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PROPERTIES OF MULTILAYER FILTERS

Interim Report

Covering the Period

March 1, 1969 to August 31, 1969

Research Grant No. NGL 33 019 003

with

National Aeronautics and

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ABSTRACT

Various parameters influencing the effect of blocking filters on the long-wavelength rejection of an ultraviolet bandpass filter are studied. The parameters include: (1) the number of blocking filters used; (2) the number of layers in the individual blocking filters; (3) the physical arrangement of the blocking filters with respect to the bandpass filter.

2. Introduction.

During the past six months we have extended our study of the use of blocking filters with a one-M filter. Although the theoretical results are applicable to any region of the spectrum, we have restricted ourselves experimentally to the following problem.

A one-M filter is used to transmit the ultraviolet mercury line at 2537 Å. A Cs-Te "solar-blind" photodetector detects the radiation transmitted by the filter. This detector is insensitive at wavelengths[1] greater than 3400 Å. Attenuation at wavelengths shorter than the passband is provided by absorbing materials. Hence our main concern is to attenuate the long-wave leak occuring at 3120 Å by the use of blocking filters.

A previous report[2] considered experimentally the use of two and four blockers with a one-M filter. A later report[3] continued this study by considering the problem of the physical placement of two blocking filters in relation to the one-M filter. Our present report will extend these results by considering the following three questions:

1. How many dielectric blocking filters can be used to increase the long-wave rejection of a one-M filter without substantially decreasing transmittance in the center of the passband?

2. How many layers should each of the blocking filters contain? I.e., what should be their reflectance?

3. What is the physical placement of the blocking filters in relation to the one-M filter to give the highest attenuation on the long-wave side of the passband?

3. Filter design and fabrication.

3.1 The one-M filter.

Previous reports have described one-M filters having an aluminum layer 200 Å in thickness[4] and 300 A in thickness[5]. The filter having the 200 Å thick aluminum layer has a transmittance of 0.62 at λ_o and 0.11 at the peak of the first long-wave leak. The filter with the 300 Å thick aluminum layer has a transmittance of 0.55 at λ_o and 0.07 at the long wave leak. We note that an increase in aluminum thickness results in decreased transmittance in both wavelength regions. A decrease is not desirable at λ_o but is beneficial in the region of the leak. The decrease in transmittance at the leak, however, is so slight that it does not justify the use of the thicker aluminum film. Also, the resulting decrease in transmittance at λ_o is not desirable in our application. Hence we adopt a compromise and choose an aluminum film of 250 Å thickness.

The computed potential transmittance[6] for this aluminum film is 0.75. The admittance matching stack is designed and then the anti-reflection coating. Two different designs were tried:

Design 1:

air $(HL)^6$ (0.72 H) Al (1.77 L) H $(LH)^6$ substrate

Design 2:

air $(HL)^7$ (0.73 H) Al (0.74 H) $(LH)^7$ substrate

The index of refraction of the quartz substrate is 1.507[7]. H and L represent layers of high and low index of optical thickness a quarter-wave at a wavelength of 2537 Å. The index of refraction of the low-index material,cryolite, is 1.36[8]. The index of refraction of the high-index material, thorium fluoride, is 1.58[8].

For both designs, it is easy to monitor the optical thickness of those layers which are a quarter-wave thick, and the fractional quarter-wave in the anti-reflection stack, since this layer is deposited onto the aluminum layer. There is an important difference between designs 1 and 2, however. The admittance-matching stack for design 1 ends with a layer which is thicker than a quarter-wave, while the corresponding layer in design 2 is less than a quarter-wave thick. It is far easier to monitor the thickness of the layer which is thicker than a quarter-wave. Hence design 1 was fabricated.

Figure 1 is a graph of the measured transmittance of the one-M filter of design 1. λ_o is 2580 Å, slightly longer than the desired 2537 Å. This is due to the angle the reflectance monitoring beam makes with the monitoring glass. This shift should in actuality be somewhat larger; it was compensated for in part by halting the quarter-waves a bit short of the desired thickness. The transmittance at λ_o is 0.60, less than the maximum attainable 0.72. The transmittance at 2537 Å is 0.20.

The transmittance at this wavelength can be increased by tilting the filter[9]; this causes the passband to shift to shorter wavelengths. The first long-wavelength leak occurs at 3120 Å; the transmittance at this wavelength is 0.082. It is this region of high transmittance which must be blocked.

3.2 The blocking filters.

We intended to increase the off-band rejection by placing the narrow-band one-M filter in series with blocking filters. The blocking filters are all-dielectric reflectors which have a high transmittance at 2537 Å and low transmittance (and therefore high reflectance) at 3120 Å.

The design of the blocking filters is

$$\text{air} \quad (H\ L)^m \quad H \quad \text{substrate} \quad ,$$

where m is an integer. The high and low index materials are thorium fluoride and cryolite; the substrate is quartz. The indices of refraction n_H, n_L, and n_S are 1.57, 1.37, and 1.507. We consider four designs by using <u>m</u>'s of 13, 14, 15 and 16; this results in filters with 27, 29, 31, and 33 layers.

The reflectance R_{2m+1} of a stack consisting of $(2m+1)$ layers is given by[10]

$$R_{2m+1} = \left(\frac{1 - n_H \dfrac{n_H}{n_S} \left(\dfrac{n_H}{n_L}\right)^{2m}}{1 + n_H \dfrac{n_H}{n_S} \left(\dfrac{n_H}{n_L}\right)^{2m}} \right)^2 \qquad (2)$$

The transmittance is then given by

$$T_{2m+1} = 1 - R_{2m+1} \qquad (3)$$

This is the transmittance into a semi-infinite medium of index n_S. We must consider multiple-internal reflections from the back surface of the substrate. This is computed via Eq. (4), which gives the transmittance $\underline{T}$ of the filter-substrate combination.

$$T = \frac{T_{2m+1} \quad T_S}{1-R_{2m+1} \; R_S} \quad , \qquad (4)$$

where

$$R_S = \left(\frac{n_S - 1}{n_S + 1} \right)^2 \quad , \qquad (5)$$

and

$$T_S = 1 - R_S \quad . \qquad (6)$$

A substrate rotator was used during the deposition of the blocking filters. This enabled us to fabricate four identical filters simultaneously. Four groups of four filters were made. The first, second, third, and fourth groups consisted of filters with 27, 29, 31, and 33 layers respectively. Table I lists the calculated and experimental transmittances at the wavelength of minimum transmittance for each group. Also listed are the calculated and experimental wavelengths of minimum transmittance.

Number of Layers	Minimum Transmittance Calculated	Minimum Transmittance Experimental	Wavelength of Minimum Transmittance Calculated (Å)	Wavelength of Minimum Transmittance Experimental (Å)
27	0.068	0.069	3120	3150
29	0.052	0.047	3120	3120
31	0.040	0.045	3120	3130
33	0.030	0.031	3120	3135

Table I.

Calculated and experimental values for the minimum transmittance and wavelength of minimum transmittance for the filters of Figs. 2 - 5.

As shown in Table I, the calculated and experimental transmittances at the center of the reflectance band are in close agreement. Also, the wavelength of minimum transmittance of the experimental filters are all reasonably close to 3120 Å. Figs. 2-5 are graphs of the transmittance as a function of wavelength for the filters of Table I.

We might conclude that if two or more identical blocking filters are to be used in conjunction with a one-M filter, it is best to use the group of blocking filters with the least transmittance in the neighborhood of 3120 Å. In other words we should use the blocking filters with the greatest number of layers. However, another criterion should be considered. The transmittance of the blocking filters must be high in the region of the passband of the one-M filter. Table II lists the calculated and experimentally measured transmittances at 2580 Å for the four types of blocking filters.

	Transmittance at 2580 Å	
Number of Layers	Calculated	Experimental
27	0.95	0.90
29	0.93	0.91
31	0.89	0.86
33	0.87	0.77

Table II.

Calculated and experimentally measured transmittances at 2580 Å for the filters of Figs. 2-5.

Outside the stopband, the transmittance oscillates between a maximum and a minimum value as the number of layers is changed, provided all other parameters remain the same[11]. This is shown by the theoretical curves of Figs. 2-5. At a given wavelength, the transmittance changes as m is altered. This explains the different values for the calculated transmittances at 2580 Å as given in Table II.

There is, however, a discrepancy between the calculated and experimental values of the transmittance for a given m, which increases as m increases. As shown in Table I and Figs. 2-5, for a given m the calculated and experimental values of the transmittance at λ_o are in excellent agreement; likewise for the values of λ_o. However, for a given m the computed curve and experimental curve are not congruent, as shown in Figs 2-5. That is to say, although λ_o and the transmittance at λ_o may agree closely, it is possible for the transmittance profiles of the filters to differ. This points out the difficulty of fabricating a filter with a large number of layers and matching all of the layers properly. This difference in profiles is attributed to imperfect control of film thickness and index.

The discrepency between experimental and computed transmittance can also be attributed to the absorption and scattering in the films. This is shown in Table III, which lists the measured values of T, R, and R' as a function of wavelength for a 31 layer blocking filter (not the same 31 layer blocking filter of Tables I and II). Also listed are the sums T+R and T+R'.

