MEMORANDUM

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FORTRAN IV PROGRAM FOR COMPUTATION OF GROUP TABLES OF FINITE GROUPS PROGRAM FOR SECOND GENERATION MACHINES
by David D. Evans and Gabriel Allen
Lewis Research Center
Cleveland, Obio 44135

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# FORTRAN IV PROGRAM FOR COMPUTATION OF GROUP TABLES OF FINITE 

 GROUPS - PROGRAM FOR SECOND GENERATION MACHINESby David D. Evans and Gabriel Allen<br>Lewis Research Center

SUMMARY

A FORTRAN program suitable for second generation machines has been written for machine computation of group tables of finite groups. The method depends on the fact that every finite group $G$, of order $n$, is isomorphic to some subgroup $T_{n}$ of the symmetric group $S_{n}$. The procedure for using the program is as follows: After finding the $T_{n}$ which is isomorphic to $G$, the user enters the elements of $T_{n}$ into the program as input data. The group table for $\mathrm{T}_{\mathrm{n}}$ is computed and printed. Using the isomorphism between $T_{n}$ and $G$, the user then translates back to the elements of $G$. The group $A_{5}$, the even permutation subgroup of $S_{5}$, is shown as an example.

## INTRODUCTION

In the past decade, there has been a greatly expanded interest in the application of group theory to physical problems. As a result, there arose a need for detailed information about fairly large finite groups. It was natural to apply computing machines to this problem and a large number of programs were written for proving theorems and algorithms about finite groups whose group tables were entered as input data (ref. 1).

Recently a further step was taken in the direction of automating computations involving finite groups when a FORTRAN program was written for the machine computation of group tables of finite groups (ref. 2). The method made use of the fact that every finite group of order $n$ is isomorphic to some subgroup of the symmetric group $S_{n}$. In this program, it was only necessary to enter the elements of the group as input data and the group table was then computed and printed as output. The only knowledge of group theory required by the user was that which established the isomorphism between the group of interest and the appropriate subgroup of $S_{n}$. This isomorphism was needed because both the input data and the output are in the form of group elements of $S_{n}$.

The program was machine-dependent and was written for use on the NASA Lewis Research Center's IBM 360-67. In many laboratories, only second generation machines are available. In order to enable people limited to such facilities to use this method, the program from reference 2 has been adapted to the language acceptable to the IBM 7094.

The adaptation to 7094 machines requires changes in the details of the program that are not trivial. However, the overall procedure is the same. Therefore, detailed descriptions of the listing are still included in this report, but the reader is frequently referred to reference 2 for extensive explanations of the reasons for the procedure. As shown in the example, the output is not quite as compact as in reference 2.

## DEFINITIONS AND CONVENTIONS

The basic idea behind the method is explained in reference 2. The symbol $S_{n}$ designates the permutation group of $n$ objects and is of order $n$ factorial. The even permutation group of $n$ objects $A_{n}$ is a proper subgroup of $S_{n}$. The group elements are expressed as cycles, each of which is broken down to a product of transpositions in executing the group operations. The numbers between commas in a cycle are called units. The convention adopted herein for describing the effect of a cycle is that in which the units denote objects and each unit is moved to the location currently occupied by the unit to its left.

The term standard configuration (SC) is used to describe an arrangement in which the $i^{\text {th }}$ object is in the $i^{\text {th }}$ location for $i=1$ to $N$ where $N$ is the number of objects and/or locations. When $\mathrm{N}=3$, for example, SC means

| Location | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- |
| Object | 1 | 2 | 3 |

The usefulness of class algebra tables for breaking up the group tables of large groups was explained sufficiently in reference 2 and will not be repeated here.

It should be noted that the first row and column of the group tables will only appear as row or column headings.

## PROGRAM DESCRIPTION

## General Description

Before the input or the form of the output is selected, an isomorphism must be
established between the group of interest $G$ and a subgroup $T_{n}$ of $S_{n}$. Often, an isomorphism can be found between $G$ and a subgroup $T_{j}$, of some $S_{j}$, where $j<n$. It will clearly be advantageous to use as small a permutation group as can be thought of for this purpose. The isomorphism between $G$ and $T_{j}$ should first be duly recorded. Then the program is used to compute the group table for $T_{j}$. From this table and the recorded isomorphism between $T_{j}$ and $G$, the group table for $G$ is obtained.

The broad outlines of the program and even most of the details are the same as in reference 2. Nevertheless, many minor changes still appear throughout the main listing, and rather extensive changes are present in subroutine SQUEZ. Therefore, a detailed description of the program is included in this report.

The program computes a group table for elements of $T_{j}$ which are read in as a series of column and a series of row operations in cycle notation. The usual convention is adopted in which a row operation refers to a group operation in the vertical heading to the left of a group table, whereas a column operation refers to a group element in the horizontal heading at the top of a group table. The general procedure consists of the following steps:
(1) A set of $K$ column operations and $L$ row operations is read into the program.
(2) Each of these operations is broken down into its equivalent sequence of transpositions. Let the group operation $P_{i j}$ represent the (group) product of the $i^{\text {th }}$ row operation $R_{i}$ by the $j^{\text {th }}$ column operation $C_{j}$. Then $P_{i j}$ will be stored as a long sequence of transpositions.
(3) The SC is rearranged in accordance with the sequence of transpositions that represent $P_{i j}$.
(4) The rearrangement is examined and a single group operation that effects the same rearrangement of the $S C$ is identified as the product $R_{i} * C_{j}$.
(5) This single group operation is entered in the $i^{\text {th }}$ row and $j^{\text {th }}$ column of the group table array.

The entry in this location is to be interpreted as being the result of the group operation $C_{j}$ followed by the group operation $R_{i}$.

An outline of the program, broken down into sections which perform recognizable functions, follows. For more detailed descriptions, the listing in appendix A can be examined.

Block 1 - set up constants. - The constants used in the program are given literal names and are declared either INTEGER or LOGICAL in TYPE statements. All the subscripted variables are dimensioned and allotted storage locations by the use of DIMENSION statements. A DATA statement is used to give literal names to the variables listed in the following table:

| Variable | Literal name | Identification |
| :---: | :---: | :--- |
| $($ | LP | Left parenthesis |
| $\prime$ | RP | Right parenthesis |
| , | CM | Comma |
|  | BLK | Blank |
| E | DNT | Identity element of group |

Block 2 - read in DATA. - The data describing the group and the group operations are read into the program. The labels to be assigned to the objects and to the locations are read in from the list for IDENT which is limited to one card. It is read in with FORMAT(80A1), right shifted, and stored in $\operatorname{INITAL}(\mathbb{N})$ in the form bbbbbX, where $b$ represents a 6 -bit blank and $X$ represents the 6 -bit location in the 36 -bit storage register (word) in which the integer from IDENT is stored.

