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PRELIMINARY ANALYSIS OF A WAKE TRAILING A SPACECRAFT

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Advanced Systems Office

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George C. Marshall Space Flight Center
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ABSTRACT

It was formulated that a region existed behind an orbiting spacecraft which was relatively drag free. Due to the interest in conserving propellant on untethered modules flying in formation with a "mother" spacecraft, consideration was given to inserting these free modules inside this region. Calculations were made to determine the dimensions of the region trailing a cylinder 600 inches long and 260 inches in diameter. Regions were also calculated for minimum diameters ranging from 200 inches to 500 inches. The results of this study are presented graphically and in pictorial form.

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

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ADVANCED SYSTEMS OFFICE
RESEARCH AND DEVELOPMENT OPERATIONS

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS Concluded

SYMBOLS AND ABBREVIATIONS

Symbol	Definition
cm	Centimeter
erf	Probability integral
$F_{10.7}$	10.7 Solar flux
ge	Gravity at Earth Surface, km/sec^2
h	height from earth surface, km
H	Hydrogen Atom
HE	Helium Atom
oK	Degrees, in thermodynamic Kelvin scale
Km.	Kilometer
M	Meter
max.	Maximum
mb	Millibar
min.	Minimum
N_2	Nitrogen
n	Number density
O	Oxygen Atom
O_2	Diatomic Oxygen
R	Universal Gas Constant
Re	Mean radius of earth, km.
S	Speed Ratio, $\frac{\text{circular satellite velocity}}{\text{most probable particle velocity}}$
sec.	Second
Tm	Molecular-scale temperature in absolute thermodynamic scale

Symbol	Definition
T_∞	Exospheric temperature
V_p	Probable particle velocity
Vs	Circular satellite velocity
Z	Geometric Altitude
θ	Divergence angle
Y[S]	$Se^{-s^2} + \sqrt{\pi}\,(S^2+1/2)\ (1 + \mathrm{erf}s)$
X[S]	$e^{-s^2} + \sqrt{\pi}\,S(1 + \mathrm{erf}s)$

FOREWORD

The high velocity motion of a space station through the rarefied atmosphere of high altitudes causes complex gas flow patterns around the orbiting station. Well known are the resulting drag forces. Little if any effort has been devoted to the wake which trails a space station.

This idea of an orbital wake behind a space station came to mind when the concept of a space station was developed flying formation with several separate sub-satellites or modules. The higher drag forces on those modules as compared to the space station would require constant expenditures of propellants in order for them to maintain their position. Over a period of a year or two this would amount to sizeable quantities.

This combination of orbital wake and independent module resulted in the idea to place the independent module in the wake of a space station and keep it there. This could result in sizeable savings of propellants, and therefore weights and complexities.

The authors undertook an investigation of this problem.

Dr. R. Oman of the Grumman Aircraft Company gave encouragement to this idea and calculated drag reductions of over 50% for bodies in the wake. We will continue this effort.

Georg von Tiesenhausen

Georg von Tiesenhausen

INTRODUCTION

A study of void regions trailing orbiting bodies could be of fundamental importance in conserving the propellant of smaller untethered modules flying in formation with a "mother" spacecraft.

The purpose of this study was to provide dimensions of void regions for typical cylinderical bodies of various sizes. These objects were considered to be in circular orbits from 200 km through 500 km about the earth with a 50° inclination.

The environment of these orbiting bodies was assumed to be of a hyperthermal free molecular origin. Thus the approach used was to determine, through theoretical assumptions, the speed ratios, divergence angles, and the configurations associated with these orbiting bodies from the most probable molecular particle velocities. (See Figure 1)

TRAILING VOID REGIONS OF AN ORBITING CYLINDER

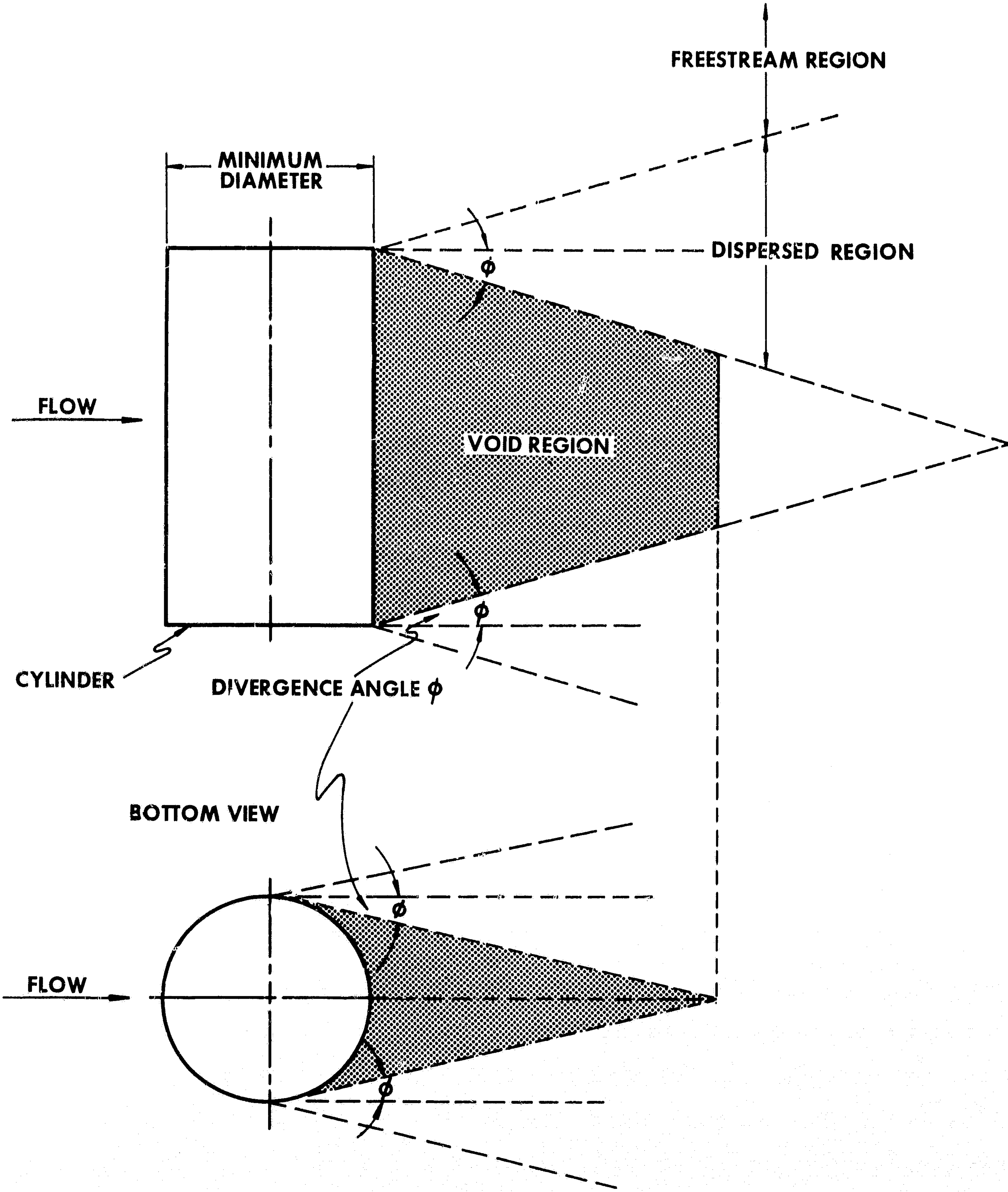


FIGURE I

ASSUMPTIONS

The following is a summary of the basic assumptions used in this study:

1. Hyperthermal free molecular flow environment.

2. Cylindrical configuration 600 inches long and 260 inches in diameter. (Approximates shape of S-IVB stage.)

3. Flow is to be steady with uniform density upstream of the surface.

4. Gas has a maxwellian velocity distribution superimposed on the mass flow velocity.

5. Dynamic and kinetic atmosphere criterion.

ANALYSIS

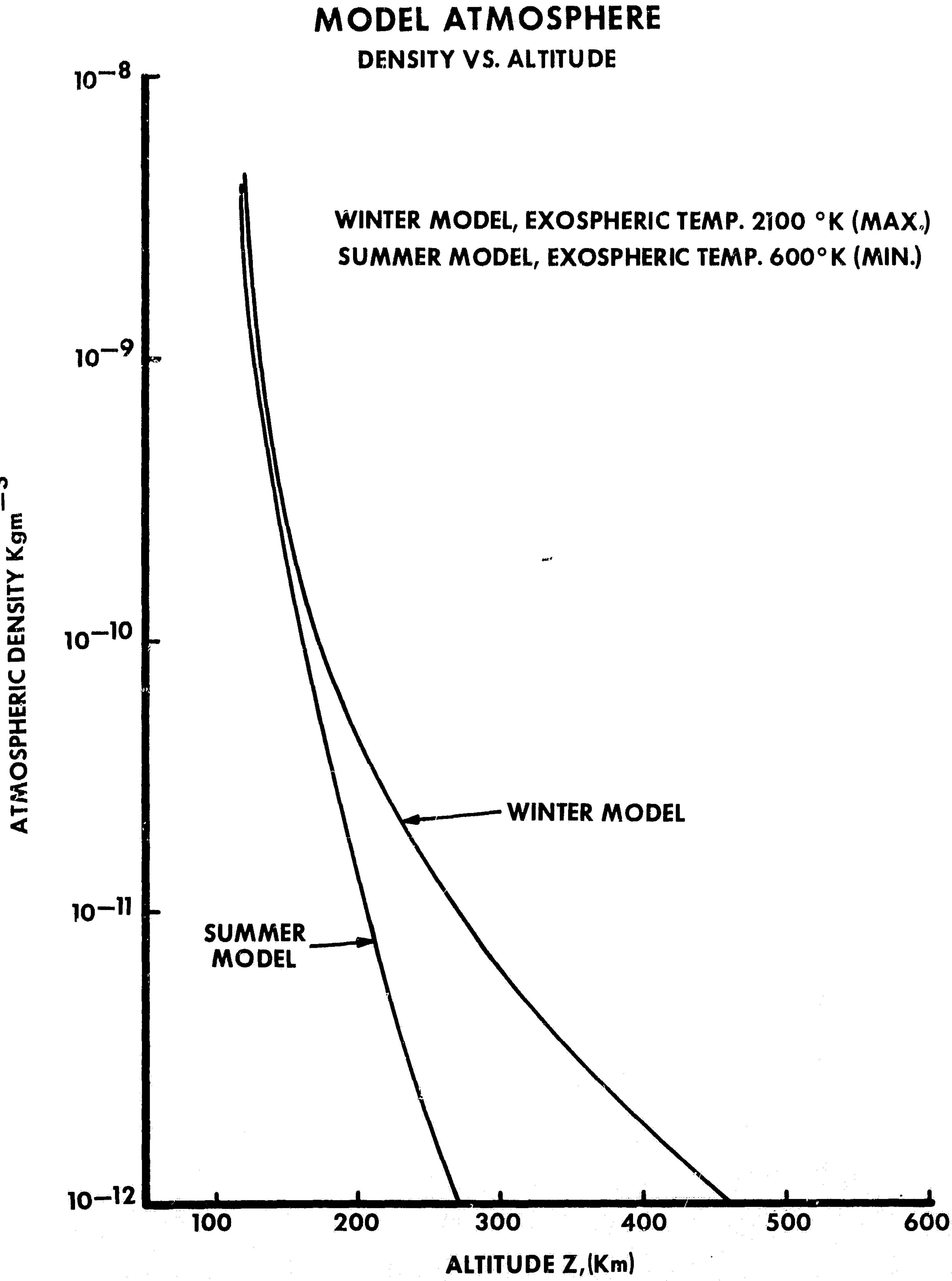
1.1 Atmosphere

The atmospheric shell or region is determined by four basic criterions. The first criterion, being a temperature distribution, is one of the most common ways of defining the layers of the earth atmosphere. The second criterion is the physicochemical process which defines the atmosphere layers through molecular reactions caused by the absorption of radiation. The third criterion defines the distinction of atmospheric layers according to photo-dissociation and gravitational separation of mean molecular weights. The fourth criterion is the dynamic and Kinetic processes which are represented by the model atmosphere used in the study. In this model, the exosphere, from which all calculations are based, is the outer-most portion of the atmosphere. The lower boundary of this model atmosphere is the critical level of escape, variously estimated at 500 km to 1000 km about the earth surface. In the exosphere, the air density is so low that the mean free path or free molecular path of individual particles is completely undirectional and depends upon their direction with respect to the local vertical, being greatest for upward moving particles. It is only from the exosphere that atmospheric gasses can, to any appreciable extent, escape into outer space.

The data used in this study which pertains to the exosphere originates from the 1966 Supplement to U.S. Standard Atmosphere 1962.

1.2 Exospheric Density

Above 120 km of the exosphere there is one basic parameter in the 1966 Supplementary Atmosphere, a set of exponential temperature curves, which were empirically derived to provide density - altitude profiles that coincide with satellite - drag densities derived for various degrees of solar and geomagnetic activity and varying solar angles. Three sets of density models above 120 km were developed from the 1966 Supplement. At lower altitudes eight supplementary atmospheres converged at 120 km to form these three sets. A single set is applicable to spring and fall conditions, a set for winter, and a set for summer. Temperature and density data were taken from these winter and summer sets. Density and number density profiles are presented in Figures 2, 2-A, and 3. Based on satellite data, variations in nitrogen number densities as a function of solar time are presented in Figure 4, 4-A, and 4-B. Variations of pressure between summer and winter models are shown in Figure 5.

