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NUMERICAL CALCULATIONS OF THE STEADY-STATE, WIND-DRIVEN CURRENTS IN LAKE ERIE

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TECHNICAL PAPER presented at Thirteenth Conference on Great Lakes Research sponsored by the International Association for Great Lakes Research
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Abstract

Solutions for the steady-state, wind-driven currents in Lake Erie have been obtained by numerical methods. A shallow lake model, which does not require the friction layers to be small by comparison with the depth of the lake, has been used. In order to obtain some of the observed features of the currents, it was necessary to use a relatively small grid (3.22 kilometers). This grid was variable in size for the mesh points adjacent to the boundaries, thus permitting an accurate approximation of the boundary.

The velocity as a function of depth and horizontal position has been determined. Results are presented for southwesterly and northeasterly winds. In both cases, narrow bands of strong currents were found near the shore. In other areas, large subsurface gyres were evident. The calculated results compare quite well with seabed drifter measurements and other observations.

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Introduction

A considerable amount of numerical work has been done in calculating the steady-state, wind-driven currents in the Great Lakes with the exception of Lake Erie. To our knowledge no detailed calculations have been done for Lake Erie.

Lake Erie differs in character from the other Great Lakes in that it is very shallow. As shown in Figure 1, the mean depths of the Western, Central, and Eastern basins are only 7.3, 18.3, and 24.4 meters respectively. The shallow depth of the lake makes the analysis somewhat different and more difficult than that for the other Great Lakes since the usual Ekman dynamics can no longer be used.

In Ekman dynamics, the bottom stress is proportional to a geostrophic velocity and can be calculated from an integrated stream function. This simplifies the analysis considerably. An example of the application of Ekman dynamics to the deeper Great Lakes is given by Rao and Murty (1969). However, Ekman dyanmics is only valid when the thickness of the friction (or Ekman) layer is negligible by comparison with the depth of the lake. In Lake Erie, for moderate winds, the thickness of the friction layer is comparable to the depth of much of the lake. The necessary extension of the Ekman analysis to the case of a shallow lake has been given by Welander (1957) and that theory has been used here with slight modifications. The shallow depth of the lake does make a steady-state analysis valid over a greater period of time since the set-up time should be shorter than for the other Great Lakes.

The solutions have been obtained by numerical methods using finite differences. The present formulation allows the calculation of the magnitude of all three components of the velocity as well as the surface water displacement. A 3.22 kilometer (2 mile) grid size has been used and work is proceeding on a 1.61 kilometer (1 mile) grid calculation. These grid sizes are necessary in order to obtain an adequate approximation to the currents near shore where the depth is shallow and is changing quite rapidly. It is hoped, by the comparison of these fairly detailed calculations with measurements, that it will be possible to better establish the range of validity of a steady-state analysis for Lake Erie.

Mathematical Model and Method of Calculation

In the present analysis, the basic assumptions are that the water density is constant, the vertical eddy viscosity is independent of depth but dependent on wind velocity (or surface wind stress), the pressure is hydrostatic, and the lateral friction and non-linear acceleration terms can be neglected. The neglect of lateral friction means that the two transverse friction terms in the Navier-Stokes equations are small compared to the vertical friction term. Lake Erie is stratified during the summer months and therefore the case presented here applies only to the fall, spring, and those periods in the winter when the lake is not iced over.

These assumptions reduce the Navier-Stokes equations to two equations containing the horizontal velocities and the surface water slope as unknowns. The boundary conditions are that, at the bottom, there is no slip and, at the free surface, the wind stress is prescribed. These equations and boundary conditions can then be solved analytically to give the velocity as a function of the depth with the surface wind stress and surface water slope as parameters. The result (Welander, 1957) is

$$u + iv = \left[\frac{\sinh \sqrt{\frac{if}{\nu}}(h + z)}{\cosh \sqrt{\frac{if}{\nu}}\, h}\right] \frac{\left(\tau_x + \tau_y\right)}{\sqrt{if\nu}}$$

$$- \frac{ig}{f}\left[\frac{\cosh \sqrt{\frac{if}{\nu}}\, z}{\cosh \sqrt{\frac{if}{\nu}}\, h} - 1\right]\left(\frac{\partial\zeta}{\partial x} + i\frac{\partial\zeta}{\partial y}\right) \qquad (1)$$

where $i = \sqrt{-1}$. Here x and y are horizontal coordinates in the East and North directions, u and v are the corresponding velocities, and z is the vertical coordinate and is positive upwards. The lake bottom is at $z = -h(x,y)$, the free surface is at $z = \zeta(x,y)$, $z = 0$ corresponds to the undisturbed surface, g is the acceleration of gravity, ν is the coefficient of vertical eddy viscosity, f is the Coriolis parameter, and τ is the surface wind stress with components τ_x and τ_y in the x and y directions. This solution for the velocities represents the sum of the drift current (proportional to wind stress) and gradient current (proportional to the slope of the free surface).

To complete the solution, the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2}$$

and Equation (1) are integrated in the vertical direction to give

$$M_x + i\, M_y = A(\tau_x + i\tau_y) + B\left(\frac{\partial \zeta}{\partial x} + i\frac{\partial \zeta}{\partial y}\right) \tag{3}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} = 0 \tag{4}$$

respectively, where M_x and M_y are the integrated mass flux components in the x and y directions, and $A = A(h)$ and $B = B(h)$ are functions of the local depth. By eliminating the mass fluxes from these equations, one arrives at an equation for the surface elevation (Welander, 1957) of the form,

$$\nabla^2 \zeta + f_1 \frac{\partial \zeta}{\partial x} + f_2 \frac{\partial \zeta}{\partial y} = f_3 \tag{5}$$

where f_1, f_2, and f_3 are functions of the depth and the derivatives of the depth with respect to x and y. In addition, f_3 is a function of the wind shear stresses.

