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BODIES OF CONSTANT STRESS EXPERIENCING FORCES OF ORBITAL FLIGHT

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16. ABSTRACT

This report attempts to analyze bodies of constant stress in orbital flight. It provides a mathematical formulation for the interrelationships between orbital parameters and variable cross section beams. A few specific examples are given.

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FOREWORD

Presented with ideas by K. E. Tsiolkovski and Yu. Artsutanov, Mr. Pierce undertook the following investigation during his three-month stay as a cooperative student in the Orbital Systems Group. A follow-up phase of Mr. Pierce's findings would analyze possible, practical implications.

Georg von Tiesenhausen

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DEFINITION OF SYMBOLS

Symbol	Definition
A	any area
A_o	initial area
C	a constant of integration
dA	differential area
dF	differential force
dx	differential distance
e	natural log base
F	force
F_c	centrifugal force on beams
F_g	gravitational force on beams
g	variable force of gravity
g'	force of gravity at earth's surface
K	gravitational constant
km	kilometers
ln	natural logarithm
m	mass
R	ratio of x/x_s
r	radius of orbit
x	distance to area A
x'	radius of earth

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
x_o	distance to area A_o
ρm	mass density
σ	stress
ϕ	the known exponential in equation (9)
ω	angular velocity

TECHNICAL MEMORANDUM X-64532

BODIES OF CONSTANT STRESS EXPERIENCING FORCES OF ORBITAL FLIGHT

SUMMARY

This report presents preliminary data accumulated from a study of the cross-sectional areas of symmetrical bodies in which a constant stress is maintained while experiencing the forces associated with orbital flight. An analysis is made of the forces associated with beams or cables extended from a satellite orbiting earth in a circular orbit. An investigation of the cross-sectional areas necessary to maintain constant stress throughout the beams is made, and the area relationships are resolved for different altitudes using a type of steel for the material of the beams.

A discussion of the problem is given; and a mathematical analysis which produces a relationship between the cross-sectional areas of the beams and distances from the center of the earth is provided. The mathematical data is supplemented with diagrams and several graphs.

INTRODUCTION

For many years, man has been making studies concerned with the phenomena of orbiting bodies. Many concentrated efforts have been made in research connected with the forces which act on a body in orbit about earth, and these forces primarily made up of centrifugal and gravitational forces have been analyzed closely. The centrifugal force depends upon the velocity of a body and the orbital radius; but gravitational force depends only on the radius of orbit or the altitude of the body.

One area of study on which little can be found is the analysis of bodies within which a constant stress is maintained while orbiting the earth in a circular path. In the case of a relatively small body in orbit, there is no apparent difference in the gravitational and centrifugal forces. For all practical purposes, these two forces are equal throughout the body. If a body several kilometers in length were placed in orbit, a difference in the centrifugal and gravitational forces along the structure could be analyzed and the resulting stress differences noted. In this paper, two beams are extended over very great distances from a hypothetical satellite, and the resulting stress differences can be easily seen. When the forces of tension, gravitational and centrifugal forces, on a beam act along the longitudinal axis of the beam and when there are no other acting forces, the stress is a function of the location of the cross-sectional areas of the beam. This force of tension on the orbiting beams varies because of the differences in the gravitational and centrifugal forces acting at different points along the beams. In order for the stress to be a constant while the tension in a beam varies, the cross-sectional areas must vary along the beam; it is with these areas that this paper deals.

REQUIREMENTS AND CONCEPTS

The basic problem to be analyzed is one of maintaining a constant stress within a body radially extended from a satellite which is in a circular orbit above the earth's equator. The circular orbit is taken to be in the plane of the equator and the equatorial radius is used as the radius of the earth. In the case of a long structure in a circular orbit, the body experiences two non-uniform forces – gravitational force and centrifugal force. The two forces are non-uniform in that they vary at different points along the structure. The gravitational force decreases as the distance from the earth's center increases, and the centrifugal force, since the angular velocity must be the same along the orbiting body, increases as the distance from the earth increases. These forces are exactly opposite in nature; the gravitational force of the earth tends to pull the body toward the center of the earth, and the centrifugal force (associated with the body's velocity) tends to push away from the earth. When these forces are in equilibrium, a satellite will remain in orbit with the orbital altitude depending upon the velocity of the satellite.

