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ESTIMATION AND PREDICTION OF THE ATTITUDE OF A PASSIVE GRAVITY-STABILIZED SATELLITE

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16. Abstract

Essential to the estimation and prediction technique is the selection of a simple observable set of satellite parameters which, if known, permits the disturbance torques to be approximated. The values of the parameters were established by analyzing a large number of imperfect observations and the attitude was subsequently calculated by integration of the equations of motion. A significant result of the study is that the parameters can be estimated solely from solar aspect information.

The technique was evaluated by a simulation study. The simulated satellite was assumed to be in a polar orbit at an altitude of 1400 km and to have solar aspect sensors digitized to have a sensitivity of 0.5° per digit. The results indicated that past attitude history could be estimated to within 0.1°. Application of the technique to long-range prediction produced the same errors in earth pointing, but the yaw errors were approximately doubled.

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NOTATION

$A(t)$	time varying matrix of partial derivatives of the attitude angles with respect to the unknown parameters
$\overline{b}_j$	unit vectors defining orthogonal reference frame fixed in main satellite body; for zero attitude errors $\overline{b}_j = \overline{o}_j$
d_j	components of the unknown vector from the center of mass to the geometrical center of the satellite; measured in the $\overline{b}_j$ frame
$e(t)$	instrument errors
$H(t)$	time varying matrix of partial derivatives of the instrument readings with respect to the unknown parameters
I_{ij}	moments and products of inertia of main satellite body relative to the $\overline{v}_j$ frame
i,j	1, 2, 3
m_j	components of the unknown magnetic dipole moment; measured in the $\overline{b}_j$ frame
$\overline{o}_j$	unit vectors defining orthogonal reference frame fixed at satellite orbit: $\overline{o}_3$ is directed toward center of earth; $\overline{o}_1$ is in the direction of motion
p	parameters describing physical characteristics of the satellite
Q	covariance matrix of the observations
q	quantization of digital sensors
r	vector function describing satellite environment
t	time
$\overline{v}_j$	unit vectors defining orthogonal reference frame coincident with principal inertia axes of entire satellite; in equilibrium $\overline{v}_j = \overline{o}_j$
x	unknown parameters composed of the elements α_i, d_i, and m_i
$y(t)$	instrument readings
z	state vector composed of attitudes and angular velocities
$\Delta(\ )$	increment
$\underline{(\ )}$	ensemble

$(\hat{\ })$	estimated quantity
α_j	unknown angles giving orientation of $\bar{b}_j$ frame relative to $\bar{v}_j$ frame
β	acute angle between boom axis and $\bar{b}_3$ axis
γ	angle the sunline makes relative to the orbit normal
ε_s	error in the subscripted quantity.
θ_{d_s}	bias of damper spring from zero position in absence of gravity torques
λ	angle of the sunline relative to the first point of Aries measured in the ecliptic plane
σ_e	variance of normally distributed errors in the edges of the region assigned to each digit of sensors
ϕ,θ,ψ	Euler angle sequence relating $\bar{b}_j$ frame to $\bar{o}_j$ frame
Ω	longitude of the ascending node of the orbit measured from the first point of Aries in the equatorial plane

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SUMMARY

For some missions of earth-oriented satellites, advance knowledge of the attitude is as useful as precise attitude control. Prediction of future attitude is possible if an accurate model is available of all disturbances to the satellite. This paper presents a study of a technique for predicting the attitude of a passive, gravity-stabilized satellite. The satellite was assumed to be in a polar orbit at an altitude of 1400 km such as might be used in a meteorological mission.

Essential to the prediction technique is the selection of a simple observable set of parameters which, if known, permits the disturbance torques to be approximated. The values of the parameters were established by analyzing a large number of imperfect observations, and the attitude was subsequently calculated by integration of the equations of motion. For the particular satellite simulated, it was found that the past attitude history could be estimated to within 0.1°. Applied to long-range prediction the technique produced the same errors in earth pointing, but the yaw errors were approximately doubled.

The technique affords the possibility of determining attitude with fewer instruments. The results cited were determined entirely from measurements from the solar aspect sensors digitized to have a sensitivity of 0.5° per digit.

INTRODUCTION

Typical past and current missions of earth satellites include communications, meteorology, and radio astronomy. Contemplated future missions include surveys of earth resources and mapping that will encompass such broad endeavors as agriculture, forestry, geology, hydrology, and oceanography. For all such missions, a reference from which to aim the sensors can be provided equally well by control of the entire satellite to within a specified error, or by relatively loose control if the attitude is known precisely. In the latter, the requirement for a precise active satellite control system is replaced by attitude information generated on the ground by processing attitude sensor measurements from the satellite.

The accuracy of any attitude determination scheme depends on the accuracy of the satellite attitude sensors and on the technique employed to process their readings. Measurements taken at a given instant contain random errors, and considerably better accuracy can be achieved when some smoothing technique is applied. An efficient smoothing, or attitude estimation, scheme can be based on the premise that the steady-state motion can be computed when the variation of the external disturbance torques with time is known. Successful implementation requires that the satellite dynamics and environment be known, and that sufficient information be obtainable from observing the attitude motion to deduce those remaining unknown quantities, such as the residual magnetic dipole, essential to the calculation of the disturbance torques.

The estimation scheme is readily applied to improving the knowledge of the attitude motion of passively controlled satellites. In theory, the scheme can be applied regardless of the type of control system in use. However, passive systems are unique in that they are subject only to forces caused by interaction with the environment. In contrast, active systems are subject to forces caused by less predictable control activity. Furthermore, since the disturbances to passive satellites are almost exclusively steady or periodic, it should be possible to predict future attitude motion from the analysis of past attitude history. If it is required that a satellite direct its sensors earthward, the natural choice of passive attitude control is gravity stabilization. In addition to long life, zero power consumption, and excellent reliability, this stabilization technique provides a vibration-free sensor platform with angular rates much lower than its active counterparts.

Attitude estimation and prediction has already been demonstrated by Scott and Rodden (ref. 1) who analyzed the motion of a gravity-stabilized satellite which employed control moment gyros to provide damping and yaw stiffness. The present study was initiated to determine the feasibility of applying estimation techniques to predict the attitude of a completely passive gravity-stabilized satellite. The aims of this study differ somewhat from those of Scott and Rodden. In particular, major objectives of the present study are to compare the estimation accuracy for various sets of instruments and to determine the simplest set of instruments sufficient to yield the necessary information.

More specifically, the estimation technique was applied to an inertially coupled satellite that relies solely on the gravitational field for damping as well as restoring torque (ref. 2). In the absence of suitable data from an actual satellite, it was necessary to analyze simulated attitude motions. The simulation assumed the satellite to be in a polar orbit at an altitude of 1400 km such as might be specified for meteorological or earth resources missions. It is, of course, not possible to determine if the model of the steady-state disturbances used in the simulation is adequate or if important errors will be caused from those nonsteady disturbances that are encountered in orbit. This type of study can, therefore, never prove that attitude estimation of an actual satellite will be successful. Conversely, if attitude estimation is not feasible for a simulated satellite, it is certain not to be feasible for an actual satellite.

ESTIMATION AND PREDICTION TECHNIQUE

The estimation technique is based on the premise that the dynamics of a gravity-stabilized satellite and all of its disturbance sources are understood. The estimation technique seeks to establish the hitherto unknown values of certain quantities through the analysis of the observed attitude sensor readings. Once the unknowns are established, a complete model of the dynamics and disturbance sources is available, and past and future attitude motion can be simulated.

The Model

All of the information, both known and unknown, that constitutes the complete model required for the simulation can be placed in three general categories; the equations of motion, the physical characteristics of the satellite, and the environment. These are discussed in the following sections.

The equations of motion.- The equations of motion must include all of the significant dynamic effects if precise estimation of future attitude motion is to be achieved. For this study, the dynamics were assumed to be adequately represented by the dynamics of a pair of connected rigid bodies. For some gravity-stabilized satellites, rigid body dynamics are not an adequate representation because the flexural modes of boom motion couple significantly with the librational motion. This is true, for instance, in the analysis of the motion of the Radio Astronomy Explorer Satellite which has boom lengths of the order of 200 m (see ref. 3). In other instances, the variation of the mass distribution with time caused by thermal distortion may invalidate the assumption of rigid body dynamics (see ref. 4). These effects will not be present if newly developed booms are used which theoretically have no thermal distortion (ref. 5), or if the booms are arranged symmetrically.