Wavelength (A)	T	R	R'	T+R	T+R'
6500	0.944	0.057	0.057	1.001	1.001
5500	0.939	0.052	0.052	1.001	1.001
4500	0.896	0.083	0.081	0.979	0.977
3500	0.832	0.135	0.135	0.967	0.967
2500	0.896	0.070	0.069	0.966	0.965
2000	0.846	0.100	0.105	0.946	0.951

Table III

Measured values of the transmittance *T*, reflectance *R*, and back surface reflectance *R'* of a 31 layer blocking filter.

It is well known that scattering from spheres having diameters much smaller than the wavelength increases inversely as the fourth power of the wavelength[12]. Thus scattering should increase as λ decreases from 4000 Å to 2000 Å. There is also absorption in the dielectric films in this ultraviolet region. This optical absorption increases with decreasing wavelength. If there were no absorption and scattering for the 31 layer filter of Table III, then the sums T+R and T+R' would be unity. Since these sums decrease with decreasing wavelength, it is likely that the discrepancy between the calculated and experimental values of the transmittance for a given value of $\underline{m}$ can be attributed to scattering and absorption.

We have discussed some of the problems which must be overcome to fabricate a filter which transmits in the vicinity of the Mercury line at 2537 Å and has minimal transmittance at wavelengths longer than the passband. Namely, we must first design the one-M filter. Then we must design the blocking filters which must have specific properties in two different wavelengths regions of the spectrum (i.e., high reflectance in one region, and high transmittance in another region). The fundamental limitation in attaining very high reflectance in the blocking region is the appearance of scattering and absorption. As layers are added to the blocking filters to increase their reflectance , the absorption and scattering increase until the transittance of the blocking filters in the region of the passband decreases to an undesirable level. When this occurs, we can no longer increase the blocking of the one-M filter. This limitation is discussed further in section 5.1. In the next two sections we discuss the arrangement of the one-M filter and blocking filters in such a way as to give increased blocking without substantially decreasing the transmittance in the passband.

4. Tandem arrays of filters.

4.1 The transmittance of a tandem array of filters.

We calculate the transmittance of a tandem array of $\underline{n}$ filters. Each filter in the array is referred to as an element of the array. Each array consists of a single one-M filter and n-1 dielectric blocking filters. For simplicity, we assure that the blocking filters have identical optical characteristics. All filters are deposited on one side of a substrate. The substrates are separated so that the transmittance of the array consists of the incoherent superposition of flux reflected from and transmitted through the separate filters.

T_i, R_i, and R_i' are respectively the radiant transmittance, reflectance, and back surface reflectance for the i th element in the array. The transmittance T, the reflectance R, and the back surface reflectance R' of the array are calculated using the matrix formulation of Diofo[13].

$$T = \frac{T_1 \ T_2 \ T_3}{1 - R_2R_1' - R_3R_2' - R_3R_1'T_2^2 + R_2R_1' \ R_3R_2'} \quad . \quad (7)$$

The expression for the transmittance of an array of more than three elements is very cumbersome. Hence, it is best to calculate T, R, and R' for the array by the matrix method.

4.2 Permutations of tandem arrays of 2, 3, 4, and 5 elements.

Consider an array of filters consisting of a single one-M filter and one, two, three, or four identical all-dielectric blocking filters. We designate the blocking filters by the symbol b . Since the blocking filters are non-absorbing, the reflectance is independent of the direction of the incident radiation. The transmittance is independent of direction, even for an absorbing filter[15]. We designate the one-M filter by the symbol <u>M</u> when the incident radiation strikes the film side of the element first, and by the symbol M' when the incident radiation strikes the substrate side of the element first. In the following sections we shall be concerned with the actual one-M filter discussed in section 3.1; its transmittance is shown in Fig. 1. The blocking filters are those discussed in section 3.2; their transmittance is depicted in Figs. 2-5.

We consider the different arrangements of the absorbing one-M filter with a single blocking filter. Upon first examination there would appear to be the following permutations:

1. M b
2. M b'
3. M' b
4. M' b'
5. b M
6. b M'
7. b' M
8. b' M'

Due to the fact that the transmittance is independent of the direction of the radiation, we see that for the following pairs of permutations the transmittances are the same for each permutation in the pair:

1 ; 8

2 ; 6

3 ; 7

4 ; 5

Thus we need only consider four permutations, 1-4. If in addition the blocking filters are nonabsorbing the permutations 1 and 2 have the same transmittance; the same is true of permutations 3 and 4. Thus we shall consider only permutations 1 and 5 (recall that permutation 5 has the same transmittance as permutation 4):

M b

b M

In the same manner we shorten the number of permutations which need to be considered in the case of a one-M filter used with 2, 3, or 4 blocking filters. The results are summarized in Table IV. In this table the first digit in the permutation designation refers to the number of blocking filters; the second digit is an identification index.

Number of blocking filters	Permutation designation	Permutation
1	(1,1)	M b
1	(1,2)	b M
2	(2,1)	M b b
2	(2,2)	b M b
2	(2,3)	b b M
3	(3,1)	M b b b
3	(3,2)	b M b b
3	(3,3)	b b M b
3	(3,4)	b b b M
4	(4,1)	M b b b b
4	(4,2)	b M b b b
4	(4,3)	b b M b b
4	(4,4)	b b b M b
4	(4,5)	b b b b M

Table IV

The permutations of tandem arrays of a one-M filter and 1, 2, 3, and 4 blocking filters to be used in computing the transmittance of the arrays.

5. Computed transmittances of tandem arrays of filters.

We wish to ascertain which permutation of Table IV, for a given number of blocking filters, gives the lowest transmittance at a wavelength of 3120 Å. We initially compute the transmittance of the various permutations using the theoretical characteristics of the blocking filters of Figs. 2-5. This is done in section 5.1. This will enable us to formulate general conclusions concerning which permutation is the best to use for blocking purposes. In section 5.2 we consider the specific case where the effect of scattering and absorption is included. This is accomplished by computing the transmittance of various permutations using the experimental characteristics of the blocking filters of Figs. 2-5.

5.1 Tandem arrays with the computed blocking filters of Figs. 2-5.

Tables V, VI, VII, and VIII list the computed transmittances at 3120 Å for the four groups of permutations of Table IV. Blocking filters of 27, 29, 31, and 33 layers are considered in each table. $T_{product}$ is the product of the transmittances of the elements of a given array, without taking into account inter-elemental reflections. T_{array} is the array transmittance taking these reflections into account; it was computed using the matrix method. The following conclusions are drawn from these tables:

(1) T_{array} is always greater than $T_{product}$ since the inter-elemental reflections direct more flux through the array.

Number of layers in blocking filter	Permutation	$T_{product}$	T_{array}
27	(1,1)	0.005 559	0.019 120
27	(1,2)	0.005 559	0.014 409
29	(1,1)	0.004 251	0.015 260
29	(1,2)	0.004 251	0.011 328
31	(1,1)	0.003 270	0.012 136
31	(1,2)	0.003 270	0.008 901
33	(1,1)	0.002 453	0.009 367
33	(1,2)	0.002 453	0.006 798

Table V.

Computed transmittances at 3120 Å for the permutations of an array containing a single one-M filter and a single blocking filter. The computations use the measured values of T, R, and R' at 3120 Å for the one-M filter, and the calculated values of T, R, and R' at 3120 Å for the blocking filters.

Number of layers in blocking filter	Permutation	$T_{product}$	T_{array}
27	(2,1)	0.000 378	0.010 826
27	(2,2)	0.000 378	0.003 554
27	(2,3)	0.000 378	0.007 901
29	(2,1)	0.000 221	0.008 415
29	(2,2)	0.000 221	0.002 243
29	(2,3)	0.000 221	0.006 086
31	(2,1)	0.000 131	0.006 555
31	(2,2)	0.000 131	0.001 409
31	(2,3)	0.000 131	0.004 707
33	(2,1)	0.000 074	0.004 968
33	(2,2)	0.000 074	0.000 834
33	(2,3)	0.000 074	0.003 546

Table VI.

Computed transmittances at 3120 Å for the permutations of an array containing a single one-M filter and two blocking filters. The computations use the measured values of T, R, and R' at 3120 Å for the one-M filter, and the calculated values of T, R, and R' at 3120 Å for the blocking filters.

Number of layers in blocking filters	Permutation	$T_{product}$	T_{array}
27	(3,1)	0.000 026	0.007 550
27	(3,2)	0.000 026	0.002 027
27	(3,3)	0.000 026	0.001 959
27	(3,4)	0.000 026	0.005 442
29	(3,1)	0.000 111	0.005 810
29	(3,2)	0.000 111	0.001 245
29	(3,3)	0.000 111	0.001 211
29	(3,4)	0.000 111	0.004 160
31	(3,1)	0.000 005	0.004 490
31	(3,2)	0.000 005	0.000 765
31	(3,3)	0.000 005	0.000 748
31	(3,4)	0.000 005	0.003 199
33	(3,1)	0.000 002	0.003 380
33	(3,2)	0.000 002	0.000 445
33	(3,3)	0.000 002	0.000 437
33	(3,4)	0.000 002	0.002 399

Table VII.

