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R E Electronics Control Div

UNIVERSITY of PENNSYLVANIA

PHILADELPHIA 19104

The Moore School of Electrical Engineering

September 1, 19'

National Aeronautics & Space Administration
Electronic Research Center
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Cambridge, Massachusetts 02139

Attn: Computer & Guidance Technology Division

Dear Sir:

Enclosed are one reproducible (and two not rec'd duplicated copies) of the final technical report, "Designer's Manual for Computer-Aided Design Control Circuits", prepared by S. D. Bedrosian and D. I. Howe under Contract NAS12-2137.

This completes the report requirements for the subject contract.

Sincerely,

Edward J. Parker

Edward J. Parker

cc: Dr. S. D. Bedrosian
Mr. R. L. Keane, ONR Resident Representative
Mr. A. Merritt, ORA
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DESIGNERS MANUAL FOR COMPUTER-AIDED DESIGN OF CONTROL CIRCUITS

FINAL TECHNICAL REPORT

Contract NAS 12-2137

National Aeronautics and Space Administration
Electronics Research Center
575 Technology Square
Cambridge, Massachusetts   02139

Prepared by

S. D. Bedrosian
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June 15, 1970

Moore School of Electrical Engineering
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Philadelphia, Pennsylvania   19104

OCT 1970

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CHAPTER I

INTRODUCTION

This manual describes the necessary preliminary electric circuit modeling and methods of computer analysis using the NASAP digital computer program to aid in the design of control circuits used in aerospace systems. The scope of design treated is limited to single-input single-output linear time invariant control systems that can be described by rational transfer functions.

Two different approaches can be followed in the design of such linear time invariant feedback control systems. In one case the designer seeks compensators for a given plant to satisfy the over-all system requirements. This approach generally involves cut and try and the well known conventional design techniques such as the Bode and Nyquist plots and use of the root locus. In the other case, the designer starts by obtaining the over-all transfer function from the given plant and the specifications. He is then in a position to determine the required compensators. The second of these two approaches seems more amenable to computer-aided design of control circuits using the NASAP program.

NASAP is an acronym for Network Analysis for Systems Applications Program. NASAP has been developed and is being maintained by the Automated Techniques Branch, NASA Electronics Research Center, Cambridge, Massachusetts. The NASAP program is based on Mason's signal-flow graph [MA 1] as extended by Happ for the closed signal-flow graph [HA 1]. The program is based upon symbol oriented techniques which with proper tagging and loop evaluation permit both the transfer function and the sensitivity to be made available.

This chapter provides brief discussions on the basic options presently available in the NASAP program and the basic procedures used in computer aided design of control circuits.

The NASAP Program

NASAP is a linear electrical circuit analysis program which computes a specified transfer function in terms of the complex frequency variable, s. The basic coding rules and description of the program are contained in the booklet "Coding Instructions for NASAP 69/I" by Gaertner Research Incorporated [GA 1] written under NASA contract NAS 12-663. A number of special options are introduced and described later in this manual. The input for NASAP is by punched card and the output is by printer tabulation of data and printer graphics. It is worth noting that the input format of NASAP is simple and that use of this program requires little knowledge of circuit theory or computer programming.

The present version of NASAP can handle linear circuits which consist of constant- value passive elements, and independent or dependent current and voltage sources. The dependent sources must be linearly related to a voltage or current in another part of the circuit. Nonlinear functional relationships (dependencies) and time-varying parameters cannot be handled.

Embodied in NASAP is the ability to give both a mathematical formulation and a numerical tabulation of the output results with some printer graphics. A brief summary of the options presently available in the program available on the RCA Spectra 70/46, at the Moore School, follows:

1. OUTPUT - The transfer function which is specified by the user is printed out as the ratio of two polynomials in the complex frequency variable, s. The poles and zeros of the transfer function are evaluated.
2. FREQ - A Bode plot of the transfer function is printed in tabular and graphical form.
3. TIME - The impulse response of the network is printed in tabular and graphical form. To facilitate control circuit design we

have also made available the step and ramp responses.

4. SENSITIVITY - The sensitivity of the transfer function is computed with respect to a designated element in the network. Furthermore the program can print out in numerical form: the sensitivity of the real part of the transfer function, of the imaginary part of the transfer function, of the magnitude of the transfer function, and of the phase of the transfer function to changes in specified circuit parameters.

Details of these options are documented in Section ID.

Procedures Used in Computer Aided Design

The major steps in computer aided circuit design are:

a. Choose the electrical circuit model for the control system.

b. Calculate element values from design equations.

c. Analyze circuit using computer.

d. Change element values if computer analysis results in discrepancies between the electrical circuit model response and desired control circuit response.

Before a control system can be analyzed using NASAP and before the necessary compensation can be determined, the plant's dynamic characteristics must be first simulated by an electric network which has an equivalent dynamic characteristic. The choice of circuit models is based on the specified response for the control circuit. Details of circuit modeling are given in chapter III. Computer analysis is amply demonstrated in chapter IV.

IA REVIEW OF CODING PROCEDURES FOR NASAP

Introduction

NASAP is a computer-aided electrical circuit analysis program which can be used by engineers without knowledge of computer programming. Using NASAP to analyze a circuit it is possible to calculate a transfer function, the sensitivity of the transfer function, the frequency response, and the impulse response. The main steps for preparing NASAP computer instructions are:

a) Obtain an electrical circuit model for the control circuit

b) Preparation of the circuit diagram in a form from which all the information required by the computer can be readily extracted.

c) The preparation of the computer instructions themselves.

The first step will be discussed in chapter III. The second step has been de cribed elsewhere [GA 1] and will be illustrated by many examples throughout this manual. The third step will be reviewed in this chapter for the convenience of the reader. For additonal information, see [GA 1]. The coding rul will be presented by an example taken virtually unchanged from [MO 1].

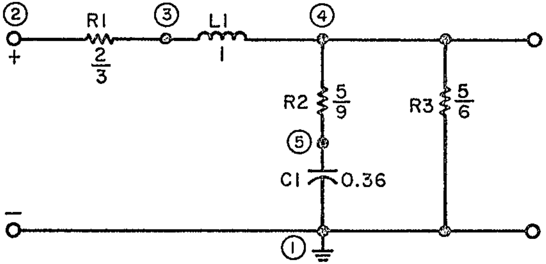
Example RLC Filter

We shall analyze the RLC filter shown in Fig. 1.1(a) to find its open circuit transfer voltage function.

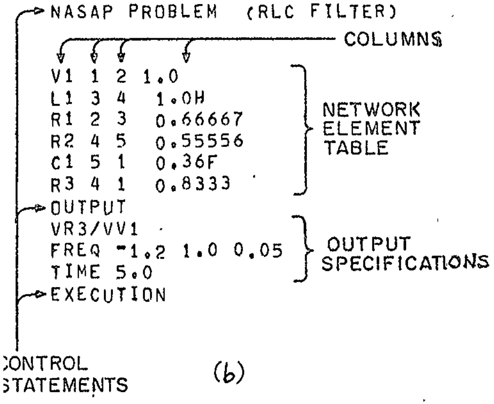
Numbering the Nodes and Elements

The first step in preparing the circuit for NASAP analysis is to number the circuit nodes and elements. The nodes must be numbered sequentially, starting with 1, without skipping any numbers.

The second step is numbering all circuit elements (resistors, capacitors, inductors, and current and voltage sources) consecutively within each categor i.e., R1, R2, . . ., L1, L2, . . ., C1, C2, etc. No two elements can have the same number designation regardless of their component values. One



(a) CIRCUIT WITH NODES AND ELEMENTS NUMBERED



```
NASAP PROBLEM  (RLC FILTER)
V1 1 2 1.0
L1 3 4  1.0H
R1 2 3  0.66667
R2 4 5  0.55556
C1 5 1  0.36F
R3 4 1  0.8333
OUTPUT
VR3/VV1
FREQ -1.2 1.0 0.05
TIME 5.0
EXECUTION
```

(b)

Fig. 1.1: RLC Filter Network and the Corresponding NASAP Input Instructions

possible set of number assignments for the RLC filter is shown in Fig. 1.1(b).

Overall Appearance of Input Instructions

Figure 1.2 shows the general appearance of the input instructions. The instructions fall into three different categories: a) Control Statements, b) Network Element Table, and c) Output Specifications. Each type of instructions will be explained briefly below.

Control Statements

The first line of the set of instruction must start with the word: NASAP which simply indicates the beginning of a circuit description. The letter N must be in the first column of the card. The word NASAP may be followed by any identifying information but only the first 50 columns of the card will be printed by the computer.

The end of the network element table is designated by the control card with the word: OUTPUT on it. Following the OUTPUT control card are the output specifications. The final control card appearing after the output specifications is the card with the word: EXECUTION.

This card signals the end of the description for the problem and starts the computer on the task of analysis. If it is desired to analyze several problems in one run, the EXECUTION card can be followed by another NASAP card, etc.

Network Element Table

General Format

The network elements and topology are described in a five column network element table. (Note that the example in Fig. 1.2 has only four columns since no dependent generators are used.) A brief description of each column follows:

Column 1: Identifies the element itself, i.e., V1, R3, etc.

Column 2 and 3: Define the nodes between which the element appears. The positive current direction is always from the first to the second node. Thus, for passive elements the first node is assumed positive with respect to the second, and for active elements the second node is assumed positive with respect to the first.

Column 4: Contains the value of the circuit component.

Column 5: Used only for dependent sources and will be described in detail later.

Input Voltage or Current

The first line after the NASAP control statement contains a description of the applied voltage or current. For example, the statement V1 1 2 1.0 in Fig. 1.2 indicates that V1 is a voltage of magnitude 1.0 applied between nodes 1 and 2 with node 2 being positive. The fourth column contains the value of the generated current or voltage and is usually set to 1.0. If any other value is used, it will act as a constant multiplier for the transfer function under consideration.

Resistors, Capacitors and Inductors

Resistors, capacitors and inductors are identified in column 1 by the letters R, C, and L, respectively. The letter is immediately followed by a one or two digit integer indicating the element number. The fourth column contains the component value. The value is a number followed by up to two letters which denote a multiplying factor according to the convention given in Table 1.1. The network element table in Fig. 1.2 follows directly from the circuit diagram of Fig. 1.1 (b).

Table 1.1. Units Following the Component Values

Letters Used in NASAP	Electrical Units	Multiplying Factor
No letter	Ohms	1
K	Kilohms	10^{3}
M	Megohms	10^{6}
F	Farads	1
UF	Microfarads	10^{-6}
PF	Picofarads	10^{-12}
H	Henries	1
MH	Millihenries	10^{-3}
UH	Microhenries	10^{-6}

Output Specifications

Once the electrical circuit model of the control system has been described, we need to specify what we want calculated. This specification always starts with a transfer function.

Transfer Function

The transfer functions are always specified as a ratio of the voltage across or current through a circuit element at the network output over the input voltage or current. The input voltage or current are specified by the letters VV1 or II1 respectively and either one must appear in the denominator of the transfer function specification. For example, VR3/VV1 used in Fig. 1.2 implies that we want to calculate the ratio of the voltage across R3 to the input voltage V1, i.e., the open circuit transfer voltage function. Since R3 was defined by the statement

R3 4 1 0.8333

node 4 is assumed to be positive with respect to node 1. If R3 has been defined by

R3 1 4 0.8333

the sign of the transfer function would be negative.

When the program is executed using the circuit description shown in Fig. 1.2 and the transfer function specification, the computer generates the output shown in Fig. 1.3. The first information printed under the heading "NUMBER OF LOOPS PER ORDER" describes the complexity of the circuit flowgraph by indicating the number of first order loops, the number of second order loops, etc. This information is only of theoretical interest and can usually be ignored during design. However, since the program prints this data, it will always be included for completeness.

Dependent Current and Voltage Sources

NASAP permits use of any of the four possible types of dependent generators (also known as controlled sources):

a) voltage controlled voltage sources (VCVS)
b) current controlled current sources (ICIS)
c) voltage controlled current sources (VCIS)
d) current controlled voltage sources (ICVS)

Each dependent generator involves two pairs of nodes and two elements: the dependent generator itself and a passive element which defines the controlling voltage or current. An example of each type of dependent source taken from [MO 1] is shown in Fig. 1.4. The node numbers are chosen to indicate that they are elements in a larger circuit and the dependent source is indicated by a diamond to distinguish it from an independent source which will be represented by a circle.

A description of the NASAP coding for dependent generators is in order. The VCVS shown in Fig. 1.4(a) is specified in NASAP by the instructions found immediately below the figure. The first instruction specifies a dependent voltage source, V3, from node 8 to node 3 (node 3 positive). The value of the source is 867.3 times the voltage across capacitor C2. The

```
NUMBER OF LOOPS PER ORDER

   1=     5
   2=     4



TRANSFER FUNCTION   VR3/VV1


                     (                                       )
                     (  5.00E 00      +1.00E 00 S            )
H(S)= 3.333E-01*---------------------------------------------------------
                     (                                                2 )
                     (  3.00E 00      +3.00E 00 S   +1.00E 00 S         )


ZEROS OF TRANSFER FUNCTION

ZEROS    REAL PART   IMAG. PART

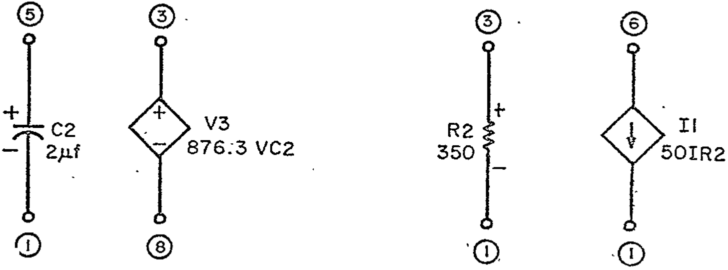
   1  -.50000E 01  0.


POLES OF TRANSFER FUNCTION

POLES    REAL PART   IMAG. PART

   1  -.15000E 01  -.86599E 00
   2  -.15000E 01   .86599E 00
```

Figure 1.3. Transfer Function for RLC Filter



V3 8 3 876.3 VC2

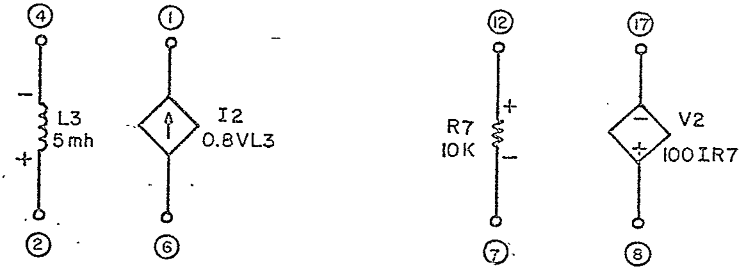
C2 5 1 2.0UF

(a) VCVS

I1 6 1 50 IR2

R2 3 1 350

(b) ICIS



L3 2 4 5.0MH

I2 6 1 0.8 VL3

(c) VCIS

R7 12 7 10K

V2 17 8 100 IR7

(d) ICVS

Figure 1.4. Examples of the Four Types of Dependent Generators and the Corresponding NASAP Instructions

second instruction specifies a capacitor from node 5 to node 1 of 2.0µf. Thus node 5 of the capacitor is assumed positive as noted in Section: Network Element Table, page 1.4. The polarity of the passive controlling element will always be shown on the circuit diagram to help insure that the nodes will be numbered in the correct sequence since the passive element may appear far from the controlle source in the network element table.

Similarly the ICIS shown in Fig. 1.4(b) is specified by the instructions below it. That is, a current source, I1, from node 6 to node 1 with a value of 50 times the current flowing through R2. R2 is defined so that node 3 is assume positive and thus the current is assumed to flow from node 3 to node 1.

The corresponding NASAP instructions for the VCIS shown in Fig. 1.4(c) and the ICVS shown in Fig. 1.4(d) are written to indicate that it does not matter whether the dependent generator or the passive controlling element is specified first in the network element table.

Sensitivity

The NASAP program is capable of determining the sensitivity of the transfer function to changes in the value of any single element. The request for a sensitivity analysis is optional and, if it is desired, is specified following the transfer function specification. This has the typical form:

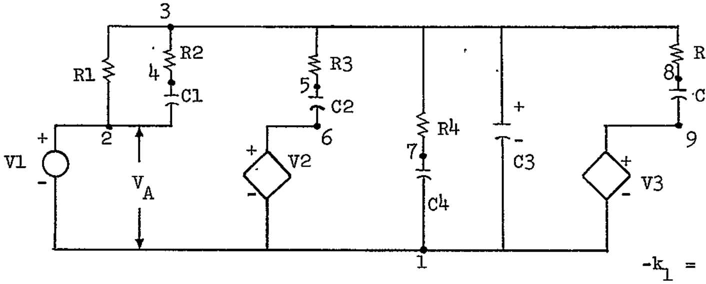
VR11/VV1/L3

This output specification requests the sensitivity of the transfer function VR11/VV1 with respect to element L3.

The frequency range over which the sensitivity is analyzed is the same as that for which the frequency response is calculated so they are both defined in the same output specification. For the RLC filter of Fig. 1.1, Moe [MO 1] tabulates and displays the response for FREQ -1.2 1.0 0.05.

Further Discussion of Dependent Sources

As stated in Gaertner's Coding manual, dependent sources can be related only to the voltages or currents of passive elements of the flowgraph. However, the controlling current or voltage in the circuit may be that of an active element. Such a situation is illustrated in Fig. 1.5 by the circuit realization for a normalized three-pole low-pass Chebyshev transfer function.



$-k_1 = k_2 = 2$

$V2 = k_1 V_A$ $V3 = k_2 V_{C3}$

Desired Transfer Function: VC3/VV1

Fig. 1.5

This circuit is an application of a general circuit introduced by Cooper and Harbourt [CO-1]. Note that the controlling voltage, V_A, of the dependent source V2, is the independent voltage source V1. However if the following card

```
V2  1  6  -2  VV1
```

where $k_1 = -2$

is included in the input list, the error message

INPUT CODING ERROR IN COLUMN 12

will result. The error is the second letter V since the program in only searching for the letter R, L, or C.

However, this difficulty is easily resolved by connecting a resistor R6 across the terminals of V1 (i.e., between nodes 2 and 1) and by making the voltage source dependent on the current through R6. The current through R6 (IR6) equals V1/R6.

Thus V1 = (R6)IR6. Since V2 = k_1 V1, then

$$V_2 = k_1 \cdot R6 \cdot IR6$$

Thus the dependency value between V2 and IR6 is $k_1 \cdot R6$. Note that R6 can have any nonzero numerical value. If, as is the case in this example, R6 is chosen to have a value of 4 ohms, input list must include the following two cards;

```
R6  2  1  4
                                   (see input list in Fig. 1.8)
V2  1  6  -8  IR6
```

The presence of this extra resistor neither increases the complexity of the flowgraph by generating additional loop sets nor does it affect any of the electrical properties of the original network.

This can be seen by comparison of Figs. 1.6 and 1.7. Fig. 1.6 is a partial flowgraph for the original network in Fig. 1.5. Fig. 1.7 is a partial flowgraph for the addition of R6 to the network.

Note that in both Figs. 1.6 and 1.7 there is only one path from the voltage node of V1 to the voltage node of V2.

The computer results for the circuit in Fig. 1.5 are given in Figure 1.8.

A similar procedure is followed when a dependent source is a function of the current of a current source. In this case a resistor R_s is added in series with

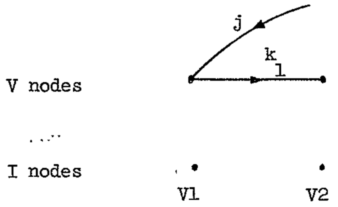


Fig. 1.6

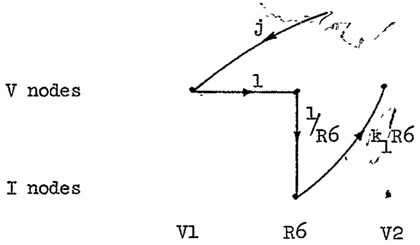


Fig. 1.7

the current source. The dependent source is then made a function of the voltage across this resistor--the dependency value being k_1/R_s where k_1 is the dependency value between the dependent source and the current source. As in the previous case addition of this resistor to the network under consideration, will in no way affect the flow graph or the electrical properties of the original network.

```
NASAP PROBLEM COOPER AND HARBOURT CCT

HERTZ
NONE

V1 1 2 1.
R1 2 3 4.08
C1 2 4  .1228F
R2 4 3 4.08
V2 1 6 -8 IR6
C2 6 5  .2455F
R3 5 3 4.08
C3 3 1 1F
R4 3 7 1.95
C4 7 1  .5125F
V3 1 9 2 VC3
R5 8 3  .307
C5 9 8 1.63F                      NUMBER OF LOOPS PER ORDER
R6 2 1 4
OUTPUT                                 1=   13
VC3/VV1                                2=   35
FREQ  -2.  -0.75  .01                  3=   37
EXECUTE                                4=   16
                                       5=    2
                                           ----
                                          103


TRANSFER FUNCTION    VC3/VV1


                   (                                        2           3 )
                   (  4.92E 03    +7.39E 03 S   +2.47E 03 S   +1.00E 00 S   )

H(S)= 1.984E-04*----------------------------------------------------------------------------------

                   (                                        2           3           4            5 )
                   (  9.77E-01    +3.93E 00 S   +6.16E 00 S   +6.19E 00 S   +3.98E 00 S   +1.00E 00 S   )


 ZERO OF TRANSFER FUNCTION            POLE OF TRANSFER FUNCTION

 ZERO   REAL PART   IMAG. PART        POLE   REAL PART   IMAG. PART

    1 -0.10006E 01  0.00000E 00          1 -0.24761E 00  0.96444E 00
    2 -0.19984E 01  0.00000E 00          2 -0.24761E 00 -0.96444E 00
    3 -0.24621E 04  0.00000E 00          3 -0.99910E 00  0.00000E 00
                                         4 -0.49402E 00  0.00000E 00
                                         5 -0.19957E 01  0.00000E 00
```

Figure 1.8

IB ERROR MESSAGES

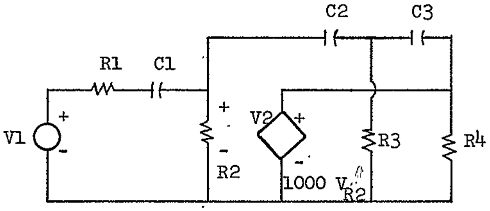
Here are provided a few comments on the meaning of selected error messages available in NASAP.

In subroutine GETCON, there are two error messages.

The message

ALGORITHM FAILURE REVIEW CIRCUIT CODING RULES

usually occurs when the NASAP program is unable to select a legitimate tree from the available resistors or capacitors. This case exists when the candidate type 1 or 2 elements form an R or C loop or when all elements connected to a node are type 6 or 7 elements; i.e., these excluded elements from a cut set. If, for example, the transfer function VR4/VV1 is desired for the circuit in Fig. 1.9.



$V2 = 1000\ VR_2$

Fig. 1.9

then the above error message will be printed out. This is necessitated by the loop formed by the elements V2 and R4 which must be in the tree. This difficulty can easily be resolved by calling for the transfer function VV2/VV1 which is identical to VR4/VV1.

The other error message is

FLOWGRAPH HAS TOO MANY CONNECTIONS EXECUTION TERMINATED

The connections of the primitive flowgraph are stored in the two dimensional connection matrix LCONC. The size of this matrix is such that it is assumed that no more than nine (edges) branches will emanate from a single flowgraph node. Thus if ten or more branches must emanate from a node, the above error message will be printed and execution stopped. This limitation is completely arbitrary and exists to restrict the size of the matrix LCONC. If one wishes to increase the maximum number of node connections, then the number 10 which appears twice on lines 4010, 5460, and 6220 and which appears once on line 5110 should be changed to a suitable integer N which is defined as the maximum number of node connections (M) plus one. Also the numeral 9 on line 6250 should be changed to the integer M. When these changes are made, the above error message will result when the number of node connections exceeds M instead of when the number of connections exceeds nine.

In subroutine FSORL, there is the error message

FLOWGRAPH FIRST ORDER LOOPS EXCEED 927.

This limitation on the actual loops of the flowgraph, the so-called first order loops, is completely arbitrary. It is necessary to store the first order loops since the higher order loops are determined from these first order loops. However, if this error message is printed out, it usually means that the tree selected was a rather poor choice. A more complete explanation of tree selection to minimize the number of loops of a flowgraph will be given later.

In subroutine BODE the error message is

PROGRAM RESTRICTS GRAPH TO 250 STEPS.

The subscript for the evaluation of the transfer function at various frequencies cannot exceed 250. Thus, if the information on the FREQ input card results in more than 250 frequency evaluations, the NASAP program will not perform any of the calculations included in subroutine BODE including the sensitivity calculations (if this has been called for). The program will jump to the time response calculation if this has been called for. Otherwise, execution will cease after the above error message.

In subroutine SENS there is the error message

SENSITIVITY PLOTS RESTRICTED TO 120 POINTS.

If the input information on the FREQ card results in more than 120 frequency points, the sensitivity results in tabular form will be printed out. However the 3-curve plot and the phase sensitivity plot will be deleted.

At present, extensive diagnostics and debugging capabilities are lacking. Input data can be written on a field-free format, and a circuit tree selection need not be specified by the user since it is done internally by the program. As is sometimes the case, when a proper tree cannot be found, a message is printed out indicating the difficulty and where it occurs in the circuit. Detailed discussion of the tree selection algorithm is found in Chapter II.

IC DOCUMENTATION OF NASAP 69/I VERSION USED AT THE UNIVERSITY OF PENNSYLVANIA

Summary

This section details the differences in input data and output results of the University of Pennsylvania version as compared to the standard NASAP 69/I as described in the Gaertner coding manual. [GA-1]

Input Cards

The NASAP version used at the Moore School of Electrical Engineering in the University of Pennsylvania (hereafter called the MSE-NASAP version) requires the user to supply two additional data cards immediately after the first data card (the NASAP PROBLEM card). The first of these cards indicates the frequency units to be used in the evaluation of the transfer function. There are four permissible entries for this card (starting in column 1)

RADIANS
HERTZ
CYCLES PER SECOND
NONE

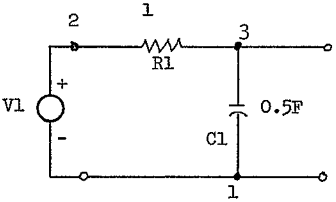
Only the first two letters (those underlined) must be correct since the NASAP program checks only these letters. Any other information can be included on this card since the program evaluates only the information contained in columns 1 and 2.

The second card indicates the type of time response desired by the user. There are four choices (starting in column 1)

IMPULSE RESPONSE
STEP RESPONSE
RAMP RESPONSE
NONE

The programs again only evaluate the data in columns 1 and 2.

Table 1.2 shows the input listing necessary on the MSE-NASAP version to find the step response for the circuit of Fig. 1.10. The response is the voltage across C1 and the excitation is a step voltage with a magnitude of 3 volts. The BODE plots of the specified transfer function are not being requested.



Transfer Function: $V_{C1}/VV1$

Fig. 1.10

Table 1.2 .

```
NASAP PROBLEM

NONE

STEP RESPONSE

V1 1 2 3.

R1 2 3 1

C1 3 1 0.5F

OUTPUT

VC1/VV1

TIME 2.0

EXECUTE
```

Printed Output Results

The MSE-NASAP version prints out much data on the primitive flowgraph which is used within the computer to determine the user-specified transfer function. The additional output data is printed between the printed listing of the input data cards and the NUMBER OF LOOPS PER ORDER table--both of which are printed by the standard NASAP package.

As an example, Fig. 1.11 gives printed output of a legitimate input listing for circuit 6 introduced in Hutton's report. [HU 1] Fig. 1.12 gives the 5 computer sheets of additional data printed out by the MSE-NASAP version based on the listing of Fig. 1.11. On the sheet immediately following the input list sheet are printed two matrices. The first matrix is the compressed cut set matrix. This matrix has $(b + 2)$ rows and $(\ell + 2)$ columns where

b = number of branches in the tree = $n - 1$

n = number of nodes in circuit

ℓ = number of links in the co-tree

It must be recalled that

$$b + \ell = \lambda$$

where λ is the number of elements in the circuit (For the circuit described in Fig. 1.11, $\lambda = 30$ and $n = 15$). This matrix is useful in that it shows which circuit elements have been selected as tree branches by the NASAP tree-selection algorithm.

As each circuit element is inputted, it is assigned an integer flag (beginning with unity). Thus for the circuit listing of Fig. 1.11.

```
NASAP PROBLEM 6 FROM HUTTTON REPORT


NONE
NONE


V1 1 2 1.
R6 7 1 390
R15 8 1 25
R1 2 3 6.8K
R2 3 4 6.8K
R3 4 5 100
R4 5 6 100K
R5 5 7 100
R7 6 1 1.2K
R8 9 1 100K
R9 9 7 100
R10 10 9 100
R11 10 12 6.8K
R12 12 11 6.8K
R13 13 1 3.6K
R14 6 8 100
R16 8 11 100K
R17 11 1 1K
C1 3 1 5UF
C2 4 6 .005UF
C3 10 1 .005UF
C4 10 12 10UF
C5 13 11 10UF
C6 12 1 20UF
C7 6 11 .012UF
I1 6 5 100 IR3
I2 1 9 100 IR10
I3 11 8 100 IR14
V2 1 14 1.0 VR7
V3 1 15 1.0 VR13
OUTPUT
VR17/VV1
EXECUTE
```

Fig. 1.11

```
          COMPRESSED CUTSET MATRIX

           TOP ROW REPRESENTS CO-TREE LINKS
        LEFT COLUMN REPRESENTS TREE BRANCHES
        INTEGERS REFER TO PLACE IN INPUT LISTING

         0  4  5  6  8 11 12 13 14 16 17 22 23 25 26 27 28  0
         1 -1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  2
         9  0 -1  0  1  0  0  0  0  1  0  0  0  1  0  0  0  2
        15  0  0  0  0  0  0  0  0  0  0 -1  1  0  0  0  0  2
        18  0  0  0  0  0  0  0 -1  0 -1  0 -1 -1  0  0  1  2
        29  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  2
        30  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  2
        19 -1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  3
        20  0 -1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  3
        21  0  0  0  0  0  1  1  0  0  0  1  0  0  0  0  0  3
        24  0  0  0  0  0  0 -1  1  0  0  0  0  0  0  0  0  3
         2  0  0  0 -1 -1  0  0  0  0  0  0  0  0  0  0  0  4
         3  0  0  0  0  0  0  0  0 -1  1  0  0  0  0  0 -1  4
         7  0  0 -1  1  0  0  0  0  0  0  0  0  0 -1  0  0  4
        10  0  0  0  0  1 -1  0  0  0  0  0  0  0  0 -1  0  4
         0  4  4  6  4  4  6  4  4  6  4  3  3  3  6  6  6  0

           FLOWGRAPH CONNECTION MATRIX

         1          4    0
         2         -9  -11    0
         3        -16   17    0
         4         19    0
         5          9  -19   20    0
         6          7  -20   26    0
         7         -6    8    0
         8          2   -7   -9    0
         9         -5    8   16   25   29    0
        10         11  -12    0
        11          2  -10    0
        12         10  -21   27    0
        13        -21   24    0
        14         18  -24    0
        15        -22   23   30    0
        16          3   -9   28    0
        17         -3   18    0
        18          1  -14  -17  -23  -25    0
        19         -4    5    0
        20         -5    6    0
        21         12   13   22    0
        22         15  -21    0
        23        -15   16    0
        24        -13   14    0
        25         -9   15    0
        26          7    0
        27         10    0
        28          3  -16    0
        29          0
        30          0
```

Fig. 1.12

```
FIRST ORDER LOOPS OF CONSECUTIVE NODES
  1    1   4  19   5   9   8   2 -11 -10 -12 -21  13  24  14  18   1
  2    1   4  19   5   9   8   2 -11 -10 -12 -21  22  15  23  18   1
  3    1   4  19   5   9  16   3  17  18   1
  4    1   4  19   5   9  16  28   3  17  18   1
  5    1   4  19   5   9  16  28 -18   1
  6    1   4  19   5   9  25  18   1
  7    1   4  19   5  20   6   7   8   2 -11 -10 -12 -21  13  24  14  18   1
  8    1   4  19   5  20   6   7   8   2 -11 -10 -12 -21  22  15  23  18   1
  9    1   4  19   5  20   6   7   8  -9  16   3  17  18   1
 10    1   4  19   5  20   6   7   8  -9  16  28   3  17  18   1
 11    1   4  19   5  20   6   7   8  -9  16  28 -18   1
 12    1   4  19   5  20   6   7   8  -9  25  18   1
 13    1   4  19   5  20   6  26   7   8   2 -11 -10 -12 -21  13  24  14  18   1
 14    1   4  19   5  20   6  26   7   8   2 -11 -10 -12 -21  22  15  23  18   1
 15    1   4  19   5  20   6  26   7   8  -9  16   3  17  18   1
 16    1   4  19   5  20   6  26   7   8  -9  16  28   3  17  18   1
 17    1   4  19   5  20   6  26   7   8  -9  16  28 -18   1
 18    1   4  19   5  20   6  26   7   8  -9  25  18   1
 19    2  -8   2
 20    2  -8  -7  -6 -20  -5   9  16   3  17  18 -14 -24 -13 -21  12  10  11   2
 21    2  -8  -7  -6 -20  -5   9  16   3  17  18 -14 -24 -13 -21  12  27  10  11
  2
 22    2  -8  -7  -6 -20  -5   9  16   3  17  18 -23 -15 -22 -21  12  10  11   2
 23    2  -8  -7  -6 -20  -5   9  16   3  17  18 -23 -15 -22 -21  12  27  10  11
  2
 24    2  -8  -7  -6 -20  -5   9  16  28   3  17  18 -14 -24 -13 -21  12  10  11
  2
 25    2  -8  -7  -6 -20  -5   9  16  28   3  17  18 -14 -24 -13 -21  12  27  10
 11    2
 26    2  -8  -7  -6 -20  -5   9  16  28   3  17  18 -23 -15 -22 -21  12  10  11
  2
 27    2  -8  -7  -6 -20  -5   9  16  28   3  17  18 -23 -15 -22 -21  12  27  10
 11    2
 28    2  -8  -7  -6 -20  -5   9  16  28 -18 -14 -24 -13 -21  12  10  11   2
 29    2  -8  -7  -6 -20  -5   9  16  28 -18 -14 -24 -13 -21  12  27  10  11   2
 30    2  -8  -7  -6 -20  -5   9  16  28 -18 -23 -15 -22 -21  12  10  11   2
 31    2  -8  -7  -6 -20  -5   9  16  28 -18 -23 -15 -22 -21  12  27  10  11   2
 32    2  -8  -7  -6 -20  -5   9  25  18 -14 -24 -13 -21  12  10  11   2
 33    2  -8  -7  -6 -20  -5   9  25  18 -14 -24 -13 -21  12  27  10  11   2
 34    2  -8  -7  -6 -20  -5   9  25  18 -23 -15 -22 -21  12  10  11   2
 35    2  -8  -7  -6 -20  -5   9  25  18 -23 -15 -22 -21  12  27  10  11   2
 36    2  -8  -9  16   3  17  18 -14 -24 -13 -21  12  10  11   2
 37    2  -8  -9  16   3  17  18 -14 -24 -13 -21  12  27  10  11   2
 38    2  -8  -9  16   3  17  18 -23 -15 -22 -21  12  10  11   2
 39    2  -8  -9  16   3  17  18 -23 -15 -22 -21  12  27  10  11   2
 40    2  -8  -9  16  28   3  17  18 -14 -24 -13 -21  12  10  11   2
 41    2  -8  -9  16  28   3  17  18 -14 -24 -13 -21  12  27  10  11   2
 42    2  -8  -9  16  28   3  17  18 -23 -15 -22 -21  12  10  11   2
 43    2  -8  -9  16  28   3  17  18 -23 -15 -22 -21  12  27  10  11   2
 44    2  -8  -9  16  28 -18 -14 -24 -13 -21  12  10  11   2
 45    2  -8  -9  16  28 -18 -14 -24 -13 -21  12  27  10  11   2
 46    2  -8  -9  16  28 -18 -23 -15 -22 -21  12  10  11   2
 47    2  -8  -9  16  28 -18 -23 -15 -22 -21  12  27  10  11   2
 48    2  -8  -9  25  18 -14 -24 -13 -21  12  10  11   2
 49    2  -8  -9  25  18 -14 -24 -13 -21  12  27  10  11   2
 50    2  -8  -9  25  18 -23 -15 -22 -21  12  10  11   2
 51    2  -8  -9  25  18 -23 -15 -22 -21  12  27  10  11   2
 52    2 -11   2
 53    2 -11 -10 -12 -21  13  24  14  18 -17  -3 -16  -9  -5  20   6   7   8   2
 54    2 -11 -10 -12 -21  13  24  14  18 -17  -3 -16  -9  -5  20   6  26   7   8
  2
 55    2 -11 -10 -12 -21  13  24  14  18 -17  -3 -16  -9   8   2
 56    2 -11 -10 -12 -21  13  24  14  18 -25  -9  -5  20   6   7   8   2
 57    2 -11 -10 -12 -21  13  24  14  18 -25  -9  -5  20   6  26   7   8   2
 58    2 -11 -10 -12 -21  13  24  14  18 -25  -9   8   2
 59    2 -11 -10 -12 -21  22  15  23  18 -17  -3 -16  -9  -5  20   6   7   8   2
 60    2 -11 -10 -12 -21  22  15  23  18 -17  -3 -16  -9  -5  20   6  26   7   8
  2
 61    2 -11 -10 -12 -21  22  15  23  18 -17  -3 -16  -9   8   2
 62    2 -11 -10 -12 -21  22  15  23  18 -25  -9  -5  20   6   7   8   2
 63    2 -11 -10 -12 -21  22  15  23  18 -25  -9  -5  20   6  26   7   8   2
 64    2 -11 -10 -12 -21  22  15  23  18 -25  -9   8   2
 65   -3 -15  -3
 66   -3 -15  -9  25  18 -17  -3
 67   -3 -15  28  -3
 68   -3 -15  28 -18 -17  -3
 69   -3  17  -3
 70    3  17  18 -25  -9  16   3
 71    3  17  18 -25  -9  16  28   3
 72   -4  19  -4
 73   -5   5  -5
 74   -5   9   8  -7  -6 -20  -5
 75   -5 -19  -5
 76   -5  20  -5
 77   -5  20   6   7   8  -9  -5
 78   -5  20   6  26   7   8  -9  -5
 79   -6   7  -6
 80    6 -20   6
 81   -6  26   7  -6
 82   -7   8  -7
 83   -8  -9  -8
 84   -9  16  -9
 85   -9  16  28 -18 -25  -9
 86   -9  25  -9
 87  -10  11 -10
 88   10 -12  10
 89   10 -12  27  10
 90   12 -21  12
 91   13 -21  13
 92  -13 -21  22  15  23  18 -14 -24 -13
 93  -13  24 -13
 94   13  24  14  16 -23 -15 -22 -21  13
 95  -14  13 -14
 96   14 -24  14
 97   15 -22  15
 98  -15  23 -15
 99  -17  18 -17
100   15 -23  15
101   16 -25  16
102  -21  22 -21
```

Fig. 1.12b

```
ORDER OF LOOPS      LOOP NUMBERS
     2=    33
     3=   143
     4=   347
     5=   500
     6=   396
     7=   124
     8=     8
     2=    33
     3=   145
     4=   368
     5=   577
     6=   500
     7=   167
     8=     8
     2=    35
     3=   174
     4=   500
     5=   821
     6=   698
     7=   232
     8=    15
        2                 5   52
     2=    53
     3=   257
     4=   734
     5=  1213
     6=  1107
     7=   500
     8=   103
     9=     8
     2=    69
     3=   406
     4=  1355
     5=  2665
     6=  2272
     7=  1562
     8=   500
     9=    27
     2=    20
     3=   500
     4=  1674
     5=  3236
     6=  3512
     7=  2072
     8=   525
     9=    32
        2                 4   17
        2                13   [illegible]
        2                15   [illegible]
     2=   209
     3=   987
     4=  2587
     5=  4137
     6=  4056
     7=  2902
     8=  1059
     9=   500
    10=   126
     2=   205
     3=   987
     4=  2592
     5=  4185
     6=  4334
     7=  3184
     8=  2222
     9=  1443
    10=   500
    11=    52
        2                22   [illegible]
        2                43   65
        2                [illegible]
        2                47   74
        2                52   [illegible]
     2=   454
     3=  2022
     4=  5629
     5= 17566
     6= 34546
     7= 46452
     8= 35015
     9= 20234
    10=  5255
    11=   532
        2                55   97
     2=   300
     3=  2440
     4=  8815
     5= 24494
     6= 49562
     7= 64759
     8= 53175
     9= 25403
    10=  6323
    11=   532
        2                [illegible]
        2                [illegible]
        2                [illegible]
        2                70   71
        2                72   [illegible]
        2                75   [illegible]
        2                77   [illegible]
        2                80   [illegible]
        2                83   [illegible]
        2                [illegible]
        2                [illegible]
```

Fig. 1.12c

```
NUMBER OF LOOPS PER ORDER
            1=    107
            2=   1060
            3=   6150
            4=  22993
            5=  53187
            6=  98838
            7=110358
            8=  78036
            9=  32424
           10=   6350
           11=    532
```

415,515

Fig. 1.12d

```
 TRANSFER FUNCTION  [illegible]17/VVI


                                              2              3              4              5 )
                  (  1.33E 16     +3.92E 15 S   +2.63E 14 S   +3.47E 12 S   -4.25E 06 S   +1.06E 00 S   )
 H(S)= 9.659E 05*--------------------------------------------------------------------------------------------------
                                              2              3              4              5              6             7 )
                  (  7.32E 21     +8.08E 20 S   +5.69E 19 S   +3.72E 18 S   +8.27E 16 S   +3.28E 12 S   +9.06E 06 S   +1.00E 00 S    )



  ZERO OF TRANSFER FUNCTION

  ZERO   REAL PART   IMAG. PART

    1 -0.56889E 02  0.00000E 00
    2  0.10997E 07  0.00000E 00
    3 -0.47090E 01  0.00000E 00
    4 -0.14284E 02  0.00000E 00
    5  0.31554E 07  0.00000E 00


  POLE OF TRANSFER FUNCTION

  POLE   REAL PART   IMAG. PART
```

Element	Integer Flag
V1	1
R6	2
R15	3
⋮	⋮
V3	30

An element in the compressed cutset matrix can have one of three values -1, 0, or +1. The top and bottom rows and the left-most and right-most columns of the compressed cutset matrix are used for identification purposes described in the following paragraphs.

The non-zero entries in the left-most column are the integer flags of those circuit elements which have been selected as tree branches. The corresponding entry in the right-most column gives the tree hierarchy value of the circuit element (see description of tree-selection algorithm). Hence in Fig. 1.12 we see that V1(flag = 1) and R7(flag = 9) are tree branches and both have a hierarchy value of 2.

Similarly the non-zero entries in the top row are the integer flags of the circuit elements that are links of the co-tree. The bottom row gives the hierarchy value of these elements. For instance, R1(flag = 4) and R2(flag = 5) are links of the co-tree and have hierarchy values of 4.

The information contained in the compressed cutset matrix is used to develop the flowgraph connection matrix which is a mathematical description of the primitive flowgraph. This matrix consists of λ (= number of elements) rows. The entries in the left-most column represent either the voltage node or the current node of the circuit elements (depending upon whether the elements are branches or links respectively) identified by the integer flags numbered consecutively from 1 to λ.

If the left-most entry of a particular row of the flowgraph connection matrix represents a voltage node of an element, then each of the other entries of this row indicates a connection from the voltage node of the left-most entry to the voltage node of the corresponding entry. There is one exception--if one of these entries refers to a voltage controlled current source, then there is a connection from the voltage node of the left-most entry to the current node of this particular entry. Conversely, if the left-most entry of a particular row of the flowgraph connection matrix represents a current node of an element, then each of the other entries of this row indicates a connection from the current node of the left-most entry to the current node of the corresponding entry. There is one exception--if one of these entries refers to a current controlled voltage source, then there is a connection from the current node of the left-most entry to the voltage node of this particular entry. A few examples from Figure 1.12 will clarify this description. The second row of the flowgraph connection matrix in Figure 1.12 contains

2 -8 -11 0

From the cutset matrix it is seen that the element with integer flag 2 (i.e. R6) is a tree branch. Thus there are two connections (with a value -1) from the voltage node of R6 - one to the voltage node of R5(flag = 8) and the other to the voltage node of R9(flag = 11). The zero indicates the end of the row.