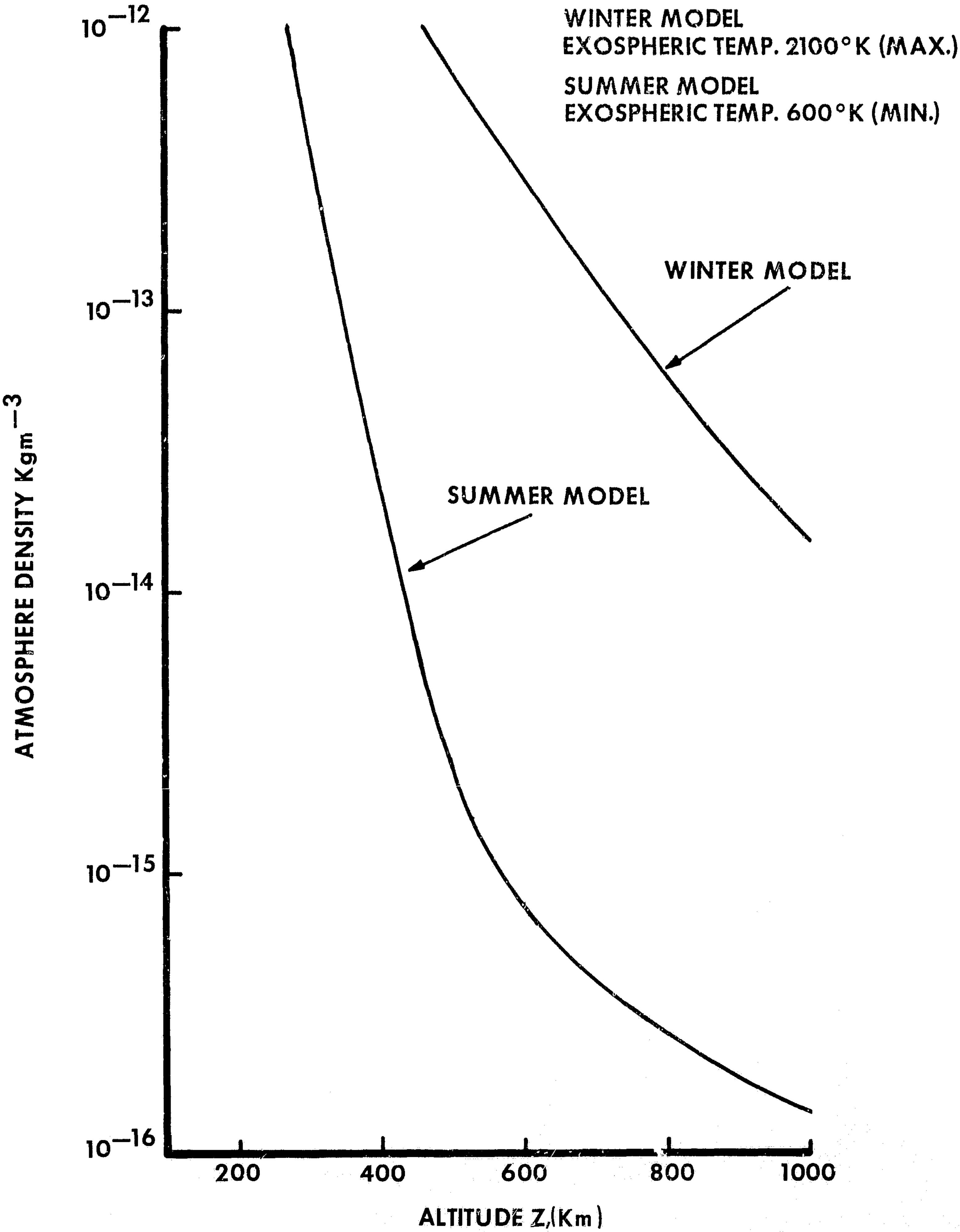


REF. 1966 SUPPLEMENT TO U.S. STANDARD ATMOSPHERE 1962

FIGURE 2

MODEL ATMOSPHERE

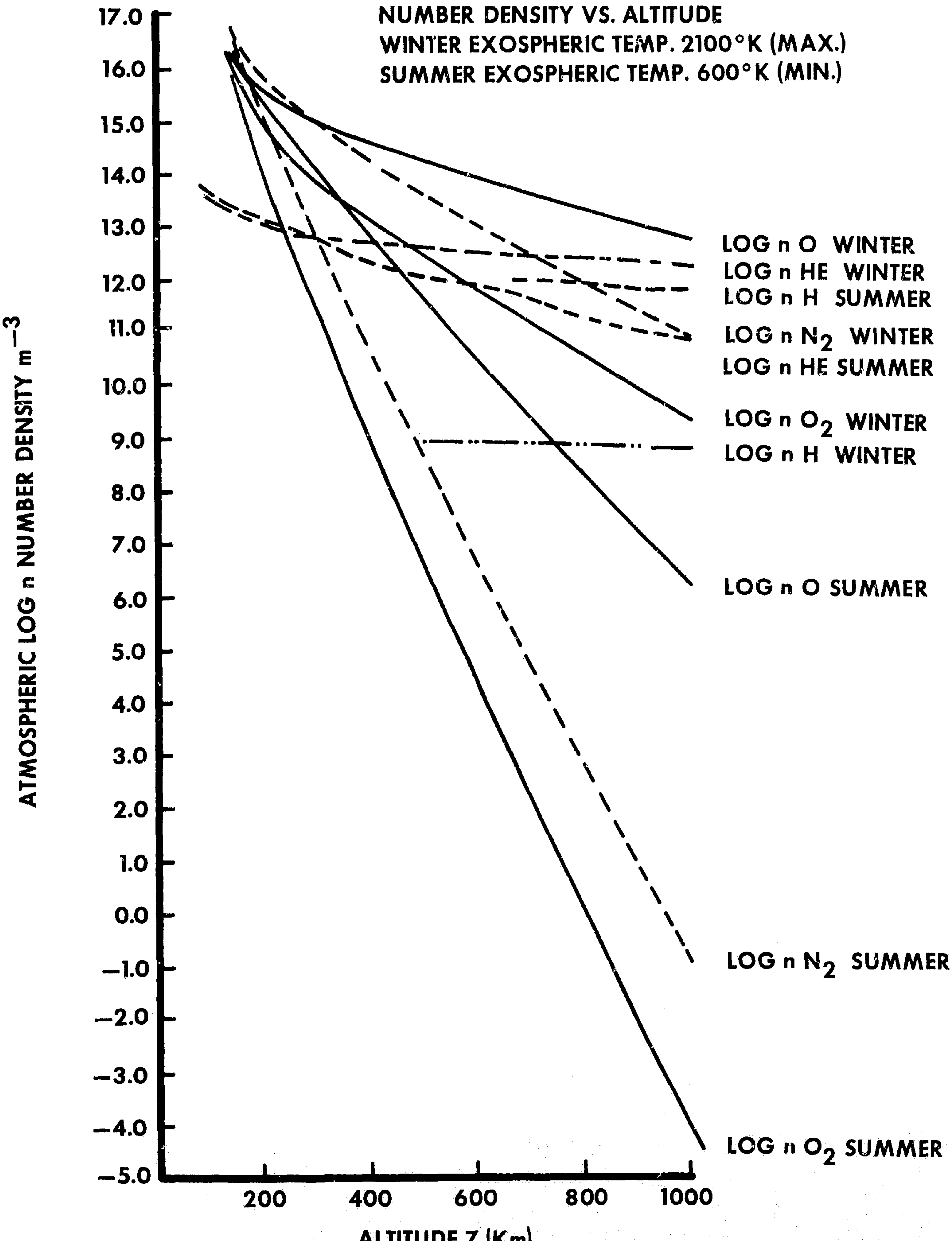
DENSITY VS. ALTITUDE



REF. 1966 SUPPLEMENT TO U.S. STANDARD ATMOSPHERE 1962

FIGURE 2a

MODEL ATMOSPHERE



REF. 1966 SUPPLEMENT TO U.S. STANDARD ATMOSPHERE 1962

FIGURE 3

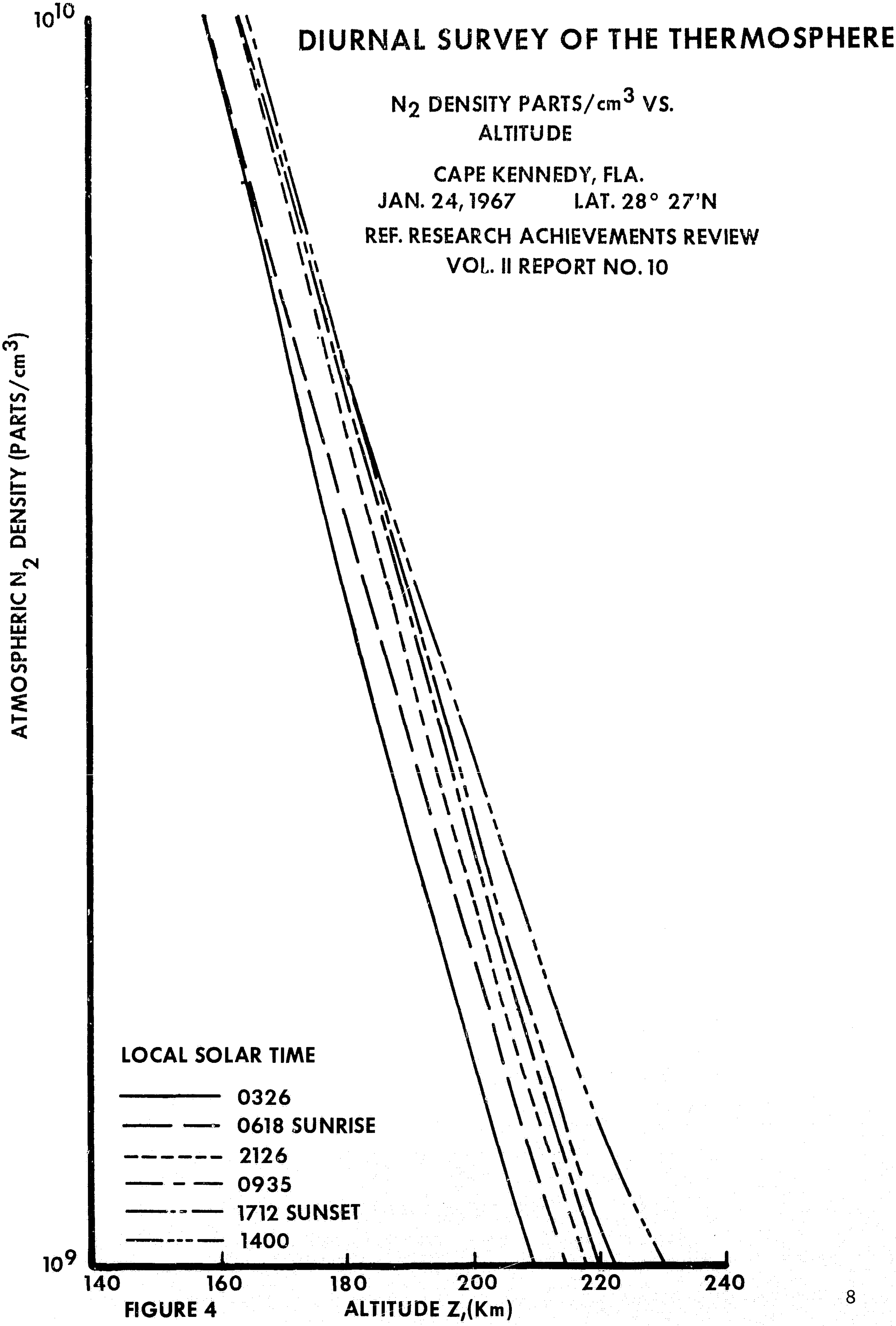

DIURNAL SURVEY OF THE THERMOSPHERE
N_2 DENSITY PARTS/cm^3 VS. ALTITUDE
CAPE KENNEDY, FLA.
JAN. 24, 1967 LAT. 28° 27'N
REF. RESEARCH ACHIEVEMENTS REVIEW
VOL. II REPORT NO. 10
10^{10}
10^9
ATMOSPHERIC N_2 DENSITY (PARTS/cm^3)
140
160
180
200
220
240
ALTITUDE Z,(Km)
LOCAL SOLAR TIME
0326
0618 SUNRISE
2126
0935
1712 SUNSET
1400