Computed transmittances at 3120 Å for the permutations of an array containing a single one-M filter and three blocking filters. The computations use the measured values of T, R, and R' at 3120 Å for the one-M filter, and the calculated values of T, R, and R' at 3120 Å for the blocking filters.

Number of layers in blocking filters	Permutation	$T_{product}$	T_{array}
27	(4,1)	0.000 001 7	0.005 797
27	(4,2)	0.000 001 7	0.001 418
27	(4,3)	0.000 001 7	0.001 118
27	(4,4)	0.000 001 7	0.001 352
27	(4,5)	0.000 001 7	0.004 151
29	(4,1)	0.000 000 5	0.004 436
29	(4,2)	0.000 000 5	0.000 862
29	(4,3)	0.000 000 5	0.000 672
29	(4,4)	0.000 000 5	0.000 829
29	(4,5)	0.000 000 5	0.003 160
31	(4,1)	0.000 000 2	0.003 414
31	(4,2)	0.000 000 2	0.000 525
31	(4,3)	0.000 000 2	0.000 406
31	(4,4)	0.000 000 2	0.000 509
31	(4,5)	0.000 000 2	0.002 423
33	(4,1)	less than 10^{-7}	0.002 562
33	(4,2)	"	0.000 303
33	(4,3)	"	0.000 233
33	(4,4)	"	0.000 296
33	(4,5)	"	0.001 813

Table VIII.

Computed transmittances at 3120 Å for the permutations of an array containing a single one-M filter and four blocking filters. The computations use the measured values of T, R, and R' at 3120 Å for the one-M filter, and the calculated values of T, R, and R' at 3120 Å for the blocking filters.

As an example suppose two blocking filters of 31 layers each are used. As shown in Table VI $T_{product}$ for 31 layers is 0.000 131. The three permutations (2,1), (2,2) and (2,3) give T_{array} of 0.006 555, 0.001 409, and 0.004 707, respectively.

(2) For a given number of blocking filters, the permutation which gives the lowest transmittance is independent of the number of layers in the blocking filters.

As an example, see Table VI. Regardless of the number of layers in the blocking filters, the permutation (2,2) has the lowest transmittance.

For convenience Table IX lists the permutations giving the lowest transmittance for a given number of blocking filters. Fig. 6 illustrates these permutations schematically. We continue with the conclusions:

(3) It is seen from Table IX that for an even number of blocking filters

Number of blocking filters	permutation designation	permutation
1	(1,2)	b M
2	(2,2)	b M b
3	(3,3)	b b M b
4	(4,3)	b b M b b

Table IX

The permutations giving the least transmittance for a given number of blocking filters.

the transmittance is least when the elements are arranged symmetrically as a sandwich, with the blocking filters serving as the "bread". This was determined in the last report for the case of two blockers[16].

(4) In the case of three blockers there is not much difference in the transmittance of the (3,2) and (3,3) permutations. This is reasonable. There can be no symmetry in the case of three blockers (i.e., four elements). The lowest transmittance is still obtained when the one-M filter is sandwiched between the blockers. Since there are two sandwiching possibilities, it is expected that they should have similar properties.

For convenience, we list in Table X the array transmittance for that permutation which provides the lowest transmittance for various numbers of blocking filters. The array transmittance is tabulated as a function of the number of layers.

Number of blocking filters	Number of layers	T_{array}
1	27	0.014 409
1	29	0.011 328
1	31	0.008 901
1	33	0.006 798
2	27	0.003 554
2	29	0.002 243
2	31	0.001 409
2	33	0.000 834
3	27	0.001 959
3	29	0.001 211
3	31	0.000 748
3	33	0.000 437
4	27	0.001 118
4	29	0.000 672
4	31	0.000 406
4	33	0.000 233

Table X

Array transmittance for the permutations of least transmittance as obtained from Tables V-VIII.

(5) For a given number of blocking filters, as the number of layers is increased, the array transmittance decreases.

(6) For a given number of layers, as the number of blocking filters is increased, the array transmittance decreases.

(7) It is possible to combine various numbers of blocking filters with various numbers of layers to achieve almost the same array transmittance.

For example, T_{array} will be almost the same for 2 blockers of 31 layers, 3 blockers of 29 layers, and 4 blockers of 27 layers; T_{array} will be almost the same for 2 blockers of 33 layers, 3 blockers of 31 layers, and 4 blockers of 29 layers.

(8) For an array containing one blocking filter (for a given number of layers) the quotient of the maximum and minimum values of T_{array} (which occur for the different permutations) is about 1.4. This quotient is between 3 and 6 for arrays containing two blocking filters; between 4 and 8 for arrays containing 3 blocking filters; and between 5 and 11 for arrays containing 4

blocking filters.

Of the foregoing conclusions, 3, 4, and 2 are perhaps the most significant. Conclusions 3 and 4 tell us in what way to arrange the filters in order to attain lowest transmittance in the wavelength region to be blocked. Conclusion 2 shows us that this arrangement is independent of the number of layers in the blocking filters; this gives the method wide applicability.

We see from conclusion 8 that the increase in blocking gained by permuting the elements of a given array depends upon the number of blocking filters in the array and upon the number of layers in these blocking filters. For one blocking filter it is hardly worth permuting the arrangement of the one-M and blocking filter; the decrease in transmittance is slight. For arrays containing two and three blocking filters about half an order of magnitude of decrease of transmittance may be gained by arranging the filters properly; this may be worth doing in some applications where highest possible blocking is to be obtained. In the case of 4 blocking filters of 33 layers, a factor of 10 can be obtained by proper arrangement.

Conclusions 5 and 6 are not suprising and are predicted from theoretical considerations. As more blocking filters are used and as the number of layers in them is increased the transmittance in the blocking region decreases. As is pointed out in 3.2, however, the accompanying decrease in transmittance in the region of the passband provides the fundamental limitation to the process of adding layers and blocking filters. We must also examine the transmittance at the wavelength of 2580 Å.

We examined $T_{product}$ and T_{array} at 2580 Å for the one-M filter and 1, 2, 3, and 4 blocking filters; each blocking filter contained 27 layers. The following conclusions are drawn.

(1) As expected, $T_{product}$ is slightly less than T_{array} for a given number of blocking filters.

(2) The foregoing implies that for a given number of layers, and for the same number of blocking filters, at wavelength of 2580 Å, various permutations do not result in substantial differences in T_{array}. This is important because it implies that for a given number

of blocking filters, we can choose the permutation which gives the lowest transmittance in the region where blocking is required. This choice does not greatly decrease the transmittance in the region of the passband.

(3) The transmittance decreases as more blocking filters are used. It is therefore best to pick the permutation for the least transmittance at 3120 Å, and then examine the transmittance at 2580 Å as a function of the number of blocking filters and the number of layers in each. This is done in Table XI. The permutations chosen are those of Table IX.

As the number of layers is increased for one blocking filter, T_{array} decreases from 0.58 to 0.53; this is not a substantial decrease. In the case of two blocking filters, the decrease is from 0.55 to 0.47; this is a slightly larger decrease. In the case of three blocking filters, the decrease is from 0.53 to 0.42. In the case of four blocking filters, the decrease is from 0.51 to 0.38; this is definitely a large decrease. How much of a decrease may be tolerated depends of course upon the application.

Number of blocking filters	Number of layers in blocking filters	Permutation	T_{array}
1	27	(1,2)	0.580 124
1	29	(1,2)	0.568 331
1	31	(1,2)	0.544 693
1	33	(1,2)	0.532 847
2	27	(2,2)	0.554 816
2	29	(2,2)	0.533 808
2	31	(2,2)	0.493 226
2	33	(2,2)	0.473 627
3	27	(3,3)	0.529 732
3	29	(3,3)	0.500 818
3	31	(3,3)	0.447 464
3	33	(3,3)	0.422 817
4	27	(4,3)	0.507 984
4	29	(4,3)	0.472 813
4	31	(4,3)	0.410 098
4	33	(4,3)	0.382 063

Table XI

Computed transmittance at 3120 Å for the permutations of least transmittance for arrays containing a single one-M filter and 1,2,3, and 4 blocking filters. The computations use the measured values of T, R, and R' at 2580 Å for the one-M filter, and the calculated values of T, R, and R' at 2580 Å for the blocking filters.

To summarize the results of this section:

The blocking filters increase the offband attenuation. Greater attenuation results from the increase of the number of blocking filters and an increase in the number of layers of the blocking filters. The offband transmittance can also be decreased by rearranging the order of the blocking filters and the one-M filter. Various permutations, for a given number of blocking filters and layers, has little effect on the transmittance in the region of the passband. The addition of extra layers and the addition of extra blocking filters decreases the transmittance in the passband. Thus there is a compromise. We should arrange the filters so that we obtain the most attenuation in the blocking region, since this does not effect the transmittance in the passband.

We next consider the effect of absorption and scattering on the previous conclusions.

5.2 Tandem arrays with the experimental blocking filters of Figs. 2-5.

We compute T_{array} for the one-M filter and the experimental data of Figs. 2-5. The reason theoretical data was used previously for the blocking filters, was the concern that some possible conclusions concerning T_{array} as a function of permutation choice might be obscured by the appearance of scattering and absorption. In that case general conclusions applicable to other cases would not have been possible.

