3,4): | (3,5,2):     | (4,3,2,5,1): | (5,4,3,2,1): | (3,1)(5,4):  | (5,3,1):     | (5,2,1):      |              |
| (1,2,4): | (5,4,3):     | (3,4,2,5,1): | (5,3,4,2,1): | (4,5,3,2,1): | (5,4,3,2,1): | (4,3,2):      |              |
| (1,2,5): | (4,3,1):     | (3,5,4,2,1): | (4,2)(5,3):  | (5,3,2,4,1): | (4,3,2):     | (4,5,2):      |              |
| (1,3,5): | (4,5,2,3,1): | (5,4,3,2,1): | (4,3,2):     | (2,4,1):     | (4,5,2):     | (2,4,5,3,1):  |              |
| (2,3,5): | (4,5,3,2,1): | (5,4,1):     | (2,4,1):     | (3,1)(4,2):  | (2,4,5,3,1): | (5,2)(4,3):   |              |
| (1,4,5): | (5,2,4,3,1): | (3,2,1):     | (5,3,2):     | (2,5,3,4,1): | (5,2)(4,3):  | (2,5,4,3,1):  |              |
| (2,4,5): | (5,4,3,2,1): | (3,4,1):     | (2,5,3,4,1): | (4,2,5,3,1): | (4,2,5,3,1): | (3,1)(5,2):   |              |
| (3,4,5): | (5,2,1):     | (4,1)(3,2):  | (3,2,5,4,1): | (2,5,4,3,1): | (4,5,2):     | (5,1)(4,2):   |              |
| (3,2,1): | (4,2,3,5,1): | (2,5,4,3,1): | (5,2,4,3,1): | (4,5,2,3,1): | (4,5,2):     | (5,4,2,3,1):  |              |
| (4,3,2): | (3,5,4,2,1): | (2,5,3,4,1): | (5,2,3,4,1): | (2,3,4,5,1): | (5,2,3,4,1): | (5,2,3,4,1):  |              |
| (4,3,1): | (3,5,2,4,1): | (5,3,2):     | (5,1)(3,2):  | (4,5,3):     | (5,1)(4,3):  | (5,1)(4,3):   |              |
| (4,2,1): | (2,4,3,5,1): | (3,1)(5,2):  | (5,3,1):     | (4,2)(5,3):  | (4,2)(5,3):  | (3,5,2,4,1):  |              |
| (5,2,1): | (4,3,2,5,1): | (3,1)(5,4):  | (2,4,5,3,1): | (2,4,3,5,1): | (5,2,4,3,1): | (3,1)(4,2):   |              |
| (5,3,1): | (4,1)(5,2):  | (3,2)(5,4):  | (3,2,4,5,1): | (5,2,4,3,1): | (5,2,4,3,1): | (3,1)(4,2):   |              |
| (5,3,2): | (4,2,1):     | (2,3,5,4,1): | (3,5,2,4,1): | (5,2,4,3,1): | (5,2,4,3,1): | (4,3,5,2,1):  |              |
| (5,4,1): | (5,2)(4,3):  | (3,2,4,5,1): | (4,5,3,2,1): | (2,1)(5,3):  | (5,3,1):     | (4,3,5,2,1):  |              |
| (5,4,2): | (2,1)(4,3):  | (3,5,2,4,1): | (4,1)(5,3):  | (2,3,1):     | (5,3,1):     | (4,3,1):      |              |
| (5,4,3): | (3,4,5,2,1): | (5,3,2,4,1): | (4,1)(3,2):  | (2,3,1):     | (4,5,2,3,1): | (4,5,2,3,1):  |              |

GROUP (1,2,3,4,5)  
K4\*K3

|          |              |              |              |              |              |              |              |
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| (1,2,3): | (1,3,5,2,4): | (1,5,4,2,3): | (1,5,3,4,2): | (1,2,5,4,3): | (1,5,2,3,4): | (1,4,5,3,2): | (5,3,1,4,2): |
| (2,3,4): | (4,2)(5,3):  | (5,4,3,2,1): | (5,1)(4,3):  | (3,2,5,4,1): | (5,3,4,2,1): | (5,3,4,2,1): | (4,3,2,5,1)  |
| (1,3,4): | (4,1)(5,3):  | (5,2,4,3,1): | (5,4,3,2,1): | (3,1)(5,2):  | (5,3,2,4,1): | (5,3,2,4,1): | (2,5,4,3,1)  |
| (1,2,4): | (4,3,5,2,1): | (5,1)(4,2):  | (5,4,2,3,1): | (2,5,1):     | (5,2,4,3,1): | (5,2,4,3,1): | (5,4,2)      |
| (1,2,5): | (3,5,4,2,1): | (5,1)(3,2):  | (5,3,1):     | (4,3,2,5,1): | (5,4,3,2,1): | (4,5,3):     | (5,3,2)      |
| (1,3,5): | (3,1)(4,2):  | (3,2)(5,4):  | (4,5,3):     | (5,4,3,2,1): | (3,4,2):     | (3,4,2):     | (4,5,3,2,1)  |
| (2,3,5): | (5,2,4,3,1): | (5,4,2):     | (3,4,2):     | (2,1)(5,4):  | (2,1)(5,4):  | (4,1)(3,2):  | (4,2,1)      |
| (1,4,5): | (5,3,2,4,1): | (3,4,2):     | (2,1)(4,3):  | (3,1)(5,4):  | (3,1)(5,4):  | (2,5,3,4,1): | (4,3,1)      |
| (2,4,5): | (3,1)(5,2):  | (3,4,2):     | (4,2)(5,3):  | (2,1)(4,3):  | (2,1)(4,3):  | (2,5,3,4,1): | (5,3,2,4,1)  |
| (3,4,5): | (3,2,5,4,1): | (2,3,1):     | (2,1)(5,3):  | (4,3,1):     | (4,3,1):     | (5,2)(4,3):  | (5,3,4,2,1)  |
| (4,3,2): | (4,1)(5,2):  | (3,1)(4,2):  | (3,5,4,2,1): | (2,3,1):     | (2,3,1):     | (3,5,2,4,1): | (5,4,2,3,1)  |
| (4,3,1): | (2,4,3,5,1): | (5,4,1):     | (5,2,3,4,1): | (5,4,2):     | (5,4,2):     | (5,1)(4,3):  | (4,3,5,2,1)  |
| (4,2,1): | (2,3,5,4,1): | (5,3,1):     | (5,2,1):     | (4,2,5,3,1): | (4,2,5,3,1): | (5,4,1):     | (3,4,5,2,1)  |
| (5,2,1): | (3,5,2):     | (5,3,4,2,1): | (5,1)(4,2):  | (2,5,3,4,1): | (2,5,3,4,1): | (5,2,1):     | (3,4,2,5,1)  |
| (5,3,1): | (3,5,1):     | (5,2,3,4,1): | (5,3,2,4,1): | (3,4,5,4,1): | (3,4,5,4,1): | (5,1)(3,2):  | (2,4,5,3,1)  |
| (5,3,2): | (3,2,4,5,1): | (2,3,5,4,1): | (2,5,3,4,1): | (2,3,5,4,1): | (2,3,5,4,1): | (2,3,4,5,1): | (4,2,5,3,1)  |
| (5,4,1): | (4,5,2):     | (2,3,5,4,1): | (3,4,5,2,1): | (2,3,5,4,1): | (2,3,5,4,1): | (3,4,5,2,1): | (4,2,3,5,1)  |
| (5,4,2): | (2,4,1):     | (3,1)(5,4):  | (3,4,5,2,1): | (5,4,2,3,1): | (5,4,2,3,1): | (3,4,1):     | (4,2,5,3,1)  |
| (5,4,3): | (3,4,5,2,1): | (4,2,3,5,1): | (4,2,3,5,1): | (5,2,4,3,1): | (5,2,4,3,1): | (4,5,2,3,1): | (4,2,5,3,1)  |
| (5,4,3): | (3,4,1):     | (4,5,2,3,1): | (4,5,3,2,1): | (5,2,4,3,1): | (5,2,4,3,1): | (4,1)(3,2):  | (2,4,5,3,1)  |
| (5,4,3): | (5,2,3,4,1): | (4,2,5,3,1): | (4,2,5,3,1): | (2,4,5,3,1): | (2,4,5,3,1): | (4,1)(5,2):  | (3,1)(4,2)   |
| (1,2,3): | (5,4,1,3,2): | (5,1,2,4,3): | (5,2,1,3,4): | (5,1,4,3,2): | (5,4,1,2,3): | (5,4,1,2,3): |              |
| (2,3,4): | (5,4,2):     | (3,5,2,4,1): | (4,5,3):     | (4,1)(5,2):  | (3,5,4,2,1): | (3,5,4,2,1): |              |
| (1,3,4): | (4,1)(5,2):  | (4,3,2,5,1): | (4,5,3,2,1): | (2,5,1):     | (3,5,2,4,1): | (3,5,2,4,1): |              |
| (1,2,4): | (4,3,2,5,1): | (2,1)(5,3):  | (4,5,2,3,1): | (5,3,2):     | (2,4,3,5,1): | (2,4,3,5,1): |              |
| (1,2,5): | (3,4,2,5,1): | (4,3,5,2,1): | (3,1)(5,4):  | (5,2)(4,3):  | (4,2,3,5,1): | (4,2,3,5,1): |              |
| (1,3,5): | (3,5,4,2,1): | (5,2,4,3,1): | (3,4,1):     | (4,3,5,2,1): | (5,4,2,3,1): | (5,4,2,3,1): |              |
| (2,3,5): | (5,4,3,2,1): | (2,4,5,3,1): | (5,2,3,4,1): | (4,5,3,2,1): | (3,5,4,2,1): | (3,5,4,2,1): |              |
| (1,4,5): | (5,4,1):     | (3,2,4,5,1): | (3,5,4,2,1): | (4,5,1):     | (4,5,1):     | (3,2,5,4,1): |              |
| (2,4,5): | (3,2,1):     | (4,3,2,5,1): | (3,5,4,2,1): | (5,1)(4,3):  | (4,1)(3,2):  | (4,1)(3,2):  |              |
| (3,4,5): | (3,4,1):     | (4,3,2,5,1): | (4,3,2,5,1): | (5,1)(3,2):  | (2,4,1):     | (2,4,1):     |              |
| (3,2,1): | (4,1)(3,2):  | (4,2)(5,3):  | (4,3,5,2,1): | (4,2,5,3,1): | (5,4,3):     | (5,4,3):     |              |
| (4,3,2): | (2,5,4,3,1): | (4,2)(5,3):  | (2,3,4,5,1): | (4,2,5,3,1): | (3,4,2,5,1): | (3,4,2,5,1): |              |
| (4,3,1): | (2,5,3,4,1): | (4,2,3,5,1): | (4,5,2):     | (3,2,5,4,1): | (2,1)(5,3):  | (2,1)(5,3):  |              |
| (4,2,1): | (3,1)(5,2):  | (5,4,3):     | (3,2,4,5,1): | (4,3,1):     | (3,2,5,4,1): | (3,2,5,4,1): |              |
| (5,2,1): | (3,1)(5,4):  | (4,3,2):     | (3,4,2,5,1): | (4,1)(3,2):  | (4,1)(3,2):  | (2,1)(5,4):  |              |
| (5,3,1): | (3,2)(5,4):  | (2,4,1):     | (3,1)(4,2):  | (4,2,3,5,1): | (5,4,1):     | (5,4,1):     |              |
| (5,3,2): | (2,3,5,4,1): | (2,3,5,4,1): | (2,1)(4,3):  | (4,3,2):     | (2,3,4,5,1): | (2,3,4,5,1): |              |
| (5,4,1): | (2,3,5,4,1): | (2,3,5,4,1): | (2,1)(4,3):  | (4,3,2):     | (4,5,2,3,1): | (4,5,2,3,1): |              |
| (5,4,2): | (3,2,4,5,1): | (2,1)(4,3):  | (3,2,1):     | (2,4,3,5,1): | (5,2,3,4,1): | (5,2,3,4,1): |              |
| (5,4,3): | (3,5,2,4,1): | (5,1)(4,3):  | (3,1)(5,2):  | (2,4,3,5,1): | (2,5,3,4,1): | (2,5,3,4,1): |              |
| (5,4,3): | (5,3,2,4,1): | (2,3,4,5,1): | (5,2,1):     | (3,2,4,5,1): | (2,5,3,4,1): | (2,5,3,4,1): |              |



