E- 5654

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ANALYSIS OF THE EFFECTS OF SEA STATE ON DOPPLER-RADAR MEASUREMENT OF AIRCRAFT GROUND SPEED

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ABSTRACT

An analysis is made of the effect of sea state on Doppler-radar measurement of aircraft ground speed. Previously published data taken at both microwave and visible-light wavelengths are used to estimate the ground-speed error in a representative situation. The magnitude of the error is determined by the sea state and by the flight direction relative to the ground-wind direction. The results indicate that a probable error of less than one percent is generally achievable after correction for the effect of sea state.

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SUMMARY

An analysis has been made of the effect of sea state on Doppler-radar measurement of aircraft ground speed. Previously published data taken at both microwave and visible-light wavelengths were used to estimate the ground-speed error in a representative situation. The magnitude of the error is determined by the sea state and by the flight direction relative to the ground-wind direction.

The results indicate that a probable error of less than one percent is generally achievable if one estimates the most probable value of the error for each heading, and for the given sea state, and then applies this estimate as a correction.

INTRODUCTION

In some meteorological experiments covering a large area over the ocean, it may be necessary to measure the wind speed and direction at several altitudes. This can be done, using instrumented aircraft, by combining Doppler radar measurement of ground speed with the aircraft's airspeed. The computed wind speed is the difference of these two much larger numbers, both of which are subject to error. The effect of sea state on the error in ground speed measurement by Doppler radar will be treated here. The analysis relies on previously-published experimental data taken at both microwave and visible-light wavelengths.

This analysis was initially performed to assist in the planning for project BOMEX (Barbados Oceanographic and Meteorological Experiment), which was conducted in May-July 1969 by ESSA with the participation and assistance of numerous governmental and university organizations.

Doppler radar measurement of aircraft ground speed over the sea is subject to two errors not found over land.* Both are due to the character of the sea surface and vary quantitatively with the sea state. The first error is due to the fact that the radar beam is reflected by the sea surface in a different manner than that in which it is reflected by land. The error, which may be termed a "calibration error," may be on the order of one percent of the indicated ground speed. The second error is due to the fact that the radar-reflecting sea surface may, under some circumstances, appear to have a velocity, relative to "true ground," whose magnitude depends on the surface wind speed. This error, which may be termed a "surface-speed error," is some fraction of the surface-wind speed. Both errors depend on the angle between the aircraft heading and the wind direction at the sea surface.

The dependence of the calibration error on the nature of the sea state appears fairly well documented by several observers (references 1-4) who have reported actual Doppler-radar data. Unfortunately, these data are given without specifying the angle between the radar beam and the surface-wind direction. The dependence of the calibration error on the angle between aircraft heading and surface wind ("crosswind" vs "upwind-downwind") is less fully documented. We have had to use the data reported by references 5 and 6, taken at visible-light wavelengths. There is no evidence that these data are inapplicable to radar wavelengths; however, the data cover a narrower range of sea states than was covered by references 1-4.

The dependence of the surface-speed error on the surface-wind speed is presented by references 1, 2, and 7. However, it is not certain that these represent independent determinations; they may partially stem from the same source. Furthermore, the magnitude of the surface-speed error is stated in a less precise form than is the calibration error. We have had to apply some intuition in order to make the best possible estimate of the surface-speed error.

* The simple error due to ocean current (1 to 2 knots) is neglected in this analysis.

ANALYSIS

In the following analysis, we first discuss the measurement of the velocity of the aircraft relative to the sea surface, and derive the corresponding calibration error. Then, we discuss the determination of the velocity of the sea surface relative to "true ground" and derive the corresponding surface-speed error.

A numerical example illustrates how these errors are combined.