In the case of the experimental curves of Figs. 2-5, the reflectance R is not in general equal to the back surface reflectance R' . Also, the sums T + R and T + R' are not quite unity. This is attributed to scattering and absorption, as is discussed in section 3.2. T, R, and R' were measured for four blocking filters each of 27 layers at 3120 Å. T_{array} and $T_{product}$ were then computed. The results are summarized in Table XII.

We see that the relations between $T_{product}$ and T_{array} as a function of the permutation are still valid despite the absorptance. Thus, in the next section where we are concerned with dispersive calculations, we shall continue to arrange the filters in the permutation which gives the lowest transmittance at 3120 Å.

Permutation	$T_{product}$	T_{array}
(1,1)	0.055 692	0.018 492
(1,2)	0.055 692	0.013 839
(2,1)	0.000 385	0.007 324
(2,2)	0.000 385	0.003 234
(2,3)	0.000 385	0.005 358
(3,1)	0.000 026	0.003 152
(3,2)	0.000 026	0.001 302
(3,3)	0.000 026	0.001 271
(3,4)	0.000 026	0.002 277
(4,1)	0.000 001 8	0.001 332
(4,2)	0.000 001 8	0.000 544
(4,3)	0.000 001 8	0.000 496
(4,4)	0.000 001 8	0.000 540
(4,5)	0.000 001 8	0.000 979

Table XII

Computed transmittance at 3120 Å for the permutations of arrays consisting of 1, 2, 3, and 4 blocking filters. The computations use the measured values of T, R, and R' at 3120 Å for all filters.

6. Experimental transmittance of tandem arrays of filters.

A cary model 14 ratio recording spectrophotometer was used to measure T, R, and R' for the one-M filter and four groups of four blocking filters each. Each group of blocking filters had the same optical properties and consisted of 27, 29, 31, and 33 layers respectively. For transmittance measurements, the half-cone angle in the Cary is about 4^{o}. For the reflectance measurements, a Strong-type V-W reflectance attachment was used in which the incident radiation is reflected twice from the sample at an angle of 8^{o}. The measurement of R and R' was corrected for this angle shift.

Transmittance was measured for four arrays from 2000 Å to 4000 Å; the permutation (4,3) was used in all cases. The values of T, R, and R' at several wavelengths were used to compute T for each array. The results are given in Figs. 7-10.

Fig. 7 shows the transmittance from 2000 Å to 4000 Å of a one-M filter and four identical blocking filters. The filters are arranged in the (4,3) permutation. The circles correspond to the computed transmittance and there is excellent argeement between theory and experiment.

The parameters of Fig. 8 are identical to those of Fig. 7 except that the blocking filters have 29 layers instead of 27. In this case the agreement between theory and experiment is good except at the points near the minimum of the blocking region.

For Fig. 9 blocking filters of 31 layers are used. For Fig. 10 blocking filters of 33 layers are used. Here, as is the case with Fig. 8, there is good agreement between theory and experiment except in the region of low transmittance.

As the number of layers in the blocking filters is increased from 27 to 33, T_{array} in the blocking region decreases from 0.000 716 to 0.000 061, which is an order of magnitude. At the same time T_{array} at 2580 Å decreases from 0.45 to 0.29, which is not a substantial decrease.

Considering the problems of scattered light in the spectrophotometer, the difficulty of making transmittance measurements at very low light levels, and the difficulty in making reflectance measurements, the agreement between theory and experiment for Figs. 7-10 is reasonable.

7. Conclusions

The attenuation on the long-wavelength side of the passband of a one-M filter can be increased by using more blocking filters and by increasing the number of layers in the individual filters. This decreases the transmittance in the region of the passband; how much can be tolerated depends upon the application. Significant further attenuation in the blocking region can be attained in the case of three or more blocking filters by arranging the blocking filters and the one-M filter in the manner described in Table IX; this will not significantly decrease transmittance in the region of the passband.

8. Personnel.

Mr. Bo Baengstrom	Graduate Research Assistant Six weeks full time
Miss Ann-Marie Eckendahl	Graduate Research Assistant Six weeks full time
Mr. Douglas Harrison	Graduate Research Assistant 50% during the academic year, full time during two summer months

9. References to the literature.

1. Technical Chart, "Typical Absolute Spectral Response Characteristics of Photoemissive Devices". Published by ITT Industrial Laboratories, Fort Wayne, Indiana.

2. "Properties of Multilayer Filters", Interim report of Research Grant NsG 308-63 (published Sept., 1968). p. 11.

3. "Properties of Multilayer Filters", Interim Report of Research Grant NGL 33 019 003 (published March, 1969). p. 8.

4. Ref. 1. p. 8, also Fig. 5 .

5. "Properties of Multilayer Filters", Final Report of Research Grant NsG 308-63 (published March, 1967). p. 18, also Figs. 19 and 20.

6. Ref. 5. p. 5.

7. Manufacturer's published data. The substrate is Suprasil quartz, manufactured by Engelhard Industries, Hillside, New Jersey.

8. Robert E. Hahn, "High Contrast Bandpass Filters for the Ultraviolet", Thesis for the Master of Science Degree in Optics, University of Rochester, 1968. p. 27.

9. M. L. Baker and V. L. Yen, "Effects of the Variation of Angle of Incidence and Temperature on Infrared Filter Characteristics", Appl. Opt. <u>6</u>, 1343 (1967).

10. M. Born and E. Wolf, "Principles of Optics", Pergamon Press, 1959, p. 69.

11. J. Arndt and P. Baumeister, "Reflectance and Phase Envelopes of an Iterated Multilayer", J. Opt. Soc. Am. 56, 1760 (1966).

12. Ref. 10 . p. 658.

13. A Diofo, Rev. d'Optique <u>47</u>, 49 (1968).

14. Ref. 3. p. 5.

15. P. H. Berning, "Theory and Calculation of Optical Thin Films", in "Physics of Thin Films", Vol. 1 (G. Hass, ed). Academic Press, 1963, p. 98.

16. Ref. 3 . p. 8.

10. Captions to the figures.

1. The measured spectral transmittance of a one-M filter which contains a 250 Å thick aluminum film. The filter design is

$$\text{air} \quad (HL)^6 \; (0.72\,H) \; Al \; (1.77L) \; H \; (LH)^6 \quad \text{quartz} \;,$$

where H and L are layers of quarterwave optical thickness at 2537 Å of thorium fluoride and cryolite, respectively.

2. The calculated and measured spectral transmittance of a 27 layer blocking filter of the design

$$\text{air} \quad (H\;L)^{13}\; H \quad \text{quartz} \quad ,$$

where H and L are layers of quarterwave optical thickness at 2537 Å of thorium fluoride and cryolite, respectively.

3. The calculated and measured spectral transmittance of a 29 layer blocking filter of the design

$$\text{air} \quad (H\;L)^{14}\; H \quad \text{quartz},$$

where H and L are layers of quarterwave

optical thickness at 2537 Å of thorium fluoride and cryolite, respectively.

4. The calculated and measured spectral transmittance of a 31 layer blocking filter of the design

$$\text{air} \quad (H\ L)^{15} \quad H \quad \text{quartz} \quad ,$$

where H and L are layers of quarterwave optical thickness at 2537 Å of thorium fluoride and cryolite, respectively.

5. The calculated and measured spectral transmittance of a 33 layer blocking filter of the design

$$\text{air} \quad (H\ L)^{16} \quad H \quad \text{quartz} \quad ,$$

where H and L are layers of quarterwave optical thickness at 2537 Å of thorium fluoride and cryolite, respectively.

6. The permutations giving the least transmittance for a one-M filter and 1, 2, 3, and 4 blocking filters. M designates the one-M filter and b the blocking filters.

