11176-H411-R0-00

PROJECT TECHNICAL REPORT
TASK KM-205

THE USE OF MAGNETIC TORQUING FOR CONTROL MOMENT GYRO DESATURATION

NAS 9-8166

5 December 1969

Prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

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TRW
SYSTEMS GROUP

NASA CR 102075

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Contents

Illustrations

1. INTRODUCTION

This report presents the results of a feasibility study of using magnetic torquing for Control Moment Gyro (CMG) desaturation and momentum dumping for the Apollo Applications (AAP) mission. In magnetic torquing, a torque is developed by placing a magnetic dipole moment in a magnetic field. This torque is applied to the vehicle, and its effect is counteracted by the CMG's. The result is to change the components of momentum in the Control Moment Gyros. The effectiveness of this technique to meet the requirements for CMG momentum dumping is the problem to be analyzed. The necessary control laws for such a system are derived, and a simple control system design is presented. The basic equations for designing either coreless or ferrite core magnetic torquers and representative design calculations are presented in a "tutorial format" to aid in understanding the problem.

This study investigates an alternative to the gravity gradient technique for CMG desaturation because the gravity gradient technique has some undesirable features and limitations. Since gravity gradient methods are not a part of this study, these problems will not be discussed and no comparison will be made between these two systems.

A review of the Marshall Space Flight Center (MSFC) and Belcomm reports on the magnetic torquing performed as a part of the Controls Analysis subtask 7 on AAP task KM205 is included in this report.

2. CONTROL LAWS

2.1 VECTORING OF THE MAGNETIC DIPOLE

To implement a magnetic torquing system, control laws must be derived to vector a magnetic moment. In this section the physical equations for the control laws will be explained and a control system adequate for CMG momentum dumping proposed.

A magnetic torquer, whether coreless or ferrite, may be represented by a simple magnetic dipole moment for the purposes of illustration and calculation (References 1, 2). The basis for the system is the phenomenon exhibited by a magnetic moment in the presence of a magnetic field. A torque vector, $\bar{T}$, will be exerted on a magnetic moment vector, $\bar{m}$, in a magnetic field vector, $\bar{B}$, according to the following relation:

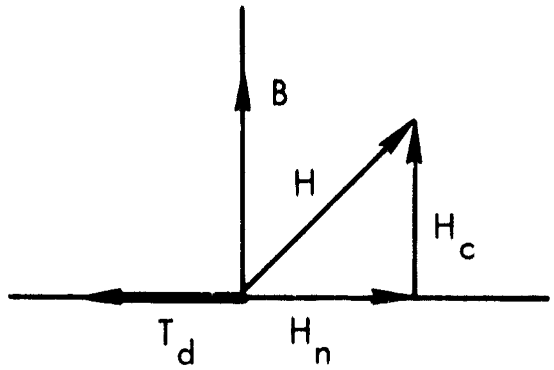
$$\bar{T} = \bar{m} \times \bar{B} \tag{1}$$

The momentum to be dumped is represented by the vector $\bar{H}$ with components $\bar{H}_n$ and $\bar{H}_c$ (Figure 1). By the definition of the cross product, Equation (1) indicates that a torque cannot be generated collinear with the earth's magnetic field. Consequently, the vector $\bar{H}_c$ represents momentum which cannot be dumped using torque derived from the earth's magnetic field vector with the instantaneous value shown in Figure 1. Therefore, $\bar{T}_d$ represents the torque required to dump the momentum $\bar{H}_n$ in some given time interval. This proportionality yields the relationship

$$\bar{T}_d = -K\bar{H}_n \tag{2}$$

where K is a time dependent proportionality constant yet to be determined. Given a constant required torque, $\bar{T}$, the proportionality constant is simply the reciprocal of time.

The value of the total dump time t is specified by the total dark side flight time and some estimate of favorable B field orientation during this time. It will be obvious in a later section that the optimum power expenditure is effected if a minimum required $\bar{m}$ is produced. To accomplish this the constant K is set equal to 1/t.



B earth's magnetic field

H momentum to be dumped

M magnetic moment which may be generated in any direction

T_d torque desired

H_c momentum colinear with earth's field

H_n momentum perpendicular with earth's field

Figure 1. Torque Vector Diagram

This system may take the form of an iterative process as follows:

$$\begin{bmatrix} \Delta H_x \\ \Delta H_y \\ \Delta H_z \end{bmatrix} = \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} - \begin{bmatrix} \int_{to}^{t} T_x dt \\ \int_{to}^{t} T_y dt \\ \int_{to}^{t} T_z dt \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \frac{1}{t_f - t} \begin{bmatrix} \Delta H_x \\ \Delta H_y \\ \Delta H_z \end{bmatrix} \tag{4}$$

where H_x, H_y, and H_z are the total momentum to be dumped. These values are determined as an average CMG momentum buildup during one orbit. The ΔH_x, ΔH_y, and ΔH_z terms represent instantaneous momentum to be dumped; T_x, T_y, and T_z represent the desired torque to be produced. The terms t_o and t_f are the beginning and end times for torquing while t represents the instantaneous time.

Since ΔH_x, ΔH_y, and ΔH_z are continually being incremented, they are time dependent. In order to simplify the calculations, the torque components T_x, T_y, and T_z should remain constant. As t approaches t_f, $\Delta\bar{H}$ should approach zero to allow a finite solution for the torque given by Equation 14. However, errors in calculations for $\Delta\bar{H}$ may require limiting the term $(t_f - t)$ to be a positive nonzero quantity. At several iterations prior to the termination time, this limitation will be imposed.

From Figure (1) it can be seen that

$$\bar{H}_c = \frac{(\bar{H} \cdot \bar{B})\,\bar{B}}{B^2} \tag{5}$$

and

$$\bar{H}_n = \bar{H} - \bar{H}_c \tag{6}$$

Substituting equations (5) and (6) into Equation (2) and using a vector identity

$$\bar{T}_d = \frac{-K}{B^2}\left(B^2\,\bar{H} - (\bar{H} \cdot \bar{B})\,\bar{B}\right)$$

$$\bar{T}_d = \frac{K}{B^2}(\bar{H} \times \bar{B}) \times \bar{B} \tag{7}$$

Relating this equation to Equation (1) and solving for $\bar{m}$

$$\bar{m} \times \bar{B} = \frac{K}{B^2}(\bar{H} \times \bar{B}) \times \bar{B}$$

$$\bar{m} = \frac{K}{B^2}(\bar{H} \times \bar{B}) \tag{8}$$

From Equation (3), the optimum magnetic moment vector dumps the momentum $\bar{H}$, given the earth's magnetic field $\bar{B}$. The instantaneous representation of Figure (1) is somewhat misleading since the orientation of the earth's magnetic field with respect to the momentum to be dumped will change along the orbital path, the momentum $\bar{H}_c$ will no longer be colinear with the earth's field. The capability to dump this momentum will then be acquired.