System Configuration

For simplicity, the system to be considered will be the Janus system (reference 9) shown in Fig. 1. The simplification is tantamount to assuming zero drift angle, so that aircraft heading coincides with the track of the aircraft over the ground, and to assuming that the plane through the forward and rearward beams includes this ground track. The aircraft heading is in the x-direction. The horizontal plane through the mean sea surface is the x-y plane. Two radar beams from the antenna at (0, 0, h) reach the sea surface and are reflected back to the antenna. Each beam is very narrow, rectangular in cross section, with included angle 2α in the x-z plane and included angle 2β in the orthogonal direction. The beams intersect the x-y plane in the two rectangles shown. The forward and rearward beams both make a mean angle θ_o with the z-axis in the x-z plane. If β lies along an isodop, the beam can be treated as two-dimensional, so that only what happens in the x-z plane need be considered.

<u>Infinitely narrow beam.</u> For the idealized condition $\alpha=0$, the basic Doppler formula holds

$$\frac{\Delta f}{f} = \frac{4v}{c} \sin \theta_o \tag{1}$$

where $\Delta f/f$ is the fractional frequency shift, c is the speed of light, and v is the component of aircraft velocity relative to the sea surface in the direction of the aircraft heading. No frequency shift is produced by relative-velocity components in the y- or z-direction.

<u>Beam of nonzero width.</u> For the realistic case $\alpha > 0$, the $\sin \theta_o$ in Eq. (1) must be replaced by some average over the interval

$$(\theta_o - \alpha) \leq \theta \leq (\theta_o + \alpha). \tag{2}$$

Assume that the antenna beam has a uniform radiated-power distribution over this interval, but that the power reflected at angle θ, per unit element of area of the sea surface, varies with the angle of incidence θ. Then, if $\sigma^{o}(\theta)$ is the back-scattering cross section of the sea surface per unit area of sea surface, Eq. (1) is replaced by

$$\frac{\Delta f}{f} = \frac{4v}{c} \cdot \frac{\int_{\theta_o-\alpha}^{\theta_o+\alpha} \sigma^{o}(\theta) \sin\theta \cdot d\theta}{\int_{\theta_o-\alpha}^{\theta_o+\alpha} \sigma^{o}(\theta) \cdot d\theta} \qquad (3)$$

This formula assumes that $\sigma^{o}(\theta)$ is the same for both forward and rearward beams. For the situations treated in this analysis, this assumption appears to be valid (references 5 and 6).

Distinction Between Land and Sea

Over land covered with reasonably dry vegetation, the value of $\sigma^{o}(\theta)$ is independent of θ (references 2 and 4), so that the ratio of the integrals, that appears in Eq. (3), is very close to $\sin\theta_o$. However, over the sea, one encounters two types of radar reflection:

Specular reflection from smooth waves. Here, the statistical distribution of wave slopes is approximately Gaussian, and $\sigma^{o}(\theta)$ is a rapidly-changing function of θ (references 1-4). The horizontal wave-propagation velocity has negligible Doppler effect in the Janus system because the water displacement is effectively a vertical oscillation (reference 1).

Scattering by whitecaps and spray. Whitecaps appear at wind speeds above 7 knots (Beaufort No. 3); spray appears at wind speeds above 17 knots (Beaufort No. 5) (reference 8). The resultant scattering produces a value of $\sigma^{o}(\theta)$ that is effectively constant over the interval Eq. (2), just as in flight over land. However, the horizontal velocity of whitecaps and spray (relative to the "ground") causes a proportional Doppler frequency shift; spray, being wind-borne, has a higher velocity than whitecaps. The component of this horizontal velocity in the direction of the aircraft heading represents the "surface-speed error."

Combination of both types of reflection. Both types of reflection can be included in Eq. (3) if we write

$$\sigma^{o}(\theta) = \sigma^{o}_{1}(\theta) + \sigma^{o}_{2}(\theta) \qquad (4)$$

where subscript 1 represents reflection by smooth waves and subscript 2 represents reflection by whitecaps and spray. Note that $\sigma^{o}_{2}(\theta)$ is a constant.