However, it is more convenient to use the integrated stream function ψ, defined by $M_x = \partial\psi/\partial y$ and $M_y = -\,\partial\psi/\partial x$, as the dependent variable because of the simplicity in stating the boundary conditions, i.e., ψ = constant. By using the definition of ψ and substituting Equation (3) into Equation (5), one obtains

$$\nabla^2\psi + g_1 \frac{\partial\psi}{\partial x} + g_2 \frac{\partial\psi}{\partial y} = g_3$$

where g_1, g_2, and g_3 are functions of the depth and its derivatives and g_3 is also a function of the wind shear stresses. The stream function must be specified on the boundary as determined by the river inflows and outflows. Details of the above formulation can be found in Welander (1957) and Gedney (1970). Once ψ is determined for a specified τ_x and τ_y, the water surface elevation and the three velocity components can be calculated.

In order to solve for the stream function, the depth and the derivatives of the depth with respect to x and y must be known. These functions were determined for Lake Erie at regular grid spacings by curve fitting the best available data. The islands in the western basin were approximated by underwater mounds with the maximum water depth over the island interior never exceeding 2.4 meters (8 feet). This is a fairly good approximation to the actual conditions since very little mass flux results when the depths are this small.

The integrated stream function ψ was solved for by finite difference methods. The ψ derivatives were expressed in terms of central difference formulas. For points adjacent to the boundaries, a non-uniform grid spacing was incorporated, i.e., a point on the boundary was used as part of the grid. The method of successive over-relaxation was employed to solve the resulting system of 5-point difference equations.

Results

The calculated Lake Erie currents for West 32 degrees South and North 40 degrees East winds are presented in Figures 2 and 3. A Detroit river inflow and a Niagara river outflow of 5,380 cubic meters/sec. has been included. These currents have been calculated using a friction layer depth d of 18.3 meters (60 feet). Here $d = \pi\sqrt{2\nu/f}$ where ν and f are as defined previously. The value of the shear stress used was that corresponding to a 2.7 meter/sec. (5 mile/hr.) velocity wind. The value of the wind speed chosen is somewhat typical of the daily resultant wind as published by the U. S. Weather Bureau. The resultant wind is the vector sum of the wind velocity observed at regular intervals divided by the number of observations. Because of the variation in wind direction, the resultant speed is typically 1/2 of the average wind speed for the day and very often is in the 2.5 to 3.0 meter/sec. range. By the use of the daily resultant wind it is hoped that some agreement can be reached between the present analysis and daily average current measurements and thus demonstrate the utility of steady-state wind analyses.

For both Figures 2 and 3, parts A through C show the horizontal currents for Lake Erie at 0, 4.27 and 7.32 meters below the surface. Part D shows horizontal currents for a constant 1.22 meters above the lake bottom. The beginning of the arrow represents the actual location of the current represented by the arrow. The magnitude of the velocity can be determined from the velocity scale indicated on the figure. Note that the velocity scale for the bottom currents in part D is different from the scale for the A, B and C parts.

The Central and Eastern Basin surface currents shown in Figures 2A and 2B are transporting mass toward the Eastern and Southern boundaries. A subsurface current returns this mass in the opposite direction as shown in Figures 2C and 2D. The Central and Eastern Basin surface currents are in general smaller in the center of the lake than near the shore. This effect is essentially due to the relatively large subsurface return current down the center of the lake which is opposite in direction to the surface current and subtracts from it. Near the shores the subsurface currents are almost normal to the shore, causing a deflection of the top surface vector and increasing its magnitude.

The Western Basin is greatly influenced by the islands, the Detroit river inflow and its shallow depth. As shown in 2B there is a subsurface return flow at a depth of 4.27 meters in the Western Basin. This return flow is not dominant in the other basins at this depth. This of course is due to the shallow depth of the Western Basin. Observations seem to indicate a clockwise surface flow around Pelee and Kelley's islands (the two most eastern islands) for a southwest wind. There is partial evidence of this as seen in Figure 2A. At a depth of 4.27 meters, Figure 2B, there is even more evidence of these clockwise flows. The strong surface current opposite to the wind direction at the Southern end of Kelley's island has been noted by measurements. In general however, the agreement with measurements is not entirely satisfactory in the island area. If the measurements are correct, this may be due to (a) treating the islands as underwater mounds, (b) using a relatively large grid compared to the size of the islands, or (c) specifying the wrong wind direction. Therefore we are proceeding to calculate the currents in the island area with a smaller mesh size and a truer geometry. In addition the effects of wind direction will be investigated.

Along portions of the North Shore in the Central Basin the bottom current as shown in Figure 2D is generally toward the shore indicating areas of upwelling. This upwelling for Southwest winds has been observed and, according to Hartley (1970), may bring nutrients up from the bottom sediment to the surface waters causing the large algae blooms in late summer.

Along other portions of the Central Basin we find parallel shore currents for all depths. These occur in fairly narrow bands and can only be calculated accurately by a fairly small variable grid of the type used here. We see that in the Eastern basin the depths are deep enough near the shore so that the bottom currents shown in Figure 2D are more perpendicular than parallel to the shore.

It is of interest to compare the bottom currents shown in Figure 2D near the shore with seabed drifter measurements reported by Hartley (1968) and given here in Figures 4 and 5. The seabed drifters were set out in the summer of 1965. In comparing these measurements with the calculated results we must make the assumption that the thermocline in the central basin does not significantly change the flow near the shore from that which occurs in the constant density lake. The thermocline in the last half of summer is approximately restricted to the interior of the 18.3 meter (60 foot) contour shown in Figure 1. It is believed that the drifters with which we are trying to compare our results were not dropped in the region of the thermocline.