GROUP (1,2,3,4,5)  
K5\*K2

|             |              |              |              |              |              |              |              |
|-------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| (1,2)(3,4): | (1,2,3,4,5): | (1,2,5,3,4): | (1,4,5,2,3): | (1,4,2,3,5): | (1,3,5,4,2): | (1,3,4,2,5): | (5,4,3,2,1): |
| (1,3)(2,4): | (4,5,2)      | (5,4,2)      | (3,2,4,5,1)  | (3,5,2,4,1)  | (4,1)(5,3)   | (4,1)(5,2)   | (5,3,1)      |
| (1,4)(2,3): | (4,5,3,2,1)  | (4,3,2,5,1)  | (2,1)(5,4)   | (2,1)(5,3)   | (3,5,2)      | (5,3,2)      | (5,2,3,4,1)  |
| (1,2)(3,5): | (3,1)(5,4)   | (3,1)(5,2)   | (4,5,3)      | (5,4,3)      | (2,4,3,5,1)  | (2,5,4,3,1)  | (5,1)(4,2)   |
| (1,3)(2,5): | (5,2)(4,3)   | (3,4,2)      | (4,3,2,5,1)  | (4,1)(5,2)   | (5,4,1)      | (5,2,3,4,1)  | (3,1)(5,4)   |
| (1,5)(2,3): | (5,3,4,2,1)  | (5,1)(4,3)   | (4,2,1)      | (4,5,3,2,1)  | (3,2)(5,4)   | (3,2)(5,4)   | (2,3,4,5,1)  |
| (1,4)(4,5): | (3,4,1)      | (3,4,5,2,1)  | (4,1)(5,3)   | (4,3,1)      | (4,5,4,3,1)  | (2,1)(4,3)   | (5,4,2)      |
| (1,5)(2,4): | (3,5,2)      | (4,2)(5,3)   | (4,2,1)      | (5,2,3,4,1)  | (3,4,1)      | (3,4,1)      | (4,3,1)      |
| (1,3)(4,5): | (5,4,2,3,1)  | (5,3,1)      | (3,4,2)      | (3,2)(5,4)   | (3,2,4,5,1)  | (3,1)(5,4)   | (2,4,3,5,1)  |
| (1,5)(2,3): | (4,1)(3,2)   | (4,5,3,2,1)  | (2,3,5,4,1)  | (2,3,1)      | (3,1)(5,2)   | (3,2,1)      | (5,2)(4,3)   |
| (1,4)(3,5): | (2,1)(5,3)   | (2,4,3,5,1)  | (5,2,1)      | (5,3,4,2,1)  | (3,4,2)      | (4,2)(5,3)   | (4,2,1)      |
| (1,5)(3,4): | (2,5,4,3,1)  | (2,3,1)      | (5,2)(4,3)   | (5,4,2)      | (5,1)(4,2)   | (5,4,2,3,1)  | (3,2,4,5,1)  |
| (2,3)(4,5): | (2,4,1)      | (2,1)(5,4)   | (3,5,2,4,1)  | (3,1)(4,2)   | (4,2,5,3,1)  | (4,2,1)      | (5,3,2)      |
| (2,4)(3,5): | (3,5,1)      | (3,5,2,4,1)  | (5,3,1)      | (5,1)(4,3)   | (2,1)(4,3)   | (2,4,3,5,1)  | (4,2,1)      |
| (2,5)(3,4): | (4,3,2,5,1)  | (4,1)(3,2)   | (2,5,4,3,1)  | (2,5,1)      | (5,2,1)      | (5,1)(3,2)   | (3,4,5,2,1)  |
|             | (5,1)(4,2)   | (5,4,1)      | (3,1)(4,2)   | (3,2,4,5,1)  | (4,5,3,2,1)  | (4,5,1)      | (2,1)(5,3)   |

|             |              |              |              |              |              |
|-------------|--------------|--------------|--------------|--------------|--------------|
| (1,2)(3,4): | (5,2,1,4,3): | (5,4,1,3,2): | (5,3,2,4,1): | (5,3,1,2,4): | (5,2,4,3,1): |
| (1,3)(2,4): | (3,5,1)      | (4,2,5,3,1)  | (5,4,2,3,1)  | (3,2)(5,4)   | (5,1)(3,2)   |
| (1,4)(2,3): | (2,3,5,4,1)  | (5,2)(4,3)   | (5,1)(4,3)   | (4,5,1)      | (5,4,1)      |
| (1,2)(3,5): | (4,2)(5,3)   | (2,5,1)      | (5,2,1)      | (3,4,5,2,1)  | (5,3,4,2,1)  |
| (1,3)(2,5): | (4,5,1)      | (5,4,2,3,1)  | (3,1)(4,2)   | (4,3,2)      | (3,2,4,5,1)  |
| (1,5)(2,3): | (4,1)(3,2)   | (2,1)(5,4)   | (4,5,2)      | (5,1)(4,2)   | (2,4,1)      |
| (1,2)(4,5): | (4,2,5,3,1)  | (5,1)(4,3)   | (4,2,5,3,1)  | (3,5,2,4,1)  | (4,2)(5,3)   |
| (1,4)(2,5): | (4,3,2)      | (3,5,1)      | (2,1)(5,3)   | (5,3,4,2,1)  | (2,1)(4,3)   |
| (1,5)(2,4): | (2,5,4,3,1)  | (3,4,5,2,1)  | (4,5,3)      | (4,1)(5,3)   | (5,4,3)      |
| (1,3)(4,5): | (5,2,3,4,1)  | (4,3,2)      | (4,3,2,5,1)  | (2,5,1)      | (4,1)(5,2)   |
| (1,4)(3,5): | (4,5,2)      | (5,1)(3,2)   | (3,2,1)      | (2,1)(4,3)   | (3,4,5,2,1)  |
| (1,5)(3,4): | (3,1)(5,2)   | (4,5,3,2,1)  | (3,2)(5,4)   | (3,5,4,1)    | (3,5,2)      |
| (2,3)(4,5): | (5,3,4,2,1)  | (2,4,1)      | (4,1)(5,2)   | (3,1)(5,2)   | (4,2,5,3,1)  |
| (2,4)(3,5): | (2,1)(5,4)   | (5,2,3,4,1)  | (3,4,1)      | (4,3,1)      | (3,1)(5,4)   |
| (2,5)(3,4): | (3,2,1)      | (4,1)(5,3)   | (2,3,5,4,1)  | (5,4,2,3,1)  | (2,3,1)      |

GROUP (1,2,3,4,5)  
K5\*K3

|             |              |              |              |              |              |              |              |
|-------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| (1,2)(3,4): | (1,3,5,2,4): | (1,5,4,2,3): | (1,5,3,4,2): | (1,2,5,4,3): | (1,5,2,3,4): | (1,4,5,3,2): | (5,3,1,4,2): |
| (1,3)(2,4): | (4,2,3,5,1)  | (5,3,2,4,1)  | (5,4,1)      | (5,3,2)      | (5,1)(4,2)   | (3,1)(5,4)   | (3,2,5,4,1)  |
| (1,4)(2,3): | (5,4,3)      | (5,2,1)      | (5,1)(3,2)   | (4,1)(5,2)   | (5,4,3,2,1)  | (2,3,4,5,1)  | (2,5,1)      |
| (1,2)(3,5): | (2,1)(5,3)   | (5,1)(4,3)   | (5,2,4,3,1)  | (3,4,2,5,1)  | (5,3,1)      | (4,5,2)      | (5,2)(4,3)   |
| (1,3)(2,5): | (4,3,2)      | (2,1)(5,4)   | (3,4,1)      | (3,2)(5,4)   | (3,4,2,5,1)  | (4,3,1)      | (4,1)(3,2)   |
| (1,5)(2,3): | (2,4,5,3,1)  | (5,4,3)      | (2,3,4,5,1)  | (5,4,1)      | (2,1)(4,3)   | (4,2,3,5,1)  | (4,5,1)      |
| (1,2)(4,5): | (3,4,2,5,1)  | (4,3,1)      | (5,2)(4,3)   | (3,5,4,2,1)  | (4,5,3)      | (4,1)(5,2)   | (4,3,5,2,1)  |
| (1,4)(2,5): | (3,2,1)      | (2,3,4,5,1)  | (4,1)(5,3)   | (4,3,2)      | (4,2,3,5,1)  | (5,3,1)      | (5,3,2,4,1)  |
| (1,5)(2,4): | (3,1)(5,4)   | (3,5,2)      | (5,3,2)      | (2,4,1)      | (2,3,1)      | (2,3,1)      | (4,5,3)      |
| (1,3)(4,5): | (5,2)(4,3)   | (4,2,1)      | (4,2,3,5,1)  | (4,3,5,2,1)  | (3,2)(5,4)   | (4,3,5,2,1)  | (2,1)(5,3)   |
| (1,4)(3,5): | (5,2,1)      | (3,4,2,5,1)  | (3,1)(4,2)   | (2,3,4,5,1)  | (4,3,5,2,1)  | (4,3,2)      | (5,1)(4,2)   |
| (1,5)(3,4): | (4,5,2,3,1)  | (4,2)(5,3)   | (4,2,1)      | (2,1)(5,3)   | (2,4,1)      | (3,2,5,4,1)  | (3,5,4,2,1)  |
| (2,3)(4,5): | (2,5,3,4,1)  | (4,3,1)      | (4,3,5,2,1)  | (3,1)(4,2)   | (4,1)(5,3)   | (5,2,1)      | (5,2,4,3,1)  |
| (2,4)(3,5): | (5,4,1)      | (3,1)(5,2)   | (3,2,1)      | (4,5,2,3,1)  | (3,2,5,4,1)  | (2,1)(4,3)   | (2,3,1)      |
| (2,5)(3,4): | (4,1)(3,2)   | (2,4,5,3,1)  | (2,1)(5,4)   | (5,3,1)      | (2,4,1)      | (3,5,4,2,1)  | (3,1)(5,4)   |