In order to put the "words" in this form, use is made of four shift functions: IALS( $\mathrm{N}, \mathrm{NWORD}$ ) and IARS(N, NWORD), which cause the integer variable NWORD to be shifted N bits to the left and to the right, respectively, and ALS(N, WORD) and ARS(N, WORD), which cause the real variable WORD to be shifted $N$ bits to the left and to the right, respectively. (The latter two functions are in subroutine SQUEZ, which is called in block 2 , but which is described in a later section.)

All of these functions are standard on the IBM 7094. The machine at Lewis Research Center has these functions in its function library, and they can be called directly. For those users who do not have direct access to these functions, a map listing for all four is provided in appendix B.

## Description of Input

The input cards are read in the following order:

| First card | TITLE | one card with FORMAT(80A1) |
| :--- | :--- | :--- |
| Second card | IDENT | one card with FORMAT(80A1) |

Note that the input for IDENT must be long enough to include the maximum number of objects to be moved in any group operation. Thus, the list for IDENT must be of the form $\left(A_{1}, A_{2}, \ldots, A_{j}\right)$ where $j$ is the subscript of $S_{j}$ and $A_{i}$ is any alphanumeric symbol.

The next set of cards contains the ACROSS (column) operations. There can be as many as four cards with FORMAT(80A1), and each operation is followed by a period.

A blank card follows the preceding set. It is needed to signal the end of the ACROSS imput.

The next set contains the DOWN (row) operations. There can be as many as four cards with FORMAT(80A1), and again each operation is followed by a period.

A blank card follows the preceding set, signalling the end of the DOWN input.
The input for both ACROSS and DOWN operations is in cycle notation.
Several error checks are run on IDENT. These include checking the maximum number of nonblank units (which is six in this version of the program), the presence of blank spaces on the card, and the illegal use of parentheses.

The group elements themselves are read in from the list for INPUT(J, K), which is also read in with FORMAT(80A1). The complete set of column operations is read first. Then the complete set of row operations is read. Each of these sets is limited to four cards and is followed by a blank card.

The example which follows is the class algebra block $K_{5} * K_{4}$ of group $A_{5}$. The symbol $\mathrm{K}_{4}$ represents the class of 20 group elements expressible as three unit cycles, while $\mathrm{K}_{5}$ is the class of 15 group elements expressible as products of two independent transpositions. (See ref. 2 for further details.)
$\mathrm{K} 5 * \mathrm{~K} 4$
( $1,2,3,4,5$ )
$(1,2,3) \cdot(2,3,4) \cdot(1,3,4) \cdot(1,2,4) \cdot(1,2,5) \cdot(1,3,5) \cdot(2,3,5) \cdot(1,4,5) \cdot(2,4,5) \cdot(3,4,5)$.
$(3,2,1) .(4,3,2) .(4,3,1) .(4,2,1) .(5,2,1) \cdot(5,3,1) .(5,3,2) \cdot(5,4,1) .(5,4,2) \cdot(5,4,3)$.
Card 5 is blank
$(1,2)(3,4) .(1,3)(2,4) .(1,4)(2,3) .(1,2)(3,5) .(1,3)(2,5) .(1,5)(2,3) .(1,2)(4,5)$.
$(1,4)(2,5) .(1,5)(2,4) .(1,3)(4,5) .(1,4)(3,5) .(1,5)(3,4) \cdot(2,3)(4,5) .(2,4)(3,5)$.
$(2,5)(3,4)$.
Card 9 is blank
Cards 3 and 4 are ACROSS (column) operations and cards 6, 7, and 8 are DOWN (row) operations.

Block 3 - store each group operation as product of transpositions. - In this block, each group operation is decomposed into its equivalent product of transpositions and then stored in $\operatorname{PAIRS}(I, N O P, K)$ (see appendix $A$ ). The third subscript $K$ is 1 or 2 according to whether the operation is a column or row operation, respectively. Thus, an entry in PAIRS(I, NOP, K) is the $I^{\text {th }}$ unit of the group element in the NOP ${ }^{\text {th }}$ column $(\mathrm{K}=1)$ or NOP ${ }^{\text {th }}$ row ( $\mathrm{K}=2$ ). Note that the four shift functions described in block 2 are used here, also.

The example used herein is the group product of $(1,3),(2,4)$, and $(5,2,1)$. This is an entry under (521) and to the right of $(1,3)(2,4)$ in the $\mathrm{K}_{5} * \mathrm{~K}_{4}$ class product of $\mathrm{A}_{5}$ (see ref. 2 and the last class product block in appendix $C$ ). The input for this block (which
was shown in the preceding section) shows that the element (521) was the 15th ACROSS (column) operation and therefore corresponds to $\mathrm{K}=1$, NOP $=15$. Before being stored in a PAIRS array, the element is considered to be of the form (52)(21). Therefore the contents of $\{\operatorname{PAIRS}(-, 15,1)\}$ is $[5221]$. The element $(1,3)(2,4)$ is the 2 nd DOWN (row) element; and it, therefore, corresponds to $K=2, N O P=2$. The contents of the four registers which constitute the complete $\{\operatorname{PAIRS}(-, 2,2)\}$ array are [1324].

Block 4 - set up arrays in standard configuration. - In this block, a number of standard configurations are set up. If $N_{C}$ and $N_{R}$ are the total number of column and row operations, respectively, then $N_{C} \times N_{R} S C$ 's are set up. A given $S C$ is set up by storing the quantity bbbbbI in location $\operatorname{ANS}(I, I 1, I 2)$ (see listing in appendix A). For a fixed I1 and I2, the complete set of $\operatorname{ANS}(I, I 1, I 2)$ over the full range of $I$ takes on the form of an SC. (See example in block 5.) Note that the second and third subscripts of ANS refer to column and row operations, respectively. On the other hand, a given PAIRS array describes either a column or a row operation, but never mixes these types.

Block 5 - perform group multiplication. - In this block, the detailed operations for all the group products, $\mathrm{P}_{\mathrm{I} 2, \mathrm{I} 1}$ (operation I2 $*$ operation I1) are actually performed. The method used is to transpose, successively, units from the SC of ANS(-, I1, I2) in accordance with the indicated operation of $\operatorname{PAIRS}(-, 11,1)$ followed by the operation of PAIRS(-, 12, 2). The operations may be understood by following the procedure on a single complete ANS(-, I1, I2) array.