The seabed drifter at the top of Figure 4 in the central basin which traveled the greatest distance was released on 7/30/65 at the origin of the arrow and beached at Long Point on 8/10/65. Note that the arrows shown in Figure 4 do not necessarily denote the paths of drifter movement.

During the time period from 7/30 to 8/10, the winds were predominantly out of the South with a resultant speed near 3 meters/sec. For the W32S wind case given in Figure 2D we have a current which agrees with the drifter movement. By using the velocity data from this calculation at the height from the bottom the drifter was supposed to act, the estimated trip time was calculated to be 5 times longer than observed. This may possibly be due to the fact that the wind direction in the calculation is not from the South. A more Southerly wind will produce higher currents in the direction of the drifter movement. For a more northerly wind the parallel shore currents in the area of Long Point are much reduced or completely eliminated. See Figure 3D for the effect of a N40E wind.

Some of the drifter returns along the South shore of the Central basin were also returned in a short time (less than 15 days) after being released during a period of predominantly Southwesterly winds. These results are in general agreement with the currents shown in Figure 3D. Many other of the drifters depicted in Figure 4 and all those depicted in Figure 5 were not recovered until a relatively long time had elapsed and winds of a variety of directions had occurred. During this time period the drifters probably moved in many directions for varying lengths of time and a comparison with data is not possible. A comparison of the bottom currents of a N40E and W32S wind indicates some of the extremes in direction the drifters could have taken.

Acknowledgments

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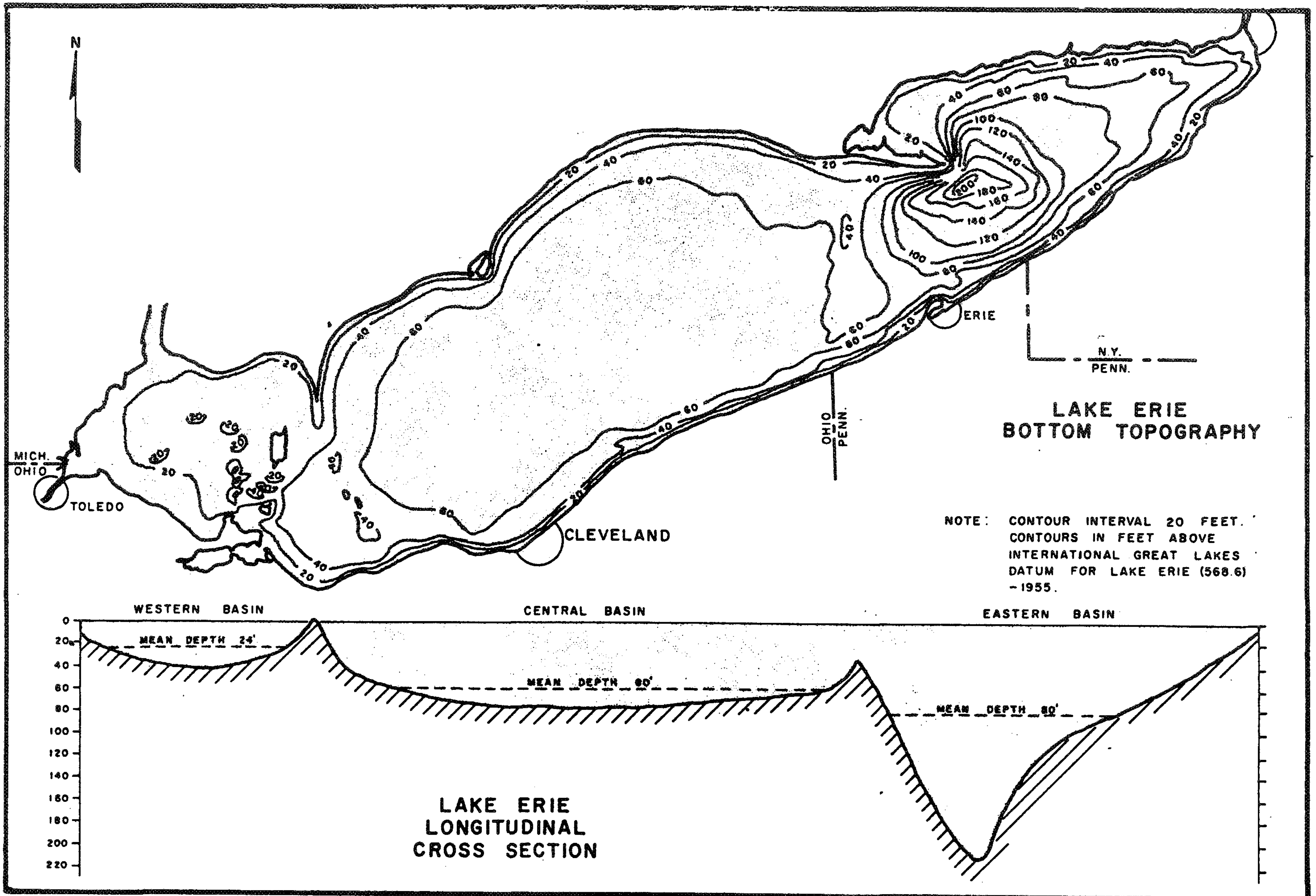
N
LAKE ERIE
BOTTOM TOPOGRAPHY
MICH.
OHIO
TOLEDO
CLEVELAND
ERIE
OHIO
PENN.
N.Y.
PENN.
NOTE: CONTOUR INTERVAL 20 FEET. CONTOURS IN FEET ABOVE INTERNATIONAL GREAT LAKES DATUM FOR LAKE ERIE (568.6) - 1955.
WESTERN BASIN
CENTRAL BASIN
EASTERN BASIN
MEAN DEPTH 24'
MEAN DEPTH 60'
MEAN DEPTH 80'
LAKE ERIE
LONGITUDINAL
CROSS SECTION

