NASA TECHNICAL NOTE

NASA TN D-5842

c. 1

NASA TN D-5842

STABILITY OF AN ELECTRON BEAM INJECTED INTO SPACE

by R. Krishan Jaggi

Manned Spacecraft Center
Houston, Texas 77058

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JULY 1970

1. REPORT NO. NASA TN D-5842	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT
4. TITLE AND SUBTITLE STABILITY OF AN ELECTRON BEAM INJECTED INTO SPACE		5. REPORT DATE July 1970
		6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) R. Krishan Jaggi, MSC		8. PERFORMING ORGANIZATION REPORT NO. MSC S-229
9. PERFORMING ORGANIZATION NAME AND ADDRESS Manned Spacecraft Center Houston, Texas 77058		10. WORK UNIT NO. 879-10-01-01-72
		11. CONTRACT OR GRANT NO.
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration Washington, D. C. 20546		13. REPORT TYPE AND PERIOD COVERED Technical Note
		14. SPONSORING AGENCY CODE
15. SUPPLEMENTARY NOTES		

16. ABSTRACT

This paper is a study of the stability of an electron beam injected into space, with the beam assumed to be confined to a cylindrical region and assumed to be in equilibrium under its electric charge and its thermal pressure. A dispersion equation, derived for surface perturbations symmetrical about the axis of the beam, shows that the beam is stable.

17. KEY WORDS (SUPPLIED BY AUTHOR)	18. DISTRIBUTION STATEMENT
· Artificial Aurora · Ionosphere · Stability · Electron Beam	Unclassified - Unlimited

19. SECURITY CLASSIFICATION (THIS REPORT)	20. SECURITY CLASSIFICATION (THIS PAGE)	21. NO. OF PAGES	22. PRICE*
None	None	12	$3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

STABILITY OF AN ELECTRON BEAM INJECTED INTO SPACE

By R. Krishan Jaggi
Manned Spacecraft Center

SUMMARY

This paper is a study of the stability of an electron beam injected into space, with the beam assumed to be confined to a cylindrical region and assumed to be in equilibrium under its electric charge and its thermal pressure. A dispersion equation, derived for surface perturbations symmetrical about the axis of the beam, shows that the beam is stable.

INTRODUCTION

The subject of this paper is the stability of a beam of energetic electrons injected into the ionosphere. A beam that consisted of 10-keV electrons was fired from a rocket at an altitude of approximately 270 kilometers above Wallops Island, Virginia (ref. 1). The beam was pulsed from an accelerator to yield pulses of 0.1- and 1-second durations. The auroras produced by the 1-second pulses were photographed by ground-based cameras. From the length of the observed rays and from the fact that the beam power penetrated the atmosphere to an altitude of approximately 100 kilometers, it may be concluded that there were no serious instabilities in the beam.

Some of the instabilities of an electron beam have been discussed by Tidman (ref. 2), Thompson (ref. 3), and Jones (ref. 4). Tidman studied the instability of whistler modes by using a Maxwellian distribution for ambient electrons with a hump caused by the high-energy beam. Thompson studied the instabilities in electrostatic modes propagating at an angle to the magnetic field. In all the aforementioned work, the beam charge was not taken into account. Furthermore, the beam was assumed to occupy infinite space.

Although the previously mentioned instabilities of an electron beam are important, it would be premature to conclude that the beam will be destroyed, because the electric field produced by the beam charge may suppress some of the instabilities. In this paper, the beam is considered to be confined to a cylindrical region of radius r_0. The problem to be studied is the stability of perturbations on the surface of such a beam. The electrostatic and whistler wave instabilities that may arise from the interaction of the beam with the ionosphere can produce a spectrum of perturbations in the beam surface.

The author wishes to thank W. N. Hess and J. D. Pierson for valuable assistance. The author is also grateful to the National Research Council and the National Academy of Sciences for the award of a senior postdoctoral resident research associateship during which this work was done.