In this study the satellite considered has symmetrically oriented booms with natural flexural frequencies that are large compared with the libration frequencies. For such a satellite the rigid body equations are considered to provide an adequate representation of the steady-state dynamics. This representation requires an eight-dimensional state space: three coordinates each for the attitude and angular velocity of the main satellite body, and two coordinates to represent the relative attitude and angular velocity of the single-degree-of-freedom damper body (see ref. 2). The state equation is

$$\dot{z} = f_1[t, z, p, r(t)] \tag{1}$$

The form of f is known explicitly; p represents the parameters defining the particular physical characteristics of the satellite; r represents the environment that reacts with the spacecraft to produce the external disturbances. The solution of equation (1) for initial conditions $z = z_o$ and $t = t_o$ will be denoted by

$$z(t) = f_2(t, t_o, z_o, p, r) \tag{2}$$

Equation (1) was used in generating the various quantities required by estimation procedure and in simulating the unknown motion to be analyzed. It is obvious, therefore, that the results of the study can provide no conclusions concerning whether or not the rigid body equations are adequate for use in the estimation procedure. Such conclusions must await application to a real, rather than to a simulated, satellite.

The environment.- The estimation procedure is based on a perfect knowledge of the environment. Essential to the knowledge of the environment is knowledge of the orbital parameters. This permits account to be taken of the effects of eccentricity on the attitude motion and establishes the relationship between satellite, solar, and earth-centered coordinates. This information, plus the known energy density of sunlight and a model of the geomagnetic field, defines the solar pressure and magnetic environment of the satellite. For near earth orbits, such as considered in this study, the assumption that the magnetic field is known perfectly appears to be reasonable. For the purpose of this study the field was assumed to be that due to a tilted dipole although more accurate models are available.

Physical characteristics of the satellite.- The attitude behavior of the satellite can be calculated only if the set of parameters defining its physical characteristics is known accurately. Many of these parameters, such as the mass distribution and geometry of the stabilized package, are known in the sense that they can be measured accurately prior to launch and are certain to remain constant thereafter. Others, such as the residual magnetic dipole, although they may be measured accurately prior to launch, vary unpredictably during the period of launch and deployment. Still others, such as solar pressure torques, cannot be measured adequately on the ground and must be estimated.

Provided the model of the satellite system is adequate, errors in the calculated attitude will result from deviations in the measured or estimated values of the unknown and poorly known parameters. An obvious approach to the attitude estimation problem is to select all the parameters with uncertain values and try to estimate better values from measured data. One difficulty with this approach is that measurements may not permit all the parameters to be uniquely distinguished from each other. Another difficulty, of a more practical nature, is that the amount of computation and numerical round-off errors increase rapidly with the number of parameters considered. A more realistic and certainly more practical approach is to limit the number of parameters to be estimated. The set of parameters selected must have two properties. First, the effect of each parameter on the attitude must be distinguishable from the effect of all the others. Second, it must be possible to find a set of values for the parameters that will produce an approximation to all the expected disturbance torques. For the particular, low altitude satellite considered, the parameters selected were the misalinement angles between the principal axes of inertia and the reference axes, the distance between the center of mass and center of area, and the magnitude and direction of the residual magnetic dipole. A total of nine quantities is needed to define these error sources.

The selection of the nine parameters to be estimated implies that all other parameters and the environment are assumed to be known perfectly. Let $p = (x, p_o)$, where x denotes the nine parameters to be estimated, and p_o denotes all those assumed to be known. The motion of the system then only depends upon t and the unknown parameters x. Thus,

$$z(t) = g_1(t, x) = f_2[t, t_o, z_o, (x, p_o), r] \tag{3}$$

Instrumentation

Attitude instrumentation for earth oriented satellites is usually selected so that the attitude can be resolved from a set of data taken at a given instant. Most earth oriented satellites have some combination of earth sensors, magnetometers, and solar aspect sensors. Each sensor can determine the coordinates of some line relative to the satellite reference system; consequently, two different types are required to determine the attitude at any given instant. With the introduction of the estimation procedure, the criterion for selecting instruments is different and less stringent. The only requirement is that it be possible to solve for the unknown parameters from some set of observations taken at selected times. The estimation procedure for determining attitude therefore affords an opportunity to simplify the instrument system. The identification of the simplest set of instruments that would yield satisfactory attitude information was one of the goals of the study.

The instruments considered were combinations of solar aspect sensors, a horizon scanner, and a damper boom angle indicator. Five solar aspect sensors are required to give spherical coverage. Three were placed 120° apart in the plane that nominally coincides with the orbital plane and the remaining two were directed in either direction normal to the orbital plane.

The general form of the equations characterizing the set of instrument readings can be expressed as:

$$y(t) = g_2(t, z) + e(t) \tag{4}$$

where e represents instrument errors. Consequently, sensor outputs are related to the unknown parameters by the composite of equations (3) and (4); that is,

$$y(t) = g_3(t, x) + e(t) \tag{5}$$

where

$$g_3(t, x) = g_2[t, g_1(t, x)]$$

Estimation of Unknown Parameters

The estimation procedure seeks to improve the knowledge of the parameters assumed to be imperfectly known. The process is similar to quasilinearization

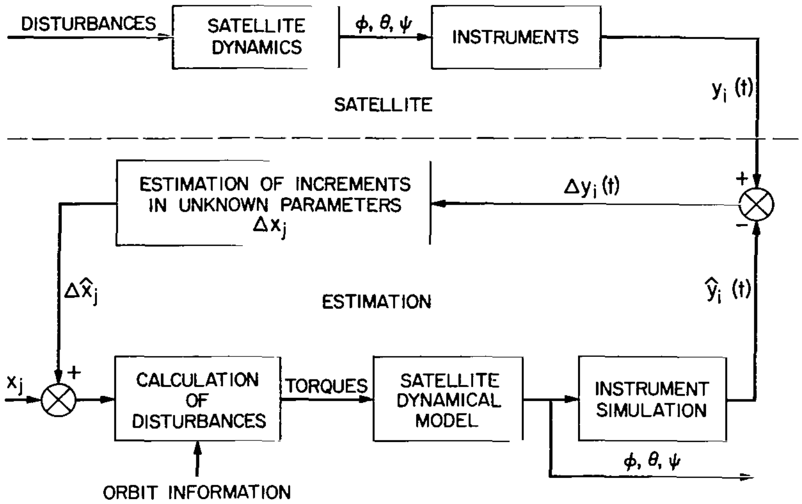


Figure 1.- Block diagram of the estimation procedure.

described in reference 6. As illustrated in the block diagram in figure 1, the process starts with an initial guess of the parameters x_j. From this guess, an initial estimate of the subsequent instrument readings is calculated from the dynamic model of the satellite and its disturbances, and compared with the actual instrument readings from the satellite. The difference between the actual and estimated instrument readings is then used to improve the knowledge of the parameters x_j. The desired quantities, the attitude angles, are obtained from the dynamic model of the satellite and its disturbances. Knowledge of x_j therefore implies the possibility of future motion as well as estimation of past motion.

The estimation technique relies upon making a large number of imperfect observations to establish the value of the unknown parameters. Since the parameters are assumed to be invariant with time, the problem is tractable and consists in finding the minimum variance solution of a system of linear equations. The system of equations can be expressed as

$$\underline{\Delta y} = \underline{H}\,\Delta x + \underline{e} \tag{6}$$

where

$$\underline{\Delta y} = \begin{pmatrix} \Delta y(t_1) \\ \cdot \\ \cdot \\ \cdot \\ \Delta y(t_n) \end{pmatrix}$$

$$\underline{H} = \begin{pmatrix} H(t_1) \\ \cdot \\ \cdot \\ \cdot \\ H(t_n) \end{pmatrix}$$

$$\underline{e} = \begin{pmatrix} e(t_1) \\ \cdot \\ \cdot \\ \cdot \\ e(t_n) \end{pmatrix}$$

and

$\Delta y(t)$ deviation of the instrument readings from those corresponding to the attitude at time t given the prior estimate of the unknowns x_p

Δx_j deviations in the unknown parameters from the prior estimate x_p

$H(t)$ matrix of partial derivations relating $\Delta y(t)$ to Δx_j

$e(t)$ instrument errors at time t

It was not feasible to calculate the elements of the matrix $H(t)$ analytically from the nonlinear equations of motion of the two-body satellite. Instead, the elements of the matrix were evaluated through simulation of the attitude motion. A reference motion based on a prior estimate of the unknown parameters x_p was first obtained. A subsequent motion was then calculated with an increment in one of the components of x. The magnitude of the increment was chosen to be equal to the expected deviation. The derivative was then evaluated assuming the variation of the instrument reading with the change in the component of x to be linear. Thus,

$$h_{ij} = \frac{g_3(t, x + \Delta x_i) - g_3(t, x)}{\Delta x_i}$$

The elements of the matrix $H(t)$ therefore depend on the prior estimate x_p and the magnitude of the increment Δx_i, as well as time. However, the primary dependence is upon time which determines the orbital position and therefore the relationship of the satellite to the sun and the magnetic field. The dependence upon x arises solely from nonlinearities.