Figure 1

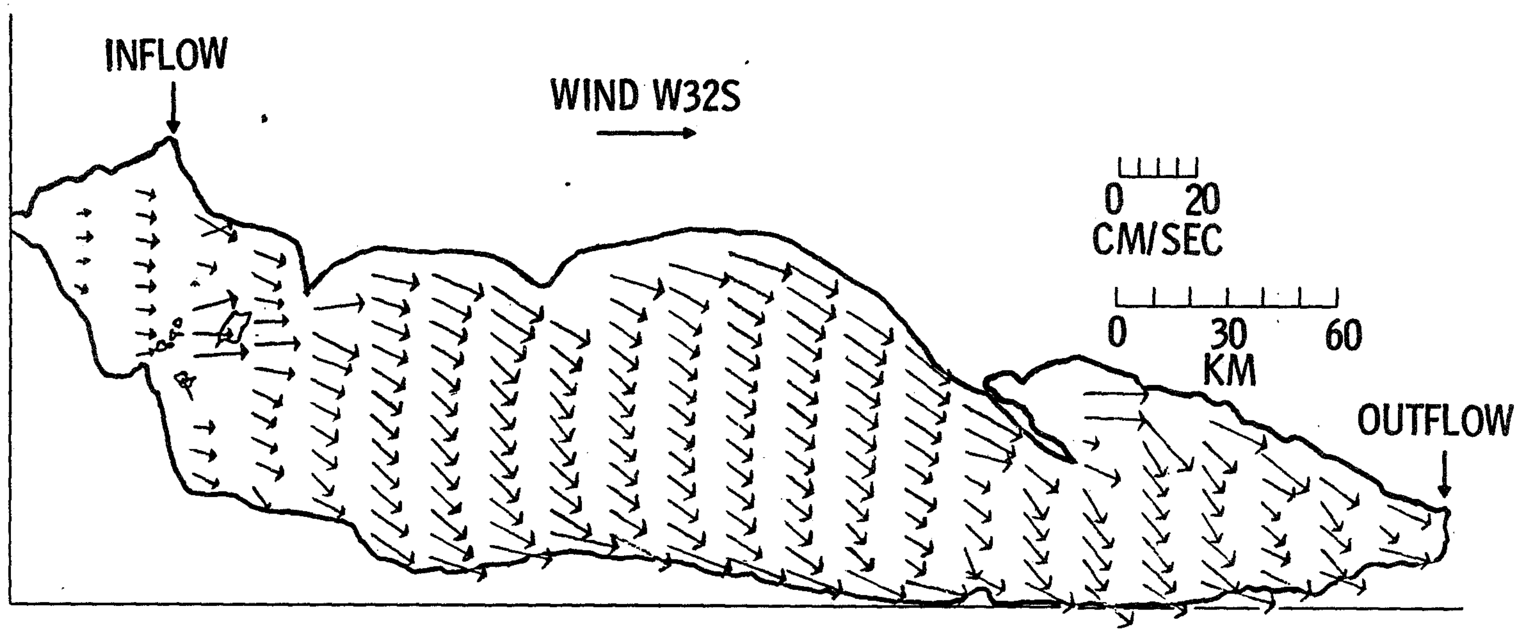

Figure 2(a). - Surface velocities.

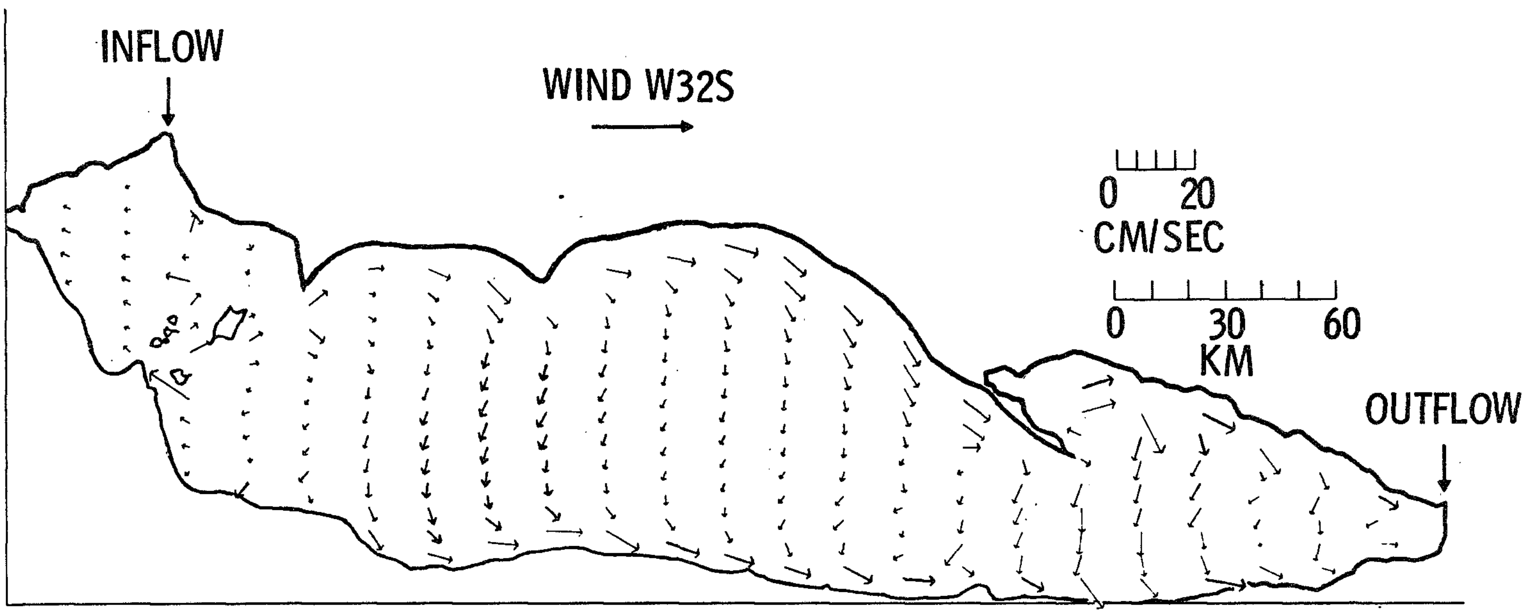

Figure 2(b). - Horizontal velocities at a constant 4.27 meters (14 feet) from surface.