SYMBOLS

A, C, D, G	constants of integration
$\vec{B}$	magnetic field vector
c	velocity of light
E	electric field
$\vec{E}$	electric field vector
E_0	electric field at the beam surface
E_r, E_z	r and z components of $\vec{E}$ inside the beam
E_r', E_z'	r and z components of $\vec{E}$ outside the beam
e	electron charge
I_0	zero-order Bessel function of the first-kind, imaginary argument
I_1	first-order Bessel function of the first-kind, imaginary argument
$K = \sqrt{k^2 + \left(\omega_p^2/c^2\right)}$	
$K' = \sqrt{k^2 + \left[\left(\omega_p'\right)^2/c^2\right]}$	
K_0	zero-order Bessel function of the second-kind, imaginary argument
K_1	first-order Bessel function of the second-kind, imaginary argument
k	wave number
l	azimuthal wave number

m	electron mass
n	electron density of the beam
n'	electron density outside the beam
$\bar{n} = \frac{n + n'}{2}$	
p_0	pressure of the electron gas at the beam axis
p_1	pressure perturbation inside the beam
p_1'	pressure perturbation outside the beam
p_e	pressure of the electron gas
r	perturbed radius
r_0	unperturbed radius
t	time
$\vec{u}$	macroscopic velocity of the electron gas
u_r, u_z	r and z components of $\vec{u}$ inside the beam
u_r', u_z'	r and z components of $\vec{u}$ outside the beam
z	coordinate along the beam axis
δ	amplitude of the perturbation
Θ	azimuthal coordinate
$\imath = \sqrt{-1}$	
Φ	electric potential
ω	perturbation frequency
$\omega_p = \sqrt{4\pi ne^2/m}$	

$\omega_p' = \sqrt{4\pi n' e^2/m}$

$[x]$	the difference between x on the two sides of the beam surface

Operator:

div( )	divergence
∇( )	gradient
Curl( )	$\nabla \times$ ( )

AN ASSUMED EQUILIBRIUM

It is assumed that the cylindrical beam is in equilibrium under its electric field and its thermal pressure. If the electron density n is uniform inside radius r_0, the electric field is radial and is given by

$$E = -E_0 \frac{r}{r_0} \text{ for } r \leq r_0 \tag{1}$$

and

$$E = -E_0 \frac{r_0}{r} \text{ for } r \geq r_0 \tag{2}$$

where $E_0 = 2\pi n e r_0$. For equilibrium, if it is assumed that the density of the ambient medium inside $r = r_0$ is negligible, the pressure p_e should vary as

$$p_e = p_0\left(1 - \frac{r^2}{r_0^2}\right) \tag{3}$$

where $p_0 = \pi n^2 e^2 r_0^2$. Outside the beam, it is assumed that the ambient medium is at zero temperature. These assumptions are made for mathematical simplicity, and the stability of the configuration is not expected to change if the outside medium is assumed to be at a definite temperature and if a positive surface charge is assumed at r_0 in order to satisfy the pressure balance. In the actual case, the beam is observed to expand. Assumption of a cylindrical beam in equilibrium at radius r_0 implies that the expansion velocity is small compared to the phase velocity of the perturbations.

THE DISPERSION EQUATION

The equations governing the motion of electrons are

$$nm \frac{d\vec{u}}{dt} = -ne\vec{E} - \vec{\nabla} p_e \tag{4}$$

$$\text{div } \vec{u} = 0 \tag{5}$$

$$\text{div } \vec{E} = -4\pi ne \tag{6}$$

$$\text{Curl Curl } \vec{E} = -\frac{4\pi}{c^2} ne \frac{\partial \vec{u}}{\partial t} \tag{7}$$

Use of equation (5) assumes the electron gas is incompressible. This assumption is justified in the present case if the phase velocity of the perturbations is small compared to the speed of sound in the beam. On physical grounds, the compressibility is expected to be important for radial pulsations that cannot take place in an incompressible medium. For the type of perturbations studied, compressibility is not required. Equation (7) is derived from Curl $\vec{B} = -(4\pi/c)ne\vec{u}$ and Curl $\vec{E} = -(1/c)(\partial\vec{B}/\partial t)$. In equation (7), the displacement current is neglected, which imposes the restriction of using only waves with phase velocity less than the velocity of light. The perturbations in the beam surface may be represented by

$$r = r_0 + \delta e^{\imath\omega t + \imath kz + \imath 1\Theta} \tag{8}$$

The present analysis is confined to the symmetrical case of $1 = 0$.

