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RESEARCH INTO ADVANCED CONCEPTS OF MICROWAVE POWER AMPLIFICATION AND GENERATION UTILIZING LINEAR BEAM DEVICES

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ABSTRACT

This is an interim report which summarizes work during the past six months on a theoretical study of some aspects of the interaction between a drifting stream of electrons with transverse cyclotron motions and an electromagnetic field. Particular emphasis is given to the possible generation and amplification of millimeter waves. The report includes a discussion of the use of a digital computer to obtain start oscillation conditions for a spiraling filamentary electron beam in a cavity for a range of beam and cavity parameters. The report also includes an extension of the previous coupled mode theory to cover the interaction of a spiraling hollow electron beam with a square waveguide. Finally the initiation of a study on the consequences of symmetry on the interaction between an electron beam and a waveguide is mentioned.

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I. INTRODUCTION

The objective of this research program is to explore theoretically some aspects of the interaction between a drifting stream of electrons having transverse cyclotron motions and an electromagnetic field; particular emphasis being given to the possible generation and amplification of millimeter waves. Because of the interest in possible applications to millimeter wavelengths, this study concentrates on electron stream - electromagnetic field interactions which involve an uniform, or fast-wave, circuit structure.

This interim report summarizes the current status of the small signal coupled mode theory for the interaction of a spiraling filamentary electron beam with the TE_{10} and TE_{01} modes of a square waveguide. A method is presented for using the Cornell IBM/360 digital computer to obtain the start oscillation conditions for a spiraling filamentary electron beam in a rectangular cavity for a wide range of electron beam and cavity parameters. An extension of the previous coupled mode theory to cover the interaction of a spiraling hollow electron beam with a square waveguide is presented, and it is shown that, to a first approximation at least, the analysis is similar to that for the spiraling filamentary electron beam. Finally, the initiation of a study on the consequences of symmetry on the interaction between an electron beam and a waveguide is mentioned.

II. SPIRALING FILAMENTARY ELECTRON BEAM INTERACTION

A. Start Oscillation Conditions from Coupled Mode Theory

A small signal, coupled mode theory for the interaction between a spiraling filamentary electron beam and the circuit waves of an uniform waveguide has been developed. Some aspects of this theory have been presented in previous Semiannual Status Reports.[1] The theory has recently been applied to the interaction of a spiraling filamentary electron beam with the fields of a cavity to obtain the start oscillation conditions. The analysis includes the effects of both the wall loss and the coupling to the external circuit on the start oscillation current and frequency.

The interaction cavity is assumed to be a rectangular box with a square cross section of width a and length L. A considerable simplification in the analysis is justified by noting that, in most cases, for normal operation the diameter of the spiraling filamentary electron beam will be much smaller than the width of the cavity. The ratio of the beam diameter to the waveguide width will be of the order of $10^{-3}\sqrt{V_o}$, where V_o is the d-c beam voltage. For values of the beam voltage less than 10,000 volts, the beam diameter will be less than 10 percent of the waveguide width. As a consequence, for the case of interaction with the TE_{10} and TE_{01} modes, one can neglect the variation of the electric and magnetic fields over the beam diameter, and also neglect the interaction of the beam with the longitudinal r-f magnetic field. That is, if the electron beam is centered in the cavity, it sees an essentially plane electromagnetic wave with transverse electric and magnetic fields.

Of the total of ten waves, six beam waves and four circuit waves, only six interact appreciably.[1] These six waves include two cyclotron-type waves (P_+' and P_-'), the longitudinal velocity and displacement waves (V and W), and the forward and reverse positive circularly polarized circuit waves (F_+' and G_+'). A schematic sketch of the oscillator configuration is shown in Figure 1. The spiraling filamentary electron beam originates at the left and terminates on a short circuit plane on the right. A positive circularly polarized input wave F_{+i}' is assumed incident on the left, with a positive circularly polarized output wave G_{+o}' for emerging from the same plane. The start oscillation conditions for the cavity are those particular sets of electron beam and cavity parameers which cause the ratio G_{+o}'/F_{+i}'