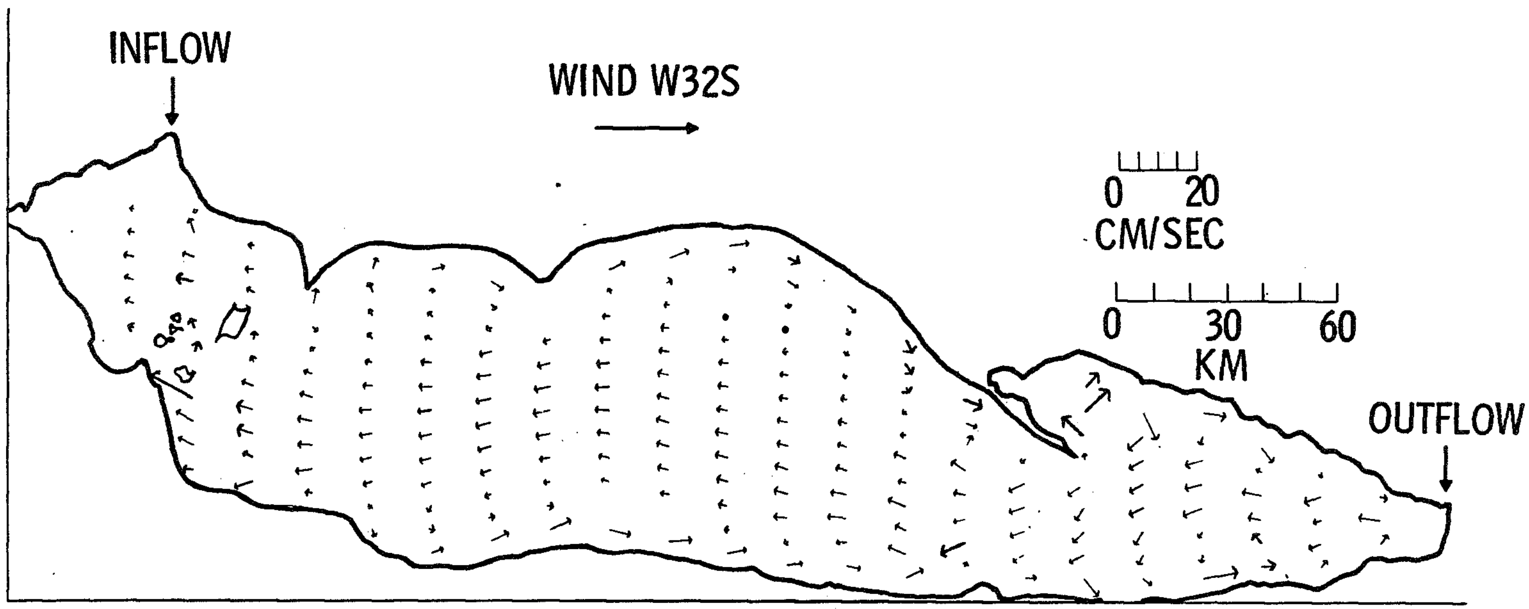

Figure 2(c). - Horizontal velocities at a constant 7.32 meters (24 feet) from surface.