FIGURE 4

DIURNAL SURVEY OF THE THERMOSPHERE

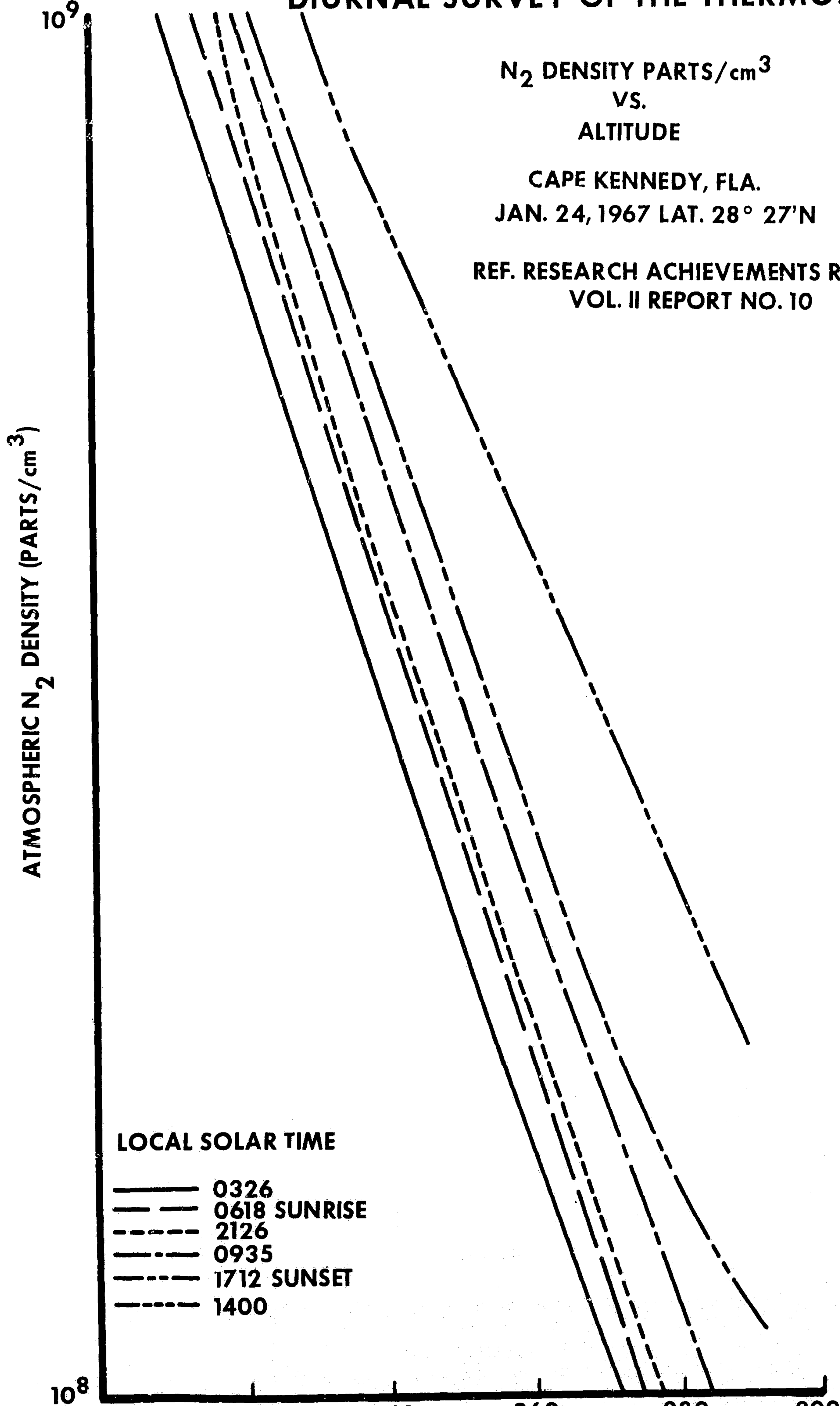
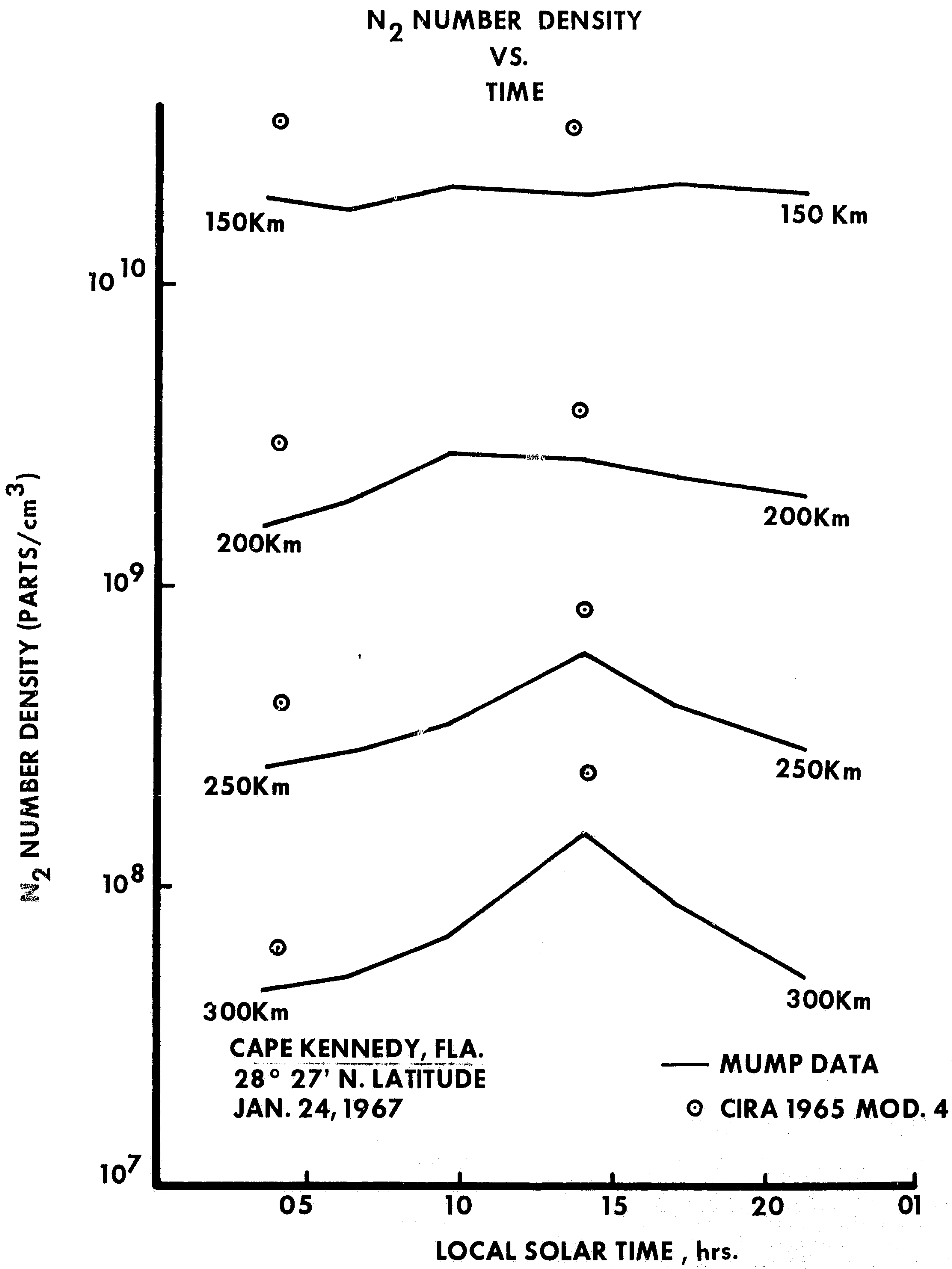