|             |              |              |              |              |              |
|-------------|--------------|--------------|--------------|--------------|--------------|
| (1,2)(3,4): | (5,4,1,3,2): | (5,1,2,4,3): | (5,2,1,3,4): | (5,1,4,3,2): | (5,4,1,2,3): |
| (1,3)(2,4): | (4,2,5,3,1)  | (3,5,2)      | (4,5,1)      | (3,1)(5,2)   | (4,2)(5,3)   |
| (1,4)(2,3): | (5,2)(4,3)   | (4,1)(5,3)   | (3,2)(5,4)   | (2,5,3,4,1)  | (4,3,5,2,1)  |
| (1,2)(3,5): | (2,5,1)      | (3,5,4,2,1)  | (2,4,5,3,1)  | (5,4,2)      | (3,5,1)      |
| (1,3)(2,5): | (5,4,2,3,1)  | (4,5,2)      | (5,1)(4,3)   | (4,5,2,3,1)  | (5,4,2)      |
| (1,5)(2,3): | (5,4,3)      | (5,3,2,4,1)  | (3,4,2)      | (4,1)(5,3)   | (5,4,3,2,1)  |
| (1,2)(4,5): | (2,1)(5,4)   | (3,1)(4,2)   | (2,5,3,4,1)  | (4,2,1)      | (3,1)(5,4)   |
| (1,4)(2,5): | (3,1)(4,2)   | (5,2)(4,3)   | (3,5,1)      | (5,2,4,3,1)  | (3,4,2)      |
| (1,5)(2,4): | (3,4,5,2,1)  | (5,4,3,2,1)  | (3,1)(4,2)   | (5,4,3)      | (5,1)(3,2)   |
| (1,3)(4,5): | (3,4,3,2)    | (4,3,1)      | (3,2,5,4,1)  | (2,1)(4,3)   | (4,5,2,3,1)  |
| (1,4)(3,5): | (5,1)(3,2)   | (2,5,4,3,1)  | (3,5,2)      | (5,3,2,4,1)  | (2,1)(4,3)   |
| (1,5)(3,4): | (5,1)(3,2)   | (2,1)(5,4)   | (5,2,4,3,1)  | (3,2)(5,4)   | (2,5,1)      |
| (2,3)(4,5): | (2,4,1)      | (3,4,2,5,1)  | (4,1)(5,2)   | (3,2,1)      | (2,4,5,3,1)  |
| (2,4)(3,5): | (2,5,3,4,1)  | (4,5,1)      | (2,1)(5,3)   | (5,1)(4,2)   | (3,4,1)      |
| (2,5)(3,4): | (4,1)(5,3)   | (5,1)(3,2)   | (4,2,1)      | (2,3,4,5,1)  | (4,1)(5,2)   |
|             |              |              |              |              | (5,3,2,4,1)  |

GROUP (1,2,3,4,5)  
K5\*K4

|             |             |             |             |             |             |             |             |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| (1,2)(3,4): | (1,2,3):    | (2,3,4):    | (1,3,4):    | (1,2,4):    | (1,2,5):    | (1,3,5):    | (2,3,5):    |
| (1,3)(2,4): | (4,3,2)     | (2,4,1)     | (4,2,1)     | (3,4,2)     | (5,2)(4,3)  | (4,3,5,2,1) | (2,4,3,5,1) |
| (1,4)(2,3): | (4,2,1)     | (3,2,1)     | (4,3,2)     | (4,3,1)     | (4,2,5,3,1) | (4,2)(5,3)  | (3,5,4,2,1) |
| (1,2)(3,5): | (3,4,1)     | (4,3,1)     | (2,3,1)     | (3,2,1)     | (3,2,5,4,1) | (2,3,5,4,1) | (4,1)(5,3)  |
| (1,3)(2,5): | (5,3,2)     | (2,5,3,4,1) | (5,3,4,2,1) | (4,2)(5,3)  | (3,5,2)     | (5,2,1)     | (2,5,1)     |
| (1,3)(2,5): | (5,2,1)     | (3,4,5,2,1) | (5,2)(4,3)  | (5,2,4,3,1) | (5,3,1)     | (5,3,2)     | (3,2,1)     |
| (1,5)(2,3): | (3,5,1)     | (5,1)(4,3)  | (2,3,4,5,1) | (3,2,4,5,1) | (3,2,1)     | (2,3,1)     | (5,3,1)     |
| (1,2)(4,5): | (3,2)(5,4)  | (2,3,5,4,1) | (3,5,4,2,1) | (5,4,2)     | (4,5,2)     | (3,4,5,2,1) | (2,3,4,5,1) |
| (1,4)(2,5): | (5,2,3,4,1) | (4,5,2,3,1) | (3,1)(5,2)  | (5,2,1)     | (5,4,1)     | (3,2,5,4,1) | (4,1)(3,2)  |
| (1,5)(2,4): | (4,2,3,5,1) | (5,1)(3,2)  | (3,2,4,5,1) | (4,5,1)     | (4,2,1)     | (3,1)(4,2)  | (5,4,2,3,1) |
| (1,3)(4,5): | (2,1)(5,4)  | (3,5,4,2,1) | (5,4,3)     | (2,5,4,3,1) | (2,4,5,3,1) | (4,5,3)     | (3,4,5,2,1) |
| (1,4)(3,5): | (2,5,3,4,1) | (4,2,5,3,1) | (5,3,1)     | (2,1)(5,3)  | (2,3,5,4,1) | (5,4,1)     | (4,1)(5,2)  |
| (1,5)(3,4): | (2,4,3,5,1) | (5,1)(4,2)  | (4,5,1)     | (2,3,4,5,1) | (2,1)(4,3)  | (4,3,1)     | (5,2,4,3,1) |
| (2,3)(4,5): | (3,1)(5,4)  | (5,4,3)     | (2,3,5,4,1) | (3,2,5,4,1) | (3,2,4,5,1) | (2,3,4,5,1) | (4,5,3)     |
| (2,4)(3,5): | (4,2,5,3,1) | (5,3,2)     | (5,3,2,4,1) | (4,1)(5,3)  | (4,2,3,5,1) | (5,1)(4,2)  | (5,4,2)     |
| (2,5)(3,4): | (5,2,4,3,1) | (4,5,2)     | (4,1)(5,2)  | (5,2,3,4,1) | (5,1)(4,3)  | (4,3,2,5,1) | (4,3,2)     |
| (1,2)(3,4): | (1,4,5):    | (2,4,5):    | (3,4,5):    | (3,2,1):    | (4,3,2):    | (4,3,1):    | (4,2,1):    |
| (1,3)(2,4): | (3,4,5,2,1) | (2,3,4,5,1) | (2,1)(5,4)  | (4,3,1)     | (2,3,1)     | (3,2,1)     | (3,4,1)     |
| (1,4)(2,3): | (2,4,5,3,1) | (3,1)(5,4)  | (3,2,4,5,1) | (3,4,2)     | (3,4,1)     | (2,4,1)     | (2,3,1)     |
| (1,2)(3,5): | (3,2)(5,4)  | (4,5,3,2,1) | (4,5,2,3,1) | (2,4,1)     | (4,2,1)     | (3,4,2)     | (4,3,2)     |
| (1,3)(2,5): | (4,3,5,2,1) | (2,4,3,5,1) | (2,1)(4,3)  | (5,3,1)     | (2,4,5,3,1) | (4,5,3,2,1) | (4,1)(5,3)  |
| (1,3)(2,5): | (4,2,5,3,1) | (3,1)(4,2)  | (3,4,2,5,1) | (3,5,2)     | (3,5,2,4,1) | (4,1)(5,2)  | (4,5,2,3,1) |
| (1,5)(2,3): | (4,1)(3,2)  | (5,3,2,4,1) | (5,2,3,4,1) | (2,5,1)     | (5,1)(4,2)  | (4,2,3,5,1) | (4,3,2,5,1) |
| (1,2)(4,5): | (5,2,1)     | (2,5,1)     | (2,1)(5,3)  | (3,1)(5,4)  | (2,5,4,3,1) | (5,4,3,2,1) | (5,4,1)     |
| (1,4)(2,5): | (5,4,2)     | (4,2,1)     | (4,2,5,3,1) | (3,5,2,4,1) | (4,3,5,2,1) | (5,2)(4,3)  | (4,5,2)     |
| (1,5)(2,4): | (2,4,1)     | (5,4,1)     | (5,3,2,4,1) | (3,4,2,5,1) | (5,1)(4,3)  | (2,4,3,5,1) | (2,5,1)     |
| (1,3)(4,5): | (5,3,1)     | (3,1)(5,2)  | (3,5,1)     | (3,2)(5,4)  | (3,2,5,4,1) | (5,4,1)     | (5,4,2,3,1) |
| (1,4)(3,5): | (5,4,3)     | (4,3,5,2,1) | (4,3,1)     | (5,3,2,4,1) | (4,5,3,2,1) | (4,5,3)     | (4,2)(5,3)  |
| (1,5)(3,4): | (3,4,1)     | (5,2,3,4,1) | (5,4,1)     | (4,3,2,5,1) | (5,1)(3,2)  | (3,5,1)     | (3,4,2,5,1) |
| (2,3)(4,5): | (5,1)(3,2)  | (5,3,2)     | (3,5,2)     | (2,1)(5,4)  | (5,4,2)     | (5,4,2,3,1) | (5,4,3,2,1) |
| (2,4)(3,5): | (2,4,3,5,1) | (5,4,3)     | (4,3,2)     | (5,3,4,2,1) | (4,5,3)     | (2,4,5,3,1) | (2,1)(5,3)  |
| (2,5)(3,4): | (3,4,2,5,1) | (3,4,2)     | (5,4,2)     | (4,3,5,2,1) | (3,5,2)     | (3,1)(5,2)  | (3,4,5,2,1) |
| (1,2)(3,4): | (5,2,1):    | (5,3,1):    | (5,3,2):    | (5,4,1):    | (5,4,2):    | (5,4,3):    |             |
| (1,3)(2,4): | (5,1)(4,3)  | (5,4,3,2,1) | (2,5,4,3,1) | (5,3,4,2,1) | (2,5,3,4,1) | (2,1)(5,3)  |             |
| (1,4)(2,3): | (5,4,2,3,1) | (5,1)(4,2)  | (3,4,2,5,1) | (5,2,4,3,1) | (3,1)(5,2)  | (3,5,2,4,1) |             |
| (1,2)(3,5): | (5,3,2,4,1) | (5,2,3,4,1) | (4,1)(5,2)  | (5,1)(3,2)  | (4,3,2,5,1) | (4,2,3,5,1) |             |
| (1,3)(2,5): | (3,5,1)     | (2,3,1)     | (2,3,1)     | (3,5,4,2,1) | (2,3,5,4,1) | (2,1)(5,4)  |             |
| (1,3)(2,5): | (2,3,1)     | (2,5,1)     | (3,5,1)     | (2,5,4,3,1) | (3,1)(5,4)  | (3,2,5,4,1) |             |
| (1,5)(2,3): | (5,3,2)     | (3,5,2)     | (5,2,1)     | (3,2)(5,4)  | (5,4,3,2,1) | (5,4,2,3,1) |             |
| (1,2)(4,5): | (4,5,1)     | (4,5,3,2,1) | (2,4,5,3,1) | (4,2,1)     | (2,4,1)     | (2,1)(4,3)  |             |
| (1,4)(2,5): | (2,4,1)     | (2,5,3,4,1) | (4,1)(5,3)  | (2,5,1)     | (4,5,1)     | (4,3,2,5,1) |             |
| (1,5)(2,4): | (5,4,2)     | (4,2)(5,3)  | (5,3,4,2,1) | (4,5,2)     | (5,2,1)     | (5,2,4,3,1) |             |
| (1,3)(4,5): | (4,5,2,3,1) | (4,5,1)     | (3,2,4,5,1) | (4,3,1)     | (3,1)(4,2)  | (3,4,1)     |             |
| (1,4)(3,5): | (3,5,2,4,1) | (3,4,1)     | (4,1)(3,2)  | (3,5,1)     | (4,2,3,5,1) | (4,5,1)     |             |
| (1,5)(3,4): | (5,2)(4,3)  | (5,4,3)     | (5,4,3,2,1) | (4,5,3)     | (5,3,4,2,1) | (5,3,1)     |             |
| (2,3)(4,5): | (4,5,3,2,1) | (4,5,2,3,1) | (4,5,2)     | (4,1)(3,2)  | (4,3,2)     | (3,4,2)     |             |
| (2,4)(3,5): | (3,5,4,2,1) | (3,1)(4,2)  | (3,4,2)     | (4,5,2)     | (3,5,2,4,1) | (4,5,2)     |             |
| (2,5)(3,4): | (2,1)(4,3)  | (2,5,4,3,1) | (5,4,3)     | (2,5,3,4,1) | (4,5,3)     | (5,3,2)     |             |