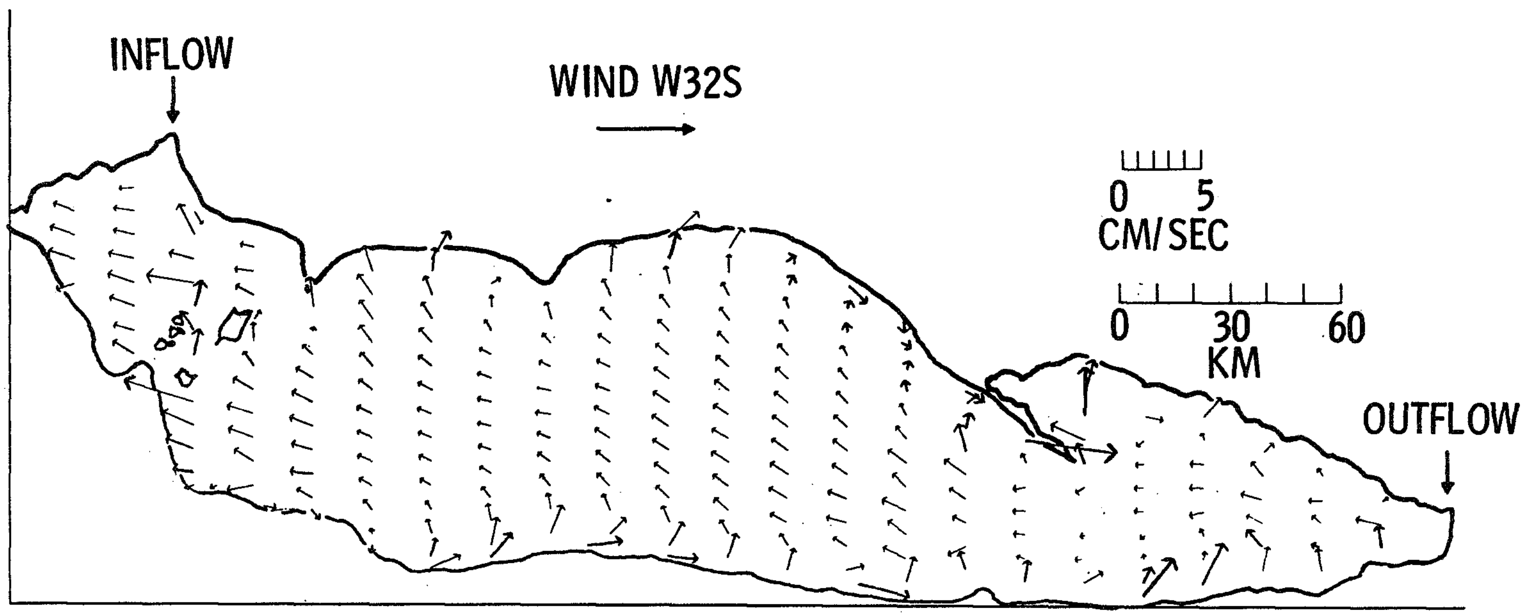

Figure 2(d) Horizontal velocities at a constant 1.22 meters (4.0 feet) from lake bottom.

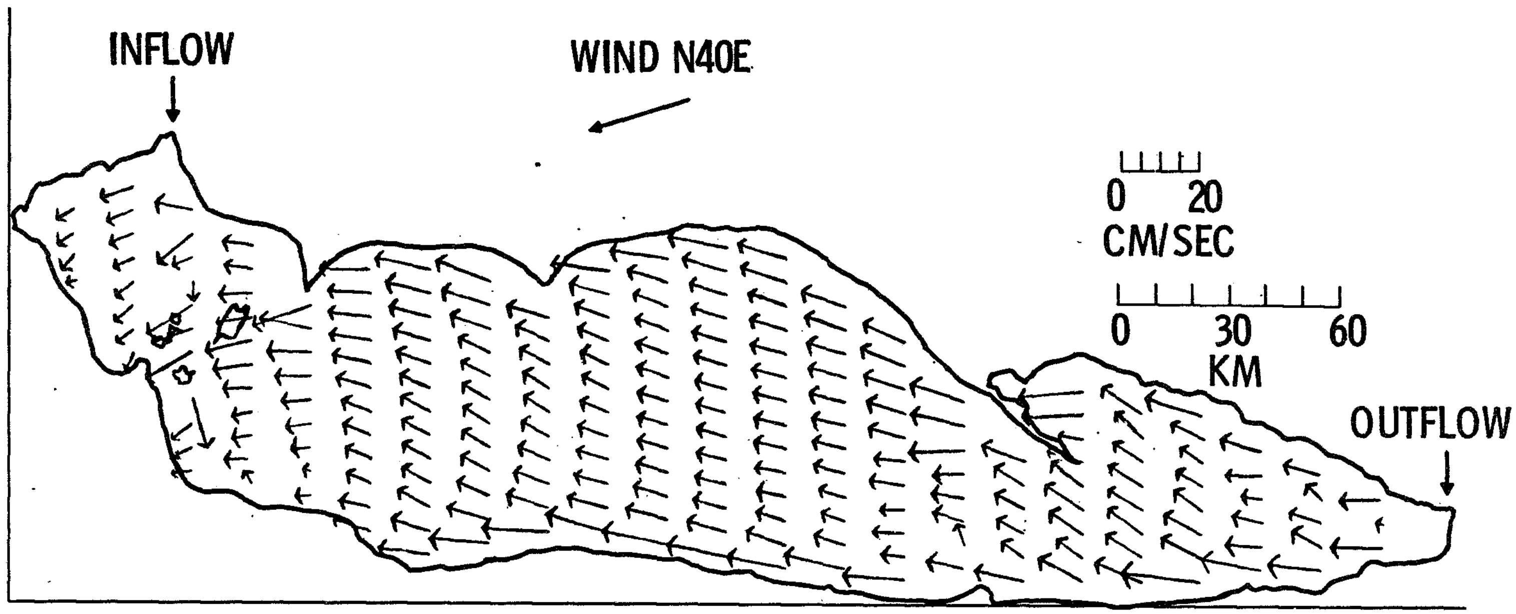

Figure 3(a) Surface velocities.