The minimum variance solution of equation (6) for the deviations of the unknown from the prior estimates, Δx, is well known (see, e.g., ref. 7) and is given as

$$\Delta\hat{x} = (\underline{H}^T\underline{Q}^{-1}\underline{H})^{-1}\underline{H}^T\underline{Q}^{-1}\ \underline{\Delta y} \tag{7}$$

where Q is the covariance of the instrument errors (i.e., $\underline{Q} = E(\underline{ee}^T)$). It is assumed in the derivation of equation (7) that the instrument errors are uncorrelated, have zero mean, and are independent of the measurements.

SIMULATION

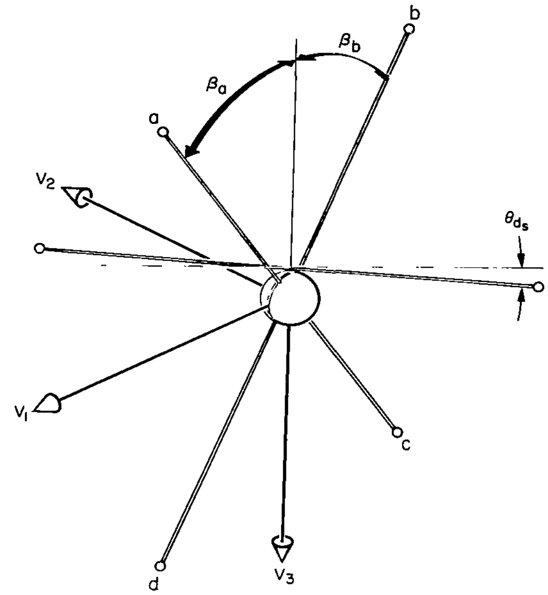
The unknown attitude motion and the corresponding instrument readings were generated by a digital computer simulation. The characteristics of the simulated satellite were obtained by assuming random deviations of its physical characteristics from a known satellite. The characteristics of the known and the unknown satellite are given in table 1. These deviations were intended to simulate the unknown error sources that might be present after the spacecraft has been launched and the booms erected. The deviations included the angle between the individual booms and the vertical reference axis, the reflectivity of the booms, the radius of curvature due to thermal distortion, the erected boom length, the damper bias angle, and the residual magnetic dipole.

A complete simulation of a satellite would require that the relationship of the orbit to the sunline change slowly with the seasons and with any motion of the line of nodes. For purposes of simulation, the orbit was assumed to be polar, and its relationship to the sunline was assumed to be fixed for any one data sample. The effect of changes in the relationship of the orbit to the sunline on the estimation procedure was studied through simulation of separate cases. These were achieved by arbitrary changes in Ω, the angle the line of nodes of the orbit makes with the first point of Aries, and λ, the angle measured in the ecliptic between the sunline to the earth and the first point of Aries. The following cases were studied:

Case	Ω	λ	Sun angle relative to orbit
I	0	0	Orbit plane contains sunline
II	60°	0	Sunline 30° from orbit normal
III	30°	0	Sunline 60° from orbit normal
IV	120°	90°	Sunline ~60° from orbit normal

All of the instruments were assumed to be representative of those currently available. Their output readings were assumed to be digital and were simulated by calculating their exact value and assigning the appropriate digital value. Each instrument was assumed to have random errors in the location of the edges of the regions assigned to each digit. These errors can arise from errors in the quantization (i.e., errors in the location of the edge of the region assigned to each digit), and from randomness in the digit assigned by the sensor electronics when the exact reading approaches the edge. In the simulation, the errors were considered to be of the latter type and were simulated by adding a random number to the exact reading prior to assigning the digital value. If quantization errors are present, the assumptions inherent in the solution of equation (7) will be violated because the error distribution associated with a particular digit will not have zero mean. Consequently, there will be a correlation between the sensor reading and its errors.

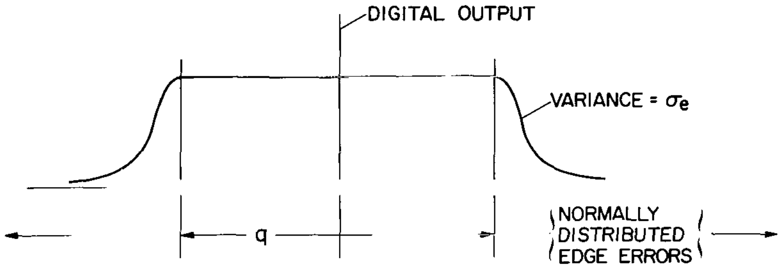
TABLE 1. CHARACTERISTICS OF ACTUAL AND ASSUMED SATELLITE



	Actual				Assumed			
Total mass, kg	223				223			
Moments of inertia, kg-m^2								
Main body, relative to V frame								
I_{11}	3005.0				3064.8			
I_{22}	3774.0				3754.7			
I_{33}	780.0				705.5			
I_{12}	65.0				58.8			
I_{13}	6.6				0			
I_{23}	-.5				0			
Damper, about hinge axis	122.3				122.3			
Boom systems								
Main booms	a	b	c	d	a	b	c	d
Diameter, cm	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14
Tip mass, kg	.727	.727	.727	.727	.727	.727	.727	.727
Length, m	30.7	30.1	30.6	29.7	30.0	30.0	30.0	30.0
Reflectivity	.88	.86	.90	.86	.92	.92	.92	.92
Minimum radius of curvature due to solar radiation, m	305	261	366	261	457	457	457	457
Angle from vertical reference, β, deg	27.1	25.6	28.2	27.3	26.7	26.7	26.7	26.7
Damper booms								
Length, m	15.0				15.0			
Tip mass, kg	.178				.178			
Spring constant, Newton-m rad	-.000536				-.000536			
Damping constant, Newton-m rad/sec	-.119				-.119			
Bias, θd_s, deg	.5				0			
Center of mass of stabilized package relative to center of sphere, m								
Δx_{body}	.16				0			
Δy_{body}	0				0			
Δz_{body}	-.04				0			
Uncompensated magnetic dipole, Weber-m^2								
Δm_1	$-.131\times10^{-5}$				0			
Δm_2	$.066\times10^{-5}$				0			
Δm_3	$-.227\times10^{-5}$				0			

The variance associated with quantization is developed in appendixes A and B. Appendix A treats the special problem peculiar to solar aspect sensors where the quantization changes somewhat from the nominal with the angles sensed. Appendix B gives the method of calculating the total variance due to quantization and the presence of the edge errors.

The characteristics of each instrument simulated is given in the sketch and table below.



PROBABILITY DENSITY FUNCTION OF INSTRUMENTS

INSTRUMENT	q	σ_e
HORIZON SCANNER	1°	0.1°
SOLAR ASPECT SENSORS	~0.5°	.1°
DAMPER BOOM ANGLE	2°	.1°

RESULTS AND DISCUSSION

Observability

The first result to be established is the identification of those sets of instruments that will permit equation (7) to be solved for the unknown parameters. A necessary condition for solving equation (7) is that all attitude angles must influence the readings of the sensors at some time during a given orbit. As in the case of determining attitude at a given instant, this condition rules out the possibility of using the horizon scanner alone, since yaw angle cannot be sensed. The additional information possibly could be supplied by a damper boom angle indicator if the damper boom responds to steady and oscillatory yawing motion. It will so respond provided either the neutral position of the damper or the hinge axis does not lie normal to the yaw axis. However, the damper boom and the hinge axis of the satellite analyzed were located in the horizontal plane. For this position, the damper does not respond to a steady yaw, thereby preventing the evaluation of the steady yaw offset, α_3. It follows that solar aspect sensors must be included in any combination of the available instruments if equation (7) is to be solved.

With the exception of a single orientation of the orbit relative to the sunline, the solar aspect sensor measurements are influenced by all the attitude errors at some time during an orbit. This is necessary, but not sufficient, to insure that a solution to equation (7) exists. To establish that a solution exists the determinant of HH^T must be nonzero for some series of measurements. With the exception of one situation described below, this condition was satisfied for solar aspect measurements alone. Hence, except for one condition, solar aspect measurements will provide sufficient information to solve for the unknown increments in the unknown parameters.

This exception occurs when the orbit plane is normal to the sunline. Then the complete list of unknowns is unobservable by any combination of instruments. The reason is that the solar pressure torques that result from the center of mass shift, d_i, produce steady angular offsets that are indistinguishable from the variables, α_i. The estimated values of either set of parameters will therefore produce both effects and one set could be eliminated if the satellite orbit were to remain normal to the sunline.

This particular orbit is also unique in that solar aspect sensors alone do not provide sufficient information to solve for even the reduced list of unknowns. The reason is that a satellite with a steady attitude error will have no change in its orientation relative to the sun as it moves along the orbital path. The sun sensors will yield information on two angles required to describe the orientation of the satellite relative to the sun. Since three variables must be evaluated to describe the steady attitude angle offset, no solution is possible. This difficulty could be resolved by using the damper boom angle for additional information if the bias of the damper from its nominal position, θ_{d_s}, is known. Otherwise, the bias must be included in the list of unknowns to be determined. In this event, one more unique equation and one more unknown are added to the system of equations and a solution does not exist.