B. Computer Calculation of the Start Oscillation Conditions

The IBM 360 digital computer at Cornell University has been programmed to determine the start oscillation conditions for a variety of electron beam and cavity parameters. First, the coupled mode equations for the six coupled waves are solved to give the six propagation constants for the coupled system. The roots are extracted using general complex root

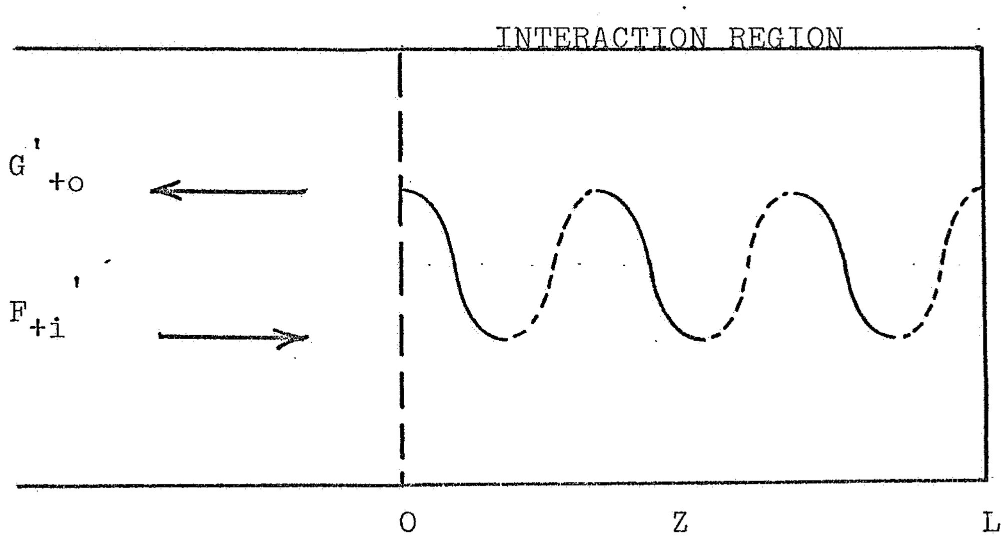

FIGURE 1. Oscillator Configuration for a Spiraling Filamentary Electron Beam in a Rectangular Cavity.

finder; the Double Precision mode is used and the roots are generated to eight place accuracy. The effect of the finite conductivity of the cavity walls is introduced at this point by accounting for the attenuation in the circuit wave equations. Copper walls for the cavity have been explicitly assumed, but any other material could be easily substituted in the calculation.

After the six perturbed propagation constants have been determined, the boundary conditions are applied to determine the relative amplitudes of the various waves of the system. These boundary conditions are:

(1) all a-c velocities and displacements of the electron beam are zero at the left plane of the cavity,

(2) the ratio of the transverse electric field to the transverse magnetic field at the right plane of the cavity is equal to the surface impedance of the wall material,

(3) the transverse electric and magnetic fields at the left plane of the cavity are continuous, and

(4) the impedance looking to the left at the left plane of the cavity is $Z_1 = U Z$, where Z is the characteristic impedance of the square waveguide which forms the cavity.

The fourth boundary condition allows the effect of the coupling to the external circuit to be varied in the calculation of the start oscillation conditions.

In this coupled wave system, there are six beam and circuit modes in the interaction region plus an output circuit wave which must be determined in terms of the input circuit wave. In the interaction region, each mode has a contribution from all six of the perturbed propagation constants, in general. Thus, in principle, a total of thirty-seven constants must be determined to solve the system. In fact, various simplifications which may be applied to this particular system reduce the number to twenty-seven. Application of the boundary conditions and the coupled wave equations lead to a linear system of twenty-seven simultaneous algebraic equations for the mode amplitudes in the interaction region and the output. These equations are written in matrix form, and the computer, programmed in the Double Precision Fortran G mode, solves for the output circuit wave using a Gauss elimination algorithm. Since the amplitude of the input circuit wave is taken as unity, the amplitude of the output circuit wave is the gain of the system considered as a one-port amplifier. If this gain exceeds the arbitrarily selected value of one hundred, a start oscillation condition is assumed to exist.