There are two alternatives to the problem of proper magnetic moment vectoring. The first method involves the use of three orthogonal torquing units. By creating a magnetic moment of the correct magnitude in each torquer, the vector sum can be adjusted to produce the desired moment vector. In the second method a single torquer is gimballed to the proper orientation which creates a moment of the necessary magnitude. The gimbal structure must have two degrees of freedom so that the magnetic dipole can be vectored in any arbitrary direction.

The development of a control law for a system of three orthogonal torquers is straightforward. The desired magnetic moment $\bar{m}$ is determined by the use of Equation (8). The magnitude of the magnetic moment produced by the torquer will be a function of the applied current $\bar{I}$. In a coreless torquer this relationship is linear; while in ferrite torquers, it may be nonlinear. The moment may then be represented by

$$\bar{m} = C\bar{I} \tag{9}$$

where C is a proportionality constant determined by the physical characteristics of the coil and $\bar{I}$ is a vector representing the vector sum of the three orthogonal moment producing currents. In the case of a coreless torquer, for instance, this would equal the turn-area product:

$$C = NA$$

where N = number of turns in coil

A = area of coil

Combining Equations (8) and (9) and expressing the cross product in scalar form, the desired relationship is expressed in Equation (10).

$$\begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} = \frac{K}{CB^2} \begin{bmatrix} \Delta H_y B_z - \Delta H_z B_y \\ \Delta H_z B_x - \Delta H_x B_z \\ \Delta H_x B_y - \Delta H_y B_x \end{bmatrix} \tag{10}$$

The ΔH terms represent the instantaneous momentum to be dumped. Since $\bar{B}$, ΔH and K will be functions of time, the current in each torquer will also be a function of time. Except for the deterministic constant K/C, the current inputs to the torquers are specified.

If a single gimballed torquer method is used, an equation of the same form as Equation (10) results. In this case however, the magnitude of the magnetic moment is required. Thus

$$\begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \frac{K}{B^2} \begin{bmatrix} \Delta H_y B_z - \Delta H_z B_y \\ \Delta H_z B_x - \Delta H_x B_z \\ \Delta H_x B_y - \Delta H_y B_x \end{bmatrix} \tag{11}$$

This equation is again time dependent. From this, the necessary relationships follow:

$$|m| = (m_x^2 + m_y^2 + m_z^2)^{1/2} \quad (12a)$$

$$I_o = \frac{|m|}{C} \quad (12b)$$

The constant C again depends on the specific torquer used, and I_o is the current requirement.

Figure (2) represents a single torquer in a two gimbal system. Two degrees of freedom allow the magnetization vector to be properly

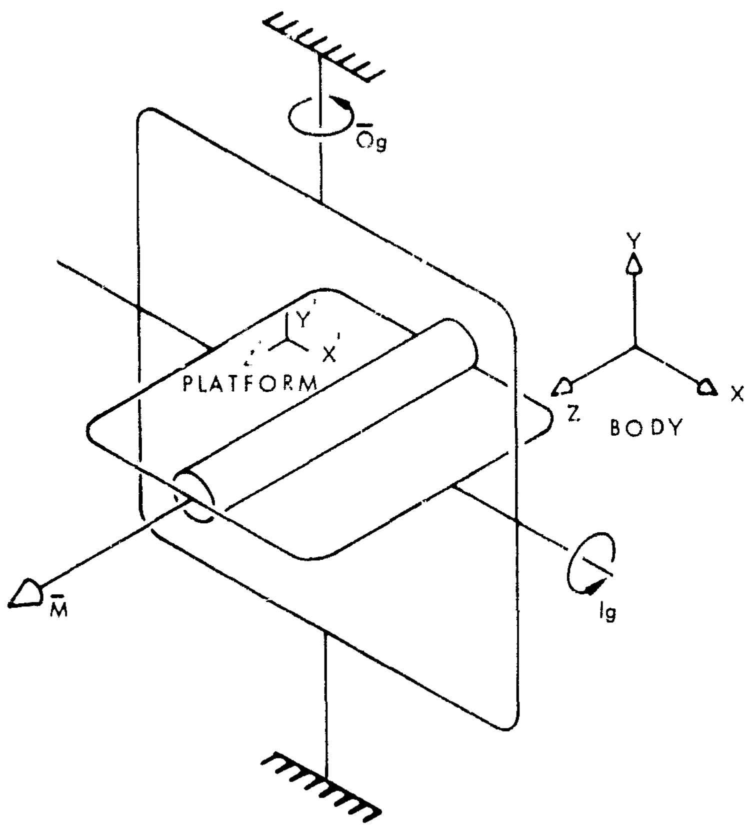


Figure 2. A Two Gimbal Torquer System

positioned. Transformation from the two gimbal system shown in figure (2) to body coordinates is accomplished by the use of Equation (13) where 0_g and I_g are the outer and inner gimbal angles respectively. The transformation takes the form:

$$\begin{bmatrix} m_{x'} \\ m_{y'} \\ m_{z'} \end{bmatrix} = \begin{bmatrix} \cos 0_g & 0 & \sin 0_g \\ -\sin 0_g \sin I_g & \cos I_g & \sin I_g \cos 0_g \\ -\cos I_g \sin 0_g & \sin I_g & \cos I_g \cos 0_g \end{bmatrix} \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

where $m_{x'} = m_{y'} = 0$, and $m_{z'} = |m|$.