Although the test for observability establishes that, with the above exception, solar aspect sensors alone yield a unique solution for the unknowns, it gives little insight into the accuracy of the estimation technique when a limited number of imperfect measurements are processed. Therefore, estimation was attempted with several instrument systems. These included solar aspect sensors alone, solar aspect sensors in combination with a damper boom angle indicator, and solar aspect sensors in combination with a horizon scanner.

Short Term Estimation

The unknown parameters were estimated from a data sample that covered about five orbits. Typical results showing the motion calculated given initial estimates of the unknowns are shown in figure 2. In this instance, the unknown parameters were initially estimated to be zero ($\bar{x} = 0$). The first iteration then seeks to minimize the variance of the difference between the actual instrument readings and the exact instrument readings corresponding to motion when $\bar{x} = 0$. As can be seen from the results, integration of the equations of motion when the disturbances are evaluated from the new estimate of the unknowns results in reasonable agreement between the estimated and the actual motion. Some improvement in the estimated motion results if a second estimate of the unknowns is made. In practice, this estimate would be based on a subsequent data sample. For purposes of this study, the same data were reprocessed.

The errors in the second iteration of the pitch, roll, and yaw for the period covering the data sample are shown in figure 3. The estimated motion matched the actual motion to within 0.1° except for a few points in yaw. This result was obtained regardless of the instrument system in use. As indicated by figure 3, no improvement in the estimate based on solar aspect sensors alone resulted when horizon scanner measurements were included. A similar conclusion was reached concerning the addition of damper angle measurements to the solar aspect sensor measurements.

An examination was made of various sources of error that can influence the accuracy of the estimation. These error sources include: insufficient number of observations, nonlinearities in the variation of the instrument readings with the unknowns, imperfections in the model, and poor observability. These errors sources will be discussed in turn.

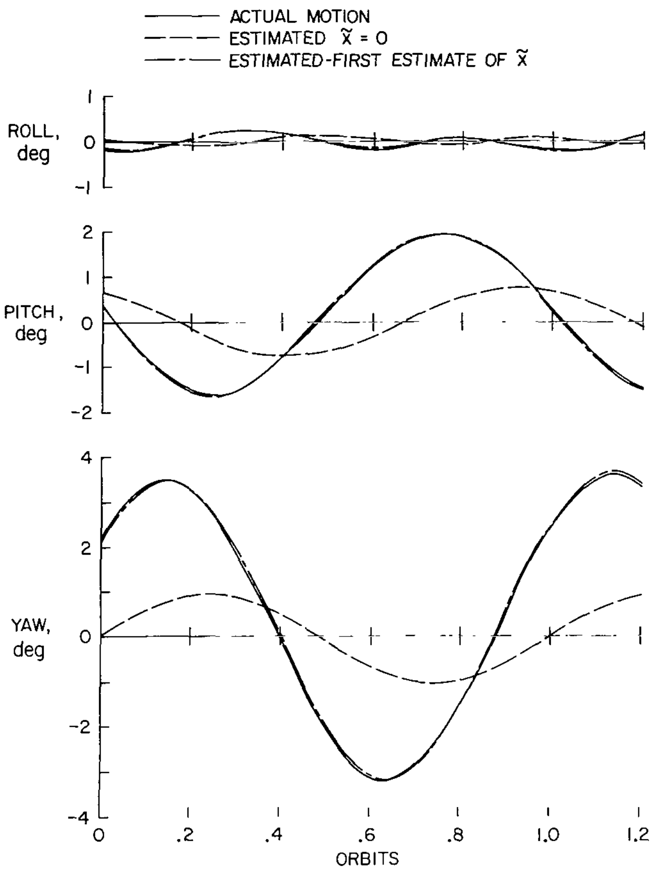


Figure 2.- Typical results of the estimation procedure.

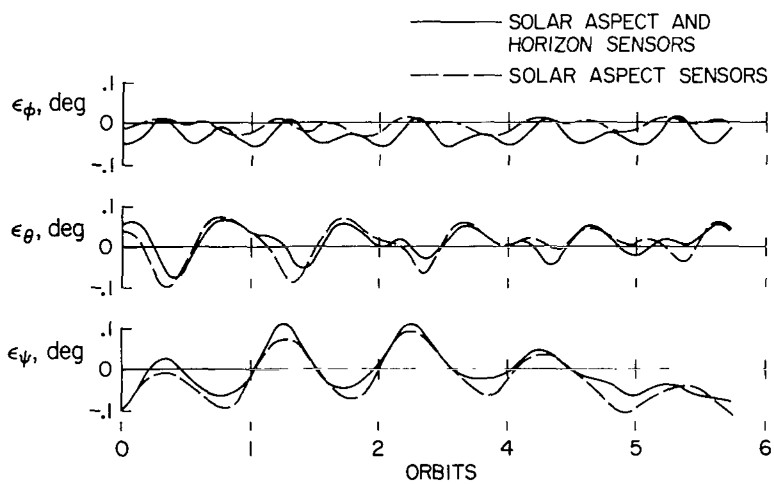


Figure 3.- Typical errors in the estimation after two iterations over the entire period of the data sample; $\Omega = 0$, $\lambda = 0$.

Effect of number of observations.- Ideally, the number of observations should be sufficient to reach the point where additional observations cause little or no change in the solution for the unknowns. The observations were arbitrarily taken 0.3 of a radian apart along the orbital path, and the maximum number of observations for any solution was limited to 100. For the set of 100 observations, the number of discrete data points varied with the set of instruments in use and with whether or not the satellite passes through the earth's shadow, thereby eliminating solar observations.

To test if 100 observations were sufficient to estimate the state accurately, estimations were made with fewer observations and the state noted as the number of observations was increased. To eliminate the possible effects of other sources of error, the data analyzed corresponded to a perfect linear model. Exact instrument readings were generated by multiplying the matrix H by an arbitrary set of values for unknowns. These readings were quantized as outlined in the section on simulation.

The error in estimating the unknown parameters for various numbers of observations is shown in figure 4. When the horizon scanner was used, little change in the estimate of the unknowns occurred after about 50 observations. When the solar aspect sensors alone were used, 100 observations appeared to be marginal in that some of the unknowns were still changing. Note, however, that the errors for 100 observations were very nearly the same for both instrumentation systems. The addition of horizon scanner measurements did nothing to improve the knowledge of the variables α_2 and α_3 which correspond to steady roll and yaw. The horizon scanner would not be expected to improve α_3 since it does not sense yaw. The reason the estimate of α_2 did not improve is that the roll angle typically never exceeded 0.5°. All readings of the digital horizon scanner, accordingly, were zero. Therefore, in effect, the horizon scanner contributed information only on the pitch motion.

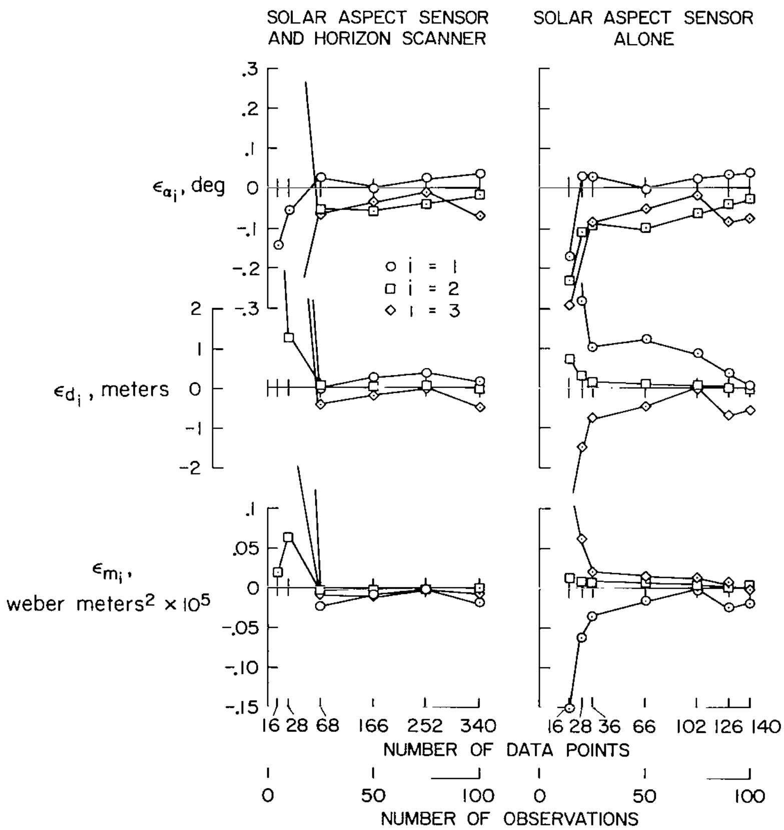


Figure 4.- The effect of the number of observations on the estimate of the unknown parameters after one iteration.