The bodies to be studied are two infinitely extendable beams which are mounted on a hypothetical satellite orbiting the earth. The beams are oriented on the satellite with one extended toward the center of the earth, and the other oppositely extended away from the earth, the most stable position for the long beams. The force of gravity is greater than the centrifugal force on the beam which extends toward earth and it is pulled toward the center of the earth. At the same time, the beam extended from earth has a greater centrifugal force acting on it which tends to pull it away from earth.

These beams are extended in such a way that the center of mass is maintained as a fixed point in the satellite which follows the orbital path. For the center of mass to be a constant point, the mass of the two beams must be equal. Maintaining a constant stress within the two beams imposes a second condition on the masses of the beams and results in a difference in the cross-sectional areas along the beams. The two beams have slightly different characteristics because of the difference in the dominating force on each beam; the centrifugal force dominates in the beam extended away from earth, and the gravitational force dominates the other. For both beams the cross-sectional area of a beam section must decrease as the section moves farther away from the satellite. As the section of beam moves away from the satellite, the force per unit of mass increases. The reason for this in the case of the beam extended toward earth is that the gravitational force increases and centrifugal force decreases. This results in a greater net force pulling on the sections of beam nearer earth; the closer the beam moves toward earth the greater this force per unit of mass becomes. If the stress is to remain a constant, the mass must decrease with the increasing force; and, therefore, the cross-sectional area of the beam must decrease. This also holds for the beam extended from earth; but the centrifugal force is dominant in that case.

The problem dealt with in this paper is the analysis of the relationship of the cross-sectional areas along the beams necessary to establish constant stress. A mathematical relationship is derived for the areas along the beams, and it is applicable to any orbital altitude of the satellite and to any type of material used for the beams. The problem of maintaining a constant center of mass, which deals with integrating the mass of one beam over different lengths and finding the lengths of the other beam which give equal masses, is not covered in this paper. This problem is discussed above only because it is an important criterion of the constant stress problem.

MATHEMATICAL ANALYSIS

The mathematical investigation of the beams begins by looking at a small cross-section of one of the beams. In this analysis the section is taken from the beam nearer earth. The section is taken small enough so that the cross-sectional area is assumed a constant, A. The length of this section is taken to be dx (Fig. 1). The force acting on this section of beam, dF, is the difference in the gravitational and the centrifugal forces. The force on this section is also given by the derivation of the following stress equation:

$$\sigma = \frac{F}{A}$$

$$F = \sigma A \tag{1}$$

Since one of the primary conditions of this problem is that stress remain a constant, the derivation of equation (1) would be as follows:

$$dF = \sigma \, dA$$

The gravitational force acting on this beam section is simply a product of the mass of the section and force of gravity at the section.