The determination of the start oscillation conditions requires the specification of most of the cavity and electron beam parameters, and a search for a pole in the gain while one or two parameters are varied. The following parameters have been fixed for an initial survey of start oscillation conditions: $\sigma = \dot{z}_o/c = 0.010$, $\mathcal{f} = \eta\omega_c r_o/c = 0.20$ (these fix $\eta = 0.9805$, and correspond to a d-c beam voltage $V_o = 5050$ volts), $a = 1.6655$ centimeters (corresponding to a waveguide cutoff frequency of 9.00 GHz). In addition, several values of the cyclotron frequency between 10 GHz and 12 GHz and of the d-c beam current between 0.1 ma and 1.0 ma will be explored. In each case the length of the cavity is chosen to be about a half wavelength at the operating frequency. For a given set of the parameters listed above, trial values of the oscillation frequency and cavity coupling factor U (which can be related to the Q_x for the cavity) are tried until a pole of the gain is located; this determines the start oscillation conditions for that parameter set.

Experience with the computer in determing some start oscillation points has shown that the gain poles can be determined with only a few trials once a little experience has been accumulated. A program of systematically exploring the start oscillation conditions has been initiated, and the results will be reported in the future.

III. SPIRALING HOLLOW ELECTRON BEAM INTERACTION

A. Model

The small signal, coupled mode theory for the interaction of a spiraling filamentary electron beam with an uniform waveguide has been extended to cover the interaction of a spiraling hollow electron beam. It is assumed that the electron beam is in the form of a sheath of infinitesimal thickness which rotates at the relativistic cyclotron frequency, $\dot{\theta}_o = \eta\,\omega_c$, and has a radius of r_o. The notation to be used in this analysis is essentially the same as in the spiraling filamentary electron beam analysis (see reference 1).

The major difference for the hollow beam case is that the possibility of azimuthal variations of the beam velocities and displacements must be allowed for, The beam quantities will, in general, be functions of x, y, z and t. In a small signal theory the dependence on x and y can be accounted for by recalling that the d-c position of an electron in the transverse plane is

$$x_o = r_o \cos(\eta\beta_c z + \phi), \qquad (1a)$$

$$y_o = r_o \sin(\eta\beta_c z + \phi), \qquad (1b)$$

where

$$\eta = \sqrt{1 - v^2/c^2},$$

$$\beta_c = \omega_c/\dot{z}_o,$$

$\dot{z}_o$ is the axial d-c velocity, and $\emptyset$ is the angular position of the electron at the entrance plane z = 0. Thus the explicit dependence on x and y can be eliminated by expressing x and y terms of z and $\emptyset$.

Since the beam quantities must be periodic in $\emptyset$, they may be written using a Fourier series in $\emptyset$. For example, the transverse velocities are given by

$$u_{\pm} (x,y,z) = \dot{x}_1 \pm j \dot{y}_1 = \sum_{n=-\infty}^{\infty} u_{\pm,n}(z)\, e^{jn\emptyset} \tag{2a}$$

$$u_{\pm,m} = \frac{1}{2\pi}\int_0^{2\pi} u_{\pm}(x,y,z)\, e^{-jm\emptyset}\, d\emptyset \quad . \tag{2b}$$

with similar expressions for the other beam quantities.

For simplicity, attention is restricted here to the interaction of a spiraling hollow electron beam with plane electromagnetic waves. This does include, however, the interaction with the TE_{10} and TE_{01} modes of a square waveguide for typical operating conditions in which the beam diameter is small compared to the waveguide cross section (see Section II above). Following the previous analysis, the positive and negative circularly polarized forward and reverse circuit wave amplitudes are defined by

$$F_{\pm} = \frac{a}{4\sqrt{Z}} \left(E_{\pm} \mp j\, Z\, H_{\pm}\right) \quad , \tag{3a}$$

$$G_{\pm} = \frac{a}{4\sqrt{Z}} \left(E_{\pm} \pm j\, Z\, H_{\pm}\right) \quad . \tag{3b}$$

Here, $E_{\pm}$ and $H_{\pm}$ are the amplitudes of the positive and negative circularly polarized electric and magnetic field components at the center of the square waveguide.