Effect of nonlinearities.- The matrix, H, was calculated with the assumption that the variation of the instrument readings with each of the unknowns was linear. The fact that several iterations are necessary to get the best fit to the actual motion indicates that the variation is not linear. This nonlinearity is clearly demonstrated in figure 5, where a comparison is made of the errors for the first and second estimates. Two curves are shown for each estimate. The curves designated as linear are the estimation errors when the attitude is assumed to vary linearly with Δx and is given by $A(t)\Delta x$ summed with the reference motion upon which the estimate was based. The curve designated as simulated is the error incurred when the estimated unknowns are used as an input to the integration of the equations of motion. The nonlinearity is evident in the first estimate given in figure 5(a). The second estimate eliminated the errors from nonlinearity, for all practical purposes, as illustrated in figure 5(b). Figure 6 shows the expected result, that a third estimate produced no further change in the unknowns.

Effect of model imperfections.- The principal source of difficulty in applying the estimation technique appears to be imperfections in the model of the unknown disturbances. These imperfections impair the accuracy of the short term estimation and degrade the prediction of future motions, particularly when seasonal variations and orbital regression change the relationship of the orbit to the sunline.

If the model is perfect it should be possible to estimate the unknowns precisely, because the effects of instrument errors can be eliminated by taking a sufficiently long data sample, and the effects of nonlinearities can be accounted for through an iterative procedure. However, consider the results shown in figure 6, which gives the estimates of the unknown parameters corresponding to an unknown motion generated by the simulation of a satellite with physical characteristics listed as "Actual" in table 1. For this simulation, the magnetic dipole is the only variable that enters directly. Other sources of attitude error, represented in the estimation by the variables α_i and d_i, are unknown solar pressure torques and rotations of the principal axes relative to the instrument package. These torques and rotations are generated by errors in boom length, boom angles, surface reflectivities, and so forth, rather than directly by changes in the variables α_i and d_i, such as used in computing the H matrix. The results of the estimation indicate that the estimated dipole is different from that known to be present (see fig. 6). The error in the estimation is not small, the largest component being underestimated by roughly 50 percent. The corresponding errors in the attitude motion are those shown in figure 3. These latter errors, however, are small, being almost always less than 0.1°.

The apparently contradictory results (i.e., good estimation of the attitude motion but poor estimation of some of the unknowns) can have two sources. One is the possible insensitivity of the motion to change in a particular unknown; the other is the possible imperfect representation of the attitude error sources by the model. The model used is an attempt to represent all the disturbances with a few distinct variables and is known to be an approximation. For instance, including the unknown distances, d_i, is an attempt to allow for unknown variations in the solar pressure torques. These

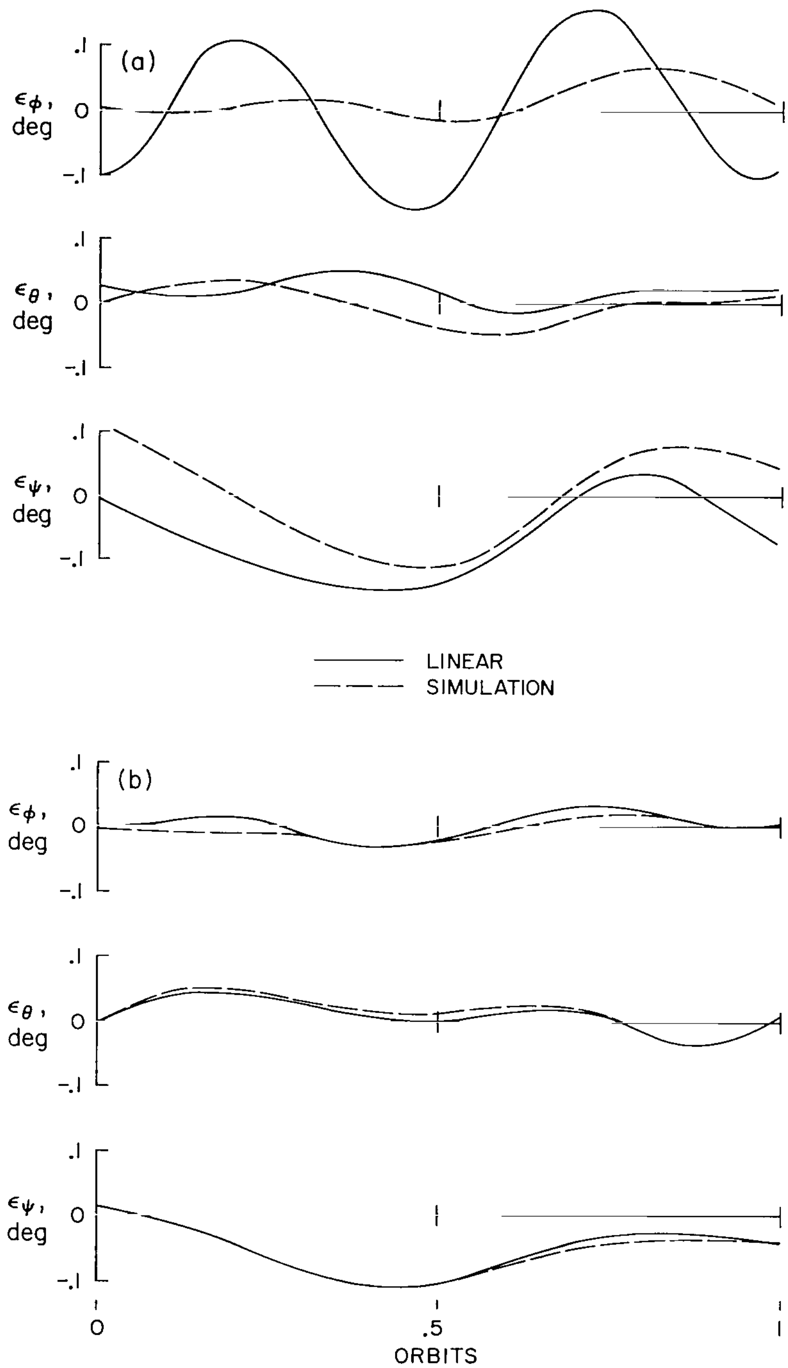


Figure 5.- Comparison of the errors in the first and second linear estimates with the errors from simulations.

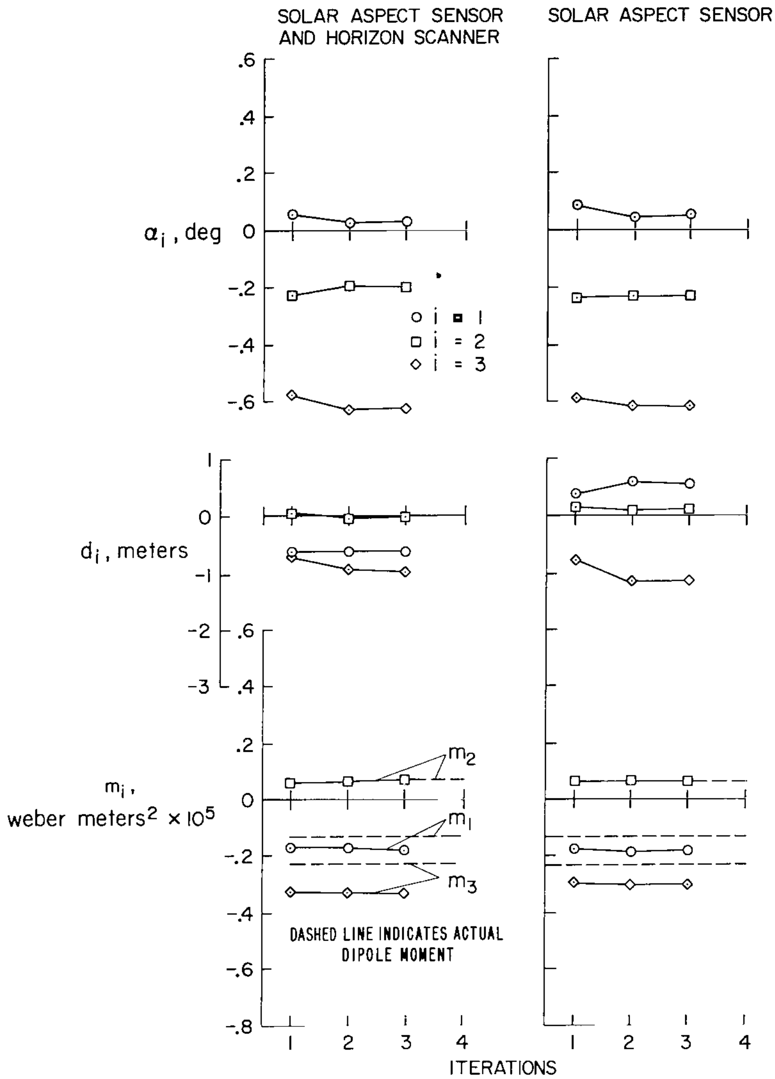


Figure 6.- Convergence of the estimated parameters with number of iterations to eliminate nonlinearities.

variations are a consequence of variations in the geometry and surface properties as well as deviations in the center of mass from its stated position. Even though deviations in the center of mass position will produce solar pressure torques with the same frequencies that actually occur, it is unlikely that a combination of the distances, d_i, will cause a solar pressure torque variation that would have the correct Fourier series representation about all axes. Also, the use of α_i, to allow for variations in the relationship between the principal axes of inertia and the instrument axes, does not allow for changes in the magnitude of the principal moments of inertia. If the moments of inertia are changed, the response of the satellite to disturbance will be different from that corresponding to the calculated H matrix.