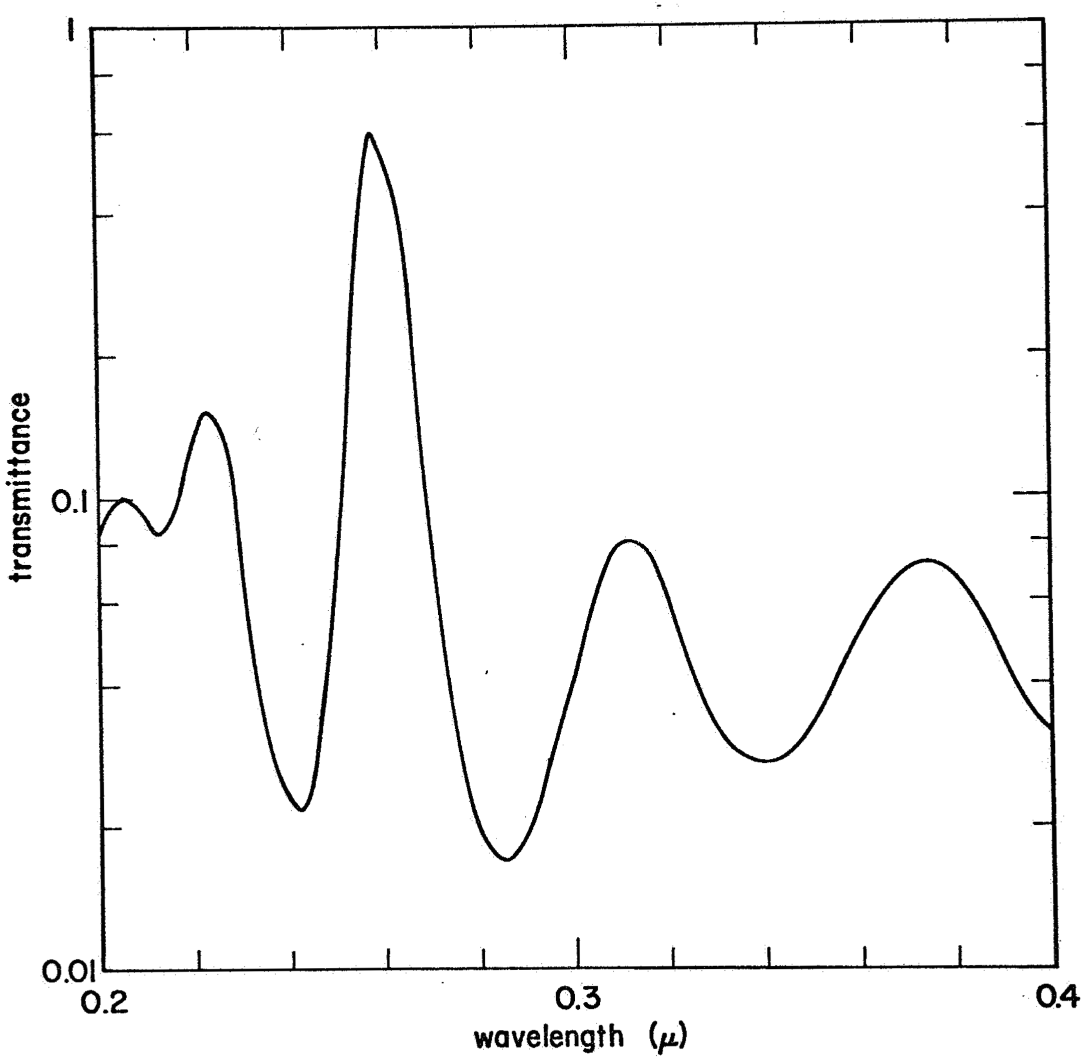
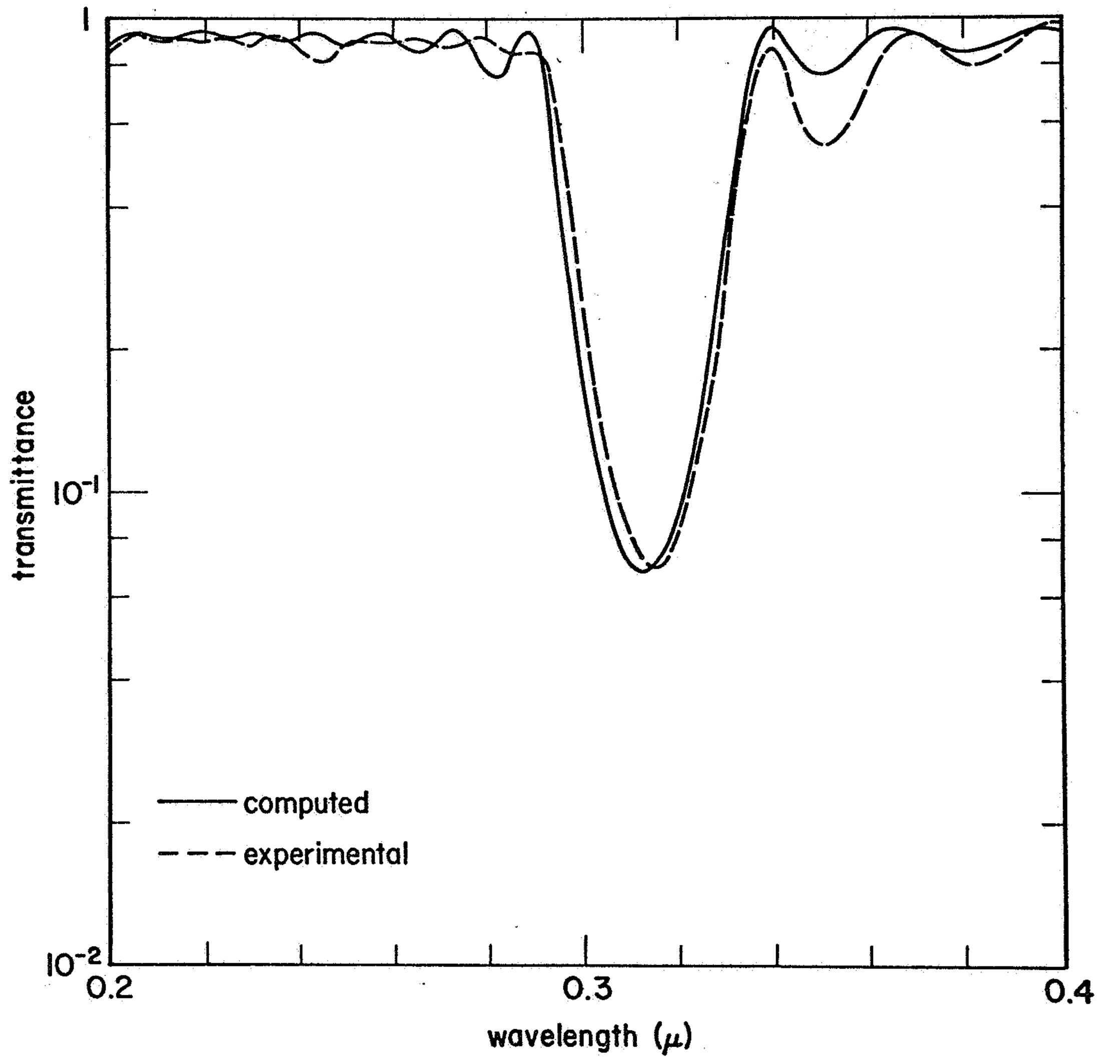
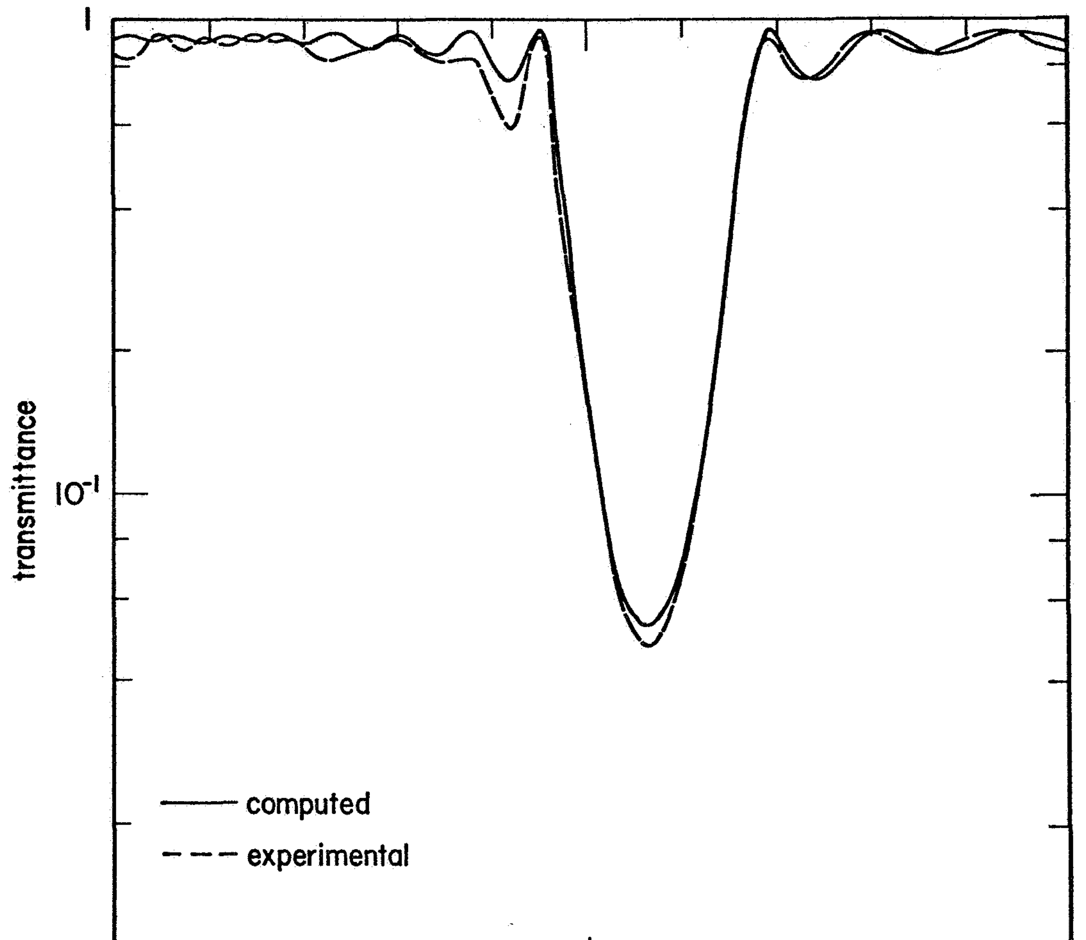