If the r and z components of the perturbation velocity and electric field are denoted by u_r, u_z, E_r, and E_z (fig. 1) and if the perturbation in the pressure is denoted by p_1, the solution of the linearized equations can be written as

$$p_1 = CI_0(kr) \qquad r < r_0 \tag{9a}$$

$$u_z = AI_0(Kr) \qquad r < r_0 \tag{9b}$$

$$u_r = \frac{lk}{K} AI_1(Kr) \qquad r < r_0 \tag{9c}$$

$$E_z = -\frac{k}{ne}\left[kCI_0(kr) + nm\omega AI_0(Kr)\right] \tag{9d}$$

$$E_r = -\frac{k}{ne}\left[CI_1(kr) - \frac{nm\omega}{K} AI_1(Kr)\right] \tag{9e}$$

where $K = \sqrt{k^2 + \left(\omega_p^2/c^2\right)}$; $\omega_p = \sqrt{4\pi ne^2/m}$; I_0 and I_1 are zero- and first-order Bessel functions of the imaginary argument, respectively; and C and A are constants of integration. If, for the outside medium, n' denotes the electron density, then the solution of the linearized equations can be written as

$$p_1' = GK_0(kr) \tag{10a}$$

$$u_z' = DK_0(K'r) \tag{10b}$$

$$u_r' = \frac{lk}{K'} K_1(K'r) \tag{10c}$$

$$E_z' = -\frac{l}{n'e}\left[kGK_0(kr) + n'm\omega DK_0(K'r)\right] \tag{10d}$$

$$E_r' = \frac{k}{n'e}\left[GK_1(kr) - \frac{n'm\omega}{K'} DK_1(K'r)\right] \tag{10e}$$

Ambient quantities	Perturbed quantities
Outside	
Density = n	Pressure = p_1'
Pressure = 0	Electric field = $\left(E_r', 0, E_z'\right)$
$E_{radial} = E_0\left(r_0 / r\right)$	Velocity = $\left(u_r', 0, u_z'\right)$
Inside	
Density = n	Pressure = p_1
Pressure = $p_0\left[1 - \left(r^2 / r_0^2\right)\right]$	Electric field = $\left(E_r, 0, E_z\right)$
$E_{radial} = E_0\left(r / r_0\right)$	Velocity = $\left(u_r, 0, u_z\right)$

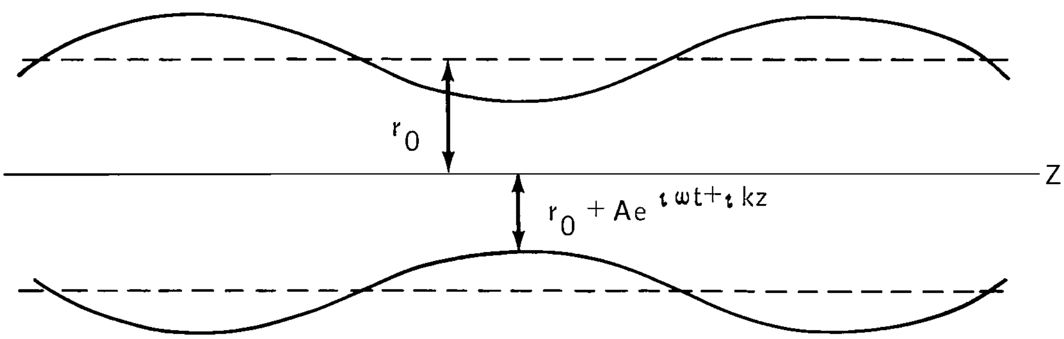


Figure 1. - Ambient and perturbed quantities inside and outside the beam plasma.