B. Coupled Mode Equations

The six beam wave equations are obtained from the three relativistic equations of motion and the three equations relating the velocity components to the displacement components. By using the orthogonality properties of the Fourier components of the beam variables, the six beam wave equations are decomposed into an infinite set of groups of six equations. The four circuit wave equations are obtained from the Maxwell curl equations.

For convenience in the coupled mode system, the following normalized Fourier components of the beam variables are defined in Equations (4). The inverse of these equations, or the beam variables in terms of the normalized values, are given in Equations (5). The normalized values are defined in this manner in order to obtain the appropriate eigenvectors of the uncoupled system.

$$U'_{+,m} = M\left[u'_{+,m} + j\frac{\sigma}{\eta\rho}\dot{z}_{1,m-1}\right], \tag{4a}$$

$$U'_{-,m} = M\left[u'_{-,m} - j\frac{\sigma}{\eta\rho}\dot{z}_{1,m+1}\right], \tag{4b}$$

$$Q'_{+,m} = M\left[u'_{+,m} - j\left(1 - \frac{\rho^2}{2}\right)\eta\omega_c\, r'_{+,m} + j\frac{\sigma\rho}{2\eta}\dot{z}_{1,m-1}\right], \tag{4c}$$

$$Q'_{-,m} = M\left[u'_{-,m} + j(1 - \frac{\rho^2}{2})\eta\omega_c r'_{-,m} - j\,\frac{\sigma\rho}{2\eta}\,\dot{z}_{1,m+1}\right] \quad , \tag{4d}$$

$$V_m = M\,\frac{2}{\eta\rho}\,\dot{z}_{1,m}, \tag{4e}$$

$$W_m = M\,\frac{2}{\eta\rho}\,\eta\omega_c z_{1,m} \quad . \tag{4f}$$

$$u'_{+,m} = \frac{1}{M}\left[U'_{+,m} - j\,\frac{\sigma}{2}\,V_{m-1}\right], \tag{5a}$$

$$u'_{-,m} = \frac{1}{M}\left[U'_{-,m} + j\,\frac{\sigma}{2}\,V_{m+1}\right], \tag{5b}$$

$$\eta\omega_c r'_{+,m} = \frac{1}{M}\left[\frac{j}{1-\rho^2/2}\,(Q'_{+,m} - U'_{+,m}) - \frac{\sigma}{2}\,V_{m-1}\right] \tag{5c}$$

$$\eta\omega_c r'_{-,m} = \frac{1}{M}\left[\frac{-j}{1-\rho^2/2}\,(Q'_{-,m} - U'_{-,m}) - \frac{\sigma}{2}\,V_{m+1}\right], \tag{5d}$$

$$\dot{z}_{1,m} = \frac{\eta\rho}{2M}\,V_m \quad , \tag{5e}$$

$$\eta\omega_c z_{1,m} = \frac{\eta\rho}{2M}\,W_m \quad . \tag{5f}$$

$$M = \sqrt{\frac{m\ I_o}{8\ e\eta}} \tag{6}$$

The physical beam variables are, of course, obtained by summing the appropriate Fourier series using the Fourier components defined in Equations (4) and (5).

Using these definitions for the normalized beam variables, and the expressions in Equations (3) for the normalized circuit waves, the ten coupled mode equations are given below.

$$\left(\frac{d}{dz} + j\beta_e + j\eta\beta_c \frac{\rho^2}{2}\right) U'_{+,m} - j\eta\beta_c \frac{\rho^2}{2} U'_{-,m-2} + K \left\{ \left[\left(1 - \frac{\sigma^2}{2} - \eta^2 \frac{\rho^2}{2}\right)(F'_+ + G'_+) \right. \right.$$

$$\left. \left. - \frac{\sigma}{2} \frac{Z_o}{Z} (F'_+ - G'_+) \right] \delta(m,o) + \left[\left(\frac{\sigma^2}{2} + \eta^2 \frac{\rho^2}{2}\right)(F'_- + G'_-) - \frac{\sigma}{2} \frac{Z_o}{Z} (F'_- - G'_-) \right] \delta(m,2) \right\} = 0, \quad (7a)$$