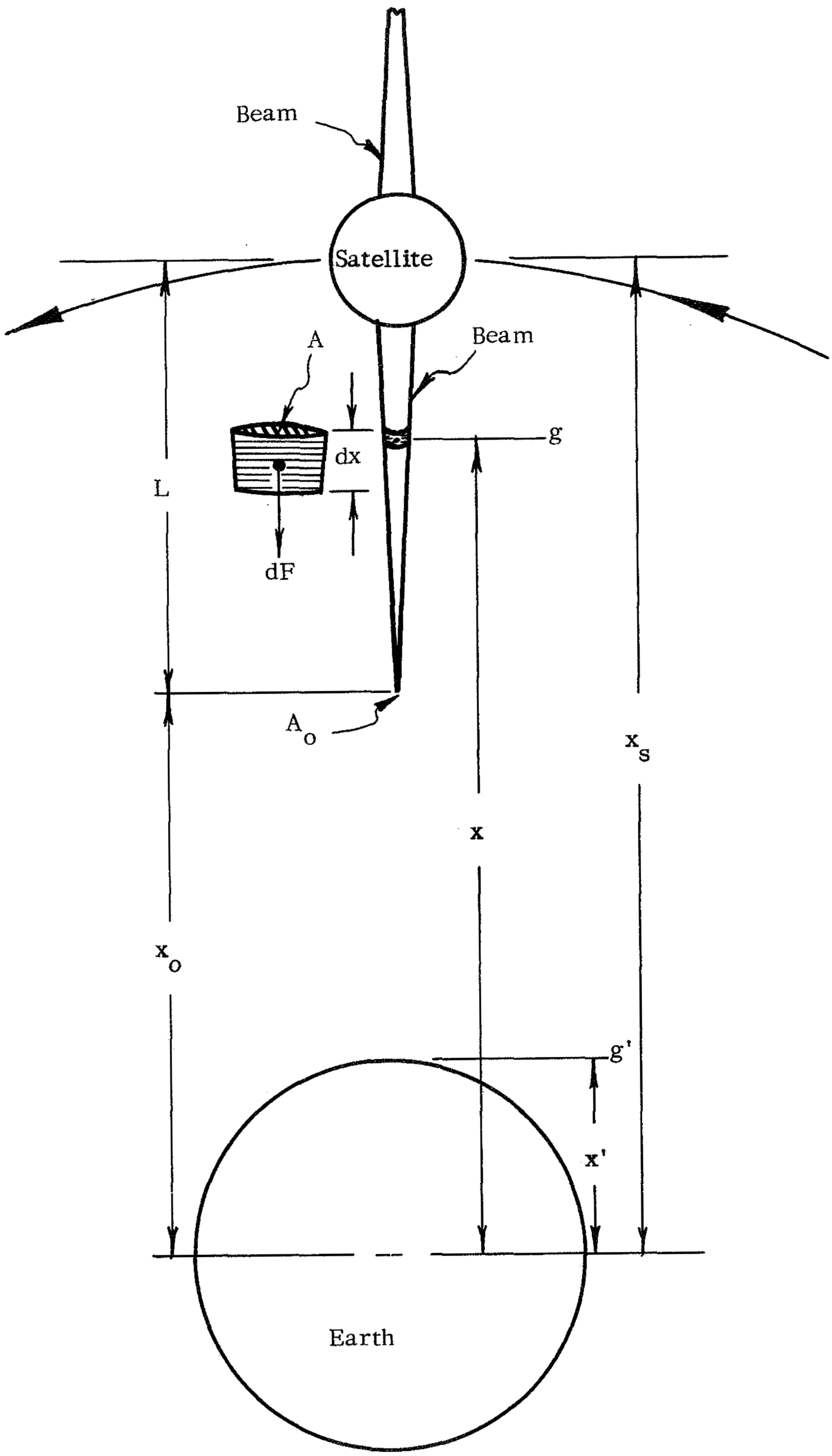

Figure 1. Mathematical diagram of orbiting satellite and beams.

$$F_g = mg$$

$$m = \rho m\, A\, dx$$

$$dF_g = \rho m\, A\, dx\, g \tag{2}$$

The gravitational force at the section is a function of the distance from the earth's center. To find this force of gravity, the following equation using a known distance and gravitational force to establish a constant is used. The value of g becomes smaller as the distance, x, increases.

$$g'(x')^2 = g(x)^2$$

$$g = \frac{g'(x')^2}{x^2}$$

$$g'(x')^2 = K = \text{constant}$$

$$g = \frac{K}{x^2}$$

This value for g is then substituted into equation (2).

$$dF_g = \rho m\, A\, dx \cdot \frac{K}{x^2} \tag{3}$$

The centrifugal force acting on this beam section is a function of its mass, its angular velocity, and its radius from the earth's center. This radius is also given by the same variable, x, used above for the gravitational force.

$$F_c = m\omega^2 r = m\omega^2 x$$

$$m = \rho m\, A\, dx$$

$$dF_c = \rho m\, A\, dx \cdot \omega^2 \cdot x \tag{4}$$

Since the beams are attached to the orbiting satellite, they must have the same angular velocity as that of the satellites. For a satellite orbiting at an altitude of x_s the following equation is applicable:

$$m x_s \omega^2 = mg$$

$$x_s \omega^2 = g$$

$$x_s \omega^2 = \frac{K}{x_s^2}$$

$$\omega^2 = \frac{K}{x_s^3} \tag{5}$$

After equation (5) is substituted into equation (4), the centrifugal force is given by the following equation:

$$dF_c = \rho m\, A\, dx \cdot \frac{K}{x_s^3} \cdot x \tag{6}$$

The total force acting on the beam section is the difference of these two forces. Equation (6) is subtracted from equation (3) and this difference is integrated to give the ratio of areas along the beam. The formula holds true for either of the two beams since the variables will apply equally well for either beam.