The estimation procedure will find the best fit to the data sample. In the example cited, the shortcomings of the model were compensated by estimating a dipole different than that known to be present. For short term estimates this compensation is of little consequence. However, the compensation required to yield the best estimate of the motion would be expected to be altered as the relationship of the orbit to the sun changes, thereby presenting some difficulty in the long term prediction of attitude motion.

Effect of poor observability.- As previously noted, when the orientation of the orbit is such that the sunline lies along the orbit normal, three components of the state become unobservable by any set of instruments. For orbits for which the sunline lies close to the orbit normal, all elements of the state are observable in that the determinant of HH^T is nonzero. However, equation (1) becomes poorly conditioned and the states differ wildly from those estimated for other orbits. This condition was found to prevail even when the orbit normal was as far as 30° from the sunline. Typical results of the first estimate are shown in the table below. The data in the first column for estimates when the sunline was 30° from the orbit normal shows quite different results from those obtained when the sunline was in the orbital plane. This is particularly true of the values of α_3 and d_1. These

Variable	$\Omega = 60°,\ \lambda = 0$			$\Omega = 0,\ \lambda = 0$
	Simulated solar aspect sensor 9 variables	Perfect solar aspect data 9 variables	Simulated solar aspect sensor 6 variables	Simulated solar aspect sensor 9 variables
α_1	0.054	0.072	0.030	0.067
α_2	-.219	-.254	-.230	-.210
α_3	-5.45	-1.24	-.227	-.575
d_1	3.94	.73	Not estimated	-.62
d_2	.07	.04	Not estimated	.04
d_3	.32	.51	Not estimated	-.66
m_1	-1.22×10^{-6}	-1.21×10^{-6}	-1.28×10^{-6}	-1.67×10^{-6}
m_2	$.67\times10^{-6}$	$.57\times10^{-6}$	$.58\times10^{-6}$	$.67\times10^{-6}$
m_3	-2.02×10^{-6}	-3.42×10^{-6}	-3.49×10^{-6}	-3.28×10^{-6}

variables produce steady motions which are compensating so that the linear fit to the unknown trajectory obtained from the product $|A(t)||\Delta\bar{x}|$ is comparable to other results of a first estimate of the unknowns. A better definition of the state might be obtained if the data sample were lengthened, thereby reducing the effect of instrument errors. Evidence of this type of improvement is given by the results shown for perfect solar aspect data.

The fact that one unknown parameter is poorly distinguished from another, or is poorly observable, can be viewed as evidence that each produces nearly the same effect on the motion. Insofar as short term estimation is concerned, it is expedient to eliminate one of the unknowns. In this instance, it was decided to eliminate all three of the distance elements, d_i. The resulting short term estimate (see column of table for estimate limited to 6 variables) yielded attitude estimation errors comparable to those shown in figure 2 for the case when the sun was in the orbit plane and all nine unknown parameters were estimated.

Prediction

The goal of the estimation procedure is to be able to predict attitude motion well into the future so that the user of the satellite will know, in advance, the geocentric coordinates of the center of the field of view of the sensors. The results discussed so far indicate that the unknown parameters can be evaluated from analysis of a short data segment such that simulation provides a fit to the attitude motion to within 0.1°. However, it is clear that the model is not perfect. There is, therefore, no assurance that the same set of parameters will provide satisfactory simulation of the motion for some future time when orbital regression and seasonal variations have changed the relationship between the solar, geomagnetic, and orbital coordinate systems.

The results of the short-term estimation were used in an attempt to predict the motion for other orbits with a different angle of the orbit normal relative to the sunline. In particular, it was attempted to predict the motion for the time of the vernal equinox ($\Omega = 0$) from the parameters evaluated when $\Omega = 60°$ and vice-versa. The results are shown in figure 7. In each case, the pointing error was predicted to within nearly 0.1°, but the prediction of the yaw motion was poor. Curiously, the predictions of yaw based on estimations when $\Omega = 60°$ were somewhat the better of the two, even though three of the unknown parameters were not estimated.

In practice, satisfactory long-term prediction might be achieved in a variety of ways. The most obvious is to determine the time period for which an estimate of the unknown parameters yields satisfactory accuracy. The duration of this period will depend on the orbit and its regression rate. After a number of successive estimates of the parameters have been made, the estimation frequency might be reduced through interpolation or extrapolation, depending upon whether one cycle of the variation of the sunline relative to the orbit has been completed. An alternative technique is to analyze data that encompass the entire range of conditions encountered in the past. After one complete cycle of these conditions has been covered, the estimate will

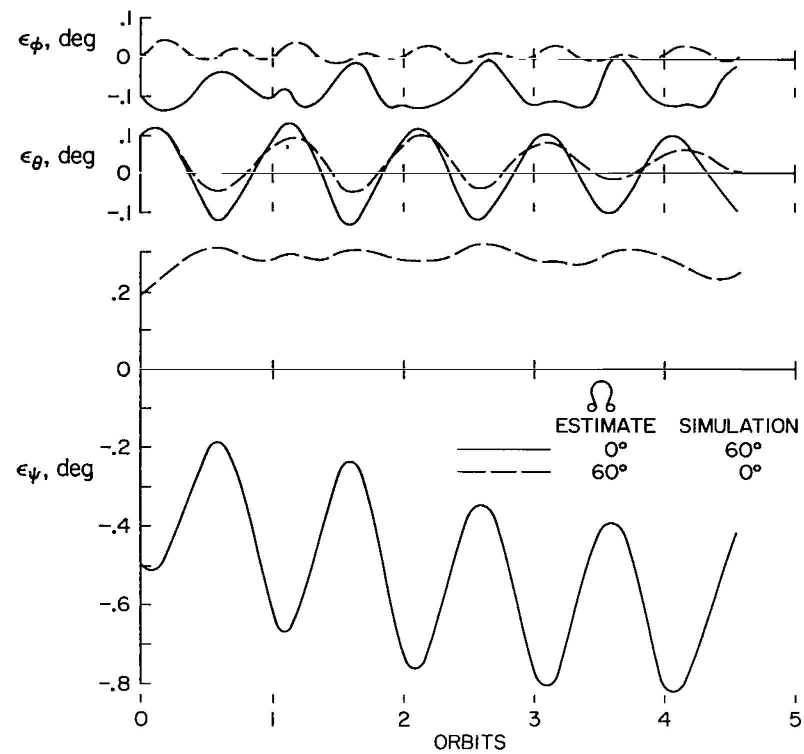


Figure 7.- Errors when a short-term estimate of the unknown parameters is used in the prediction of attitude for a different orbit-sun relationship.