$$\left(\frac{d}{dz} + j\beta_e - j\eta\beta_c \frac{\rho^2}{2}\right) U'_{-,m} + j\eta\beta_c \frac{\rho^2}{2} U'_{+,m+2} + K \left\{ \left[\left(1 - \frac{\sigma^2}{2} - \eta^2 \frac{\rho^2}{2}\right)(F_-^{+} + G'_-) \right. \right.$$

$$\left. \left. - \frac{\sigma}{2} \frac{Z_o}{Z} (F'_- - G'_-) \right] \delta(m,o) + \left[\left(\frac{\sigma^2}{2} + \eta^2 \frac{\rho^2}{2}\right)(F'_+ + G'_+) - \frac{\sigma}{2} \frac{Z_o}{Z} (F'_+ - G'_+) \right] \delta(m,-2) \right\} = 0, \quad (7b)$$

$$\left(\frac{d}{dz} + j\beta_e + j\eta\beta_c\right) Q'_{+,m} - j\eta\beta_c \frac{\rho^2}{2} U'_{-,m-2} + K \left\{ \left[\left(1 - \frac{\sigma^2\rho^2}{4} - \eta^2 \frac{\rho^2}{2}\right)(F'_+ + G'_+) \right. \right.$$

$$\left. - \sigma\frac{Z_o}{Z} \left(1 - \frac{\rho^2}{4}\right)(F'_+ - G'_+) \right] \delta(m,o) + \left[\left(\frac{\sigma^2\rho^2}{4} + \eta^2 \frac{\rho^2}{2}\right)(F'_- + G'_-) \right.$$

$$\left. \left. - \sigma\frac{Z_o}{Z} \frac{\rho^2}{4} (F'_- - G'_-) \right] \delta(m,2) \right\} = 0 \quad , \qquad (7c)$$

$$\left(\frac{d}{dz} + j\beta_e - j\eta\beta_c\right) Q'_{-,m} + j\eta\beta_c \frac{\rho^2}{2} U'_{+,m+2} + K \left\{ \left[\left(1 - \frac{\sigma^2\rho^2}{4} - \eta^2 \frac{\rho^2}{2}\right)(F'_- + G'_-) \right. \right.$$

$$\left. - \sigma\frac{Z_o}{Z} \left(1 - \frac{\rho^2}{4}\right)(F'_- - G'_-) \right] \delta(m,o) + \left[\left(\frac{\sigma^2\rho^2}{4} + \eta^2 \frac{\rho^2}{2}\right)(F'_+ + G'_+) \right.$$

$$\left. \left. - \sigma\frac{Z_o}{Z} \frac{\rho^2}{4} (F'_+ - G'_+) \right] \delta(m,-2) \right\} = 0 \qquad (7d)$$

$$\left(\frac{d}{dz} + j\beta_e\right) V_m - jK\left\{\left[\frac{Z_o}{Z}\,(F_+' - G_+') - \sigma\,(F_+' + G_+')\right]\delta\,(m, -1) - \left[\frac{Z_o}{Z}\,(F_-' - G_-') - \sigma\,(F_-' + G_-')\right]\delta\,(m,1)\right\} = 0, \tag{7e}$$

$$\left(\frac{d}{dz} + j\beta_e\right) W_m - \eta\beta_c V_m = 0 , \tag{7f}$$

$$\left(\frac{d}{dz} + j\,\xi + j\,\eta\,\beta_c\right) F_+' - 2K\left[U_{+,o}' - j\,\frac{\sigma}{2} V_{-1}\right] = 0 , \tag{7g}$$

$$\left(\frac{d}{dz} + j\,\xi - j\,\eta\beta_c\right) F_-' - 2K\left[U_{-,o}' + j\,\frac{\sigma}{2} V_{1}\right] = 0 , \tag{7h}$$

$$\left(\frac{d}{dz} - j\,\xi + j\,\eta\beta_c\right) G_+' + 2K\left[U_{+,o}' - j\,\frac{\sigma}{2} V_{-1}\right] = 0 , \tag{7i}$$