$$dF = dF_g - dF_c$$

$$dF = \rho m\, A\, dx \cdot \frac{K}{x^2} - \rho m\, A\, dx \cdot x \cdot \frac{K}{x_s^3}$$

$$\sigma\, dA = \rho m\, A\, dx \cdot \frac{K}{x^2} - \rho m\, A\, dx \cdot x \cdot \frac{K}{x_s^3}$$

$$\int \frac{dA}{A} = \frac{\rho m K}{\sigma} \int \frac{dx}{x^2} - \frac{x\,dx}{x_s^3}$$

$$\ln A = \frac{\rho m K}{\sigma}\left(-\frac{1}{x}-\frac{x^2}{x_s^3}\right)+\ln C$$

$$\frac{A}{C} = e^{-\left[\frac{K\rho m}{\sigma}\left(\frac{1}{x}-\frac{x^2}{2x_s^3}\right)\right]} \tag{7}$$

So that a ratio of areas along the beam may be established, the area, A_o, and distance, x_o, are substituted into equation (7).

$$\frac{A_o}{C} = e^{-\left[\frac{K\rho m}{\sigma}\left(\frac{1}{x_o}+\frac{x_o^2}{2x_s^3}\right)\right]} \tag{8}$$

By dividing equation (8) into equation (7), the following ratio is given:

$$\frac{A}{A_o} = \left[e^{\left(\frac{K\rho m}{\sigma}\frac{1}{x_o}+\frac{x_o^2}{2x_s^3}\right)}\right]\left\{e^{-\left[\frac{K\rho m}{\sigma}\left(\frac{1}{x}+\frac{x^2}{2x_s^3}\right)\right]}\right\} . \tag{9}$$

To simplify equation (9), let ϕ symbolize the first exponential which can be easily resolved once an orbiting altitude and x_o are assumed, and by setting x/x_s equal to R.

$$\frac{A}{A_o} = \phi\, e^{-\left[\frac{K\rho m}{\sigma}\left(\frac{1}{R}+\frac{R^2}{2}\right)\right]} \tag{10}$$

This is the final form of the exponential relationship which describes the areas of the two beams. The relationship is in the form of a ratio; and after a distance, x_o, and an orbital altitude is assumed, any area of the beam from the center of the earth to infinity can be compared with the initial area. Since this relationship is in the form of a ratio, the relative size of the beams depends on the size of this initial area chosen.

DISCUSSION OF RESULTS

The mathematical equation derived in the preceding section describes the cross-sectional areas based upon distances from the center of the earth and type of material used for the beams. The strength of material plays as important a role in this relationship as does the orbiting altitude of the satellite with beams. Once the material is chosen, the mass density and stress strength of the material are applied to equation (10). The stress of a material is taken as a value under the yielding strength depending on the desired safety factor.

Figures 2 through 5 show the data assembled from beams composed of a grade of steel with a mass density of 7.85 g/cm^3 and a stress of 200 000 psi. The ratio of any area, A, to an initial area, A_O, is plotted against the ratio, $x/x_S m$, which gives the distance of A from the earth's center. The ratio of areas is a maximum at the point on the graph at which x/x_S is equal to one; this is the point on the graph which represents the area, A, that is attached to the satellite at orbital altitude. The ratio of the areas then decreases on both sides of this maximum point – one side for the beam near earth and one for the beam away from earth. By looking closely at the graphs, the difference in the beam nearer earth and the one farther from earth can be seen; the area of the beam nearer earth decreases at a faster rate as it moves away from the satellite than the area of the beam directed away from earth. This substantiates the difference postulated in the earlier discussion.

Four different orbiting altitudes, which are selected as representative of the situation, are shown in Figures 2 through 5. For each altitude, a convenient initial, x_O, is used to give approximately equal maximum values of A/A_O. From this it should be noted that the cross-sectional area of the beams decreases at a faster rate for lower altitudes than for the higher altitudes. This can be seen from the fact that the values for x_O are nearer the satellite for the lower altitudes to give approximately equal maximum values of A/A_O for the other altitudes. The four orbiting altitudes used in these graphs are 10 000 km, 20 000 km, 30 000 km, and 42 172 km (synchronous orbit). These altitudes are over a broad enough range to give an adequate indication of the differences in the cross sections established within the beams.