FIG. 1



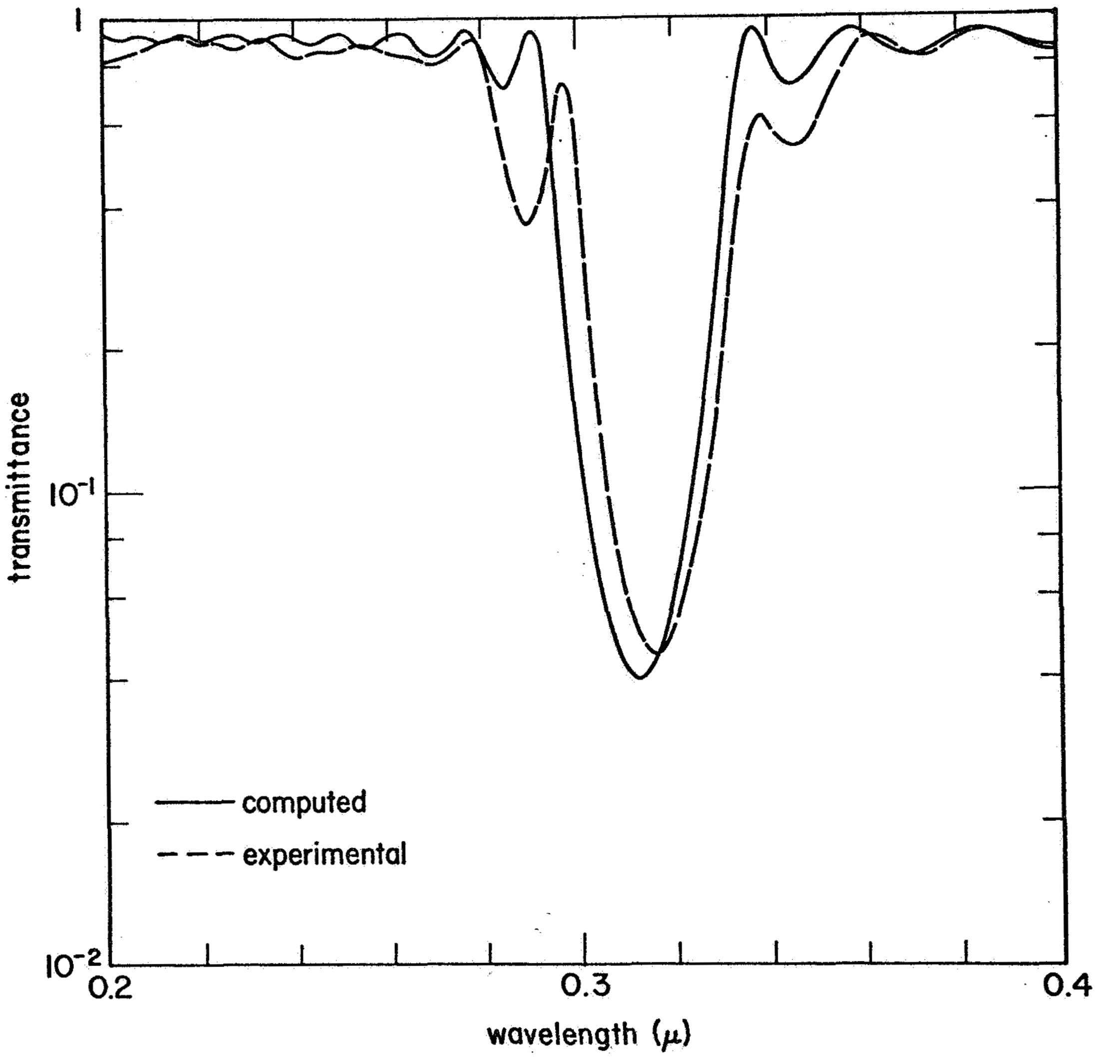
1
10^{-1}
10^{-2}
transmittance
0.2
0.3
0.4
wavelength (μ)
computed
experimental

FIG. 2



1
10^{-1}
10^{-2}
transmittance
0.2
0.3
0.4
wavelength (μ)
computed
experimental

FIG. 3



1
10^{-1}
10^{-2}
transmittance
0.2
0.3
0.4
wavelength (μ)
computed
experimental

FIG. 4

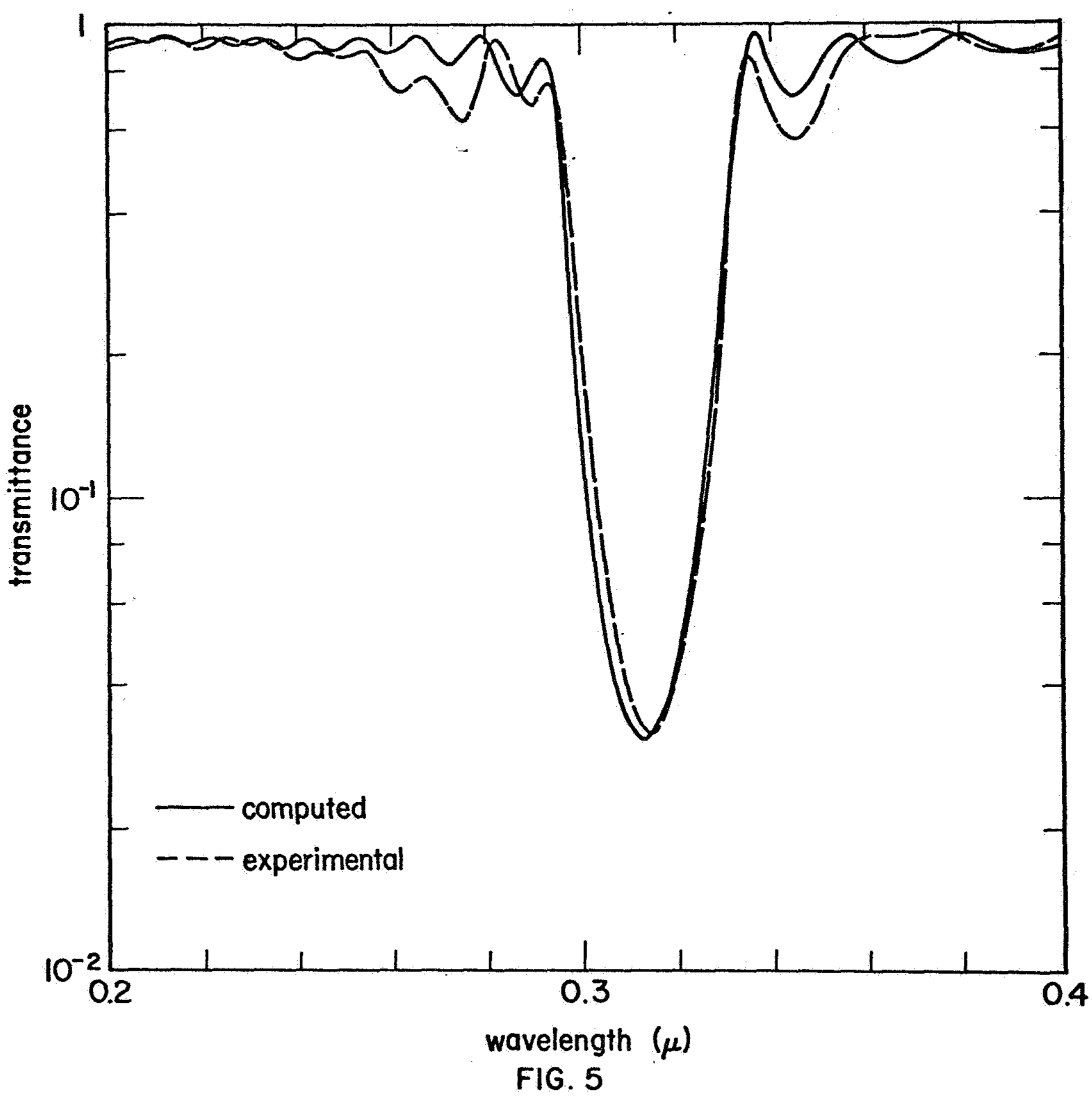

1
10^{-1}
10^{-2}
transmittance
computed
experimental
0.2
0.3
0.4
wavelength (μ)