# PROGRAM DESCRIPTION

## General Description

The program computes a group table for elements of  $S_n$  which are read in as a series of column and a series of row operations in cycle notation. The general procedure consists of the following steps:

- (1) A set of  $K$  column operations and  $L$  row operations is read into the program.
- (2) Each of these operations is broken down into its equivalent sequence of transpositions. Let the group operation  $P_{ij}$  represent the (group) product of the  $i^{\text{th}}$  row operation  $R_i$  by the  $j^{\text{th}}$  column operation  $C_j$ . Then  $P_{ij}$  will be stored as a long sequence of transpositions.
- (3) The SC is rearranged in accordance with the sequence of transpositions which represent  $P_{ij}$ .
- (4) The rearrangement is examined and a single group operation which effects the same rearrangement of SC is identified as the product  $R_i * C_j$ .
- (5) This single group operation is entered in the  $i^{\text{th}}$  row and  $j^{\text{th}}$  column of the group table array.

The manner in which the machine performs steps (2) to (5) is far from standard. For this reason, an illustrative example is carried through in some detail. The example chosen is the entry in  $A_5$  corresponding to the group multiplication (14235) (521).

For future reference, the notation  $\{\text{SYMBOL}(-, N, M)\}$  is used to denote a complete set of entries for the triply subscripted quantity  $\text{SYMBOL}(L, N, M)$  over the full range of the first subscript  $L$  for fixed  $N$  and  $M$ . Thus,  $\{\text{ANS}(-, I1, I2)\}$  is the collection of  $\text{ANS}(I, I1, I2)$  values with  $I1^{\text{th}}$  column and  $I2^{\text{th}}$  row fixed and with  $I$  allowed to range from its minimum to its maximum value.

The notation  $[A_1, A_2, \dots, A_n]$  is used to display the actual contents of an array. This notation signifies that the content of the first storage location of the array is  $A_1$ , that of the second storage location is  $A_2$ , etc.

An outline of the program, broken down into sections which perform recognizable functions, follows. (The flow charts and listings in appendixes A and B contain more detailed descriptions.)

Block 1 - set up constants. - The constants used in the program are given literal names and are declared either INTEGER or LOGICAL in TYPE statements. All the subscripted variables are dimensioned and allocated storage locations by the use of a DIMENSION statement. A DATA statement is used to give literal names to the following variables:

| Variable | Literal name | Identification            |
|----------|--------------|---------------------------|
| (        | LP           | Left parenthesis          |
| )        | RP           | Right parenthesis         |
| ,        | CM           | Comma                     |
|          | BLK          | Blank                     |
| E        | IDNT         | Identity element of group |

Block 2 - read in DATA. - The data describing the group and the group operations are read into the program. The labels to be assigned to the objects and to the locations are read from the list for IDENT which is limited to one card. It is read in with FORMAT(80A1) and stored in INITAL(N) in the form bbbX, where b represents an 8-bit blank and X represents the 8-bit location in the 32-bit storage register in which the integer from IDENT is stored. This storing arrangement is accomplished by sub-routine SQUEZ, which is described later.

## Description of Input

The input cards are read in the following order:

|             |       |                            |
|-------------|-------|----------------------------|
| First card  | TITLE | one card with FORMAT(80A1) |
| Second card | IDENT | one card with FORMAT(80A1) |

Note that the input for IDENT must be long enough to include the maximum number of objects to be moved in any group operation. Thus, the list for IDENT must be of the form  $(A_1, A_2, \dots, A_n)$ , where  $n$  is the subscript in  $S_n$  and  $A_i$  is any alphanumeric symbol.

The next set of cards contains the ACROSS (column) operations. There can be as many as four cards with FORMAT(80A1), and each operation is followed by a colon.

A blank card follows the preceding set. It is needed to signal the end of the ACROSS input.

The next set contains DOWN (row) operations. There can be as many as four cards with FORMAT(80A1), and again each operation is followed by a colon.

A blank card follows the preceding set. It is needed to signal the end of the DOWN input.

The input for both ACROSS and DOWN operations is in cycle notation.

Several error checks are run on IDENT. These include checking the maximum number of nonblank units (which is six in this version of the program), the presence of

blank spaces on the card, and the illegal use of parentheses.

The group elements themselves are read from the list for INPUT(J, K), which is also read in with FORMAT(80A1). The complete set of column operations is read first. The upper limit of this input is four cards followed by a blank card. Then the complete set of row operations is read in. This set is also limited to four cards followed by a blank card.

### Example of Input - Class Algebra Block $K_2 * K_4$ of $A_5$

$K_2 * K_4$   
 (1,2,3,4,5)  
 (1,2,3):(2,3,4):(1,3,4):(1,2,4):(1,2,5):(1,3,5):(2,3,5):(1,4,5):(2,4,5):(3,4,5):  
 (3,2,1):(4,3,2):(4,3,1):(4,2,1):(5,2,1):(5,3,1):(5,3,2):(5,4,1):(5,4,2):(5,4,3):  
 Card 5 is blank.  
 (1,2,3,4,5):(1,2,5,3,4):(1,4,5,2,3):(1,4,2,3,5):(1,3,5,4,2):(1,3,4,2,5):  
 (5,4,3,2,1):(5,2,1,4,3):(5,4,1,3,2):(5,3,2,4,1):(5,3,1,2,4):(5,2,4,3,1):  
 Card 8 is blank.

Cards 3 and 4 are ACROSS (column) operations, and cards 6 and 7 are DOWN (row) operations.

Block 3 - store each group operation as product of transpositions. - In this block, each group operation is decomposed into its equivalent product of transpositions and then stored in PAIRS(I, NOP, K) (see appendix B). The 3rd subscript K is 1 or 2 according to whether the operation is a column or row element, respectively. Thus, an entry in PAIRS(I, NOP, K) is the  $I^{\text{th}}$  unit of the  $\text{NOP}^{\text{th}}$  column ( $K = 1$ ) or row ( $K = 2$ ) group element.

The example used herein is an entry in the  $K_2 * K_4$  class product block of  $A_5$ . The input for this block, which was given in the preceding section, shows that the element (521) was the 15<sup>th</sup> ACROSS (column) operation and therefore corresponds to  $K = 1$ ,  $\text{NOP} = 15$ . Before being stored in a PAIRS array, the element is considered to be of the form (52)(21). The contents of {PAIRS(-, 15, 1)} is [5 2 2 1] or

|                 |                 |                 |                 |
|-----------------|-----------------|-----------------|-----------------|
| PAIRS(1, 15, 1) | PAIRS(2, 15, 1) | PAIRS(3, 15, 1) | PAIRS(4, 15, 1) |
| 5               | 2               | 2               | 1               |

The element (14235) is the 4<sup>th</sup> DOWN (row) element, and it therefore corresponds to  $K = 2$ ,  $\text{NOP} = 4$ . The contents of the eight registers which constitute the complete {PAIRS(-, 4, 2)} array are [1 4 4 2 2 3 3 5].