CONCLUSIONS

The mathematical analysis and the graphs show that at a particular earth orbital altitude the cross-sectional area and the mass of beams under constant stress decrease as the distance from the satellite increases. They also show that the cross-sectional area and the mass diminish at an even faster rate at low earth orbital altitudes. Therefore, for long structures extended from orbiting space satellites, it would be advantageous to design these structures to fit the constant stress criterion. This would mean a reduction in the weight of such structures. This weight reduction would be effective only on long structures, and would not have practical application to shorter objects.

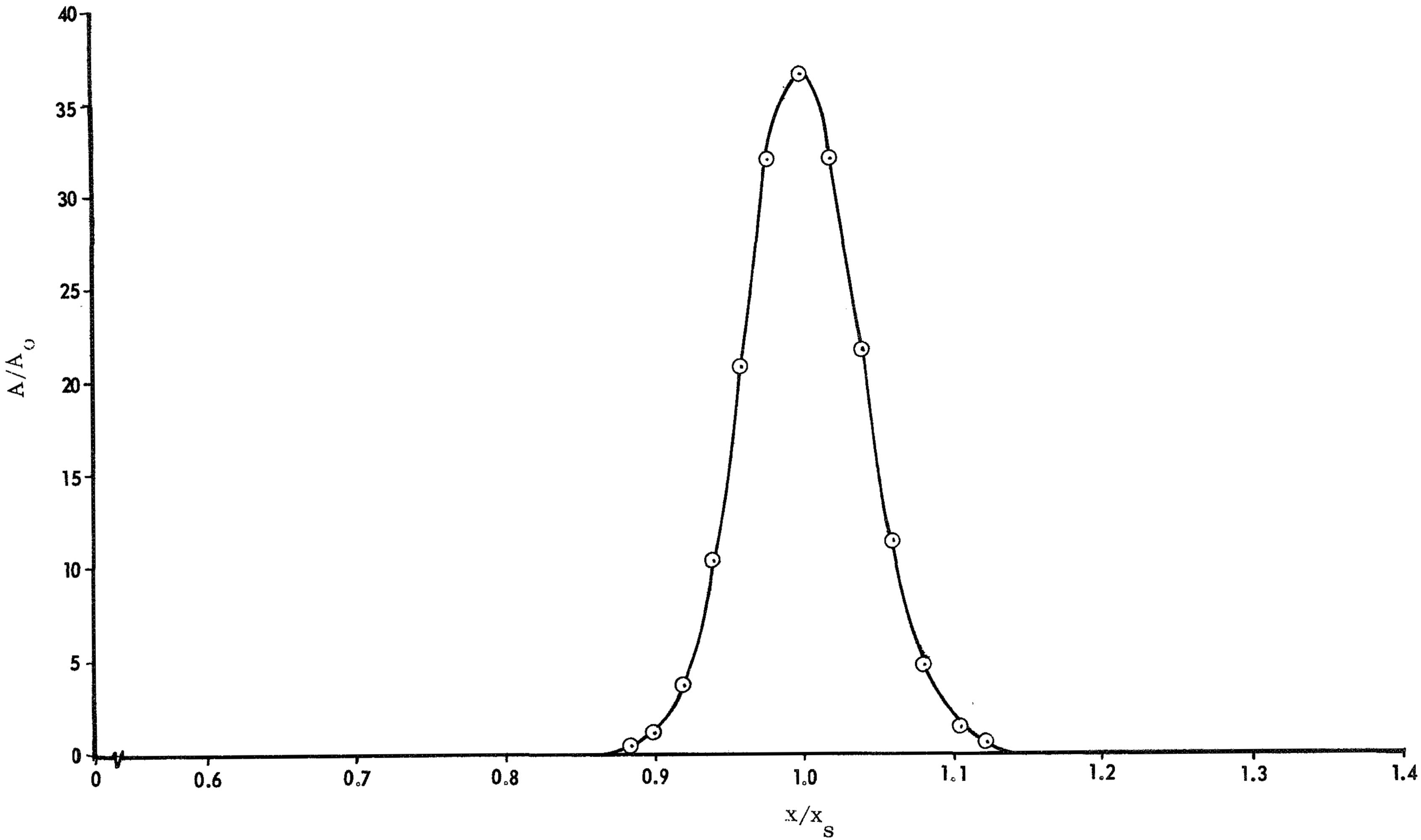

Figure 2. Ratio of areas for $x_s = 10\ 000$ km, $x_o = 9000$ km.