As a second example, the fifth row of this connection matrix contains

5 9 -19 20 0

Since R2(flag = 5) is a link in the co-tree, there is a connection with a value of +1 from current node of R2 to the current node of R7(flag = 9). Also there is a connection (value = -1) to the current node of C1(flag = 19) and one (with value = -1) to the current node of C2(flag = 20). Use of Ohm's Law yields the passive element (R,L,C) joining the voltage and current nodes, since the voltage across an element is related to the current flowing through the element by means of the element's impedance.

The next printed output consists of all the closed loops (first order loops) contained in the flowgraph determined by means of the flowgraph connection matrix. The heading of this output block of data is

FIRST ORDER LOOPS BY CONSECUTIVE LOOPS

Each loop is defined by the nodes contained in the loop. This output shows the order in which the nodes have been found by NASAP for each loop. Each loop is given an identification integer starting with unity. This identification integer is shown in the left-most columns. The remaining integers in a given row refer to the integer flags of the circuit elements.

In Figure 1.12 we see that the flowgraph under discussion has a total of 102 first order loops. Let us examine one of these loops more closely--say loop 81 defined by

-6 26 7 -6

The first and last integers are always identical since it is the starting point of the loop. From the input listing we have

Element	Integer Flag
R3	6
I1	26
R4	7

Each loop is found by the path-finding procedure on the flowgraph connection matrix (for details see the Potash-McNamee User's Manual). From the cutset matrix we note that R4 is a tree branch while I1 and R3 are co-tree links.

To illustrate this we now show how this loop 81 can be found from the flowgraph connection matrix. Start at the current node of R3 (R3 is a link). There is a connection from this node to the current node of the current source I1 (i.e., an integer 26 in row 6 of the connection matrix). There is a connection from the current node of I1 to the current node of R4 (i.e., an integer 7 in the 26th row of the connection matrix). Since R4 is a passive element, there is a connection (with a value = R4) from the current node of R4 to the voltage node of R4 (recall that R4 is a tree branch). From the 7th row of the connection matrix (integer flag of R4 = 7) we observe that -6 is an entry. Thus there is a connection from the voltage node of R4 to the voltage node of R3 (with a value of -1). Since R3 is a passive element that is a co-tree link, there is a connection (with value = $^1/R3$) from the voltage node of R3 to the current node of R3 (the starting point)--thus completing the loop.

The higher order loops, the sets of non-touching first order loops, are easily found by assigning each first order loop an integer value based upon the nodes contained in the loop (each node is identified by the circuit element integer flag). This integer value is stored in the one-dimensional array LOOP defined by

$$\text{LOOP}(J) = \sum_{k=1}^{n} 2^{|N_k| - 1}$$

where J is the loop identification integer

N_k refers to the kth flowgraph node in the Jth loop

n = number of flowgraph nodes in the Jth loop

As an example, for the loop numbered 79 in Fig. 1.12, we have

$$\text{LOOP}(79) = 2^{|-6| -1} + 2^{|7| -1} = 2^5 + 2^6$$

$$\text{LOOP}(79) = (96)_{10}$$

$$= (110000)_2$$

(Note: see Potash-McNamee Manual for details of how higher-order loop are found by use of the array LOOP).

Finally the sheet with the heading

ORDER OF LOOP LOOP NUMBERS

is printed out to assist the user in locating the point of termination if the number of flowgraph loops is so large that the allowable computer time is used up before completion of the analysis.

The information of this sheet is presented in this manner:

1. Every 50th second order loop is printed out with the identification integers of the first order loop comprising the second order loop.
2. When the number of some jth order loop ($j = 2,3,4,\ldots$) equals 500, the number of all loops of all order at this point of the loop enumeration procedure. (Note: see the Potash-McNamee Manual for details of the HIGORL subroutine which determines the higher order loops from the first order loops).

From Figure 1.12 we see that the output data

```
2     18     88
```

is the fourth of this type in the output data. Consequently the second order loop consisting of loop 18 and loop 88 is the 200th second order loop found by the NASAP subroutine called HIGORL. (Note: from the list of first order loops, loop 88 consists of nodes 10 and 12 which do not appear as nodes in loop 18). Immediately following this line of output, we observe that the number of 9th order loops found by NASAP equals 500. When this occurred, NASAP had found 209 2nd order loops, 987 3rd order loops, etc.

It should be noted that, if a flowgraph has less than 50 second order loops and the number of each of the nth ($n = 3,4,5,\ldots$) order loops of this flowgraph is less than 500, only the heading

```
ORDER OF LOOP          LOOP NUMBERS
```

will be printed in the MSE-NASAP version.

In order to find all the higher order loops and at the same time to avoid repeating any of these loops, the HIGORL subroutine selects each first order loop in order of its identification integer and determines all the higher order loops formed by this selected first order loop and those loops having identification integers greater than that of the selected loop (see Potash-McNamee manual for details).

With this procedure in mind, we can use the higher-order loop data printed out by the MSE-NASAP version to determine approximately where the HIGORL subroutine was in the higher order loop finding process if execution is terminated before the desired transfer function is obtained. This will enable the user to determine whether the problem should be executed with a longer running time specified or should be cancelled since the generated flowgraph is too complicated (with regard to running time) for the computer in use.

As an example, let us assume that the printed output of Figure 1.12 ended with the line

2 68 88

There was no sheet with the heading

NUMBER OF LOOPS PER ORDER

and no sheet listing the specified transfer function. From the list of first order loops, we see that this flowgraph contains 102 loops. From the above output line, we see that the loop under selection by HIGORL is the loop numbered 68. In other words, when this line was printed loops 69 through 101 have not yet been selected as the starting loop in the higher order loop finding process.

Since the number of higher order loops generated by the starting loop decreases as the identification integer of the starting loop increases, knowledge that loop 68 (in a group of 102 loops) is the present starting loop enables the user to decide whether or not to rerun the problem. It must be noted that the last line printed does not mean that this was the last loop set found by the HIGORL subroutine before termination. It is possible that as many as 49 more

2nd order loops and many more nth (n = 3,4,5...) order loops have been found since the printing of the last output line and before the termination.

Following this output data, the output results of the MSE-NASAP version conform with the output data of NASAP 69/I. One remaining minor difference is the order of the sensitivity output data.

The order for NASAP 69/I is:

Table of Sensitivities of ReH, ImH, |H|, Phase H as a function of frequency.

2. Table of the Logarithm of the above sensitivities as a function of frequency.
3. Plot of 3 Sensitivity Expressions.
4. Plot of the Logarithm of the Sensitivity.
5. Table of the Sensitivity Function as a function of frequency.
6. Plot of the Logarithm of the Absolute Value of the Sensitivity function.
7. Plot of the Phase of the Sensitivity Function.
8. Pole and Zero Sensitivities.

On the other hand, the order of the MSE-NASAP version is:

1. Table of Sensitivity Function as a function of frequency.
2. Plot of Logarithm of Absolute Value of Sensitivity Function.
3. Plot of Phase of Sensitivity Function.
4. Table of Sensitivities of ReH, ImH, |H|, Phase H as a function of frequency.
5. Table of Logarithms of the above sensitivities as a function of frequency
6. Plot of 3 Sensitivity Expressions.
7. Plot of the Logarithm of the Sensitivity of the Phase H.
8. Pole and Zero Sensitivities.

CHAPTER II

NASAP TREE SELECTION ALGORITHM--USER OPTIONS

IIA GENERAL DESCRIPTION

Although the graph representation of an electric network usually has a large number of possible trees (i.e., a structure containing n-1 branches which interconnect the n nodes of the circuit without forming any closed paths), NASAP has an algorithm that selects only a particular tree configuration. This tree is the basis for subsequent circuit analysis.

Each electrical circuit element is assigned a Type number as shown.

Independent voltage source	1
Dependent voltage source	2
Capacitor	3
Resistor	4
Inductor	5
Dependent current source	6
Independent current source	7

Elements of type 1 and 2 are always included in the tree while type 6 and 7 elements are never included in branches of a tree. If there are not enough elements of type 1 and 2 to form a tree, a search is made of type 3 elements (i.e. capacitors) starting with the first capacitance listed in the input data and working down the input list. If a tree is still not found after searching through all the type 3 elements, a similar search is made of all type 4 elements (selecting those type 4 elements that do not form closed paths and neglecting those that do). If a tree does not result, a search is made of type 5 elements. If a tree is not found after this search, an error message will be printed.

The element type categories 2 and 6 need a further explanation. The elements in category 2 include not only dependent voltage sources, but also those elements whose voltages control the voltage or current of some dependent source or whose voltage is the required output variable or input variable. For example, the following is a legal NASAP input record.

I3 3 4 5.2 VR3

Since the voltage across resistor R3 controls the dependent current source I3, resistor R3 will be assigned element type 2 not element type 4. Also the following is a legal NASAP output record

VL5/VV1/R2

Since the voltage across inductor L5 is the desired output variable, Element type 2 not element type 5 will be assigned to inductor L5. Similarly since V1 is the input, it will be assigned element type 2 not element type 1.

Similarly the elements in category 6 include not only dependent current sources but also those elements whose currents control the voltage or current of some dependent source or whose current is the required output variable or input variable. In

I3 3 4 5.2 IR3

Resistor R3 is assigned element type 6 since the current through R3 controls the dependent current source I3. In

IC5/II2

capacitor C5 is assigned element type 6 since the current through C5 is the required output current. Also, I2 will be a type 6 element (not type 7) since it is the input variable.

Due to the search technique that reads down the input list looking for elements to be branches of a tree, the actual tree selected by NASAP can be varied simply by rearranging the order in which the elements are placed in the input list (NOTE: There is no restriction as to the order in which the elements are listed in NASAP). However, not all of the possible tree configurations can be selected by NASAP due to the requirement that all type 1 and 2 elements must be included in the tree while all type 6 and 7 elements cannot be included in the tree.

IIB ILLUSTRATIVE EXAMPLE

The complexity of the flowgraph is greatly dependent on the particular tree. By complexity is meant the number of loop sets present in the flowgraph. The values of these loop sets are used in NASAP to calculate the transfer function by means of Mason's formula as extended by Happ for the closed signal-flow graph. Since much time is consumed by the NASAP algorithms in finding the loop sets, selection of the tree that minimizes the number of loop set can save computer time as well as increase the accuracy of the coefficients of the polynomials in the transfer function.

In Fig. 2.1 is shown a transistor with known h-parameters. The input resistance of the circuit is to be calculated.

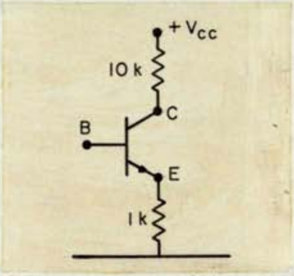


Fig. 2.1. A Common Emitter Transistor

The NASAP equivalent circuit model is given in Fig. 2.2.

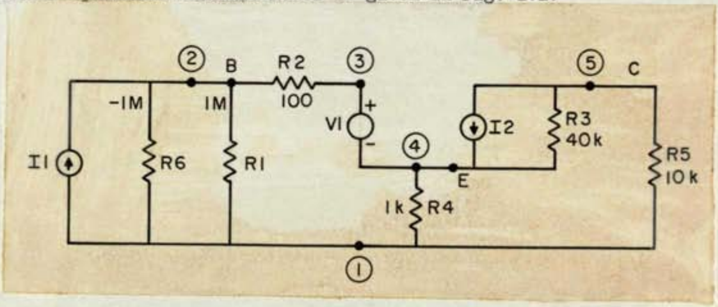


Fig. 2.2. Equivalent Circuit for Transistor in Fig. 2.1.

I1 is the independent current source. R1 and R6 are positive and negative resistances of equal numberical value. These resistances, when added to the network, yield an element whose voltage equals the input voltage and at the same time does not load the circuit (the parallel connection of a positive and negative resistance yields an infinite resistance). Thus the required transfer function for the input impedance is

VR1/II1

The network in Fig. 2.2 has five nodes; thus the tree for this circuit must have four elements. The elements R1, V1, and R3 are type 2 elements and must be included in the tree while I1, I2, and R2 are type 6 elements and cannot be part of the tree. Thus one more element must be selected to form the tree. The partially completed tree structure is shown in Fig. 2.3.

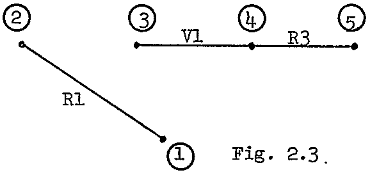


Fig. 2.3.

R6 cannot be used as a tree element since it forms a closed path with R1 which is an element in tree. However either R5 or R4 can be used to form a tree. Thus the NASAP tree selection algorithm will pick that resistance which is listed first in the input data. Thus if the input data is listed as follows,

```
I1  1  2  1.
I2  5  4  100   IR2
V1  4  3  0.0001  VR3
R1  2  1  1M
R2  2  3  1K
R3  5  4  40K
R5  5  1  10k
R4  4  1  1K
R6  2  1  -1M
OUTPUT
VR1/II1
```

with ±1M resistors

```
NASAP   IIT PROBLEM

NONE
NONE

I1 1 2 1.
I2 5 4 100 IR2
V1 4 3 0.0001 VR3
R1 2 1 1M
R2 2 3 1K
R3 5 4 40K
R4 4 1 1K
R6 2 1 -1M
R5 5 1 10K
OUTPUT
VR1/II1
EXECUTE

NUMBER OF LOOPS PER ORDER

    1=     9
    2=    16
    3=     4

TRANSFER FUNCTION    VR1/II1


                      (
                      (   1.00E 00       )
H(S)= 8.033E 04*-----------------
                      (                  )
                      (   1.00E 00       )


 ZERO OF TRANSFER FUNCTION

          NONE


 POLE OF TRANSFER FUNCTION

          NONE
```

Fig. 2.7

```
NASAP   IIT PROBLEM
```

with ± 1M resistors

```
NONE
NONE

I1 1 2 1.
I2 5 4 100 IR2
V1 4 3 0.0001 VR3
R1 2 1 1M
R2 2 3 1K
R3 5 4 40K
R5 5 1 10K
R4 4 1 1K
R6 2 1 -1M
OUTPUT
VR1/II1
EXECUTE

NUMBER OF LOOPS PER ORDER

     1=     14
     2=     29
     3=     10

TRANSFER FUNCTION    VR1/II1

                         (                    )
                         (   1.00E 00         )
H(S)= 8.333E 04*------------------
                         (                    )
                         (   1.00E 00         )

 ZERO OF TRANSFER FUNCTION

          NONE

 POLE OF TRANSFER FUNCTION

          NONE
```

Fig. 2.5

R5 will be included in the tree elements. The chosen tree is shown in Fig. 2.4.

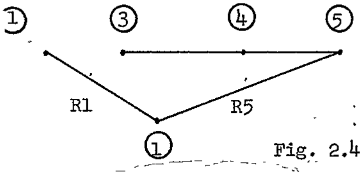


Fig. 2.4

The output results are given in Fig. 2.5.

However, if the input listing format is given as

```
11  1  2  1.
I2  5  4  100  IR2
V1  4  3  0.0001  VR3
R1  2  1  IM
R2  2  3  IK
R3  5  4  40k
R4  4  1  1K
R6  2  1  -1M
R5  5  1  10K
OUTPUT
VR1/II1
```

(Note that R5 is now the last element described)

the tree selected by NASAP will now contain R4 as shown in Fig. 2.6.

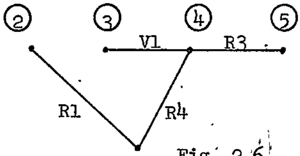


Fig. 2.6

The output results are shown in Fig. 2.7.

Note that both coding formats yield the same transfer function, as expected. However, the tree with R4 yields a flowgraph with 29 loop sets as compared with 53 loop sets in the flowgraph obtained from the tree with R5. Thus the number of loop sets is almost halved by the proper choice of a tree for this circuit configuration. This reduction of loop sets is apparent in the execution time. The tree with R5, NASAP required an execution time of 24.3 seconds on the RCA Spectra 70/46 while the execution time for the tree with R4 was 22.6 seconds, a saving of 1.7 seconds.

IIC THE OPTIMUM TREE

There is a procedure to select the optimum tree (optimum in the sense that such a tree will minimize the total number of loops in the primitive flowgraph) that does not require a great deal of effort on the part of the NASAP user. The details and proof of this procedure are given in reference [ZO-1]. The procedure begins by putting all type 1 and 2 elements in the tree. Next all capacitors (type 3 elements) are included in the tree. If a capacitor forms a loop with some type 1 or 2 elements, it is removed from the tree. If two capacitors form a loop with some other tree branches, then the user arbitrarily picks one of these capacitors to be a tree branch--realizing that the capacitor picked to be the tree branch must precede the other capacitor in the input lists. All type 6 and 7 elements and any element that forms a loop with the chosen tree branches are put in the co-tree. Note that if the sum of the type 1, 2, and 3 elements equals the number of nodes minus one and if these elements do not form any loops, then the NASAP tree selection algorithm will have picked a tree after a search of all capacitors. The user thus will not be able to vary the tree.

If, on the other hand, the number of resistors in the circuit is greater than the number of elements necessary to complete the tree, the NASAP user will have some flexibility in the type of tree selected by NASAP by permutting the resistor input cards.

Once the user has selected a tree, the optimum tree search procedure goes as follows:

Each link forms a loop with some of the branches of the tree. For each link, the number of tree branches in each loop is recorded as well as the specific branches that form the loop. The Branch Count is the sum of the number of tree branches in each loop. The Circuit Count is the sum of the number of loops for a specified tree branch. The Branch Count will always equal the Circuit Count. However the tree that yields a smaller Branch Count will yield a flowgraph with fewer loops.

As an example let us reconsider the input impedance circuit given above. Recall that it was shown that V1, R1, and R3 must be included in the tree while I1, I2, R2 and R6 must be links in the co-tree. However either R4 or R5 can be in the tree depending on the order of the input cards. If R4 is a branch of the tree, then there are five links in the co-tree - I1, I2, R2, R6, and R5.

The link I1 forms a loop with branch R1

The link I2 forms a loop with branch R3

The link R2 forms a loop with branches R1, V1, R4

The link R6 forms a loop with branch R1

Finally, the link R5 forms a loop with branches R3, R4. Table 2.1 summarizes this information. ("Branch" refers to a tree branch.)

Table 2.1

Links	I1	I2	R2	R6	R5	branch count
Branches/Loop	1	1	3	1	2	8
Branches		V1	R1	R3	R4	circuit count
Loops/Branch		1	3	2	2	8

Note that the Branch Count equals the Circuit Count, as required.

However, if R5 is made a tree branch, then the co-tree links are I1, I2, R2, R6, R4.

The link I1 forms a loop with branch R1

The link I2 forms a loop with branch R3

The link R2 forms a loop with branches R1, V1, R3, and R5

The link R6 forms a loop with branch R1

The link R4 forms a loop with branches R3 and R5

Table 2.2 summarizes this information.

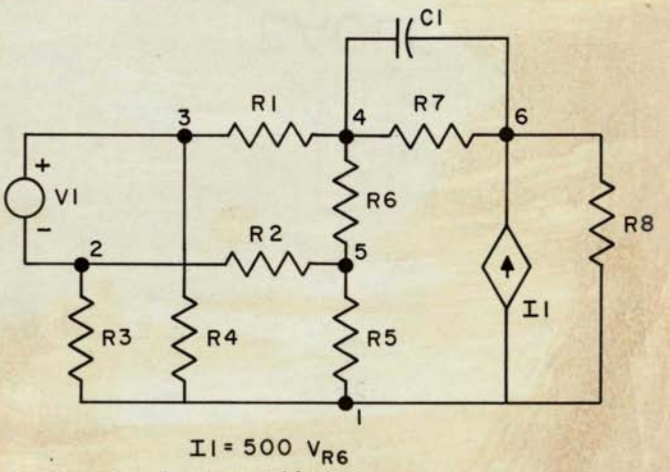
Table 2.2

Links	I1	I2	R2	R6	R4	Branch count
Branches/Loop	1	1	1	4	2	9
Branches		V1	R1	R3	R5	Circuit count
Loops/Branch		1	3	3	2	9

Both the Branch Count and Circuit Count equal nine. Since the Branch Count for the tree containing R4 is less than that for the tree with R5, then the number of loops for the flowgraph formed from the tree with R4 will be less than that obtained from the tree with R5. As noted above, there are 29 loop sets including 9 first order loops in the flowgraph formed from the R4 tree as opposed to the 53 loop sets including 14 first order loops in the flowgraph formed from the R5 tree.

Illustrative Example

The circuit shown in Fig. 2.8 from [MO-1] illustrates the case when different trees yield the same branch count.



Desired transfer function VR8/VV1

Fig. 2.8. An EOG Filter from [MO-1]

It is apparent that V1, R6, and R8 (type 2 elements) and C1 must be branches of the tree while I1, R5, and R7 must be links in the co-tree since they form loop with the above mentioned tree branches. For the six node circuit in Fig. 2.8 a tree has five branches. Since four of these branches have already been designated the last branch can be chosen from resistor R1, R2, R3 or R4. Thus four different trees can be formed by the NASAP algorithm. The question remains which of these four trees will yield the fewest loops in the flowgraph.

If R2 is selected as the fifth tree branch, then the links of the co-tree are I1, R5, R7, R1, R3, and R4. Table 2.3 gives the pertinent information on the number of branches in each loop formed by each link and the number of loops that each branch is a part of.

Table 2.3: Tree with R2

Links	I1	R5	R7	R1	R3	R4
Branches/Loop	1	3	1	3	4	5
Branches	V1	R6	R8	C1	R2	
Loops/Branch	2	4	4	4	3	

The Branch Count for the R2 tree is 17.

If instead we let R4 be a branch of the tree; then the links become I1, R5, R7, R1, R2, and R3. Table 2.4 gives the corresponding branch and loop information.

Table 2.4: Tree with R4

Links	I1	R5	R7	R1	R2	R3
Branches/Loop	1	3	1	3	5	2
Branches	V1	R6	R8	C1	R4	
Loops/Branch	2	2	4	4	3	

Here the Branch Count is 15. Since the branch count for the R4 tree is smaller, then this tree is a better choice than the R2 tree in terms of fewer loops for the flowgraph.

If we next let R3 be a tree branch, the co-tree links become I1, R5, R7, R1, R2, and R4. Table 2.5 gives the branch and loop data for this case.

Table 2.5: Tree with R3

Link	I1	R5	R7	R1	R2	R4
Branches/Loop	1	3	1	4	4	2
Branch	V1	R6	R8	C1	R3	
Loop/Branch	2	2	4	4	3	

Note that the branch count is 15, the same as that obtained for the R4 tree.

Finally Table 2.6 gives the branch and loop data for the case when the tree contains the element R1.

Table 2.6: Tree with R1

Link	I1	R5	R7	R2	R3	R4
Branches/Link	1	3	1	3	4	3
Branch	V1	R6	R8	C1	R1	
Loops/Branch	2	2	4	4	3	

The Branch Count for the R1 tree is also 15.

Thus three of the four trees have the same low Branch count. Thus the other criteria, the Branch product and the Loop Product, must be used to determine which of these three trees will yield the flowgraph with the fewest loops. The Loop Product, defined as the product of the number of loops involving each branch, is the same for the three trees; namely 192. However, the Branch Product, defined as the product of the number of tree branches in each loop formed by each co-tree link, is different in each case. The Branch Product is 1x3x1x3x5x2 = 90 for the R4 tree while the Branch Product for the R3 tree is 1x3x1x4x4x2 = 96. The Branch Product for the R1 tree is 1x3x1x3x4x3 = 108. It is found that the tree having the smallest Branch Product (R4) yields the flowgraph with the fewest

Note that the Branch Counts and Branch Products of both trees are equal. Since the Loop Product of the C2-C5 tree is smaller, it would seem that this tree would yield the flowgraph with the fewer loops.

However, the modified Branch count of the C3-C6 tree is smaller than that of the C2-C5 tree. This criterion indicates that the C3-C6 tree gives the fewer flowgraph loops. This, in fact, is the case as the computer results of Figs. 2.18, 2.19, 2.20, 2.21 show. The tree with C3 and C6 generates a flowgraph of 969 loops including 19 first order loops while there are 1529 loops including 25 first order loops in the flowgraph formed from the C2-C5 tree.

From Dunn and Chan, [DU 1] it has been shown that the star tree (a tree in which all the branches have a common node) yields the minimum number of flowgraph loops. The more star-like the tree structure, the fewer the number of loops in the corresponding flowgraph.

In the circuit of Fig. 2.17, all of the type 2 elements are connected together at node 1. Thus 5 of the 9 branches are joined at a single node. Examination of this circuit reveals that two other elements (R4 and R8) are also connected to node 1. If these resistors were branches of the tree, then 7 of the 9 tree branches would be connected to a single node--a tree structure that is definitely more star-like than the two trees described above. However, since R4 and R8 are resistors and type 4 elements, they are not considered for eligibility as tree branches by the NASAP tree selection algorithm until all type 3 elements (i.e. capacitors) are considered. As is shown above, the capacitors are so connected in the circuit that they do form legitimate trees (in fact, 4 trees depending upon the input listing). Thus R4 and R8, as type 4 elements, can never be tree branches.. We next indicate how to overcome this problem.

If somehow the voltages across R4 and R8 controlled some dependent sources, they would become type 2 elements and would therefore be branches of the

```
NASAP FROM 1-STAGE EOG FILTER MOE


NONE
NONE


V1 2 3 1.0
R4 3 1 1K
R2 2 5 10K
R3 2 1 1K
R1 3 4 10K
R5 5 1 500K
R6 5 4 1M
R7 4 6 500K
R8 6 1 200
C1 4 6 0.015UF
I1 1 6 500 VR6
OUTPUT
VR8/VV1                 NUMBER OF LOOPS PER ORDER
EXECUTE
                            1=   72
                            2=  151
                            3=   95
                            4=   14
                               ------
                                332


  TRANSFER FUNCTION    VR8/VV1



                    (                                )
                    (   1.24E 04       +1.00E 00 S   )

  H(S)=-5.382E-01*-----------------------------------

                    (                                )
                    (   1.33E 02       +1.00E 00 S   )



   ZERO OF TRANSFER FUNCTION

   ZERO    REAL PART    IMAG. PART


     1 -0.12388E 05   0.00000E 00



   POLE OF TRANSFER FUNCTION

   POLE    REAL PART    IMAG. PART


     1 -0.13346E 03   0.00000E 00
```

Fig. 2.9 and 2.10: R4 in tree

```
NASAP FROM 1-STAGE EOG FILTER MOE


NONE
NONE


V1 2 3 1.0
R3 2 1 1K
R2 2 5 10K
R1 3 4 10K
R4 3 1 1K
R5 5 1 500K
R6 5 4 1M
R7 4 6 500K
R8 6 1 200
C1 4 6 0.015UF
I1 1 6 500 VR6
OUTPUT
VR8/VV1                    NUMBER OF LOOPS PER ORDER
EXECUTE
                              1=    74
                              2=   151
                              3=    93
                              4=    16
                                   334


TRANSFER FUNCTION   VR8/VV1



                  (                               )
                  (  1.24E 04      +1.00E 00 S    )

H(S)=-5.382E-01*----------------------------------

                  (                               )
                  (  1.33E 02      +1.00E 00 S    )



 ZERO OF TRANSFER FUNCTION

 ZERO   REAL PART    IMAG. PART


    1 -0.12388E 05  0.00000E 00



 POLE OF TRANSFER FUNCTION

 POLE   REAL PART    IMAG. PART


    1 -0.13346E 03  0.00000E 00
```

Figs. 2.11 and 2.12: R3 in tree

```
NASAP FROM 1-STAGE EOG FILTER MOE


NONE
NONE


V1 2 3 1.0
R1 3 4 10K
R2 2 5 10K
R3 2 1 1K
R4 3 1 1K
R5 5 1 500K
R6 5 4 1M
R7 4 6 500K
R8 6 1 200
C1 4 6 0.015UF
I1 1 6 500 VR6
OUTPUT
VR8/VV1                    NUMBER OF LOOPS PER ORDER
EXECUTE
                                  1=    80
                                  2=   163
                                  3=   104
                                  4=    19
                                      ----
                                       366


 TRANSFER FUNCTION    VR8/VV1



                  (                                 )
                  (   1.24E 04        +1.00E 00 S   )
 H(S)=-5.382E-01*-----------------------------------
                  (                                 )
                  (   1.33E 02        +1.00E 00 S   )



  ZERO OF TRANSFER FUNCTION

  ZERO   REAL PART    IMAG. PART

    1 -0.12388E 05  0.00000E 00



  POLE OF TRANSFER FUNCTION

  POLE   REAL PART    IMAG. PART

    1 -0.13345E 03  0.00000E 00
```

Figs. 2.13 and 2.14: R1 in tree

```
NASAP FROM 1-STAGE EOG FILTER MOE

NONE
NONE

V1 2 3 1.0
R2 2 5 10K
R1 3 4 10K
R3 2 1 1K
R4 3 1 1K
R5 5 1 500K
R6 5 4 1M
R7 4 6 500K
R8 6 1 200
C1 4 6 0.015UF
I1 1 6 500 VR6
OUTPUT
VR8/VV1
EXECUTE
                          NUMBER OF LOOPS PER ORDER

                             1=    186
                             2=    369
                             3=    210
                             4=     31
                                   796

TRANSFER FUNCTION    VR8/VV1



                    (                                 )
                    (  1.24E 04      +1.00E 00 S      )
H(S)=-5.382E-01*-----------------------------------
                    (                                 )
                    (  1.33E 02      +1.00E 00 S      )



  ZERO OF TRANSFER FUNCTION

  ZERO   REAL PART   IMAG. PART

    1 -0.12387E 05  0.00000E 00



  POLE OF TRANSFER FUNCTION

  POLE   REAL PART   IMAG. PART

    1 -0.13346E 03  0.00000E 00
```

Figs. 2.15 and 2.16: R2 in tree

loops. Conversely, the tree having the largest Branch Product (R1) yields the flowgraph with the greatest number of loops when compared with the flowgraphs generated by the R4 and R3 trees. However, the flowgraph generated by the R1 tree will give fewer loops than the flowgraph generated by the R2 tree since the R1 tree yields a smaller branch count.

The circuit in Table 2.1 was analyzed using the NASAP program for the four possible trees available from the NASAP tree selection algorithm. The computer results are given in Figs. 2.9-2.16. Fig. 2.9 gives an input listing that includes R4 in the tree. Note that this is not the only listing order that will cause R1 to be a branch of the tree--the only requirement on the input listing is that the card describing R4 must precede those describing R1, R2, and R3. Note that there are 332 flowgraph loops including 72 first order loops Fig. 2.12 shows an input listing that makes R3 a tree branch and show that there are 334 flowgraph loops including 74 first order loops. In Fig. 2.14 is a listing with R1 in the tree. The flowgraph loops generated by the R1 tree total 366 including 80 first order loops. Finally Fig. 2.16 gives a listing with R2 a tree branch. The flowgraph loops number 796 including 186 first order loops. Figs. 2.10, 2.11, 2.14, 2.15 give the transfer functions obtained for the different trees.

Note that the R4 tree does indeed yield the fewest flowgraph loops. However there is almost no difference between the number of loops generated by the R3 tree and the R4 tree. The R1 tree yields about 10% more loops than either the R4 tree or the R3 tree. The R2 tree generates more than twice as many loops as either of the other trees.

A Needed Modification

It should be noted that the primitive flowgraph as developed by the NASAP program does differ slightly from the actual primitive flowgraph developed from Kirchhoff's voltage and current laws and Ohm's Law. In the NASAP flowgraph there can be no connections to the current node of an independent or dependent voltage source as well as no connections to the voltage node of an independent or dependent current source. However such connections may exist in a true primitive flowgraph. If there are no connections emanating from these nodes, then these nodes will not be a part of any loop and no information will be lost in determining the transfer function. This is the reason why one is not able to call for the current of a voltage source or the voltage of a current source as an output variable in the NASAP program. Furthermore, this difference between the NASAP and true primitive flowgraph affects the procedure for determining the optimum tree.

Since there can be no connection between the voltage and current nodes of current and voltage sources (either dependent or independent), let us modify the branch count by omitting the branch count of those loops which are formed from independent and dependent current sources and by deleting from the branch count of each remaining loop those branches representing independent or dependent voltage sources. Similarly the Loop Count will be modified by omitting the loop count of those tree branches which represent independent or dependent voltage sources and by deleting those loops, which are formed by independent and dependent current sources from the loop count of the remaining tree branches. In other words, the modified branch count is the sum of the passive element tree branches that are part of those loops formed from the passive element links while the modified loop count is the sum of the loops, formed from passive element links, that pass through passive element tree branches. Note that the modified loop count will always equal the modified branch count.

The need for the modified Branch Count, Branch Product, and Loop Product is demonstrated from the analysis of the Butterworth filter circuit in Fig. 2.17. [SA-1]

Desired Transfer Function: VV3/VV1

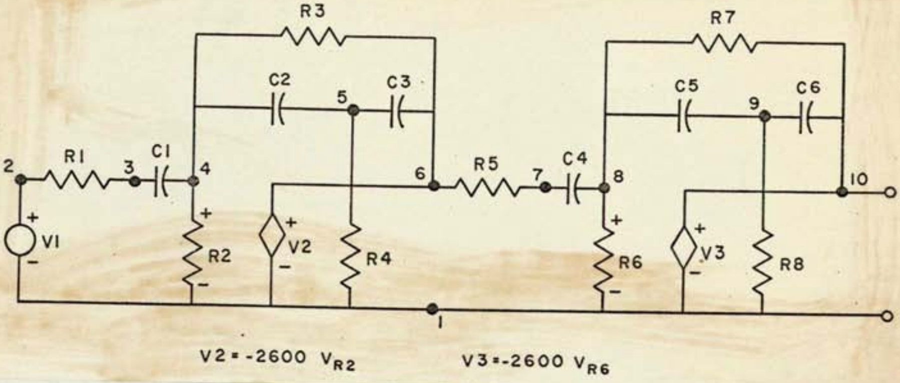


Fig. 2.17: Equivalent Circuit of a Butterworth Filter

The type 2 elements, V1-R2-V2-R6-V3 will be tree branches as will C1 and C4 due to the topology of the network. Either C2 or C3 but not both can be branches and the same situation holds for C5 and C6. There are four trees to choose from, namely

1. with C2 and C5
2. with C3 and C6
3. with C2 and C6
4. with C3 and C5

We will concern ourselves only with the first two choices. Table 2.7 gives branch-loop data for the C3-C6 tree while the same data for the C2-C5 tree is shown in Table 2.8.

Table 2.7: Tree with C3 and C6

Link	R1	R3	R4	C2	R5	R7	R8	C5
	3	2	2	3	3	2	2	3

Branch	V1	V2	V3	C1	R2	C3	C4	R6	C6
	1	4	3	1	3	2	1	3	2

Branch Count = 20
Branch Product = 1296
Loop Product = 432

Passive Link	R1	R3	R4	C2	R5	R7	R8	C5
	2	1	1	2	2	1	1	2

Passive Branch	C1	R2	C3	C4	R6	C6
	1	3	2	1	3	2

Modified Branch Count = 12
Modified Branch Product = 16
Modified Loop Product = 36

Table 2.8: Tree with C2 and C5

Link	R1	R3	R4	C3	R5	R7	R8	C6
	3	2	2	3	3	2	2	3

Branch	V1	V2	V3	C1	R2	C2	C4	R6	C5
	1	3	2	1	4	2	1	4	2

Branch Count = 20
Branch Product = 1296
Loop Product = 384

Passive Link	R1	R3	R4	C3	R5	R7	R8	C6
	2	1	2	2	2	1	2	2

Passive Branch	C1	R2	C2	C4	R6	C5
	1	4	2	1	4	2

Modified Branch Count = 14
Modified Branch Product = 64
Modified Loop Product = 64

```
        NASAP   FRENCH #1 BUTTERWORTH

        NONE
        NONE

        V1 1 2 1.
        R1 2 3 7.414K
        C1 3 4 .202UF
        R2 4 1 10M
     5  C2 4 5 20UF
     6  C3 5 6 20UF
        R3 4 6 10.510K
        R4 5 1 .0523K
        V2 1 6 -2600 VR2
        R5 6 7 8.542K
        C4 7 8 .282UF                  NUMBER OF LOOPS PER ORDER
        R6 8 1 10M
    13  C5 8 9 20UF                          1=   25
    14  C6 9 10 20UF                         2=  186
        R7 8 10 12.108K                      3=  474
        R8 9 1 .06025K                       4=  529
        V3 1 10 -2600 VR6                    5=  266
        OUTPUT                               6=   49
        VV3/VV1
        EXECUTE                               1529



        TRANSFER FUNCTION    VV3/VV1


                               (                                         2               3              4 )
                               (  0.00E 00      +0.00E 00 S    +1.98E 05 S    +8.93E 02 S    +1.00E 00 S    )

        H(S)= 1.578E 02*-------------------------------------------------------------------------------------------

                               (                                         2               3              4              5              6 )
                               (  4.32E 12      +3.71E 10 S    +2.33E 09 S    +1.39E 07 S    +3.05E 05 S    +1.10E 03 S    +1.00E 00 S    )



         ZERO OF TRANSFER FUNCTION                   POLE OF TRANSFER FUNCTION

         ZERO   REAL PART    IMAG. PART              POLE   REAL PART    IMAG. PART

           1  0.00000E 00  0.00000E 00                 1 -0.42975E 01  0.58384E 02
           2  0.00000E 00  0.00000E 00                 2 -0.42975E 01 -0.58384E 02
           3 -0.41494E 03  0.97890E-09                 3 -0.49350E 01  0.67260E 02
           4 -0.47801E 03 -0.79611E-08                 4 -0.49350E 01 -0.67260E 02
                                                       5 -0.41509E 03 -0.17377E-05
                                                       6 -0.66777E 03  0.38203E-06
```

Figs. 2.18 and 2.19: Tree with C2 and C5

```
NASAP  FRENCH  1 BUTTERWORTH

NONE
NONE

     V1 1 2 1.
     R1 2 3 7.414K
     C1 3 4 .202UF
     R2 4 1 10M
   5 C3 5 6 20UF
   6 C2 4 5 20UF
     R3 4 6 10.510K
     R4 5 1 .0523K
     V2 1 6 -2600 VR2
     R5 6 7 8.542K
     C4 7 8 .282UF                  NUMBER OF LOOPS PER ORDER
     R6 8 1 10M
  13 C6 9 10 20UF                        1=   19
  14 C5 8 9 20UF                         2=  115
     R7 8 10 12.108K                     3=  286
     R8 9 1 .06025K                      4=  333
     V3 1 10 -2600 VR6                   5=  180
     OUTPUT                              6=   36
     VV3/VV1                           ----------
     EXECUTE                               969


  TRANSFER FUNCTION    VV3/VV1


                    (                                            2                3                4 )
                    (  0.00E 00     +0.00E 00 S    +1.98E 05 S    +8.93E 02 S    +1.00E 00 S     )

  H(S)= 1.578E 02*-----------------------------------------------------------------------------------------------

                    (                                            2                3                4                5                6 )
                    (  4.32E 12     +3.71E 10 S    +2.33E 09 S    +1.39E 07 S    +3.05E 05 S    +1.10E 03 S    +1.00E 00 S    )


     ZERO OF TRANSFER FUNCTION                      POLE OF TRANSFER FUNCTION

     ZERO   REAL PART   IMAG. PART                  POLE   REAL PART   IMAG. PART

       1  0.00000E 00  0.00000E 00                    1 -0.42956E 01  0.58382E 02
       2  0.00000E 00  0.00000E 00                    2 -0.42956E 01 -0.58382E 02
       3 -0.41494E 03  0.97890E-09                    3 -0.49374E 01  0.67261E 02
       4 -0.47891E 03 -0.79611E-08                    4 -0.49374E 01 -0.67261E 02
                                                      5 -0.41512E 03  0.10178E-04
                                                      6 -0.41512E 03  0.10862E-04 ← pole 6 equals pole 5
                                                            run on 10/7/69 gives pole 6 as -.66777 E 03
```

Figs. 2.20 and 2.21: Tree with C3 and C6 tree. The problem remains of selecting these dependent sources such that they do not affect the circuit being analyzed. This is easily accomplished by making these sources dependent voltage sources with one node of the source connected to any node of the original circuit and with the other node left unconnected. Each unconnected node is actually an additional node in the modified circuit. Since a tree is an interconnection of all nodes and since the only connection to these hanging nodes is the source itself, then each of these sources must be a tree branch and thus a voltage source. However since one node of these voltage source is left floating, these sources will in no way affect the original circuit because no current can flow through these sources. The zero-load feature of this type of "dummy" source is also quite apparent in the flowgraph. There is a connection to the voltage node of the voltage source from the voltage node of the controlling element. However no connection emanates from the voltage node of the voltage source since this "dummy" voltage source is not involved in any of the network loops formed by the links of the co-tree. A node can be part of a flowgraph loop only if there is a connection leading to and away from the node. Since these "dummy" voltage sources are not part of any flowgraph loop, then they cannot affect the determination of the transfer function.

Thus the addition of the following two cards

```
V4    1    11    1.0    VR4
V5    1    12    1.0    VR8
```

to the input lists of either Fig. 2.18 or Fig. 2.20 will make R4 and R8 tree branches without affecting the electrical properties of the original circuit. Nodes 11 and 12 are the floating nodes. Note that the dependency value (1.0) in this case) is completely arbitrary and can have any value except 0.0. In essence, the above "trick" simply is an artificial means to tell the NASAP program what elements we wish to have in the tree. It does have the drawback in that elements are needlessly wasted but this only becomes a factor when the number of elements in the circuit is near the limit set by the length of the computer word. Note also that if an element which one desires to put into the tree by the above "trick" is also declared a type 6 element somewhere else in the input list (i.e., its current controls some source or is the desired output variable), then an error message will result.

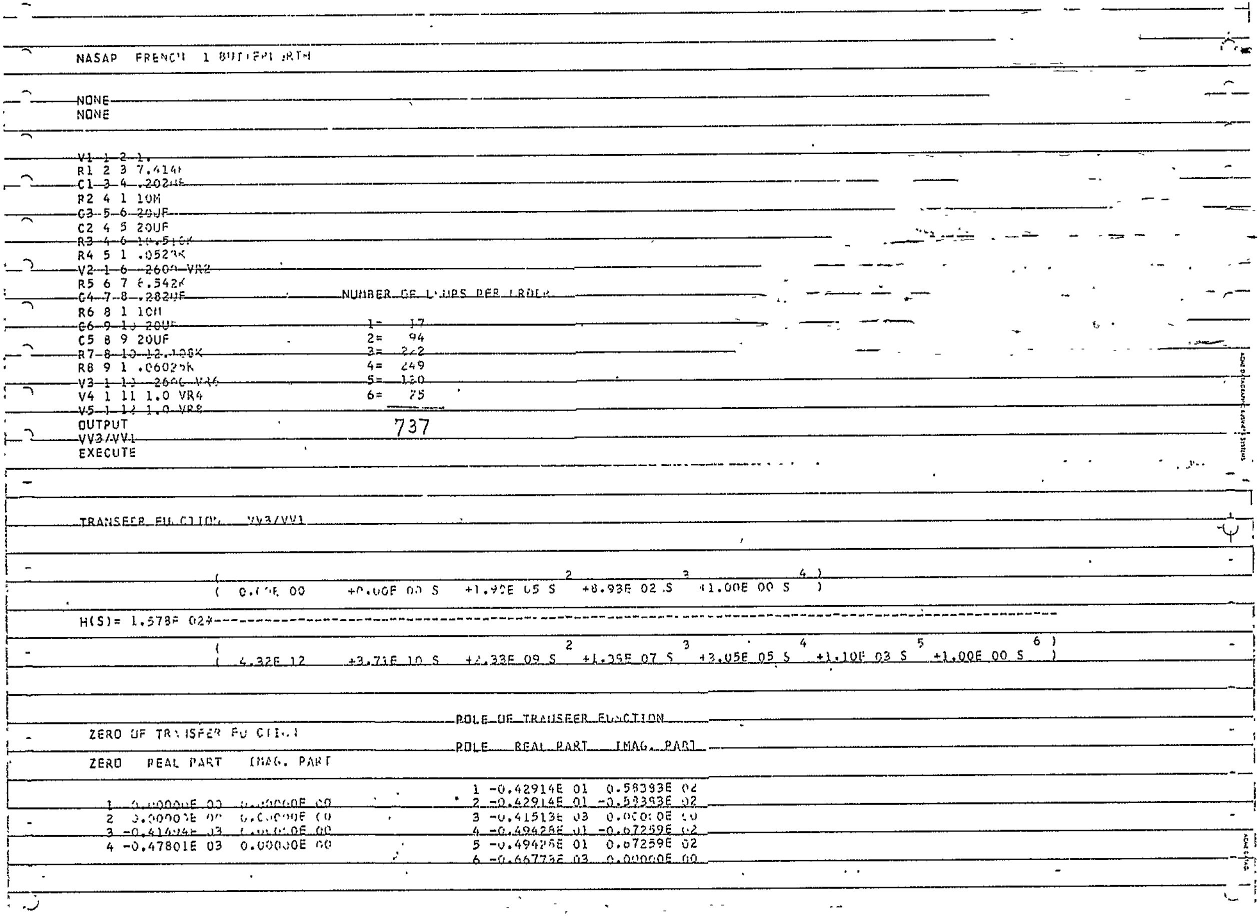
Fig. 2.22 gives the necessary branch data for the R4-R8 tree. Take note that the branch count, branch product, and loop product were determined without regard to the "dummy" sources V4 and V5.