Error Due to Reflection by Sea

Comparison of Eq. (3) for sea reflection with the corresponding formula for land reflection (where σ^{o} is a constant) leads to the fractional error in the fractional Doppler frequency shift.

$$\frac{\delta\left(\frac{\Delta f}{f}\right)}{\left(\frac{\Delta f}{f}\right)} = \left[\frac{\int_{\theta_o-\alpha}^{\theta_o+\alpha} \sigma^{o}(\theta)\,\sin\theta \cdot d\theta}{\int_{\theta_o-\alpha}^{\theta_o+\alpha} \sigma^{o}(\theta) \cdot d\theta} \Bigg/ \frac{\int_{\theta_o-\alpha}^{\theta_o+\alpha} \sin\theta \cdot d\theta}{\int_{\theta_o-\alpha}^{\theta_o+\alpha} d\theta} \right] - 1 \qquad (5)$$

References 1-4 present radar data on $\sigma^{o}(\theta)$ for various values of θ_o and various sea states. For the interval, Eq. (2), in which $\alpha \ll \theta_o$, the value of σ^{o} can be approximated by an equation of the form

$$\sigma^{o} = A + B\theta \qquad (6)$$

where $B = (d\sigma^{o}/d\theta)_{\theta=\theta_o}$

Substitution of Eq. (6) into Eq. (5) yields

$$\frac{\delta\left(\frac{\Delta f}{f}\right)}{\frac{\Delta f}{f}} = \frac{1}{\tan\theta_o} \cdot \left(1 - \frac{\alpha}{\tan\alpha}\right) \cdot \frac{1}{\sigma^{o}(\theta_o)} \cdot \left(\frac{d\sigma^{o}}{d\theta}\right)_{\theta=\theta_o} \qquad (7)$$

$$\approx \frac{\alpha^{2}}{3\tan\theta_o} \cdot \frac{1}{\sigma^{o}(\theta_o)} \cdot \left(\frac{d\sigma^{o}}{d\theta}\right)_{\theta=\theta_o} \qquad (8)$$

References 1 and 7 indicate that this calibration error can be partially compensated by a land/sea switch on a radar set. Such compensation reduces the probable calibration error by a factor of 2 (or more) through the expedient of inserting a correction equal to the most probable value of the calibration error, corresponding to the most probable sea state.

Error Due to Surface Wind

The calibration error just discussed affects the determination of the velocity v_a of the airplane relative to the sea surface. There remains to be determined the velocity v_s of the sea surface relative to "ground." This latter velocity can be estimated from two data:

(1) observation of the sea state

(2) independently-measured or estimated surface wind velocity.

If (1) shows the sea to have only smooth waves (few or no whitecaps, Beaufort No. less than 4), the velocity v_s of the sea relative to "ground" may be assumed to be zero.

If (1) shows the sea to have numerous whitecaps (Beaufort No. 4 or higher), the velocity v_s of the sea relative to "ground" may be taken to be between $\frac{1}{10}$ and $\frac{1}{5}$ of the surface wind velocity, according to references 1, 2, and 7. Presumably the fraction will be larger at higher surface-wind speeds.

The component of v_s in the direction of aircraft heading, when added to v_a, yields the aircraft ground speed in the direction of aircraft heading.

Calculations of Errors

Radar scattering cross section of the sea surface. Graphs of σ^o against sea state are shown for X-band (3 cm) waves by references 1 and 2 in terms of Beaufort number, and by references 3 and 4 in terms of wind speed. Measurements of reference 4 were on a river instead of open sea. Angle between wind direction at the water surface and the radar-beam or aircraft heading is not stated.

To indicate the order of magnitude of errors that might be encountered, computations will be made for the case

$$\theta_o = 20^o; \quad \alpha=1^o$$

A linear approximation of the form $\sigma^o = A + B\theta$ to the published graphs, at $\theta = 20$ degrees, gives the values of

$$\frac{1}{\sigma^o(\theta_o)} \cdot \left(\frac{d\sigma^o}{d\theta}\right)_{\theta=\theta_o} ,$$

that are listed in Table I. The fractional error $\delta(\Delta f/f)/(\Delta f/f)$ for $\alpha = 1$ degree, computed by Eq. (8), is also listed in Table I and is plotted in Fig. 2.