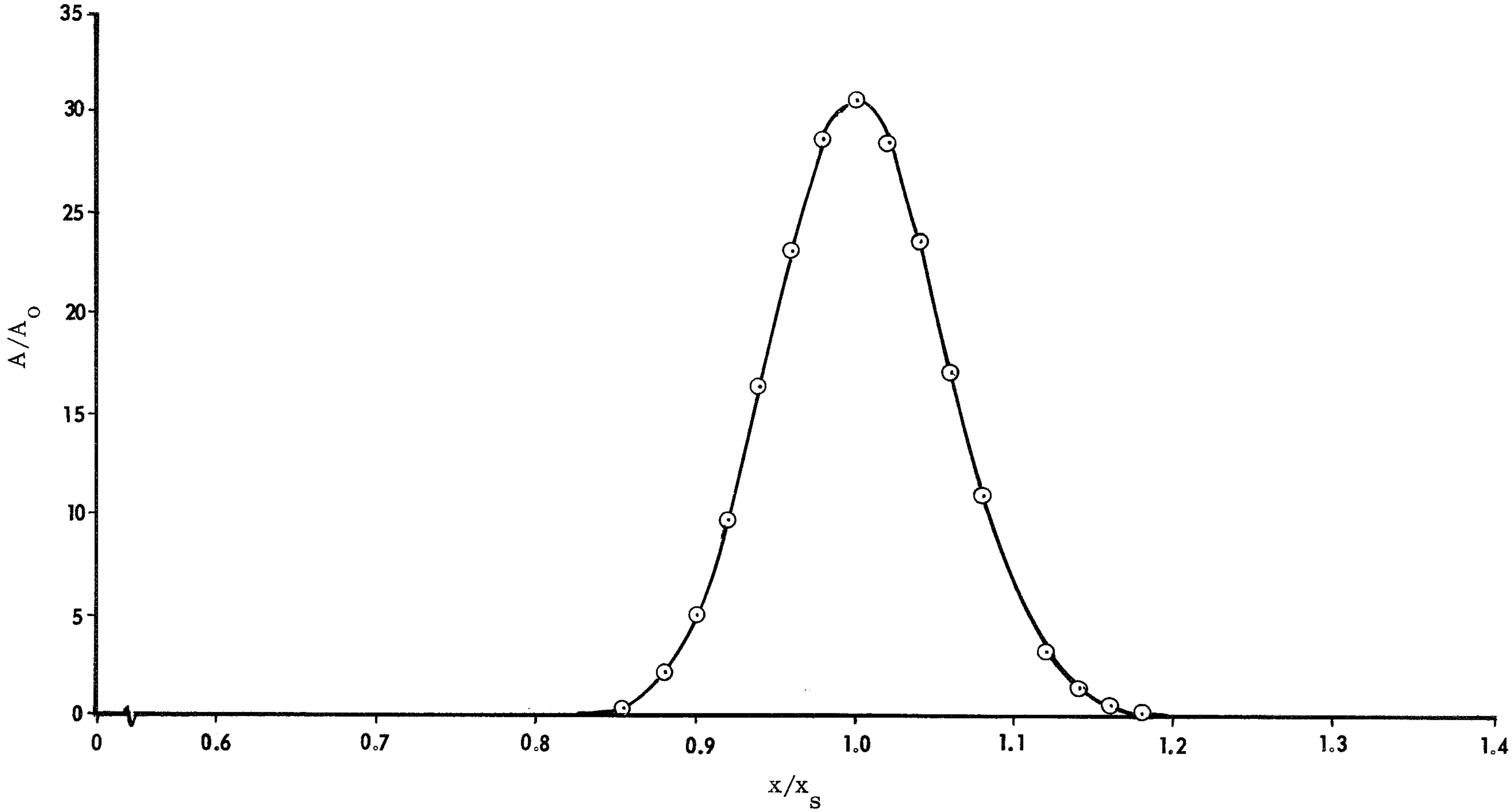

Figure 3. Ratio of areas for $x_s = 20\ 000$ km, $x_o = 17\ 300$ km.