Fig. 2.22: Tree with R4 and R8

Branch count = 18
Branch product = 576
Loop product = 216

Modified Branch count = 12
Modified Branch product = 16
Modified Loop product = 36

Note that the modified data is identical to that obtained for the C3-C6 tree. However the unmodified branch count is two less than that of either the C2-C5 tree or the C3-C6 tree. Figs. 2.24 and 2.25 give the computer results. The R4-R8 tree generates a flowgraph of 737 loops (including 17 first order loops). This is less than one-half the loops generated by the C2-C5 tree and more than 200 loops less than the number of loops derived from the C3-C6 tree--a substantial reduction.



```
NASAP  FRENCH  1 BUTTERWORTH

NONE
NONE

V1 1 2 1.
R1 2 3 7.414K
C1 3 4 .202UF
R2 4 1 10M
C3 5 6 20UF
C2 4 5 20UF
R3 4 6 10.510K
R4 5 1 .0523K
V2 1 6 -2600 VR2
R5 6 7 6.542K
C4 7 8 .282UF                  NUMBER OF LOOPS PER ORDER
R6 8 1 10M
C6 9 10 20UF                         1=   17
C5 8 9 20UF                          2=   94
R7 8 10 12.108K                      3=  222
R8 9 1 .06025K                       4=  249
V3 1 10 -2606 VR6                    5=  130
V4 1 11 1.0 VR4                      6=   75
V5 1 12 1.0 VR8                        ----
OUTPUT                                  737
VV3/VV1
EXECUTE


TRANSFER FUNCTION   VV3/VV1

                                                          2                3                 4 )
                 (  0.00E 00     +0.00E 00 S    +1.90E 05 S    +8.93E 02 S     +1.00E 00 S      )

H(S)= 1.578E 02*----------------------------------------------------------------------------------------
                                                          2                3                 4                5                6 )
                 (  4.32E 12     +3.71E 10 S    +2.33E 09 S    +1.39E 07 S     +3.05E 05 S     +1.10E 03 S    +1.00E 00 S     )


                                          POLE OF TRANSFER FUNCTION
  ZERO OF TRANSFER FUNCTION
                                          POLE    REAL PART    IMAG. PART
  ZERO    REAL PART    IMAG. PART

                                             1 -0.42914E 01  0.58393E 02
     1  0.00000E 00  0.00000E 00             2 -0.42914E 01 -0.58393E 02
     2  0.00000E 00  0.00000E 00             3 -0.41513E 03  0.00000E 00
     3 -0.41494E 03  0.00000E 00             4 -0.49428E 01 -0.67259E 02
     4 -0.47801E 03  0.00000E 00             5 -0.49428E 01  0.67259E 02
                                             6 -0.66773E 03  0.00000E 00
```

Figs. 2.24 and 2.25

CHAPTER III

MODELING A CONTROL SYSTEM FOR NASAP

IIIA GENERAL DISCUSSION OF CONTROL SYSTEMS

The preliminary discussion of feedback control systems is kept brief on the assumption that the reader already is generally acquainted with feedback control theory. Familiarity with introductory textbooks such as Dorf's Modern Control Systems [DO-1] and Perkins and Cruz Engineering of Dynamic Systems [PE-1] would be particularly helpful in that they use aerospace feedback control system problems as illustrative examples.

The major subdivisions of feedback control systems usually are:

1. A plant, process or controlled system wherein the position or state is being regulated or monitored.
2. The controller consisting of a sensor and control elements.
3. A comparator or error-sensing device to detect the difference between the input reference and the output signal.

Next we itemize the major steps involved in the design of a feedback control system. These steps are:

a. Establish performance specifications for the system (e.g. type of control, tolerance on accuracy, speed of response, overshoot, etc.)

b. Interpret the specification data in terms of design parameters and components of the control system. (give due consideration to reliabilit space, cost, etc.)

c. Formulation of the transfer functions of the components and analysis of the preliminary design.

d. Improve the performance of the preliminary design by suitable compensation to meet the specifications.

The scope of computer-aided design in this manual only covers aspects of steps c and d.

Control system design can be carried out either in the frequency or in the time domain. This is important in considering the possible role of NASAP in such applications. Control engineers have found it convenient, in the analysis of linear feedback control systems, to use the transfer function concept and the block diagram representation of the system. The transfer function concept is basic in the application of the frequency response method of analysis. In this approach the steady state response of the system to a sinusoidal input is used. The output/input signal relationship for each component of a control system is described by a transfer function. The operations of these components are represented by noninteracting blocks which are interconnected to form the block diagram or the corresponding signal flow graph of the overall system. Thus one obtains a functional representation of the feedback control system equivalent to the set of simultaneous differential equations that relate the variables of the physical system.

The basic procedures usually followed to analyze and design a feedback control system by the frequency response method are:

i) Determine transfer functions for each of the components used in the system (from the differential equations via transforms or from physical measurements).

ii) Formulate the signal flow graph from the system block diagram.

iii) Reduce the complicated block diagram of the system to a simple single loop configuration having a transfer function for the forward and the feedback branch if the open loop transfer function and output transform of the control system are to be used.

iv) Determine the system characteristics using the Bode plot or Nyquist diagram (an alternative graphical method uses the Nichols chart).

v) To have the system meet the prescribed performance specifications, design the necessary compensators that will reshape the plots obtained in step iv. This may involve cut and try.

In addition to the above, the designer often has to investigate sensitivity of parameters to variation of individual elements. The final step may include analog simulation or physical model tests.

As an alternative to the frequency response approach, the analysis and synthesis of feedback control systems determination of the system stability and the evaluation of the output of the system in response to impulse, step or ramp input functions. Here again there is often the need for compensation of the system so that it will meet specs.

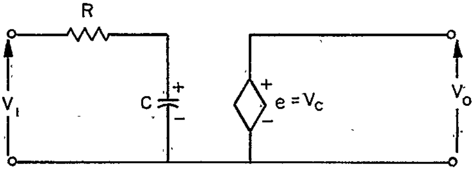
In the remaining chapters of this manual we shall indicate how NASAP can assist the design engineer to accomplish some of this work. It should be noted that some of these procedures are best carried out with a hybrid computer. Dr. C. H. Beck [BE-1] has developed a hybrid NASAP module for such applications as part of this cooperative development of the NASAP program.

IIIB1 Equivalent Electrical Networks for Transfer Functions

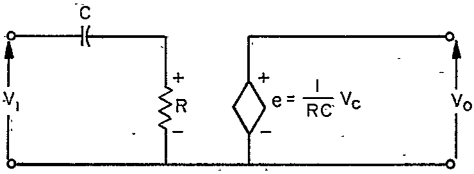
Copes + l.c.

Before a control system can be analyzed using NASAP and before any necessary compensation can be determined, the dynamic characteristics of the plant must first be simulated by an electric network which has an equivalent dynamic characteristic. The transfer function of a lumped linear plant can be expressed as a ratio of two polynomials. The problem of modeling the plant transfer function can be simplified if the polynomials are put in factored form. The individual factors or group of factors can be modeled by using simple RLC circuits. Then for a complicated transfer function these circuits are connected in cascade with suitable isolation between each circuit to prevent loading that would result in a change in the modeled transfer function. This necessary isolation is obtained by using ideal dependent voltage or current sources (which are available in NASAP). Table 3.1 gives a list of some elementary circuits with isolation and their corresponding transfer functions.

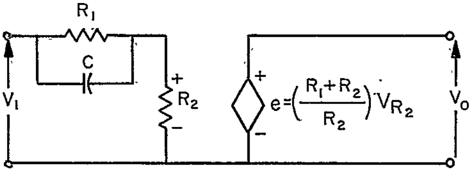
Table 3.1



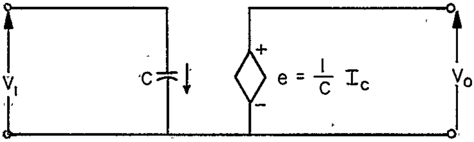
$$\frac{V_o}{V_1} = \frac{1}{\tau S+1} \qquad \tau = RC$$



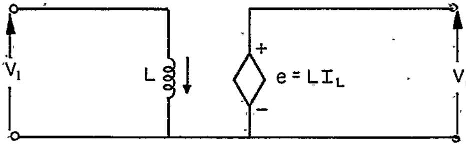
$$\frac{V_o}{V_1} = \frac{S}{\tau S+1} \qquad \tau = RC$$



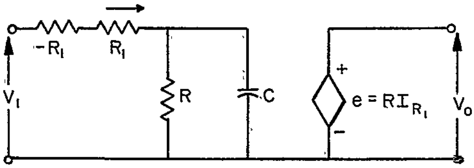
$$\frac{V_o}{V_1} = \frac{\tau_1 S+1}{\tau_2 S+1} \qquad \tau_1 = R_1C \qquad \tau_2 = \frac{R_2}{R_1+R_2} R_1C$$



$$\frac{V_o}{V_1} = S$$

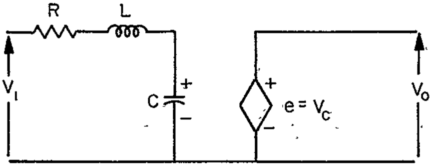


$$\frac{V_o}{V_1} = \frac{1}{S}$$

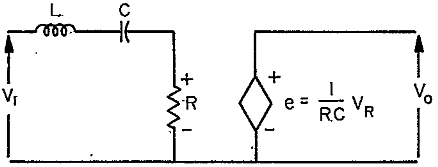


$$\frac{V_o}{V_1} = \tau S+1 \qquad \tau = RC$$

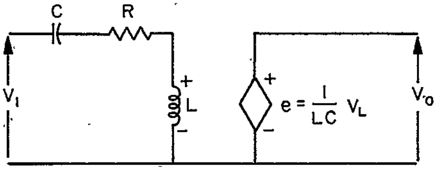
Table 3.I (continued)



$$\frac{V_O}{V_I} = \frac{1}{S^2 LC + SRC + 1}$$



$$\frac{V_O}{V_I} = \frac{S}{S^2 LC + SRC + 1}$$



$$\frac{V_O}{V_I} = \frac{S^2}{S^2 LC + SRC +}$$

IIIB2 Cascade Interconnection of Transfer Function Models

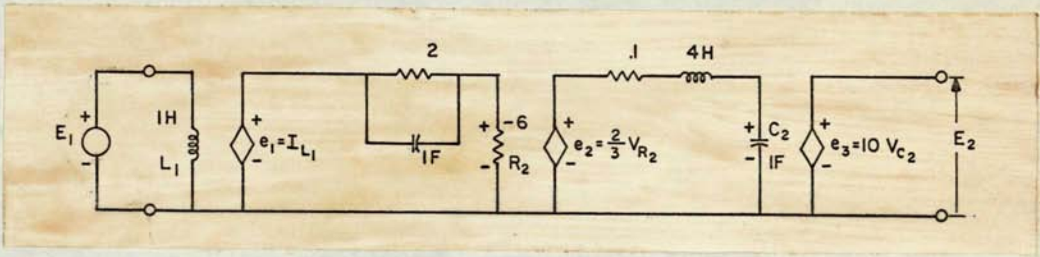
As indicated earlier the simple transfer function models can be cascaded. Consider a plant transfer function given as:

$$\frac{E_2(s)}{E_1(s)} = \frac{10(2s + 1)}{s(3s + 1)(4s^2 + 0.1s + 1)} \qquad (3.1)$$

This can be rewritten as:

$$\frac{E_2(s)}{E_1(s)} = \frac{1}{s} \cdot \left(\frac{2s + 1}{3s + 1}\right) \cdot \left(\frac{10}{4s^2 + 0.1s + 1}\right) \qquad (3.2)$$

Thus a cascade interconnection of three circuits from Table 3.1 can be used to model the above transfer function.



Note that R_2 is a negative resistance and that the gain factor (for this example, 10) is included in the dependency ralation for the third dependent voltage source (e_3). The gain factor could just as easily be included in the dependency relations of e_1 or e_2 .

Many of the subsequent examples used to illustrate various aspects of computer-aided control system design will incorporate techniques for modeling the pertinent transfer function.

IIIC ADDITIONAL EQUIVALENT NETWORK MODELS

IIIC1 Use of Negative R, L or C

Since negative element values of R, L, and C are permitted in the NASAP input coding, rational transfer functions of control systems can be modeled simply by application of the continued fraction expansion procedure. Accordingly the rational function will be represented in general by the input admittance or impedance of a ladder structure consistings of positive or negative R, L, and C elements. It is emphasized that this approach works because physical realizability as a passive network is not a consideration. Only the equivalent dynamic characteristic matters.

As an example, let us consider the biquadratic all-pass function

$$F(s) = \frac{s^2 - as + b}{s^2 + as + b} \tag{3.3}$$

Performing the continued fraction expansion of F(s) yields

$$\begin{array}{lllll}
 & \underline{\;1\;} & & & \\
s^2 + as + b & \big)\, s^2 - as + b & & & \\
 & \underline{s^2 + as + b - \frac{1}{2a} s} & & & \\
 & -2as \;\big)\, s^2 + as + b & & & \\
 & & \underline{s^2 \qquad\qquad -2} & & \\
 & & as + b \;\big)\, -2as & & \\
 & & & \underline{-2as \; -2b \quad \frac{a}{2b} s} & \\
 & & & 2b \;\big)\, as + b & \\
 & & & & \underline{as \qquad 2} \\
 & & & & b \;\big)\, 2b
\end{array} \tag{3.4}$$

If F(s) is assumed to be an admittance Y_1 , then the resulting ladder network is shown in Fig. 3.1.

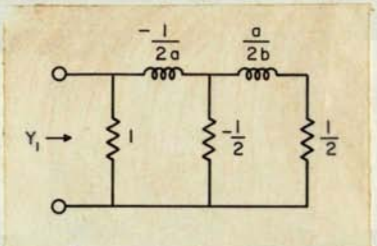


Fig. 3.1

In other words, the input admittance Y_1 of the circuit of Fig. 3.1 is a representation of the specified rational function F(s).

To utilize this input admittance model with the NASAP program, two additional elements, a voltage source and a resistance of very small value, must be included in the circuit of Fig. 3.1 (see Fig. 3.2).

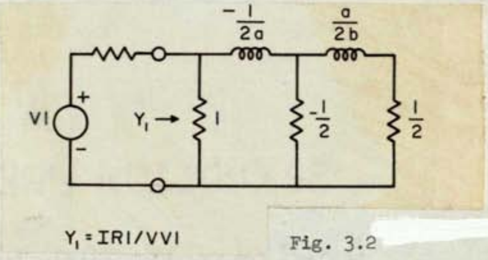


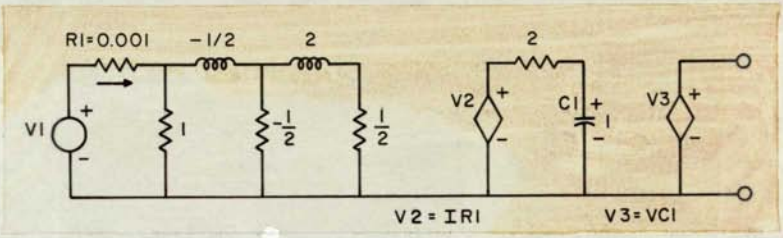
Fig. 3.2

The voltage source V1 represents the excitation for the input admittance $Y_1 = \frac{I_1}{V_1}$. The response is the current I_1 . However the current flowing through a voltage source cannot be specified with the NASAP program. Since no element of the circuit of Fig. 3.1 is in series with the source V1, it is necessary to include the small resistor R1. Thus the input admittance of this circuit can be specified as

$$Y_1 = \mathrm{IR1/VV1} \qquad (3.5)$$

The circuit of Fig. 3.2 can be easily cascaded with other isolated circuits to model more complex rational functions. As an example, the transfer voltage ratio of the circuit of Fig. 3.3 models the function.

$$\frac{s^2 - s + 4}{(s^2 + s + 4)(2s + 1)} \qquad (3.6)$$



$$VV3/VV1 = \frac{s^2 - s + 4}{(s^2 + s + 4)(2s + 1)}$$

Fig. 3.3

Alternatively the function F(s) given above can be assumed to be an input impedance Z_1. From the continued fraction expansion, we obtain the NASAP applicable circuit of Fig. 3.4.

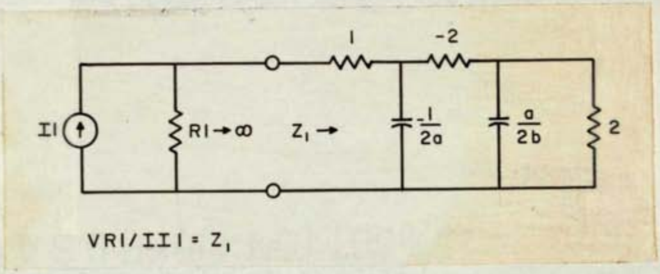


Fig. 3.4

The current source I1 represents the excitation necessary for the input impedance. The response is the voltage across the very large resistance R1 which must be added to the circuit since no single element obtained from the continued fraction expansion is connected across the ideal current source terminals.

However it is possible to avoid the use of the small series resistor of Fig. 3.2 and the large shunt resistor of Fig. 3.4 by taking the reciprocal of (i.e. inverting) the rational function that is to be modelled by NASAP before performing the continued fraction expansion.

As an example, let us desire to model the biquadratic all-pass function given above by the input impedance of some ladder circuit. Thus we desire a circuit whose input impedance is defined by

$$Z_2(s) = \frac{s^2 - as + b}{s^2 + as + b} \tag{3.7}$$

Inverting this expression we obtain

$$Y_2(s) = \frac{s^2 + as + b}{s^2 - as + b} \tag{3.8}$$

Performing the continued fraction expansion this time yields the NASAP circuit is shown in Fig. 3.5.

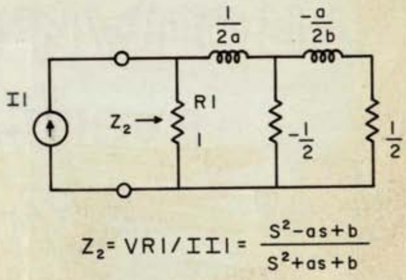


Fig. 3.5

Note that now there is no need to include a large shunt resistor across the I1 current source terminals (as in Fig. 3.4) since the resistor R1 already shunts these terminals.

Similarly, we can model the biquadratic all-pass function by the input admittance of a ladder network defined by

$$Y_3 = \frac{s^2 - as + b}{s^2 + as + b} \qquad (3.9)$$

Inverting this expression and then performing the continued fraction expansion (given above), we obtain the ladder circuit of Fig. 3.6.

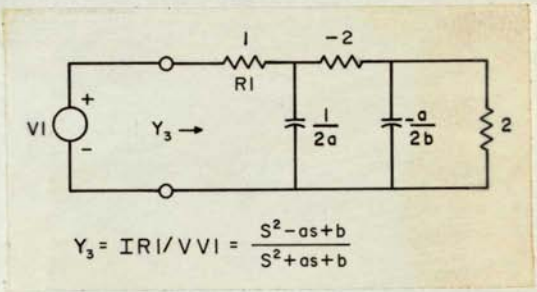


Fig. 3.6

Note that addition of a small series resistor (as in Fig. 3.2) is not needed since the current flowing in the resistor R1 is from the voltage source V1.

In summary, by use of the continued fraction expansion procedure, we have obtained four NASAP-codable circuits to model the specified all-pass function F(s). The circuits of Figs. 3.5 and 3.6 are more desirable as models than those of Figs. 3.2 and 3.4 since they require one less element. It should be further noted that each of these four circuits has another desirable feature. The NASAP tree-selection algorithm will automatically select the shunt elements of each circuit as tree branches. This will lead to a star tree which generates the flowgraph with the fewest loops.

This will always occur if the polynomials of the specified rational function are arranged in descending powers of s. The resulting continued fraction expansion will either make the series elements inductors or the shunt elements capacitors.

IIIC2 Illustrative Examples

Sometimes it is necessary to arrange the polynomials in ascending powers to attain a ladder network with both positive and negative elements. For example, let us assume we wish to model the function

$$F_1(s) = \frac{1}{s^3 + 2s^2 + s + 1} \tag{3.10}$$

as the input impedance of a ladder network. We cannot perform a continued fraction expansion on $F_1^{-1}(s) = s^3 + 2s^2 + s + 1$. Consequently we will need a large shunt resistor in the NASAP model. Furthermore we must rearrange the denominator of $F_1(s)$ in ascending powers of s before performing the continued fraction expansion. The resulting NASAP model with shunting resistor R1 is shown in Fig. 3.7. Note that, since the shunt elements are inductors, the NASAP tree-selection algorithm will pick a linear tree consisting of resistor R1 and three of the other four resistances (the particular R's depending upon their location in the input list). This type of tree yields the largest number of loops in the corresponding primitive flowgraph.

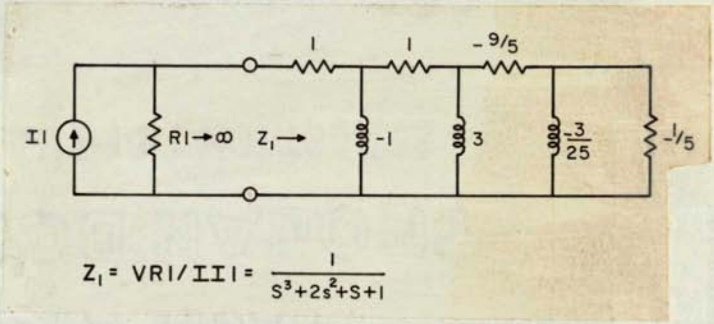


Fig. 3.7

Sometimes in the continued fraction expansion procedure more than one term is eliminated by subtraction. This may necessitate a rearranging of the remaining polynomials to achieve a NASAP model. Such a case exists for the function

$$F_2(s) = \frac{1}{s^3 + s^2 + s + 1} \tag{3.11}$$

The first three steps of the continued fraction expansion process is based on ascending order of the denominator. Then three terms become zero. This necessitates reversing the polynomial $-s-s^2-s^3$ to $-s^3-s^2-s$.

In Fig. 3.8 is given a NASAP-codable ladder network whose input impedance equals the desired function $F_2(s)$.

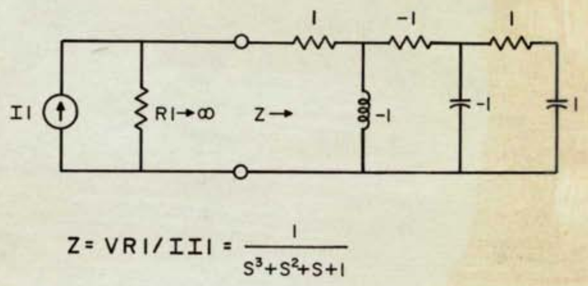


Fig. 3.8

Another example is the biquadratic function

$$F_3(s) = \frac{s^2 + as + b}{s^2 + as + c} \tag{3.12}$$

Let us model this function as an input impedance. Inverting $F_3(s)$ and then performing the continued fraction expansion yields

$$\begin{array}{lllll}
 & 1 & & & \\
s^2 + as + b \;\big) & s^2 + as + c & & & \\
 & s^2 + as + b & \frac{b}{c-b} & & \\
\cline{2-3}
 & (c-b) \;\big) & b + as + s^2 & \frac{c-b}{as} & \\
 & & b & & \\
\cline{3-4}
 & & as + s^2 \;\big) & c - b & \frac{-a^2}{c-b} \\
 & & & c - b + \frac{c-b}{a}s & \\
\cline{4-5}
 & & & \frac{c-b}{a}s \;\big) & as + s^2
\end{array} \tag{3.13}$$

$$\begin{array}{r|l} as & -\frac{c-b}{as} \\ \hline s^2 & -\frac{c-b}{a}\,s \\ & -\frac{c-b}{a}\,s \\ \hline \end{array} \qquad \begin{array}{l}(3.13)\\ \text{cont.}\end{array}$$

The resulting network is shown in Fig. 3.9.

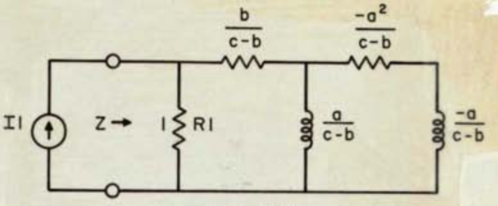


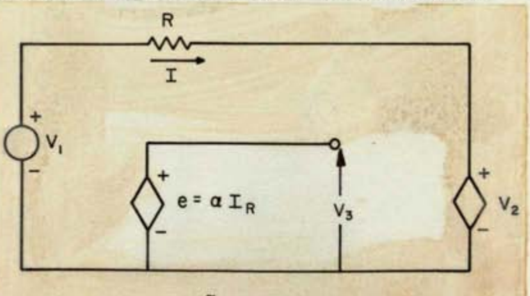
Fig. 3.9

$$Z = VR1/II1 = \frac{S^2+as+b}{S^2+as+c}$$

Note the large shunt resistor is again avoided by use of the initial inversion. Note also that in the underlined section of the continued fraction process it was necessary to reverse the polynomical $s^2 + as + b$ since two terms were eliminated in the preceding subtraction.

IIIC3 Equivalent Networks for Summing Element

Since most control systems require some sort of feedback loop, an electrical network equivalent to the summing (or subtracting) element that is compatible with NASAP must be used. Such networks are shown in Figs. 3.10a and 3.10b.



$$V_3 = \frac{\alpha}{R}(V_1 - V_2)$$

Fig. 3.10a

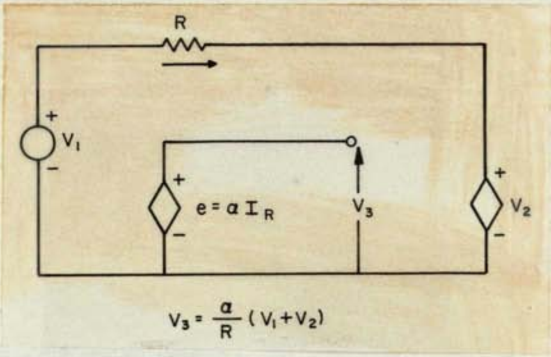


Fig. 3.10b

In Fig. 3.10a if $\alpha = R = 1$, the output voltage is equal to the difference of the two input voltage. Similarly if $\alpha = R = 1$ in Fig. 3.10b, the output voltage is equal to the sum of the two input voltages.

IIID MODELS OF FEEDBACK CONTROL SYSTEMS

IIID1 Examples of System Models

Thus we now have all the elements necessary to model a feedback control system with an electric network that is compatable with NASAP. As an example, the unity feedback control system shown in Fig. 3.11.

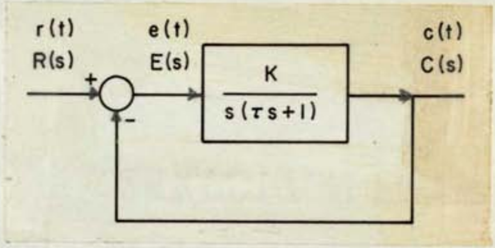


Fig. 3.11

has the following equivalent electric network

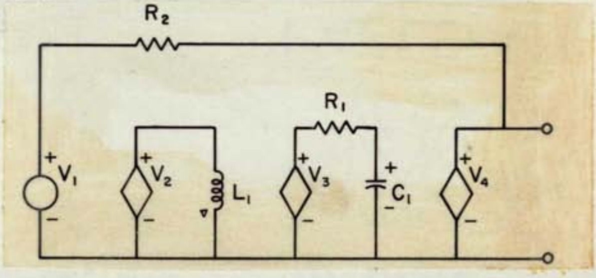


Fig. 3.12

where V_1 is equivalent to r(t)

V_2 is equivalent to e(t)

V_4 is equivalent to c(t)

A second example shows a NASAP model for a non-unity feedback system (Fig. 3.13).

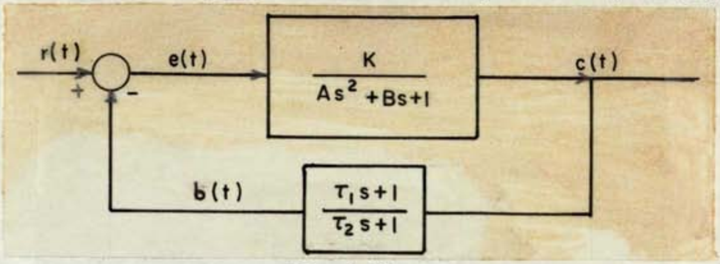


Fig. 3.13

Fig. 3.14 is the equivalent electric network.

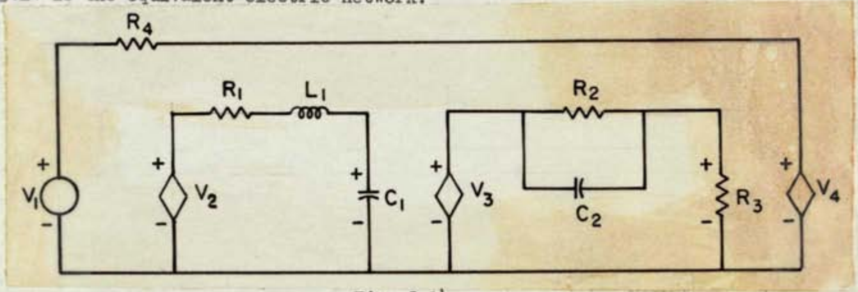


Fig. 3.14

where V_1 is equivalent to r(t)

V_2 is equivalent to e(t)

V_3 is equivalent to c(t)

V_4 is equivalent to b(t)

IIID2 Control System Model and Its Step Response

Using a control system design problem adapted from D'Azzo and Houpis, pp. 408-411 [DA-1] we illustrate modeling of the system and use of NASAP to tabulate and plot the step response.

Consider the unity feedback control system with cascade lead compensation shown in Fig. 3.15. To determine the step response when K = 10 we first obtain the NASAP

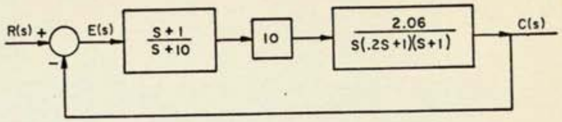


Fig. 3.15

circuit model shown in Fig. 3.16. In this model

V5 represents the input R(s)

V4 represents the output C(s)

V6 represents 2.06 times the error signal; 2.06 E(s)

Note that the system gain, K = 10, is included in the dependency relation of V1. Also in Fig. 3.16 we have NASAP output, namely the transfer function zeros and poles based on a simple flowgraph having only right first order loops. The desired step response is given in Fig. 3.17. Figure 3.17a is the response function and 3.17b gives 51 discrete time values from zero to six seconds in equal increments. Finally in Fig. 3.18 we have the plot of this time response.

These NASAP outputs can be used to determine the key step response characteristics. Note in particular that the steady state value is unity since Fig. 3.17a we see that the residue of the pole at the origin is unity. Furthermore in Fig. 3.17b the peak overshoot occurs at t = 1.44 seconds and is 9.15%. This response settles (for 2% tolerance) in 2.04 seconds.

NASAP MODEL FOR CONTROL SYSTEM

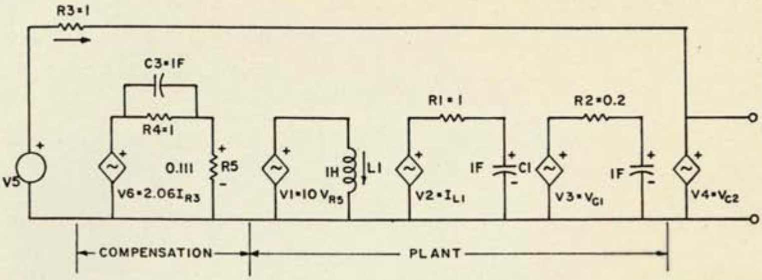


Fig. 3.16

Resistance Values in Ohms

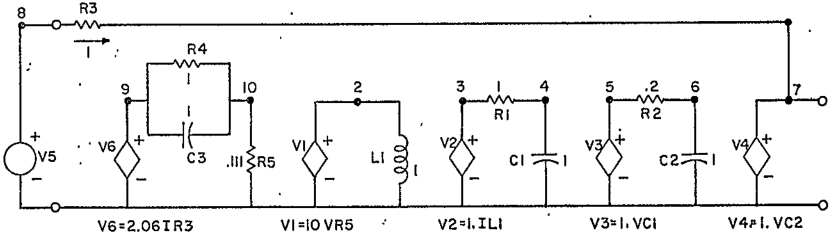
V5 represents the input R(s)

V4 represents the output C(s)

V6 represents 2.06 times the error signal; 2.06 E(s)

```
NASAP PROBLEM  LEAD  COMPENSATION

     V1 1 2 10. VR5
     L1 2 1 1.H
     V2 1 3 1.0 IL1
     R1 3 4 1.
     C1 4 1 1F
     V3 1 5 1.0 VC1
     R2 5 6 0.2
     C2 6 1 1.F
     V4 1 7 1.0 VC2
     R3 8 7 1.0
     V5 1 8 1.0
     V6 1 9 2.06 IR3
     C3 9 10 1.F
     R4 9 10  1.000
     R5 10 1  0.111
     OUTPUT
     VV4/VV5
     FREQ -1.0 1.0 0.05
     TIME 6.0
     EXECUTE
```



```
     NUMBER OF LOOPS PER ORDER

          1=     8
          2=     5
          3=     2
```

```
     TRANSFER FUNCTION    VV4/VV5


                        (                               )
                        (  1.00E 00     +1.00E 00 S     )
     H(S)= 1.030E 02*-----------------------------------------------------------------------------
                        (                                              2               3              4 )
                        (  1.03E 02     +1.55E 02 S    +6.51E 01 S    +1.60E 01 S    +1.00E 00 S      )



                                                   POLE OF TRANSFER FUNCTION

                                                   POLE    REAL PART   IMAG. PART
      ZERO OF TRANSFER FUNCTION
                                                     1 -0.17968E 01  0.24072E 01
      ZERO    REAL PART    IMAG. PART                2 -0.17968E 01 -0.24072E 01
                                                     3 -0.10000E 01 -0.48388E-11
        1 -0.10000E 01  0.00000E 00                  4 -0.11415E 02  0.80302E-09
```

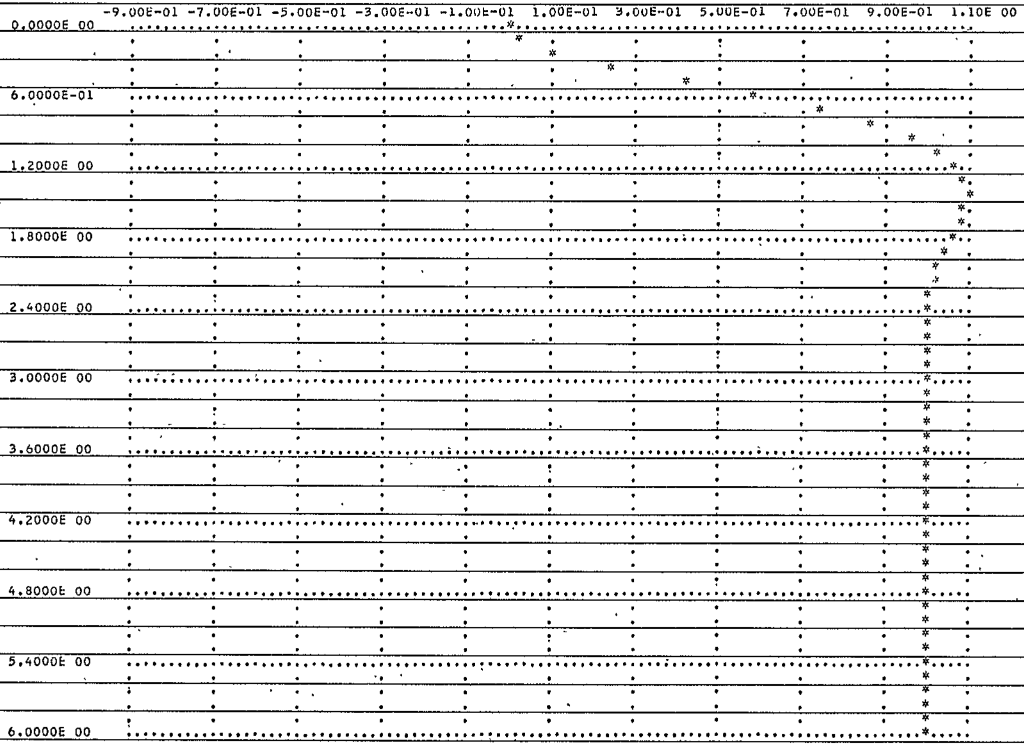
```
STEP     RESPONSE FUNCTION

F(T) =
                                         (-0.1797E 01  J 0.2407E 01 ) T
          (-0.4541E 00  J 0.5566E 00 ) E
                                         (-0.1797E 01  J-0.2407E 01 ) T
          (-0.4541E 00  J-0.5566E 00 ) E
                                         (-0.1000E 01  J-0.4839E-11 ) T
          ( 0.2954E-05  J 0.7443E-11 ) E
                                         (-0.1142E 02  J 0.8030E-09 ) T
          (-0.9178E-01  J-0.6499E-11 ) E
                                         ( 0.0000E 00  J 0.0000E 00 ) T
          ( 0.1000E 01  J 0.6551E-10 ) E
```

```
STEP     RESPONSE

TIME                       VV4/VV5
 0.0000E 00                0.00000000E 00
 0.1200E 00                0.19343615E-01
 0.2400E 00                0.10478669E 00
 0.3600E 00                0.24626642E 00
 0.4800E 00                0.41497654E 00
 0.6000E 00                0.58518541E 00
 0.7200E 00                0.73395842E 00
 0.8400E 00                0.86608970E 00
 0.9600E 00                0.96266139E 00
 0.1080E 01                0.10293007E 01
 0.1200E 01                0.10695372E 01
 0.1320E 01   peak         0.10884190E 01
→0.1440E 01 <              0.10914555E 01 ← 9%
 0.1560E 01   overshoot    0.10838785E 01
 0.1680E 01                0.10701990E 01
 0.1800E 01                0.10539808E 01
 0.1920E 01   settling     0.10378084E 01
→0.2040E 01 < time       > 0.10233526E 01 ← 2%  tolerance
 0.2160E 01                0.10115232E 01
 0.2280E 01                0.10026340E 01
 0.2400E 01                0.99658346E 00
 0.2520E 01                0.99300891E 00
 0.2640E 01                0.99141634E 00
 0.2760E 01                0.99127769E 00
 0.2880E 01                0.99209821E 00
 0.3000E 01                0.99345618E 00
 0.3120E 01                0.99502158E 00
 0.3240E 01                0.99655831E 00
 0.3360E 01                0.99791598E 00
 0.3480E 01                0.99901557E 00
 0.3600E 01                0.99983263E 00
 0.3720E 01                0.10003805E 01
 0.3840E 01                0.10006971E 01
 0.3960E 01                0.10008297E 01
 0.4080E 01                0.10008287E 01
 0.4200E 01                0.10007420E 01
 0.4320E 01                0.10006075E 01
 0.4440E 01                0.10004568E 01
 0.4560E 01                0.10003109E 01
 0.4680E 01                0.10001831E 01
 0.4800E 01                0.10000811E 01
 0.4920E 01                0.10000067E 01
 0.5040E 01                0.99995738E 00
 0.5160E 01                0.99992949E 00
 0.5280E 01                0.99991864E 00
 0.5400E 01                0.99991995E 00
 0.5520E 01                0.99992901E 00
 0.5640E 01                0.99994224E 00
 0.5760E 01                0.99995673E 00
 0.5880E 01                0.99997056E 00
 0.6000E 01                0.99998248E 00
```

STEP RESPONSE



CHAPTER IV

CONTROL SYSTEM ANALYSIS IN THE FREQUENCY DOMAIN

IVA ANALYSIS OBJECTIVES

As mentioned in Chapter III control system design is usually aimed at a set of specifications. Therefore we start with a summary of typical performance specifications as general background. For convenience we adapt the tabulations given by Grabbe, Ramo, and Woodruff [GA 2] and list some of them for transient response in Table 4.1a and for frequency response in Table 4.1b. These are sufficient to indicate the sort of specifications that can be expected for the class of linear time-invariant single input single output control systems being considered in this manual.

There are several approximations or rules of thumb which were developed by control engineers for use when time or facilities are not available for a more exact analysis of this class of systems. They are also useful as rough checks on the results of a computer analysis. The more common of these rules of thumb are presented, virtually unchanged from [GA 2] in Table 4.2. They must, however, be used with caution since being approximations, they do not apply with equal validity to all control systems. Note that the approximations for transient response are applicable only for step inputs.

Following Table 4.2 we have Fig. 4.1 which shows typical step and frequency response curves to help pin down the definition of some of the terms used in these tables.

Table 4.1a

COMMON PERFORMANCE SPECIFICATIONS Transient response

Type	Definition	General Remarks
1. Transient overshoot	Usually taken as ratio of peak of transient to final value for a step input.	Convenient when transient solution is available. Can be estimated from root locus or frequency response. Useful for nonlinear systems System must be excited by step input and be underdamped
2. Settling time	Defined as time to reach and remain within a specified percentage of final value (often as 5% or 2%) after a step input.	See 1 above Used for systems which require rapid synchronization, e g., fire control system.
3. Steady-state error	Final error existing between desired and actual output.	See 1 above. Easily calculated from static characteristics or final value theorem. Useful when input is simple aperiodic function. Can include frequency components which arise in nonlinear systems.
4. Rise time	Defined as (a) time to ½ the final value, or (b) slope at ½ the final value, or (c) time between 10% and 90% of final value after a step input.	Easily estimated from frequency response or root locus and is indicative of band pass of system. Used for overdamped systems. Has found application in process controls where characteristics (1) or (2) may not be easily recognized.
5. Dead time	Defined as (a) time for output action to be initiated, or (b) for output to reach a given level (10% or 50%), or (c) time to the intersection of the slope of the transient at ½ the final value and the initial value after a step input.	See 1 above. Easily estimated from frequency response and is indicative of phase shift near gain crossover in systems. Useful when delay times exist in system. Used for overdamped systems. Both rise and delay time derive from filter theory.
6. Absolute damping, decrement factor	Defined as the real part of the roots of a quadratic system and as such determines the rate of decay of transient.	Convenient method of interpreting more complex systems in terms of quadratic systems. Valuable in combination with relative damping in work with root locus analysis. Has had extensive use in systems demanding prescribed transient performance, particularly when the time decay is important, e.g., in autopilots.
7. Damping ratio	Damping ratio is defined as ζ in the quadratic $s^2 + 2\omega_0\zeta s + \omega_0^2$ and indicates the decay per cycle of the natural frequency.	Useful because it is a parameter in nondimensional plot of quadratic response. Used in combination with 6 above in root locus analysis. Used when number and size of overshoot are important In combination with 6 above defines decay of oscillatory component of transient.