FIG. 5

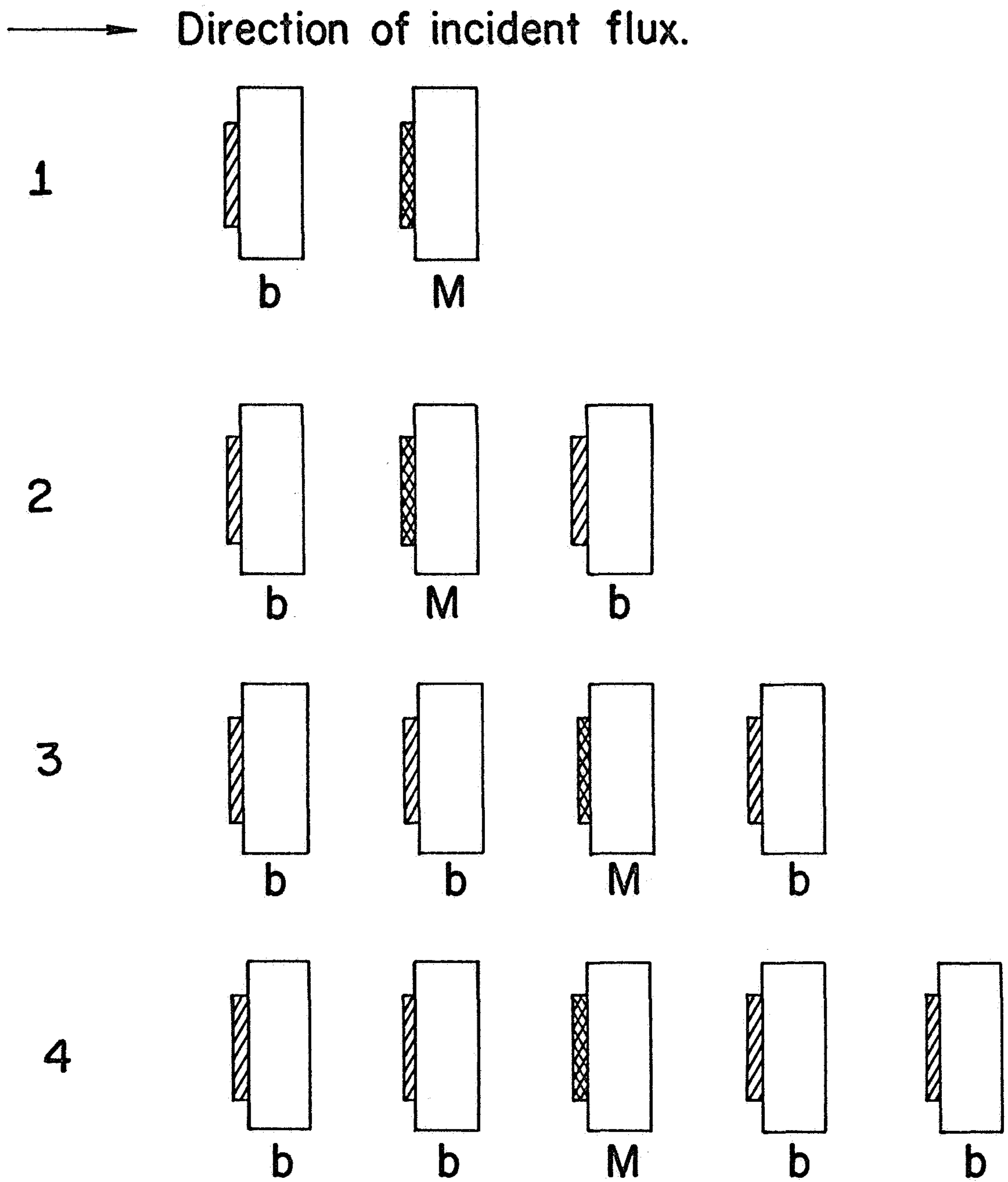
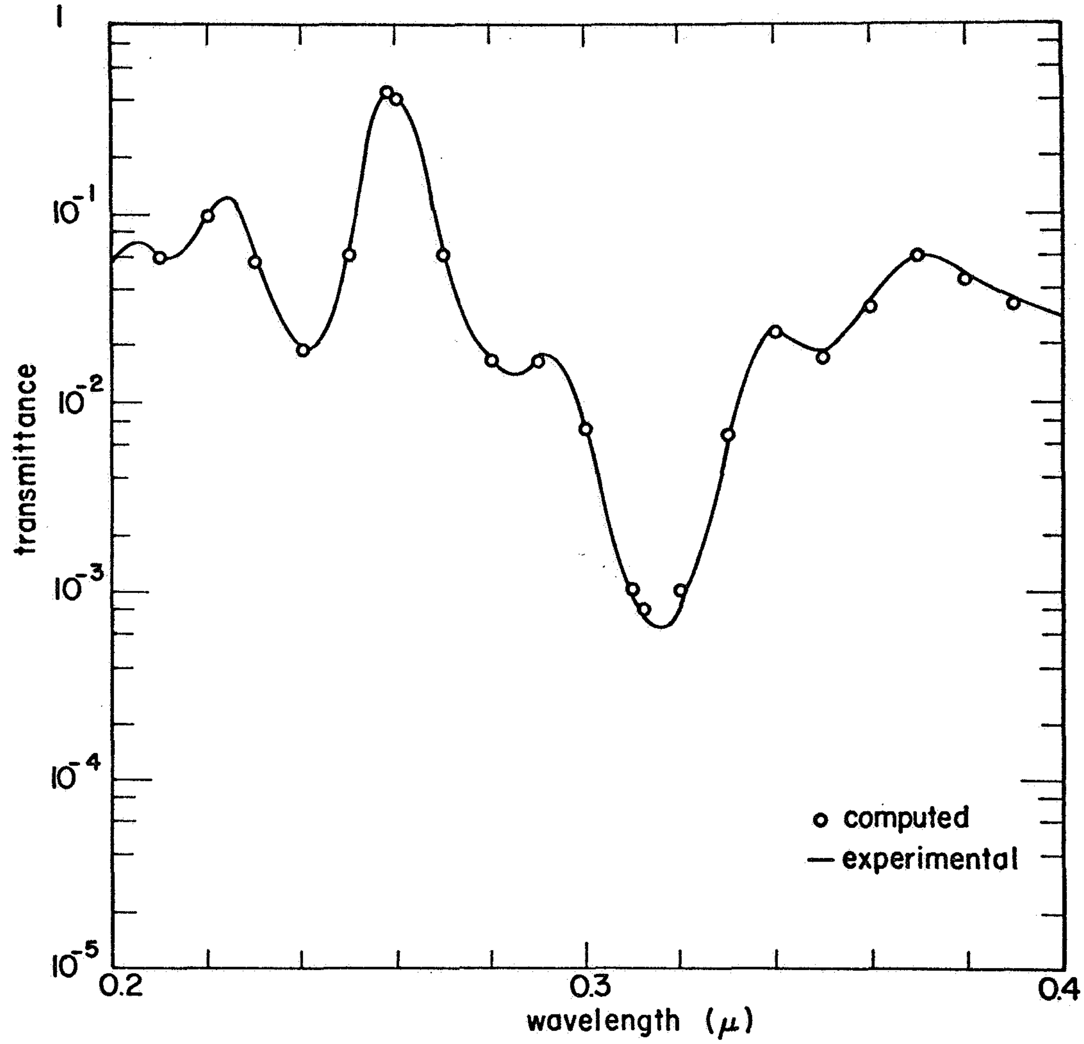


FIG. 6



1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
transmittance
0.2
0.3
0.4
wavelength (μ)
o computed
— experimental