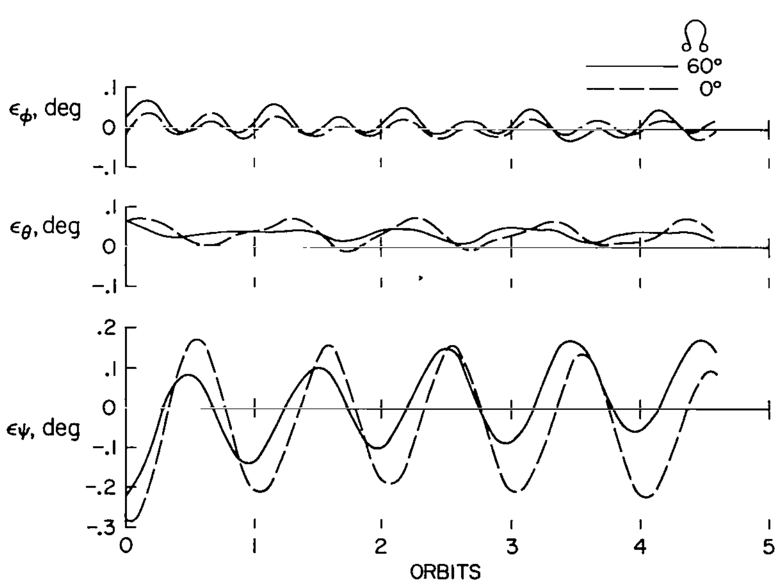
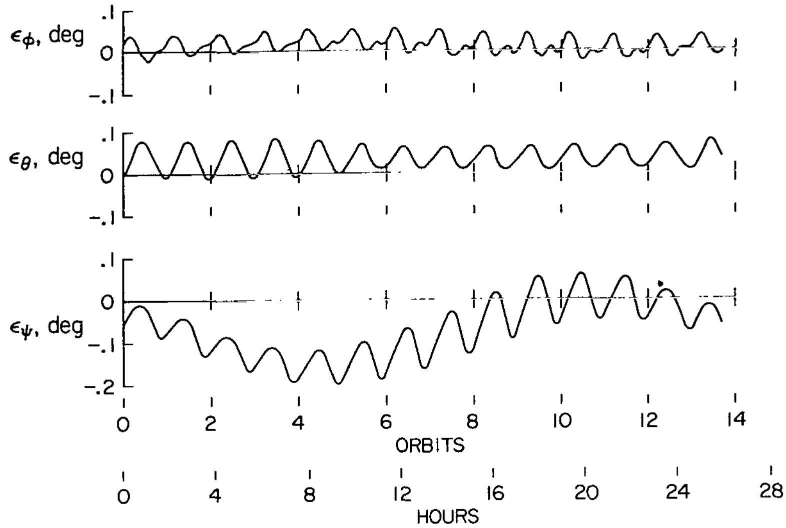
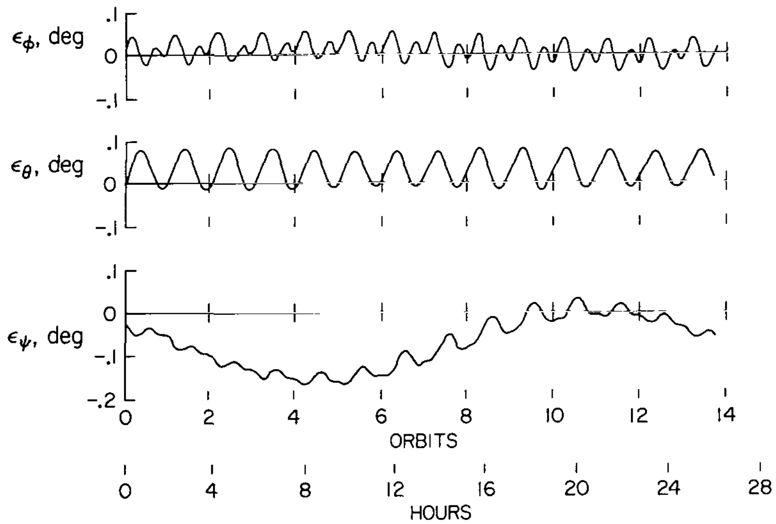


Figure 8.- Errors when the solar aspect measurements for two different orbit-sun relationships are analyzed as one data sample, $\lambda = 0$.

yield a set of parameters for a best fit to all the attitude motion. This possibility was explored in a rudimentary fashion by analyzing a data segment composed of 50 observations when $\Omega = 60°$ and 50 when $\Omega = 0°$. The results are shown in figure 8. The estimation of roll and pitch for either condition was equivalent to the estimate when all 100 pieces of data were taken from a single condition. The yaw error, though about doubled, was considerably smaller than the error encountered under the condition of figure 7. The parameters estimated from a combination of the two data



(a) Ω = 30°, λ = 0°



(b) Ω = 90°, λ = 120°

Figure 9.- Errors in attitude prediction when $\gamma \sim 60°$ from parameters estimated from data when $\gamma = 90°$ and $\gamma \approx 30°$.

segments were used to predict the motion for other orbits. Results of predictions when the orbit-sun relationship was midway between the two data samples are shown in figure 9 for 24-hour periods. Two cases are shown, one corresponding to the time of the equinox, and one at the time of the solstice. In each case, the errors in pitch and roll are the same magnitude as estimated for a particular data sample. The error in yaw is roughly twice that estimated for a short term data sample and shows a 24-hour variation indicative of an error in estimation of the components of the magnetic dipole.

The results presented are limited in that they are based entirely on simulation, rather than flight results, and in that a single example satellite confined to a particular orbit was studied. If the results can be considered typical, indications are that the pitch and roll attitude can be predicted with an accuracy about five times better than the accuracy of a single measurement of an attitude sensor. For the particular satellite studied, the prediction, along with an accurate satellite ephemeris, should yield the geographic location of the earth-oriented instrument axes to within 3 km. The errors in the long-term prediction of the yaw errors was about twice that for roll and pitch. Better yaw information might be obtained through estimation with advance prediction limited to periods encompassing only a small change of the orbit relative to the sunline.

CONCLUSIONS

The steady-state motion of a passive gravity-stabilized satellite is determined solely by environmental disturbances. This fact can be exploited to predict future attitude motion. Simulation studies of one prediction technique have been made in which measurements of the attitude instruments were analyzed to evaluate unknown parameters in the disturbance torque model. The prediction then consists in calculating the steady-state attitude motion caused by the disturbances. The studies established the following:

1. Attitude estimation and prediction are considerably more accurate than the attitude determined from a single set of instrument readings taken at a given instant.

2. Attitude estimation and prediction for a gravity-stabilized satellite can be accomplished from solar aspect measurements alone.

3. A simple observable set of parameters was found from which a good approximation to the disturbance torques can be calculated to permit estimation of the current attitude motion. However, the set of parameters does not model the disturbance torques perfectly so that prediction of future attitude deteriorates as orbital regression changes the plane of the orbit.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., 94035, Sept. 5, 1969

APPENDIX A

VARIANCE DUE TO DIGITIZATION OF THE SOLAR ASPECT SENSOR

The solar aspect sensor was chosen to be identical with the Digital Solar Aspect System installed in the Applications Technology satellites designed for gravity-gradient stabilization. Each sensor has a 64° field of view in any plane containing the normal to the surface upon which it is mounted. Full spherical coverage is provided by mounting three sensors 120° apart in a plane normal to the pitch axis and two sensors viewing in opposite directions along the pitch axis. When the sun is in view of more than one sensor, the reading is selected from the sensor for which the sun is more nearly centered in the field of view.

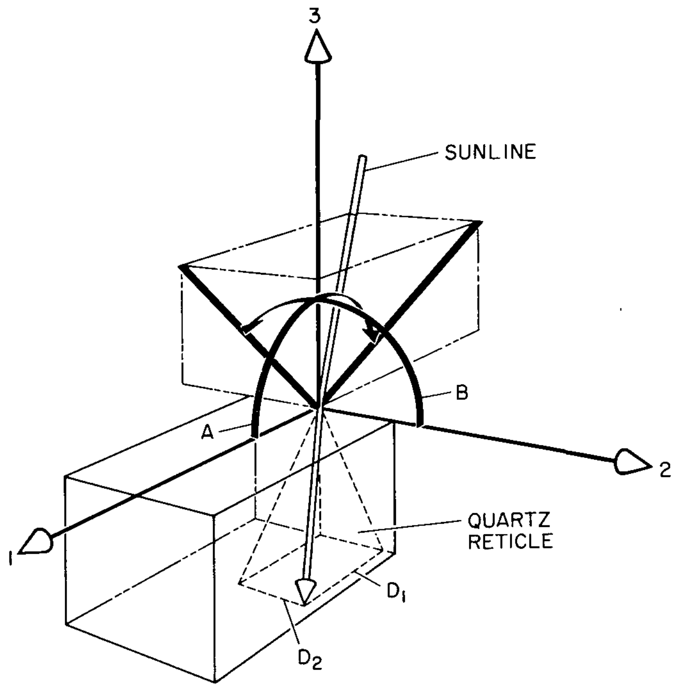


Figure 10.- Geometry of a solar aspect sensor head.

The geometry of an individual sensor head is shown schematically in figure 10. Each sensor has two units, each with a quartz reticle with a slit on its upper surface and a binary coded pattern on the lower surface. One of the units measures the angle A, and the other, the angle B. The measurements of angles A and B indicated in the figure will depend on the distances D_1 and D_2, respectively. The distances depend on the index of refraction of the quartz reticle and the angle that the sunline makes with its surface. Since the latter angle depends on both angles A and B, the reading of either of the two units depends on angles A and B.