$$\left(\frac{d}{dz} - j\,\xi - j\,\eta\beta_c\right) G_-' + 2K\left[U_{-,o}' + j\,\frac{\sigma}{2} V_{1}\right] = 0 . \tag{7j}$$

$$K = \frac{1}{a\,\dot{z}_o}\sqrt{\frac{\eta_e\, I_o\, Z}{2m}} \tag{8}$$

In these equations, ξ and Z are the propagation constant and characteristic impedance, respectively, for the uncoupled waveguide, $Z_o = \sqrt{\mu_o/\epsilon_o}$, and $\delta(m,p)$ is one for $m = p$ and zero for $m \neq p$.

C. Discussion of the Coupled Mode Equations

Examination of the ten coupled wave equations given in (7a) - (7j) shows that there is coupling between some of the different Fourier components of some of the beam waves. This coupling is caused by relativistic effects, since in the nonrelativistic limit, $\sigma = \mathcal{J} = 0$, the coupling disappears.

The coupling between the different Fourier components of the beam waves divides the Fourier components into two groups. In one group, $U'_{+,m}$, $U'_{-,m}$, $Q'_{+,m}$, $Q'_{-,m}$ for m even and V_m, W_m for m odd all couple together; while in the other group, the remaining Fourier components couple together. It is also seen that the circuit waves couple only to the former group of beam wave Fourier components, at least for TEM circuit waves. Thus, for the interaction of a spiraling hollow electron beam with the TE_{10} and TE_{01} modes of a square waveguide (with the small beam diameter approximation made above), then only the first group of beam wave Fourier components will be excited and the second group can be taken to be zero.

Although an infinite set of Fourier components for the beam waves will be generated, in general, a reasonable first approximation for the analysis of the spiraling hollow electron beam interaction is to take $U'_{+,o}$, $U'_{-,o}$, $Q'_{+,o}$, $Q'_{-,o}$, V_1, V_{-1}, W_1, and W_{-1} as the only non-zero components for the beam waves. When this is done, the $\omega - \beta$ diagrams for the uncoupled beam and circuit waves are given in Figure 2. The region of possible interaction where amplification or oscillation might occur involves $U'_{+,o}$, F'_{+}, G'_{+}, V_{-1}, and W_{-1} in the neighborhood of the intersection of the $\omega - \beta$ curves of the first two of these waves.

With the approximations and simplifications which have been made above, the coupled mode analysis for the spiraling hollow electron beam is essentially similar in form to that for the spiraling filamentary electron beam. Therefore, one would expect that the results of the analysis would also be similar. In addition, one should be able to use the same computer techniques for the investigation of gain, and of start oscillation conditions, with only minor modifications.