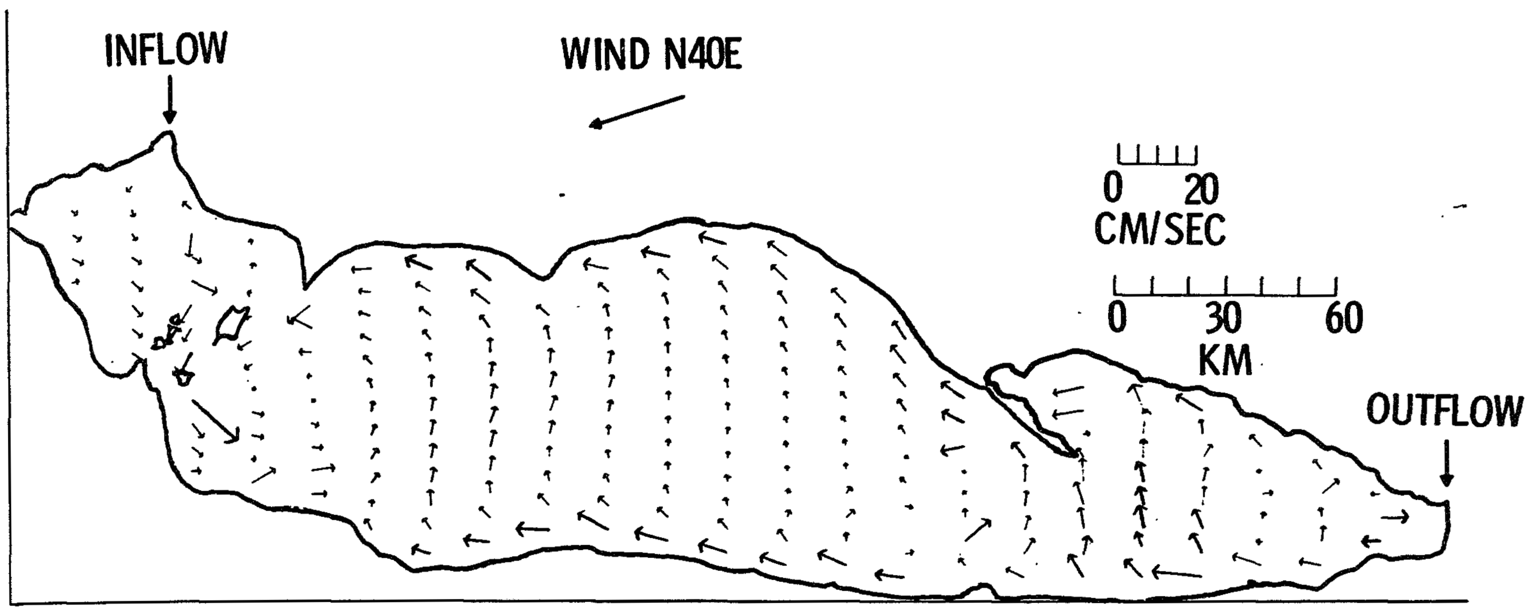

Figure 3(b) Horizontal velocities at a constant 4. 27 meters (14 ft) from surface.

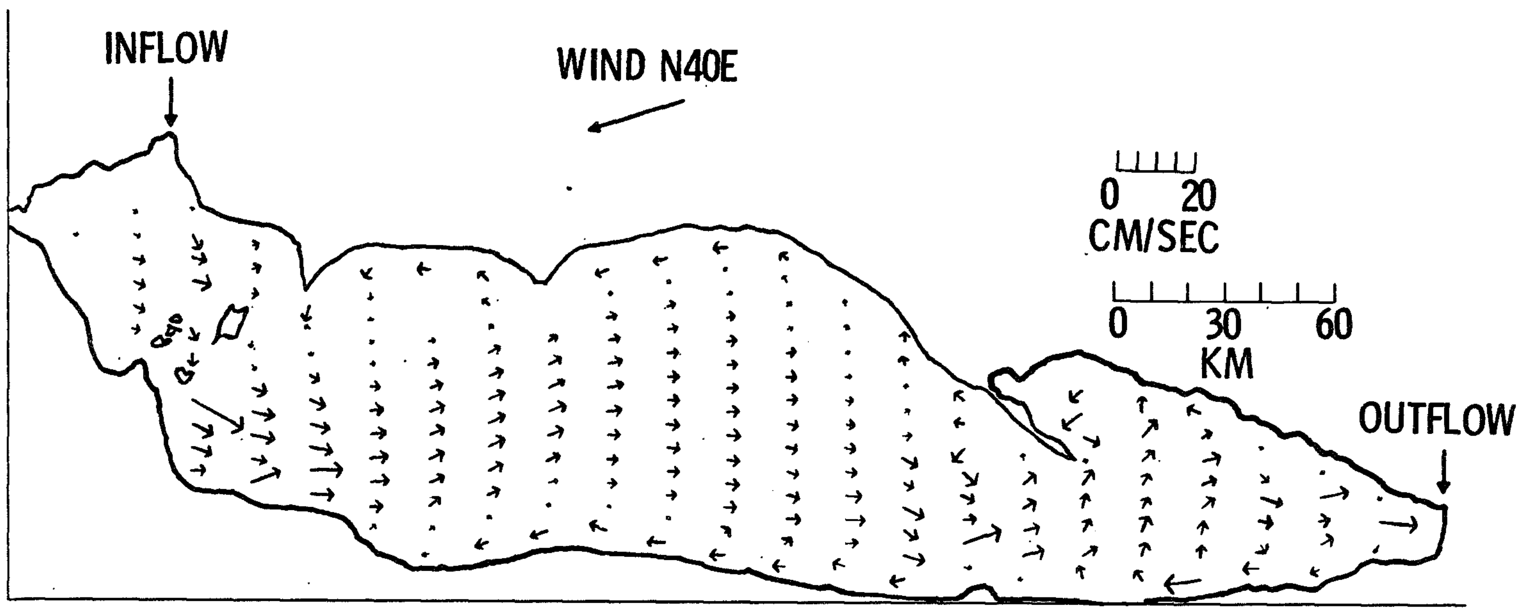

Figure 3(c) Horizontal velocities at a constant 7.32 meters (24 ft) from surface.