Table 4.1b

COMMON PERFORMANCE SPECIFICATIONS		Frequency response
8. Phase margin	Defined as 180° + phase shift at unity gain of the open loop frequency response.	Used as a rule of thumb in frequency response analysis to indicate stability and performance. Easy to use and to obtain directly from frequency response diagram.
9. Gain margin	Gain margin is ratio of maximum stable gain to actual gain, i e., gain at phase crossover.	Same as 8. Indicates relative sensitivity of system to gain variations. Can be calculated by Routh's criterion. Not as good a criterion for performance as 8. Little used.
10. M_m peak	Ratio of maximum of closed loop frequency response to a low frequency value.	Used with Nyquist and frequency response analysis. Rules of thumb relate M_m and transient overshoot. Easy to calculate from frequency response diagram.
11. Band width	Defined variously (*a*) usually as frequency where closed loop response falls to $\sqrt{\frac{1}{2}}$ or 3 db of its low frequency value, or (*b*) sometimes as the frequency at the significant peak M_m, or (*c*) the crossover of the open loop response.	Used with frequency response analysis and is related to speed of response of system. Used also when definite frequency bandpass is needed for fidelity. M_m, bandpass, and the phase shift at these valves give a good indication of the closed loop response and are often used when a number of closed loops are operated in tandem as system.
12. Static error coefficient	Defined as the final error resulting from a continuous input of position, or velocity, or acceleration, etc. The magnitude of the input and the maximum tolerable error must be specified.	Used to set low-frequency gain of open loop frequency response. Useful where steady inputs are encountered.
13. Dynamic error coefficients (or steady-state error coefficients)	Defined as the steady-state error resulting from the derivatives of the input function. The time function and/or its derivatives must be specified as well as the maximum tolerable error.	Relates system gain and time constants to errors arising from higher derivatives of input. Used to estimate error resulting from varying input to given system and conversely to determine closed loops pole-zero location to give desired error. Accurate where input varies at slow rate compared to bandpass. Becomes poorer as input varies more rapidly because of transient effects.

Table 4.2

RULE-OF-THUMB APPROXIMATIONS

Parameter	Approximation	Remarks
Time to peak	$t_p \approx \pi/\omega_c$ where t_p = time from step input to peak value of response transient, seconds ω_c = open loop crossover frequency, radians/second	In cases with a dominant complex pair of closed loop poles, the open loop crossover frequency, ω_c, times the time to peak, t_p, is about 3 or π. In other words, the time to peak is about half the period corresponding to the open loop crossover frequency.
Peak overshoot	$C/R\|_p \approx 0.85M_m$ where $C/R\|_p$ = peak value of transient response to a step input M_m = maximum value of closed loop frequency response	The peak value of the transient response, $C/R\|_p$, to a unit step input is generally less than the maximum steady-state value, M_m, of the closed loop frequency response. The maximum value of $C/R\|_p$ generally approaches 2.0 while the maximum value of M_m approaches infinity. For many applications "good" servos are those with the values of M_m between 1.3 and 1.5.
Damping ratio	$\zeta \doteq 1/(2M_c)$ where ζ = damping ratio M_c = value of closed loop frequency response at the corner frequency	The damping ratio may be approximated from the value of the closed loop frequency response of the system at the corner frequency, ω_c (the frequency at which the lines asymptotic to the log magnitude curve intersect). This is exact for a second order system.
Settling time	$t_{s(5\%)} \approx 3\sqrt{1 - \zeta^2}/\zeta\omega_d$ $t_{s(2\%)} \approx 5\sqrt{1 - \zeta^2}/\zeta\omega_d$ $t_{s(5\%)} \approx 3T_{eq}$ where t_s = time for response to step input to settle to within some per cent of final value, seconds T_{eq} = time for response to reach 63% of final value ω_d = damped natural frequency, radians/second ζ = damping ratio	The settling time, t_s, is generally defined as the time for the system to settle to within 5 or sometimes 2% of the final value. In either case it is quite difficult to predict t_s for an underdamped system because it is subject to fluctuations of about one-half the period of oscillation for only small changes in system parameters.

Table 4.2

RULE-OF-THUMB APPROXIMATIONS (*Continued*)

Parameter	Approximation	Remarks
Rise time	$t_r\omega_t \approx t_r\omega_m \approx 1.3$ where t_r = rise time (10 to 90%) ω_t = (defined above) ω_m = (defined above)	The system's rise time, t_r, which is here considered to be the time for the response to a step input to go from 10 to 90% of its final value may be approximated as indicated for systems with a M_m value of about 1 3 to 1.5.
Phase margin at crossover frequency	$\gamma_c \geqq 40°$ where γ_c = open loop phase margin at the crossover frequency	A phase margin of 40° at the unity gain (crossover) frequency generally corresponds to a M_m ratio of approximately 1.5. Since this value of M_m is the maximum ordinarily considered feasible, the phase margin should be 40° or greater.
Oscillation frequency	$\omega_t \approx \omega_m \approx 0.75\omega_c$ where ω_t = oscillation frequency of transient response, radians/second ω_m = frequency at which M_m occurs, radians/second ω_c = open loop gain crossover frequency, radians/second	The frequency of oscillation of the transient response, ω_t, is generally about equal to the frequency, ω_m, at which the frequency response peak, M_m, occurs. Both ω_m and ω_t are usually less than ω_c, the open loop crossover frequency. For the "good" servos with M_m = 1.3 to 1.5 an approximate relationship is as indicated. In this approximation ω_t is used to mean essentially the same thing as ω_d, the damped natural frequency, previously defined for a system with a dominant complex pair of poles.

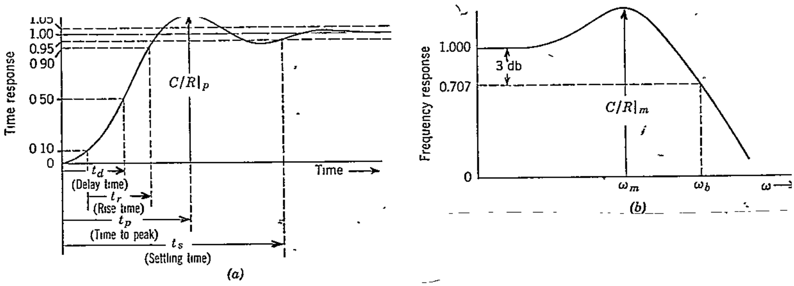


Figure 4.1. (a) Typical servo system response to unit step input.
(b) Typical system frequency response (closed loop).

IVB BODE AND ROOT LOCUS PLOTS

Bode showed that the phase angle of a (minimum phase) transfer function G could be related to the rate at which the magnitude of G decreases with increasing frequency. This is the basis for the "frequency response" method of analysis. In this method, the magnitude of G in decibels and the phase angle, are plotted on semilog paper as functions of the frequency ω (plotted on the log scale) with jω substituted for s in G. The value of $|G|$ in decibels (dB) can be found from the value of $|G|$ by the following equation.

$$|G|_{dB} = 20 \log_{10} |G| \tag{4.1}$$

For preliminary estimation of compensating networks to insure stability or improve performance, it is often possible to omit the phase angle plot and to make use of an approximate attenuation plot (or plot of $|G|$). Exact plots are needed for a final check after selection of the proposed stabilizing transfer functions, however. The method of drawing approximate attenuation plots is described in most textbooks on control theory. Before we discuss and illustrate how NASAP obtains exact plots of the magnitude and phase of G, we provide a collection of typical transfer functions with their corresponding Bode plots in Table 4.3. These plots indicate the gain and phase margins.

Another important tool for analysis and synthesis of linear control systems, usually attributed to Evans, is known as the "Root Locus" method. As with the frequency response methods, its importance derives from the fact that it helps provide insight into the significant aspects of any particular system. It is not restricted to direct feedback systems nor to systems with open loop poles and zeros in the left half plane.

It is recalled that for any closed-loop system with "degenerative feedback" see Fig. 4.2, the closed-loop transfer function is

$$\frac{C}{R} = \frac{G}{1 + GH} \qquad (4.2)$$

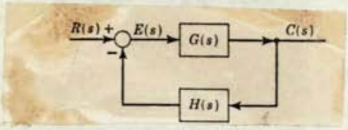


Fig. 4.2: Block diagram of a basic feedback control system.

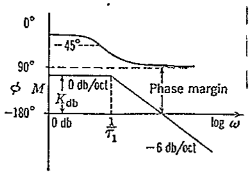
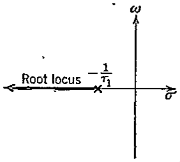
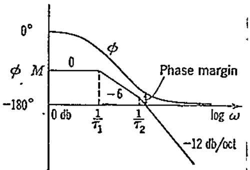
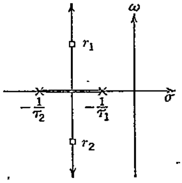
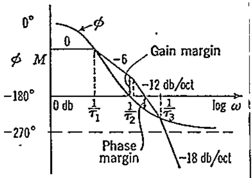
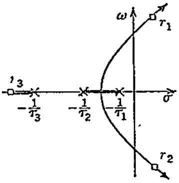
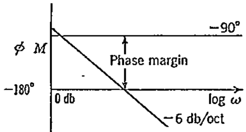
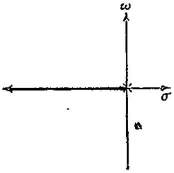
The closed-loop response is determined by the roots of the denominator, i.e., the characteric equation. The roots must all be in the left half plane in order that the system be stable. Furthermore the time-domain design parameters, such as peak time maximum overshoot, damping factor, and settling time, are initimately related to the s-plane location of the roots of the characteristic equation of the control system, which are the poles of the closed-loop system function. Accordingly, a knowledge of how the roots of 1 + GH vary when the gain constant of GH varies should be of considerable assistance in understanding the system. A plot of the locus of the characteristic roots of the control system with the system loop gain as a parameter is commonly known as the root locus.

The design of feedback control systems by use of the root-locus method involves the reshaping of the root-locus plots by shifting or introducing open-loops poles and zeros. As a preliminary to the discusssion of the design aspects, the effects of shifting open-loop poles and zeros are first indicated with the aid of examples in Table 4.3.

Gain Adjustment. When the preliminary analysis of a control system indicates that the system is unstable or that the over-all performance is inadequate, steps must be taken to improve the system performance. The most direct and simplest way of changing the performance is by the adjustment of the system gain.

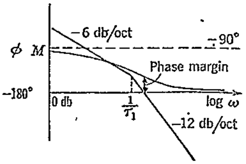
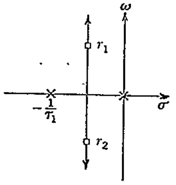
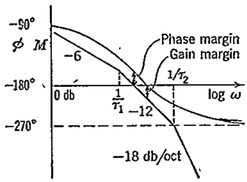
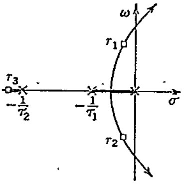
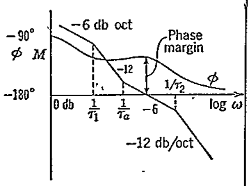
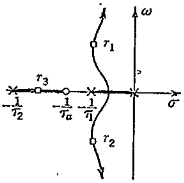
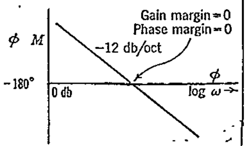
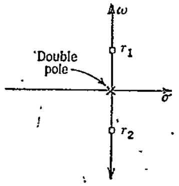
Table 4.3

PLOTS FOR TYPICAL TRANSFER FUNCTIONS

$G(s)$	Bode diagram	Root locus	Comments
$\frac{K}{s\tau_1 + 1}$			Stable, gain margin = ∞
$\frac{K}{(s\tau_1 + 1)(s\tau_2 + 1)}$			Stable; gain margin = ∞
$\frac{K}{(s\tau_1 + 1)(s\tau_2 + 1)(s\tau_3 + 1)}$			Shown, unstable, but can be made stable by reducing gain
$\frac{K}{s}$			Ideal integrator; stable

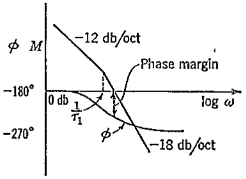
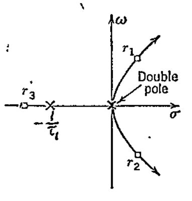
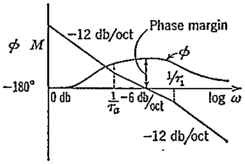
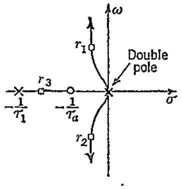
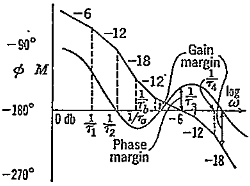
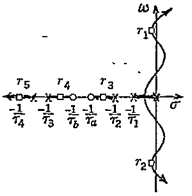
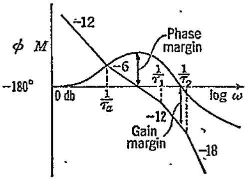
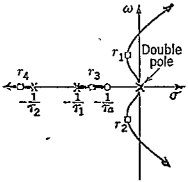
(Continued)

Table 4.3 (con't.)

$G(s)$	Bode diagram	Root locus	Comments
$\frac{K}{s(s\tau_1+1)}$			Stable, gain margin = ∞
$\frac{K}{S(S\tau_1+1)(s\tau_2+1)}$			System is; stable as shown, but will become unstable with increased gain
$\frac{K(s\tau_a+1)}{S(S\tau_1+1)(s\tau_2+1)}$			Above control system with phase-lead (derivative) compensator; stable
$\frac{K}{s^2}$			Inherently unstable; must be compensated

(Continued)

Table 4.3 (con't.)

$G(s)$	Bode diagram	Root locus	Comments
$\frac{K}{s^2(s\tau_1+1)}$			Inherently unstable must be compensated
$\frac{K(s\tau_a+1)}{s^2(s\tau_1+1)}$			Stable for all gains
$\frac{K(s\tau_a+1)(s\tau_b+1)}{(s\tau_1+1)(s\tau_2+1)(s\tau_3+1)(s\tau_4+1)}$			Conditionally stable; stable at low gain, becomes unstable as gain is raised, again becomes stable as gain is further increased, and becomes unstable for very high gains
$\frac{K(s\tau_a+1)}{s^2(s\tau_1+1)(s\tau_2+1)}$			Conditionally stable; becomes unstable at high gain

Adapted from George J. Thaler and Robert G. Brown, *Analysis* *Design of Feedback Control Systems*, 2nd Ed., McGraw-Hill, New York, 1960.

However, for most control systems the design specifications cannot be met by gain adjustment alone. The usual alternative is the introduction of compensating devices into the control system.

The adequate gain setting for a control system can be determined from the gain-phase plot of the system or from the Bode diagram of the system. A change in system gain usually affects practically all of the system design parameters. For instance, an increase in system gain may cause a reduction of the system error, may increase the speed of response of the control system, and may make the system more oscillatory. The effects of gain variations upon the behavior of a control system are conveniently observed on the root locus plot of the control system also.

NASAP Output

For a given transfer function G with all the coefficients known, the frequency response is calculated by setting $s = j\omega$ and simplifying the expression to a linear combination of real and imaginary terms:

$$G = A(\omega) + jB(\omega) \ . \tag{4.3}$$

The magnitude of G and the angle $\theta(\omega)$ are computed according to the equations,

$$|G(j\omega)| = A^2(\omega) + B^2(\omega) \ , \tag{4.4}$$

and

$$\theta(\omega) = \tan^{-1} \frac{B(\omega)}{A(\omega)} \ . \tag{4.5}$$

Now if ω is made to vary, then for each value of ω the $|G(j\omega)|$ and $\theta(\omega)$ can be obtained over the frequency range of interest and thus can be made available for plotting. With the complex arithmetic capability of FORTRAN IV, these computations are easily done in NASAP. The Bode plot consists of the $|G(j\omega)|$ in decibel units and $\theta(\omega)$ in degrees versus the $\log_{10}\omega$, taken over the frequency range specified by the user.

To obtain a root locus plot for a control system, Fig. 4.2, it is required to find the values of s for which GH = -1 (or 1 + GH = 0). For this it is necessary that the angle of the complex number, GH, be 180 degrees and the magnitude of GH be unity. Thus, the complex number, s, must be selected so that the angle of the complex number, GH, is 180 degrees. When such a complex number for s is determined, a value of gain K can then be found which will make the magnitude of GH unity, although this value of K might not necessarily be the same as the value specified in the transfer function. However, after a locus of values of s for which GH = 180 degrees has been found, somewhere along this locus one can find a number that yields $|GH| = 1$ for the specified value of K. NASAP furnishes the necessary data in tabulated form to obtain the locus of points for which GH = 180 degrees. To obtain the root locus plot directly by the computer requires an extra program that will not be described here. Such programs are available in the literature as illustrated by Program D91RTL by Vernon [Appendix I in VE 1] and another by Krall and Fornaro [see KR 1 or KR2].

An alternative approach is to use the root sensitivity data available from NASAP to approximate the root locus plot. Such sensitivity data for an aerospace control problem is given in Chapter VI.

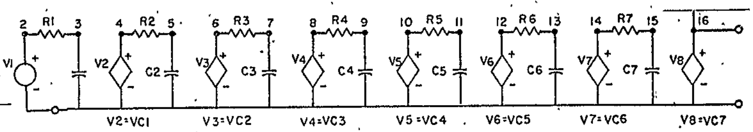
As an illustration of the open loop Bode plot output of NASAP we use a control system plant whose transfer function has a seventh degree polynomial denominator. The NASAP print out is shown in Fig. 4.3. This Eisenberg control problem [EI 1] is discussed further in Chapters V and VII.

```
NASAP PROBLEM  EISENBERG CONTROL SYSTEM

RADIANS
NONE

V1 1 2 1.
R1 2 3  6.
C1 3 1 1F
V2 1 4 1. VC1
R2 4 5  2.
C2 5 1 1F
V3 1 6 1. VC2
R3 6 7 2.
C3 7 1 1F
V4 1 8 1. VC3
R4 8 9 2.
C4 9 1 1F
V5 1 10 1. VC4
R5 10 11 1.
C5 11 1 1F
V6 1 12 1. VC5
R6 12 13 1.
C6 13 1 1F
V7 1 14 1. VC6
R7 14 15 1.
C7 15 1 1F
V8 1 16 1. VC7
OUTPUT                  NUMBER OF LOOPS PER ORDER
VV8/VV1/V6
FREQ -2.0 1.0 0.05         1=     8
EXECUTE                    2=    21
                           3=    35
                           4=    35
                           5=    21
                           6=     7
                           7=     1
```

$$\boxed{\frac{1}{(6s+1)(2s+1)^3(s+1)^3}}$$



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```
TRANSFER FUNCTION   VV8/VV1/V6


                    (                  )
                    (  1.00E 00        )
H(S)= 2.083E-02*-----------------------------------------------------------------------------------------------------------
                                                              2                 3                 4                 5                 6              7 )
                    (  2.08E-02      +3.12E-01 S    +1.81E 00 S    +5.44E 00 S    +9.25E 00 S    +9.00E 00 S    +4.67E 00 S    +1.00E 00 S    )

                                  POLE OF TRANSFER FUNCTION

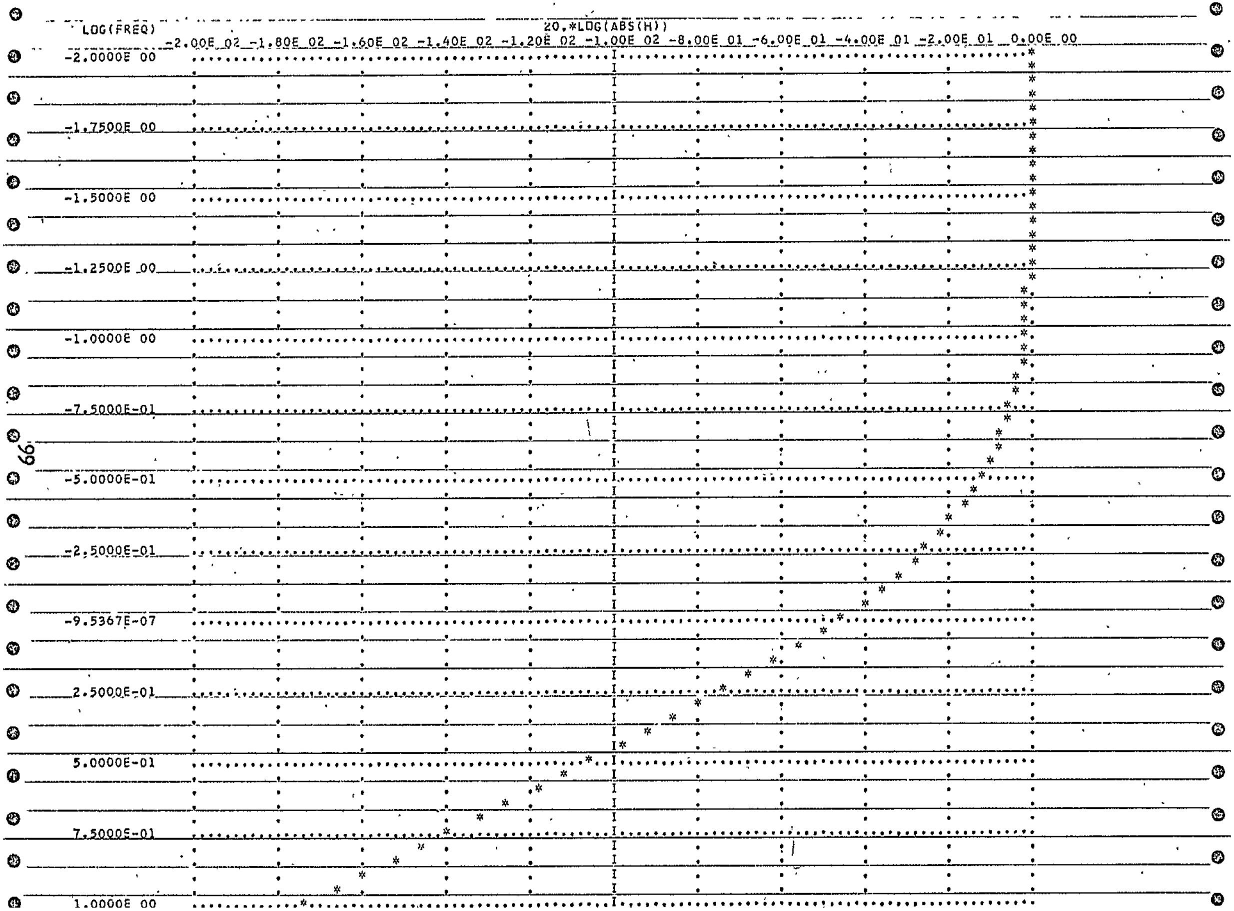
                                  POLE   REAL PART   IMAG. PART
    ZERO OF TRANSFER FUNCTION
                                    1 -0.97827E 00 -0.17633E-02
             NONE                   2 -0.97827E 00  0.17633E-02
                                    3 -0.51017E 00 -0.95019E-02
                                    4 -0.51017E 00  0.95019E-02
                                    5 -0.48502E 00 -0.17578E-02
                                    6 -0.48502E 00  0.17578E-02
                                    7 -0.16667E 00  0.21177E-12
```

```
    LOG(FREQ)          FREQ           20.*LOG(ABS(H))       PHI(H)           ABS(H)          LOG(ABS(H))
 -0.1999999E 01    0.1000002E-01    -0.2211870E-01    -0.8589749E 01    0.9974567E 00    -0.1105935E-02
 -0.1949999E 01    0.1122021E-01    -0.2783884E-01    -0.9636519E 01    0.9968001E 00    -0.1391942E-02
 -0.1899999E 01    0.1258929E-01    -0.3502743E-01    -0.1081046E 02    0.9959754E 00    -0.1751372E-02
 -0.1849998E 01    0.1412543E-01    -0.4407636E-01    -0.1212687E 02    0.9949384E 00    -0.2203818E-02
 -0.1799999E 01    0.1584896E-01    -0.5545091E-01    -0.1360279E 02    0.9936363E 00    -0.2772545E-02
 -0.1749999E 01    0.1778283E-01    -0.6975049E-01    -0.1525727E 02    0.9920018E 00    -0.3487526E-02
 -0.1699999E 01    0.1995267E-01    -0.8771539E-01    -0.1711147E 02    0.9899522E 00    -0.4385769E-02
 -0.1649999E 01    0.2238728E-01    -0.1102796E 00    -0.1918886E 02    0.9873838E 00    -0.5513981E-02
 -0.1599999E 01    0.2511889E-01    -0.1386065E 00    -0.2151541E 02    0.9841697E 00    -0.6930023E-02
 -0.1549999E 01    0.2818388E-01    -0.1741254E 00    -0.2411990E 02    0.9801527E 00    -0.8706272E-02
 -0.1499999E 01    0.3162285E-01    -0.2186347E 00    -0.2703383E 02    0.9751429E 00    -0.1093174E-01
 -0.1449999E 01    0.3548143E-01    -0.2743350E 00    -0.3029163E 02    0.9689096E 00    -0.1371675E-01
 -0.1400000E 01    0.3981075E-01    -0.3439417E 00    -0.3393065E 02    0.9611753E 00    -0.1719739E-01
 -0.1349999E 01    0.4466842E-01    -0.4307985E 00    -0.3799130E 02    0.9516124E 00    -0.2153993E-01
 -0.1299999E 01    0.5011881E-01    -0.5389174E 00    -0.4251649E 02    0.9398410E 00    -0.2694562E-01
 -0.1249999E 01    0.5623426E-01    -0.6731448E 00    -0.4755144E 02    0.9254282E 00    -0.3365724E-01
 -0.1200000E 01    0.6309575E-01    -0.8392789E 00    -0.5314361E 02    0.9078959E 00    -0.4196395E-01
 -0.1150000E 01    0.7079464E-01    -0.1044142E 01    -0.5933934E 02    0.8867330E 00    -0.5220710E-01
 -0.1099999E 01    0.7943290E-01    -0.1295652E 01    -0.6618787E 02    0.8614247E 00    -0.6478262E-01
 -0.1049999E 01    0.8912528E-01    -0.1602863E 01    -0.7373538E 02    0.8314877E 00    -0.8014417E-01
 -0.1000000E 01    0.1000000E 00    -0.1976027E 01    -0.8202527E 02    0.7965235E 00    -0.9880137E-01
 -0.9500008E 00    0.1122016E 00    -0.2426394E 01    -0.9109807E 02    0.7562759E 00    -0.1213197E 00
 -0.9000006E 00    0.1258924E 00    -0.2956266E 01    -0.1009892E 03    0.7107006E 00    -0.1483133E 00
 -0.8500004E 00    0.1412537E 00    -0.3608796E 01    -0.1117283E 03    0.6600246E 00    -0.1804398E 00
 -0.8000002E 00    0.1584892E 00    -0.4367934E 01    -0.1233394E 03    0.6047881E 00    -0.2183967E 00
 -0.7500010E 00    0.1778275E 00    -0.5258370E 01    -0.1358402E 03    0.5458602E 00    -0.2629185E 00
 -0.7000008E 00    0.1995259E 00    -0.6295592E 01    -0.1492426E 03    0.4844180E 00    -0.3147796E 00
 -0.6500006E 00    0.2238718E 00    -0.7495926E 01    -0.1635502E 03    0.4218943E 00    -0.3747963E 00
 -0.6000004E 00    0.2511884E 00    -0.8876655E 01    -0.1787578E 03    0.3598872E 00    -0.4438333E 00
 -0.5500011E 00    0.2818375E 00    -0.1045691E 02     0.1651522E 03    0.3000539E 00    -0.5228004E 00
 -0.5000010E 00    0.3162271E 00    -0.1225315E 02     0.1482119E 03    0.2439733E 00    -0.6126574E 00
 -0.4500008E 00    0.3548127E 00    -0.1428750E 02     0.1304735E 03    0.1930300E 00    -0.7143748E 00
 -0.4000006E 00    0.3981066E 00    -0.1657837E 02     0.1120113E 03    0.1482802E 00    -0.8289167E 00
 -0.3500013E 00    0.4466827E 00    -0.1914375E 02     0.9292386E 02    0.1103600E 00    -0.9571881E 00
 -0.3000011E 00    0.5011859E 00    -0.2199997E 02     0.7333247E 02    0.7943296E-01    -0.1099998E 01
 -0.2500010E 00    0.5623401E 00    -0.2516006E 02     0.5338127E 02    0.5520728E-01    -0.1258003E 01
 -0.2000008E 00    0.6309562E 00    -0.2863329E 02     0.3323123E 02    0.3701133E-01    -0.1431664E 01
 -0.1500015E 00    0.7079433E 00    -0.3242444E 02     0.1305595E 02    0.2392086E-01    -0.1621222E 01
 -0.1000013E 00    0.7943258E 00    -0.3653371E 02    -0.6966206E 01    0.1490436E-01    -0.1826686E 01
 -0.5000114E-01    0.8912486E 00    -0.4095615E 02    -0.2665607E 02    0.8957598E-02    -0.2047808E 01
 -0.9537743E-06    0.9999978E 00    -0.4568188E 02    -0.4584215E 02    0.5198810E-02    -0.2284095E 01
  0.4999828E-01    0.1122014E 01    -0.5069650E 02    -0.6436623E 02    0.2918595E-02    -0.2534825E 01
  0.9999847E-01    0.1258921E 01    -0.5598169E 02    -0.8209105E 02    0.1588233E-02    -0.2799085E 01
  0.1499987E 00    0.1412533E 01    -0.6151573E 02    -0.9890337E 02    0.8398707E-03    -0.3075787E 01
  0.1999989E 00    0.1584889E 01    -0.6727481E 02    -0.1147180E 03    0.4327705E-03    -0.3363741E 01
  0.2499981E 00    0.1778272E 01    -0.7323381E 02    -0.1294793E 03    0.2179258E-03    -0.3661691E 01
  0.2999983E 00    0.1995255E 01    -0.7936798E 02    -0.1441615E 03    0.1075475E-03    -0.3968399E 01
  0.3499985E 00    0.2238713E 01    -0.8565311E 02    -0.1557636E 03    0.5216073E-04    -0.4282656E 01
  0.3999987E 00    0.2511879E 01    -0.9206674E 02    -0.1673068E 03    0.2492653E-04    -0.4603337E 01
  0.4499979E 00    0.2818369E 01    -0.9858833E 02    -0.1778292E 03    0.1176476E-04    -0.4929417E 01
  0.4999981E 00    0.3162264E 01    -0.1052001E 03     0.1726171E 03    0.5495335E-05    -0.5260005E 01
  0.5499983E 00    0.3548120E 01    -0.1118862E 03     0.1639730E 03    0.2545011E-05    -0.5594310E 01
  0.5999985E 00    0.3981058E 01    -0.1186333E 03     0.1561741E 03    0.1170401E-05    -0.5931664E 01
  0.6499977E 00    0.4466812E 01    -0.1254299E 03     0.1491543E 03    0.5351847E-06    -0.6271496E 01
  0.6999979E 00    0.5011848E 01    -0.1322670E 03     0.1428479E 03    0.2435850E-06    -0.6613348E 01
  0.7499981E 00    0.5623388E 01    -0.1391364E 03     0.1371911E 03    0.1104513E-06    -0.6956822E 01
  0.7999983E 00    0.6309548E 01    -0.1460323E 03     0.1321236E 03    0.4993214E-07    -0.7301615E 01
  0.8499975E 00    0.7079417E 01    -0.1529490E 03     0.1275888E 03    0.2251877E-07    -0.7647449E 01
  0.8999977E 00    0.7943240E 01    -0.1598827E 03     0.1235337E 03    0.1013584E-07    -0.7994136E 01
  0.9499979E 00    0.8912466E 01    -0.1668299E 03     0.1199103E 03    0.4555130E-08    -0.8341494E 01
  0.9999981E 00    0.9999956E 01    -0.1737879E 03     0.1166740E 03    0.2044577E-08    -0.8689394E 01
```

Phase Margin ≈ (180−85.9) ≈ 94.1°

since largest ABS(H) does not reach 1 but is .997

→ Gain Margin = $\frac{1}{.3598672}$ = 8.876065 db.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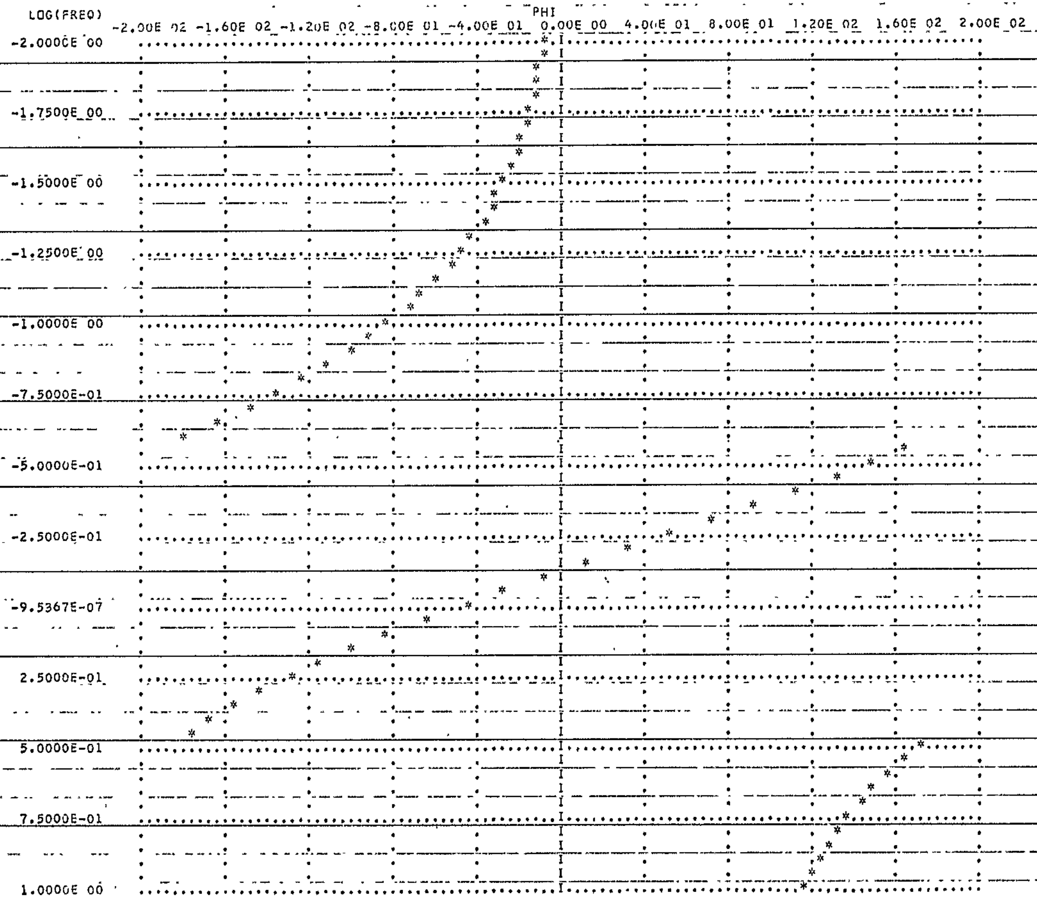
20.*LOG(ABS(H))
-2.00E 02
-1.80E 02
-1.60E 02
-1.40E 02
-1.20E 02
-1.00E 02
-8.00E 01
-6.00E 01
-4.00E 01
-2.00E 01
0.00E 00
-2.0000E 00
-1.7500E 00
-1.5000E 00
-1.2500E 00
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
-9.5367E-07
2.5000E-01
5.0000E-01
7.5000E-01
1.0000E 00


LOG(FREQ)
PHI
-2.00E 02
-1.60E 02
-1.20E 02
-8.00E 01
-4.00E 01
0.00E 00
4.00E 01
8.00E 01
1.20E 02
1.60E 02
2.00E 02
-2.0000E 00
-1.7500E 00
-1.5000E 00
-1.2500E 00
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
-9.5367E-07
2.5000E-01
5.0000E-01
7.5000E-01
1.0000E 00

IVC USE OF COMPENSATION

When the desired behavior of a control system cannot be obtained by the gain adjustment alone, compensation techniques must be used. Compensation means to improve the system performance by reshaping the open-loop transfer function characteristic of the system. The compensation of a control system can often be accomplished either by an element in series with other components as shown in Fig. 4.4a or by an element in parallel with one or more components to form a subsidiary loop, as shown in Fig. 4.4b. The former arrangement is referred to as cascade or series compensation and the latter is called feedback or minor-loop compensation. A compensator or compensating device can stabilize a system which is unstable for all values of gain; it can improve both the transient and the steady-state performance of a system; and it can reduce the system error. Compensators are classified according to their operating characteristics into phase-lead (differentiating) type, phase-lag (integrating) type, and lag-lead (integro-differentiating) type. An example of the latter was given at the end of Chapter 3. The phase-lead type of compensator is generally used to modify the high-frequency portion of the open-loop transfer function plot and to improve the transient behavior of the system,

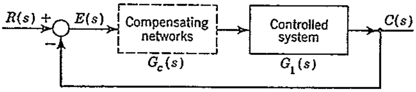


Fig. 4.4a Cascade compensation

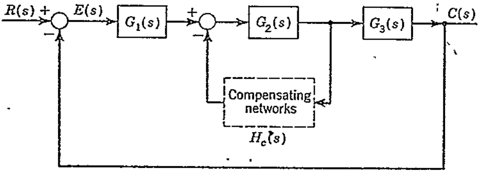


Fig. 4.4b Feedback compensation

whereas the phase-lag type of compensator is often used to alter the low-frequency portion of the open-loop plot and to improve the steady-state performance of the system. The lag-lead compensation gives results intermediate between these two extremes. By proper adjustment of time constants, considerable flexibility of control system characteristics may be obtained.

Th choice of a method of compensation generally depends upon the specific system involved, the available components, cost considerations and the designers's experience and judgment.

The most commonly used configuration for compensation is shown in Fig. 4.4a 4a The procedure to compute $G_c(s)$ is to first compute the open loop transfer function and then compute $G_c(s)$. There are two disadvantages in using this configuration. First if the overall open loop transfer function $G_f(s)$ is not properly chosen, the compensator computer in this manner may not be realizable as a RC network. Second, it generally requires pole-zero concellation. In system theory terminology, it means that some poles of the overall system are uncontrollable and/or unobservable. This poses a design problem in that these poles are dictated by the given plant $G_1(s)$ and cannot be controlled by the designer.

The feedback compensation, shown in Fig. 4.4b, is sometimes superior to cascade compensation in that variation of the parameters of the system components bridged by the feedback elements of the minor loop have less effect upon system performance if the minor-loop gain is made sufficiently large and if the parameters of the feedback compensator do not vary. Although the freedom in choosing $G_1(s)$ and $H_c(s)$ is greatly increased, the difficulties encountered in Fig. 4.4a may still occur. Therefore the series and shunt compensation are not always applicable in practice.

Consider the configuration of compensators introduced by Chen [CH 1]. This is shown in Fig. 4.5, where k is a constant gain, $C_1(s) = \frac{N_1(s)}{D_1(s)}$ and

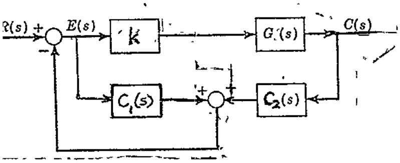


Fig. 4.5 Chen's Control System Configuration

$C_2(s) = \frac{N_2(s)}{D_2(s)}$ are proper rational functions with the dgree of polynomials $D_1(s)$, $D_2(s)$ and $N_2(s)$ equal to n-1 and that of $N_1(s) = n-2$. This control system configuration has the overall transfer function

$$\begin{aligned} G_f(s) &= \frac{kG(s)}{1 + C_1(s) + kC_2(s)\ G(s)} \\ &= \frac{kND_2\ D_1}{D\ D_1\ D_2 + N_1\ D_2\ D + kN_2\ N\ D_1} \end{aligned} \tag{4.6}$$

If the denominators of $C_1(s)$ and $C_2(s)$ are chosen to be the same, that is $D_1(s) = D_2(s)$, then the last equation reduces to

$$G_f(s) = \frac{kND_1}{DD_1 + N_1D + kN_2N} \tag{4.7}$$

It can be shown that by using this system configuration the compensators can be always chosen to be realizable by RC networks and the cancelled poles can also be controlled by the designer [see CH 1]. It is worth noting that the complexity of compensators in the system of Fig. 4.5 is comparable to that required for the compensator of the corresponding control system based on the configuration of Fig. 4.4a.

Examples

An example of cascade compensation was included at the end of Chapter III (refer to Figs. 3.16 and 3.17). Here we provide the Bode plots in Figs. 4.6 and 4.7 for the uncompensated as well as the compensated case respectively to facilitate comparisons.

Next we illustrate basic feedback compensation with the aid of two examples having the configuration shown in Fig. 4.8. The feedback function H_c is used to modify the characteristics of the plant G, while the cascade function G_c is provided to aid in adjusting the performance of the major loop. Often the cascade compensation G_c is a simple gain factor used to adjust the degree of stability of the system. The burden of modifying the transfer function G is placed on the feedback compensation H_c. In addition, the feedback function is usually provided with an adjustable gain factor to permit setting the degree of stability of the minor loop.

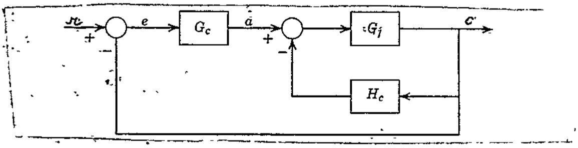


Fig. 4.8 General arrangement for feedback compensation.

The procedure for adjusting the feedback compensation can be based on the magnitude asymptotes of the minor loop. In the frequency ranges where the open-minor-loop frequency-response magnitude $|G(j\omega)\ H_c(j\omega)|$ is very large, the closed-minor-loop response $|c(j\omega)/\alpha(j\omega)|$ behaves like the reciprocal of the feedback compensation. When the open-minor-loop response magnitude is very small, the closed-minor-loop response behaves like the plant.

The feedback compensation is used primarily to improve the dynamic behavior of the plant in the mid-frequency range without altering the high gain of the plant at low frequencies. A high-frequency boundary will always exist since in any practical case the magnitude of $G(j\omega)\ H_c(j\omega)$ will become less than unity as frequency increase.

The procedure for adjusting the feedback function H_c and the cascade gain factor $[G_c(s) = K_c]$ can thus be roughed out by means of asymptotic plots of the pertinent responses and then carried out in detail by means of the gain-phase or Bode plots. The asymptotic plots enable one to examine the form of the closed-minor-loop response as the feedback compensation is adjusted.

Since there are usually several parameters to adjust in the feedback compensation procedure, the process of design is one of trial and error.

OPEN LOOP BODE PLOT.