The storage location ascribed to $\operatorname{ANS}(\mathrm{J}, \mathrm{I} 1, \mathrm{I} 2)$ is considered to be the $J^{\text {th }}$ location of the SC. The quantity stored in $\operatorname{ANS}(J, I 1, I 2)$ is considered to be the "thing" which moved to the $J^{\text {th }}$ location as a result of the operation (operation $\mathrm{I} 2 *$ operation I 1 ) on the SC. Schematically, upon entering block 5 , the $\{\operatorname{ANS}(-, I 1, I 2)\}$ array is in the SC. Thus, for $\mathrm{A}_{5},\{\operatorname{ANS}(-, \mathrm{II}, \mathrm{I} 2)\}=[12345]$.

Continuing with the example of $(13)(24)(521)$, when $\mathrm{K}=1, \mathrm{II}=15$, and when $\mathrm{K}=2$, $\mathrm{I} 2=2$. Therefore, $\{\operatorname{ANS}(-, \mathrm{I} 1, \mathrm{I} 2)\}=\{\operatorname{ANS}(-, 15,2)\}$. The first rearrangement of the SC of this ANS array is the one effected by $\{\operatorname{PAIRS}(-, 15,1)\}$. After being operated on by $\{\operatorname{PAIRS}(-, 15,1\}$, the $\{\operatorname{ANS}(-, 15,2)\}$ array has the content [51342]. Following this rearrangement by $\{\operatorname{PAIRS}(-, 15,1)\}$, the $\operatorname{I2}$ th row operation $\{\operatorname{PAIRS}(-, 2,2)\}$ is brought into play. The content of $\{\operatorname{ANS}(-, 15,2]\}$ upon leaving block 5 and entering block 6 is thus [53124].

Block 6 - identification of the group product element. - In effect, the permuted stored values in each ANS array leaving block 5 are examined and a determination is made of the single group operation which would have permuted the SC to this ANS array in a single step. In block 6 , this single operation is determined and stored in \{OUT(-, I1, I2)\}. Thus, an array \{OUT(-, I1, I2)\} represents the one group operation which has the same effect on the standard ANS array (SC) as does the product of the two operations (operation I2 * operation I1).

In order to understand how the identification is made, reference may be made to the content of $\{$ ANS $(-, 15,2)\}$ upon entering block 6 . The content of a given register of ANS is the same as the label of the location in the SC. Since, in the SC, the location and object have the same label, an examination of the content of a particular register in ANS provides (partial) information about the rearrangement of two objects. Referring to the ANS $(-, 15,2)$ array shows the content of register $(1,15,2)$ to be the integer 5 . This is taken to mean that the object 5 now occupies the position originally held by object 1 . In cycle notation, this fact is indicated by placing a 1 to the left of 5 ; thus: 1,5 .

The next step is to examine the content of register 5 , which is 4 . Therefore, the next entry in the cycle has been found and the result is $1,5,4$.

Since the content of register 5 was 4 , the content of register 4 is examined next. This is found to be a 2 , so that the cycle chain is now $1,5,4,2$ and register 2 is examined next. A 3 is found there and the chain is now complete: $(1,5,4,2,3)$. The completion of the chain is tantamount to concluding that the operation (521) followed by the operation (13)(24) is equivalent to the single operation (15423). This cycle is the same group operation as (54231) which is the actual entry at the intersection of the column headed by (521) and the row headed by (13)(24) in the class product block $\mathrm{K}_{5} * \mathrm{~K}_{4}$ (see appendix C). As explained in reference 2 all equivalent cycles are printed in a unique manner in which the smallest integer in the cycle is at the extreme right.

Block 7 - output and error messages. - The group table is listed in this block. The error messages are also contained here.

Subroutine SQUEZ. - The usual manner of storing a single datum on the IBM 7094 is in a word of 36 bits. All the data in this program are integer type of such a size that only one byte in each word reserved for a datum is utilized. This results in a very inefficient use of storage. SQUEZ packs six pieces of data into one storage word. It also eliminates extraneous blanks.

The three variables from the main program to which SQUEZ is applied are: IDENT, INPUT, and OUT. It should be noted that the stored entries in OUT are of mixed form. At the time that SQUEZ is called, the punctuation marks in OUT are stored in the usual form Xbbbbb; whereas the numbers are stored in the right-shifted form bbbbbX. SQUEZ left-shifts either of such forms or any combination thereof until every byte in a given stored word contains useful information.

An example of the effect of SQUEZ can be seen by examining the form of the group element ( $1,2,3$ ) before and after being "SQUEZed." If this element were part of an OUT array in the main program, then the seven words required to store it would have the following form:
(bbbbbbbbbb1, bbbbbbbbbbb2, bbbbbbbbbb3) bbbbb


Each grouping of six typed symbols shows a single stored word. Upon leaving SQUEZ, the group element is stored in only two words of the following form:

$$
(1,2,3) \mathrm{bbbbb}
$$

A flow chart for this subroutine is given in figure 1.

## SUMMARY OF PROCEDURE

A FORTRAN program for use on second generation (7094) computing machines has been written for the computation of group tables for finite groups. Use is made of the existence of an isomorphism between any finite group of order $n$ and some subgroup $T_{n}$ of the symmetric group $S_{n}$. The elements of $T_{n}$ are entered as input data. The program then performs the group operations on these elements and identifies the products of these operations.

Each element entered as input data is expressed as a product of transpositions. The indicated interchanges for a group product of two such elements are then applied to a standard configuration. The resulting configuration is then identified with that configuration which a single group element would cause if applied to the standard configuration. This group element is called the product of the first two and entered in the row and column headed by the original two elements of $T_{n}$.

The complete table of $A_{5}$, the group of even permutations of five objects, is shown as a worked out example in appendix $C$.

Lewis Research Center,
National Aeronautics and Space Administration, Cleveland, Ohio, March 11, 1970, 129-04.

## APPENDIXA

## PROGRAM LISTING

## Main Program

C TITLE IS NAME OF GROUP OR PART OF GROUP
REAC $(5,66)$ IDENT

C ICENT= THE NUMBERS OR LETTERS USED IN THE PERMUTATICN GRDUP

WRITE(6,80) TITLE
WRITE 6,81 I IDENT
$\triangle L F=C$
$1 R P=C$
$\mathrm{N}=\mathrm{C}$
CHAR $=$ ELK

C BLK IS STCREC IN DCTAL FORM AS 606060606060
C A NCN-BLANK CHARACTER HAS THE FORM XX6060606060
DC $5 \quad 1=1.8 \mathrm{C}$
TENP = IDENT(I)
IF (TEMP.EG BLK) GO TO 5
IF (IEMP.EG.LP) GO TO 2
IF (TEMP.EGORP) GO TO 3
IF ITEMP.EG.CM GO TO 4
C SEE CCMMENTS AND SYMBOL LIST SECTION FOLLOWING FORMAY STAYEMENTS
$C A R=C R(I A L S(6, C H A R)$ IARS 30, TEMP) $)$
6C TC 5
$I L P=I L P+1$
GC. TC 5
$I R P=I R P * 1$
$N=N+1$
IF (N.GT.6) GOTO 61
IF (CHAR.EG.BLK) GO TO 62
ANTAL(N)=CFAR