Block 4 - set up arrays in standard configuration. - In this block, a number of standard configurations are set up. If  $N_C$  and  $N_R$  are the total number of column and row operations, respectively, then  $N_C \times N_R$  SC's are set up. A given SC is set up by storing the quantity  $\text{bbbI}$  in location  $\text{ANS}(I, \text{II}, \text{I2})$  (see appendix B). For a fixed  $\text{II}$



and I2, and SC is the complete set of ANS(I, I1, I2) over the full range of I (see example in block 5).

It is very important to note the differences in the roles of the second and third subscripts here and in PAIRS. In ANS, the second subscript always denotes the I1<sup>th</sup> column entry and the third subscript always denotes the I2<sup>th</sup> row entry. In PAIRS, on the other hand, the third subscript is used to denote whether a row (K = 2) or a column (K = 1) operation is involved. The second subscript labels which row or column. However, an entry in PAIRS never mixes row and column operations: it refers either to one or to the other, depending on whether the third subscript is 1 or 2.

Block 5 - perform group multiplication. - In this block, the indicated operations for all the group products operation I2 \* operation I1 (= P<sub>I2, I1</sub>) are actually performed. The method used is to transpose successively units from the SC of {ANS(-, I1, I2)} in accordance with the indicated operation of {PAIRS(-, I1, 1)} followed by the operation of {PAIRS(-, I2, 2)}. The way in which this is accomplished may be understood by following the procedure on a single, complete {ANS(-, I1, I2)} array.

The storage location ascribed to ANS(J, I1, I2) is considered to be the J<sup>th</sup> location of the SC. The quantity stored in ANS(J, I1, I2) is considered to be the "thing" which moved to the J<sup>th</sup> location as a result of the operation (operation I2 \* operation I1) on the SC. Schematically, upon entering block 5, the {ANS(-, I1, I2)} array is in the SC. Thus, for A<sub>5</sub>, {ANS(-, I1, I2)} = [1 2 3 4 5].

Continuing with the example of (14235)(521), when K = 1, I1 = 15, and when K = 2, I2 = 4. Therefore, {ANS(-, I1, I2)} = {ANS(-, 15, 4)}. The first rearrangement of the SC of this ANS array is the one effected by {PAIRS(-, 15, 1)}. After being operated on by PAIRS(-, 15, 1), the {ANS(-, 15, 4)} array is [5 1 3 4 2]. After this rearrangement by {PAIRS(-, 15, 1)}, the I2<sup>th</sup> row operation {PAIRS(-, 4, 2)} is brought into play. After the two successive rearrangements effected by the two PAIRS arrays, the {ANS(-, 15, 4)} array is [1 4 5 2 3]. This is the content of {ANS(-, 15, 4)} upon leaving block 5 and entering block 6.

Block 6 - identification of single group product element. - In effect, the permuted stored values in each ANS array leaving block 5 are examined and a determination is made of the single group operation which would have permuted the SC to this final ANS array in one step. It should be recalled that the permuted ANS array which enters block 6 is the result of the successive operations - operation I1 acting on the SC, followed by operation I2 acting on this result.

The same final ANS array can always be obtained by the action of a single group operation on the SC. In block 6, this single operation is determined and stored in {OUT(-, I1, I2)}. Thus, an array {OUT(-, I1, I2)} represents the single group operation which has the same effect on the standard ANS array (SC) as does the product of the two operations (operation I2 \* operation I1).

In order to understand how the program makes the identification, reference may be made to the contents of the  $\{ANS(-, 15, 4)\}$  array upon entering block 6. The content of a given register of ANS is the same as the label of the location in the SC. Since, in the SC, the location and the object had the same label, an examination of the content of a particular ANS register provides partial information about the rearrangement of two objects. Referring to the  $\{ANS(-, 15, 4)\}$  array shows the content of register (2, 15, 4) to be the integer 4. This means that object 4 now occupies the position originally held by object 2. In cycle notation, this state is indicated by placing a 2 to the left of 4 - thus, 2, 4.

The next step is to examine the contents of register 4. Since a 2 is found there, it is clear that a simple transposition of objects 2 and 4 has occurred, so that one part of the group element is the two unit cycle (24).

An examination of the content of register 3 will show a 5. This then leads to an examination of register 5. The content of register 5 is 3, so that another complete cycle has been found. An examination of register 1 (which is actually made first) shows a 1 to be there. This fact is correctly interpreted as evidence that object 1 has not moved, and no 1 will appear in the group element. The examination of all the registers of  $\{ANS(-, 15, 4)\}$  is now complete and the conclusion is reached that the two operation (521) followed by (14235) is equivalent to the single operation (42)(53). Therefore,  $OUT(-, 15, 4) = (4, 2)(5, 3)$ . This agrees with the entry in the table at the intersection of the column headed by (521) and the row headed by (14235).

It would be confusing to examine group table which contained cycles representing the same group operation but written in a different way. Such cycles are called equivalent cycles. A trivial example would be a transposition. Thus, (24) and (42) certainly represent the same operation. Less trivial examples are (123) and its cyclic permutation (231) as well as (312). It should be noted that equivalent cycles cannot be constructed in this program. To understand this, it is sufficient to note two facts. First, the configuration resulting from the operation of a given element of  $S_n$  on the SC is unique. Since equivalent cycles represent the same group element, they all result in the same unique final array. Therefore, two final ANS arrays leaving block 5 can only differ from one another if they are associated with two distinct group elements. Secondly, block 6 follows the same procedure in examining each ANS array and in storing the findings in the corresponding OUT array. Therefore, if  $\{ANS(-, I1, I2)\}$  has the same content as  $\{ANS(-, I3, I4)\}$ , then  $\{OUT(-, I1, I2)\}$  is identical to  $\{OUT(-, I3, I4)\}$ .

Block 7 - output and error messages. - In this block, the group table is listed. The error messages are also contained here.

Subroutine SQUEZ. - The usual manner of storing data on the IBM 360 series is in a word of 32 bits. SQUEZ packs four pieces of data in one storage word. It also elimi-

nates extraneous blanks. Before calling SQUEZ an entry in the group multiplication table would look like the following:

bbb(bbb1bbb, bbb2bbb, bbb3bbb)

After calling SQUEZ we have the following:

(1, 2, 3)

## SUMMARY OF PROCEDURE

A FORTRAN program for use on the IBM 360-67 has been written for the computation of group tables for finite groups. Use is made of the existence of an isomorphism between any finite group of order  $n$  and some subgroup of the symmetric group  $S_n$ . Those elements of  $S_n$  which are isomorphic to the group of interest are fed into the program as input data. The program then performs the group operations on these elements of  $S_n$  and identifies the products of these operations.

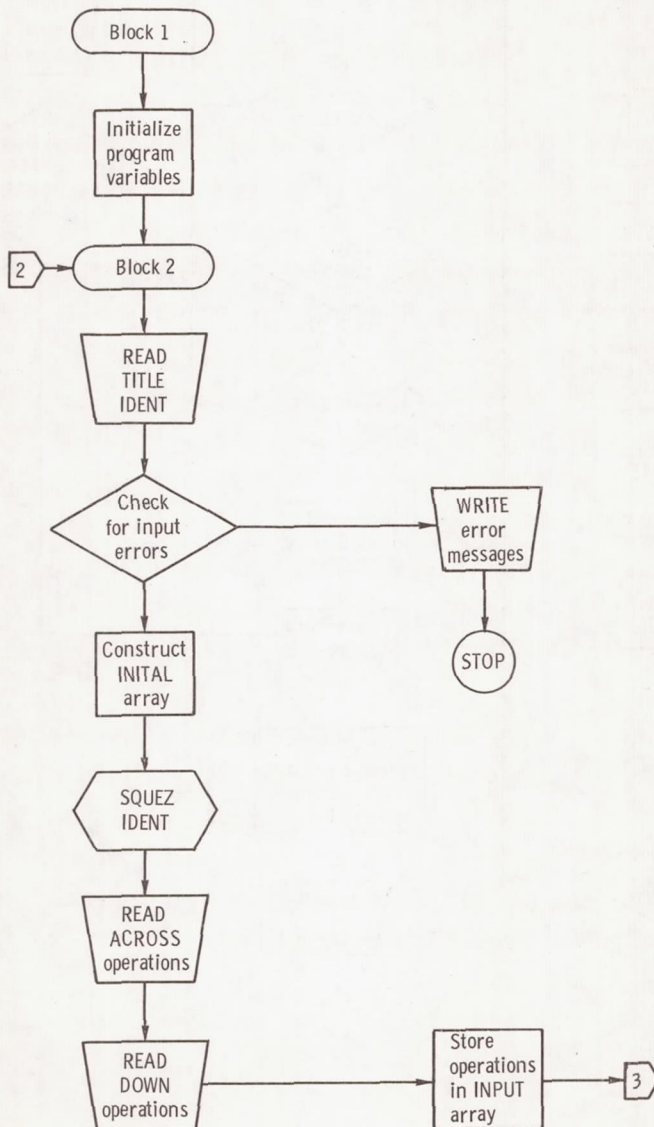
The procedure is based on the fact that every element of  $S_n$  can be expressed as a product of transpositions or simple interchanges of two objects. The program expresses each element given as input data in this form and performs the indicated interchanges for a group product of two such elements on a standard configuration. The resulting configuration is then identified with that configuration which a single group element would cause if applied to the standard configuration. This group element is called the product of the other two and entered in the row and column headed by the original two elements of  $S_n$ .