```
NASAP PROBLEM UNCOMPENSATED PLANT

RADIANS
NONE

V1 1 2 0.83
L1 2 1 1.H
V2 1 3 1.0 IL1
R1 3 4 1.
C1 4 1 1F
V3 1 5 1.0 VC1
R2 5 6 0.2
C2 6 1  1F
V4 1 7 1.0 VC2
OUTPUT
VV4/VV1/V1                          NUMBER OF LOOPS PER ORDER
FREQ -2. 1.0 0.05
EXECUTE                                  1=     3
                                         2=     1

TRANSFER FUNCTION    VV4/VV1/V1


                   (                 )
                   (  1.00E 00       )
H(S)= 4.150E 00*---------------------------------------------------------------
                   (                                         2              3 )
                   (  0.00E 00      +5.00E 00 S   +6.00E 00 S   +1.00E 00 S    )



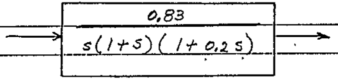
 ZERO OF TRANSFER FUNCTION

          NONE


 POLE OF TRANSFER FUNCTION

 POLE   REAL PART   IMAG. PART

   1  0.00000E 00  0.00000E 00
   2 -0.10000E 01  0.00000E 00
   3 -0.50000E 01  0.00000E 00
```



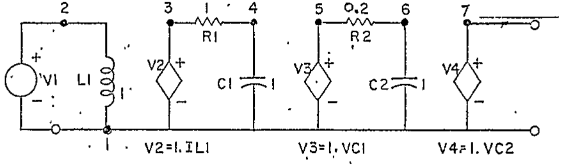


Fig 4.6

```
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 -0.1950000E 01   0.1122019E-01    0.3738094E 02    -0.9077148E 02    0.7396873E 02    0.1869047E 01
 -0.1900000E 01   0.1258926E-01    0.3636080E 02    -0.9086565E 02    0.6592366E 02    0.1819040E 01
 -0.1849999E 01   0.1412539E-01    0.3538063E 02    -0.9097119E 02    0.5875325E 02    0.1769032E 01
 -0.1800000E 01   0.1584892E-01    0.3438039E 02    -0.9108969E 02    0.5236253E 02    0.1719020E 01
 -0.1750000E 01   0.1778279E-01    0.3338010E 02    -0.9122261E 02    0.4666656E 02    0.1669005E 01
 -0.1700000E 01   0.1995263E-01    0.3237971E 02    -0.9137177E 02    0.4158981E 02    0.1618986E 01
 -0.1650000E 01   0.2238723E-01    0.3137926E 02    -0.9153906E 02    0.3706499E 02    0.1568963E 01
 -0.1600000E 01   0.2511884E-01    0.3037868E 02    -0.9172685E 02    0.3303200E 02    0.1518934E 01
 -0.1550000E 01   0.2818382E-01    0.2937793E 02    -0.9193741E 02    0.2943729E 02    0.1468897E 01
 -0.1500000E 01   0.3162277E-01    0.2837700E 02    -0.9217369E 02    0.2623318E 02    0.1418850E 01
 -0.1450000E 01   0.3548135E-01    0.2737582E 02    -0.9243874E 02    0.2337721E 02    0.1368792E 01
 -0.1400001E 01   0.3981066E-01    0.2637437E 02    -0.9273607E 02    0.2083145E 02    0.1318719E 01
 -0.1350000E 01   0.4466832E-01    0.2537251E 02    -0.9306957E 02    0.1856210E 02    0.1268626E 01
 -0.1300000E 01   0.5011870E-01    0.2437016E 02    -0.9344357E 02    0.1653903E 02    0.1218509E 01
 -0.1250000E 01   0.5623413E-01    0.2336726E 02    -0.9386304E 02    0.1473547E 02    0.1168364E 01
 -0.1200001E 01   0.6309563E-01    0.2236359E 02    -0.9433340E 02    0.1312746E 02    0.1118179E 01
 -0.1150001E 01   0.7079446E-01    0.2135896E 02    -0.9486073E 02    0.1169361E 02    0.1067948E 01
 -0.1100000E 01   0.7943273E-01    0.2015312E 02    -0.9545187E 02    0.1041494E 02    0.1017656E 01
 -0.1050000E 01   0.8912504E-01    0.1934579E 02    -0.9611426E 02    0.9274496E 01    0.9672902E 00
 -0.1000001E 01   0.9999979E-01    0.1833659E 02    -0.9685640E 02    0.8257155E 01    0.9168300E 00
 -0.9500017E 00   0.1122014E 00    0.1732504E 02    -0.9768747E 02    0.7349417E 01    0.8662525E 00
 -0.9000015E 00   0.1258921E 00    0.1631052E 02    -0.9861774E 02    0.6539225E 01    0.8155259E 00
 -0.8500013E 00   0.1412533E 00    0.1529229E 02    -0.9965829E 02    0.5815876E 01    0.7646147E 00
 -0.8000011E 00   0.1584889E 00    0.1426945E 02    -0.1008215E 03    0.5169791E 01    0.7134727E 00
 -0.7500019E 00   0.1778272E 00    0.1324087E 02    -0.1021203E 03    0.4592444E 01    0.6620438E 00
 -0.7000017E 00   0.1995254E 00    0.1220513E 02    -0.1035690E 03    0.4076210E 01    0.6102565E 00
 -0.6500015E 00   0.2238714E 00    0.1116050E 02    -0.1051826E 03    0.3614305E 01    0.5580248E 00
 -0.6000013E 00   0.2511879E 00    0.1010490E 02    -0.1069763E 03    0.3200702E 01    0.5052448E 00
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 -0.4500017E 00   0.3548120E 00    0.6844790E 01    -0.1135944E 03    0.2199073E 01    0.3422396E 00
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 -0.3500023E 00   0.4466813E 00    0.4556966E 01    -0.1191745E 03    0.1689851E 01    0.2278483E 00
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 -0.1500025E 00   0.7079417E 00   -0.4689473E 00    -0.1333551E 03    0.9474425E 00   -0.2344712E-01
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 -0.5000210E-01   0.8912466E 00   -0.3293255E 01    -0.1418157E 03    0.6844428E 00   -0.1646628E 00
 -0.1907349E-05   0.9999956E 00   -0.4799035E 01    -0.1463098E 03    0.5755038E 00   -0.2399518E 00
  0.4999733E-01   0.1122011E 01   -0.6370761E 01    -0.1509385E 03    0.4802439E 00   -0.3185381E 00
  0.9999752E-01   0.1258918E 01   -0.8009747E 01    -0.1556711E 03    0.3976606E 00   -0.4004874E 00
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  0.2499971E 00   0.1778268E 01   -0.1332894E 02    -0.1702271E 03    0.2155524E 00   -0.6664468E 00
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  0.5499973E 00   0.3548112E 01   -0.2572151E 02     0.1603796E 03    0.5175161E-01   -0.1266076E 01
  0.5999975E 00   0.3981049E 01   -0.2801645E 02     0.1555733E 03    0.3973529E-01   -0.1400823E 01
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  0.6999969E 00   0.5011837E 01   -0.3280843E 02     0.1462162E 03    0.2288638E-01   -0.1640422E 01
  0.7499971E 00   0.5623376E 01   -0.3530406E 02     0.1417252E 03    0.1717111E-01   -0.1765201E 01
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  0.8499966E 00   0.7079402E 01   -0.4048213E 02     0.1332728E 03    0.9460025E-02   -0.2024107E 01
  0.8999968E 00   0.7943223E 01   -0.4315671E 02     0.1293645E 03    0.6952863E-02   -0.2157836E 01
  0.9499969E 00   0.8912447E 01   -0.4588158E 02     0.1256950E 03    0.5080663E-02   -0.2294079E 01
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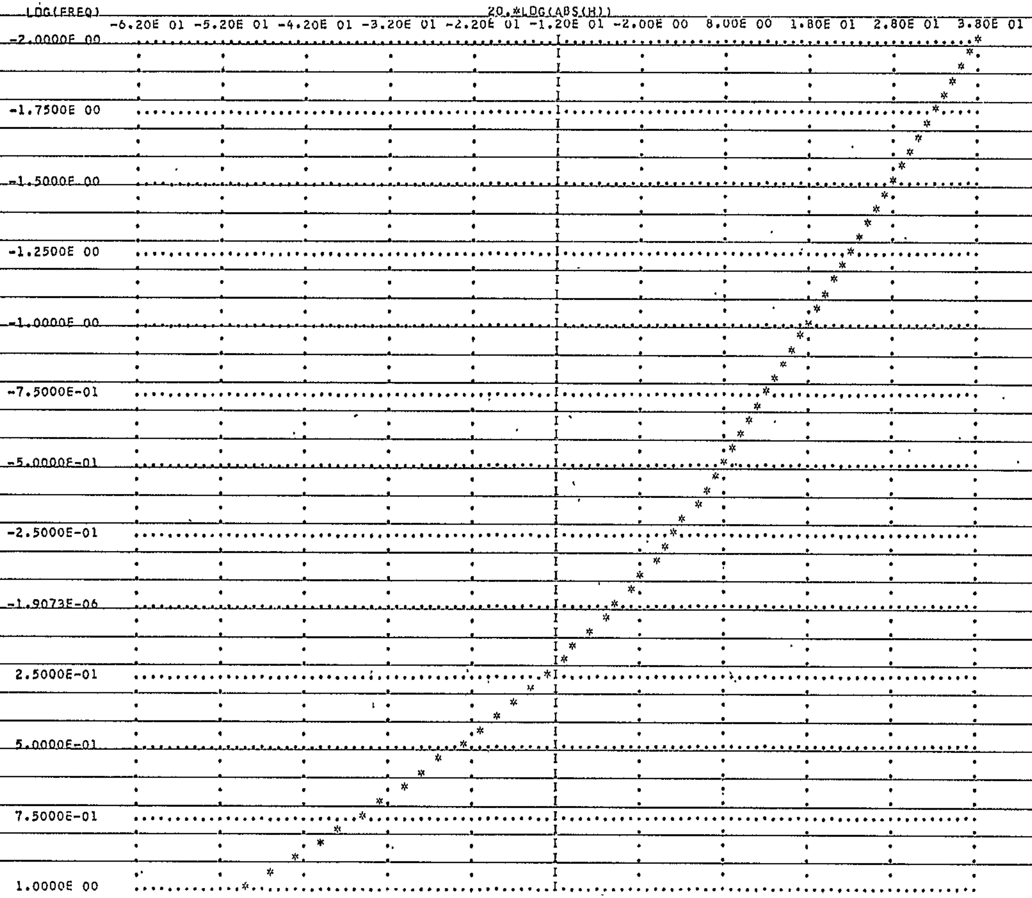
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-6.20E 01
-5.20E 01
-4.20E 01
-3.20E 01
-2.20E 01
-1.20E 01
-2.00E 00
8.00E 00
1.80E 01
2.80E 01
3.80E 01
-2.0000E 00
-1.7500E 00
-1.5000E 00
-1.2500E 00
-1.0000E 00
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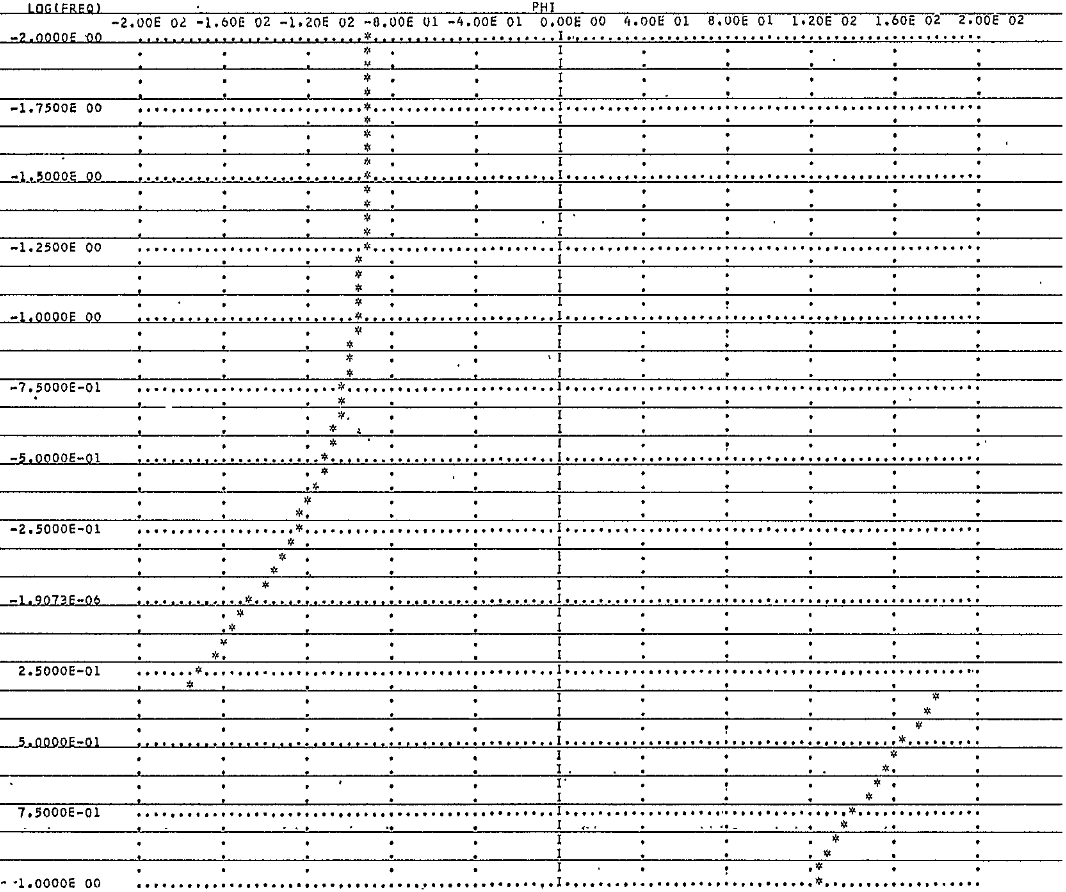

ACME DATAGRAPHIC SYSTEMS INC

```
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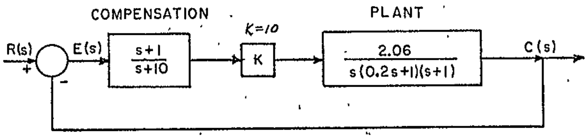
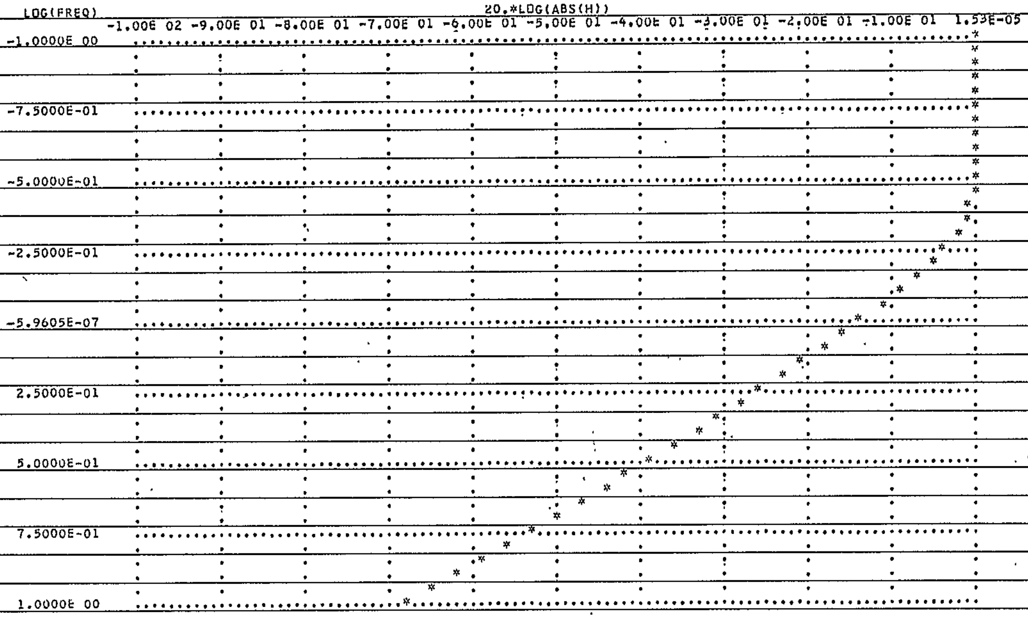
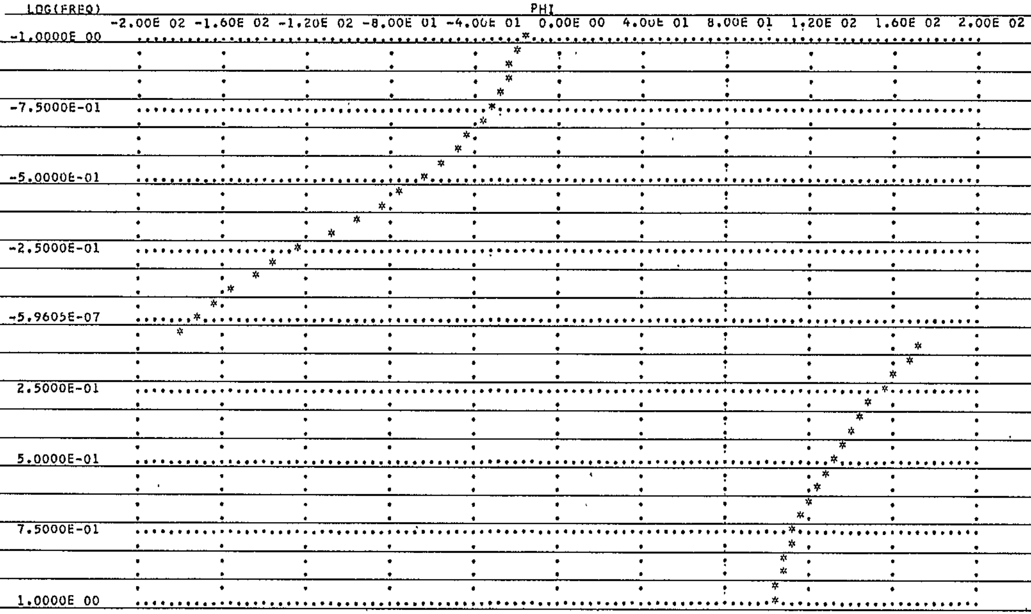
  -0.9499997E 00    0.1122019E 00    0.1081116E 00    -0.2008299E 02    0.1012525E 01    0.5405582E-02
  -0.8999997E 00    0.1258926E 00    0.1322612E 00    -0.2266541E 02    0.1015344E 01    0.6613061E-02
  -0.8499998E 00    0.1412538E 00    0.1603050E 00    -0.2561649E 02    0.1018627E 01    0.8015253E-02
  -0.7999998E 00    0.1584894E 00    0.1918001E 00    -0.2900237E 02    0.1022327E 01    0.9590007E-02
  -0.7499999E 00    0.1778280E 00    0.2252238E 00    -0.3290480E 02    0.1026269E 01    0.1126119E-01
  -0.6999999E 00    0.1995263E 00    0.2571684E 00    -0.3742506E 02    0.1030050E 01    0.1285842E-01
  -0.6500000E 00    0.2238721E 00    0.2808114E 00    -0.4268726E 02    0.1032858E 01    0.1404057E-01
  -0.6000000E 00    0.2511886E 00    0.2836820E 00    -0.4883908E 02    0.1033199E 01    0.1418410E-01
  -0.5500001E 00    0.2818382E 00    0.2441469E 00    -0.5604611E 02    0.1028507E 01    0.1220735E-01
  -0.5000001E 00    0.3162277E 00    0.1271771E 00    -0.6446948E 02    0.1014750E 01    0.6358854E-02
  -0.4500002E 00    0.3548133E 00   -0.1194394E 00    -0.7421626E 02    0.9863431E 00   -0.5971968E-02
  -0.4000002E 00    0.3981070E 00   -0.5638217E 00    -0.8525583E 02    0.9371495E 00   -0.2819109E-01
  -0.3500003E 00    0.4466833E 00   -0.1277848E 01    -0.9733092E 02    0.8631923E 00   -0.6389242E-01
  -0.3000003E 00    0.5011868E 00   -0.2312773E 01    -0.1099373E 03    0.7662337E 00   -0.1156387E 00
  -0.2500004E 00    0.5623409E 00   -0.3675883E 01    -0.1224468E 03    0.6549464E 00   -0.1837942E 00
  -0.2000004E 00    0.6309568E 00   -0.5328104E 01    -0.1343181E 03    0.5414953E 00   -0.2664052E 00
  -0.1500005E 00    0.7079450E 00   -0.7204965E 01    -0.1452428E 03    0.4362663E 00   -0.3602483E 00
  -0.1000005E 00    0.7943273E 00   -0.9242250E 01    -0.1551485E 03    0.3450542E 00   -0.4621125E 00
  -0.5000055E-01    0.8912498E 00   -0.1139055E 02    -0.1641129E 03    0.2694461E 00   -0.5695277E 00
  -0.5960464E-06    0.9999986E 00   -0.1361802E 02    -0.1722763E 03    0.2084963E 00   -0.6809011E 00
   0.4999864E-01    0.1122015E 01   -0.1590686E 02    -0.1797832E 03    0.1601979E 00   -0.7953428E 00
   0.9999383E-01    0.1258922E 01   -0.1824892E 02     0.1732430E 03    0.1223359E 00   -0.9124460E 00
   0.1499790E 00    0.1412535E 01   -0.2064151E 02     0.1667096E 03    0.9288013E-01   -0.1032076E 01
   0.1999992E 00    0.1584890E 01   -0.2308481E 02     0.1605530E 03    0.7010663E-01   -0.1154241E 01
   0.2499985E 00    0.1778273E 01   -0.2557968E 02     0.1547348E 03    0.5260354E-01   -0.1278984E 01
   0.2999986E 00    0.1995256E 01   -0.2812703E 02     0.1492349E 03    0.3923260E-01   -0.1406352E 01
   0.3499988E 00    0.2238715E 01   -0.3072656E 02     0.1440467E 03    0.2908516E-01   -0.1536328E 01
   0.3999990E 00    0.2511881E 01   -0.3337682E 02     0.1391703E 03    0.2143671E-01   -0.1668841E 01
   0.4499983E 00    0.2818372E 01   -0.3607518E 02     0.1346091E 03    0.1571231E-01   -0.1803760E 01
   0.4999985E 00    0.3162267E 01   -0.3881822E 02     0.1303652E 03    0.1145746E-01   -0.1940911E 01
   0.5499986E 00    0.3548122E 01   -0.4160159E 02     0.1264381E 03    0.8316088E-02   -0.2080080E 01
   0.5999988E 00    0.3981061E 01   -0.4442075E 02     0.1228238E 03    0.6011199E-02   -0.2221038E 01
   0.6499981E 00    0.4466816E 01   -0.4727100E 02     0.1195136E 03    0.4329611E-02   -0.2363550E 01
   0.6999983E 00    0.5011852E 01   -0.5014796E 02     0.1164955E 03    0.3108854E-02   -0.2507399E 01
   0.7499985E 00    0.5623393E 01   -0.5304749E 02     0.1137542E 03    0.2226512E-02   -0.2652374E 01
   0.7999986E 00    0.6309554E 01   -0.5596584E 02     0.1112729E 03    0.1591134E-02   -0.2798292E 01
   0.8499979E 00    0.7079423E 01   -0.5889972E 02     0.1090334E 03    0.1135043E-02   -0.2944986E 01
   0.8999981E 00    0.7943247E 01   -0.6184648E 02     0.1070167E 03    0.8084897E-03   -0.3092324E 01
   0.9499983E 00    0.8912474E 01   -0.6480367E 02     0.1052044E 03    0.5751951E-03   -0.3240184E 01
   0.9999985E 00    0.9999965E 01   -0.6776936E 02     0.1035783E 03    0.4088187E-03   -0.3388469E 01
```



For the above unity feedback control system with lead compensation, get Bode plot.

Fig 4.9


LOG(FREQ)
20.*LOG(ABS(H))
-1.00E 02
-9.00E 01
-8.00E 01
-7.00E 01
-6.00E 01
-5.00E 01
-4.00E 01
-3.00E 01
-2.00E 01
-1.00E 01
1.53E-05
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
-5.9605E-07
2.5000E-01
5.0000E-01
7.5000E-01
1.0000E 00


LOG(FREQ)
PHI
-2.00E 02
-1.60E 02
-1.20E 02
-8.00E 01
-4.00E 01
0.00E 00
4.00E 01
8.00E 01
1.20E 02
1.60E 02
2.00E 02
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
-5.9605E-07
2.5000E-01
5.0000E-01
7.5000E-01
1.0000E 00

This example, shown in Fig. 4.9 along with its NASAP circuit model, is taken from Newton Gould and Kaiser [NE 1 pp. 326-333]. One finds that the magnitude plot of the closed-loop transfer function shows no sign of resonance effects so that a reasonable closed-loop performance may be expected. The corresponding phase curve is given in Fig. 4.10.

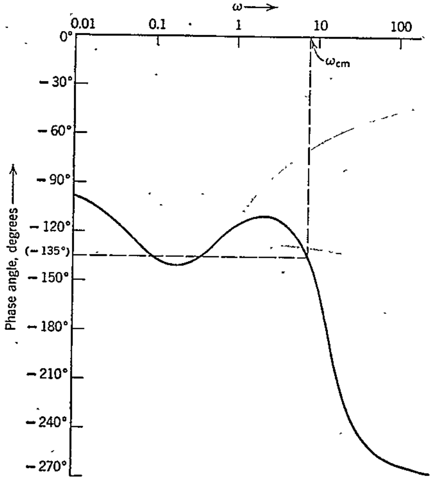


Fig. 4.10 Closed-minor-loop phase-angle response.

Using the 45° phase-margin criterion to adjust the cascade compensation, the magnitude crossover frequency of the major loop is found to be

$$\omega_{cm} = 7.7 \text{ rad sec}^{-1} \tag{4.8}$$

and the corresponding gain factor of the cascade compensation is

$$K = 38 \tag{4.9}$$

If this control system exhibits undesirable performance because of the lag-compensation effect in the open-major-loop response at low frequencies, some improvement can be expected by decreasing the feedback compensation time constant T_c and by some increase in the minor-loop gain factor K_c. The degree of improvement achievable must be ascertained by further NASAP runs. Further discussion of this example is found in Chapter V including step response and error response for the control system with its gain K = 38.

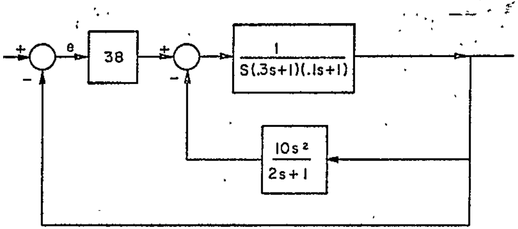
NASAP ~~[illegible]~~ *OUTPUT RESPONSE*

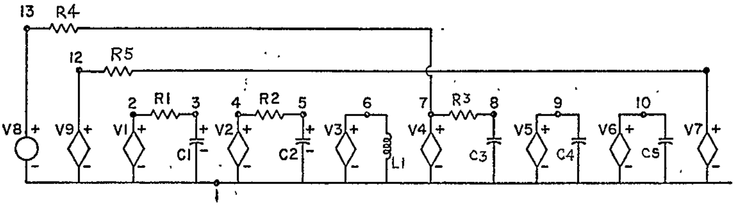
NEWTON - GOULD - KAISER

P. 326 - 333

```
RADIANS
STEP

V8 1 13 1.
R4 13 7 1.
V9 1 12 38 IR4
R5 12 11 1.
V1 1 2 1. IR5
R1 2 3 0.3
C1 3 1 1F
V2 1 4 1. VC1
R2 4 5 0.1
C2 5 1 1F
V3 1 6 1. VC2
L1 6 1 1H
V4 1 7 1. IL1
R3 7 8 2.
C3 8 1 1F
V5 1 9 1. VC3
C4 9 1 1F
V6 1 10 10 IC4
C5 10 1 1F
V7 1 11 1. IC5
OUTPUT
VV4/VV8
FREQ -1.0 2.0 0.05
TIME 2.0
EXECUTE
```





```
TRANSFER FUNCTION    VV4/VV8


                    (                            )
                    (  5.00E-01     +1.00E 00 S  )
H(S)= 1.267E-03*-------------------------------------------------------------     NUMBER OF LOOPS PER ORDER
                    (                          2             3             4 )              1=    6
                    (  6.33E 02     +1.28E 03 S   +2,07E 02 S   +1.38E 01 S   +1.00E 00 S   )              2=    5
                                                                                                        3=    1


  ZERO OF TRANSFER FUNCTION                    POLE OF TRANSFER FUNCTION
  ZERO   REAL PART    IMAG. PART               POLE   REAL PART    IMAG. PART

     1 -0.50000E 00  0.00000E 00                 1 -0.53860E 00  0.00000E 00
                                                 2 -0.28810E 01  0.12157E 02
                                                 3 -0.28810E 01 -0.12157E 02
                                                 4 -0.75327E 01  0.00000E 00
```

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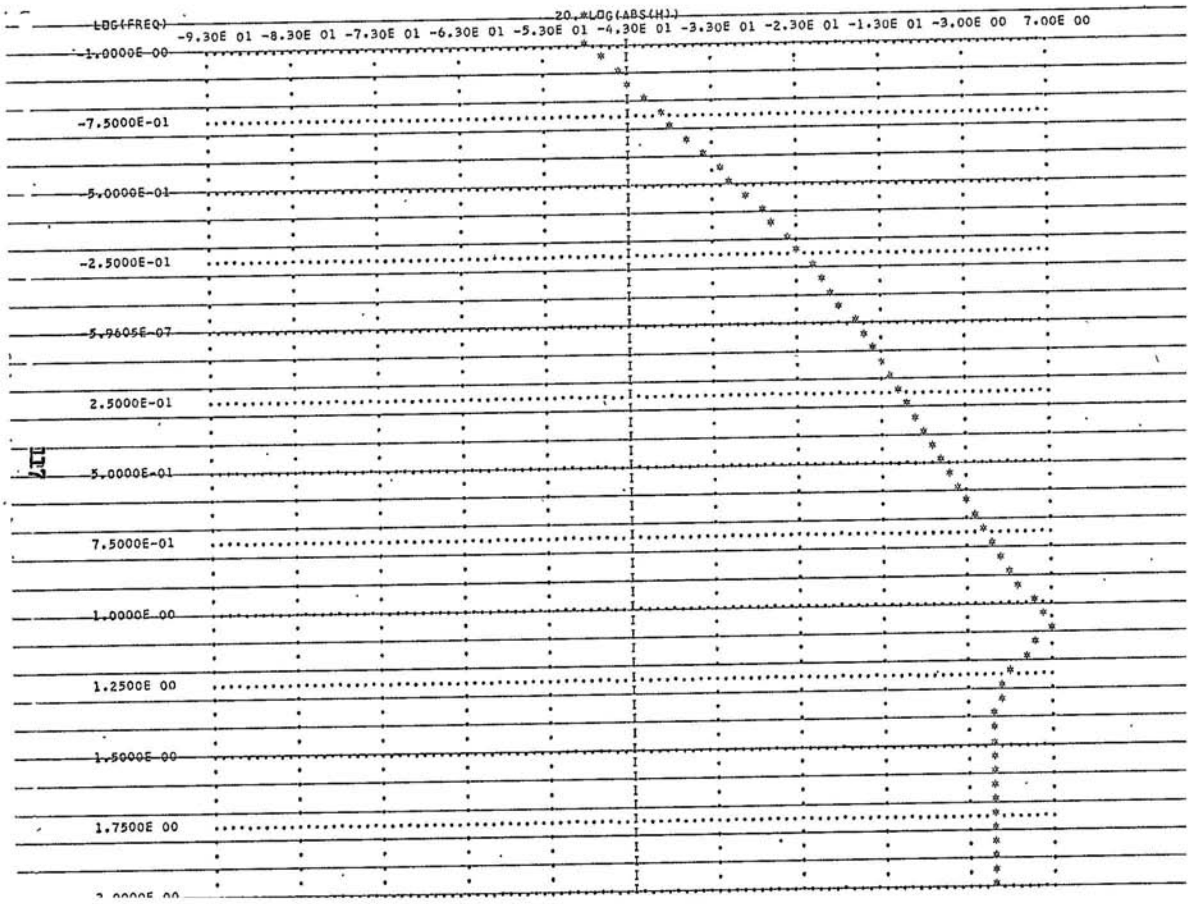
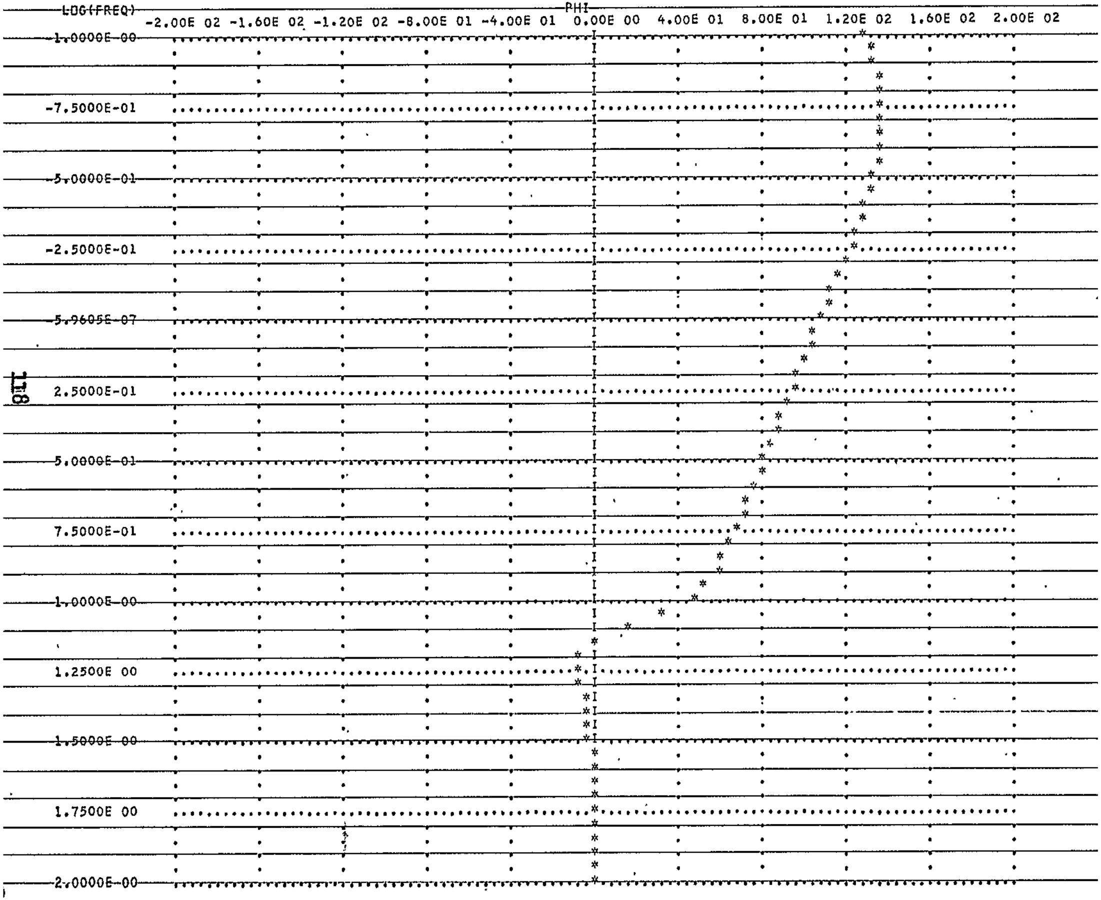
Fig 49 a

```
     LOG(FREQ)          FREQ         20.*LOG(ABS(H))      PHI(H)           ABS(H)        LOG(ABS(H))
  -0.9999996E 00    0.1000001E 00    -0.4772751E 02    0.1298571E 03    0.4107933E-02    -0.2386375E 01
  -0.9499997E 00    0.1122019E 00    -0.4613454E 02    0.1317182E 03    0.4934825E-02    -0.2306727E 01
  -0.8999997E 00    0.1258926E 00    -0.4449901E 02    0.1331188E 03    0.5957287E-02    -0.2224951E 01
  -0.8499998E 00    0.1412538E 00    -0.4282719E 02    0.1346163E 03    0.7221695E-02    -0.2141360E 01
  -0.7999998E 00    0.1584894E 00    -0.4112643E 02    0.1355759E 03    0.8783691E-02    -0.2056322E 01
  -0.7499999E 00    0.1778280E 00    -0.3940480E 02    0.1361707E 03    0.1070923E-01    -0.1970241E 01
  -0.6999999E 00    0.1995263E 00    -0.3767090E 02    0.1363828E 03    0.1307537E-01    -0.1883545E 01
  -0.6500000E 00    0.2238721E 00    -0.3593353E 02    0.1362026E 03    0.1597067E-01    -0.1796677E 01
  -0.6000000E 00    0.2511886E 00    -0.3420165E 02    0.1356290E 03    0.1949489E-01    -0.1710078E 01
  -0.5500001E 00    0.2818382E 00    -0.3248387E 02    0.1346695E 03    0.2375774E-01    -0.1624194E 01
  -0.5000001E 00    0.3162277E 00    -0.3078896E 02    0.1333401E 03    0.2887696E-01    -0.1539448E 01
  -0.4500002E 00    0.3548133E 00    -0.2912479E 02    0.1316652E 03    0.3497519E-01    -0.1456240E 01
  -0.4000002E 00    0.3981070E 00    -0.2749858E 02    0.1296771E 03    0.4217642E-01    -0.1374929E 01
  -0.3500003E 00    0.4466833E 00    -0.2591649E 02    0.1274155E 03    0.5060278E-01    -0.1295825E 01
  -0.3000003E 00    0.5011868E 00    -0.2438327E 02    0.1249259E 03    0.6037195E-01    -0.1219164E 01
  -0.2500004E 00    0.5623409E 00    -0.2290211E 02    0.1222572E 03    0.7159674E-01    -0.1145106E 01
  -0.2000004E 00    0.6309568E 00    -0.2147447E 02    0.1194587E 03    0.8438689E-01    -0.1073724E 01
  -0.1500005E 00    0.7079450E 00    -0.2010020E 02    0.1165779E 03    0.9885269E-01    -0.1005011E 01
  -0.1000005E 00    0.7943273E 00    -0.1877763E 02    0.1136571E 03    0.1151111E 00    -0.9388824E 00
  -0.5000055E-01    0.8912498E 00    -0.1750392E 02    0.1107315E 03    0.1332917E 00    -0.8751966E 00
  -0.5960464E-06    0.9999986E 00    -0.1627541E 02    0.1078284E 03    0.1535427E 00    -0.8137707E 00
   0.4999864E-01    0.1122015E 01    -0.1508801E 02    0.1049658E 03    0.1760352E 00    -0.7544003E 00
   0.9999884E-01    0.1258922E 01    -0.1393749E 02    0.1021536E 03    0.2009673E 00    -0.6968746E 00
   0.1499990E 00    0.1412535E 01    -0.1281991E 02    0.9939413E 02    0.2285621E 00    -0.6409953E 00
   0.1999992E 00    0.1584890E 01    -0.1173173E 02    0.9668375E 02    0.2590677E 00    -0.5865864E 00
   0.2499985E 00    0.1778273E 01    -0.1066996E 02    0.9401411E 02    0.2927532E 00    -0.5334982E 00
   0.2999986E 00    0.1995256E 01    -0.9632131E 01    0.9137375E 02    0.3299083E 00    -0.4816065E 00
   0.3499988E 00    0.2238715E 01    -0.8616458E 01    0.8874956E 02    0.3708318E 00    -0.4308229E 00
   0.3999990E 00    0.2511881E 01    -0.7621623E 01    0.8612885E 02    0.4158328E 00    -0.3810812E 00
   0.4499983E 00    0.2818372E 01    -0.6646667E 01    0.8349980E 02    0.4652288E 00    -0.3323334E 00
   0.4999985E 00    0.3162267E 01    -0.5690676E 01    0.8085335E 02    0.5193572E 00    -0.2845338E 00
   0.5499986E 00    0.3548122E 01    -0.4752439E 01    0.7818417E 02    0.5785994E 00    -0.2376220E 00
   0.5999988E 00    0.3981061E 01    -0.3829819E 01    0.7549117E 02    0.6434414E 00    -0.1914909E 00
   0.6499981E 00    0.4466816E 01    -0.2918818E 01    0.7277647E 02    0.7145935E 00    -0.1459409E 00
   0.6999983E 00    0.5011852E 01    -0.2012150E 01    0.7004272E 02    0.7932178E 00    -0.1006075E 00
   0.7499985E 00    0.5623393E 01    -0.1097363E 01    0.6728380E 02    0.8813163E 00    -0.5486818E-01
   0.7999986E 00    0.6309554E 01    -0.1538576E 00    0.6446570E 02    0.9824424E 00    -0.7692881E-02
   0.8499979E 00    0.7079423E 01     0.8508540E 00    0.6148209E 02    0.1102917E 01     0.4254270E-01
   0.8999981E 00    0.7943247E 01     0.1964594E 01    0.5805446E 02    0.1253804E 01     0.9822971E-01
   0.9499983E 00    0.8912474E 01     0.3245031E 01    0.5350861E 02    0.1452953E 01     0.1622516E 00
   0.9999985E 00    0.9999965E 01     0.4707366E 01    0.4633690E 02    0.1719366E 01     0.2353683E 00
   0.1049997E 01    0.1122012E 07     0.6098736E 01    0.3395134E 02    0.2018073E 01     0.3049368E 00
   0.1099998E 01    0.1258918E 02     0.6524124E 01    0.1588383E 02    0.2119368E 01     0.3262062E 00
   0.1149998E 01    0.1412530E 02     0.5379307E 01    0.6764114E-01    0.1857656E 01     0.2689654E 00
   0.1199998E 01    0.1584886E 02     0.3663709E 01   -0.7256633E 01    0.1524704E 01     0.1831855E 00
   0.1249997E 01    0.1778268E 02     0.2276337E 01   -0.8603992E 01    0.1299622E 01     0.1138169E 00
   0.1299997E 01    0.1995250E 02     0.1364489E 01   -0.7533766E 01    0.1170104E 01     0.6822443E-01
   0.1349998E 01    0.2238708E 02     0.8104062E 00   -0.5898328E 01    0.1097793E 01     0.4052031E-01
   0.1399998E 01    0.2511873E 02     0.4825823E 00   -0.4389949E 01    0.1057132E 01     0.2412912E-01
   0.1449997E 01    0.2818362E 02     0.2894176E 00   -0.3187364E 01    0.1033881E 01     0.1447063E-01
   0.1499997E 01    0.3162257E 02     0.1749709E 00   -0.2285169E 01    0.1020349E 01     0.8748546E-02
   0.1549997E 01    0.3548112E 02     0.1065980E 00   -0.1627623E 01    0.1012348E 01     0.5329899E-02
   0.1599998E 01    0.3981049E 02     0.6536704E-01   -0.1155313E 01    0.1007554E 01     0.3268354E-02
   0.1649997E 01    0.4466803E 02     0.4031325E-01   -0.8186165E 00    0.1004652E 01     0.2015662E-02
   0.1699997E 01    0.5011838E 02     0.2496372E-01   -0.5795133E 00    0.1002878E 01     0.1248186E-02
   0.1749997E 01    0.5623376E 02     0.1551770E-01   -0.4100884E 00    0.1001788E 01     0.7758853E-03
   0.1799997E 01    0.6309535E 02     0.9678025E-02   -0.2901447E 00    0.1001115E 01     0.4839012E-03
   0.1849997E 01    0.7079402E 02     0.6044853E-02   -0.2052789E 00    0.1000696E 01     0.3022428E-03
   0.1899997E 01    0.7943224E 02     0.3776457E-02   -0.1452416E 00    0.1000435E 01     0.1888229E-03
   0.1949997E 01    0.8912447E 02     0.2368760E-02   -0.1027696E 00    0.1000273E 01     0.1184380E-03
   0.1999997E 01    0.9999934E 02     0.1482621E-02   -0.7272267E-01    0.1000171E 01     0.7413107E-04
```


20.*LOG(ABS(H))
LOG(FREQ)
-9.30E 01
-8.30E 01
-7.30E 01
-6.30E 01
-5.30E 01
-4.30E 01
-3.30E 01
-2.30E 01
-1.30E 01
-3.00E 00
7.00E 00
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
-5.9605E-07
2.5000E-01
5.0000E-01
7.5000E-01
1.0000E 00
1.2500E 00
1.5000E 00
1.7500E 00
2.0000E 00


LOG(FREQ)
PHI
-2.00E 02
-1.60E 02
-1.20E 02
-8.00E 01
-4.00E 01
0.00E 00
4.00E 01
8.00E 01
1.20E 02
1.60E 02
2.00E 02
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
-5.9605E-07
2.5000E-01
5.0000E-01
7.5000E-01
1.0000E 00
1.2500E 00
1.5000E 00
1.7500E 00
2.0000E 00