ENTRIES IN INITALINI ARE STORED AS BBBBBX
CHAR=ELK
COMPINUE

```
        IF (ILP.NE.1.OR.IRP.NE.1) GO TO 63
        10=8C
        CALL SQUEZ (IDENY.ID:
        CC 8 K=1,2
        DC 7 I=1,5
        JS=(1-1)*8C+1
        JE=I*&0
        REAE (5,66) (INPUT(J,K),J=JS,JE)
        WRIME(6,90) (INPUT(J.K),J=JS:JE)
        DC G J=JS;JE
    IF (IAPUY(J.K).NE.RLK) GO TO 7
    continue
    C NINPLT(K) IS THE SL IN TYPE K INPUT. IT IS SET EG TO 8O(NUMBER OF INPUT
C
NPAIR(NOP,K)=NPAIR(NOP,K)+1
IF (AFAIR(NCP,K),GT.I7) GO TC 59
IF (CHAR.EG.ELK) GO TO 56
IJ=NPAIR(NCP,K)
```

C

## CC 27 IL $=1, \mathrm{NCPI}$ <br> $K 1=N F A I R(I 1,1)$

CC 27 I2 $=1, ~ \cap C P 2$
$K 2=A F A I R(I 2,2)$
CC $23 \quad 1=1, K 1,2$
$J=K 1-1+1$
$\mathrm{J} 1=\mathrm{C}$
$\mathrm{J} 2=\mathrm{C}$
CC $22 L=1, N$
IF (ANSIL:II,I2).NE,PAIRS(J,I1,1)) GC TC 21
$\mathrm{J}=\mathrm{L}$
IF $(A A S(L, I 1, I 2), N E . P A I R S(J-1, I 1,1) \mid$ GO TO 22
$\mathrm{J} 2=\mathrm{L}$
ccatinue
IF (J1.EG.C.OR.J2.EQ.0) CO TC 55
$I T N P=A N S(J 1, I l, 12)$
ANS(J1, 11,12)=ANS(J2,11,12)
ANS (12, 11,12)=ITMP
centinue

```
        CC 2t I=1,K2.2
        J=k2-1+1
        Jl=C
        J2=C
        CC 25 L=1,N
        IF (ANS(L,II,I2).NE.PAIRSIJ.I2,2|) GO PO 24
        Jl=L
        IF (ANS(L.Il,I2).NE.PAIRS(J-1,I2,2)) GO T0 25
        J2=L
C kn(K) eg o means the content of the kih location in the ans array
C HAS EEEN IncentIFIED.
KN(K)=0
J=J+1
CUT(J.11,12)=ANS(K.11,12)
J=J&1
OUP(J,11,12)=CM
GC TC }3
KA(K)=0
CC 32 KK=1,N
K=KN(KK)
IF (K.NE.O) GO TO 33
ccatinue
Kl=K
IF (CLY(J-2.11,I2).NE.LP) GO TO 34
IF (K.EG.0) GO TC 36
CUY(J-1,11;12)=ANS(K;11,12)
GC IC 35
```

$J=J R F$
IF (J.EG.O) GO TC 37
CALL SQUEZ (CUT(1.11.12):J)
C NCLT(II,I2) IS THE NUMBER OF SYMBOLS IN IHE I 1,12 CYCLE (SEE MAX)
C NCYE IHAT AFYER RETURN FROM SQUEZ.jEg THE TOTAL NUMBER DF HORDS REQURED


C title anc ICent isee symbol listi are hritten betheen here and do 54 state

```
        NC=21
```

$\operatorname{HAX}=\mathrm{MAX}+1$
$N L A=A C / N A X-1$
IF (ALM.LT.2) GO TO 64
NN = (ACP $1+N(N-1) / N U M$
WRITE $(6,67)$ (IDENT(I), $1=1,1[$ )
WRite (e.68) title
C
CLT(J.11,12)=RP
JRF=J
IF (K.EG.C) CO YC 36
$J=J+1$
ClT(J.IL,Ie)=LP
$J=J+1$
CUT(J.11,12)=ANS (K,11.12)
$J=5+1$
CLI(J, 11,12)=CM
FCR (LT $\{-11,12$ ) ARRAY IN SqUEEZEC FORM
NCLT(11.12)=J
IF (J.LE.MAXI GO TO 38
$N A X=J$
ccatiaue
ELCCK(T) SETS UP THE OUTPLY FORMAT AND WRIYES ERRCR MESSAGES
CC $41 \mathrm{k}=1.2$
C THE [C $4 I$ lCCP DOES SOME LOGISTICAL PRELIMINARIES FCR THE ROW AND CCLUMN
heaciags cf the grolp rable
$\mathrm{J}=\mathrm{C}$
$\mathrm{J} 1=1$
IE=AIAPLT(K)
CC $4 \mathrm{C} \quad \mathrm{I}=1, \mathrm{IE}$
IF (IAPUY(I.K).NE.PER) GC 1040
THE JTH GRCLP OPERATION OF TYPE K
$\mu x=1-J 1+1$
CALL SQUEZ (INPUT(JI,K), MX)
If (NX.LE.NAX) GC TO 39
$\mathrm{FAX}=\mathrm{FX}$
$J 1=N x+J 1$
$J=J+1$
NIA (J.K) $=N X$
ccatiaue
ccailaue
ment。