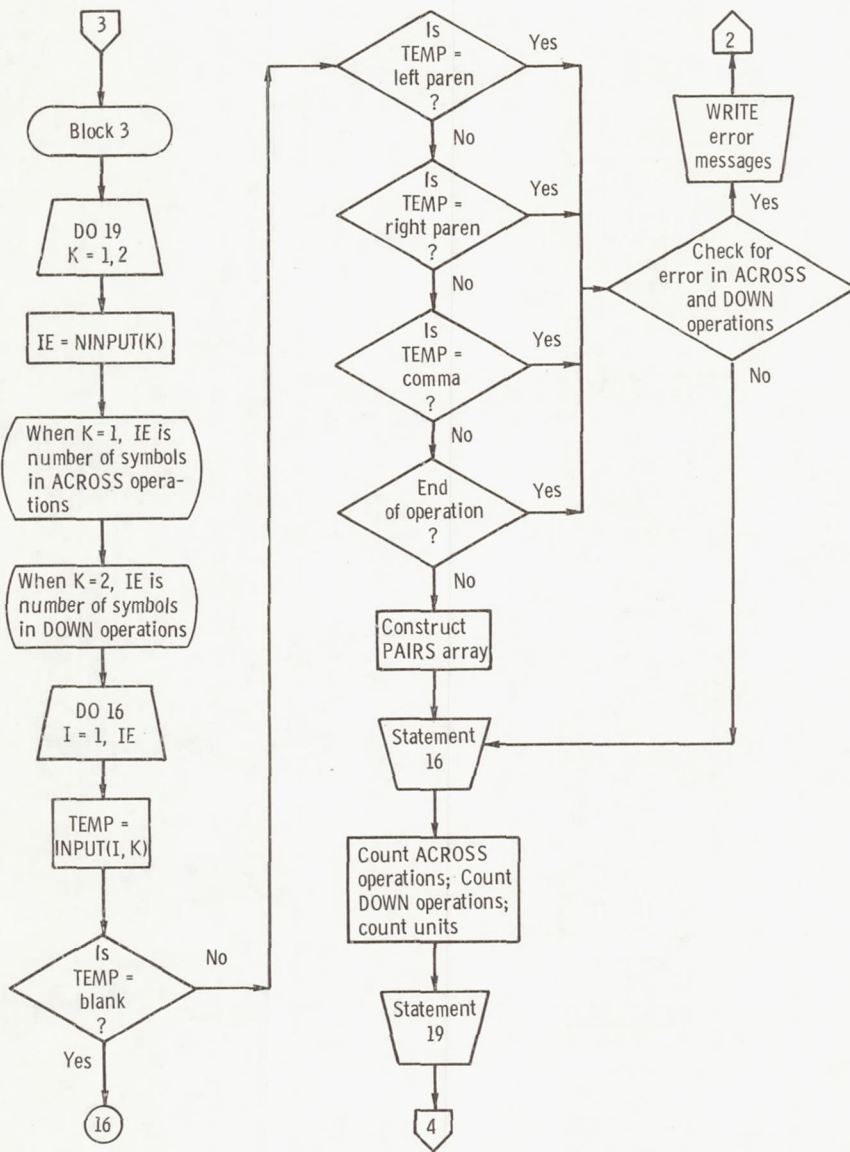
Two examples are shown worked out in the report. One of these is the complete table of  $S_4$ , the other is  $A_5$ . For the latter, the class algebra table is presented. This table, which is of interest in itself, does not appear to have been previously published.

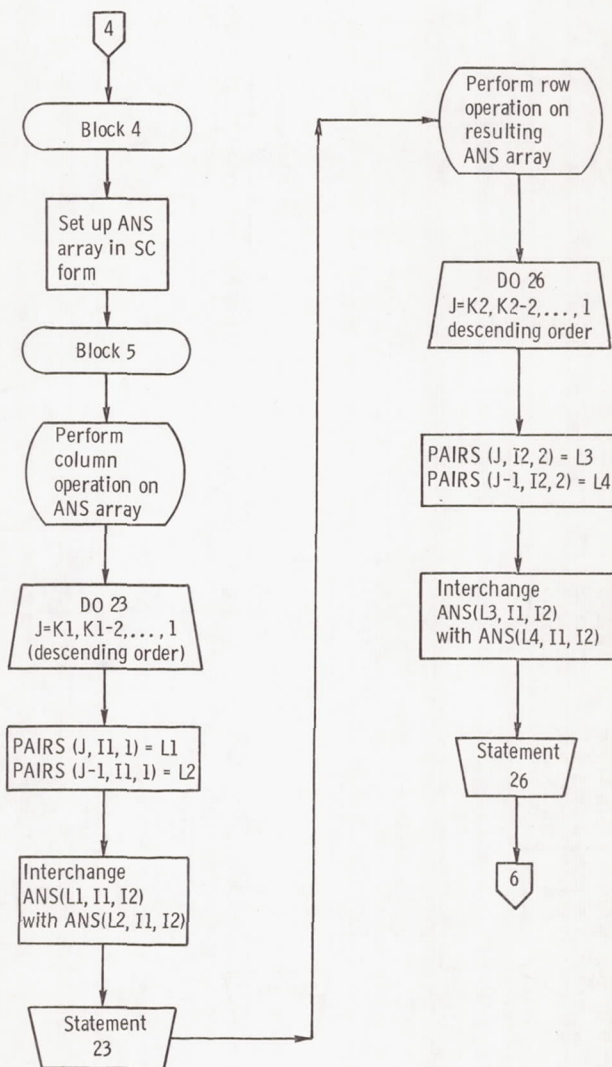
Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, June 2, 1969,  
124-09-12-01-22.