It therefore follows that

$$\begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = |m| \begin{bmatrix} \sin O_g \cos I_g \\ -\sin I_g \\ \cos O_g \cos I_g \end{bmatrix} \quad (13)$$

From Equation (13) the gimbal angles may be determined from the specified components of required magnetic moments in body coordinates:

$$\frac{m_x}{m_z} = \frac{|m| (\sin O_g \cos I_g)}{|m| (\cos O_g \cos I_g)} = \tan O_g$$

$$O_g = \tan^{-1} \left(\frac{m_x}{m_z} \right) \quad (14a)$$

$$\frac{m_y}{m_z} = \frac{|m| (-\sin I_g)}{|m| (\cos I_g \cos O_g)} = -\tan I_g \sec O_g$$

$$I_g = -\tan^{-1} \left[\left(\frac{m_y}{m_z} \right) \cos O_g \right] \quad (14b)$$

2.2 DETERMINATION OF THE EARTH'S MAGNETIC FIELD

The earth's magnetic field vector may be determined in several ways. The geomagnetic field may be stored in the onboard computer in the form of a table, with the independent variables identified as longitude and latitude and the dependent variables as the components of the earth's field in a local vertical reference frame. A fixed altitude would be assumed. This method would require a two-dimensional (latitude - longitude) interpolation. The stored table may require a storage of approximately 300 fixed values for the earth's magnetic field and an undetermined amount of storage capacity for the interpolation routine. Another possible method is to use a spherical harmonic model for the geomagnetic field. Again large storage capacity may be required to store the fundamental harmonic equations and coefficients. As a third alternative, three orthogonal magnetometers could be used as sensors to provide the scalar components of the field vector in body coordinates. The software requirements for this method are much simpler than the other two mentioned, requiring only that the magnetometer measurements be inputs to the onboard computer. However, the major problem with this method is to provide adequate isolation of the additional hardware, namely the three magnetometers, from the magnetic torquers.

Any one of these methods appears feasible as a method of determining the earth's field using onboard computations or sensing. However, the tradeoffs among these methods is beyond the scope of this investigation.

3. GENERATION OF MAGNETIC MOMENTS

3.1 ASSUMPTIONS

Several assumptions are necessary before the design criteria are discussed so that a complete understanding of the design is achieved. To limit the complexity of the calculations, the value of the earth's magnetic field is taken to be constant at a value of 0.3 gauss. This value represents the average magnetic field magnitude at 230 nautical miles based on a dipole model of the earth's magnetic field. A momentum value of 1000 foot-pound-seconds is taken for the CMG dump requirement and it is assumed that one half orbit (45 minutes) is available for dumping. This dictates the torque requirement of 0.5 Newton meters necessary to dump accumulated momentum. These values are assumed constant throughout.

3.2 CORELESS TORQUERS

The coreless torquer is considered as a current loop. Conceptually, the force acting on a current loop is caused by the magnetic dipole moment of the loop, but, in actuality, it results from the force acting on a current element:

$$\bar{F} = q(\bar{V} \times \bar{B}) = \bar{i} \times \bar{B} \qquad (15)$$

where

$\bar{F}$ represents force

q represents charge

$\bar{v}$ is the velocity of the charge

and $\bar{i}$ represents current

The torque acting on the loop may be developed from this equation. Figure (3) is a representation of a current loop. Referring to Figure (3), Equation (15) may be written in the form:

$$\overline{dF} = \overline{I d\ell} \times \bar{B} \qquad (16)$$

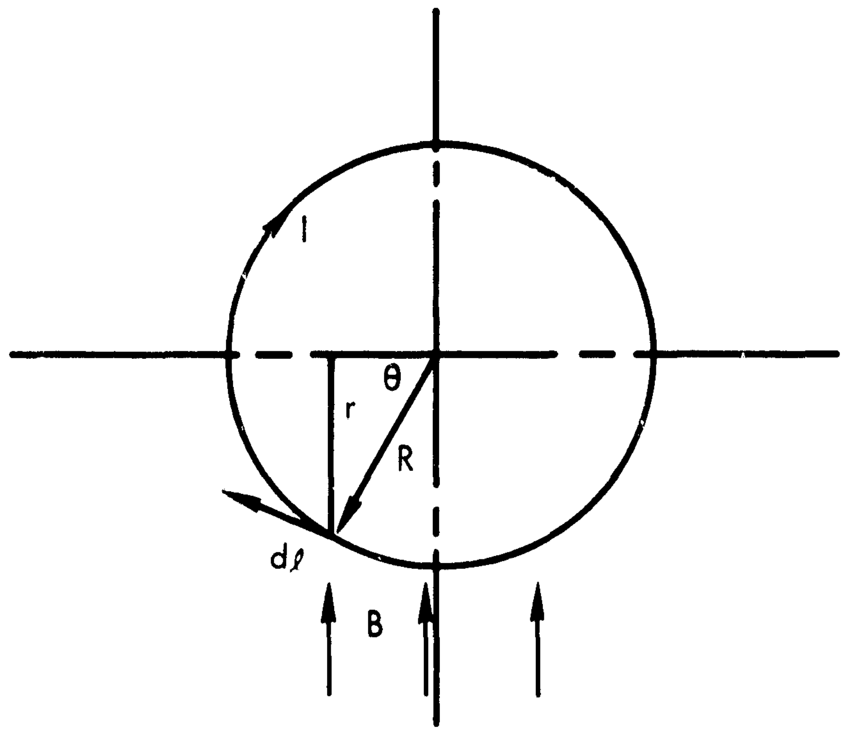


Figure 3. Current Loop

where I is the current flowing in the loop and $\overline{d\ell}$ is an incremental element of the loop. From the definition of torque,

$$d\,\bar{T} = \bar{r} \times \overline{df} \tag{17}$$

From figure (2) the magnitude of

$$d\ell = R d\theta \tag{18}$$

and

$$r = R \sin\theta \tag{19}$$

Combining Equations (16), (17), (18), and (19), the resulting magnitude equation represents the present system:

$$dT = IR^2\, B\, \sin^2\theta\, d\theta \tag{20}$$

Integrating this equation and evaluating the integral, the resultant torque is

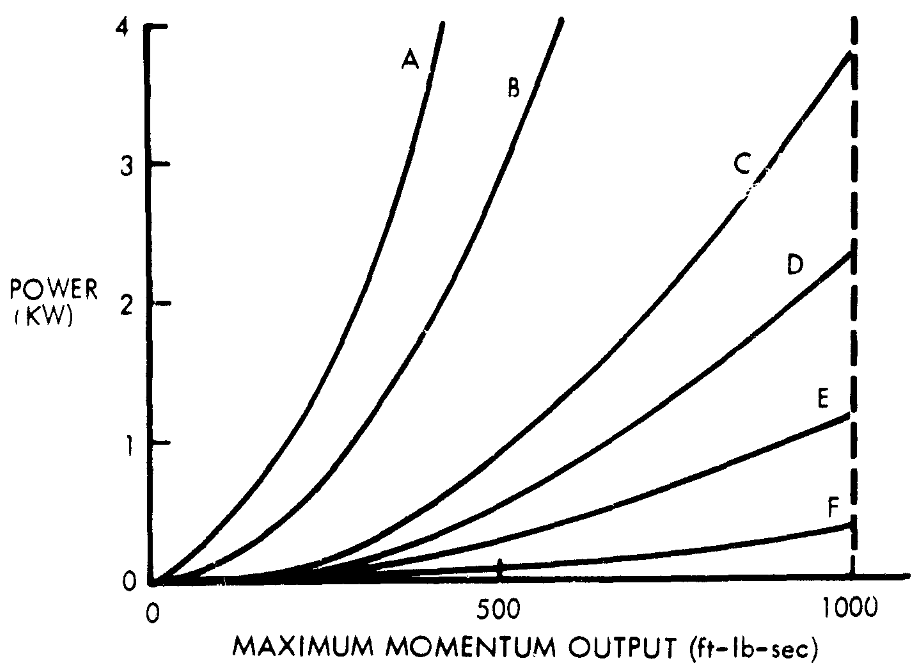
$$T = \pi R^2\, I\, B \tag{21}$$

Substituting $A = \pi R^2$ into the equation and $I = Ni$, a more familiar form results:

$$T = A\, Ni\, B \tag{22}$$

where the product ANi is defined as the magnitude of the magnetic moment $\bar{m}$.

Based upon the assumed values for T and B, Equation (22) may now be used to calculate the magnetic dipole moment necessary from the coreless torquer, and a value for $\bar{m}$ of 0.166 X 10^5 amp - m^2 is determined. Since the magnitude of $\bar{m}$ is equal to the product ANi, it is obvious that some tradeoffs are available in the design of the loop. Since A = πR^2, the magnetic dipole moment is a function of the radius R squared, and it is apparent that the larger the loop's radius, (and thus the area) the lower the current requirement. The power requirement is solely a function of the ohmic loss and, therefore, proportional to the radius of the loop and the square of the current in the loop. Larger loops, consequently, dissipate less power than smaller loops. By means of the same reasoning, it may be shown that smaller loops have more mass for fixed torque requirement. The various tradeoffs between radius, power, and weight are indicated in the curves given in Figure 4.



LEGEND

CURVE	RADIUS (ft)	NUMBER OF TURNS	WEIGHT (lb)
A	1.6	2 X 10^4	67
B	3.3	5 X 10^3	33
C	10.0	6 X 10^2	25
D	1.6	2 X 10^6	670
E	3.3	5 X 10^5	334
F	10.0	6 X 10^3	246

ASSUMPTIONS

- B = 300 MILLIGAUSS (CONSTANT EARTH MAGNETIC FIELD)
- DUMP TIME = 45 MIN
- WINDINGS CONSIST OF NUMBER 30 WIRE THROUGHOUT

Figure 4. Coreless Magnetic Torquer Performance

Example calculations will be performed for a 2 meter and a 10 meter diameter loop. The windings consist of number 24 annealed copper wire with an electrical resistance of 84.21 ohms per kilometer and a mass of 1.82 kilograms per kilometer. A current value of 0.10 ampere will be used in each case.

In the case of the 2-meter diameter loop,

$$NAI = 0.166 \times 10^5 \text{ amp} - m^2$$

$$A = \pi R^2 = 3.1416 \ m^2$$

Therefore, the number of turns necessary is

$$N = 5.29 \times 10^4 \text{ turns}$$

The total length of wire, ℓ, follows:

$$\ell = N2\pi R = 33.25 \times 10^4 \text{ meters}$$

The resistance of the coil is 28.0 kilohms and therefore the power dissipated is

$$P_{dis} = 280 \text{ watts}$$

The weight of the loop is

$$W = 605 \text{ kilograms} = 1320 \text{ pounds}$$

In the case of the 10 meter diameter loop, the area is given by

$$A = \pi R^2 = 78.5 \ m^2$$

The number of turns necessary in the coil is

$$N = 2115 \text{ turns}$$

The total length of wire necessary, ℓ, is

$$\ell = N2\pi R = 66.4 \times 10^3 \text{ meters}$$

The resistance of the wire comprising the coil is 5,590 ohms, and the power dissipated in ohmic losses is

$$P_{dis} = 55.9 \text{ watts}$$

The weight of the loop is

$$W = 121 \text{ kilograms (266 pounds)}$$

These calculations indicate the obvious tradeoffs which can be made between the weight, radius, and power parameters. For example, if weight is of more concern than power consumption, lower weight can be achieved through smaller diameter wire. For the 10-meter air loop, a number 30 annealed copper wire will decrease the weight to 30.2 kilograms (66.5 pounds) with a power consumption of 225 watts.

Although the derivation will not be carried out here, it may be shown (Reference 2) that the magnitude of the magnetic field resulting from a current loop, along the axis of the loop is

$$B = \mu_o \frac{I}{2} \frac{a^2}{(Z^2 + a^2)^{2/3}} \tag{23}$$

where a is the radius of the loop and z is the distance from the center of the loop along the axis. Substituting the appropriate values, the magnitude of the magnetic field in the plane of the loop, for the 10 meter diameter loop will be

$$B = 0.534 \text{ gauss}$$

At a distance of one meter, this value is

$$B = 0.502 \text{ gauss}$$

The field strength is only about twice that of the earth's field and represents no difficulty to any electronic equipment onboard.

3.3 FERROMAGNETIC MATERIAL

All materials exhibit some magnetic properties. Depending on their magnetic behavior, substances can be classified as diamagnetic, paramagnetic, and ferromagnetic. If a magnetic field is applied to these materials, the magnetization in the diamagnetic materials is opposed to the applied field. In the paramagnetic materials, the magnetization is in the same direction as in the applied field. In both cases, however, the effect is weak. Materials in the ferromagnetic group, on the other hand, exhibit very strong magnetic effects. It is these materials which will be considered for addition as a core to the torquer with the expectation of obtaining some benefit from their inclusion.

The characteristic of ferromagnetic material is to produce a magnetization vector proportional to the field applied to it. This magnetization vector may be viewed as a volume aggregate of atomic magnets and is defined as the magnetic dipole moment per unit volume. It is then possible to define the magnetic dipole moment of a volume of ferromagnetic material as the product of the volume of the material v and the magnetization vector $\bar{M}$. Thus,

$$\bar{m} = \bar{M}v \tag{24}$$

For a given magnetization vector and specified magnetic moment, the required volume of material can be determined. The magnetization vector will be a function of the material considered and the magnitude of the applied field.