SEI LF FCR THE FIRST ROW (COLUMN HFACINGS) CF GRCLF TARLE
$k 1=1$
DC 54 $I=1,1, N$

```
C MAKE FIRST 21 wORCS bLANK. NOTE THAT EACH SQUEEZED hORD TAKES UP 6 SPACES
C TC PRINT OLT
    CC 42 L=1.21
    LINE(L)=RLK
    LI=NRX
    DC 44 J=1.NLN
    II=(I-1)*NLM+J
    IF (II.GT.ACPI) GC YC 45
    K2=N1N(11,1)+K1-1
    OC 42 K=K1,K2
    LI=LI+l
    LINE(LI)=IAPUT(K,H)
    Ll=(J+1) #NAX
    K1=kz+1
    WRITE (6,65) LINE
    statement 45 has writien the first line cf the table (column headings)
    the maln bcey of the table is writyen line by line beymeen the statement
    K3 EG 1 ANC STATEMENT NUMBER 52.
    k3=1
    CC 52 12=1,NCP2
    L1=C
    THE FIRST 21 WORCS BLANKED DLT (SEE COMMENT ON DO 42)
    CC 4E L=1,21
    LIAE(L)=ELK
    K4=A1A(12,2)+K3-1
    CC 47 K=K3,K4
    Ll=\1+1
    LINE(LI)=INPUT(K,2)
    K3=K441
    LI=NAX
    CC El J=1,NlN
    1!=(1-1)*N(N+J
    IF (II.GT.NCPI) GO TO 52
    K5=NCLY(11,12)
    IF (K5.EG.C) GO TO 49
    CC 4E K=1,K5
    LI=LI+1
    LINE(L1)=CLY(K,I1,12)
    GC TC 5C
    Ll=Ll+1
    LIAE(LI)=ICAT
    Ll=(J+l)*MAX
    ccayINuE
    WRITE (6,7C) LINE
    cCATINUE
    ccatinue
    write statenents for errgr messages
    CC IC 1
    kRIYE (6,71) (IDENT(I),I=1,IC)
    WRIYE(6.79) TITLE
    GC IC 1
    WRITE (6,72)
    GC TE I
    WRITE (6,73)
    CC IC 1
```

    WRITE (6.74)
    GC TC I
    WRIIE (6.75)
    GC TC 1
    WRITE \((6,76)\)
    GC YC 1
    WRITE \((6,77)\)
    GC IE 65
    WRITE \((6,72)\)
    GC TC 65
    WRITE ( 6,72\()\)
    6C IC 65
    WRITE (6,78)
    GC YC 1
    RETLRA
    FCRNAT (80A1)
    FCRFAT(1FJ,10X,GHGROUP ,11AG)
    FCRMAT(1FJ,5CX,3CA11
    FCRMAT(1HL, 21AG)
    FCRNAY(1FJ.21A6)
    FCRMATS FOR ERROR MESSAGES
    FCRNAI(IHJ,44HILIEGAL ELEMENT IN GROLP. IDENTITY GRCUP IS , IIA6)
    FCRNAY(1FJ.29HBLANK IS NCY A VALID ELEMENT.l
    FCRNETIHJ, 22 HILLEGAL USE OF PARENS.I
    FCRNAT( 1 HJ . 14 HILLEGAL GROUP.)
    FCRNAY(IHJ. 34 HTHE PAIRS ARRAY HAS BEEN EXCEEDED.)
    FCRNATIHJ.34HTOC MANY OPERATIONS. LIMIT IS 24.1
    FCRNAY(IHJ,39HMORE THAN 6 ELEMENTS IN IDENTITY GRCLP.)
    FCRMAY(IHJ,45HNOT ENCUGH ROON ON PRINT LINE TO PRINY TABLE.)
    FCRNAY(1HJ.27HCHECK INPUT CARDS IN GROUP, 80A1)
    FCRNAT(1+1,80A1)
    FCRMAT(1FJ, ROA1)
    FCRMAT(IHJ.8OAI)
    the fellching variables are sqeeled ddent, input, and out.
    STCRAGE FCR A WORD ON THE 7094 IS ALLOYYED 36 BITS. A WORD CAN BE 6
    alphanumeric characters long. each character is represented by one byte
    (6 EITS LCRE).
    A EYTE REPRESENTING A BLANK IS CODED IN CCTAL AS 6C
    A FLLL BLANK WORC IS CODED IN DCYAL AS 6C6C60606C6C
    A SIACLE NCN-BLANK CHARACTER IS STORED AS \(\times 66060606060\)
    IALS(E.CHAR) SHIFTS THE CHARACTERS IN THE hORD CHAR 6 BITS IOR ONE BYTE)
    to ite left. the last byte is then replaced by zercesinot blanks). a blank
    WLRC CPERATEC ON BY IALS(6.CFAR) WOULD ASSLME THE FORM 606060606060.
    IARS ( 30, TENP) SHIFTS THE CHARACTERS IN TEMP 30 BITS ( 5 BYTES) TO THE RIGHT
    AGAIA. THE 5 bYTES ARE REPLACED BY ZEROES THE FORM CF A NON-BLANK WORD
    OPERATEC ON RY IARS (30,YEMP) WOLLD BE CCOCOOOOOOXX.
    the legical or between two qlantites a and b stores i in a given bit
    LCCATION IF EITHER A OR B HAS I IN THAY LCCATICN O IS STORED IN A
    given bit lccation only if rcth a and b mave o in ihat location
    THE EFFECT CF OR(IALS(G,CHAR) IARS(3C,TENP) ON A NCN-BLANK TEMP OF THE
    FCRN \(X X 6 C 6 C 606060\) IS TO CHANGE IT TO THE FORM \(606 C 606060 \times X\)
    ENE
    
## Subroutine SQUEZ

```
    SUERCLTINE SQUEZ(OUT.N)
    CIMEASICN CUT(80)
    DATA MASK,ELANK,ZERO,BLANK1/
*C770CCOOCOCOO,06C000CCOOCOO.COOOCCOOOCOOC.C6060606C6060 /
    IA=C
    IE=C
    TERF=2ERC
    DC 1 I=1,N
    CC 1 J=1,6
    PART=ANC(MASK,ALS(6*(J-1),OUT(1))
    IF(PART.EG.RLANK) GO TO I
    IA = IA+1
    TEFP=CR(TEFP,ARS(6*(IA-1),PART))
    IF(IA.NE.G) GC TC l
    IE=IE+1
    CUT(IE)=TENP
    IA=C
    TENP=ZERC
1 continue
    IF(IA.EG.O) CO TC 2
    IE=IE+1
    CLT(IE)=TENP
    IA= [A+1
    C[ 2 J J=1A,E
    OUT(IR)=CR(CUT(IB),ARS(6*(J-1),BLANK))
    IF(N.CT.N) CO TO 4
    CC 5 J=M,N
    OUT(JI= ELANKI
    N=IE
    RETLRA
    ENC
```