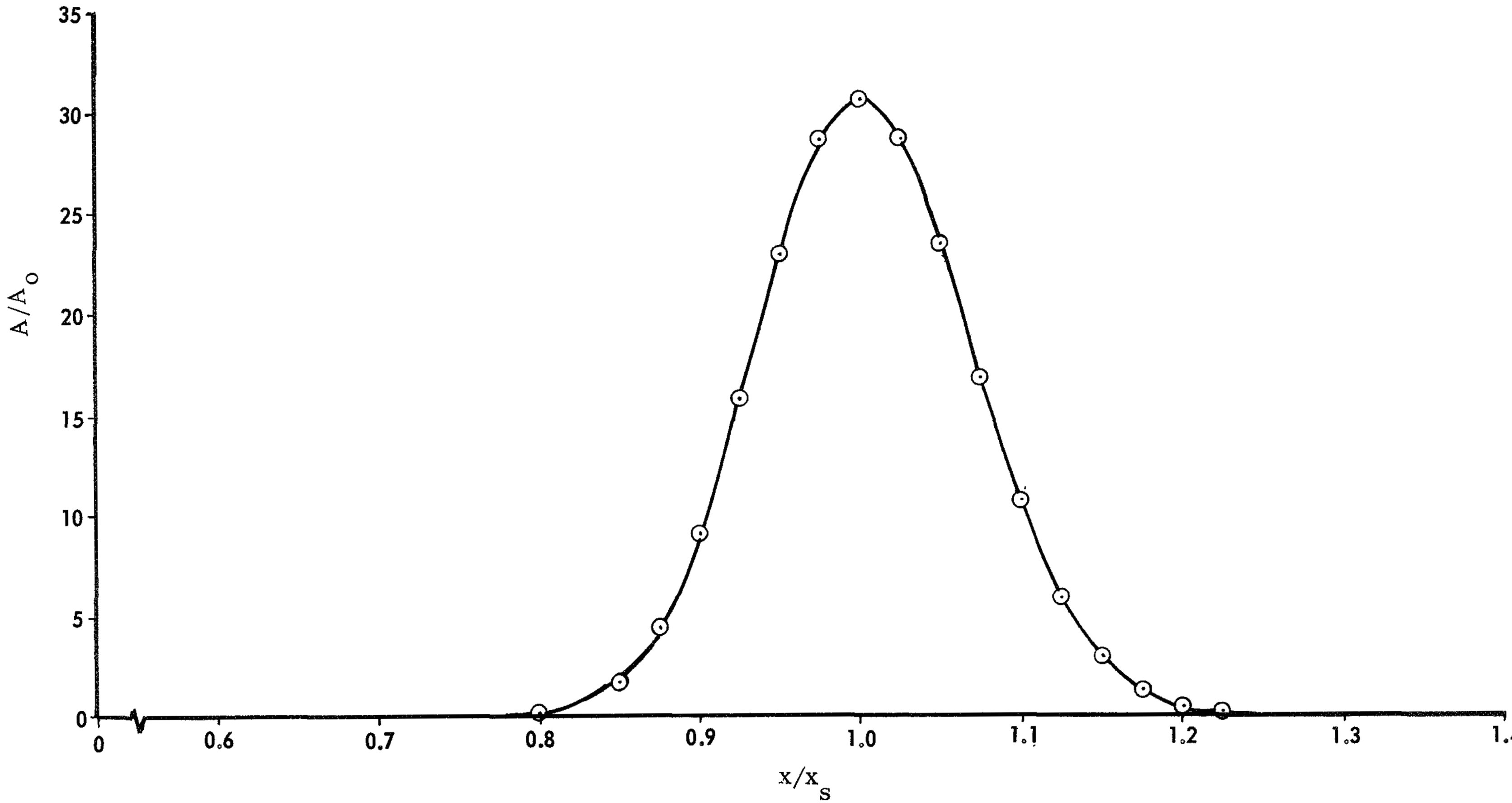

Figure 4. Ratio of areas for $x_s = 30\ 000$ km, $x_o = 25\ 100$ km.