The configuration of the second multiloop example is also similar to that shown in Fig. 4.8. In this case however the G_c block is simply a direct connection as shown in Fig. 4.11. Figures 4.11a through 4.11f include the NASAP printout of the Bode plots and the step response.

```
    NASAP PROBLEM MULTIPLE FEEDBACK SYSTEM

    V1 1 2 1. IR3
    R1 2 3 0.3
    C1 3 1 1F
    V2 1 4 1.0 VC1
    R2 4 5 0.8
    C2 5 1 1.F
    V3 1 6 1.0 VC2
    L1 6 1 1H
    V4 1 7 0.042 IL1
    C3 7 1 1F
    V5 1 8  30 IC3
    V6 1 9  95.7 IR4
    R3 9 8 1.0
    V7 1 10 1.
    R4 10 7 1.
    OUTPUT
    VV4/VV7
    FREQ  -1.0  1.0  0.05
    TIME 5.0
    EXECUTE

  NUMBER OF LOOPS PER ORDER

       1=     5
       2=     1

  TRANSFER FUNCTION    VV4/VV7

                        (                )
                        (  1.00E 00      )
    H(S)= 1.675E 01*----------------------------------------------------
                        (
                        (  1.67E 01     +9.42E 00 S   +4.58E 00 S 2  +1.00E 00 S 3 )

  ZERO OF TRANSFER FUNCTION

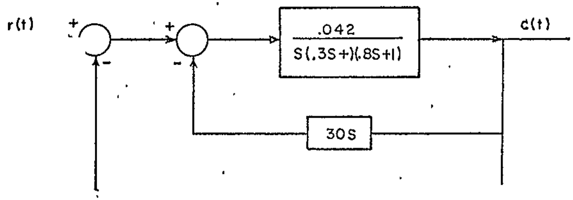
               NONE

   POLE OF TRANSFER FUNCTION

   POLE    REAL PART    IMAG. PART
      1 -0.65674E 00  0.21657F 01
      2 -0.65674E 00 -0.21657E 01
      3 -0.32699E 01 -0.44516E-11
```

} -.658 ± j 2.166

-3.27



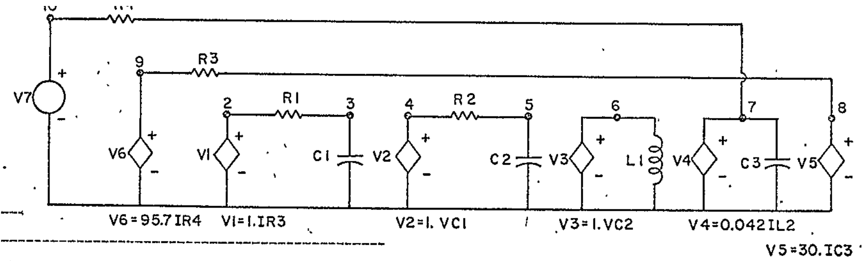


Fig 4.11

```
          LOG(FREQ)         FREQ          20.*LOG(ABS(H))     PHI(H)            ABS(H)          LOG(ABS(H))
        -0.9999996E 00   0.1000001E 00   0.4088327E 00   -0.2078052E 02   0.1048194E 01   0.2044164E-01
        -0.9499996E 00   0.1122020E 00   0.5185683E 00   -0.2348901E 02   0.1061521E 01   0.2592842E-01
        -0.8999997E 00   0.1258928E 00   0.6587968E 00   -0.2661081E 02   0.1078797E 01   0.3293984F-01
        -0.8499997F 00   0.1412539E 00   0.8384077E 00   -0.3024033E 02   0.1101337E 01   0.4192039E-01
        -0.7999997E 00   0.1584895F 00   0.1068852E 01   -0.3451056E 02   0.1130948E 01   0.5344265F-01
        -0.7499997E 00   0.1778282E 00   0.1364452E 01   -0.3961685E 02   0.1170099E 01   0.6822264F-01
        -0.6999997E 00   0.1995266F 00   0.1741681E 01   -0.4586024E 02   0.1222036E 01   0.8708405E-01
        -0.6499997E 00   0.2238724E 00   0.2214771E 01   -0.5372380E 02   0.1290442E 01   0.1107386F 00
        -0.5999997F 00   0.2511889E 00   0.2779410E 01   -0.6400209E 02   0.1377116E 01   0.1389706E 00
        -0.5499998E 00   0.2818387E 00   0.3361508E 01   -0.7794807E 02   0.1472569E 01   0.1680754E 00
        -0.4999998E 00   0.3162283E 00   0.3691287F 01   -0.9706577E 02   0.1529552E 01   0.1845644E 00
        -0.4499998E 00   0.3548137E 00   0.3186799E 01   -0.1213195E 03   0.1443245E 01   0.1593400E 00
        -0.3999998F 00   0.3981074E 00   0.1365776E 01   -0.1464752E 03   0.1170278E 01   0.6828880E-01
        -0.3499998E 00   0.4466839E 00   -0.1468303E 01  -0.1674146E 03   0.8444711E 00   -0.7341516E-01
        -0.2999998E 00   0.5011875E 00   -0.4692690E 01   0.1768610E 03   0.5825933E 00   -0.2346345E 00
        -0.2499998E 00   0.5623416E 00   -0.7964831E 01   0.1650078E 03   0.3997224E 00   -0.3982416F 00
        -0.1999998E 00   0.6309577E 00   -0.1118003E 02   0.1556891E 03   0.2760568E 00   -0.5590015E 00
        -0.1499999E 00   0.7079461E 00   -0.1432597E 02   0.1480431E 03   0.1921771E 00   -0.7162983E 00
        -0.9999985F-01   0.7943286E 00   -0.1741570E 02   0.1415611E 03   0.1346525E 00   -0.8707854E 00
        -0.4999989E-01   0.8912512E 00   -0.2046562E 02   0.1359417E 03   0.9478039E-01  -0.1023281E 01
         0.1192093F-06   0.1000000E 01   -0.2348933E 02   0.1310002E 03   0.6691647E-01  -0.1174467E 01
         0.4999959E-01   0.1122017E 01   -0.2649677E 02   0.1266174E 03   0.4733269E-01  -0.1324839E 01
         0.9999979E-01   0.1258924E 01   -0.2949487E 02   0.1227107E 03   0.3351625E-01  -0.1474744E 01
         0.1500000E 00   0.1412538E 01   -0.3248805E 02   0.1192196E 03   0.2374632E-01  -0.1624403E 01
         0.1999992E 00   0.1584890E 01   -0.3547902F 02   0.1160962E 03   0.1682860E-01  -0.1773952E 01
         0.2499994E 00   0.1778276F 01   -0.3846954E 02   0.1133009E 03   0.1192675F-01  -0.1923477E 01
         0.2999996E 00   0.1995260E 01   -0.4146046E 02   0.1107996E 03   0.8452315E-02  -0.2073024E 01
         0.3499998E 00   0.2238720E 01   -0.4445229E 02   0.1085622E 03   0.5989432F-02  -0.2222614E 01
         0.4000000E 00   0.2511886E 01   -0.4744510E 02   0.1065618E 03   0.4243694F-02  -0.2372255E 01
         0.4499992E 00   0.2818375E 01   -0.5043889E 02   0.1047741E 03   0.3006455E-02  -0.2521945E 01
         0.4999994F 00   0.3162272E 01   -0.5343379E 02   0.1031771E 03   0.2129658E-02  -0.2671690F 01
         0.5499996E 00   0.3548129E 01   -0.5642957E 02   0.1017511E 03   0.1508416E-02  -0.2821479E 01
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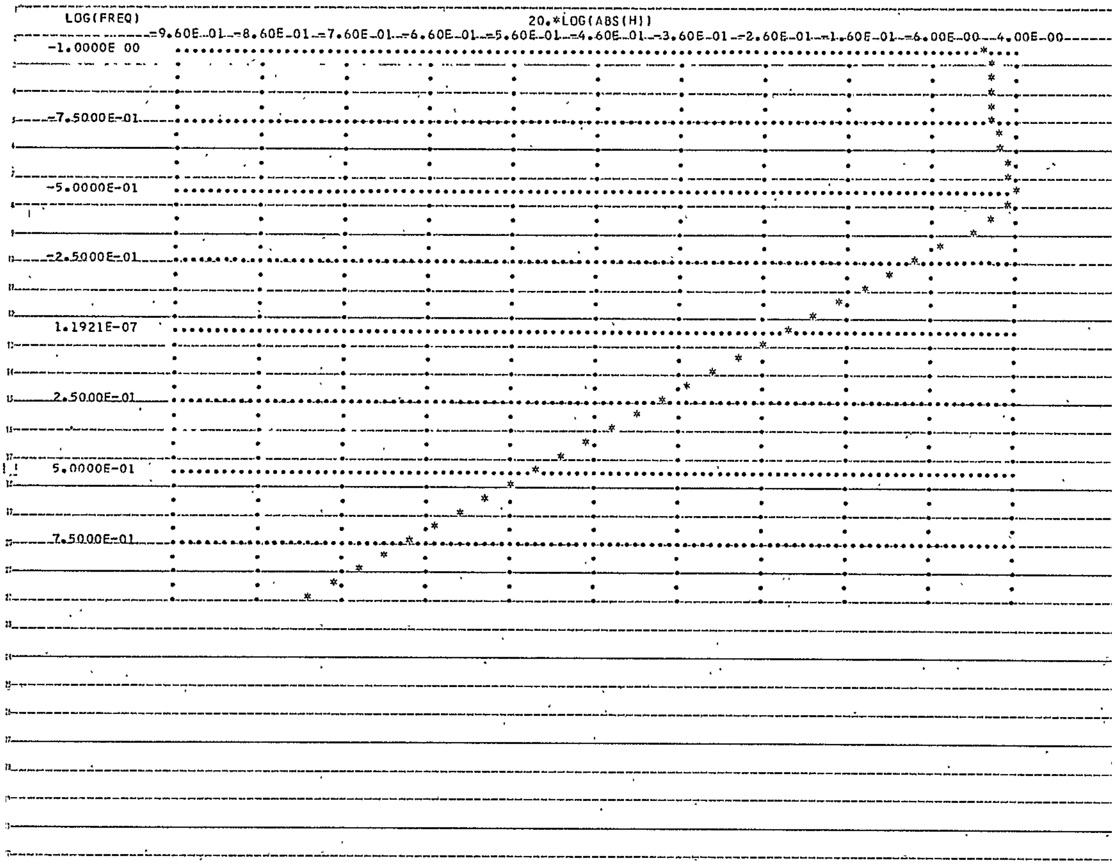
         0.7999996E 00   0.6309566E 01   -0.7141765F 02   0.9661897E 02   0.2686067E-03  -0.3570883E 01
         0.8499998E 00   0.7079450E 01   -0.7441646E 02   0.9590010E 02   0.1901853E-03  -0.3720823E 01
         0.8999990E 00   0.7943260F 01   -0.7741547E 02   0.9525916E 02   0.1346562E-03  -0.3870773E 01
         0.9499992E 00   0.8912486E 01   -0.8041469E 02   0.9468771E 02   0.9533772E-04  -0.4020735E 01
```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-7.60E-01
-6.60E-01
-5.60E-01
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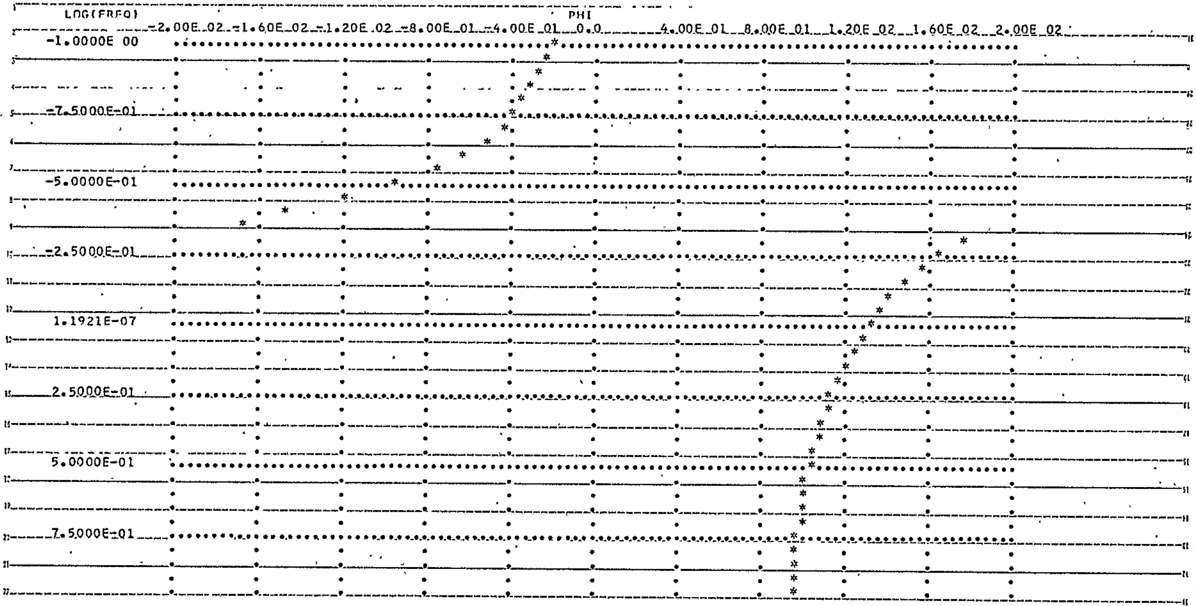
-3.60E-01
-2.60E-01
-1.60E-01
-6.00E-00
-4.00E-00
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
1.1921E-07
2.5000E-01
5.0000E-01
7.5000E-01


LOG(FREQ)
PHI
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-1.60E 02
-1.20E 02
-8.00E 01
-4.00E 01
0.0
4.00E 01
8.00E 01
1.20E 02
1.60E 02
2.00E 02
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
1.1921E-07
2.5000E-01
5.0000E-01
7.5000E-01

A04118

```
STEP    RESPONSE FUNCTION

    E(T) =
                                       (-0.6567E 00   J 0.2166E 01 ) T
          (-0.2777E 00   J 0.4199E 00 ) E

                                       (-0.6567E 00   J-0.2166E 01 ) T
          (-0.2777E 00   J-0.4199E 00 ) E

                                       (-0.3270E 01   J-0.4452E-11 ) T
          (-0.4446E 00   J 0.6053E-12 ) E

                                       ( 0.0          J 0.0         ) T
          ( 0.1000E 01   J-0.1361E-11 ) E
```

$r(t) = \text{unit step}$

$\lim_{t\to\infty} c(t) = 1.0 \quad \therefore e_{ss} = 0$

```
STEP    RESPONSE

    TIME                    VV4/VV7
     0.0                    0.0
     0.1000E 00             0.24863482E-02
     0.2000E 00             0.17693222E-01
     0.3000E 00             0.52990913E-01
     0.4000E 00             0.11118782E 00
     0.5000E 00             0.19170851F 00
     0.6000E 00             0.29158026E 00
     0.7000E 00             0.40626097E 00
     0.8000E 00             0.53033084E 00
     0.9000E 00             0.65806329F 00
     0.1000E 01             0.78388274E 00
     0.1100E 01             0.90271705E 00
     0.1200E 01             0.10102568E 01
     0.1300E 01             0.11031199E 01
     0.1400E 01             0.11789398F 01
     0.1500E 01             0.12363815E 01
     0.1600E 01             0.12750998E 01
     0.1700E 01             0.12956476E 01
     0.1800E 01             0.12993526E 01
     0.1900E 01             0.12881603E 01
     0.2000E 01             0.12644787F 01
     0.2100E 01             0.12310028E 01
     0.2200E 01             0.11905565F 01
     0.2300E 01             0.11459455E 01
     0.2400E 01             0.10998249E 01
     0.2500E 01             0.10545912E 01
     0.2600E 01             0.10123024F 01
     0.2700E 01             0.97462213E 00
     0.2800E 01             0.94278491E 00
     0.2900E 01             0.91759592E 00
     0.3000E 01             0.89944178E 00
     0.3100E 01             0.88837527E 00
     0.3200E 01             0.88391238F 00
     0.3300E 01             0.88558936E 00
     0.3400F 01             0.89252687F 00
     0.3500E 01             0.90374583E 00
     0.3600F 01             0.91818368E 00
     0.3700E 01             0.93475419E 00
     0.3800E 01             0.95240295F 00
     0.3900E 01             0.97015399E 00
     0.4000E 01             0.98714637E 00
     0.4100E 01             0.10026608E 01
     0.4200E 01             0.10161390F 01
     0.4300E 01             0.10271864E 01
     0.4400E 01             0.10355749F 01
     0.4500E 01             0.10412312E 01
     0.4600E 01             0.10442200F 01
     0.4700E 01             0.10447283E 01
     0.4800E 01             0.10430355E 01
     0.4900E 01             0.10394945F 01
     0.5000E 01             0.10345020E 01
```

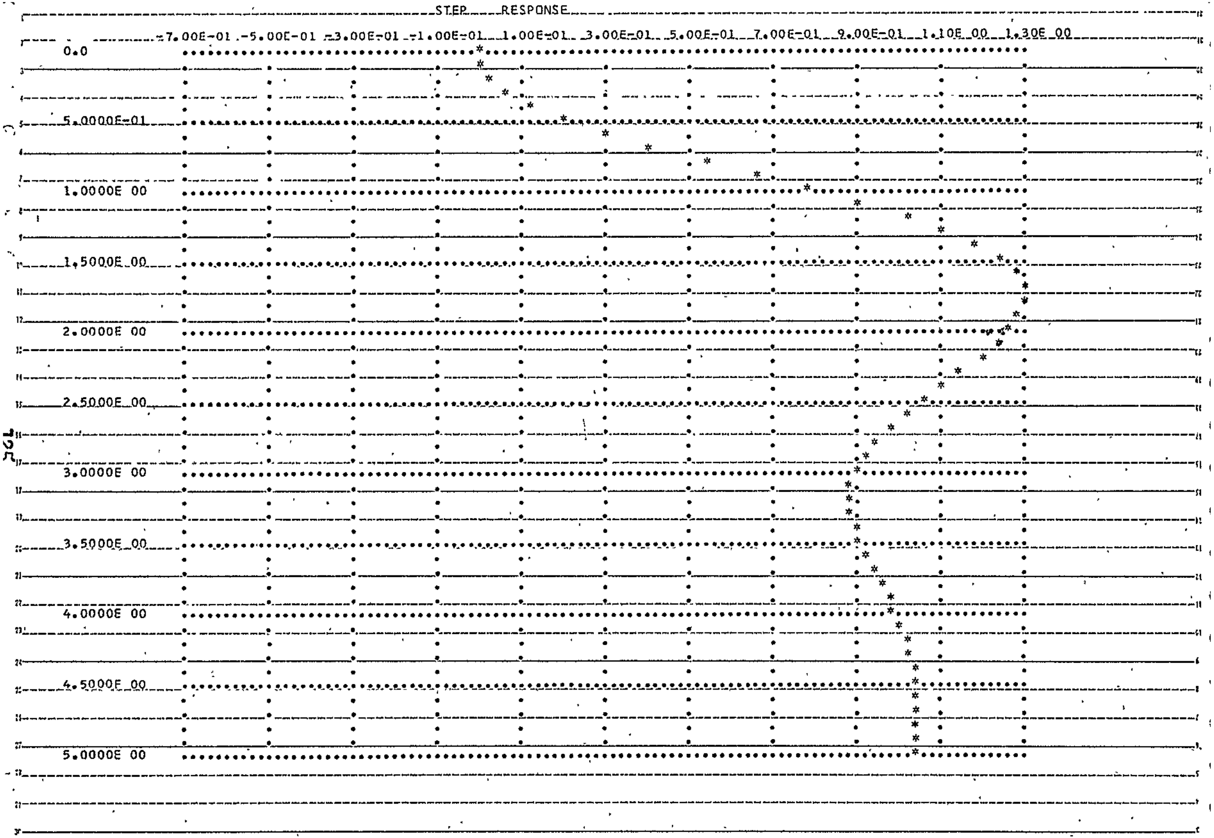
peak of overshoot (0.1800E 01, 0.12993526E 01)

30%

valley of undershoot (0.3200E 01, 0.88391238F 00)

12%

A04122

STEP RESPONSE
-7.00E-01
-5.00E-01
-3.00E-01
-1.00E-01
1.00E-01
3.00E-01
5.00E-01
7.00E-01
9.00E-01
1.10E 00
1.30E 00
0.0
5.0000E-01
1.0000E 00
1.5000E 00
2.0000E 00
2.5000E 00
3.0000E 00
3.5000E 00
4.0000E 00
4.5000E 00
5.0000E 00

CHAPTER V

CONTROL SYSTEMS ANALYSIS IN THE TIME DOMAIN

VA INPUT SIGNALS FOR TIME RESPONSE

The impulse function is the basic tool for analysis and synthesis of linear systems. However specifically for the study of linear control systems the unit step function response is the most widely used with the unit impulse and the unit ramp functions also commonly used as test inputs.

We note in passing that these three functions are related to each other by one or more integrations or differentiations. For example the unit ramp as a function of time is the integral of the unit step function. On the other hand the unit impulse may be regarded as the derivative of the unit step function (this concept is adequate for the purposes of this manual even though this derivative does not exist in the sense of calculus).

One of the first steps in adapting NASAP to control system design was to investigate the feasibility of incorporating additional input functions. This is discussed next.

VB. ADDITIONAL INPUTS FOR CONTROL APPLICATIONS

Although the NASAP 69/I package provides the time response of the output only for an impulse excitation, the time response subroutine INV of NASAP can easily be extended to the other basic input signals often used in control theory. The necessary additions to NASAP incorporate the step and ramp excitations are given in Appendix A.

The algorithm used by NASAP to determine the residues of the poles of the transfer function assumes that the poles are simple. This poses no problem since the root-finding subroutine of NASAP will not find double or higher roots but will locate a number of simple roots in the neighborhood of the actual higher order root location. Thus for an actual double root on the negative σ-axis, say at

$$s_{1,2} = -a,$$

the root finding algorithm of NASAP will indicate complex root pair at

$$s_{1,2} = a \pm jb$$

where the imaginary parts of $s_{1,2}$ are extremely small in comparison to the real part. Thus while the analytic time response for a system with higher order poles will not be correct, strictly speaking, the table and plot of discrete time values will be sufficiently accurate for practical purposes.

Given a rational transfer function

$$H(s) = \frac{C(s)}{R(s)} \tag{5.1}$$

and the Laplace transform of the excitation R(s), the Laplace transform of the output can be expressed

$$C(s) = H(s)R(s).$$

Then, by finding the residues of the poles of H(s)R(s), the time response of the output c(t) is readily obtained.

If the excitation is an impulse function, R(s) = 1, then the Laplace transform of the output C(s) is numerically equal to the transfer function H(s). This is method used by NASAP to find the impulse response. Once the transfer function and its poles have been determined, all that is necessary to find the corresponding time response of the output is to calculate the residues of the poles of the transfer function.

However, if the excitation is a unit step, $R(s) = \frac{1}{s}$, then the poles of the Laplace transform of the output are the poles of transfer function H(s) in addition to a pole at the origin. If H(s) has no higher order poles and no pole at the origin, then the analytic expression and the tabular values of the output will be correct. Note that the residue of the pole at the origin is the steady-state value of the output. If H(s) has higher order poles but no pole at the origin, then even though the analytic expressio for the output as determined by NASAP will be somewhat in error, the tabular values will be correct. If H(s) does have a pole at the origin, then the results obtained from NASAP for a step response will be in error and will probably cause premature termination of the execution due to division by zero. This can be seen as follows:

Given
$$H(s) = \frac{K(s+a_1)(s+a_2) \ldots (s+a_m)}{s(s+b_1)(s+b_2) \ldots (s+b_n)} \tag{5.3}$$

Then
$$C(s) = \frac{1}{s} \frac{K(s+a_1)(s+a_2) \ldots (s+a_m)}{s(s+b_1)(s+b_2) \ldots (s+b_n)} \tag{5.4}$$

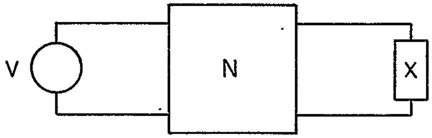
when $R(s) = \frac{1}{s}$.

The residue at the pole at $s = b_1$ can be found by

$$\mathrm{Res}(s - b_1) = (s + b_1)\, C(s) \Big|_{s=b_1}$$

$$= \frac{1}{-b_1} \frac{K(a_1-b_1)(a_2-b_1) \ldots (a_m-b_1)}{-b_1(b_2-b_1) \ldots (b_n-b_1)} \tag{5.5}$$

Since the algorithm used in NASAP cannot indicate double poles, it assures two distinct poles at the origin. When the algorithm proceeds to determine the residue of one of these poles at the origin, the presence of the other pole at the origin causes the denominator of (3) to go to zero and thus the residue approaches infinity. This situation can be avoided by the addition of some elements to the original circuit to put a zero at the origin and thereby determining the ramp response of this new network. In Fig. 5.1 is given a two port network N with a source V (either an independent current or voltage source) and an element x whose voltage is the required output quantity. The transfer function of this network has a pole at the origin and it is required to find the response of the voltage across the element x to a unit step excitation.



$$H(s) = \frac{V_x(s)}{V(s)} = \frac{P(s}{sQ(s}$$

Fig. 5.1

Now consider the modification of the network in Fig. 5.1 shown in Fig. 5.2.

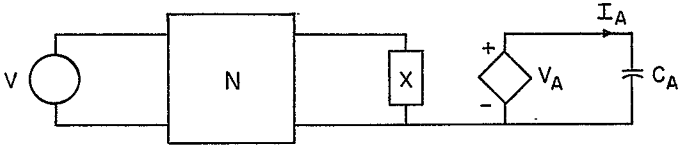


Fig. 5.2

where V_A is a dependent voltage source whose voltage equals that across the element x and C_A is a one farad capacitor. Now

$$\frac{I_A(s)}{V(s)} = \frac{sC_AV_A(s)}{V(s)} = \frac{sV_A(s)}{V(s)} \tag{5.6}$$

But $$V_A(s) = V_x(s)$$

so that $$\frac{I_A(s)}{V(s)} = \frac{sV_x(s)}{V(s)} = \frac{sP(s)}{sQ(s)} = \frac{P(s)}{Q(s)} \tag{5.7}$$

Thus the pole at the origin is effectively eliminated. The response of $i_a(t)$ to a ramp input is equivalent to the response of $v_x(t)$ to a step input.

A similar method can be used if the output quantity is a current through an element x. In this case V_A is a dependent voltage source whose voltage equals the current through the element x.

If the excitation is a ramp, r(t) = t, then $R(s) = \frac{1}{s^2}$. Thus the poles of the Laplace transform of the output are the poles of H(s) in addition to the double pole at the origin. As noted earlier the algorithms of NASAP can only approximate double poles. To illustrate this assume that the double pole at the origin is approximated by a complex pole pair located on the j axis a distance α from the origin. Thus

$$R(s) = \frac{1}{(s+j\alpha)(s-j\alpha)} \tag{5.8}$$

partial fraction expansion yields

$$R(s) = \frac{1}{2j\alpha}\left[\frac{1}{s-j\alpha} - \frac{1}{s+j\alpha}\right] . \tag{5.9}$$

Taking the inverse Laplace transform yields

$$\begin{aligned} r(t) &= \frac{1}{2j\alpha}(e^{j\alpha t} - e^{-j\alpha t}) = \frac{1}{\alpha} \sin \alpha t \\ &= -j\frac{1}{2\alpha} e^{j\alpha t} + j\frac{1}{2\alpha} e^{-j\alpha t} \end{aligned} \tag{5.10}$$

However if αt is small, $\sin \alpha t$ can be approximated by αt. Thus

$$i(t) \approx t \qquad \text{if } \alpha t << 1 \quad \text{or} \quad t << \frac{1}{\alpha} .$$

Thus the smaller α is, the larger the range of time values for which the approximation closely resembles a ramp input.

Note that the coefficients of the exponential terms in (5.10) are conjugate imaginary. However, in general, these coefficients will be complex conjugate. The significance of this is now demonstrated. Let us assume that the coefficient of $e^{j\alpha t}$ is A-jB while that of $e^{-j\alpha t}$ is A+jB. Thus we have

$$(A-jB)e^{j\alpha t} + (A+jB)e^{-j\alpha t} = A(e^{j\alpha t} + e^{-j\alpha t}) - jB(e^{j\alpha t} - e^{-j\alpha t})$$

$$= 2A\cos\alpha t + 2B\sin\alpha t \qquad (5.11)$$

Now let

$$2A = M\sin\theta \qquad (5.12)$$

$$2B = M\cos\theta \qquad (5.13)$$

Substituting (5.12) and (5.13) into (5.11) and then simplifying yields

$$(A-jB)e^{j\alpha t} + (A+jB)e^{-j\alpha t} = M\sin(\alpha t + \theta) \qquad (5.14)$$

where

$$M = 2\sqrt{A^2 + B^2} \qquad (5.15)$$

and

$$\Theta = \tan^{-1}\frac{A}{B} \qquad (5.16)$$

Now if $A << B$, then (5.15) and (5.16) can be approximated by

$$M \approx 2B \qquad (5.17)$$

$$\theta \approx \frac{A}{B} \qquad (5.18)$$

Since θ and α are both small, there is a range of values of t where the approximation

$$\sin(\alpha t + \theta) \approx \alpha t + \theta$$

is valid. Thus

$$(A-jB)e^{j\alpha t} + (A+jB)e^{-j\alpha t} \approx 2\alpha Bt + 2A \qquad (5.19)$$

for $A << B$ which represents a ramp with slope $2\alpha B$ plus a step function of magnitude 2A.

If the constant A is zero, then the above equations approximates only a ramp of slope $2\alpha B$. This agrees with equation (5.10). On the other hand, if $A >> B$, then (5.15) and (5.16) can be approximated by

$$M \approx 2A$$

$$\theta \approx \frac{\pi}{2} \quad .$$

Thus $\sin(\alpha t+\phi) \cong \sin(\alpha t + \frac{\pi}{2})$ $\cos \alpha t$. Since αt is quite small for a wide range of time values, then

$$\cos \alpha t \approx 1$$

This results in

$$(A-jB)e^{j\alpha t} + (A+jB)e^{-j\alpha t} \approx 2A \tag{5.20}$$

where $A >> B$ which represents a step function of magnitude 2A.

A simple example will illustrate these points. In Fig. 5.3 is the block diagram of a plant whose transfer function is known. It is required to determine the output c(t) for a ramp input of unit slope.

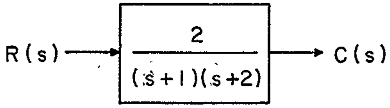


Fig. 5.3

Thus

$$C(s) = \frac{2}{s^2(s+1)(s+2)} \tag{5.21}$$

By partial fraction expansion and the inverse Laplace transform, one can obtain the exact solution

$$c(t) = t - \frac{3}{2} + 2e^{-t} - \frac{1}{2}e^{-2t} \tag{5.22}$$

In Fig. 5.4 is the equivalent circuit model for the plant given in Fig. 5.3 for computer analysis with NASAP.

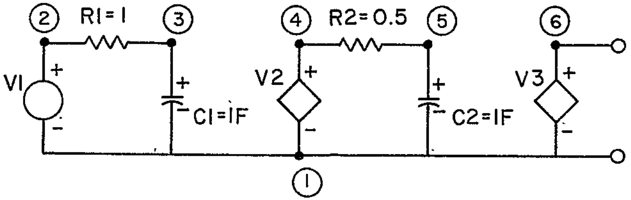


Fig. 5.4: Circuit Model for Fig. 5.3

V1 = input r(t)   V2 = VC1   V3 = VC2 = output c(t)

Desired Transfer Function VV3/VV1.

Since the ramp response is desired, the NASAP program will evaluate the residues of the poles of the function

$$C(s) = \frac{2}{(s+j\alpha)(s-j\alpha)(s+1)(s+2)} \qquad (5.23)$$

$$\text{Res}(s = -1) = \frac{+2}{1+\alpha^2} \approx +2$$

$$\text{Res}(s = -2) = \frac{-2}{4+\alpha^2} \approx -\frac{1}{2}$$

$$\text{Res}(s = j\,\alpha) = \frac{-j}{\alpha(1+j\alpha)(2+j\alpha)} \approx -\frac{j}{2\alpha}$$

$$\text{Res}(s = -j\alpha) = \frac{j}{\alpha(1+j\alpha)(2+j\alpha)} \approx \frac{j}{2\alpha}$$

The computer results for the transient analysis of the circuit in Fig. 5.4 are given in Fig. 5.5. Note that for $\alpha = 0.001$, the coefficient A and B are -0.75 and 500 respectively. Indeed $B >> A$ and thus by use of (5.18) we obtain

$$c(t) = t-1.5 \text{ for large } t$$

which agrees with the exact results obtained from (5.22).

```
NASAP RAMP RESPONSE

NONE
RAMP RESPONSE


V1 1 2 1
R1 2 3 1
C1 3 1 1F
V2 1 4 1 VC1
R2 4 5 0.5
C2 5 1 1F
V3 1 6 1 VC2
OUTPUT
VV3/VV1
TIME  1.5
EXECUTE
```

```
TRANSFER FUNCTION   VV3/VV1


                           (                             )
                           (  1.00E 00                   )

H(S)= 2.000E 00*-----------------------------------------------

                           (                                           2 )
                           (  2.00E 00     +3.00E 00 S    +1.00E 00 S    )



 ZERO OF TRANSFER FUNCTION

          NONE



 POLE OF TRANSFER FUNCTION

 POLE   REAL PART   IMAG. PART                   NUMBER OF LOOPS PER ORDER

                                                         1=    3
   1 -0.10000E 01  0.00000E 00                          2=    1
   2 -0.20000E 01  0.00000E 00



RAMP     RESPONSE FUNCTION

F(T) =
                                        (-0.1000E 01   J 0.0000E 00 ) T
      ( 0.2000E 01   J 0.4210E-09 ) E

                                        (-0.2000E 01   J 0.0000E 00 ) T
      (-0.5000E 00   J-0.5262E-10 ) E

                                        ( 0.0000E 00   J 0.1000E-02 ) T
      (-0.7500E 00   J-0.5000E 03 ) E

                                        ( 0.0000E 00   J-0.1000E-02 ) T
      (-0.7500E 00   J 0.5000E 03 ) E
```

For $\alpha = 0.001$

$A = -0.75$ $B >> A$

$B = 500$

thus $2\alpha B t + 2A = t - 1.5$

```
RAMP     RESPON..

TIME                 VV3/VV1
 0.0000E 00        -0.22649765E-05
 0.3000E-01         0.57220459E-05
 0.6000E-01         0.66041946E-04
 0.9000E-01         0.22459030E-03
 0.1200E 00         0.52416325E-03
 0.1500E 00         0.10040998E-02
 0.1800E 00         0.16986132E-02
 0.2100E 00         0.26420355E-02
 0.2400E 00         0.38610697E-02
 0.2700E 00         0.53819418E-02
 0.3000E 00         0.72265863E-02
 0.3300E 00         0.94183683E-02
 0.3600E 00         0.11973619E-01
 0.3900E 00         0.14907360E-01
 0.4200E 00         0.18234968E-01
 0.4500E 00         0.21967888E-01
 0.4800E 00         0.26117563E-01
 0.5100E 00         0.30690789E-01
 0.5400E 00         0.35696208E-01
 0.5700E 00         0.41138589E-01
 0.6000E 00         0.47023952E-01
 0.6300E 00         0.53353786E-01
 0.6600E 00         0.60132444E-01
 0.6900E 00         0.67359865E-01
 0.7200E 00         0.75038612E-01
 0.7500E 00         0.83165944E-01
 0.7800E 00         0.91742098E-01
 0.8100E 00         0.10076451E 00
 0.8400E 00         0.11023188E 00
 0.8700E 00         0.12014079E 00
 0.9000E 00         0.13048774E 00
 0.9300E 00         0.14126885E 00
 0.9600E 00         0.15248007E 00
 0.9900E 00         0.16411662E 00
 0.1020E 01         0.17617333E 00
 0.1050E 01         0.18864465E 00
 0.1080E 01         0.20152563E 00
 0.1110E 01         0.21481055E 00
 0.1140E 01         0.22849298E 00
 0.1170E 01         0.24256706E 00
 0.1200E 01         0.25702637E 00
 0.1230E 01         0.27186441E 00
 0.1260E 01         0.28707469E 00
 0.1290E 01         0.30265069E 00
 0.1320E 01         0.31858605E 00
 0.1350E 01         0.33487368E 00
 0.1380E 01         0.35150689E 00
 0.1410E 01         0.36847913E 00
 0.1440E 01         0.38578355E 00
 0.1470E 01         0.40341347E 00
 0.1500E 01         0.42136174E 00
```

Fig 5.5a

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RAMP RESPONSE

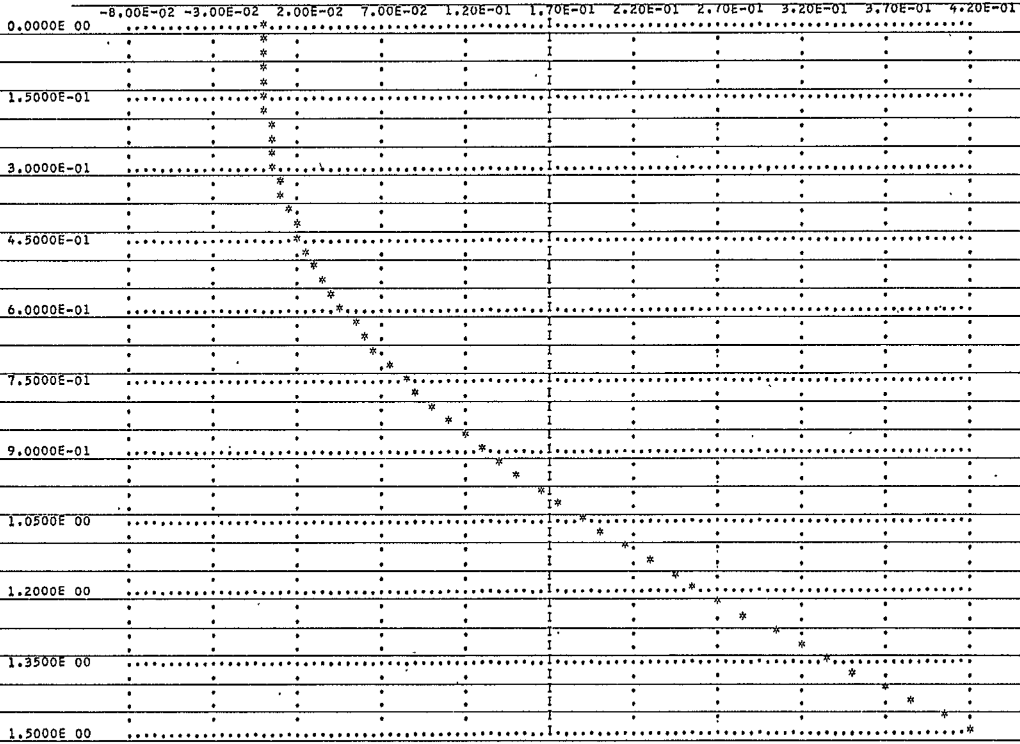


Fig 5.56

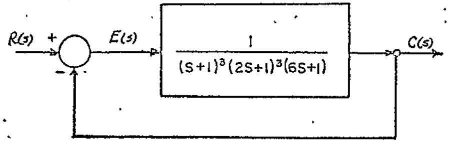
Although a few examples of NASAP printout of step response appeared earlier in this manual we include a specific example here. This step response is for the Eisenberg control problem shown previously in Fig. 4.3. The NASAP printout is shown in Fig. 5.6. We shall refer to this problem again in Chapter VII.

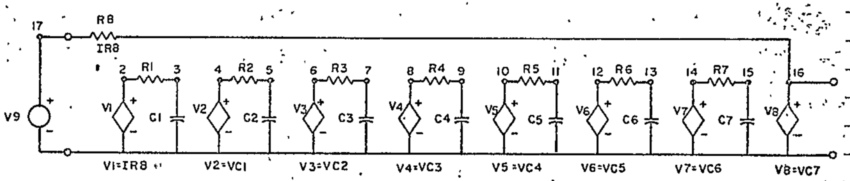
We are not including any examples of impulse response since all the previous references to NASAP only show such printouts.