$2 \quad \mu=1 E+1$

## APPENDIX B

## MAP LISTING OF SHIFT ROUTINES

|  | entry | ALS 5 |
| :---: | :---: | :---: |
|  | ENTRY | ALS |
|  | tentry | ARSF |
|  | ENTRY | ARS |
|  | ENTRY | ALGRF |
|  | Entry | ALGR |
|  | ENTRY | ALGLF |
|  | ENTRY | ALGL |
|  | ENTRY | EXORF |
|  | ENTRY | EXUR |
|  | Entry | IEXOR |
|  | entry | IALS |
|  | ENTRY | IALSF |
|  | Entry | IARS |
|  | ENTRY | IARSF |
|  | ENTRY | LRS |
|  | ENTRY | LRSF |
|  | ENTRY | LGR |
|  | ENTRY | LGRF |
|  | ENTRY | LLSF |
|  | ENTRY | LLS |
|  | ENTRY | LGI |
|  | ENTRY | LGLF |
|  | ENTRY | XLRS ${ }^{\text {P }}$ |
|  | ENTRY | XLRS |
|  | ENTRY | XLLSF |
|  | ENTRY | XLLS |
| I ARS | NULL |  |
| ARSF | NULL |  |
| IARSF | NULL |  |
| ARS | CLA* | 3.4 |
|  | STA | * +2 |
|  | CAL; | 4.4 |
|  | ARS | 䒼 |
|  | KCL |  |
|  | XCA |  |
|  | TRA | 1.4 |
| IALS | null |  |
| [ ALSF | NULL |  |
| ALSF | Null |  |
| ALS | CLA* | 3.4 |
|  | STA | * +2 |
|  | CAL* | 4.4 |
|  | ALS | ** |
|  | XCL |  |
|  | XCA |  |
|  | TRA | 1.4 |
| XLRS | NULL |  |
| XLRSF | NULL |  |
| LRSF | NULL |  |
| LRS | CLA | 3.4 |
|  | STA | $4+3$ |
|  | CAL* | 4.4 |
|  | LDQ | TEMP |
|  | LRS | ** |


|  | STO | TEMP |
| :---: | :---: | :---: |
|  | XCL |  |
|  | XCA |  |
|  | TRA | 1.4 |
| XLLS | NULL |  |
| LLSF | NULL |  |
| XLLSF | NULL |  |
| LLS | CLA* | 3,4 |
|  | STA | + 4 |
|  | CAL* | 4.4 |
|  | LDQ | TEMP |
|  | LLS | * |
|  | STQ | TEMP |
|  | XCL |  |
|  | XCA |  |
|  | TRA | 1,4 |
| EXORF | NULL |  |
| IEXOR | NULL |  |
| EXOR | CAL* | 3.4 |
|  | ERA* | 4,4 |
|  | XCL |  |
|  | XCA |  |
|  | rRA | 1.4 |
| ALGL | NULL |  |
| LGLF | NULL |  |
| ALGLF | NULL |  |
| LGL | CLA* | 3.4 |
|  | STA | \#+3 |
|  | CAL* | 4.4 |
|  | LDQ | TEMP |
|  | LGL | ** |
|  | STQ | TEMP |
|  | XCL |  |
|  | XCA |  |
|  | TRA | 1.4 |
| ALGR | NULL |  |
| ALGRF | NULL |  |
| LGRF | NULL |  |
| LGR | ClA | 3.4 |
|  | STA | \#+3 |
|  | CAL* | 4.4 |
|  | LDQ | TEMP |
|  | LGR | ** |
|  | STQ | TEMP |
|  | XCL |  |
|  | XCA |  |
|  | TRA | 1.4 |
| TEMP | OCr |  |
|  | END |  |

## APPENDIX C

## EXAMPLE OF COMPUTER OUTPUT－GROUP $\mathrm{A}_{5}$

KてもKで<br>$(1,2,3,4,5),(1,2,5,3,4)-(1,4,5,2,3),(1,4,2,3,5),(1,3,5,4,2),(1,3,4,2,5)$ ．<br>$(5,4,3,2,1),(5,2,1,4,3),(5,4,1,3,2),(5,3,2,4,1),(5,3,1,2,4),(5,2,4,3,1)$.<br>$(1,2,3,4,5),(1,2,5,3,4),(1,4,5,2,3),(1,4,2,3,5),(1,3,5,4,2),(1,3,4,2,5)$.<br>$(5,4,3,2,1),(5,2,1,4,3),(5,4,1,3,2),(5,3,2,4,1),(5,3,1,2,4),(5,2,4,3,1)$ 。

|  | （1，2，3，4，5）． | $(1,2,5,3,4)$ ． | （1，4，5，2，3）． | （1，4，2，3，5）． | $(1,3,5,4,2)$ ． | $(1,3,4,2,5)$ ． |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| （1，2，3，4，5）． | $(3,5,2,4,1)$ | $(3,5,4,2,1)$ | $(5,3,2,4,1)$ | $(5,2,4,3,1)$ | $(4,3,1)$ | $(4,3,5,2,1)$ |
| $(1,2,5,3,4)$ ． | $(5,2,4,3,1)$ | $(5,4,2,3,1)$ | $(4,3,2)$ | $(4,5,2)$ | $(4,5,1)$ | $(4,5,2,3,1)$ |
| （1，4，5，2，3）． | $(3,5,4,2,1)$ | $(3,5,1)$ | $(5,3,4,2,1)$ | $(5,4,3,2,1)$ | $(4,3,2)$ | （5，4，3） |
| （1，4，2，3，5）． | $(3,2,5,4,1)$ | $(3,2,1)$ | $(2,5,3,4,1)$ | $(2,5,4,3,1)$ | $(5,2,4,3,1)$ | $(5,4,3,2,1)$ |
| （1，3，5，4，2）． | $(5,3,2)$ | $(4,3,2)$ | （2，5，3） | $(2,5,3,4,1)$ | $(5,2,3,4,1)$ | $(5,3,2,4,1)$ |
| $(1,3,4,2,5)$ ． | $(5,3,2,4,1)$ | $(5,4,3,2,1)$ | （2，4，1） | $(2,4,5,3,1)$ | $(4,5,2,3,1)$ | $(4,5,3,2,1)$ |
| （5，4，3，2，1）． | E | $(4,5,2)$ | $(3,5,1)$ | （3，4，1） | $(2,5,3,4,1)$ | （2，4，1） |
| $(5,2,1,4,3)$ ． | $(5,4,2)$ | E | $(3,4,2,5,1)$ | $(3,2,5,4,1)$ | $(5,3,2,4,1)$ | （5，4，1） |
| $(5,4,1,3,2)$ ． | $(5,3,1)$ | （5，2，4，3，1） | E | $(4,5,3)$ | （2，3，4，5，1） | $(2,4,5,3,1)$ |
| $(5,3,2,4,1)$ ． | （4，3，1） | $(4,5,2,3,1)$ | $(5,4,3)$ | E | （2，5，1） | （2，3，1） |
| $(5,3,1,2,4)$ ． | $(4,3,5,2,1)$ | $(4,2,3,5,1)$ | $(5,4,3,2,1)$ | （5，2，1） | E | $(3,5,2)$ |
| （5，2，4，3，1）． | （4，2，1） | （4，5，1） | $(3,5,4,2,1)$ | （3，2，1） | $(5,3,2)$ | $\varepsilon$ |
|  | $(5,4,3,2,1)$ ． | $(5,2,1,4,3)$ ． | （5，4，1，3，2）． | （5，3，2，4，1）． | （5，3，1，2，4）． | （5，2，4，3，1）． |
| $(1,2,3,4,5)$. | E | （5，3，1） | （4，2，1） | （5，4，2） | $(3,2,5,4,1)$ | $(5,3,2)$ |
| $(1,2,5,3,4)$ ． | （3，5，1） | E | $(4,2,3,5,1)$ | $(3,5,4,2,1)$ | $(5,4,3,2,1)$ | $(3,2,1)$ |
| $(1,4,5,2,3)$ ． | $(2,4,1)$ | $(5,3,2,4,1)$ | E | $(2,5,1)$ | $(3,4,2,5,1)$ | $(2,5,3,4,1)$ |
| （1，4，2，3，5）． | $(4,5,2)$ | $(2,4,5,3,1)$ | （5，2，1） | E | （3，4，1） | $(4,5,3)$ |
| （1，3，5，4，2）． | $(4,5,2,3,1)$ | $(2,3,4,5,1)$ | $(5,2,4,3,1)$ | （4，3，1） | E | $(4,5,1)$ |
| （1，3，4，2，5）． | $(3,5,2)$ | $(2,3,1)$ | $(4,3,5,2,1)$ | $(5,4,3)$ | （5，4，1） |  |
| （5，4，3，2，1）． | $(4,2,5,3,1)$ | $(3,4,2,5,1)$ | $(2,4,5,3,1)$ | $(4,5,2,3,1)$ | $(3,5,2)$ | $(4,2,3,5,1)$ |
| $(5,2,1,4,3)$ ． | $(2,4,5,3,1)$ | （3，2，4，5，1） | （5，3，1） | （2，3，1） | （3，4，2） | $(2,3,4,5,1)$ |
| （5，4，1，3，2）． | $(4,2,3,5,1)$ | （3，4，2） | $(2,4,3,5,1)$ | $(4,3,5,2,1)$ | （5，2，1） | $(4,2,1)$ |
| $(5,3,2,4,1)$ ． | （3，4，2，5，1） | （5，4，2） | \｛2，3，4，5，1） | $(3,4,5,2,1)$ | $(4,3,5,2,1)$ | $(3,5,4,2,1)$ |
| $(5,3,1,2,4)$ ． | （3，4，1） | （5，4，1） | $(3,4,2)$ | $(3,4,2,5,1)$ | $(4,3,2,5,1)$ | $(3,2,5,4,1)$ |
| （5，2，4，3，1） ． | $(2,5,3,4,1)$ | （3，2，5，4，1） | $(4,5,3)$ | $(2,3,4,5,1)$ | 14，2，3，5，11 | $(2,3,5,4,1)$ |