<u>Visible-light reflection by the sea surface.</u> Because of the sparseness of data on σ^o at radar wavelengths, it is useful also to use optical measurements of wave-surface slope and to derive an equivalent value of $\frac{1}{\sigma^o} \cdot \frac{d\sigma^o}{d\theta}$ therefrom. It is reasonable to assume that radar waves and light waves are reflected in similar manner, since the wave length of either is very short compared to the wavelengths of the sea waves.

References 5 and 6 report on specular reflection of light waves from the sea surface. They show that the statistical distribution of sea-wave slopes is Gaussian to a first approximation, with a smaller variance in the crosswind direction than in the upwind-downwind direction (Fig. 3). If the probability dP that wave slope lies in the interval $d\psi$ is given by

$$dP = p(\psi) \cdot d\psi \tag{9}$$

where

$$p(\psi) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\psi^2}{2\overline{\psi}^2}} \tag{10}$$

and $\overline{\psi}^2$ is the variance of the distribution, then the reflectance $R(\psi_o)$ of the surface at a mean angle ψ_o is proportional to the probability

$$p(\psi_o) \cdot \Delta\psi \tag{11}$$

where $\Delta\psi$ is the very small angle subtended by the transmitter-receiver (antenna) at the sea surface. A linear approximation to the curve of $R(\psi)$ versus ψ, in the vicinity of ψ_o, and in the narrow interval between $\psi_o + \frac{\Delta\psi}{2}$ and $\psi_o - \frac{\Delta\psi}{2}$, yields

$$R(\psi_o) = c_1 \cdot p(\psi_o) \cdot \Delta\psi \qquad (12)$$

$$\left(\frac{dR}{d\psi}\right)_{\psi=\psi_o} = c_1 \cdot \left(\frac{dp}{d\psi}\right)_{\psi=\psi_o} \cdot \Delta\psi \qquad (13)$$

$$\frac{1}{R(\psi_o)} \cdot \left(\frac{dR}{d\psi}\right)_{\psi=\psi_o} = -\frac{\psi_o}{\bar{\psi}^2} \qquad (14)$$

where c_1 is a constant of proportionality that disappears from the final result (14).

The quantity $\frac{1}{R(\psi_o)} \cdot \left(\frac{dR}{d\psi}\right)_{\psi=\psi_o}$ can replace the quantity $\frac{1}{\sigma^o(\theta_o)} \cdot \left(\frac{d\sigma^o}{d\theta}\right)_{\theta=\theta_o}$ in Eq. (8), if $\theta_o = \psi_o$.

Figure 2 includes the errors so computed, using values of $\bar{\psi}$ from Fig. 3. The range of wind speeds covered is the range over which references 5 and 6 consider the data reliable. The maximum wind speed is 15 knots, where whitecaps become numerous; the minimum wind speed, imposed by experimental limitations, is that where $\bar{\psi}$ becomes less than $0.4\ \psi_o$.

Comparison between crosswind and upwind-downwind directions. The only reported data, presented in Fig. 2, that unequivocally indicates a distinction between the crosswind and the upwind-downwind directions of flight, are the light-wave data of references 5 and 6 at 15 knots. The error in crosswind flight is about 1.5 times the error in upwind-downwind flight.

Based on this reported data, we make the following intuitive assumptions concerning radar waves:

(1) At wind speeds higher than 15 knots, where whitecaps and spray provide stronger reflections, the ratio between crosswind and upwind-downwind directions should be closer to unity.

(2) The radar data of references 1, 2, 3, and 4 were probably taken in the upwind-downwind direction.

Numerical example. Table II presents numerical calculations of the errors under the following assumptions:

(1) Aircraft speed is 300 knots

(2) $\theta_o = 20^\circ$, $\alpha = 1^\circ$

(3) Data of references 1 and 3 are used for the upwind-downwind calibration error.

(4) Crosswind calibration error is assumed 1.5 times the upwind-downwind calibration error at wind speeds up to 15 knots; the multiplier is assumed to be 1.25 at 26 knots.

(5) Magnitude of surface-speed error in direction of the surface wind is assumed to be $\frac{1}{8}$ of surface-wind speed at 15 knots and to be $\frac{1}{5}$ of surface-wind speed at 26 knots.