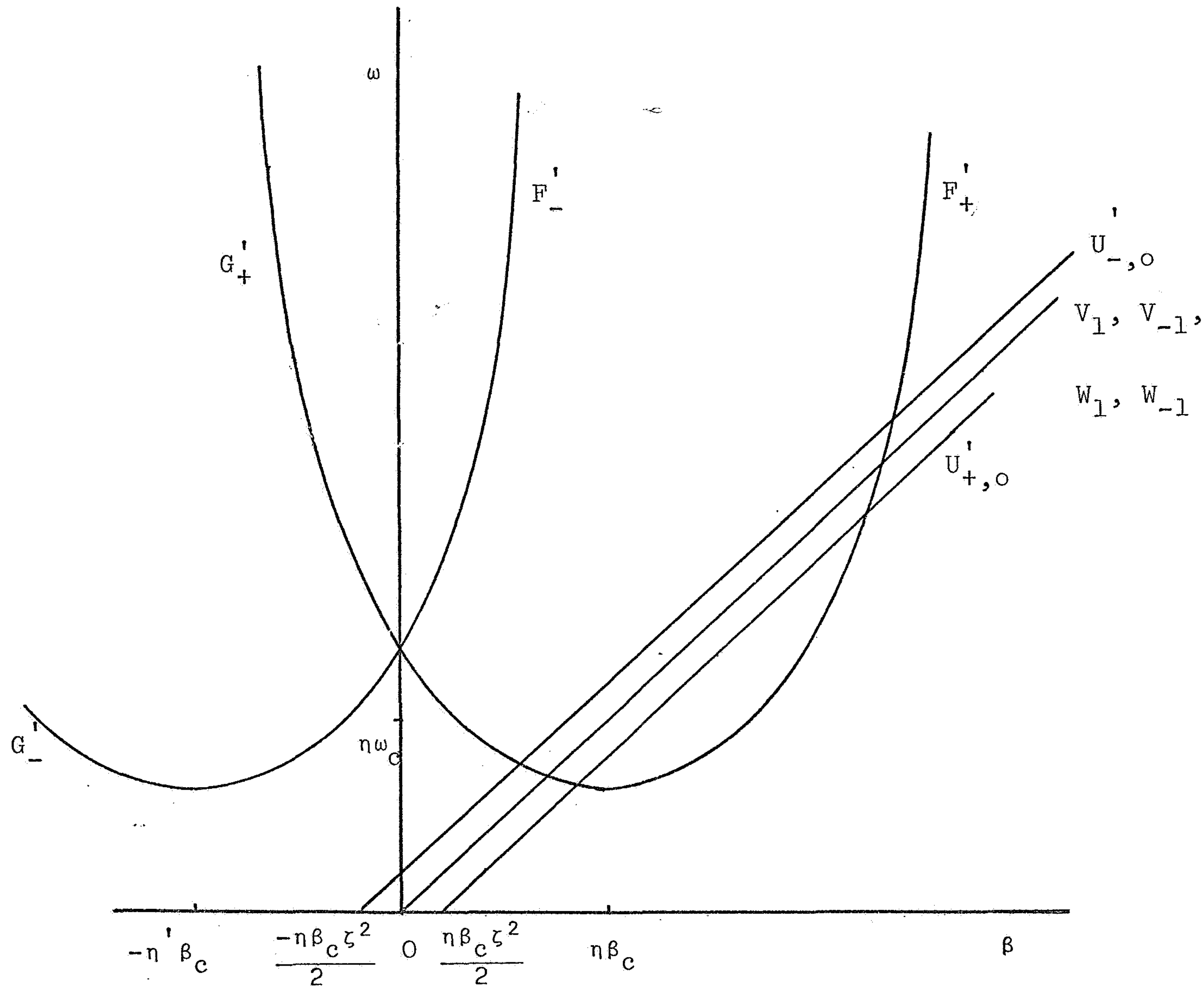

FIGURE 2. ω–β Diagram for Uncoupled Waves of a Spiraling Hollow Electron Beam and a Square Waveguide.

IV. CONSEQUENCES OF SYMMETRY

A study of the consequences of symmetry for the interaction between electron beams and microwave circuits has been initiated. Since this is a field in which almost no research has been done and few results are available, the study is proceeding slowly to establish a firm foundation.

According to its geometric symmetry, any structure (either microwave circuit or electron beam) may be classified in a symmetry group. This symmetry group contains the set of all symmetry operations (e.g., reflections, rotations, inversions) for which the structure is invariant. This symmetry group may be identified with an abstract group[2] containing the same number of group operations, but which are not necessarily associated with geometric or physical transformations.

To each abstract group one can assign matrices chosen so that they are invariant under the set of group operations. These matrices are called representations of the group. It is shown in group theory that for each abstract group there is a particular set of matrices (i.e., a particular set of representations) which is simpler than any other, and in terms of which any other allowed representations can be expressed. These particular representations are called the irreducible representations for the abstract group.

It is known from the theory of linear operators that the irreducible representations of the symmetry groups play an important role in characterizing the eigenvalues and the eigenvectors of physical systems and in determining the possibility of coupling between two systems. The study currently in progress has considered some waveguides belonging to particular symmetry groups, and the possible consequences of perturbations with certain symmetry properties. These perturbations, for example, might be due to an electron beam. It is hoped to exploit the theory of group representations to obtain information concerning the interaction between an electron beam and a waveguide.

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