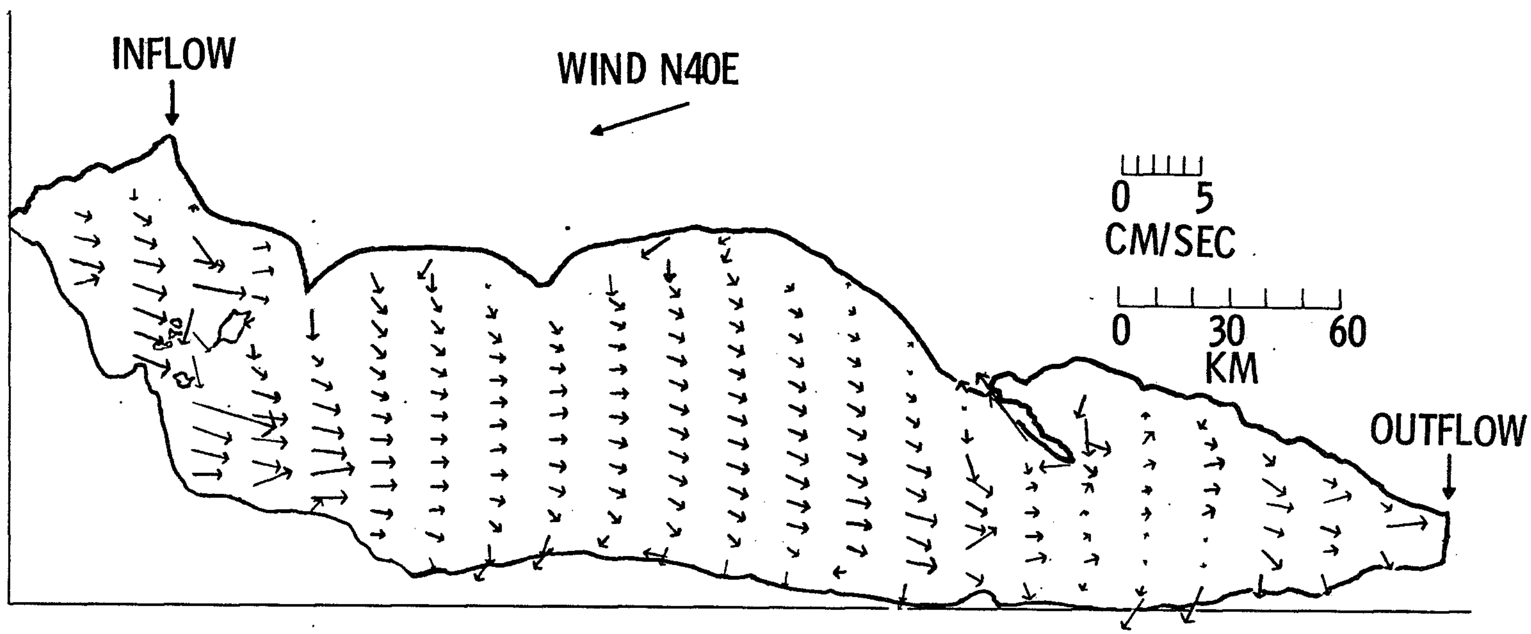

Figure 3(d). - Horizontal velocities at a constant 1.22 meters (4.0 ft) from lake bottom.

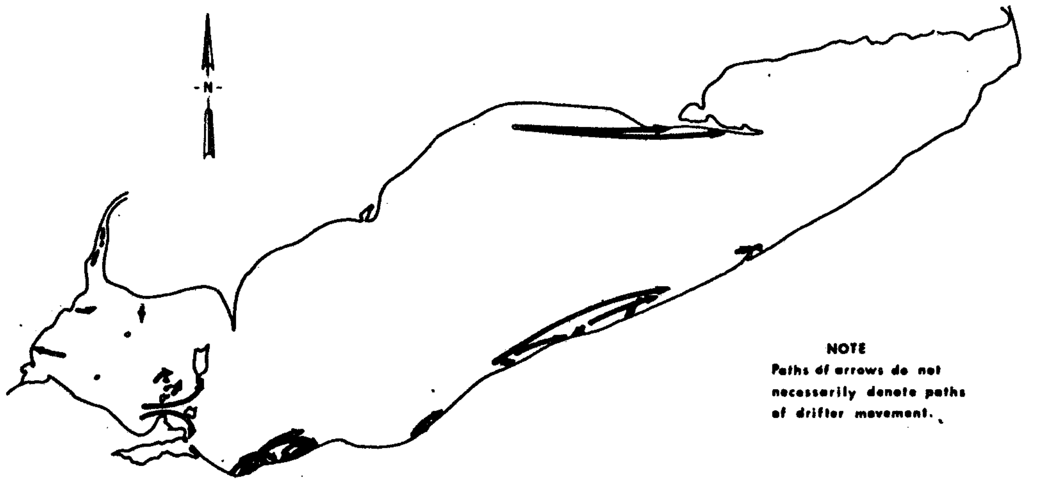

Figure 4. - Seabed drifter returns during last half of 1965.

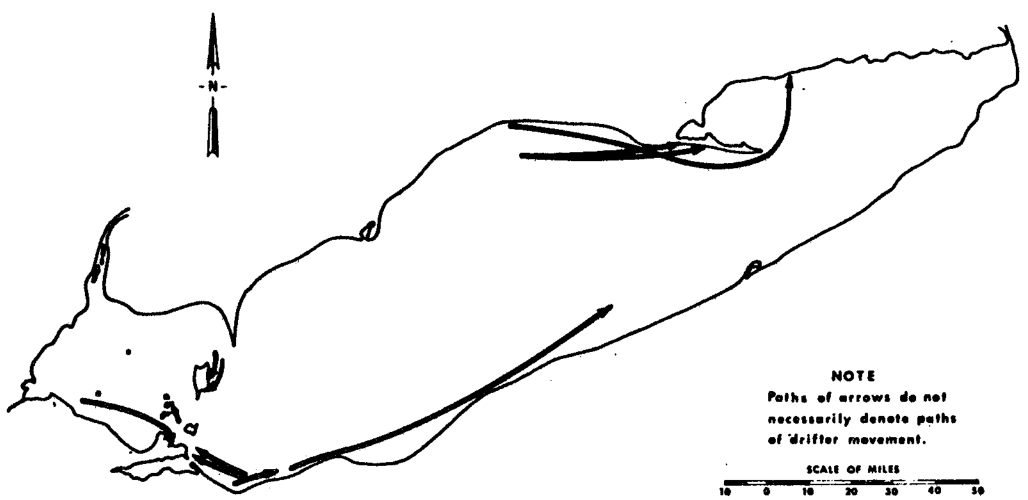

Figure 5. - Seabed drifter returns during first half of 1966.