where $K' = \sqrt{k^2 + \left[\left(\omega_p'\right)^2 / c^2\right]}$; $\omega_p' = \sqrt{4\pi n' e^2 / m}$; K_0 and K_1 are zero- and first-order Bessel functions of the imaginary argument, respectively; and G and D are constants of integration. Four constants of integration (A, C, D, and G) are used, and four boundary conditions are required in order to obtain the dispersion equation that gives ω in terms of k. The continuity of the normal components of both $\vec{u}$ and $\vec{E}$ gives

$$D = -A \frac{K'}{K} \frac{I_1\left(Kr_0\right)}{K_1\left(K'r_0\right)} \tag{11}$$

and

$$G = -C \frac{n'}{n} \frac{I_1(kr_0)}{K_1(kr_0)} \tag{12}$$

The continuity of the tangential component of E gives

$$C = -A \frac{r_0 nm\omega K_1(kr_0)}{K_1(K'r_0)} \left[K_1(K'r_0) I_0(Kr_0) + \frac{K'}{K} I_1(Kr_0) K_0(K'r_0)\right] \tag{13}$$

The pressure balance equation for electrons requires some explanation. The equation of motion for electrons is

$$nm \frac{d\vec{u}}{dt} = -ne\vec{E} - \vec{\nabla} p_e = n_e \vec{\nabla}\Phi - \vec{\nabla} p_e \tag{14}$$

where Φ is the electric potential defined by $\vec{E} = -\vec{\nabla}\Phi$. Integration across a boundary gives rise to the continuity of $-\bar{n}e\Phi + p_e$, which can be written in the conventional notation

$$\left[-\bar{n}e\Phi + p_e\right] = 0 \tag{15}$$

In equation (15), $\bar{n}$ is the average electron density on the boundary surface and is represented by the arithmetic mean of n and n'. This boundary condition and equations (11), (12), and (13) yield the dispersion equation

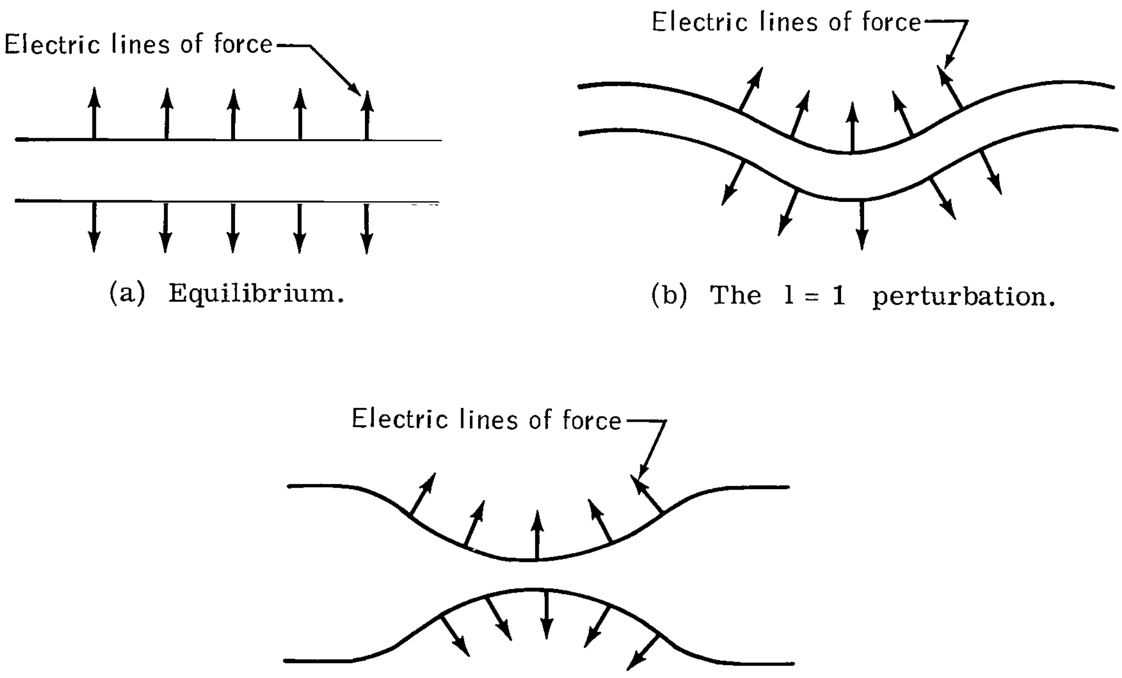
$$\omega^2 = \frac{\pi ne^2 (n + n') I_0(kr_0) K_1(K'r_0)}{Knm\left[I_0(kr_0) K_1(kr_0) + \frac{n'}{n} I_0(kr_0) K_0(kr_0)\right] \left[I_0 K_1 + \frac{K'}{K} I_0 K_0\right]} \tag{16}$$