Two distinct groups of ferromagnetic material are considered for use as torquer cores. The first group (high μ ferrites) is represented by two materials, Supermalloy and Vanadium Permendur. In general, these materials exhibit a high permeability with little magnetic retention; that is, when the applied field is reduced to zero, the magnetic field within the material returns to approximately zero. The second group (the permanent magnetic ferrites) is represented by Alnico V. This material exhibits a high permeability as well as the ability to retain a strong magnetic field within the material as the applied field is returned to zero. This ability is called magnetic remanence, and its value is dependent upon the material used and the past history (hysteresis) of applied magnetic fields. Pertinent data for these materials are listed in Table 1.

Table 1. Ferromagnetic Material *

Name	Br remanence, (gauss)	Maximum permeability (gauss/oersted)	Hc Coercive Force, (oersted)	Bs=ΔπMs Saturation Indication (gauss)
Alnico V	12,500	15	660	nil
Supermalloy	nil	1,000,000	0.002	7,900
Vanadium Permendur	nil	4,500	2.0	24,000

* See Reference 5

3.4 DEMAGNETIZATION

With the introduction of ferromagnetic material, it is necessary to discuss the concept of a demagnetizing field. This demagnetizing field accounts for a noncontinuous ferromagnetic path (Reference 5). When a rod is magnetized by an applied field H_a, the ends carry magnetic poles which cause magnetic fields in all parts of the rod. Normally, these fields are directed in the opposite direction to the applied fields and are, therefore, called demagnetizing fields. The true field acting on a given section of the bar, (e.g., its middle) is then the resultant of the applied field and the demagnetizing field ΔH

$$H = H_a - \Delta H \quad (25)$$

The demagnetizing field is approximately proportional to the magnetization

$$\Delta H = NM \quad (26)$$

where N is the demagnetization factor. It is apparent from Equation (26) that a large demagnetization factor will give a large demagnetizing field. For the ferrites, the demagnetization factor is a function of the geometry considered. Equations to determine demagnetization factors have been published. Tables and charts based on the solutions of these equations for cylinders and ellipsoids of revolution have also been published (Reference 5, 6, 7). Table 2, for example, lists demagnetization factors for various length to diameter (L/D) ratio cylinders.

Table 2. Demagnetization Factors for Cylinders

L/D	Demagnetization Factor
1	0.27
2	0.14
5	0.04
10	0.0172
20	0.00617
50	0.00129

A large L/D ratio for the solenoid core and a corresponding low value of demagnetization factor are desirable. How large L/D may be is a function of space limitations. For the purpose of calculations, a ratio of 20 is chosen and the resulting demagnetization factor is 0.006.

3.5 SOLENOIDAL TORQUERS

A helical coil, or solenoid, is often used to produce a magnetic field. The coil would consist of N turns of wire carrying a current I. Although the proof will not be carried out here, it may be shown (Reference 4) that a torque is exerted on a solenoid in an external field B so that

$$T = NIAB \tag{27}$$

where A is the area of a cross section of the solenoid. This is the same expression derived for the loop (Equation 22, Section 3.2). Torque produced for the loop is a limiting case of that for the solenoid. A ferrite core was not considered for use with the loop because the loop is an inefficient method to produce a field in a long cylinder due to the field pattern of the loop. If a thin cylinder is considered, restricting it to the thickness of the loop and its location to the center of the loop, the demagnetization factor is approximately unity (1), and, therefore the ferrite is ineffective.

The magnetomotive force at the center of a long solenoid i.e., where the length of the helix is much larger than the mean radius of the helix, can be shown (Reference 4) to be

$$H_a = NI \tag{28}$$

where a is the radius of the loop. The field necessary to produce the desired magnetization vector is given by

$$H = H_a + \Delta H$$

where H_a represents the field as seen by the ferrite, ΔH represents the demagnetizing field and H is the field applied to the ferrite. The magnetic field is given by the relationship

$$B_{ferrite} = \mu(H)H$$

where $\mu(H)$ is the nonlinear relationship known as permeability.

The solenoid is used to apply a magnetic field to a cylinder of ferrite material. It will be wound on the surface of the cylinder and be restricted to this area. Calculations of the physical parameters of the torquer will be accomplished for each ferromagnetic material. As was discussed earlier, a torquing system using the solenoidal torquers may consist of three stationary orthogonal torquers or one gimballed torquer.

3.5.1 Supermalloy

Consulting the magnetization curves for Supermalloy (Reference 5), an operating point must be chosen. In this instance,

$$M = 557 \times 10^3 \text{ amps/m}$$

was chosen below saturation to avoid inefficient utilization of the ferrite.

Therefore, with the desired torque and assumed magnitude of the geomagnetic field, the volume of material necessary is

$$V = 0.0299 \text{ m}^3$$

With an L/D ratio of 20, this results in the radius of the cylinder equal to 0.619 meter (2.43 inches) and the length of the cylinder equal to 2.479 meters (97.4 inches). The resultant demagnetizing field using a demagnetizing factor of 0.006 is

$$\Delta H = 3.34 \times 10^3 \text{ amps/m}$$

From the magnetization curves, a resultant field of 0.06 oersted or 4.78 amps/m is necessary to produce M. Therefore, the total applied field necessary is the sum

$$H = H_a + \Delta H = 3.344 \times 10^3 \text{ amp turns}$$

The physical quantities of a solenoid are related to the field by Equation (28). Now, since the solenoid is wound uniformly over the cylinder, and the cylinder is 2.479 meters long,

$$NIL = nI = 8.28 \times 10^3 \text{ amp turns}$$

where n equals the total number of turns. If the current I is 0.5 ampere, then

$$n = 16.56 \times 10^3 \text{ turns}$$

and the length of wire necessary for a cylinder of this size is

$$\ell = 2\pi rn = 6.44 \text{ km}$$

Using number 24 annealed copper wire, the coil will weigh 11.75 kg (25.9 pounds) and have a resistance of 544 ohms. The power loss will then be an I^2R loss and amounts to 136 watts. The total weight of a single torquer will consist of the core weight of 573 pounds and the coil weight of 25.9 pounds, totaling 598.9 pounds. There is virtually no hysteresis loss encountered with Supermalloy.