FIGURE 4a

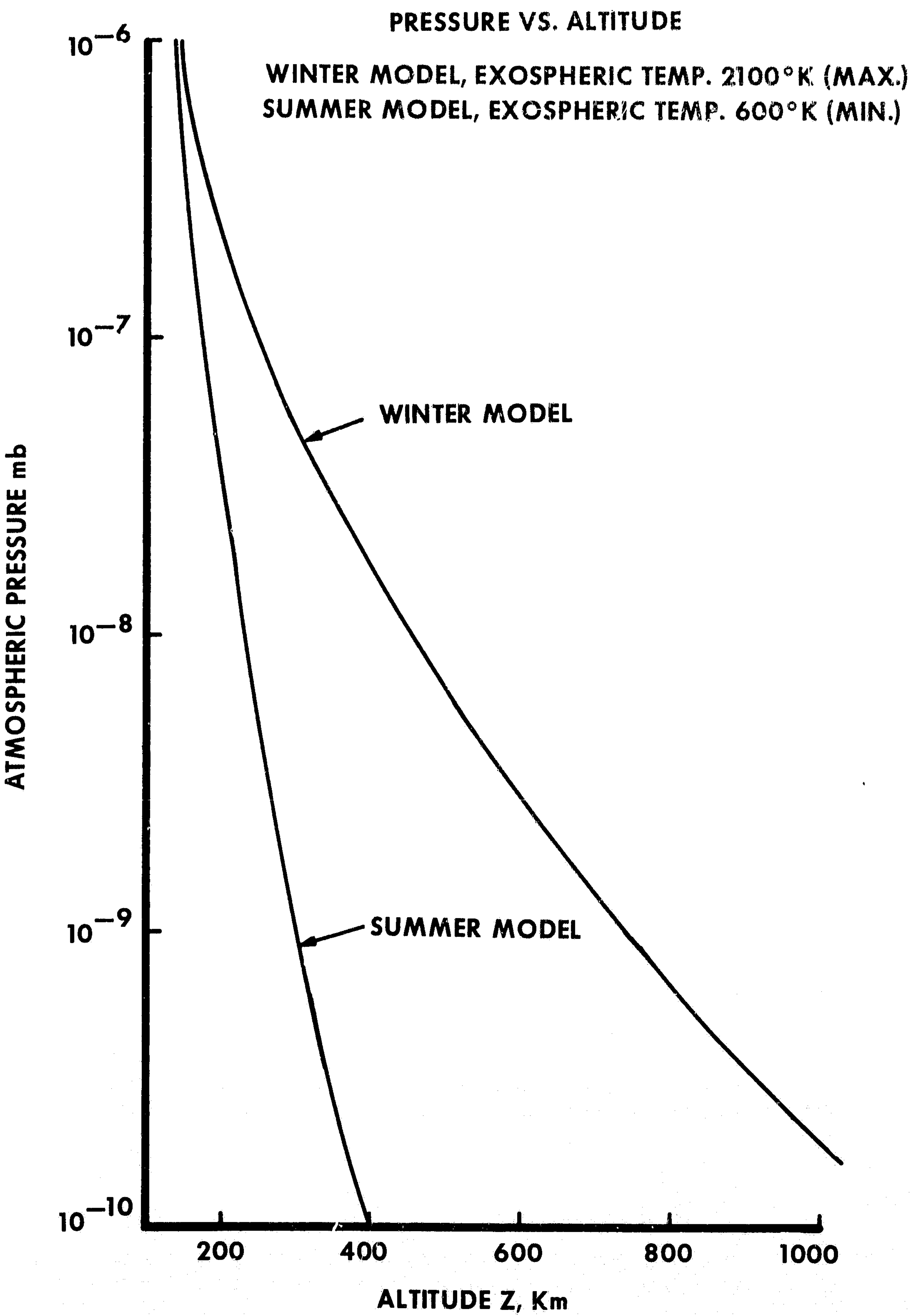
DIURNAL SURVEY OF THE THERMOSPHERE



REF. RESEARCH ACHIEVEMENTS REVIEW VOL. II REPORT NO. 10

FIGURE 4b

MODEL ATMOSPHERE



REF. 1966 SUPPLEMENT TO U.S. STANDARD ATMOSPHERIC 1962

FIGURE 5

1.3 Exospheric Temperature

Four types of temperature variations have been recognized at heights greater than 200 km, namely:

1. A variation with solar activity
2. A semiannual variation
3. A diurnal variation, and
4. A variation with geomagnetic activity.

Empirical formulas have been constructed to compute the exospheric temperature when these parameters are known. Once the exospheric temperature T_∞ is computed, atmospheric densities and related quantities can be found for any given height.

Temperatures used in the calculations of the most probable velocity are:

1. Winter exospheric temperature at
   a. 600° K minimum model
   b. 2100° K maximum model
2. Summer exospheric temperature at
   a. 600° K minimum model
   b. 2100° K maximum model

Temperatures from the winter model (max) and summer model (min) as a function of altitude are presented in Figure 6.

1.4 Computations

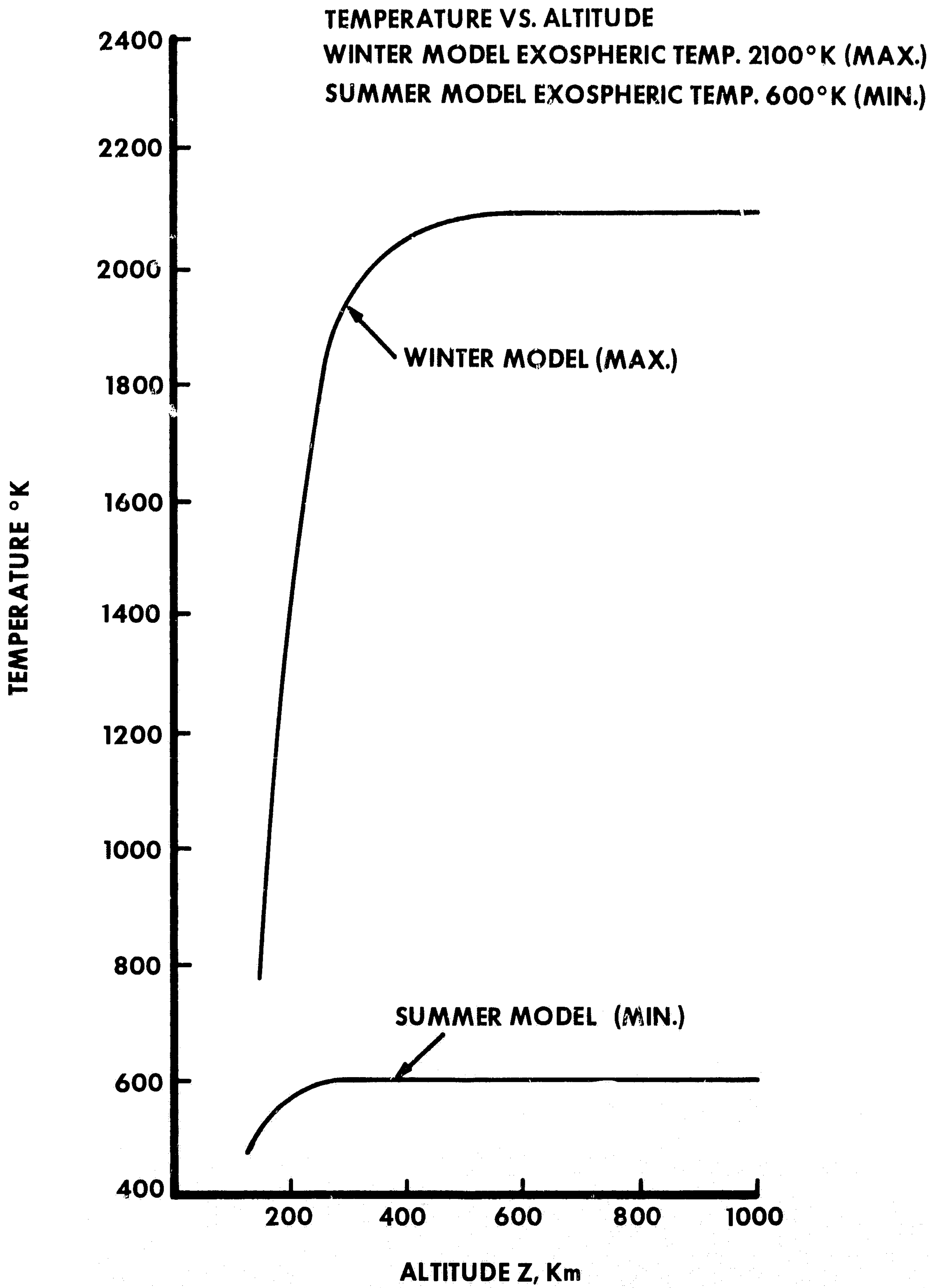
Most probable velocities were calculated from 200 km to 500 km with 50 km increments. The following equation form was used:

$$V_p = \sqrt{\frac{2RTm}{M}}$$

Circular satellite velocities were calculated from 200 km to 500 km with 50 km increments. The following equation was used:

$$Vs = R_e \sqrt{\frac{ge}{R_e + h}}$$

MODEL ATMOSPHERE



REF. 1966 SUPPLEMENT TO U.S. STANDARD ATMOSPHERE 1962

FIGURE 6

A speed ratio for each altitude of 50 km increments between 200 km and 500 km was calculated as shown in Figure 7. The following equation was used and the results are tabulated in Table 1.

Speed Ratio S = Circular satellite velocity/most probable particle velocity.