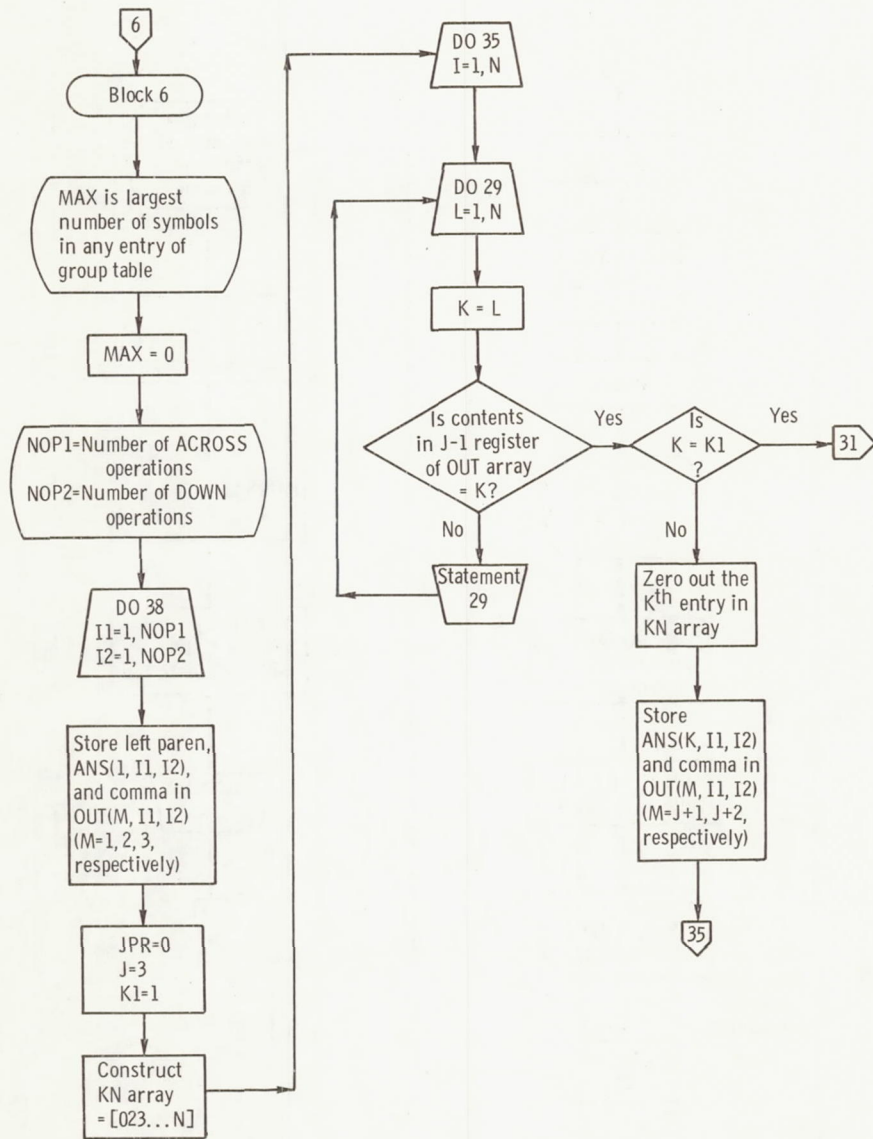
# APPENDIX A

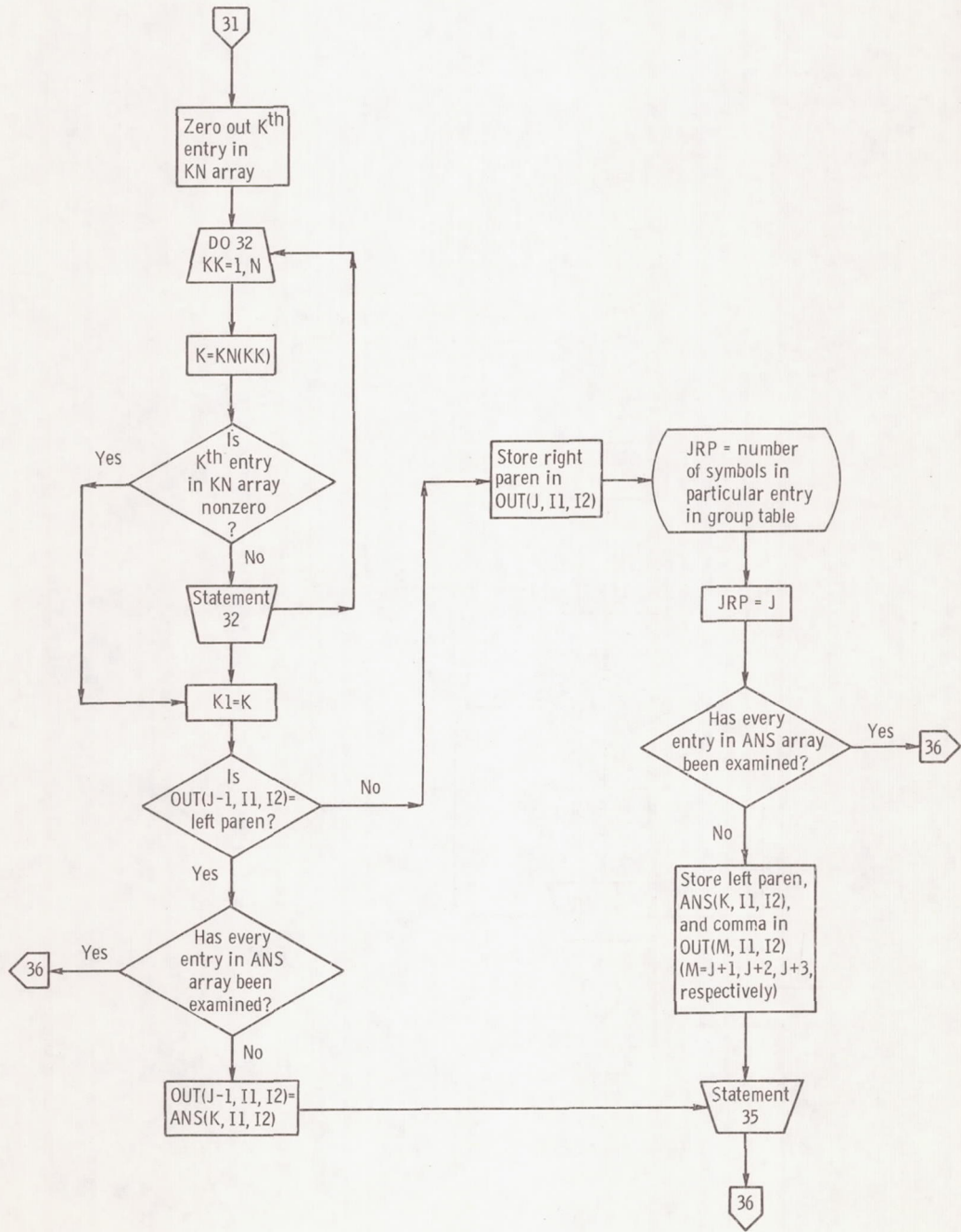
## FLOW OF PROGRAM LOGIC



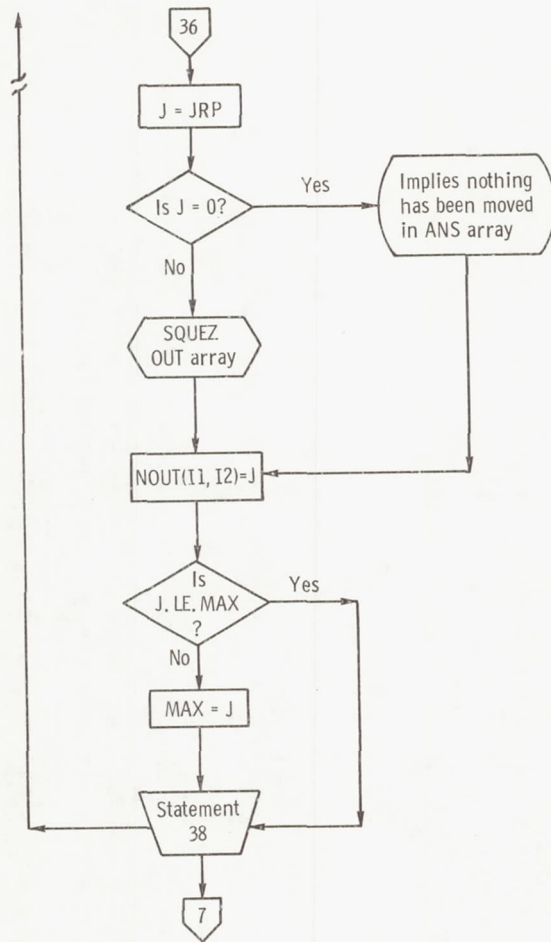


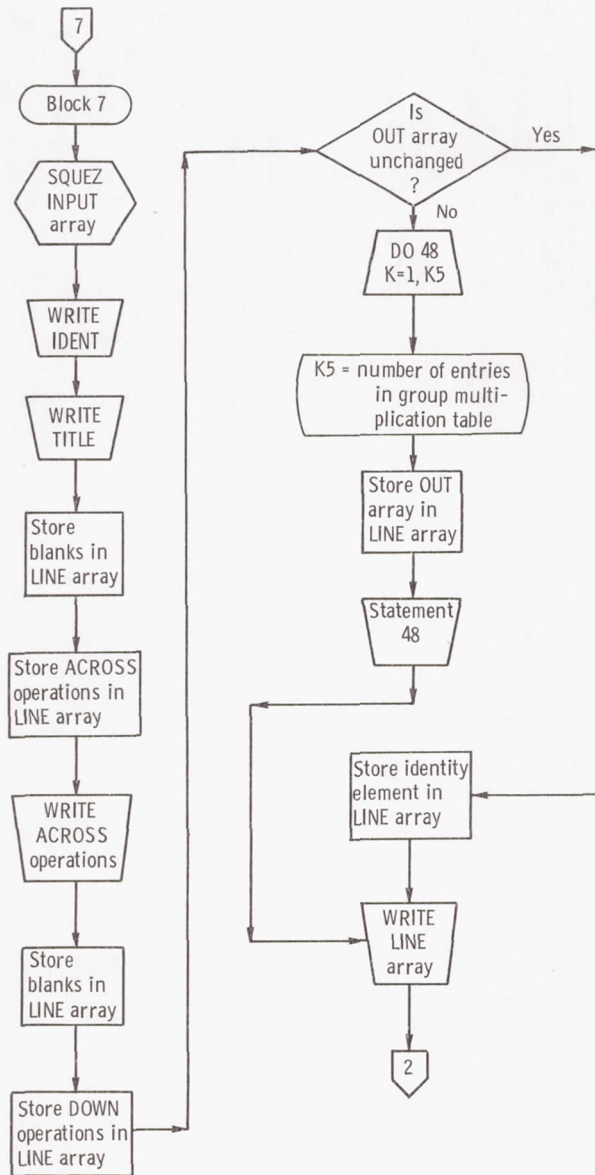


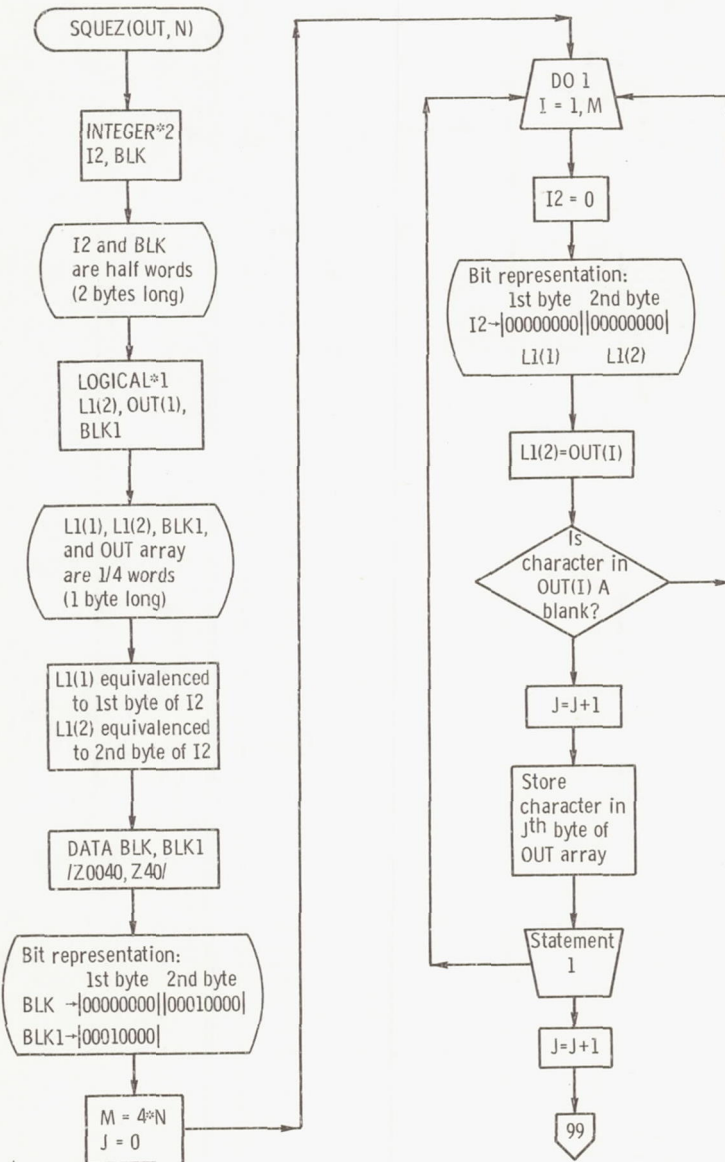


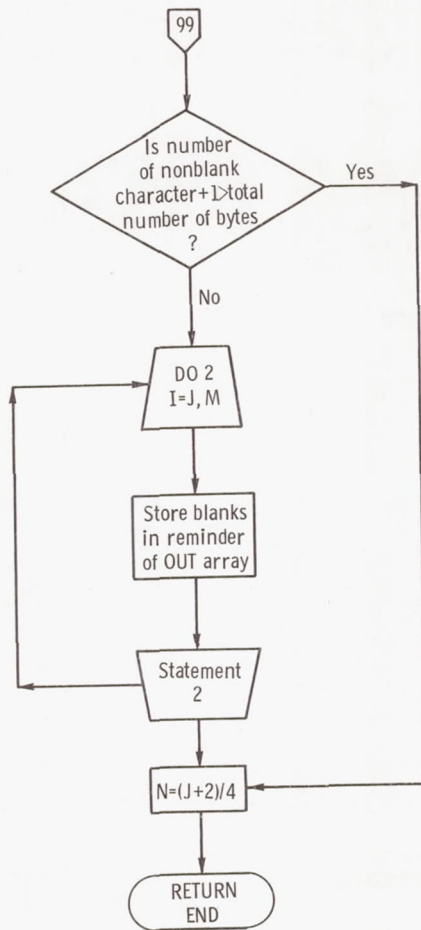












## APPENDIX B

### LISTING

```
C      BLOCK (1)  SETS UP CONSTANTS
C
      DIMENSION INPUT(440,2), INITAL(6), IDENT(80), NPAIR(24,2), NCUT(24
1,24), LINE(32), NIN(24,2), NINPUT(2), NOPA(2), KN(6)
      INTEGER RP,CM,TEMP,BLK,CHAR,COL,PAIRS(18,24,2),ANS(6,24,24),CUT(15
1,24,24),TITLE(80)
      LOGICAL COMMA,CNE
      DATA LP,RP,CM,BLK,COL,IDENT/'(',')',',','.',':','E'/'

C
C      BLOCK (2)  READS IN DATA
C
1      READ (5,66) TITLE
      READ (5,66) IDENT
      ILP=0
      IRP=0
      N=0
      CHAR=BLK
      DO 5 I=1,80
      TEMP=IDENT(I)
      IF (TEMP.EQ.BLK) GO TO 5
      IF (TEMP.EQ.LP) GO TO 2
      IF (TEMP.EQ.RP) GO TO 3
      IF (TEMP.EQ.CM) GO TO 4
      CHAR=ISLL(8,CHAR)+ISRL(24,TEMP)
      GO TO 5
2      ILP=ILP+1
      GO TO 5
3      IRP=IRP+1
4      N=N+1
      IF (N.GT.6) GO TO 61
      IF (CHAR.EQ.BLK) GO TO 62
      INITAL(N)=CHAR
      CHAR=BLK
5      CONTINUE
      IF (ILP.NE.1.OR.IRP.NE.1) GO TO 63
      ID=80
      CALL SQUEZ (IDENT,ID)
      DO 8 K=1,2
      DO 7 I=1,5
      JS=(I-1)*80+1
      JE=I*80
      READ (5,66) (INPUT(J,K),J=JS,JE)
      DO 6 J=JS,JE
      IF (INPUT(J,K).NE.BLK) GO TO 7
6      CONTINUE
      NINPUT(K)=JS
      GO TO 8
7      CONTINUE
8      CONTINUE
```