where $I_0 = I_0(Kr_0)$; $I_1 = I_1(Kr_0)$; $K_0 = K_0(K'r_0)$; and $K_1 = K_1(K'r_0)$.

The right-hand side of equation (16) is a positive quantity; that is, ω is real for all wave numbers k. Therefore, no instability occurs in the equilibrium configuration considered.

DISCUSSION

The physical cause of stability can be understood by examining figures 2(a) to 2(c). Figure 2(a) shows the equilibrium configuration, and figures 2(b) and 2(c) show the $l = 1$ and $l = 0$ perturbations, respectively. From figures 2(b) and 2(c), it is clear that on the concave side the electric lines of force "crowd" together and that on the convex side the lines of force are separated from each other. Thus, additional forces are produced that evidently decrease the bending illustrated in figures 2(b) and 2(c). Mathematically stated, the magnitude of the electric field that results from the charge on the beam varies inversely with the distance of the electric field outside the beam. A perturbation that moves the beam surface inward provides greater electric force than a perturbation that moves the beam surface outward (fig. 2(b)). That is, since the direction of the electric force is outward from the beam, the resultant of the forces on the concave and convex sides in figure 2(b) is a force that decreases the bending.



(a) Equilibrium.

(b) The $l = 1$ perturbation.

(c) The $l = 0$ perturbation.

Figure 2. - A column of the charged beam in equilibrium and perturbed. The arrows indicate the electric lines of force.

CONCLUSION

From analysis of the stability of an electron beam injected into space, with the beam assumed to be confined to a cylindrical region and assumed to be in equilibrium under its electric charge and its thermal pressure, it is concluded that the beam charge produces a stabilizing effect.

Manned Spacecraft Center
National Aeronautics and Space Administration
Houston, Texas, January 27, 1970
879-10-01-01-72

REFERENCES

1. Hess, Wilmot N.: Generation of an Artificial Aurora. Science, vol. 164, no. 3887, June 27, 1969, pp. 1512-1513.

2. Tidman, D. A.; Helliwell, R. A.; Hess, W. N.; et al.: Report of Ad Hoc Committee on Environment Modification Experiments in Space. The Institute for Fluid Dynamics and Applied Mathematics, Univ. of Maryland, Mar. 1968.

3. Thompson, J. R.: A Brief Look at Plasma Beam Instabilities Relative to the Wallops Island Electron Accelerator Rocket Experiment. CPPT-7 Center for Plasma Physics, Thermonuclear Research, Univ. of Texas at Austin, Apr. 1968.

4. Jones, T. W.: Calculations of Instabilities Produced by a Beam of 40 kev Electrons in the Ionosphere. M.S. Thesis, Univ. of Minnesota, Mar. 1969.

NASA-Langley, 1970 —— 13 S-229

TIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546

OFFICIAL BUSINESS

FIRST CLASS MAIL

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AN
SPACE ADMINISTRATION

03U 001 38 51 3DS 70185 00903
AIR FORCE WEAPONS LABORATORY /WLOL/
KIRTLAND AFB, NEW MEXICO 87117

ATT E. LOU BOWMAN, CHIEF,TECH. LIBRARY

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546