RUNNING TIME : 48.65 SEC. on SPECTRA 70

```
NASAP PROBLEM  EISENBERG CONTROL SYSTEM

V9 1 17 1.
R8 17 16 1.
V1 1 2 1.  IR8
R1 2 3  6.
C1 3 1 1F
V2 1 4 1. VC1
R2 4 5  2.
C2 5 1 1F
V3 1 6 1. VC2
R3 6 7 2.
C3 7 1 1F
V4 1 8 1. VC3
R4 8 9 2.
C4 9 1 1F
V5 1 10 1. VC4
R5 10 11 1.
C5 11 1 1F
V6 1 12 1. VC5
R6 12 13 1.
C6 13 1 1F
V7 1 14 1. VC6
R7 14 15 1.
C7 15 1 1F
V8 1 16 1. VC7
OUTPUT
VV8/VV9
TIME 50.
EXECUTE
```





V9 corresponds to input R(s)
V8 " " output C(s)
V1 " " error signal E(s)

```
NUMBER OF LOOPS PER ORDER

     1=    9
     2=   21
     3=   35
     4=   35
     5=   21
     6=    7
     7=    1
```

```
TRANSFER FUNCTION   VV8/VV9


                  (                   )
                  (  1.00E 00         )
H(S)= 2.083E-02*----------------------------------------------------------------------------------------------------------------
                  (                                          2              3              4              5              6              7 )
                  (  4.17E-02     +3.12E-01 S    +1.81E 00 S    +5.44E 00 S    +9.25E 00 S    +9.00E 00 S    +4.67E 00 S    +1.00E 00 S    )


 ZERO OF TRANSFER FUNCTION
                                                    POLE OF TRANSFER FUNCTION
          NONE
                                                    POLE   REAL PART    IMAG. PART

                                                      1 -0.52223E 00  0.48111E 00
                                                      2 -0.52223E 00 -0.48111E 00
                                                      3 -0.72132E-01 -0.20595E 00
                                                      4 -0.72132E-01  0.20595E 00
                                                      5 -0.10790E 01 -0.38799E 00
                                                      6 -0.10790E 01  0.38799E 00
                                                      7 -0.13200E 01 -0.92015E-12
```

```
STEP    RESPONSE FUNCTION

F(T) =

                                (-0.5222E 00   J 0.4811E 00 ) T
          (-0.7046E-01   J 0.1088E 00 ) E

                                (-0.5222E 00   J-0.4811E 00 ) T
          (-0.7046E-01   J-0.1088E 00 ) E

                                (-0.7213E-01   J-0.2060E 00 ) T
          (-0.9572E-01   J-0.3404E 00 ) E

                                (-0.7213E-01   J 0.2060E 00 ) T
          (-0.9572E-01   J 0.3404E 00 ) E

                                (-0.1079E 01   J-0.3880E 00 ) T
          (-0.5657E-01   J-0.4702E-01 ) E

                                (-0.1079E 01   J 0.3880E 00 ) T
          (-0.5657E-01   J 0.4702E-01 ) E

                                (-0.1320E 01   J-0.9202E-12 ) T
          (-0.5450E-01   J-0.6949E-08 ) E

                                ( 0.0000E 00   J 0.0000E 00 ) T
          ( 0.5000E 00   J 0.9109E-07 ) E
```

$e_{ss} \equiv c_{ss} = 0.5$

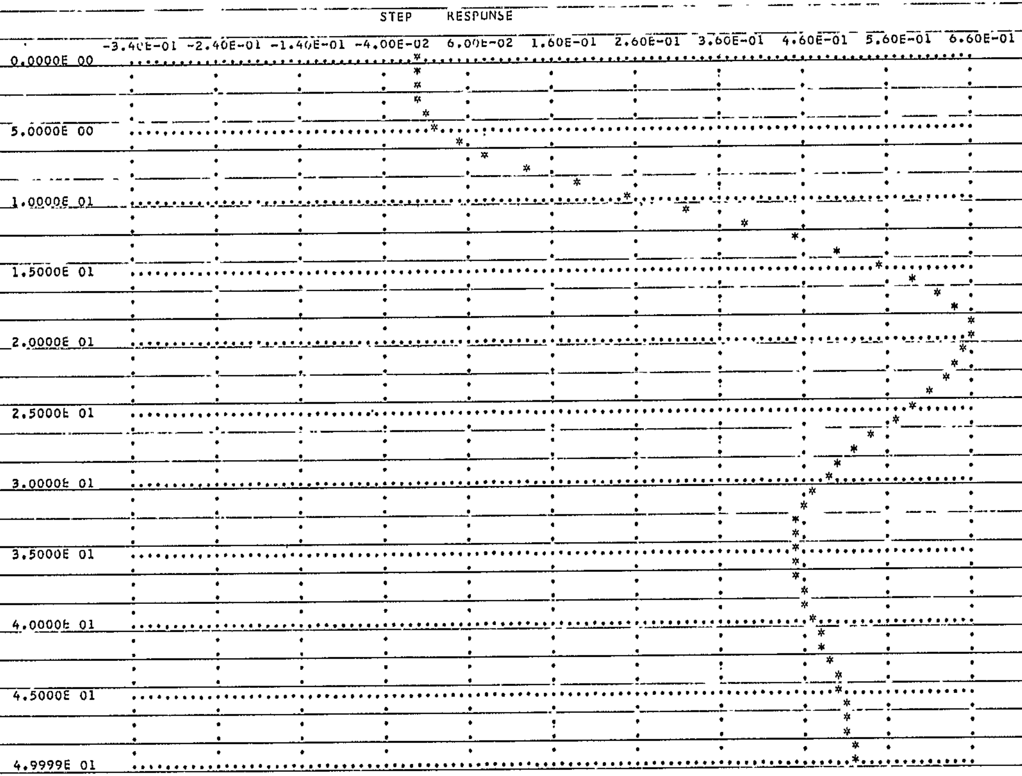
```
STEP     RESPONSE

TIME                     VV8/VV9
 0.0000E 00              0.11920929E-06
 0.1000E 01              0.26226044E-05
 0.2000E 01              0.16987324E-03
 0.3000E 01              0.16793013E-02
 0.4000E 01              0.73934197E-02
 0.5000E 01              0.21032572E-01
 0.6000E 01              0.45632005E-01
 0.7000E 01              0.82479000E-01
 0.8000E 01              0.13089931E 00
 0.9000E 01              0.18865520E 00
 0.1000E 02              0.25258356E 00
 0.1100E 02              0.31920141E 00
 0.1200E 02              0.38515985E 00
 0.1300E 02              0.44752681E 00
 0.1400E 02              0.50393325E 00
 0.1500E 02              0.55262429E 00
 0.1600E 02              0.59245241E 00
 0.1700E 02              0.62283635E 00
 0.1800E 02              0.64370114E 00
 0.1900E 02              0.65540731E 00
 0.2000E 02              0.65867203E 00
 0.2100E 02              0.65448737E 00
 0.2200E 02              0.64404070E 00
 0.2300E 02              0.62863541E 00
 0.2400E 02              0.60961848E 00
 0.2500E 02              0.58831459E 00
 0.2600E 02              0.56597197E 00
 0.2700E 02              0.54371685E 00
 0.2800E 02              0.52252042E 00
 0.2900E 02              0.50317657E 00
 0.3000E 02              0.48629081E 00
 0.3100E 02              0.47227865E 00
 0.3200E 02              0.46137297E 00
 0.3300E 02              0.45363820E 00
 0.3400E 02              0.44898993E 00
 0.3500E 02              0.44721884E 00
 0.3600E 02              0.44801658E 00
 0.3700E 02              0.45100296E 00
 0.3800E 02              0.45575190E 00
 0.3900E 02              0.46181697E 00
 0.4000E 02              0.46875346E 00
 0.4100E 02              0.47613794E 00
 0.4200E 02              0.48358405E 00
 0.4300E 02              0.49075466E 00
 0.4400E 02              0.49737012E 00
 0.4500E 02              0.50321269E 00
 0.4600E 02              0.50812829E 00
 0.4700E 02              0.51202422E 00
 0.4800E 02              0.51486546E 00
 0.4900E 02              0.51666820E 00
 0.5000E 02              0.51749223E 00
```

t_p = 20 sec.

peak overshoot = 31.7%

at t=50 sec overshoot = 3.5%


STEP RESPONSE
-3.40E-01
-2.40E-01
-1.40E-01
-4.00E-02
6.00E-02
1.60E-01
2.60E-01
3.60E-01
4.60E-01
5.60E-01
6.60E-01
0.0000E 00
5.0000E 00
1.0000E 01
1.5000E 01
2.0000E 01
2.5000E 01
3.0000E 01
3.5000E 01
4.0000E 01
4.5000E 01
4.9999E 01

ERROR RESPONSE

```
NASAP RAMP RESPONSE

NONE
RAMP RESPONSE

V1 1 2 1
R1 2 3 1
C1 3 1 1F
V2 1 4 1 VC1
R2 4 5 0.5                              NUMBER OF LOOPS PER ORDER
C2 5 1 1F
V3 1 6 1 VC2                                 1=     4
R3 2 6 1                                     2=     3
OUTPUT                                       3=     1
IR3/VV1
TIME  1.5
EXECUTE


TRANSFER FUNCTION   IR3/VV1


                 (                                          2 )
                 (  0.00E 00      +3.00E 00 S    +1.00E 00 S    )

H(S)= 1.000E 00*----------------------------------------------

                 (                                          2 )
                 (  2.00E 00      +3.00E 00 S    +1.00E 00 S    )


  ZERO OF TRANSFER FUNCTION                 POLE OF TRANSFER FUNCTION

  ZERO   REAL PART   IMAG. PART             POLE   REAL PART   IMAG. PART

     1  0.00000E 00  0.00000E 00               1 -0.10000E 01  0.00000E 00
     2 -0.30000E 01  0.00000E 00               2 -0.20000E 01  0.00000E 00


RAMP     RESPONSE FUNCTION

F(T) =
                                   (-0.1000E 01   J 0.0000E 00 ) T
        (-0.2000E 01  J-0.4210E-09 ) E

                                   (-0.2000E 01   J 0.0000E 00 ) T
        ( 0.5000E 00  J 0.5262E-10 ) E

                                   ( 0.0000E 00   J 0.1000E-02 ) T
        ( 0.7500E 00  J-0.8750E-03 ) E

                                   ( 0.0000E 00   J-0.1000E-02 ) T
        ( 0.7500E 00  J 0.8750E-03 ) E
```

Fig 5.7a

```
RAMP      RESPONSE

TIME                  IR3/VV1
 0.0000E 00          0.10728836E-05
 0.3000E-01          0.29993057E-01
 0.6000E-01          0.59932828E-01
 0.9000E-01          0.89774251E-01
 0.1200E 00          0.11947465E 00
 0.1500E 00          0.14899468E 00
 0.1800E 00          0.17830014E 00
 0.2100E 00          0.20735681E 00
 0.2400E 00          0.23613775E 00
 0.2700E 00          0.26461649E 00
 0.3000E 00          0.29277182E 00
 0.3300F 00          0.32058012E 00
 0.3600E 00          0.34802485E 00
 0.3900E 00          0.37509120E 00
 0.4200E 00          0.40176356E 00
 0.4500E 00          0.42803073E 00
 0.4800E 00          0.45388103E 00
 0.5100E 00          0.47930789E 00
 0.5400E 00          0.50430220E 00
 0.5700E 00          0.52885991E 00
 0.6000E 00          0.55297452E 00
 0.6300E 00          0.57664478E 00
 0.6600E 00          0.59986609E 00
 0.6900E 00          0.62263888E 00
 0.7200E 00          0.64495999E 00
 0.7500E 00          0.66683275E 00
 0.7800E 00          0.68825656E 00
 0.8100E 00          0.70923400E 00
 0.8400E 00          0.72976661E 00
 0.8700E 00          0.74985778E 00
 0.9000E 00          0.76951081E 00
 0.9300E 00          0.78872955E 00
 0.9600E 00          0.80751842E 00
 0.9900E 00          0.82588184E 00
 0.1020E 01          0.84382498E 00
 0.1050E 01          0.86135292E 00
 0.1080E 01          0.87847179E 00
 0.1110E 01          0.89518672E 00
 0.1140E 01          0.91150403E 00
 0.1170E 01          0.92742980E 00
 0.1200E 01          0.94297022E 00
 0.1230E 01          0.95813203E 00
 0.1260E 01          0.97292125E 00
 0.1290E 01          0.98734486E 00
 0.1320E 01          0.10014086E 01
 0.1350E 01          0.10151215E 01
 0.1380E 01          0.10284872E 01
 0.1410E 01          0.10415154E 01
 0.1440E 01          0.10542107E 01
 0.1470E 01          0.10665808E 01
 0.1500E 01          0.10786314E 01
```

For $\alpha = 0.001$

$A = 0.75$ $\quad A \gg B$

$B = 0.875 \times 10^{-3}$

$2\alpha Bt + 2A = 1.5$

RAMP RESPONSE

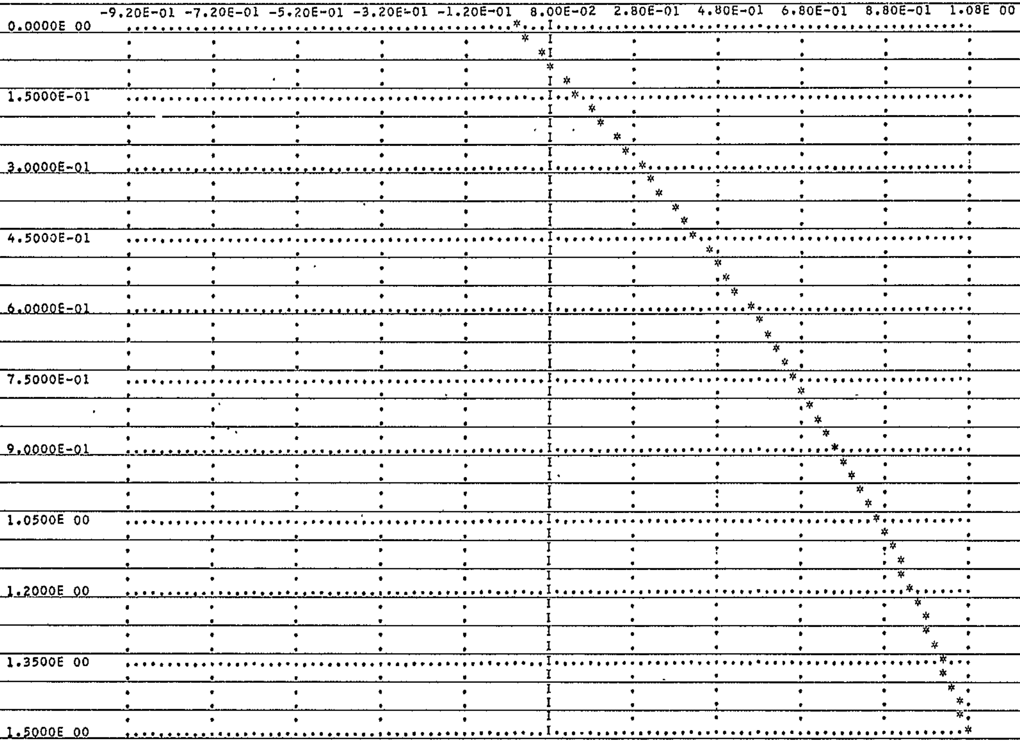


Fig 5.7b

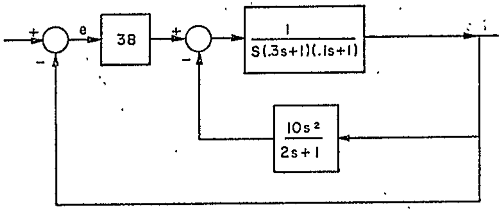
We continue with the Newton Gould and Kaiser problem, see Fig. 4.9, to illustrate error responses. In Fig. 5.8 gives the step error. Figure 5.9 gives the step response. The latter is included here to emphasize that only a single NASAP computer card need be changed to get the alternative response output. Finally in Fig. 5.10 we show the ramp error response for this same control system.

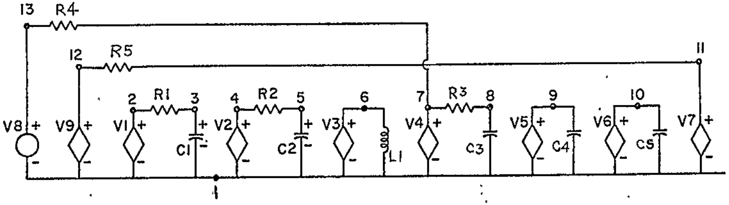
ERROR RESPONSE

```
NASAP   NEWTON-G-K

RADIANS
STEP

V8 1 13 1.
R4 13 7 1.
V9 1 12 38 IR4
R5 12 11 1.
V1 1 2 1. IR5
R1 2 3 0.3
C1 3 1 1F
V2 1 4 1. VC1
R2 4 5 0.1
C2 5 1 1F
V3 1 6 1. VC2
L1 6 1 1H
V4 1 7 1. IL1
R3 7 8 2.
C3 8 1 1F
V5 1 9 1. VC3
C4 9 1 1F
V6 1 10 10 IC4
C5 10 1 1F
V7 1 11 1. IC5
OUTPUT
IR4/VV8/V9
FREQ -1.0 2.0 0.05
TIME 2.0
EXECUTE.
```





```
TRANSFER FUNCTION   IR4/VV8/V9


                        (                                                  2                3                4 )
                        (  0.00E 00     +1.67E-01 S    +2.07E-02 S    +1.38E 01 S    +1.00E 00 S     )
H(S)= 1.000E 00*--------------------------------------------------------------------------------------
                        (                                                  2                3                4 )
                        (  6.33E 02     +1.28E.03 S    +2.07E 02 S    +1.38E 01 S    +1.00E 00 S     )


ZERO OF TRANSFER FUNCTION
                                                  POLE OF TRANSFER FUNCTION
ZERO   REAL PART    IMAG. PART
                                                  POLE   REAL PART    IMAG. PART

   1  0.00000E 00  0.00000E 00
   2 -0.81083E-01  0.00000E 00                       1 -0.53860E 00  0.00000E 00
   3 -0.68761E 01  0.12581E 02                       2 -0.28810E 01  0.12157E 02
   4 -0.68761E 01 -0.12581E 02                       3 -0.28810E 01 -0.12157E 02
                                                     4 -0.75327E 01  0.00000E 00
```

```
STEP     RESPONSE FUNCTION

F(T) =

                                   (-0.5386E 00   J 0.0000E 00 ) T
       (-0.8468E-01   J 0.1472E-07 ) E

                                   (-0.2881E 01   J 0.1216E 02 ) T
       ( 0.4340E-01   J-0.3176E 00 ) E

                                   (-0.2881E 01   J-0.1216E 02 ) T
       ( 0.4340E-01   J 0.3176E 00 ) E

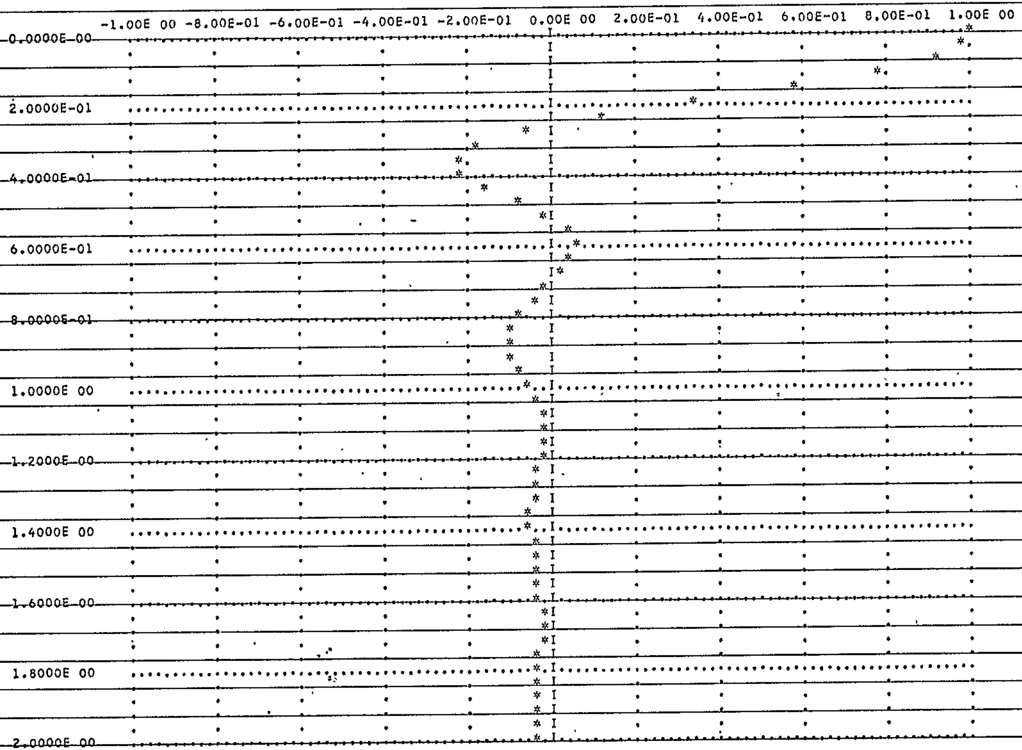
                                   (-0.7533E 01   J 0.0000E 00 ) T
       ( 0.9979E 00   J-0.2777E-07 ) E

                                   ( 0.0000E 00   J 0.0000E 00 ) T
       ( 0.0000E 00   J 0.0000E 00 ) E
```

```
STEP     RESPONSE

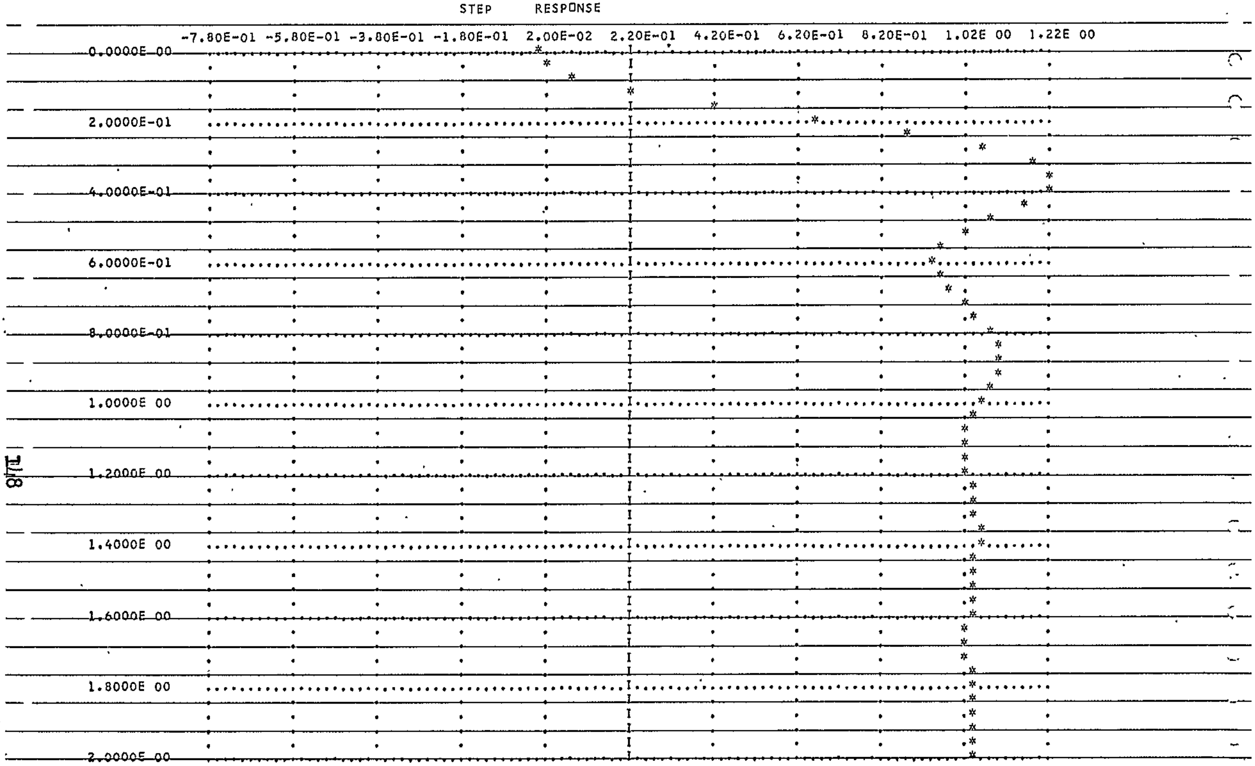
TIME                    IR4/VV8
 0.0000E 00              0.99999970E 00
 0.4000E-01              0.98830140E 00
 0.8000E-01              0.92671491E 00
 0.1200E 00              0.77826107E 00
 0.1600E 00              0.57408595E 00
 0.2000E 00              0.34088778E 00
 0.2400E 00              0.11746454E 00
 0.2800E 00             -0.62708139E-01
 0.3200E 00             -0.17894578E 00
 0.3600E 00             -0.22616476E 00
 0.4000E 00             -0.21346736E 00
 0.4400E 00             -0.15973860E 00
 0.4800E 00             -0.87881029E-01
 0.5200E 00             -0.19262429E-01
 0.5600E 00              0.30433368E-01
 0.6000E 00              0.53309571E-01
 0.6400E 00              0.49267378E-01
 0.6800E 00              0.24339374E-01
 0.7200E 00             -0.11951964E-01
 0.7600E 00             -0.49438279E-01
 0.8000E 00             -0.79693377E-01
 0.8400E 00             -0.97540200E-01
 0.8800E 00             -0.10153145E 00
 0.9200E 00             -0.93511105E-01
 0.9600E 00             -0.77544510E-01
 0.1000E 01             -0.58580298E-01
 0.1040E 01             -0.41192196E-01
 0.1080E 01             -0.28661814E-01
 0.1120E 01             -0.22530064E-01
 0.1160E 01             -0.22621151E-01
 0.1200E 01             -0.27435813E-01
 0.1240E 01             -0.34752585E-01
 0.1280E 01             -0.42262293E-01
 0.1320E 01             -0.48091702E-01
 0.1360E 01             -0.51122818E-01
 0.1400E 01             -0.51082287E-01
 0.1440E 01             -0.48427440E-01
 0.1480E 01             -0.44095926E-01
 0.1520E 01             -0.39202157E-01
 0.1560E 01             -0.34757502E-01
 0.1600E 01             -0.31470507E-01
 0.1640E 01             -0.29654402E-01
 0.1680E 01             -0.29240042E-01
 0.1720E 01             -0.29869609E-01
 0.1760E 01             -0.31034324E-01
 0.1800E 01             -0.32216437E-01
 0.1840E 01             -0.33004209E-01
 0.1880E 01             -0.33160023E-01
 0.1929E 01             -0.32636251E-01
 0.1960E 01             -0.31546686E-01
 0.2000E 01             -0.30108280E-01
```

STEP RESPONSE

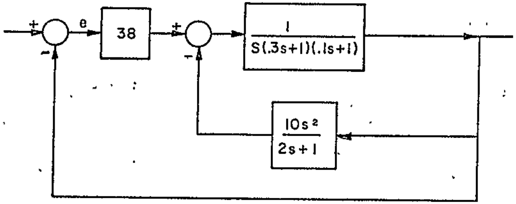


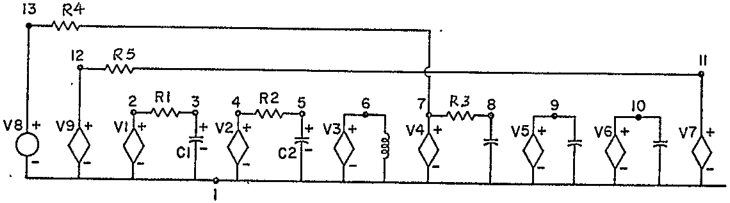
```
      STEP      RESPONSE FUNCTION                                        STEP      RESPONSE

      F(T) =                                                             TIME                  VV4/VVB
                                                                           0.0000E 00            0.00000000E 00
                                           (-0.5386E 00  J 0.0000E 00 ) T  0.4000E-01            0.11698127E-01
                (-0.8466E-01  J 0.0000E 00 ) E                             0.8000E-01            0.79284728E-01
                                                                           0.1200E 00            0.22173876E 00
                                           (-0.2881E 01  J 0.1216E 02 ) T  0.1600E 00            0.42591405E 00
                (-0.4340E-01  J 0.3176E 00 ) E                             0.2000E 00            0.65911257E 00
                                                                           0.2400E 00            0.88253599E 00
                                           (-0.2881E 01  J-0.1216E 02 ) T  0.2800E 00            0.10627079E 01
                (-0.4340E-01  J-0.3176E 00 ) E                             0.3200E 00            0.11789455E 01
                                                                           0.3600E 00            0.12261648E 01 <-- peak 22.6%
                                           (-0.7533E-01  J 0.0000E 00 ) T  0.4000E 00            0.12134676E 01
                (-0.9979E 00  J 0.3546E-07 ) E                             0.4400E 00            0.11597385E 01
                                                                           0.4800E 00            0.10878811E 01
                                           ( 0.0000E 00  J 0.0000E 00 ) T  0.5200F 00            0.10192623E 01
                (-0.1000E 01  J 0.6523E-07 ) E                             0.5600E 00            0.96956694E 00
                                                                           0.6000E 00            0.94669074E 00
                                                                           0.6400E 00            0.95073295E 00
                                                                           0.6800E 00            0.97566098E 00
                                                                           0.7200E 00            0.10119514E 01
                                                                           0.7600E 00            0.10494365E 01
                                                                           0.8000E 00            0.10796938E 01
                                                                           0.8400E 00            0.10975399E 01
                                                                           0.8800E 00            0.11015310E 01
                                                                           0.9200E 00            0.10935106E 01
                                                                           0.9600E 00            0.10775442E 01
                                                                           0.1000E 01            0.10585804E 01
                                                                           0.1040E 01            0.10411921E 01
                                                                           0.1080E 01            0.10286617E 01
                                                                           0.1120E 01            0.10225296E 01
                                                                           0.1160E 01            0.10226212E 01
                                                                           0.1200E 01            0.10274353E 01
                                                                           0.1240E 01            0.10347528E 01
                                                                           0.1280E 01            0.10422621E 01
                                                                           0.1320E 01            0.10480919E 01
                                                                           0.1360E 01            0.10511227E 01
                                                                           0.1400E 01            0.10510817E 01
                                                                           0.1440E 01            0.10484276E 01
                                                                           0.1480E 01            0.10440960E 01
                                                                           0.1520E 01            0.10392017E 01
                                                                           0.1560E 01            0.10347576E 01
                                                                           0.1600E 01            0.10314703E 01
                                                                           0.1640E 01            0.10296545E 01
                                                                           0.1680E 01            0.10292397E 01
                                                                           0.1720E 01            0.10298691E 01
                                                                           0.1760E 01            0.10310345E 01
                                                                           0.1800E 01            0.10322161E 01
                                                                           0.1840E 01            0.10330038E 01
                                                                           0.1880E 01            0.10331602E 01
                                                                           0.1920E 01            0.10326357E 01
                                                                           0.1960E 01            0.10315466E 01
                                                                           0.2000E 01            0.10301085E 01
```


STEP RESPONSE
-7.80E-01
-5.80E-01
-3.80E-01
-1.80E-01
2.00E-02
2.20E-01
4.20E-01
6.20E-01
8.20E-01
1.02E 00
1.22E 00
0.0000E 00
2.0000E-01
4.0000E-01
6.0000E-01
8.0000E-01
1.0000E 00
1.2000E 00
1.4000E 00
1.6000E 00
1.8000E 00
2.0000E 00

ERROR RESPONSE





```
NASAP      CLARK [illegible]
                NEWTON -G-K

RADIANS
RAMP RESPONSE

V8 1 13 1.
R4 13 7 1.
V9 1 12 38 IR4
R5 12 11 1.
V1 1 2 1. IR5
R1 2 3 0.3
C1 3 1 1F
V2 1 4 1. VC1
R2 4 5 0.1
C2 5 1 1F
V3 1 6 1. VC2
L1 6 1 1H
V4 1 7 1. IL1
R3 7 8 2.
C3 8 1 1F
V5 1 9 1. VC3
C4 9 1 1F
V6 1 10 10 IC4
C5 10 1 1F
V7 1 11 1. IC5
OUTPUT
IR4/VV8/V9
TIME 2.0
EXECUTE


TRANSFER FUNCTION   IR4/VV8/V9                                                                  NUMBER OF LOOPS PER ORDER

                                                                                                          1=     6
                                                                                                          2=     8
                           (                                       2           3            4 )             3=     4
                           (  0.00E 00      +1.67E 01 S   +2.07E 02 S   +1.38E 01 S   +1.00E 00 S   )       4=     1
H(S)= 1.000E 00*--------------------------------------------------------------------------
                           (                                       2           3            4 )
                           (  6.33E 02      +1.28E 03 S   +2.07E 02 S   +1.38E 01 S   +1.00E 00 S . )


ZERO OF TRANSFER FUNCTION
                                                     POLE OF TRANSFER FUNCTION
ZERO    REAL PART    IMAG. PART
                                                     POLE    REAL PART   IMAG. PART

   1  0.00000E 00  0.00000E 00
   2 -0.81083E-01  0.00000E 00                          1 -0.53860E 00  0.00000E 00
   3 -0.68761E 01  0.12581E 02                          2 -0.28810E 01  0.12157E 02
   4 -0.68761E 01 -0.12581E 02                          3 -0.28810E 01 -0.12157E 02
                                                        4 -0.75327E 01  0.00000E 00
```

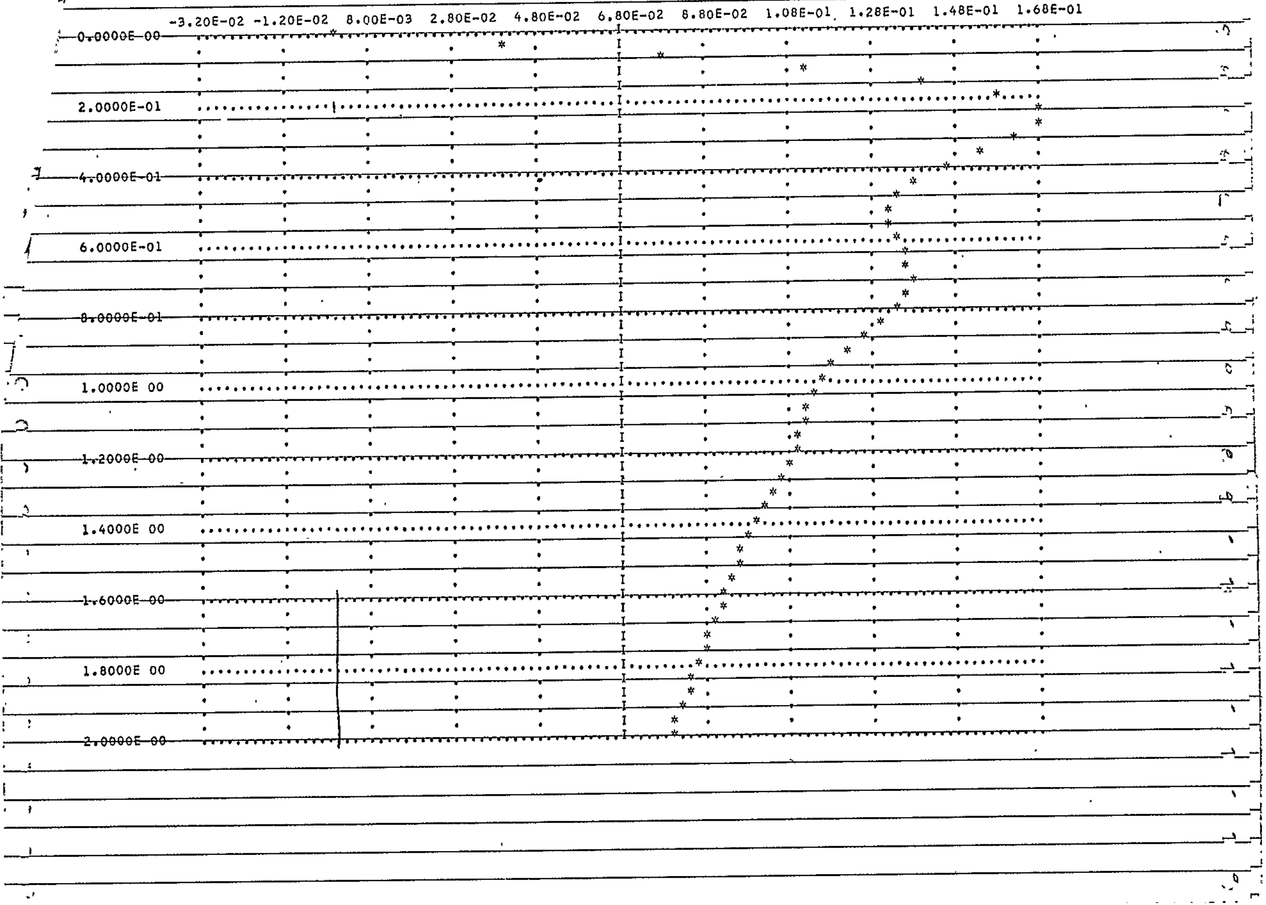
```
RAMP      RESPONSE FUNCTION

F(T) =
                                     (-0.5386E 00  J 0.0000E 00 ) T
        ( 0.1572E 00  J 0.2739E-07 ) E
                                     (-0.2881E 01  J 0.1216E 02 ) T
        (-0.2553E-01  J 0.2481E-02 ) E
                                     (-0.2881E 01  J-0.1216E 02 ) T
        (-0.2553E-01  J-0.2481E-02 ) E
                                     (-0.7533E-01  J 0.0000E 00 ) T
        (-0.1325E 00  J 0.3688E-08 ) E
                                     ( 0.0000E 00  J 0.1000E-02 ) T
        (-0.1316E-01  J 0.1365E-03 ) E
                                     ( 0.0000E 00  J-0.1000E-02 ) T
        ( 0.1316E-01  J-0.1365E-03 ) E
```

```
RAMP      RESPONSE

TIME                IR4/VV8
 0.0000E 00          0.91956387E-07
 0.4000E-01          0.39879285E-01
 0.8000E-01          0.78300536E-01
 0.1200E 00          0.11252379E 00
 0.1600E 00          0.13972944E 00
 0.2000E 00          0.15806085E 00
 0.2400E 00          0.16713339E 00
 0.2800E 00          0.16804075E 00
 0.3200E 00          0.16297650E 00
 0.3600E 00          0.15465176E 00
 0.4000E 00          0.14568663E 00
 0.4400E 00          0.13812327E 00
 0.4800E 00          0.13314849E 00
 0.5200E 00          0.13104695E 00
 0.5600E 00          0.13135147E 00
 0.6000E 00          0.13312036E 00
 0.6400E 00          0.13525462E 00
 0.6800E 00          0.13678169E 00
 0.7200E 00          0.13705003E 00
 0.7600E 00          0.13581055E 00
 0.8000E 00          0.13219290E 00
 0.8400E 00          0.12960213E 00
 0.8800E 00          0.12557584E 00
 0.9200E 00          0.12164086E 00
 0.9600E 00          0.11820133E 00
 0.1000E 01          0.11547697E 00
 0.1040E 01          0.11349326E 00
 0.1080E 01          0.11211604E 00
 0.1120E 01          0.11111414E 00
 0.1160E 01          0.11023003E 00
 0.1200E 01          0.10924119E 00
 0.1240E 01          0.10800177E 00
 0.1280E 01          0.10645854E 00
 0.1320E 01          0.10464364E 00
 0.1360E 01          0.10264891E 00
 0.1400E 01          0.10059488E 00
 0.1440E 01          0.98597368E-01
 0.1480E 01          0.96743226E-01
 0.1520E 01          0.95077157E-01
 0.1560E 01          0.93600869E-01
 0.1600E 01          0.92280924E-01
 0.1640E 01          0.91063499E-01
 0.1680E 01          0.89889765E-01
 0.1720E 01          0.88710368E-01
 0.1760E 01          0.87493122E-01
 0.1800E 01          0.86227357E-01
 0.1840E 01          0.84921122E-01
 0.1880E 01          0.83595574E-01
 0.1920E 01          0.82277477E-01
 0.1960E 01          0.80992222E-01
 0.2000E 01          0.79758406E-01
```

RAMP RESPONSE



VD. FIGURES OF MERIT BASED ON ERROR SIGNAL

The conventional criteria on this basis are the transient performance and the steady state performance. These were briefly summarized in Chapter IV.

The overall response of a control system is determined by the poles and zeros of its transfer function $G_f(s) = N_f(s) / D_f(s)$; it is not possible to tell it from the degree of $D_f(s)$ and the difference of the degrees of $D_f(s)$ and $N_f(s)$ alone. However it is found that for the same δN_f an optimal system with a smaller δD_f is better than such a system with a larger δD_f; where we have let $\delta(\ )$ denote the degree of the polynomials. Similarly for the same δD_f, an optimal system with a smaller $(\delta D_f - \delta N_f)$ is better than such a system with a larger $(\delta D_f - \delta N_f)$. Therefore, given a plant, for which we can choose the degree of δD_f and δN_f, it is desirable to choose a smaller δD_f and a smaller $(\delta D_f - \delta N_f)$. Now the smallest possible $(\delta D_f - \delta N_f)$ is governed by the given plant.

It can be shown [CH 1] that the absolute minimum of δD_f is $\delta D - \delta N$; where D and N refer to the uncompensated system. To achieve this minimum, δN_f is required to be zero. In general $\delta N_f \geq \delta N$, unless some pole-zero cancellations are employed in the design.

The transient performance of a system may be specified by percentage overshoot, rise time and settling time. These specifications are dictated by the poles and zeros of the transfer function. Since we do not have control over the zeros of a system, usually we just try to put the poles of the over-all system in some desired location. For a second order transfer function with a constant numberator, the desired pole locations can be readily determined from the transient specifications. For high order transfer functions, the concept of dominant poles can often be used. Then a pair of complex conjugate poles is located as in the second order transfer function and the rest of the poles are located in the far left half plane with real parts at least ten times as large as the real parts of the conjugate poles. In choosing these poles, the steady state performance should be kept in mind.

The steady-state performance of a control system $G_f(s)$ depends only on the coefficients of $G_f(s)$. Let

$$G_f(s) = \frac{b_o + b_1 s + b_2 s^2 + \cdots + b_{n-1} s^{n-1}}{a_o + a_1 s + a_2 s^2 + \cdots + a_{n-1} s^{n-1} + s^n},$$

If the steady-state error due to a step input and a ramp input are required to be smaller than k $^a/_b$, then we need, respectively,

$$\left| \frac{b_o - a_o}{a_o} \right| \leq k/100; \qquad \text{and} \qquad a_o = b_o, \ \left| \frac{b_1 - a_1}{a_o} \right| \leq k/100.$$

Hence the steady-state performance of a system can be rather easily controlled. When using these criteria it is necessary to check the response of the chosen overall transfer function with an analog or a digital computer to be sure that it is satisfactory before continuing the design.

In addition to the conventional criteria just discussed, we shall mention a few other criteria based on the error signal that serve to make the control system "optimum" in some sense.

(i) ITAE criterion (Integral of time-multiplied absolute-value of error): This criterion was first introduced by Graham and Lathrop [GA 1]. For a given plant, the problem is to design an overall system which minimizes

$$\int_o^\infty t|r(t) - c(t)|dt$$

where r is the reference or desired signal and c is the output of the over-all system. It is clear that $|r(t) - c(t)|$ is the error between the desired signal and the actual output. The multiplication of t on $|r(t) - c(t)|$ provides an increasingly heavy penalty for a sustained error. Using step functions as reference inputs, Graham and Lathrop obtained, by analog computer simulation a set of optimal transfer functions [GA 1].

To choose an optimal transfer function $G_f(s)$ for a given plant $G(s)$ note that if all the forward paths from r to c pass through the plant, then the zeros of the plant (the roots of $N(s)$) will be independent of how the compensators are introduced. Consequently if the numerator of $G_p(s)$ does not contain all the zeros of $G(s)$, the missing zero must be cancelled by a pole. The advantage of this criterion is that it is very selective; however it cannot be studied analytically. This criterion is not widely used, because a complete list of optimal transfer functions is not available.

(ii) Quadratic criterion: The optimal system is the one which minimizes

$$\int_0^\infty [u^2(t) + (r(t) - c(t))^2] dt$$

where u is the input to the plant. In this criterion, if u^2 is not included, the optimal transfer function will always be unity and the required compensators may not be physically realizable; furthermore the magnitude of u may be large and the system will be saturated. The optimal transfer function for a given plant and a given reference input r can be obtained by applying Chang's root-square-locus method [CH 1] as well as by using the dynamical equation description [see AT 1]

The design by using the quadratic criterion can be solved rigorously. However there are three arguments against using this criterion. First, it is not very selective [GR 1]. Second, the criterion is chosen mainly for mathematical convenience rather than practical reasons. Finally and most seriously, the resulting optimal transfer function may not be realizable in practice. If all the zeros of the transfer function of the plant $G(s) = \frac{N(s)}{D(s)}$ have negative real parts and if the reference input is a step function, the optimal transfer function is of the form $\frac{kN(s)}{D_f(s)}$ and $\delta D_f(s) = \delta D$. In this case, it is easily realized. However for plants with nonminimum-phase transfer function, the design by using quadratic criterion might have difficulty. Another difficulty in using this criterion occurs when the reference input is a ramp.

(iii) ISE criterion (the integral of the error squared): Here one seeks to minimize

$$\int_0^t e^2(t)\, dt.$$

To obtain any of these criteria from NASAP use of an external integration subroutine is required. To illustrate this we follow Beck [BE 1] and use the integral of the squared error as the performance index although other criteria can be applied with equal ease since an analytical solution is not required. The formation of the chosen performance index is indicated in Fig. 5.7 taken from [BE 1] where it is shown that the

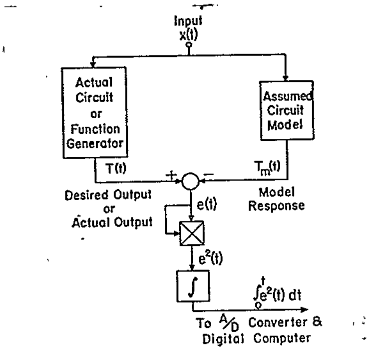


Fig. 5.7: Index of performance formation

ISE is generated in the analog computer and returned to the digital computer through the analog to digital converters. In the hybrid computer application an optimization algorithm operates upon this output.

CHAPTER VI

SENSITIVITY ANALYSIS

VIA INTRODUCTION TO SENSITIVITY

The NASAP sensitivity results, given in tabular and graphical form, can be used to predict the percent change, absolute change, and modified value of the transfer function to changes in a particular network parameter. By definition the sensitivity of some real function α (in NASAP, ReH, ImH $|H|$ and ϕ) to a change in a real parameter x (in NASAP, resistance, capacitance, inductance and dependency value) is defined as

$$S_x^{\alpha} \triangleq \frac{x}{\alpha}\frac{d\alpha}{dx} = \frac{\frac{d\alpha}{\alpha}}{\frac{dx}{x}} = \frac{d\ln\alpha}{d\ln x} \tag{6.1}$$

By rearranging (6.1), the differential dα can be related to the differential dx,

$$d\alpha = \alpha S_x^{\alpha} \frac{dx}{x} \tag{6.2}$$

For an incremental change, Δx, in the parameter x the incremental change, Δα, in α can be approximated from (6.2)

$$\Delta\alpha \approx \alpha S_x^{\alpha} \frac{\Delta x}{x} \tag{6.3a}$$

If the change in x is expressed as some percentage of x, then (6.3a) can be expressed as

$$\Delta\alpha \approx \alpha S_x^{\alpha} \frac{Px}{100} \tag{6.3b}$$

where Px is the percent change in the parameter x. The percent change in α is easily found by dividing both sides of (6.3b) by α, thus

$$P\alpha \approx S_x^{\alpha} Px \tag{6.4}$$

where P_α is the percent change in the function α. The modified value of α, called α', can be expressed as

$$\alpha' \triangleq \alpha + \Delta\alpha \tag{6.5}$$

$$\alpha' \approx \alpha\left(1 + \mathcal{S}_x^\alpha \frac{\Delta x}{x}\right) \tag{6.6a}$$

or

$$\alpha' \approx \alpha\left(1 + \mathcal{S}_x^\alpha \frac{Px}{100}\right) \tag{6.6b}$$

Naturally the smaller Δ_x or P_x, the more accurate $\Delta\alpha$, α', and P_α will be. The accuracy will also be enhanced if α is roughly a linear function of x over the range of x under investigation.

If indeed α is a linear function of x of the form

$$\alpha = Kx, \tag{6.7}$$

then from (6.1) it is seen that

$$\mathcal{S}_x^\alpha = 1 \tag{6.8}$$

Furthermore from (6.1) if α is independent of x, then

$$\mathcal{S}_x^\alpha = 0 \tag{6.9}$$

or the function α is insensitive to changes in x.