3.5.2 Vanadium Permendur

For Vanadium Permendur, a point is chosen on the magnetization curve where

$$M = 1.595 \times 10^6 \text{ amps/m}$$

This point, far below saturation, was chosen to take maximum advantage of the slope of the magnetization curve. The volume of material required is

$$v = 0.0104 \text{ m}^3$$

and, for an L/D ratio of 20, results in a cylinder with a radius equal to 0.0436 meter (1.74 inches) and a length of 1.746 meters (68.5 inches). The resultant demagnetizing field using a demagnetization factor of 0.006 is

$$\Delta H = 9.55 \times 10^3 \text{ amps/m}$$

From the magnetization curve a resultant field of

$$H_a = 1.2 \times 10^3 \text{ amps/m}$$

is necessary, and, therefore, the applied field consists of the sum

$$H = H_a + \Delta H = 10.75 \times 10^3 \text{ amp-turns}$$

The cylinder, being 1.746 meters long, requires an amp-turns figure of

$$NIL = nI = 18.75 \times 10^3 \text{ amp-turns}$$

For a current value of 1 ampere

$$n = 18.75 \times 10^3 \text{ turns}$$

and a length of 5.11 kilometers of wire is necessary. Again using number 24 annealed copper wire, the weight of the wire is 9.37 kilograms (20.65 pounds) and the resistance 434 ohms. Power dissipation in the wire is 434 watts. Vanadium Permendur has a discernable hysteresis loss of 600 joules/m^3 and thus, an additional power dissipation of 6.26 joules (watt/sec) is suffered during each turn-on and turn-off sequence. The weight of single torquer, solenoid windings and core material, amounts to 94.97 kilograms (209.15 pounds).

For the purpose of comparison, if the L/D ratio is now changed to 50 for a Vanadium Permendur torquer, the cylinder would have a radius of 1.46 inches and a length of 12.4 feet. Because of the increased L/D ratio, the demagnetization factor decreases by a factor of 4.8 to 0.00129. Recalculating the pertinent quantities, if the current is decreased to 0.5 ampere, the power dissipation in the wire is now 106.5 watts with the same torquer weight of 209.15 pounds. This is a substantial savings in power.

3.5.3 Alnico V

The permanent magnetic material, Alnico V, exhibits the phenomenon known as remanence. When a field of sufficient magnitude to bring the material into or near saturation is applied and subsequently removed, a strong magnetization vector will be retained within the material. Saturation in this material is represented by a inductance of

$$B = 12,500 \text{ gauss}$$

Repeating the type of calculations carried out above

$$M = 962 \times 10^3 \text{ amps/m}$$
$$\Delta H = 5.772 \times 10^3 \text{ amps/m}$$

Therefore, the required applied field is

$$H = \Delta H + H_a = 77.46 \times 10^3 \text{ amps/m}$$

The volume of required material is

$$V = 0.01722 \text{ m}^3$$

and therefore, for an L/D ratio of 20, the cylinder has the dimension of

$$r = 0.0515 \text{ meters (2.025 inches)}$$
$$L = 2.102 \text{ meters (83.6 inches)}$$

For these values, the required coil will consist of

$$nI = 162.5 \times 10^3 \text{ amp-turns}$$

Since this is an exceedingly large number, a current of 5 amperes is chosen to avoid astronomical coil weights. A coil wire length of 10.52 kilometers is necessary with a resistance of 886 ohms and a weight of 19.15 kilograms (42.2 pounds). The power dissipated through ohmic losses is 22.2 kilowatts. The hysteresis loss for this material during a torquing on-off cycle is approximately 330 joules. The total torquer weight is 144.75 kilograms (319.2 pounds).

Although the power requirement is high, this represents the instantaneous peak power necessary. If a current pulse of finite width is used, as is possible with a permanent magnetic material, the average power over a single torquing period will be much lower. If, for example, a 6-second turn-on and turn-off pulse is used, an average power requirement of 50 watts is necessary.

3.5.4 Tradeoff Factors

The amount of material necessary for a torquer is a constant for a specific material. The weight and power requirements of the solenoidal coil however, are variable, and tradeoffs are possible. For example, if a number 20 annealed copper wire is used with the Vanadium Permendur core instead of the number 24 wire, with a L/D ratio of 50 at a current of 1.0 ampere, the power dissipation will be 85.5 watts, and the coil will weigh 26.2 pounds. Also, for a given wire size the power-weight product is constant; for example, if the current is decreased by a factor of 10, the power will decrease and the weight will increase, each by a factor of 10.

3.5.5 Field Quantities

The magnetic field exterior to the torquer is an important quantity because of instrumentation onboard the vehicle. For a sufficiently long solenoid, the field along the side of the solenoid will be negligible (Reference 2). The only field of consequence will be from the end faces.

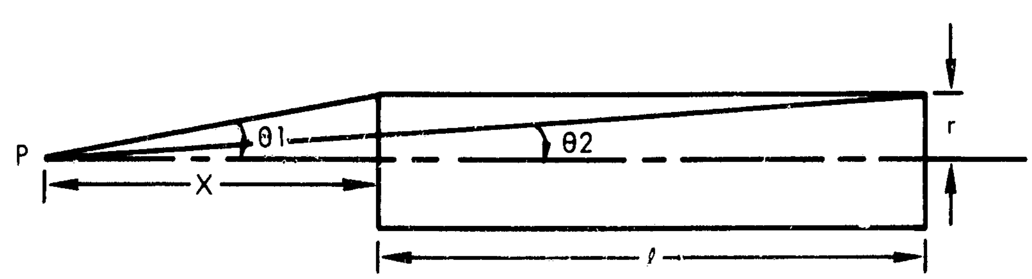


Figure 5. Solenoidal Torquers

In Figure (5), the expression for the magnetic field at point P along the axis of the solenoid will be

$$B = \tfrac{1}{2}\mu_o NI(\cos\theta_2 - \cos\theta_1)$$

$$B = \frac{\mu_o M}{2}\left[\frac{\ell + x}{\sqrt{r^2+(\ell+x)^2}} + \frac{x}{\sqrt{r^2+x^2}}\right]$$

Carrying through calculations for Vanadium Permendur with

$$x = 1.0 \text{ meter}$$
$$r = 0.0436 \text{ meter}$$
$$\ell = 1.746 \text{ meters}$$
$$B_i = \mu_o (M + H) = 2.02 \text{ webers/m}^2$$

we have:

$$B = \frac{B_i}{2}\left[\frac{\ell + x}{\sqrt{r^2 + (\ell+x)^2}} - \frac{x}{\sqrt{r^2 + x^2}}\right]$$

then the field at one meter is

$$B \text{ (1 meter)} = 1.01\left[\frac{2.746}{\sqrt{0.0019 + 7.5405}} - \frac{1}{\sqrt{0.0019 + 1}}\right]$$
$$= 8.484 \text{ gauss}$$

Figure 5 is representative of the solenoidal torquers with ferrite cores as a group. At three meters, the same calculations show that the field is reduced to 0.404 gauss or slightly more than the magnitude of the earth's field.