[^0]$(1,3,5,2,4)$.
$(1,5,4,2,3)$.
$(1,5,3,4,2)$.
$(1,2,5,4,3)$.
$(1,5,2,3,4)$.
$(1,4,5,3,2)$.
$(5,3,1,4,2)$.
$(5,1,3,2,4)$.
$(5,1,2,4,3)$.
$(5,2,1,3,4)$.
$(5,1,4,3,2)$.
$(5,4,1,2,3)$.
$(1,3,5,2,4)$
$(3,4,3,2,1)$
$(4,5,3)$
$(4,5,1)$
$(3,4,2)$
$(4,5,3,2,1)$
$(2,5,1)$
$(6,5,4,3,1)$
$(2,5,4,3)$
$(5,4,2,3,1)$
$(4,3,2,5,1)$
$(2,3,1)$
$(5,3,4,2,11$

$(5,3,1,4,2)$
$E$
$(2,4,3,5,1)$
$(2,3,5,4,1)$
$(3,2,4,5,1)$
$(5,4,3)$
$(5,2,3,4,1)$
$(2,3,4,5,1)$
$(5,2,1)$
$(3,2,1)$
$(5,4,1)$
$(3,4,5,2,1)$
$(4,3,2)$

| $(1,5,4,2,3)$ | $(1,5,3,4,2)$ |
| :--- | :--- |
| $(2,5,1)$ | $(2,3,1)$ |
| $(4,3,5,2,1)$ | $(4,3,2,5,1)$ |
| $(3,5,2,4,1)$ | $(3,2,5,4,1)$ |
| $(4,5,3,2,1)$ | $(4,5,1)$ |
| $(2,4,3,5,1)$ | $(2,5,4,3,1)$ |
| $(3,4,1)$ | $(3,5,2,4,1)$ |
| $(3,4,5,2,1)$ | $(3,2,4,5,1)$ |
| $E$ | $(3,5,2)$ |
| $(5,3,2)$ | $(2,3,5,4,1)$ |
| $(2,4,1)$ | $(4,5,2)$ |
| $(4,5,3)$ | $(4,3,11$ |
| $(4,3,2,5,11$ |  |
|  |  |
|  |  |
| $(5,1,3,2,4)$ | $(5,1,2,4,3)$ |
| $(5,3,4,2,1)$ | $(4,5,3,2,1)$ |
|  | $(3,4,1)$ |
| $(4,3,1)$ | $(5$ |
| $(3,5,2)$ | $(5,2,3,4,1)$ |
| $(4,2,1)$ | $(3,2,1)$ |
| $(2,5,4,3,1)$ | $(5,4,2)$ |
| $(5,4,3)$ | $(5,4,11$ |
| $(2,5,3,4,1)$ | $(4,2,5,3,1)$ |
| $(5,2,3,4,1)$ | $(4,5,2,3,1)$ |
| $(4,2,5,3,1)$ | $(5,3,2)$ |
| $(2,3,5,4,1)$ | $(5,4,2,3,2)$ |
| $(5,2,1)$ | $(3,4,5,2,1)$ |

$(1,2,5,4,3)$
$(4,5,1)$
$(3,5,2,4,1)$
$(3,5,2)$
$(5,3,2,4,1)$
$(3,5,1)$
$(3,4,2)$
$(5,2,3,4,1)$
$(4,2,1)$
$(4,5,3,2,1)$
$E$
$(5,3,4,2,1)$
$(3,2,4,5,1)$

$(5,2,1,3,4)$
$(5,4,2,3,1)$
$(5,3,2)$
$(4,3,2,5,1)$
$E$
$(4,2,5,3,1)$
$(2,4,3,5,1)$
$(4,3,2)$
$(2,3,5,4,1)$
$(5,4,1)$
$(4,2,3,5,1)$
$(2,4,1)$
$(5,3,1)$
$(1,5,2,3,4)$
$(2,5,4,3,1)$
$(4,5,3,2,1)$
$(3,2,4,5,1)$
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$(2,4,5,3,1)$
$(3,5,1)$
$(3,2,1)$
$(5,4,3)$
$(5,4,2)$
$(2,4,3,5,1)$
$(4,2,5,3,1)$