Note that the calibration errors would vary proportionately if aircraft speed were other than 300 knots, but that the surface-speed errors would not change.

SUMMARY OF RESULTS

The error in ground-speed measurement is the sum of a calibration error and a surface-speed error. The absolute magnitude of the calibration error at 300 knots is listed in Table II; this absolute error would vary in proportion to the speed. The surface-speed error, as listed in Table II would not vary with speed.

Analysis of the data in Table II then shows that for the conditions $\theta_o = 20^\circ$, $\alpha = 1^\circ$, airspeed 300 knots, a probable error of less than one percent of the speed would be achieved if one estimated the most probable value of the ground-speed error from Table II and then applied this estimate as a correction.

REFERENCES

1. Fried, W. R.: Principles and Performance Analysis of Doppler Navigation Systems. IRE Trans. on Aeron. Navigational Elec., vol. ANE-4, no. 4, Dec. 1957, pp. 176-196.

2. Berger, F. B.: The Design of Airborne Doppler Velocity Measuring Systems. IRE Trans. on Aeron. Navigational Elec., vol. ANE-4, no. 4, Dec. 1957, pp. 157-175.

3. Wiltse, J. C.; Schlesinger, S. P.; and Johnson, C. M.: Back-Scattering Characteristics of the Sea in the Region from 10 to 50 KMC. IRE Proc., vol. 45, no. 2, Feb. 1957, pp. 220-228.

4. Grant, C. R.; and Yaplee, B. S.: Back-Scattering from Water and Land at Centimeter and Millimeter Wavelengths. IRE Proc., vol. 45, no. 7, July 1957, pp. 976-982.

5. Schooley, A. H.: A Simple Optical Method for Measuring the Statistical Distribution of Water Surface Slopes. J. Opt. Soc. Am., vol. 44, no. 1, Jan 1954, pp. 37-40.

6. Cox, C.; and Munk, W.: Measurement of the Roughness of the Sea Surface from Photographs of the Sun's Glitter. J. Opt. Soc. Am., vol. 44, no. 11, Nov. 1954, pp. 838-850.

7. Gray, Trevor: Airborne Doppler Navigation Techniques. Radar Techniques for Detection Tracking and Navigation. AGARDograph-100, W. T. Blackband, ed., Gordon and Breach Science Publ., 1966, pp. 213-234.

8. Bowditch, Nathaniel: American Proactical Navigator. U. S. Navy Hydrographic Office, 1966, p. 1059.

9. McMahon, F. A.: The AN/APN-81 Doppler Navigation System. IRE Trans. on Aeron. Navigational Elec., vol. ANE-4, no. 4, Dec. 1957, pp. 202-211.

Table I

Calibration Error Over Sea Surface,
Relative to Land Surface Calibration,
For $\theta_o = 20^\circ$, $\alpha = 1^\circ$

Reference	Beaufort Number	Surface-wind speed, knots	$\frac{1}{\sigma^o(\theta_o)} \cdot \left(\frac{d\sigma^o}{d\theta}\right)_{\theta=\theta_o}$ Degree^{-1}	$\frac{\delta\left(\frac{\Delta f}{f}\right)}{\frac{\Delta f}{f}}$
1	1	1-3	-0.40	-0.0064
	2	4-6	-0.30	-0.0048
	3	7-10	-0.26	-0.0042
	4	11-16	-0.24	-0.0039
2	1	1-3	-0.32	-0.0051
	2-3	7	-0.22	-0.0035
3	--	16	-0.22	-0.0035
	--	26	-0.13	-0.0021
4	--	0-5	-0.30	-0.0048
	--	5-10	-0.40	-0.0064
	--	10-15	-0.20	-0.0032
	--	15-20	-0.23	-0.0037

Table II

Estimated Ground Speed Errors

$\theta_o = 20^\circ$, $\alpha = 1^\circ$

Beaufort No.	Sea State	Surface-wind speed, knots	Type of error	Error in stated flight direction, knots		
				Upwind	Downwind	Crosswind
1	smooth waves	2	Calibration[a]	-1.9	-1.9	-2.9
			Surface-speed	0	0	0
			Total	-1.9	-1.9	-2.9
4	whitecaps	15	Calibration[a]	-1.1	-1.1	-1.7
			Surface-speed	1.9*	-1.9*	0
			Total	0.8	-3.0	-1.7
6	spray	26	Calibration[a]	-0.6	-0.6	-0.75*
			Surface-speed	5.2*	-5.2*	0
			Total	4.6*	-5.8*	-0.75*

*Two significant figures are not warranted.