3.6 SPHERICAL TORQUERS

A spherical core wound with three orthogonal windings generates a general magnetic dipole with the advantage of a weight and space savings over systems using three separate cores. Although the weight and space saving seems attractive, the demagnetization factor of 0.333 for the sphere represents a distinct disadvantage. The implications of this will become more obvious after later calculations.

The spherical torquer will be excited with a coil wound on the sphere with uniform axial density (Reference 1). Referring to Figure 6, this will be represented by

$$\bar{K} = \bar{i} \frac{3m}{4\pi R^3} \sin\theta \qquad (30)$$

The field inside the sphere will have the form

$$\bar{H}_i = \frac{m}{2\pi R^3}(\bar{i}_r \cos\theta + \bar{i}_\theta \sin\theta) \qquad (31)$$

and outside the sphere, the field will be that of an ideal magnetic dipole

$$H_o = \frac{m}{2\pi R^3} (\bar{i}_r \cos\theta + \bar{i}_\theta \tfrac{1}{2} \sin\theta) \qquad (32)$$

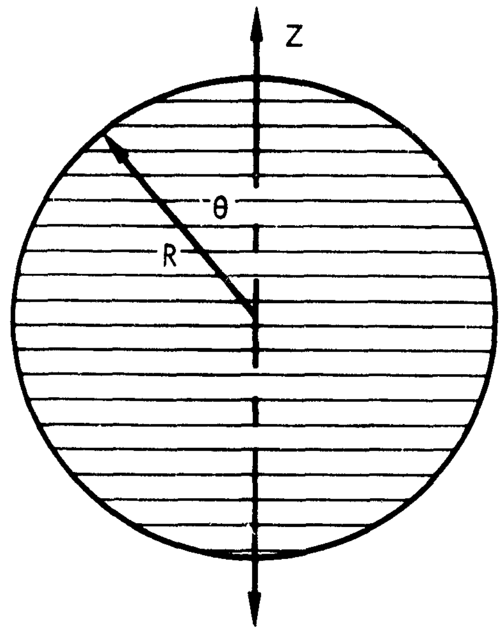


Figure 6. Spherical Torquer

Using Supermalloy, due to its extremely high permeability and negligible remanence field,

$$M = 557 \times 10^3 \text{ amps/m}$$

Now, with a demagnetization factor of 0.333, the demagnetizing field is equal to

$$\Delta H = 185.481 \times 10^3 \text{ amps/m}$$

With a necessary field of 4.78 amps/m, the applied field must now obtain the value

$$H = 185.486 \times 10^3 \text{ amps/m}$$

The coil windings add a significant torque as a result of the magnetic dipole moment they produce. This torque results from the large demagnetization factor (0.333 for the sphere versus 0.006 for a L/D = 20 solenoid). To calculate the dimensions of the material necessary, it will be noted that the total magnetic moment from the core and the winding takes the following form

$$Mv + NIA = m \qquad (33)$$

where v is the volume and A is the area of the sphere. The coil necessary to produce the necessary magnetic field is

$$NI = \frac{3m}{2\pi R^2} = H \cdot 3R$$

or approximately

$$NI = M \cdot R$$

Equation (33) then takes the form

$$R^3 = \frac{3m}{16\pi M}$$

Substituting the appropriate values and carrying out the calculations the value for the radius is obtained:

$$R = 0.1694 \text{ meters} = 4.48 \text{ inches}$$

The volume of the sphere is then

$$V = 0.02036 \text{ m}^3$$

This corresponds to a weight of 178.51 kilograms (394 pounds) for the core material.

The coil necessary is now

$$\begin{aligned} NI &= H \cdot 3R \\ &= 3\ (185.486 \times 10^3)\ (0.1694) \\ &= 94.29 \times 10^3 \text{ amp-turns} \end{aligned}$$

Since the same tradeoffs among weight and power made earlier for the solenoid are also made here, a current equal to 1.0 ampere is chosen. To calculate the necessary length of wire for a sphere, the average radius must be known. This is determined as follows:

$$\begin{aligned} R_{av} &= \frac{1}{Rm} \int_o^{Rm} (R_m^{\ 2} - x^2)\ dx \\ &= \frac{1}{Rm} \left[\frac{x}{2} \sqrt{Rm^2 - x^2} + \frac{Rm^2}{2} \sin^{-1} \left(\frac{x}{Rm}\right) \right]_o^{Rm} \\ &= Rm\ (0.7854) = 1330 \text{ meters} \end{aligned}$$

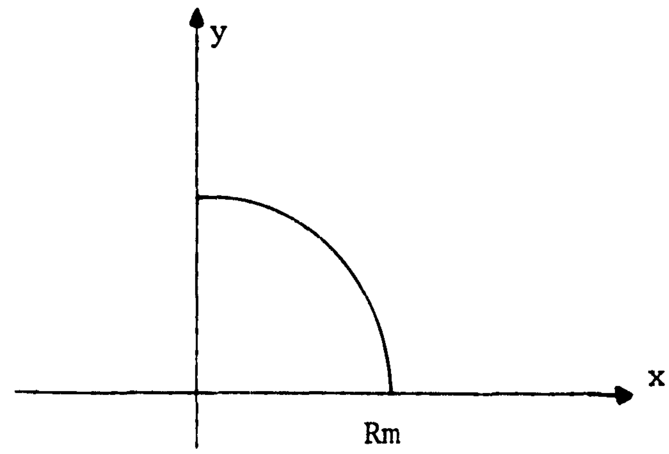


Figure 7. Radius Calculation

Then

$$\ell = 2\pi R_{av} N = 78.82 \text{ kilometers}$$

Again using number 24 wire, the resistance of the wire is 6.448 kilohms and the weight of a single coil is 143.3 kilograms (316 pounds). The power dissipation is now

$$P_{dis} = I^2 R = 6448 \text{ kilowatts}$$

The total weight of the torquer for one axis momentum generation capability is 321.8 kilograms (710 pounds).