$(5,1,4,3,21)$
$(4,5,3)$
$(2,4,1)$
$(2,3,1)$
$(3,5,2,4,1)$
6
$(5,4,2,3,1)$
$(2,3,5,4,1)$
$(5,3,4,2,1)$
$(3,4,5,2,1)$
$(5,3,1)$
$(3,5,4,2,1)$
$(4,5,2)$
（1，4，5，3，21．
$(3,4,2)$
（2，5，1）
$(2,5,4,3,1)$
$(3,5,1)$
$(5,4,2)$
$\{5,2,4,3,1)$
$(2,4,3,5,1)$
$(5,2,3,4,1)$
（3，4，1）
$(5,4,2,3,1)$
$(3,5,2,4,1)$
$\stackrel{13}{=}$
$(5,4,1,2,3)$ ．
$(4,3,2,5,1)$
$(4,3,2,5,1)$
$(3,4,5,2,1)$
（1＋，5，2）
$(5,3,4,2,1)$
$13,2,4,5,1$
$(5,2,1)$
$(5,2,1)$
$(4,2,5,3,1)$
$(4,2,5,3,1$
$(4,3,2)$
$(5,3,1)$
$(3,4,2,5,1)$

$\left(\begin{array}{l}1,2,3),(2,3,4) \cdot(1,3,4),(1,2,4),(1,2,5) \cdot(1,3,5)=(2,3,5) \cdot(1,4,5),(2,4,5),(3,4,5), \\ (3,2,1),(4,3,2) \cdot(4,3,1) \cdot(4,2,1) \cdot(5,2,1),(5,3,1), 15,3,2),(5,4,1),(5,4,2),(5,4,3)\end{array}\right.$ GROUP $(1,2,3,4,5)$
G $1.40,1) \cdot(4,2,1) \cdot(5,2,4) \cdot(5,3,1) \cdot(5,3,2) \cdot(5,4,1) \cdot(5,4,2) \cdot(5,4,3) \cdot$







[^1] $1,4)(2,4)$.
$2,5)(3,4)$.
GROUP $(1,2,3,4,5)$



## 人2卒K3

$\left(\begin{array}{l}(2,5,2,4)-(1,5,4,2,3),(1,5,3,4,2),(1,2,5,4,3),(1,5,2,3,4),(1,4,5,3,2), \\ (5,3,1,4,2) \cdot(5,1,3,2,4),(5,1,2,4,3),(5,2,1,3,4),(5,1,4,3,2),(5,4,1,2,3),\end{array}\right.$
$\left(\begin{array}{l}1,2,3,4,5)-11,2,5,3,4) \cdot(1,4,5,2,3)-(1,4,2,3,5) \cdot(1,3,5,4,21,(1,3,4,2,5), \\ (5,4,3,2,11,(5,2,1,4,3) \cdot(5,4,1,3,21,(5,3,2,4,1),(5,3,1,2,4),(5,2,4,3,1),\end{array}\right.$ GHOUP $(1,2,3,4,5)$ $(1,3,5,2,4)^{\circ}$ $(1,5,4,2,3)$.
$(1,2,5,4,3)$
$(3,2,1)$
$(5,1)(3,2)$
$(3,4,1)$
$(3,4,5,2,1)$
$(4,5,2)$
$(5,2,1)$
$(4,2)(5,3)$
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## $(1,2,3,4,5)$ $(1,2,3,4,5),(1,2,5,3,4)-(1,4,5,2,3)-(1,4,2,3,5),(1,3,5,4,2),(1,3,4,2,5)$, $(5,4,3,2,1) \cdot(5,2,1,4,3)-(5,4,1,3,2),(5,3,2,4,1),(5,3,1,2,4),(5,2,4,3,1)$. <br> $(1,3,5,2,4) \cdot(1,5,4,2,3),(1,5,3,4,2) \cdot(1,2,5,4,3),(1,5,2,3,4),(1,4,5,3,2)$, $(5,3,1,4,2)=(5,1,3,2,4),(5,1,2,4,3),(5,2,1,3,4)=(5,1,4,3,2),(5,4,1,2,3)$, GROUP $11,2,3,4,5 i \quad k 3 * K 2$



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$(5,3,1,4,2),(5,1,3,2,4) \cdot(5,1,2,4,3) \cdot(5,2,1,3,4) \cdot(5,1,4,3,2),(5,4,1,2,3)$, $k 5, k 3$
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$(5,3,1,4,2) \cdot(5,1,3,2,4) \cdot(5,1,2,4,3) \cdot(5,2,1,3,4) \cdot(5,1,4,3,2),(5,4,1,2,3)$.
$(1,2)(3,4) \cdot(1,3)(2,4) \cdot(1,4)(2,3) \cdot(1,2)(3,5),(1,3)(2,5) \cdot(1,5)(2,3),(1,2)(4,5)$,
$(1,4)(2,5) \cdot(1,5)(2,4) \cdot(1,3)(4,5) \cdot(1,4)(3,5),(1,5)(3,4) \cdot(2,3)(4,5),(2,4)(3,5)$. $(2,5)(3,4)$. GROUP (1,2,3.4.5)

# (1,2,5,4,3). 

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$(2,4,1)$ - $\left(2^{\circ} \varepsilon^{\circ} \cos ^{\circ} T^{\circ} 5\right)$






## REFERENCES

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Figure 1. - Flowchart for subroutine sQuEZ.
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[^0]:    K3＊K3
    $(1,2,3,4,5)$
    $(1,3,5,2,4),(1,5,4,2,3),(1,5,3,4,2),(1,2,5,4,3) .(1,5,2,3,4),(1,4,5,3,2)$.
    $(1,3,5,2,4),(1,5,4,2,3),(1,5,3,4,2),(1,2,5,4,3),(1,5,2,3,4),(1,4,5,3,2)(1)$
    $(5,3,1,4,2),(5,1,3,2,4),(5,1,2,4,3),(5,2,1,3,4),(5,1,4,3,2),(5,4,1,2,3)$.

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[^1]:    $(1,2)(3,4),(1,3)(2,4),(1,4)(2,3) \cdot(1,2)(3,5) .(1,3)(2,5) .(1,5)(2,3) .(1,2)(4,5)$.
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[^2]:    $1,3,5,2,4) \cdot(1,5,4,2,3),(1,5,3,4,2),(1,2,5,4,3),(1,5,2,3,4,1,(1,4,5,3,2)$,
    $(5,3,1,4,2),(5,1,3,2,4) \cdot(5,1,2,4,3) \cdot(5,2,1,3,4) \cdot(5,1,4,3,2) \cdot(5,4,1,2,3)$