Divergency angles were determined for altitudes corresponding to the altitudes used in the previous calculations. Angles were determined from the equation.

$$\theta = \tan^{-1} \frac{1}{\pi} \frac{X[S]}{\psi[S]}$$

Divergence angles determined from this equation are presented in Fig. 8 as a function of speed ratio, S. The results are tabulated in Table 1.

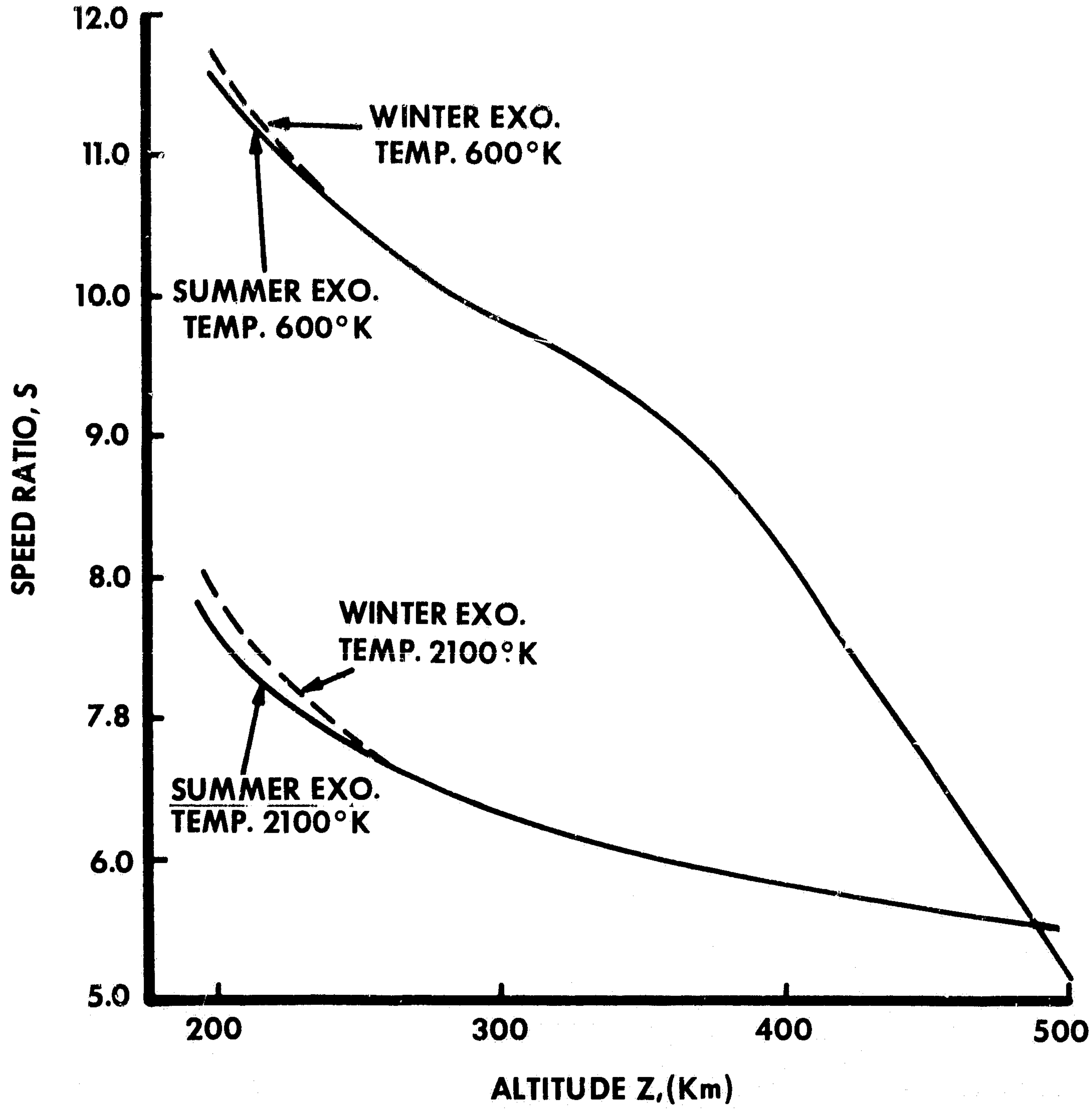
Since the minimum diameter and divergence angles are known, trigonometric relations were used to find the length of the void regions. The lengths are presented in Figure 9 as a function of the minimum diameter with divergence angles as a parameter.

Figures 10, 11 and 12 illustrate the shape of the void regions with angles of attack 90°, 45° and 0° respectively. For zero angles of attack the void region is a cone of $\theta°$ half angle and for all other angles of attack greater than zero the void region is wedge shaped.

MODEL ATMOSPHERE

SPEED RATIO
VS.
ALTITUDE (KM)

WINTER & SUMMER EXOSPHERIC TEMP. 600°K (MIN.)
WINTER & SUMMER EXOSPHERIC TEMP. 2100°K (MAX.)



REF. 1966 SUPPLEMENT TO U.S. STANDARD ATMOSPHERE 1962

FIGURE 7

TABLE 1

Speed Ratio, S, and Divergence angles at various circular Orbit Altitudes

		Altitudes, km						
		200	250	300	350	400	450	500
2100°K Summer	Divergence Ø	4.2	4.7	5.1	5.3	5.5	5.7	5.8
Exo. Temp. (max.)	Speed Ratio, S	7.63	6.75	6.32	6.03	5.82	5.63	5.48
2100° K Winter	Speed Ratio, S	7.89	6.81	-	-	-	-	-
Exo. Temp. (max.)	Divergence Ø	4.1	4.7	-	-	-	-	-
600° K Summer	Divergence Ø	2.8	3.1	3.3	3.5	3.9	4.8	6.1
Exo. Temp. (min.)	Speed Ratio, S	11.50	10.45	9.80	9.23	8.16	6.64	5.17
600° K Winter	Speed Ratio, S	11.61	-	-	-	-	-	-
Exo. Temp. (min.)	Divergence Ø	2.75	-	-	-	-	-	-