[a] At 300 knots aircraft speed.

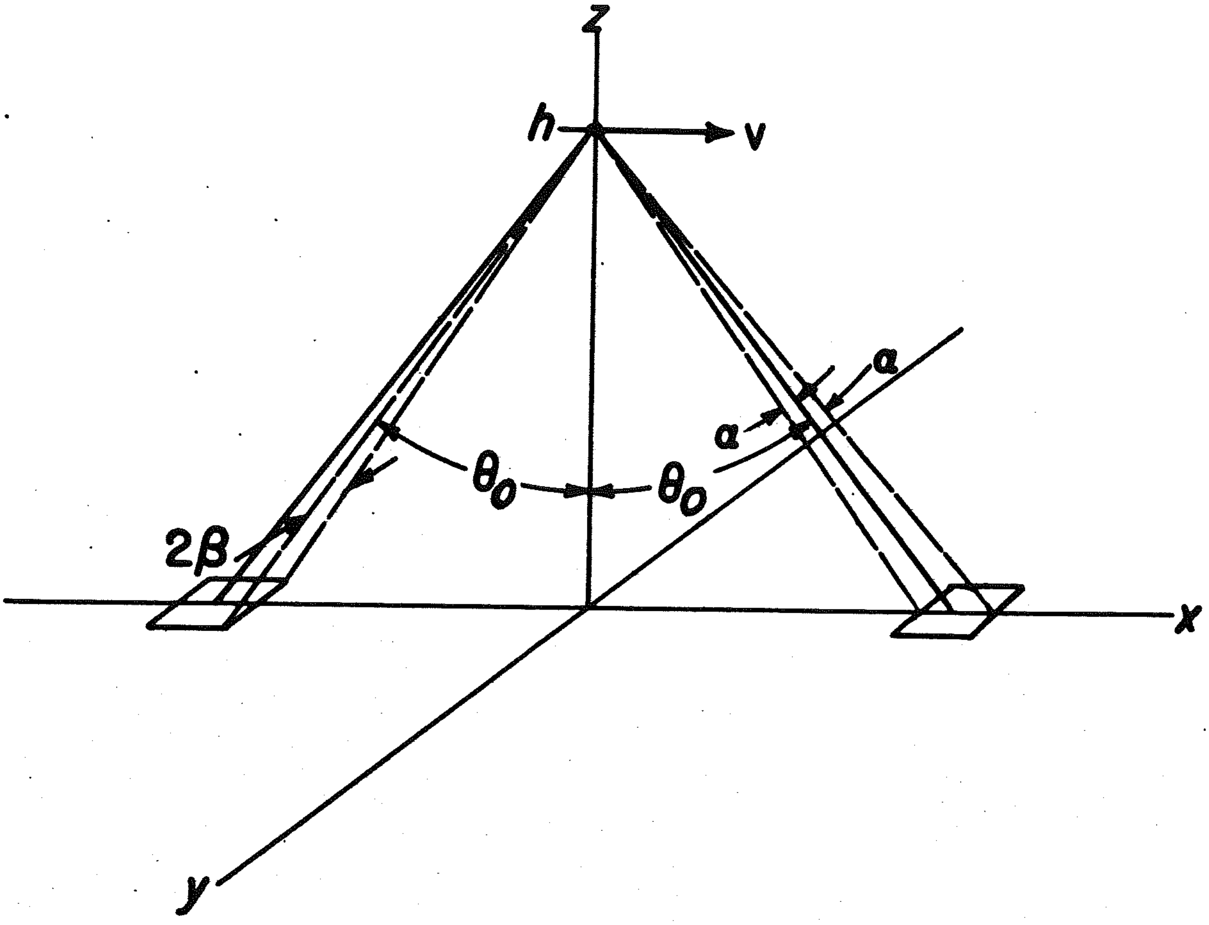

Figure 1. Doppler-radar beams