C  
C BLOCK (3) STORES EACH OPERATION IN PAIRS ARRAY AS A PRODUCT OF  
C TRANSPOSITIONS  
C

```
DO 19 K=1,2
NPAIR(1,K)=0
NOP=1
IPRN=0
CHAR=BLK
II=C
ONE=.TRUE.
COMMA=.FALSE.
IE=NINPUT(K)
DO 16 I=1,IE
TEMP=INPUT(I,K)
IF (TEMP.EQ.BLK) GO TO 16
IF (TEMP.EQ.LP) GO TO 9
IF (TEMP.EQ.RP) GO TO 10
IF (TEMP.EQ.CM) GO TO 11
IF (TEMP.EQ.COL) GO TO 14
CHAR=ISLL(8,CHAR)+ISRL(24,TEMP)
GO TO 15
9 IPRN=IPRN+1
ONE=.TRUE.
IF (IPRN.LT.0.OR.IPRN.GT.1) GO TO 57
GO TO 15
10 IPRN=IPRN-1
IF (.NOT.COMMA) GO TO 58
COMMA=.FALSE.
ONE=.TRUE.
IF (IPRN.LT.0.OR.IPRN.GT.1) GO TO 57
GO TO 12
11 COMMA=.TRUE.
12 NPAIR(NOP,K)=NPAIR(NOP,K)+1
IF (NPAIR(NOP,K).GT.17) GO TO 59
IF (CHAR.EQ.BLK) GO TO 56
IJ=NPAIR(NOP,K)
PAIRS(IJ,NOP,K)=CHAR
IF (ONE) GO TO 13
NPAIR(NOP,K)=NPAIR(NOP,K)+1
IJ=NPAIR(NOP,K)
PAIRS(IJ,NOP,K)=CHAR
13 CHAR=BLK
ONE=.FALSE.
GO TO 15
14 NOP=NOP+1
IF (NOP.GT.24) GO TO 60
NPAIR(NOP,K)=0
15 II=II+1
INPUT(II,K)=TEMP
16 CONTINUE
NINPUT(K)=II+1
IF (INPUT(II,K).NE.COL) GO TO 17
NOP=NOP-1
GO TO 18
17 INPUT(II+1,K)=COL
18 IF (NPAIR(1,K).EQ.0) GO TO 1
NPAIR(K)=NOP
19 CONTINUE
```

```

C
C   BLOCK (4)  SETS UP ANS ARRAY IN STANDARD FORM OR CONFIGURATION
C
NOP1=NOPA(1)
NOP2=NOPA(2)
DO 20 I1=1,NOP1
DO 20 I2=1,NOP2
DO 20 I=1,N
20  ANS(I,I1,I2)=INITAL(I)
C
C   BLOCK (5)  PERFORMS THE PRODUCT OPERATIONS
C
DO 27 I1=1,NOP1
K1=NPAIR(I1,1)
DO 27 I2=1,NOP2
K2=NPAIR(I2,2)
DO 23 I=1,K1,2
J=K1-I+1
J1=0
J2=0
DO 22 L=1,N
IF (ANS(L,I1,I2).NE.PAIRS(J,I1,1)) GO TO 21
J1=L
21  IF (ANS(L,I1,I2).NE.PAIRS(J-1,I1,1)) GO TO 22
J2=L
22  CONTINUE
IF (J1.EQ.0.OR.J2.EQ.0) GO TO 55
ITMP=ANS(J1,I1,I2)
ANS(J1,I1,I2)=ANS(J2,I1,I2)
ANS(J2,I1,I2)=ITMP
23  CONTINUE
DO 26 I=1,K2,2
J=K2-I+1
J1=0
J2=0
DO 25 L=1,N
IF (ANS(L,I1,I2).NE.PAIRS(J,I2,2)) GO TO 24
J1=L
24  IF (ANS(L,I1,I2).NE.PAIRS(J-1,I2,2)) GO TO 25
J2=L
25  CONTINUE
IF (J1.EQ.0.OR.J2.EQ.0) GO TO 55
ITMP=ANS(J1,I1,I2)
ANS(J1,I1,I2)=ANS(J2,I1,I2)
ANS(J2,I1,I2)=ITMP
26  CONTINUE
27  CONTINUE

```

C  
C BLOCK (6) IDENTIFIES THE SINGLE GROUP ELEMENT WHICH HAS THE SAME  
C EFFECT AS BLOCK (5)  
C

```
MAX=0
DO 38 I1=1,NOP1
DO 38 I2=1,NOP2
OUT(1,I1,I2)=LP
OUT(2,I1,I2)=ANS(1,I1,I2)
OUT(3,I1,I2)=CM
JRP=0
J=3
K1=1
DO 28 KK=1,N
28 KN(KK)=KK
KN(1)=0
DO 35 I=1,N
DO 29 L=1,N
K=L
IF (OUT(J-1,I1,I2).EQ.INITAL(K)) GO TO 30
29 CONTINUE
30 IF (K.EQ.K1) GO TO 31
KN(K)=0
J=J+1
OUT(J,I1,I2)=ANS(K,I1,I2)
J=J+1
OUT(J,I1,I2)=CM
GO TO 35
31 KN(K)=0
DO 32 KK=1,N
K=KN(KK)
IF (K.NE.0) GO TO 33
32 CONTINUE
33 K1=K
IF (OUT(J-2,I1,I2).NE.LP) GO TO 34
IF (K.EQ.0) GO TO 36
OUT(J-1,I1,I2)=ANS(K,I1,I2)
GO TO 35
34 OUT(J,I1,I2)=RP
JRP=J
IF (K.EQ.0) GO TO 36
J=J+1
OUT(J,I1,I2)=LP
J=J+1
OUT(J,I1,I2)=ANS(K,I1,I2)
J=J+1
OUT(J,I1,I2)=CM
35 CONTINUE
36 J=JRP
IF (J.EQ.0) GO TO 37
CALL SQUEZ (OUT(1,I1,I2),J)
37 NOUT(I1,I2)=J
IF (J.LE.MAX) GO TO 38
MAX=J
38 CONTINUE
```



```

C      BLOCK (7)  SETS UP THE OUTPUT FORMAT AND WRITES ERROR MESSAGES
C
      DO 41 K=1,2
      J=0
      J1=1
      IE=NINPUT(K)
      DO 40 I=1,IE
      IF (INPUT(I,K).NE.COL) GO TO 40
      MX=I-J1+1
      CALL SQUEZ (INPUT(J1,K),MX)
      IF (MX.LE.MAX) GO TO 39
      MAX=MX
39     J1=MX+J1
      J=J+1
      NIN(J,K)=MX
40     CONTINUE
      IF (J.EQ.NOPA(K)) GO TO 41
41     CONTINUE
      NC=32
      MAX=MAX+1
      NUM=NC/MAX-1
      IF (NUM.LT.2) GO TO 64
      NM=(NOP1+NUM-1)/NUM
      WRITE (6,67) (IDENT(I),I=1,10)
      WRITE (6,68) TITLE
      K1=1
      DO 54 I=1,NM
      DO 42 L=1,32
42     LINE(L)=BLK
      L1=MAX
      DO 44 J=1,NUM
      I1=(I-1)*NUM+J
      IF (I1.GT.NOP1) GO TO 45
      K2=NIN(I1,1)+K1-1
      DO 43 K=K1,K2
      L1=L1+1
43     LINE(L1)=INPUT(K,1)
      L1=(J+1)*MAX
44     K1=K2+1
45     WRITE (6,69) LINE
      K3=1
      DO 53 I2=1,NOP2
      L1=0
46     DO 46 L=1,32
      LINE(L)=BLK
      K4=NIN(I2,2)+K3-1
      DO 47 K=K3,K4
      L1=L1+1
47     LINE(L1)=INPUT(K,2)
      K3=K4+1
      L1=MAX
      DO 51 J=1,NUM
      I1=(I-1)*NUM+J
      IF (I1.GT.NOP1) GO TO 52
      K5=NGOUT(I1,I2)
      IF (K5.EQ.0) GO TO 49
      DO 48 K=1,K5
      L1=L1+1
48     LINE(L1)=OUT(K,I1,I2)
      GO TO 50

```

```

49  LI=LI+1
    LINE(LI)=IDNT
50  LI=(J+1)*MAX
51  CONTINUE
52  WRITE (6,70) LINE
53  CONTINUE
54  CONTINUE
    GO TO 1
55  WRITE (6,71) (IDENT(I),I=1,ID)
    GO TO 1
56  WRITE (6,72)
    GO TO 1
57  WRITE (6,73)
    GO TO 1
58  WRITE (6,74)
    GO TO 1
59  WRITE (6,75)
    GO TO 1
60  WRITE (6,76)
    GO TO 1
61  WRITE (6,77)
    GO TO 65
62  WRITE (6,72)
    GO TO 65
63  WRITE (6,73)
    GO TO 65
64  WRITE (6,78)
    GO TO 1
65  RETURN
66  FORMAT (80A1)
67  FORMAT ('1',10X,'GROUP ',11A4)
68  FORMAT (' ',80A1)
69  FORMAT ('-',32A4)
70  FORMAT (' ',32A4)
71  FORMAT (' ILLEGAL ELEMENT IN GROUP. IDENTITY GROUP IS ',11A4)
72  FORMAT (' BLANK IS NOT A VALID ELEMENT.')
73  FORMAT (' ILLEGAL USE OF PARENS.')
74  FORMAT (' ILLEGAL GROUP.')
75  FORMAT (' THE PAIRS ARRAY HAS BEEN EXCEEDED.')
76  FORMAT (' TOO MANY OPERATIONS. LIMIT IS 24.')
77  FORMAT (' MORE THAN 6 ELEMENTS IN IDENTITY GROUP.')
78  FORMAT (' NOT ENOUGH ROOM ON PRINT LINE TO PRINT TABLE.')
    END

```

```
SUBROUTINE SQUEZ(OUT,N)
INTEGER*2 I2,BLK
LOGICAL*1 L1(2),OUT(1),BLK1
EQUIVALENCE (L1(1),I2)
DATA BLK,BLK1/Z0040,Z40/
M= 4*N
J=0
DO 1 I=1,M
I2= 0
L1(2)= OUT(I)
IF(I2.EQ.BLK) GO TO 1
J= J+1
OUT(J)= OUT(I)
1 CONTINUE
J = J+1
IF(J.GT.M) GO TO 3
DO 2 I=J,M
2 OUT(I)= BLK1
3 N= (J+2)/4
RETURN
END
```

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