To check the accuracy of these calculations, the magnetic dipole moment must be calculated for the core and the windings.

$$\begin{aligned} m_c &= Mv \\ &= (557 \times 10^3)\ (0.02036) \\ &= 0.05259 \times 10^5 \text{ amp} - m^2 \end{aligned}$$

Summing these:

$$\begin{aligned} m_c + m_w &= 0.16613 \times 10^5 \text{ amp} - m^2 \\ &= m \text{ as desired} \end{aligned}$$

where m_c equals the magnetic moment of the core and m_w equals the magnetic moment of the windings. It is fairly obvious by looking at the physical parameters for the windings that construction of this unit is virtually impossible. If the amount of wire necessary is reduced by a factor of 10 to a more reasonable figure, the power requirement increases to 64.48 kilowatts.

As can be seen from Equation (32), the form of the magnetic field is that of a magnetic dipole. Taking the derivative with respect to θ of Equation (32) and setting the result equal to zero, the maximum value of the external field occurs at an angle of approximately 26 degrees from the moment axis. At this angle and using the above values, the magnetic field at a distance of one meter from the surface of the sphere was calculated to be

$$B(1 \text{ meter}) = 23.4 \text{ gauss}$$

This value is a factor of 2 larger than the field values for the solenoidal torquers

3.7 TRADEOFF DISCUSSION

Three orthogonal coreless loops wound around the S-IVB represents the minimum weight and power configuration. The magnetic field vector developed by the torquer has a magnitude approximately twice that of the earth's magnetic field at the center of the loop. This appears to be at a safe level although no data are presently available on the magnetic susceptability of onboard equipment. This is one area which must be further investigated before final qualifications can be considered. If the coreless loops are not acceptable because of space, the alternative methods are represented by the ferrite torquers. The magnetic field produced by the ferrite torquers is many times the magnitude of the earth's magnetic field, and its acceptability is questionable. Considering only weight and power, a tradeoff comparison can be made using the results presented in Table 3.

4. CONCLUSIONS

Other reports issued by Marshall Space Flight Center, Bellcomm and NASA have documented the results of analyses made to determine the possible uses of magnetic torquing. Most of these reports are concerned with continuous attitude stabilization for AAP. However, Marshall Space Flight Center published a memorandum dated November 8, 1968, in which CMG desaturation is considered (Reference 9). This report concludes that magnetic torquing for CMG momentum dumping should be considered. The Bellcomm letter dated February 11, 1969 (Reference 10) states that the feasibility of the concept of attitude stabilization has been established. The Marshall report which concludes that the magnetic torquing method is feasible, is the only analysis dealing with the use of magnetic torquing exclusively for CMG desaturation. Discussions presented earlier in this report support this conclusion within certain stated qualifications. However, to arrive at an unqualified conclusion a more detailed investigation in the form of a simulation is necessary to determine dumping capability. It appears that Bellcomm is pursuing this, although no results have as yet been published. Weight and power figures for magnetic torquers are included in Table 4 as published by Marshall and Bellcomm. They are included here for the purpose of comparison. Since no indication of computational assumptions accompanied the Marshall and Bellcomm results, it is not possible to do more than note the discrepancies between power and weight when compared to those given Table 4.

TABLE 3 COMPARISON BETWEEN CORELESS AND FERRITE CORE TORQUERS

CORE TYPE	COIL WEIGHT	CORE WEIGHT	TOTAL WEIGHT	CONTROL* TORQUE	POWER
Aircore 10 m dia	30.2 kg (66.5 lb)		30.2 kg (66.5 lb)	0.5 Nm	226.5 watts
Alnico 5 Solenoid 4.05 in dia 83.6 in long	19.15 kg (42.2 lb)	125.6 kg (277 lb)	144.75 kg (319.2 lb)	0.5 Nm	22.2 kw 330 joules Hyst. loss
Supermalloy Solenoid 4.87 in dia 97.4 in long	11.75 kg (25.9 lb)	260 kg (573 lb)	271.11 kg (598.9 lb)	0.5 Nm	136 watts
Vanadium Permendur Solenoid 3.48 in dia 68.5 in long	9.37 kg (20.65 lb)	85.6 kg (188.5 lb)	94.97 kg (209.15 lb)	0.5 Nm	434 watts 6.26 joules Hyst. loss
Supermalloy Sphere 15.2 in dia	139.5 kg (307.5 lb)	183 kg (404.5 lb)	322.5 kg (712 lb)	0.5 Nm	0.448 kw

* Earth's Magnetic Field Equal To 3.0×10^{-5} Webers/M^2

TABLE 4. COMPARISON DATA FROM OTHER ANALYSES

MARSHALL SPACE FLIGHT CENTER REPORT (REFERENCE 9)

CORE TYPE	COIL WEIGHT	CORE WEIGHT	TOTAL WEIGHT	CONTROL* TORQUE	POWER
Aircore 3.56 m dia.	210 lbs	------	210 lbs	.5 Nm	400 watts for 30 min 1000 Nms
Same	53.6	------	53.6 lbs	.166 Nm	150 watts for 90 min 1000 Nms
Aircore 6.6 m dia.	17.4 lbs		17.4 lbs	.166 Nm	150 watts for 90 min 1000 Nms
	52.3 lbs		52.3 lbs	.166 Nm	50 watts for 90 min 1000 Nms
Alnico 5 4.5 in dia 90 in long	1 lb	356 lbs	357 lbs	.5 Nm	16 watts for 30 min 1000 Nms
Alnico 5 3 in dia 60 in long	1 lb	116.5 lbs	117.5 lbs	.166 Nm	2.42 watts for 90 min 1000 Nms

* Torque developed in a magnetic field of 3.7×10^{-5} W/M^2

BELLCOMM (Reference 11)

CORE TYPE	COIL WEIGHT	CORE WEIGHT	TOTAL WEIGHT	CONTROL TORQUE	POWER
15 ft long 12.8 in dia	unknown	unknown	7860 lbs	unknown	Peak power 520 watts Average power 310 watts
8 ft long 3-1/8 in dia	unknown	unknown	233 lbs	unknown	Peak power 48 watts Average power 31 watts
5 ft long 2-3/8 in dia	unknown	unknown	91 lbs	unknown	Peak power 31 watts Average power 20 watts

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