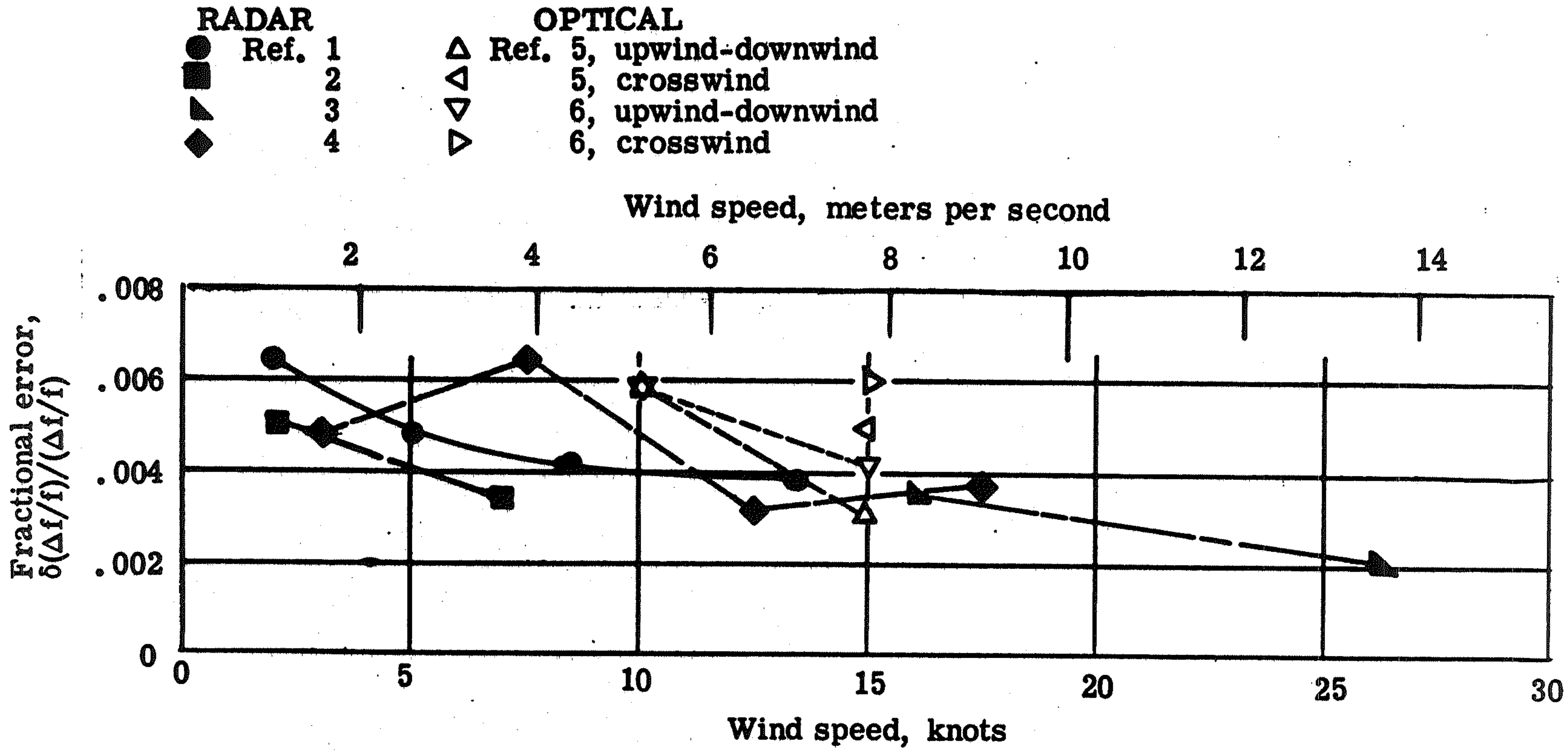

Figure 2. Error of Doppler frequency shift. $\theta_o = 20^\circ$, $\alpha = 1^\circ$