The instrument senses D_1 and D_2. These distances, and the number of bits in the instrument output define two parameters α and β which can be related to the angles A and B as follows:

$$\left.\begin{aligned} \alpha &= \frac{128\ D_1}{(D_1)_{max}} = \frac{-L \cos A}{\sqrt{1 - R^2 \cos^2 A + (1 - R^2)\cot^2 B \sin^2 A}} \\ \beta &= \frac{128\ D_2}{(D_2)_{max}} = \frac{-L \cos B}{\sqrt{1 - R^2 \cos^2 B + (1 - R^2)\cot^2 A \sin^2 B}} \end{aligned}\right\} \qquad (A1)$$

where

L $\left(128\sqrt{1 - R^2 \sin^2 64°}\right)/\sin 64°$

R reciprocal of the index of refraction of the reticle

64° maximum view angle of instrument

The output of the instrument E_1 and E_2 is digital and is related to the parameters α and β

$$\left.\begin{aligned} E_1 &= n_1 \operatorname{sgn} \alpha \qquad n_1 \leq |\alpha| < n_1 + 1 \\ E_2 &= n_2 \operatorname{sgn} \beta \qquad n_2 \leq |\beta| < n_2 + 1 \end{aligned}\right\} \tag{A2}$$

$n_1, n_2 = 0, 1, 2, 3, . . ., 128.$

In terms of the digital outputs, E_1 and E_2, the angles indicated by the instrument are as follows:

$$\left.\begin{aligned} A_o &= \cot^{-1} \frac{E_1}{\sqrt{L^2 - (1 - R^2)(E_1^2 + E_2^2)}} \\ B_o &= \cot^{-1} \frac{E_2}{\sqrt{L^2 - (1 - R^2)(E_1^2 + E_2^2)}} \end{aligned}\right\} \tag{A3}$$

The possible deviation of the true angles A and B from the indicated angles A_o and B_o is dependent upon A and B. The variance of the solar aspect sensor therefore must be calculated for each data point for use in the minimum variance solution of equation (1). The variance was calculated as follows:

The actual value of the angle A is dependent upon α and β.

By definition the variance of the angle A is:

$$\sigma_A^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (A - A_o)^2 f(\alpha, \beta) d\alpha\, d\beta \tag{A4}$$

where $f(\alpha, \beta)$ is the probability density function. Since α and β are independent variables,

$$f(\alpha, \beta) = f(\alpha) f(\beta)$$

A Maclaurin series expansion of $(A - A_o)$ as a function of two variables yields:

$$(A - A_o) = \frac{\partial A}{\partial \alpha}(\alpha - E_1) + \frac{\partial A}{\partial \beta}(\beta - E_2)$$

$$+ \frac{1}{2!}\left[\frac{\partial^2 A}{\partial \alpha^2}(\alpha - E_1)^2 + 2\frac{\partial^2 A}{\partial \alpha\, \partial \beta}(\alpha - E_1)(\beta - E_2) + \frac{\partial^2 A}{\partial \beta}(\beta - E_2)^2\right]$$

$$+ \text{third and higher order terms in } (\alpha - E_1) \text{ and } (\beta - E_2)$$

and

$$(A - A_o)^2 = \left(\frac{\partial A}{\partial \alpha}\right)^2(\alpha - E_1)^2 + \left(\frac{\partial A}{\partial \beta}\right)^2(\beta - E_2)^2 + 2\frac{\partial A}{\partial \alpha}\frac{\partial A}{\partial \beta}(\alpha - E_1)(\beta - E_2)$$

$$+ \text{third and higher order terms in } (\alpha - E_1) \text{ and } (\beta - E_2) \qquad \text{(A5)}$$

By definition:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha)f(\beta)d\alpha\ d\beta = 1 \qquad \text{(A6)}$$

and

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\alpha - E_1)^2 f(\alpha)f(\beta)d\alpha\ d\beta = \sigma_{E_1}^2 \qquad \text{(A7)}$$

For a given digital output, E_1 is considered to be the mean value of α and the distribution is assumed to be symmetrical. Therefore,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\alpha - E_1)^n f(\alpha)f(\beta)d\alpha\ d\beta = 0 \quad \text{for } n \text{ odd} \qquad \text{(A8)}$$

The distribution functions can be expressed as:

$$f(\alpha) = 1 \qquad |(\alpha - E_1)| < \frac{1}{2}$$

$$f(\alpha) = 0 \qquad |(\alpha - E_1)| > \frac{1}{2}$$

and

$$\sigma_A^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (A - A_o)^2 f(\alpha)f(\beta)d\alpha\ d\beta \qquad \text{(A9)}$$

Within the region where $f(\alpha) = 1$, the derivatives $\partial A/\partial\alpha$ and $\partial A/\partial\beta$ are evaluated at the point A_o and treated as constants. Therefore, equation (A5) when substituted into (A9), using equations (A7) and (A8) yields

$$\sigma_A^2 = \left(\frac{\partial A}{\partial \alpha}\right)^2 \sigma_{E_1}^2 + \left(\frac{\partial A}{\partial \beta}\right) \sigma_{E_2}^2 \tag{A10}$$

Similarly,

$$\sigma_B^2 = \left(\frac{\partial B}{\partial \alpha}\right) \sigma_{E_1}^2 + \left(\frac{\partial B}{\partial \beta}\right)^2 \sigma_{E_2}^2$$

A similar calculation will yield an expression for σ_{AB} that represents the unknown distortion of the interval caused by the variation of α and β within the interval. This covariance is small and was ignored in the analysis of the data.

The partial derivatives in equations (A10) are evaluated at the center of the interval from equations (A1) as follows:

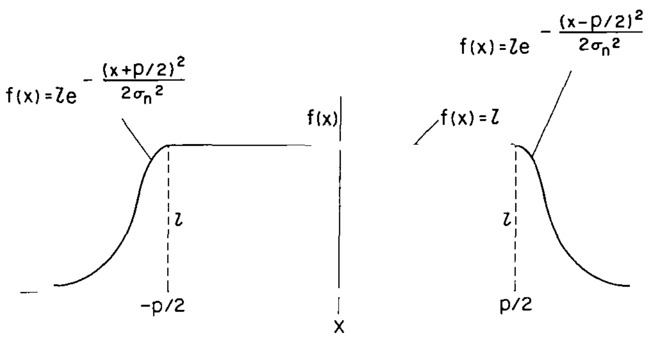
$$\left.\begin{aligned}
\frac{\partial A}{\partial \alpha} &= \frac{-L^2 - (1 - N^2)\beta^2 \cos^3 A}{\alpha^3 \sin A} \\
\frac{\partial A}{\partial \beta} &= \frac{-\beta(1 - N^2)\cos^3 A}{\alpha^2 \sin A} \\
\frac{\partial B}{\partial \alpha} &= \frac{-\alpha(1 - N^2)\cos^3 B}{\beta^2 \sin B} \\
\frac{\partial B}{\partial \beta} &= \frac{-L^2 - (1 - N^2)\alpha^2 \cos^3 B}{\beta^3 \sin B}
\end{aligned}\right\} \tag{A11}$$

APPENDIX B

VARIANCE OF A DIGITAL INSTRUMENT WITH BOUNDARY ERRORS

Deviations between the output of a digital instrument and the true value of the quantity being measured can arise from digitization and from uncertainties in the boundaries separating regions corresponding to adjacent digits. In the following, the variance is calculated for digital instruments for which the errors at the boundary are normally distributed.

The assumed distribution function f(x) is illustrated in the sketch.



By definition, $\int_{-\infty}^{\infty} f(x)dx = 1$; therefore,

$$lp + l\int_{-\infty}^{-p/2} e^{-[x+(p/2)]^2/2\sigma_n^2}\,dx + l\int_{p/2}^{\infty} e^{-[x-(p/2)]^2/2\sigma_n^2}\,dx = 1 \qquad \text{(B1)}$$

where σ_n is the variance of the normal distribution. The solution of equation (B1) yields

$$l = \frac{1}{p + \sigma_n\sqrt{2\pi}} \qquad \text{(B2)}$$

The variance of the distribution f(x) is

$$\sigma^2 = l\left\{\int_{-\infty}^{-p/2} x^2\, e^{-[x+(p/2)]^2/2\sigma_n^2}\, dx + \int_{-p/2}^{p/2} x^2\, dx + \int_{p/2}^{\infty} x^2\, e^{-[x-(p/2)]^2/2\sigma_n^2}\, dx\right\}$$

$$= l\left(\sqrt{2\pi}\,\sigma_n^3 + 2p\sigma_n^2 + \frac{p^2}{4}\sqrt{2\pi}\,\sigma_n + \frac{p^3}{12}\right) \tag{B3}$$

Substituting equation (B2) in equation (B3) yields

$$\sigma^2 = \frac{\frac{p^3}{12} + \frac{p^2}{4}\sqrt{2\pi}\,\sigma_n + 2\sigma_n^2 + \sqrt{2\pi}\,\sigma_n^3}{p + \sigma_n\sqrt{2\pi}} \tag{B4}$$

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