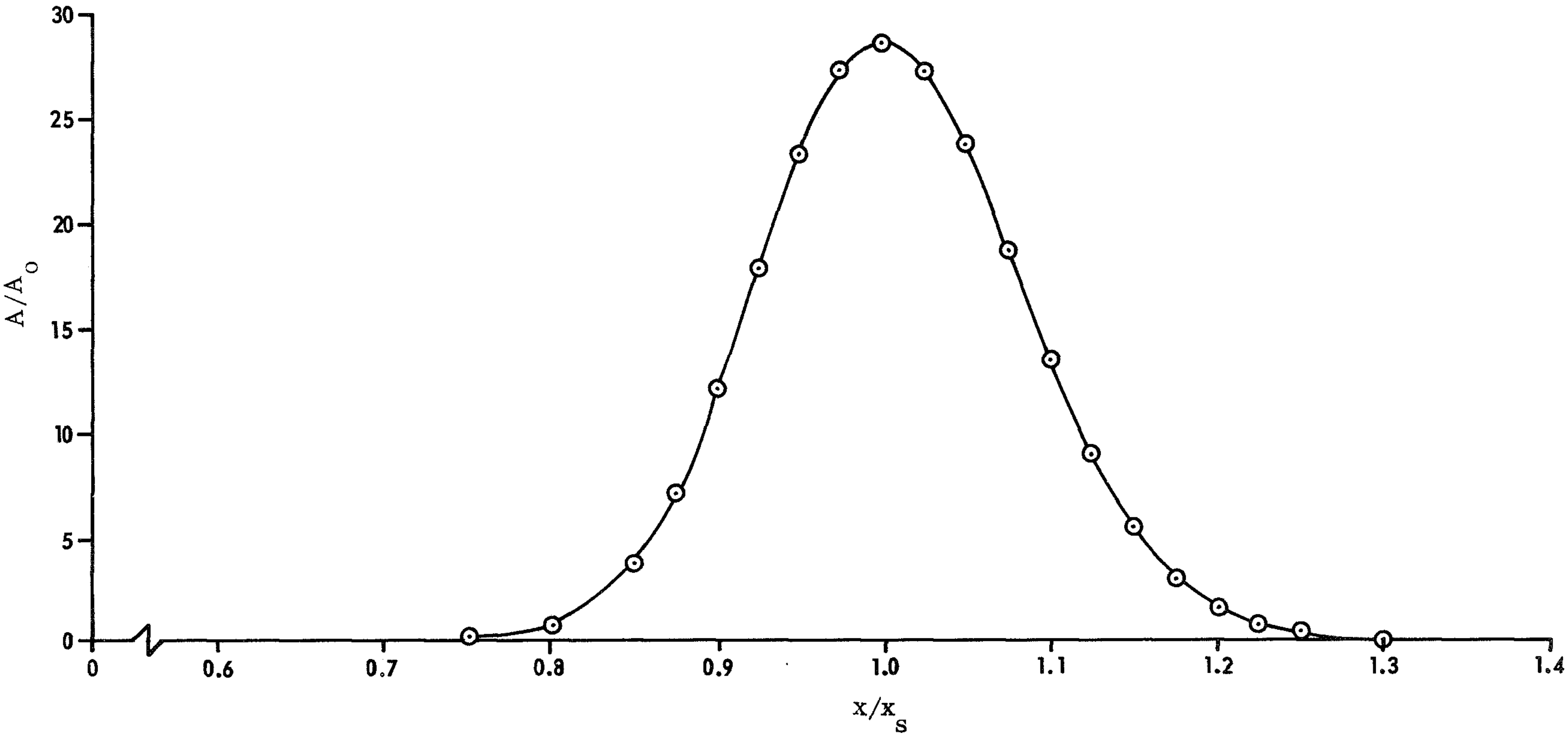

Figure 5. Ratio of areas for synchronous orbit $x_S = 42\ 172$ km, $x_O = 34\ 172$ km.

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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