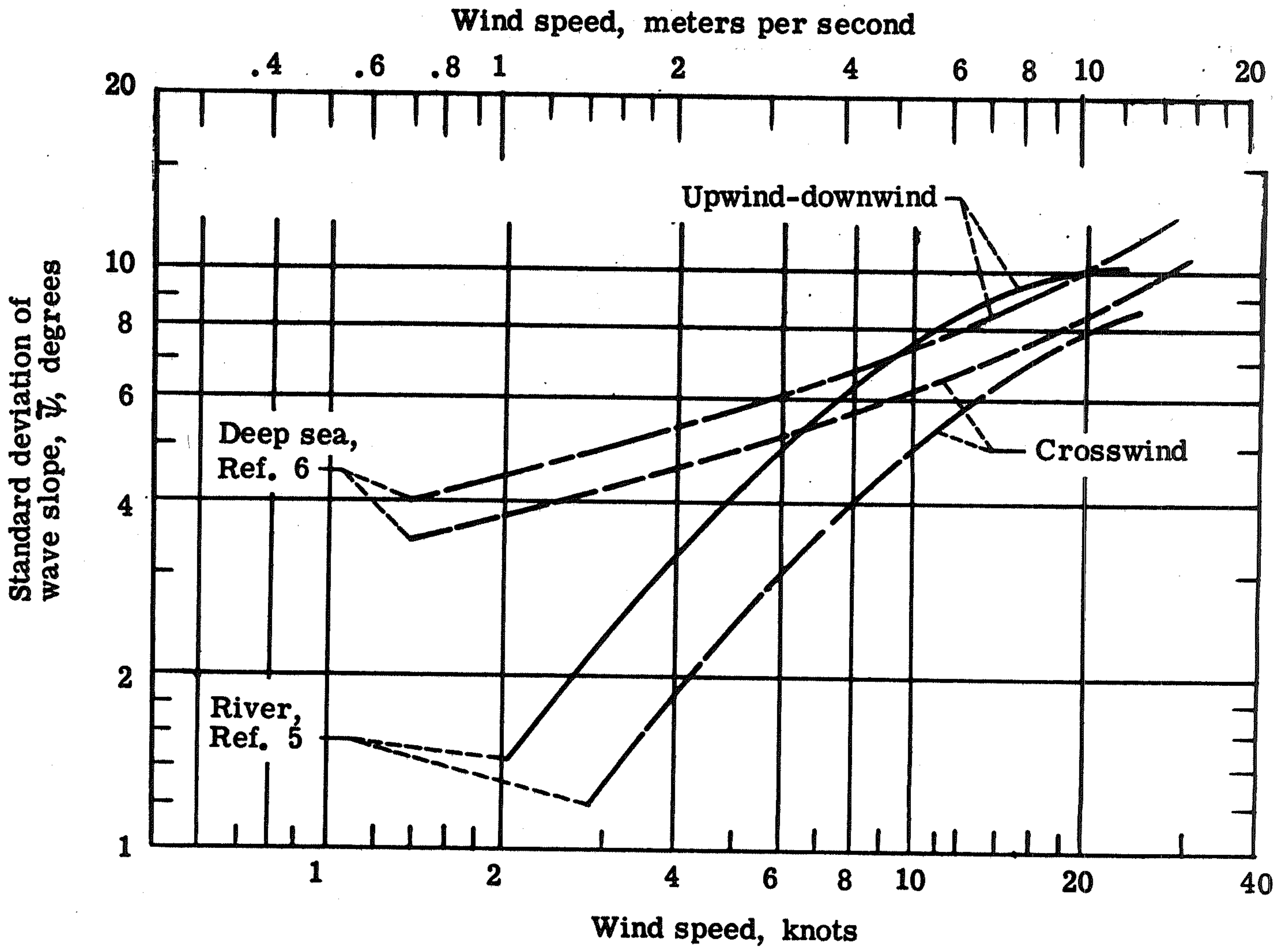

Figure 3. Optically-measured wave slope dispersion