DIVERGENCE ANGLE ϕ AS A FUNCTION OF SPEED RATIO, S

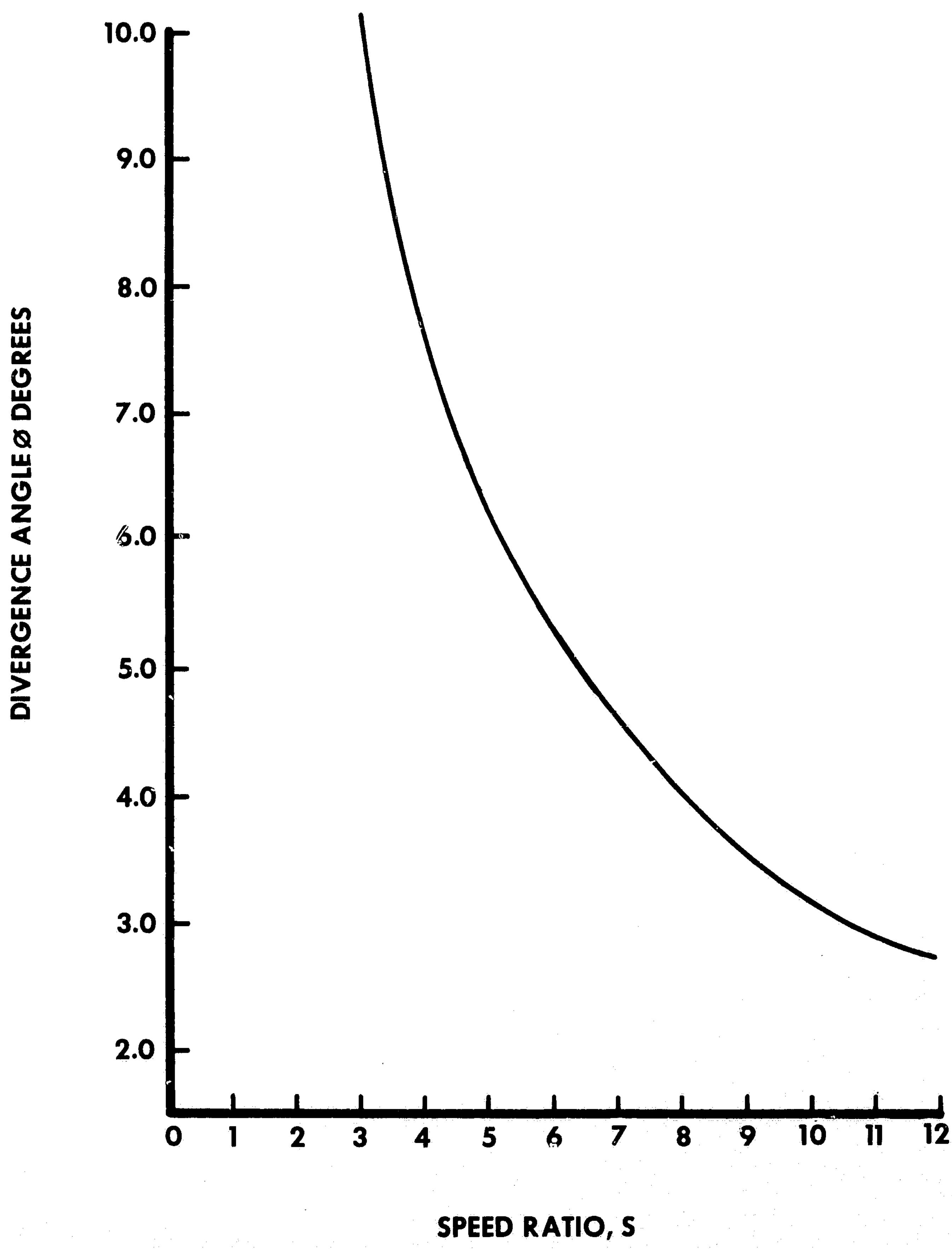


FIGURE 8

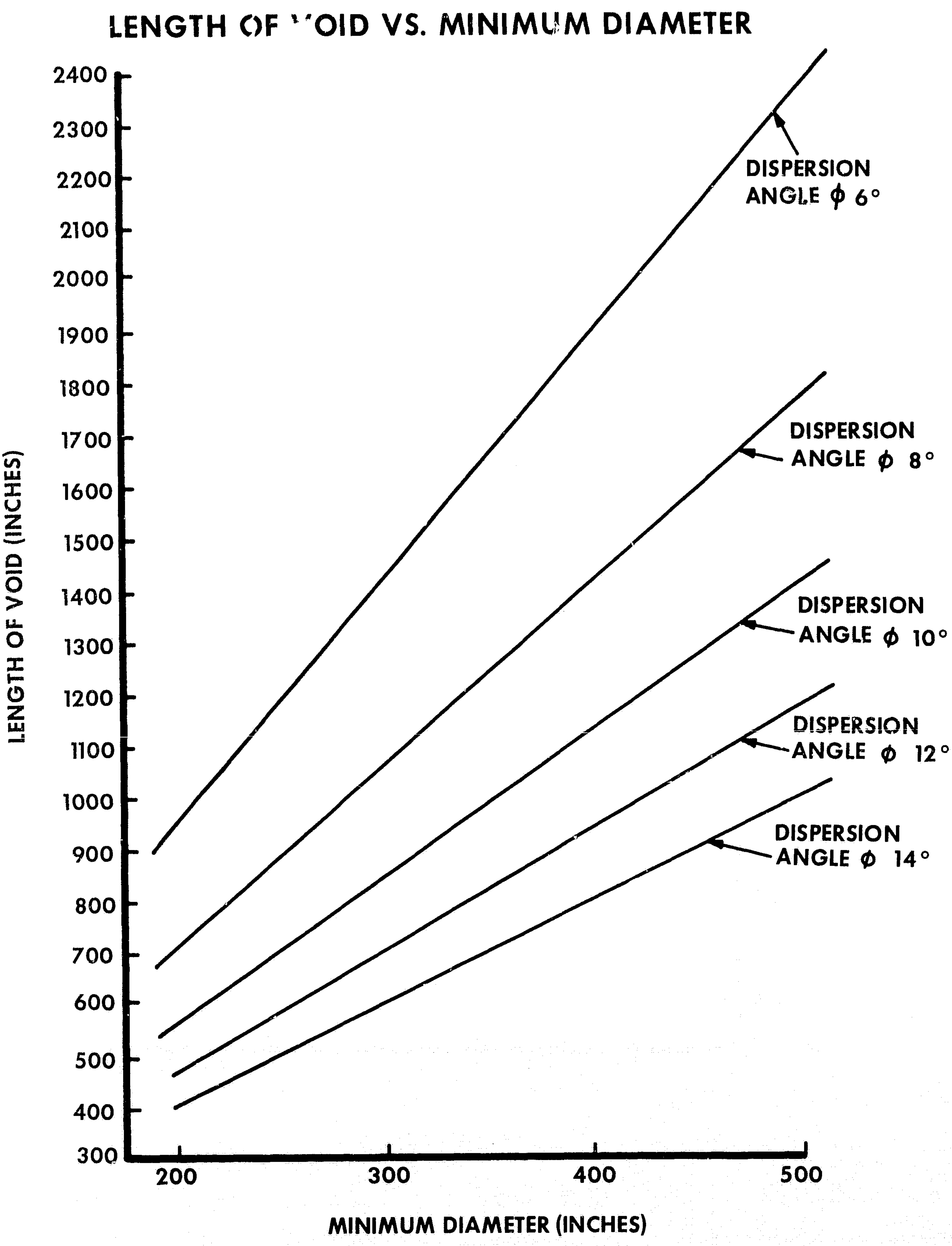

LENGTH OF VOID VS. MINIMUM DIAMETER
LENGTH OF VOID (INCHES)
2400
2300
2200
2100
2000
1900
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
300
400
500
DISPERSION ANGLE ϕ 6°
DISPERSION ANGLE ϕ 8°
DISPERSION ANGLE ϕ 10°
DISPERSION ANGLE ϕ 12°
DISPERSION ANGLE ϕ 14°
MINIMUM DIAMETER (INCHES)