FIG. 7

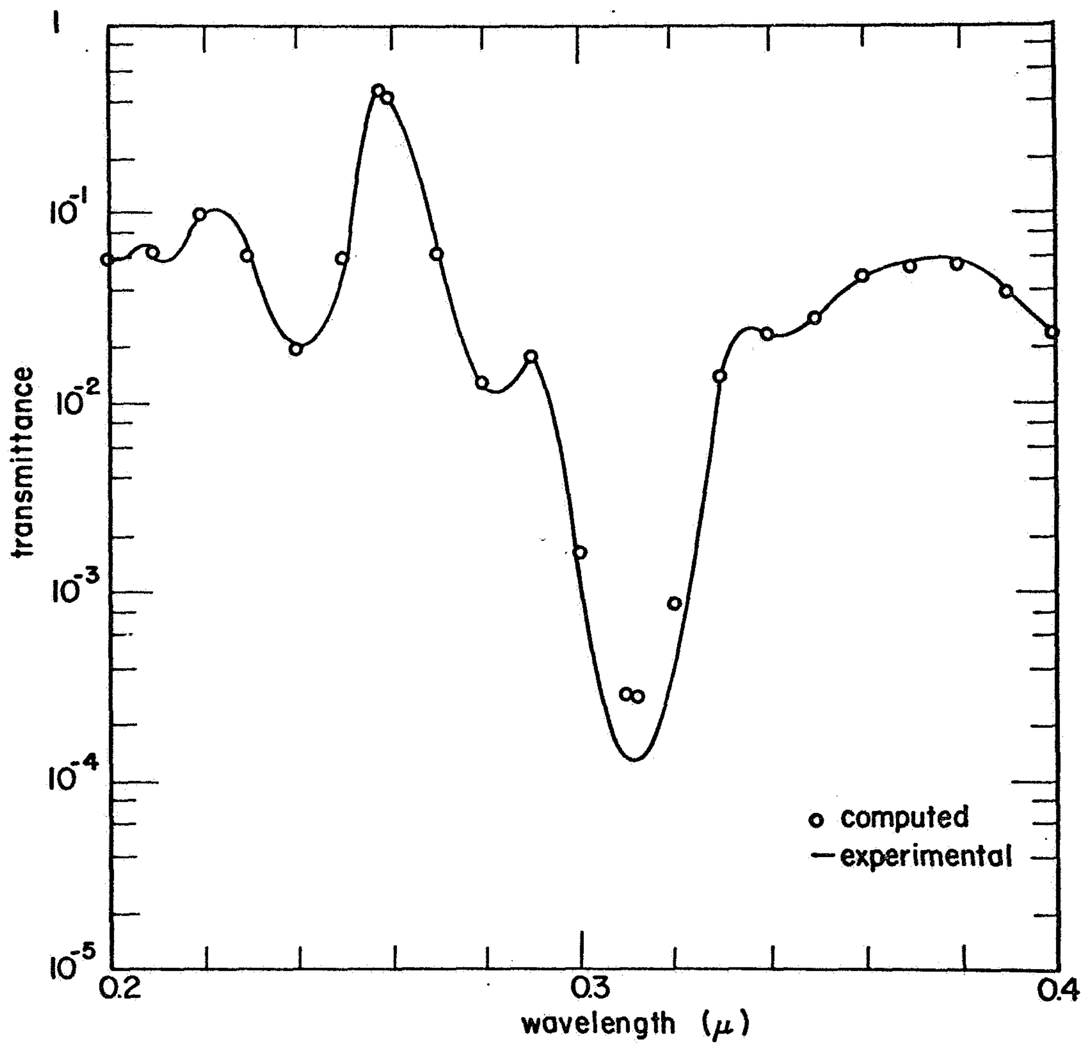

1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
transmittance
0.2
0.3
0.4
wavelength (μ)
o computed
— experimental

FIG. 8

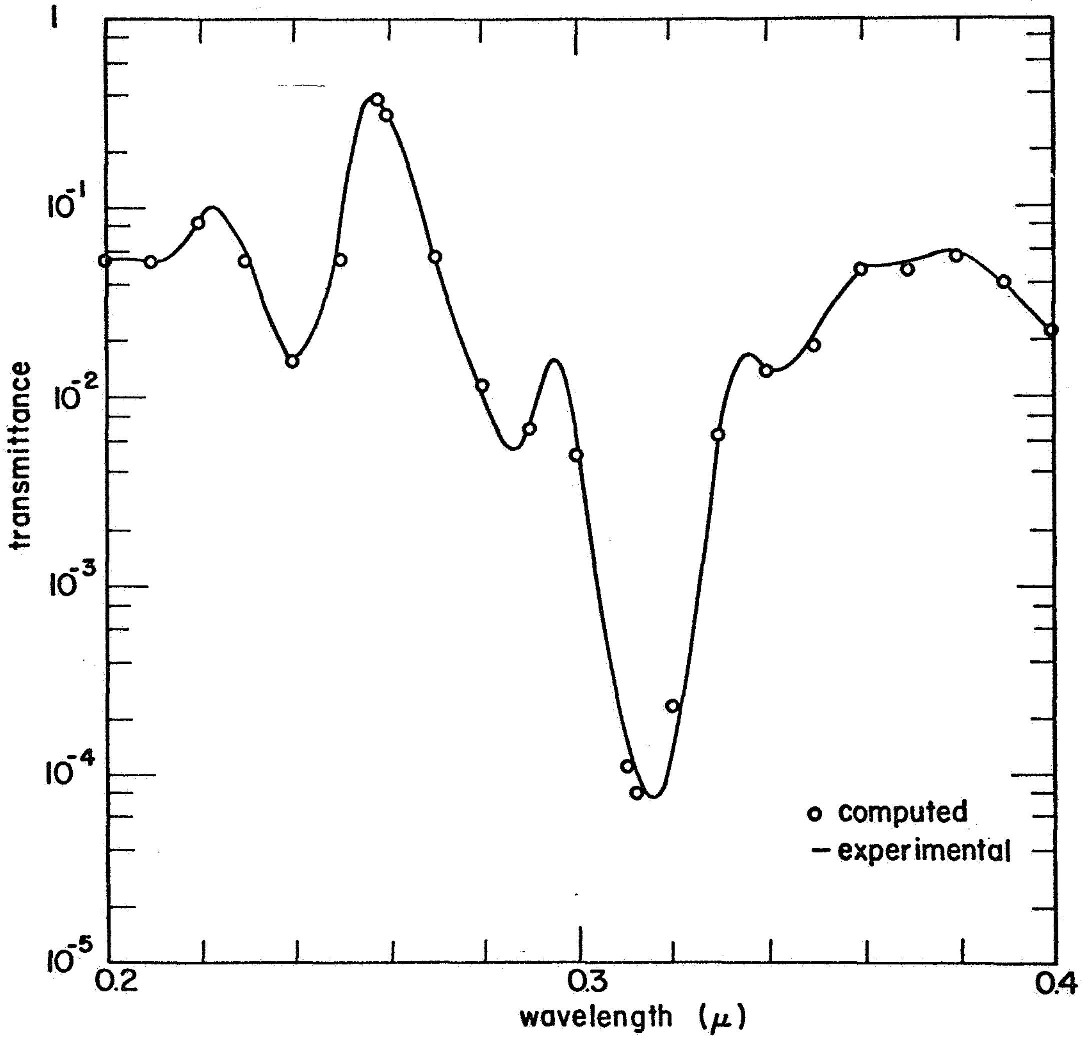
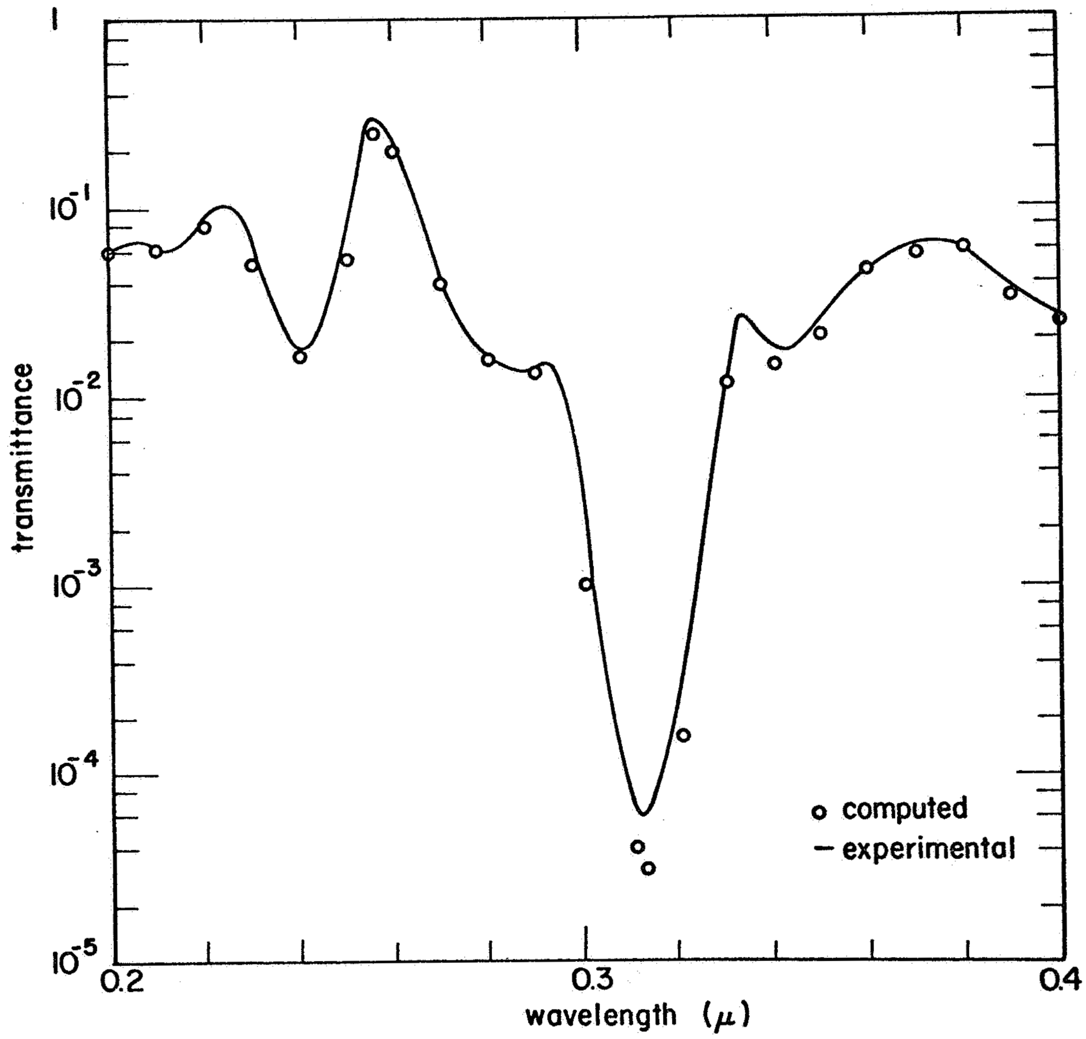

transmittance
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.2
0.3
0.4
wavelength (μ)
o computed
— experimental

FIG. 9



1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
transmittance
0.2
0.3
0.4
wavelength (μ)
o computed
— experimental

FIG. 10