FIGURE 9

TRAILING VOID REGIONS OF AN ORBITING CYLINDER
ANGLE OF ATTACK α 90°

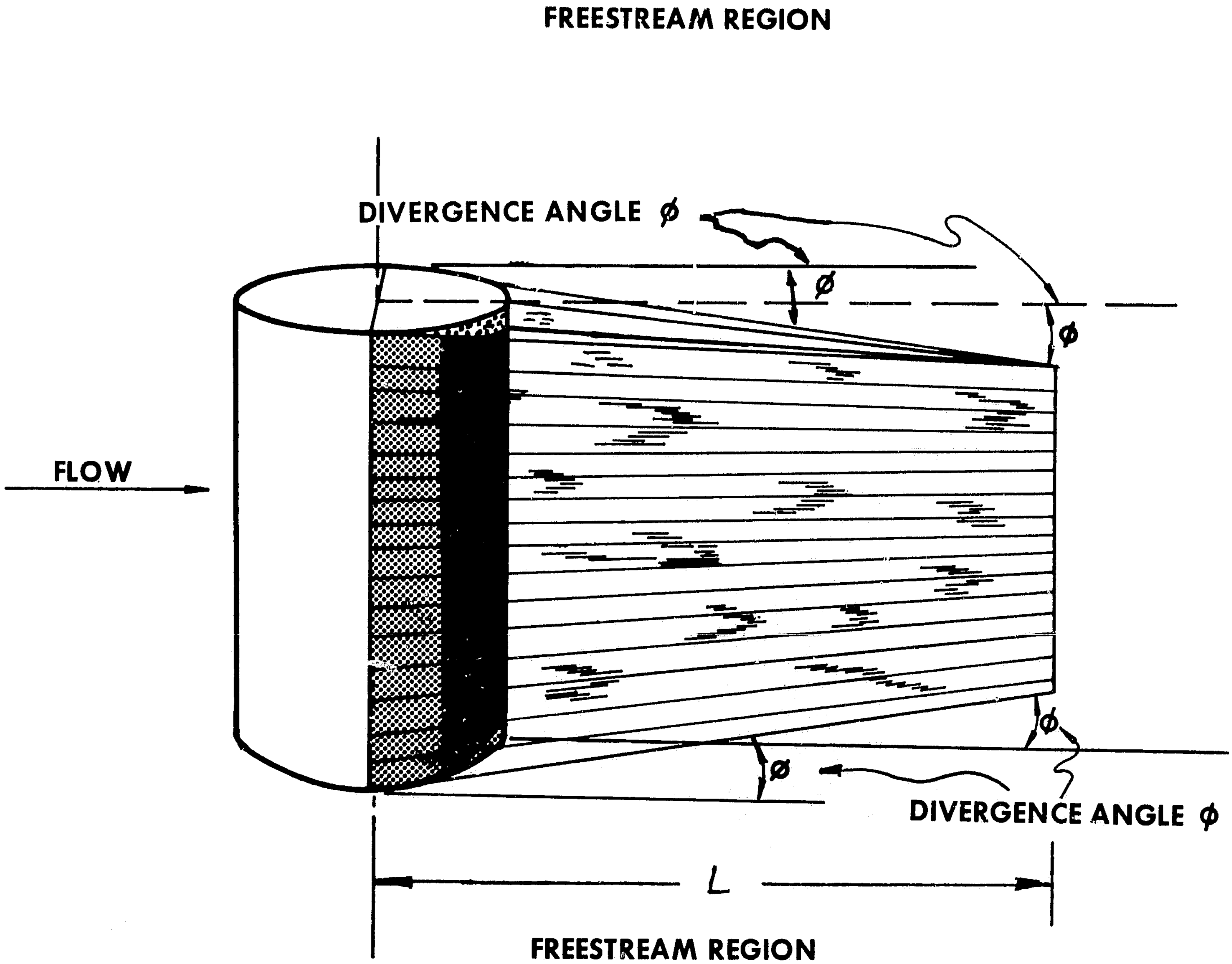


FIGURE 10

TRAILING VOID REGIONS OF AN ORBITING CYLINDER
ANGLE OF ATTACK $\alpha = 45°$

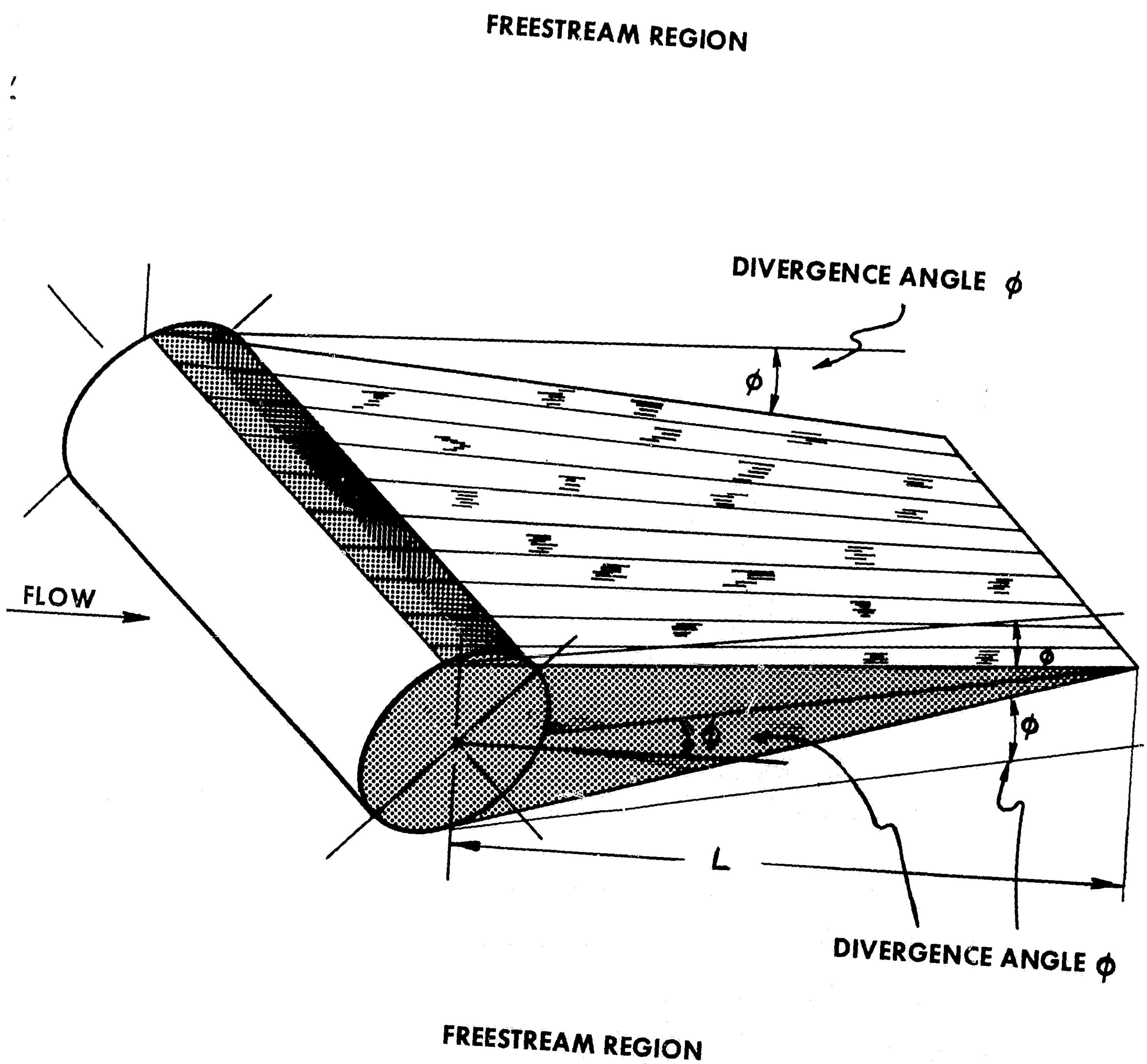


FIGURE 11.

TRAILING VOID REGION OF AN ORBITING CYLINDER
ANGLE OF ATTACK α O°

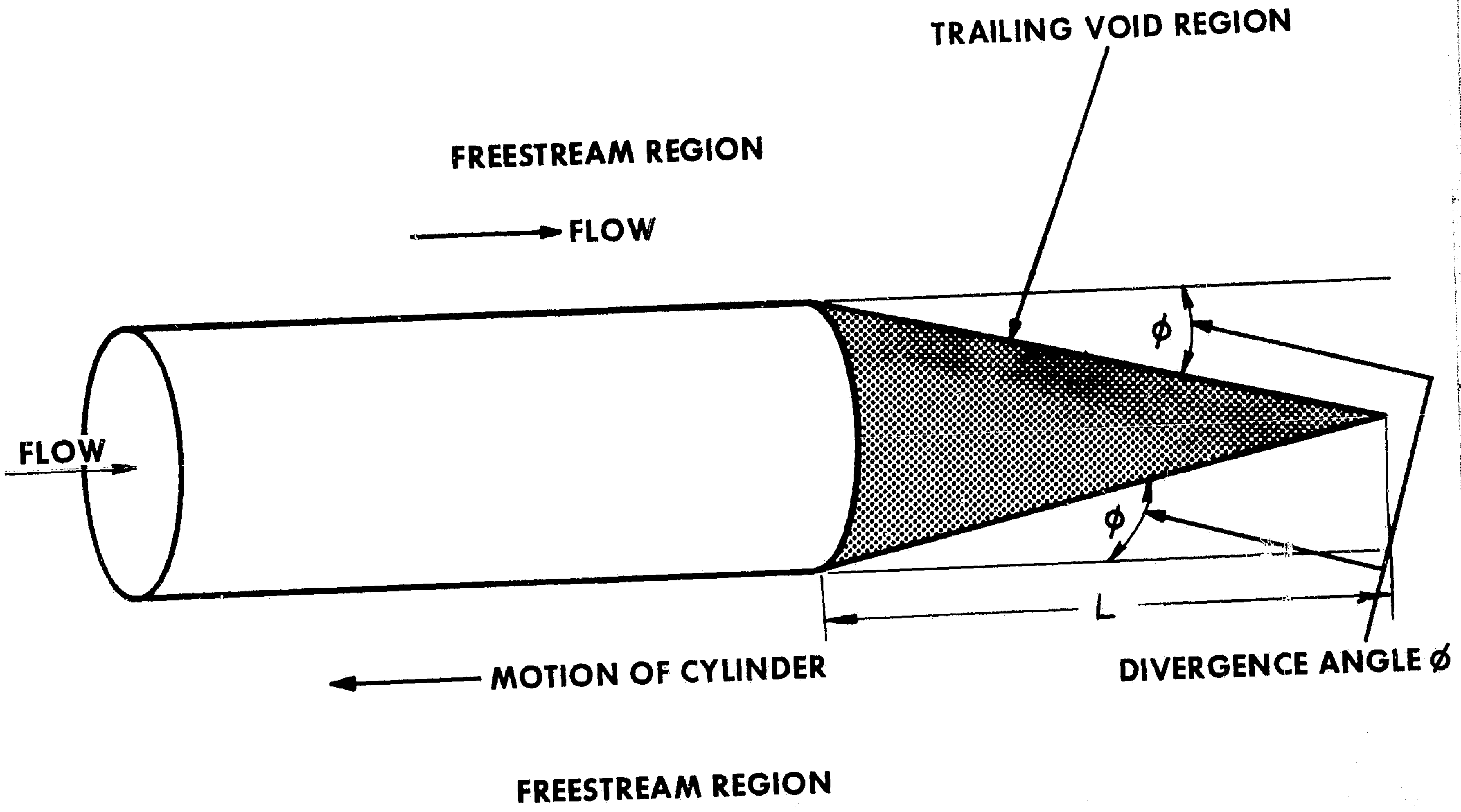


FIGURE 12.

CONCLUSION

There exists a sizable wake trailing a typical orbiting body which is dependent, to a large extent, on the speed ratio and minimum diameter of the module. The wake is considered large enough to accommodate reasonably large modules.

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