Similarly the root sensitivity printed out by NASAP can be used to predict the new pole and zero locations for a small change in some network parameter. The root sensitivity is defined as

$$\sigma_x^p \triangleq x \frac{dp}{dx} \tag{6.10}$$

where p is a zero or pole of the given transfer function. The differential dp can be expressed in terms of the root sensitivity as

$$dp = \sigma_x^p \frac{dx}{x} = \sigma_x^p \, d\ln x \tag{6.11}$$

Note that dp will be complex since σ_x^p is also a complex number while $\frac{dx}{x}$ is a real quantity.

The incremental change in p, Δp, for an incremental change in x is derived from (6.11)

$$\Delta p \cong \sigma_x^p \frac{\Delta x}{x} \tag{6.12}$$

If Δx is expressed as some percentage of x then (6.12) becomes

$$\Delta p \cong \sigma_x^p \frac{Px}{100} \tag{6.13}$$

where Px is the percent change in x.

VIB DERIVATION OF SENSITIVITY FORMULAS

Although the formulas used in NASAP to calculate the transfer function sensitivity were derived from a tagging technique on the loops of the flowgraph (see [MA 1]), these sensitivity formulas can be obtained by using simple calculus. Suppose the transfer function is given

$$H(s) = \frac{N(s)}{D(s)} \tag{6.14}$$

where N(s) and D(s) are polynomials in s.

Since H(s) is the transfer function of a linear circuit it can be expressed as a bilinear function of any element in the circuit. That is, both

N(s) and D(s) can be expressed as the sum of two polynomials in s where one polynomial does and the other does not contain the specified element. Thus

$$H(s) = \frac{N(s)}{D_1(s)} = \frac{A(s) + x\,B(s)}{C(s) + x\,D(s)} \tag{6.15}$$

where x is the specified element in the circuit and A,B,C, and D are polynomials in s.

The sensitivity of the transfer function H(s) to some parameter x is defined as

$$S_x^{H(s)} \triangleq \frac{dH/H}{dx/x} = \frac{d\ln H}{d\ln x} \tag{6.16}$$

Thus to find the sensitivity of H(s) to changes in x in (6.15), one obtains

$$S_x^{H(s)} = S_x^{N(s)} - S_x^{D_1(s)} \tag{6.17}$$

but

$$\begin{aligned} S_x^{N(s)} &= \frac{x}{N(s)} \frac{d\,N(s)}{dx} \\ &= \frac{x}{A+Bx} \frac{d}{dx}(A+Bx) \\ &= \frac{Bx}{A+Bx} \end{aligned} \tag{6.18}$$

Similarly

$$S_x^{D_1(s)} = \frac{Dx}{C+Dx} \tag{6.19}$$

Substituting (6.18) and (6.19) into (6.17) yields

$$S_x^{H(s)} = \frac{Bx}{A+Bx} - \frac{Dx}{C+Dx} \tag{6.20}$$

This expression remains unchanged if 1 is added and subtracted from the righthand side. Then we can write

$$\mathcal{S}_x^{H(s)} = \frac{Bx}{A+Bx} - \frac{A+Bx}{A+Bx} - \frac{Dx}{C+Dx} + \frac{C+Dx}{C+Dx}$$

$$= - \frac{A}{A+Bx} + \frac{C}{C+Dx}$$

$$= - \frac{A(s)}{N(s)} + \frac{C(s)}{D_1(s)} \tag{6.21}$$

This equation can also be derived from the tagging technique on the loops of the flowgraph and is the one used in subroutine SENSS of NASAP to determine the transfer function sensitivity.

VIC DISCUSSION OF SENSITIVITY FORMULAS IN SENS

In subroutine SENS of the NASAP program are calculated the sensitivity expressions $\mathcal{S}_x^{ReH}$, $\mathcal{S}_x^{ImH}$, $\mathcal{S}_x^{|H|}$ and $\mathcal{S}_x^{\phi}$

where $H(j\omega) = ReH(j\omega)+jIm\ H(j\omega)$

and $H(j\omega) = |H(j\omega)|e^{j\phi}$.

Again the tagging techniques of [MA 1] are used to determine the sensitivity expressions. The basis for the tagging procedure is that the transfer function H(s) can be written as in (6.15) where x is the sensitivity parameter. Since the polynomials A, B, C, and D are, in general, complex quantities for s = jω, (6.15) can be rewritten as

$$H(j\omega) = \frac{(Re\ A + x\ Re\ B) + j\ (Im\ A + x\ Im\ B)}{(Re\ C + x\ Re\ D) + j\ (Im\ C + x\ Im\ D)} \tag{6.22}$$

where $A(j\omega) = ReA(j\omega) + j\ Im\ A\ (j\omega)$ etc.

After some mathematical manipulation, the right side of (6.22) can be separated into its real and imaginary parts. Thus

$$\mathrm{ReH}(j\omega) = \frac{N_R}{D} = \frac{(\mathrm{ReA\ ReC} + \mathrm{ImA\ ImC}) + x(\mathrm{ReB\ ReC} + \mathrm{ReA\ ReD} + \mathrm{ImB\ ImC} + \mathrm{ImA\ ImD}) + x^2(\mathrm{ReB\ ReD} + \mathrm{ImB\ ImD})}{\left[(\mathrm{ReC})^2 + (\mathrm{ImC})^2\right] + 2x(\mathrm{ReC\ ReD} + \mathrm{ImC\ ImD}) + x^2\left[(\mathrm{ReD})^2 + (\mathrm{ImD})^2\right]} \qquad (6.23)$$

and

$$\mathrm{ImH}(j\omega) = \frac{N_I}{D} = \frac{(\mathrm{ImA\ ReC} - \mathrm{ReA\ ImC}) + x(\mathrm{ImA\ ReD} - \mathrm{ReA\ ImD} + \mathrm{ImB\ ReC} - \mathrm{ReB\ ImC}) + x^2(\mathrm{ImB\ ReD} - \mathrm{ReB\ ImD})}{\left[(\mathrm{ReC})^2 + (\mathrm{ImC})^2\right] + 2x(\mathrm{ReC\ ReD} + \mathrm{ImC\ ImD}) + x^2\left[(\mathrm{ReD})^2 + (\mathrm{ImD})^2\right]} \qquad (6.24)$$

Thus by use of (6.17) one obtains

$$\mathcal{S}_x^{\mathrm{ReH}} = \mathcal{S}_x^{N_R} - \mathcal{S}_x^{D}$$

$$= \frac{x}{N_R}\frac{d}{dx}N_R - \frac{x}{D}\frac{d}{dx}D$$

or

$$\mathcal{S}_x^{\mathrm{ReH}} = \frac{x}{N_R D}\left(D\frac{d}{dx}N_R - N_R\frac{dD}{dx}\right) \qquad (6.25)$$

where $$\frac{dN_R}{dx} = (\mathrm{ReB\ ReC} + \mathrm{ReA\ ReD} + \mathrm{ImB\ ImC} + \mathrm{ImA\ ImD}) + 2x(\mathrm{ReB\ ReD} + \mathrm{ImB\ ImD}) \qquad (6.26)$$

and $$\frac{dD}{dx} = 2(\mathrm{ReC\ ReD} + \mathrm{ImC\ ImD}) + 2x\left[(\mathrm{ReD})^2 + (\mathrm{ImD})^2\right] \qquad (6.27)$$

and N_R and D are defined in (6.23).

By a similar use of (6.17) one arrives at the expression

$$\mathcal{S}_x^{\mathrm{ImH}} = \frac{x}{N_I D} D\left(\frac{dN_I}{dx} - N_I \frac{dD}{dx}\right) \qquad (6.28)$$

where $\frac{dN_I}{dx} = (\mathrm{ImA\ ReD} - \mathrm{ReA\ ImD} + \mathrm{ImB\ ReC} - \mathrm{ReB\ ImC}) + 2x(\mathrm{ImB\ ReD} - \mathrm{ReB\ ImD})$ (6.29

and $\frac{dD}{dx}$ is given, (6.27), and N_I and D are defined in (6.24).

Equations (6.25) and (6.28) are used in subroutine SENS to evaluate the sensitivities of the real part and imaginary part of the transfer function to changes in the parameter x.

The sensitivities $\mathcal{S}_x^{|H|}$ and $\mathcal{S}_x^{\phi}$ are evaluated in terms of the sensitivities found in (6.25) and (6.28). By definition

$$|H| = \sqrt{(\mathrm{ReH})^2 + (\mathrm{ImH})^2} \qquad (6.30)$$

Differentiating this with respect to the sensitivity parameter x yields

$$\frac{d}{dx}|H| = \frac{\mathrm{ReH}\ \frac{d\mathrm{ReH}}{dx} + \mathrm{ImH}\ \frac{d\mathrm{ImH}}{dx}}{\sqrt{(\mathrm{ReH})^2 + (\mathrm{ImH})^2}} \qquad (6.31)$$

By definition

$$\mathcal{S}_x^{|H|} = \frac{x}{|H|} \frac{d|H|}{dx} \qquad (6.32)$$

Substituting (6.30) and (6.31) into (6.32) results in

$$\mathcal{S}_x^{|H|} = \frac{x\mathrm{ReH}\ \frac{d\mathrm{ReH}}{dx} + x\mathrm{ImH}\ \frac{d\mathrm{ImH}}{dx}}{|H|^2} \qquad (6.33)$$

Recalling that

$$\mathcal{S}_x^{\mathrm{reH}} \triangleq \frac{x}{\mathrm{ReH}} \frac{d\mathrm{ReH}}{dx} \qquad (6.34)$$

and

$$\mathcal{S}_x^{\mathrm{ImH}} \triangleq \frac{x}{\mathrm{ImH}} \frac{d\mathrm{ImH}}{dx}, \qquad (6.35)$$

one can simplify (6.33) to

$$\mathcal{S}_x^{|H|} = \frac{(\mathrm{ReH})^2\ \mathcal{S}_x^{\mathrm{ReH}} + (\mathrm{ImH})^2\ \mathcal{S}_x^{\mathrm{ImH}}}{|H|} \tag{6.36}$$

The sensitivity of the phase of the transfer function to changes in x, $\mathcal{S}_x^{\phi}$, is also easily obtained. By definition

$$\tan\phi = \frac{\mathrm{ImH}}{\mathrm{ReH}} \tag{6.37}$$

By implicit differentiation of (6.37) with respect to x, one obtains

$$\sec^2\phi\ \frac{d\phi}{dx} = \frac{\mathrm{ReH}\ \frac{d\mathrm{ImH}}{dx} - \mathrm{ImH}\ \frac{d\mathrm{ReH}}{dx}}{(\mathrm{ReH})^2} \tag{6.38}$$

Since

$$\mathcal{S}_x^{\phi} \triangleq \frac{x}{\phi}\ \frac{d\phi}{dx} \tag{6.39}$$

one obtains by substituting (6.38) into (6.39)

$$\mathcal{S}_x^{\phi} = \frac{\cos^2\phi}{\phi\,(\mathrm{ReH})^2}\left(x\mathrm{ReH}\ \frac{d\mathrm{ImH}}{dx} - x\mathrm{ImH}\frac{d\mathrm{ReH}}{dx}\right) \tag{6.40}$$

By use of the definitions in (6.34) and (6.35) and the relation

$$\mathrm{ReH} = |H|\ \cos\phi\,, \tag{6.41}$$

the expression in (6.40) can be simplified to

$$\mathcal{S}_x^{\phi} = \frac{1}{\phi\,|H|^2}\left(\mathrm{ReH}\ \mathrm{ImH}\right)\left(\mathcal{S}_x^{\mathrm{ImH}} - \mathcal{S}_x^{\mathrm{ReH}}\right) \tag{6.42}$$

Equations (6.36) and (6.42) are used in subroutine SENS to determine

$\mathcal{S}_x^{|H|}$ and $\mathcal{S}_x^{\phi}$, respectively.

This is possible since the quantities ϕ, $|H|$, ReH, and ImH have been previously calculated and stored during generation of the Bode tables and plots while S_x^{ReH} and S_x^{ImH} have been determined earlier in subroutine SENS.

VID DISCUSSION OF REVISED SENSITIVITY SUBROUTINE

In Appendix B is a revised version of subroutine SENSS which does the calculations of SENS and SENSS in a more efficient manner. This version of SENSS requires only 3/4 of the core storage required by the present SENS and SENSS and it utilizes a simpler algorithm that greatly reduces the number of mathematical operations required. This saves execution time and should increase the accuracy of the sensitivity calculations.

The version of SENSS given in Appendix B uses the same tagging procedure that is used in the present SENSS and calculates $S_x^{H(j\omega)}$ with the use of equation (6.25) as does the present SENSS. However, the outputs of the present SENS, that is $S_x^{|H|}$, S_x^{ϕ}, S_x^{ReH}, and S_x^{ImH} are related to the real and imaginary parts of $S_x^{H(j\omega)}$. Thus the rather complication sensitivity expressions now used in the present SENS and described in equations (6.23) through (6.42) are completely avoided.

Since, in general, the sensitivity expression $S_x^{H(j\omega)}$ is a complex quantity, it can be written as

$$S_x^{H(j\omega)} = \text{Re}S_x^{H(j\omega)} + j\text{Im}\, S_x^{H(j\omega)} \tag{6.43}$$

But

$$H(j\omega) = |H(j\omega)|\ e^{j\phi} \tag{6.44}$$

where ϕ is the phase of the transfer function $H(j\omega)$.

Thus one has

$$S_x^{H(j\omega)} = S_x^{|H|e^{j\phi}}$$

$$= S_x^{|H|} + S_x^{e^{j\phi}} \tag{6.45}$$

Let us examine the rightmost term of (6.45) more closely. By definition

$$S_x^{e^{j\phi}} \triangleq \frac{x}{e^{j\phi}} \frac{d}{dx} e^{j\phi} . \tag{6.46}$$

By use of the differentiation chain rule, the derivative expression in (6.46) can be simplified to

$$\frac{d}{dx} e^{j\phi} = je^{j\phi} \frac{d\phi}{dx} . \tag{6.47}$$

Substituting (6.46) and (6.47) into (6.45) yields

$$S_x^{H(j\omega)} = S_x^{|H|} + jx \frac{d\phi}{dx} . \tag{6.48}$$

Using the definition given in (6.39) this equation can be rewritten as

$$S_x^{H(j\omega)} = S_x^{|H|} + j\phi S_x^{\phi} . \tag{6.49}$$

If the sensitivity parameter is a real quantity, then the expressions $S_x^{|H|}$ and S_x^{ϕ}, will also be real. Thus if (6.49) is compared with (6.43) and the real and imaginary parts equated (under the assumption that x is real) then one obtains

$$S_x^{|H|} = \mathrm{Re}\, S_x^{H(j\omega)} \tag{6.50}$$

and

$$\phi S_x^{\phi} = \mathrm{Im}\, S_x^{H(j\omega)} \tag{6.51a}$$

or

$$S_x^{\phi} = \frac{1}{\phi} \mathrm{Im}\, S_x^{H(j\omega)} \tag{6.51b}$$

where x is a real variable.

The sensitivity of the real part of H(jω) to changes in x, S_x^{ReH}, can also be obtained in terms of the real and imaginary parts of $S_x^{H(j\omega)}$. By definition

$$ReH(j\omega) = |H(j\omega)| \cos\phi . \tag{6.52}$$

Thus

$$\begin{aligned} S_x^{ReH} &= S_x^{|H|\cos\phi} \\ &= S_x^{|H|} + S_x^{\cos\phi} \end{aligned} \tag{6.53}$$

But

$$\begin{aligned} S_x^{\cos\phi} &= \frac{x}{\cos\phi} \frac{d}{dx} \cos\phi \\ &= -x \tan\phi \frac{d\phi}{dx} \quad . \end{aligned} \tag{6.54}$$

Equation (6.54) can be rewritten as

$$S_x^{\cos\phi} = -\phi \tan\phi S_x^{\phi} \tag{6.55}$$

by use of the definition in (6.39). Thus equation (6.53) becomes, after the substitution of (6.55),

$$S_x^{ReH} = S_x^{|H|} - \phi \tan\phi S_x^{\phi} . \tag{6.56}$$

However, after (6.50), (6.51a) and (6.37) are substituted into (6.56), the expression becomes

$$S_x^{ReH} = Re S_x^{H(j\omega)} - \frac{ImH}{ReH} Im S_x^{H(j\omega)} \tag{6.57}$$

A similar expression can be derived for the sensitivity of the imaginary part of H(jω) with respect to changes in x, S_x^{ImH}. By definition

$$ImH(j\omega) = |H(j\omega)| \sin\phi \tag{6.58}$$

Thus

$$\mathcal{S}_x^{\mathrm{ImH}} = \mathcal{S}_x^{|H| \sin \phi}$$

$$\mathcal{S}_x^{|H|} + \mathcal{S}_x^{\sin \phi} \quad . \tag{6.59}$$

By a similar mathematical technique, it can be shown that

$$\mathcal{S}_x^{\sin \phi} = \frac{\phi}{\tan \phi} \mathcal{S}_x^{\phi} \tag{6.60}$$

Substituting this expression into (6.59) gives

$$\mathcal{S}_x^{\mathrm{ImH}} = \mathcal{S}_x^{|H|} + \frac{\phi}{\tan \phi} \mathcal{S}_x^{\phi} \tag{6.61}$$

which can be further simplified by the substitution of (6.50), (6.51a) and (6.37)

$$\mathcal{S}_x^{\mathrm{ImH}} = \mathrm{Re}\, \mathcal{S}_x^{H(j\omega)} + \frac{\mathrm{ReH}}{\mathrm{ImH}} \mathrm{Im}\, \mathcal{S}_x^{H(j\omega)} \tag{6.62}$$

Equations (6.50), (6.51b), (6.57) and (6.62) are used in the version of SENSS given in Appendix B. This is possible since the real quantities ϕ, ReH, and ImH have been calculated earlier in subroutine BODE while the complex quantity $\mathcal{S}_x^{H(j\omega)}$ is calculated in the Appendix B version of SENSS.

Note that (6.57) involves only 3 arithmetic operations while (6.25) involves 8 arithmetic operations plus the numerous operations involved in equations (6.23) and (6.26). The same comparison can be made between (6.62) and (6.28). With regard to $\mathcal{S}_x^{|H|}$, (6.50) involves no arithmetic operations while (6.36) uses 8 operations. Similarly in determining $\mathcal{S}_x^{\phi}$ equation (6.51b) requires one arithmetic operation while (6.42) involves 6 operations. Since the sensitivity calculations must be redone for each frequency value, the number of arithmetic operations is quite substantially reduced with the version of SENSS given in Appendix B.

VIE ROOT SENSITIVITY

The sensitivites of the poles and zeros of the transfer function to changes in some specified network parameter are also determined by NASAP by use of the tagging technique and the formulas given in [KU1, PA1] which will be summarized here.

Suppose we have a polynomial in the complex frequency variable s, P(s), which can also be expressed as

$$P(s) = \alpha(s) + x\beta(s) = \sum_{k=o}^{N} a_k s^k \tag{6.63}$$

where the degree of P(s) is N and P(s) has only positive powers of s and x is the sensitivity parameter. Furthermore the roots of the above polynomial are known, i.e., N values of s are known such that

$$P(r_i) = 0 \qquad i = 1,2,\cdots N \tag{6.64}$$

where r_i is the i^{th} root of P(s).

If the order of the root r_i is m, then the root sensitivity, s_x^{ri}, as defined by

$$s_x^{ri} \triangleq x \frac{dr_i}{dx} \tag{6.65}$$

can be expressed in terms of the given polynomials $\alpha(s)$ and $\beta(s)$

$$s_x^{r_i} = \left. \frac{\dfrac{d^{m-1}}{ds^{m-1}}\left(x\beta(s)\right)}{\dfrac{d^m}{ds^m}\left(\alpha(s) + x\beta(s)\right)} \right|_{s = r_i} \qquad i=1,2,\cdots N \tag{6.66}$$

If r_i is a simple root of P(s), then

$$s_x^{r_i} = \left. \frac{-x\beta(r_i)}{\dfrac{d}{ds}\left(\alpha(s) + x\beta(s)\right)} \right|_{s = r_i} \tag{6.67}$$

As was shown above, the transfer function H(s) can be written as

$$H(s) = \frac{N(s)}{D_1(s)} = \frac{A(s) + xB(s)}{C(s) + xD(s)} \qquad (6.68)$$

where x is the sensitivity parameter.

If Z_i is a simple zero of H(s), then the zero sensitivity with respect to x can be expressed as

$$S_x^{Z_i} = \left.\frac{xB(Z_i)}{\frac{d}{ds}(A(s) + xB(s))}\right|_{s = Z_i} = \left.\frac{-xB(Z_i)}{\frac{d}{ds} N(s)}\right|_{s = Z_i} \qquad i=1,2,\cdots M \qquad (6.69)$$

where M is the number of distinct zeros and A(s), B(s), and N(s) are defined in (6.23).

Similarly, if p_i is a simple pole of H(s), then the pole sensitivity with respect to x can be written as

$$S_x^{p_i} = \left.\frac{xD(p_i)}{\frac{d}{ds}\,(C(s) + xD(s))}\right|_{s=p_i} = \left.\frac{-xD(p_i)}{\frac{d}{ds}\,D_1(s)}\right|_{s=p_i} \qquad i=1,2,\cdots, N \quad (6.70)$$

where N is the number of distinct poles of H(s) and C(s), D(s), and $D_1(s)$ are defined in (6.68).

Since the polynomials A(s), xB(s), C(s), and xD(s) have been determined by the tagging process during the evaluation of the loops in subroutines FLGRPH and HIGORL, the sensitivities of the poles and zeros are easily obtained by differentiating the denominator and numberator polynomials respectively and by evaluating the resultant polynomial and the appropriate tagged polynomial at the given pole or zero. These calculations are performed in subroutine ROOTSS.

VIF EXAMPLES

In the foregoing sections of this chapter we have indicated many possibilities for sensitivity analysis with the aid of NASAP. We shall illustrate a few of these as follows:

1. Taking advantage of the background developed in Chapters IV and V on the unity feedback control system with lead cascade compensation see Fig. 6.1a (also Figs. 3.16 and 4.7) we obtain the sensitivity S_x^H of the transfer function VV4/VV5 with respect to resistor R1. This is a judicious choice of circuit element since one of the time constants of the control system plant is

$$\tau_1 = (R1)\ (C1)$$

Hence if C1 is constant in value the sensitivity determined for R1 is the same as that for the time constant τ_1. The NASAP printout for this example is shown in Fig. 6.1. The corresponding zero and pole sensitivities are given in Fig. 6.2.

2. The second example Fig. 6.2 follows up with the uncompensated unity feedback control system for which we obtain the ramp response in Fig. 6.3 and the sensitivity function S_K^H. Thus we seek the sensitivity of the transfer function VV4/VV5 with respect to system gain K, in this case K = 0.83. The NASAP printout is shown in Fig. 6.4 with the corresponding zero and pole sensitivities given in Fig. 6.5.

3. The third example furnishes sensitivity data, specifically S_K^H for the multiloop feedback compensated control system with K = 38. Since the pertinent NASAP model and transfer function print out were given in Fig. 5, they are not repeated here. The corresponding gain sensitivity print outs are given in Fig. 6.6 and the zero-pole sensitivity in Fig. 6.7.

4. For the final example we use one of the control subsystem of the Ranger space vehicle taken from Dorf [DOl, Problem 4.5]. The mission in 1965 was to scan the lunar surface with TV and other sensors. The requirement on the altitude control subsystem was to stabilize and control the Ranger spacecraft from second stage separation to lunar impact. Briefly a high gain antenna furnished input signals to the earth horizon sensor which in turn fed the gyro loop. The latter consisted of the spacecraft, as the plant, with the altitude gyro in cascade as shown in Fig. 6.8z along with its NASAP model. To give some feel for the cut and try process in determining the gyro loop gain K that will keep the step response overshoot under the required 5%, we show for three values of gain the print outs of S_K^H, zero-pole sensitivities $S_H^{p_i}$ and the corresponding step response. For convenience in examining the print outs we tabulate the pertinent figure numbers.

K	step response	S_K^H	$S_H^{p_i}$
6	6.8	Fig. 6.9	6.10
20	6.11	Fig. 6.12	6.13
75	6.14	Fig. 6.15	6.16

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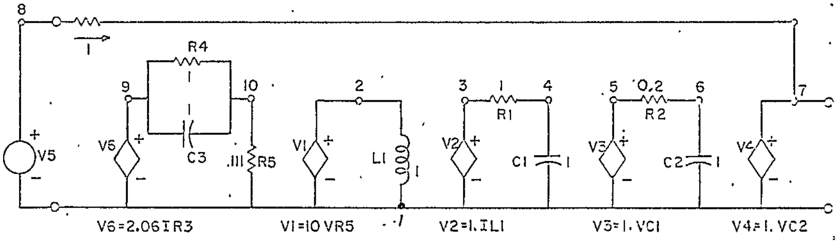
```
    NASAP OF BLF   LEAD -  COMPENSATION

    HERTZ
    STEP RESPONSE

    V1 1 2 10. VR5
    L1 2 1 1.H
    V2 1 3 1.0 IL1
    R1 3 4 1.
    C1 4 1 1.F
    V3 1 5 1.0 VC1
    R2 5 6 0.2
    C2 6 1 1.F
    V4 1 7 1.0 VC2
    R3 8 7 1.0
    V5 1 8 1.0
    V6 1 9 2.06 IR3
    C3 9 10 1.F
    R4 9 10  1.000
    R5 10 1  0.111
    OUTPUT
    VV4/VV5/R1
    FREQ -1.0 1.  0.05
    TIME 6.0
    EXECUTE
```



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```
    TRANSFER FUNCTION    VV4/VV5/R1

                                                                                                 NUMBER OF LOOPS PER ORDER

                          (                      )                                                    1=     8
                          (  1.00E 00     +1.00E 00 S    )                                            2=     5
                                                                                                      3=     2
    H(S)= 1.030E 02*-----------------------------------------------------------------------
                          (                                       2            3            4 )
                          (  1.03E 02     +1.55E 02 S    +6.51E 01 S    +1.60E 01 S    +1.00E 00 S    )


     ZERO OF TRANSFER FUNCTION

     ZERO    REAL PART    IMAG. PART

        1 -0.10000E 01   0.00000E 00


     POLE OF TRANSFER FUNCTION

     POLE    REAL PART    IMAG. PART

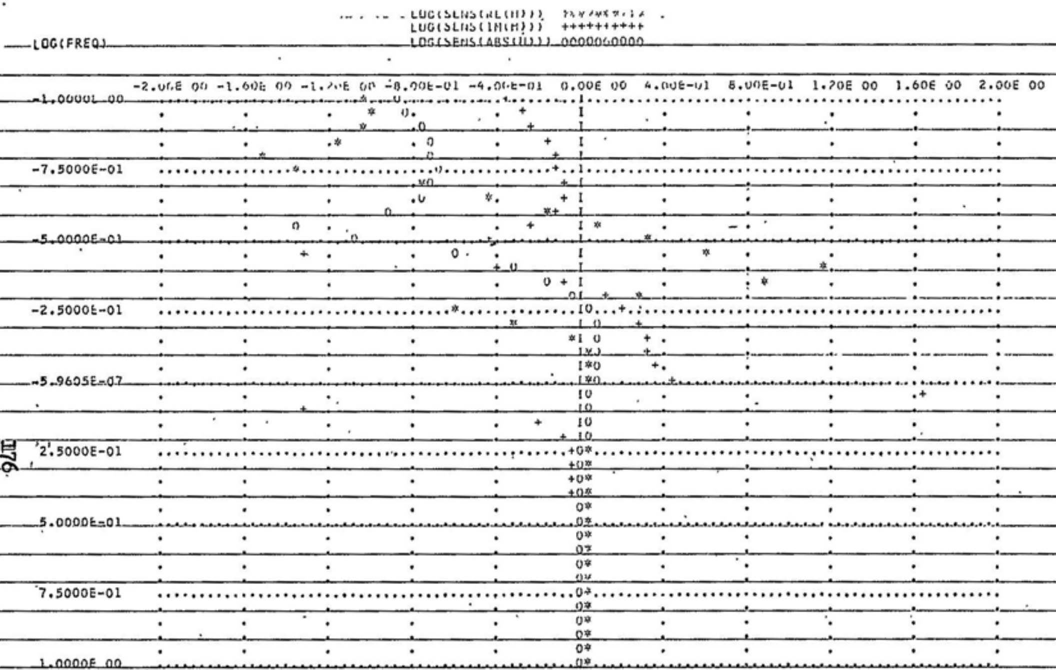
        1 -0.17946E 01   0.24072E 01
        2 -0.17945E 01  -0.24072E 01
        3 -0.10000E 01  -0.48348E-11
        4 -0.11415E 02   0.89302E-09
```

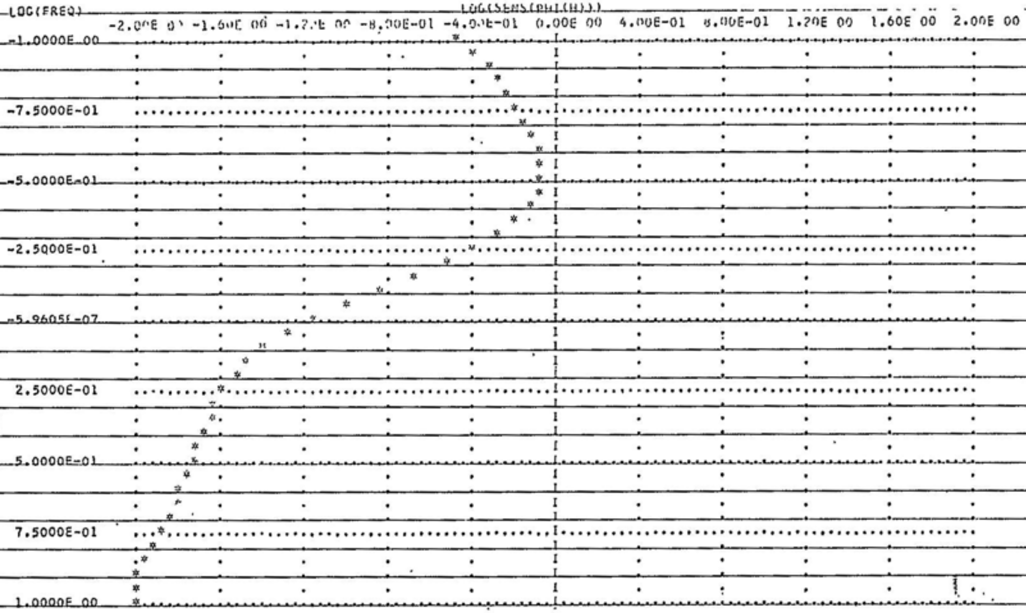
```
SENSITIVITY ANALYSIS #1

     LOG(FREQ)         FREQ         ABS(SENS(H))     PHI(SENS(H))    LOG(ABS(SENS(H)))
  -0.9999998E 00   0.1000001E 00   0.1856611E 00   -0.3925294E 02   -0.7307220E 00
  -0.9499997E 00   0.1122019E 00   0.2023157E 00   -0.4321170E 02   -0.6939700E 00
  -0.8999997E 00   0.1258926E 00   0.2458671E 00   -0.4750122E 02   -0.6092993E 00
  -0.8499998E 00   0.1412538E 00   0.2972979E 00   -0.5207359E 02   -0.5268080E 00
  -0.7999999E 00   0.1584894E 00   0.3576479E 00   -0.5693672E 02   -0.4465441E 00
  -0.7499999E 00   0.1778280E 00   0.4260353E 00   -0.6211020E 02   -0.3705704E 00
  -0.6999999E 00   0.1995263E 00   0.5096315E 00   -0.6763199E 02   -0.2927437E 00
  -0.6500000E 00   0.2238721E 00   0.6035404E 00   -0.7356511E 02   -0.2192936E 00
  -0.6000000E 00   0.2511886E 00   0.7104592E 00   -0.8000124E 02   -0.1484424E 00
  -0.5500001E 00   0.2818382E 00   0.8301610E 00   -0.8705687E 02   -0.8083761E-01
  -0.5000001E 00   0.3162277E 00   0.9599916E 00   -0.9485393E 02   -0.1774255E-01
  -0.4500002E 00   0.3548133E 00   0.1093459E 01   -0.1034699E 03    0.3861202E-01
  -0.4000002E 00   0.3981070E 00   0.1218934E 01   -0.1128564E 03    0.8597642E-01
  -0.3500003E 00   0.4466833E 00   0.1320599E 01   -0.1227497E 03    0.1207708E 00
  -0.3000003E 00   0.5011868E 00   0.1384347E 01   -0.1326521E 03    0.1412449E 00
  -0.2500004E 00   0.5623409E 00   0.1404862E 01   -0.1419534E 03    0.1476337E 00
  -0.2000004E 00   0.6309558E 00   0.1388241E 01   -0.1501430E 03    0.1424648E 00
  -0.1500005E 00   0.7079450E 00   0.1347578E 01   -0.1569545E 03    0.1294694E 00
  -0.1000005E 00   0.7943273E 00   0.1295344E 01   -0.1623679E 03    0.1123836E 00
  -0.5000055E-01   0.8912493E 00   0.1241673E 01   -0.1665227E 03    0.9400636E-01
  -0.5963464E-06   0.9999986E 00   0.1191928E 01   -0.1696256E 03    0.7624926E-01
   0.4999364E-01   0.1122015E 01   0.1148697E 01   -0.1718967E 03    0.6020543E-01
   0.9999383E-01   0.1258922E 01   0.1112724E 01   -0.1735302E 03    0.4638646E-01
   0.1499931E 00   0.1412535E 01   0.1083743E 01   -0.1746913E 03    0.3492633E-01
   0.1999924E 00   0.1584870E 01   0.1061136E 01   -0.1755158E 03    0.2572587E-01
   0.2499965E 00   0.1778273E 01   0.1043644E 01   -0.1761135E 03    0.1855077E-01
   0.2999986E 00   0.1995256E 01   0.1030622E 01   -0.1765516E 03    0.1310263E-01
   0.3499978E 00   0.2238715E 01   0.1021110E 01   -0.1768894E 03    0.9072448E-02
   0.3999954E 00   0.2511863E 01   0.1014227E 01   -0.1771647E 03    0.6158510E-02
   0.4499730E 00   0.2818572E 01   0.1009436E 01   -0.1774018E 03    0.4100382E-02
   0.4999785E 00   0.3162267E 01   0.1006154E 01   -0.1776123E 03    0.2677243E-02
   0.5499736E 00   0.3548122E 01   0.1003951E 01   -0.1778132E 03    0.1712548E-02
   0.5999388E 00   0.3981006E 01   0.1002469E 01   -0.1779921E 03    0.1070979E-02
   0.6499751E 00   0.4467616E 01   0.1001503E 01   -0.1781749E 03    0.6522504E-03
   0.6999738E 00   0.5011552E 01   0.1000886E 01   -0.1783464E 03    0.3845985E-03
   0.7499725E 00   0.5623492E 01   0.1000498E 01   -0.1784935E 03    0.2161457E-03
   0.7999706E 00   0.6309554E 01   0.1000260E 01   -0.1786400E 03    0.1130551E-03
   0.8499779E 00   0.7079423E 01   0.1000117E 01   -0.1787736E 03    0.5094058E-04
   0.8999381E 00   0.7943247E 01   0.1000036E 01   -0.1788945E 03    0.1573838E-04
   0.9499583E 00   0.8912474E 01   0.9999917E 00   -0.1790088E 03   -0.3598164E-05
   0.9999825E 00   0.9999595E 01   0.9999693E 00   -0.1791110E 03   -0.1333148E-04
```

```
   LOG(FREQ)          FREQ          SENS(RE(H))      SENS(IM(H))      SENS(ABS(H))     SENS(PHI(H))
-0.9999996E 00   0.1000001E 00    0.9473562E-01    0.4542611E 00    0.1263896E 00    0.3367962E 00
-0.9499997E 00   0.1122019E 00    0.9680724E-01    0.5263335E 00    0.1474535E 00    0.3952041E 00
-0.8999997E 00   0.1258926E 00    0.9040082E-01    0.6001911E 00    0.1661015E 00    0.4562461E 00
-0.8499997E 00   0.1412539E 00    0.7649327E-01    0.6711322E 00    0.1827346E 00    0.5245203E 00
-0.7999998E 00   0.1584894E 00    0.7875878E-01    0.7358609E 00    0.1951202E 00    0.5921402E 00
-0.7499999E 00   0.1778280E 00   -0.4467111E-01    0.7649208E 00    0.2062231E 00    0.6587504E 00
-0.6999999E 00   0.1995263E 00   -0.1667102E 00    0.8098001E 00    0.1939422E 00    0.7215140E 00
-0.6500000E 00   0.2238721E 00   -0.5631811E 00    0.7983633E 00    0.1707567E 00    0.7769867E 00
-0.6000000E 00   0.2511886E 00   -0.6769993E 00    0.7350565E 00    0.1233604E 00    0.8208535E 00
-0.5500001E 00   0.2818382E 00   -0.1118654E 01    0.7008559E 00    0.4262423E-01    0.8475517E 00
-0.5000001E 00   0.3162277E 00   -0.2013935E 01    0.3756453E 00   -0.8122915E-01    0.8511109E 00
-0.4500002E 00   0.3548133E 00   -0.4046785E 01    0.4587835E-01   -0.2547082E 00    0.8209603E 00
-0.4000002E 00   0.3981070E 00   -0.1400783E 02   -0.3802397E 00   -0.4734553E 00    0.7548524E 00
-0.3500003E 00   0.4466833E 00    0.7918202E 01   -0.6572225E 00   -0.7144040E 00    0.6538234E 00
-0.3000003E 00   0.5011868E 00    0.1878963E 01   -0.1307275E 01   -0.9379563E 00    0.5366337E 00
-0.2500004E 00   0.5623409E 00    0.5551514E 00   -0.1656801E 01   -0.1100242E 01    0.4051384E 00
-0.2000004E 00   0.6309568E 00   -0.4962066E 00   -0.1878839E 01   -0.1203979E 01    0.2948095E 00
-0.1500004E 00   0.7079450E 00   -0.7718463E 00   -0.1999960E 01   -0.1239549E 01    0.2010696E 00
-0.1000005E 00   0.7943273E 00   -0.1052759E 01   -0.2081637E 01   -0.1234487E 01    0.1446988E 00
-0.5000055E-01   0.8912498E 00   -0.1125115E 01   -0.2224243E 01   -0.1207479E 01    0.1010312E 00
-0.5960464E-06   0.9999936E 00   -0.1143356E 01   -0.2754840E 01   -0.1172447E 01    0.7137913E-01
 0.4999564E-01   0.1122015E 01   -0.1126615E 01   -0.4392862E 02   -0.1137228E 01    0.5160255E-01
 0.9999463E-01   0.1258942E 01   -0.1110495E 01   -0.4738998E-01   -0.1105636E 01   -0.4146710E-01
 0.1499995E 00   0.1412535E 01   -0.1102775E 01   -0.6547648E 00   -0.1079099E 01   -0.3444892E-01
 0.1999992E 00   0.1584870E 01   -0.1087048E 01   -0.8230433E 00   -0.1057784E 01   -0.2957765E-01
 0.2499935E 00   0.1778273E 01   -0.1074626E 01   -0.8913554E 00   -0.1041241E 01   -0.2619349E-01
 0.2999926E 00   0.1995256E 01   -0.1065671E 01   -0.9240281E 00   -0.1028767E 01   -0.2380097E-01
 0.3499958E 00   0.2238715E 01   -0.1059793E 01   -0.9432107E 00   -0.1019606E 01   -0.2203948E-01
 0.3999919E 00   0.2511861E 01   -0.1056394E 01   -0.9549750E 00   -0.1013041E 01   -0.2065541E-01
 0.4499943E 00   0.2818372E 01   -0.1054845E 01   -0.9643079E 00   -0.1008450E 01   -0.1947839E-01
 0.4999785E 00   0.3162267E 01   -0.1054566E 01   -0.9697254E 00   -0.1005313E 01   -0.1840059E-01
 0.5499286E 00   0.3548122E 01   -0.1055110E 01   -0.9749365E 00   -0.1003221E 01   -0.1736032E-01
 0.5999988E 00   0.3981061E 01   -0.1056120E 01   -0.9792617E 00   -0.1001859E 01   -0.1632749E-01
 0.6499981E 00   0.4466816E 01   -0.1057343E 01   -0.9829318E 00   -0.1000995E 01   -0.1529206E-01
 0.6999983E 00   0.5011652E 01   -0.1058619E 01   -0.9860165E 00   -0.1000466E 01   -0.1425711E-01
 0.7499985E 00   0.5623493E 01   -0.1059864E 01   -0.9885928E 00   -0.1000154E 01   -0.1323100E-01
 0.7999986E 00   0.6309554E 01   -0.1060956E 01   -0.9907355E 00   -0.9999768E 00   -0.1222457E-01
 0.8499979E 00   0.7079423E 01   -0.1061928E 01   -0.9925042E 00   -0.9998588E 00   -0.1124847E-01
 0.8999781E 00   0.7943247E 01   -0.1062782E 01   -0.9939563E 00   -0.9998509E 00   -0.1031174E-01
 0.9499983E 00   0.8912474E 01   -0.1063496E 01   -0.9951404E 00   -0.9996420E 00   -0.9421561E-02
 0.9999985E 00   0.9999965E 01   -0.1064089E 01   -0.9961015E 00   -0.9998489E 00   -0.8582644E-02
```

```
      LOG(FREQ)          FREQ        LOG(SENS(RS(H)))  LOG(SENS(IM(H)))  LOG(SENS(ABS(H)))  LOG(SENS(PHI(H)))
  -0.9999996E 00   0.1000001E 00   -0.1023486E 01   -0.3426944E 00   -0.8914692E 00   -0.4726326E 00
  -0.9499997E 00   0.1122016E 00   -0.1014691E 01   -0.2757342E 00   -0.8313446E 00   -0.4031765E 00
  -0.8999997E 00   0.1258926E 00   -0.1043637E 01   -0.2217104E 00   -0.7796761E 00   -0.3389012E 00
  -0.8499998E 00   0.1412538E 00   -0.1154017E 01   -0.1772665E 00   -0.7281106E 00   -0.2802377E 00
  -0.7999993E 00   0.1584894E 00   -0.1254071E 01   -0.1332397E 00   -0.7056775E 00   -0.2275754E 00
  -0.7499999E 00   0.1778280E 00   -0.1350985E 01   -0.1051819E 00   -0.6964854E 00   -0.1812791E 00
  -0.6999999E 00   0.1995263E 00   -0.7780377E 00   -0.9162211E-01   -0.7123275E 00   -0.1417552E 00
  -0.6500006E 00   0.2238721E 00   -0.4398767E 00   -0.9779936E-01   -0.7676221E 00   -0.1095864E 00
  -0.6000006E 00   0.2511886E 00   -0.1694117E 00   -0.1336792E 00   -0.5068240E 00   -0.8573431E-01
  -0.5500016E 00   0.2818382E 00    0.7535514E-01   -0.2212224E 00   -0.1370442E 01   -0.7183379E-01
  -0.5000001E 00   0.3162277E 00    0.3158841E 00   -0.4252220E 00   -0.1090287E 01   -0.7052439E-01
  -0.4500002E 00   0.3548133E 00    0.6046781E 00   -0.1348491E 01   -0.5939570E 00   -0.8567780E-01
  -0.4000002E 00   0.3981070E 00    0.1146370E 01   -0.4199425E 00   -0.3247210E 00   -0.1221380E 00
  -0.3500006E 00   0.4466833E 00    0.8926667E 00   -0.6687045E-01   -0.1460561E 00   -0.1845395E 00
  -0.3000003E 00   0.5011868E 00    0.2716054E 00    0.1163669E 00   -0.2781737E-01   -0.2752051E 00
  -0.2500006E 00   0.5623409E 00   -0.5925832E 00    0.2192703E 00    0.4328953E-01   -0.3923966E 00
  -0.2000004E 00   0.6309564E 00   -0.3043354E 00    0.2738696E 00    0.6061904E-01   -0.5304585E 00
  -0.1500005E 00   0.7079455E 00   -0.5856493E-01    0.3010212E 00    0.9336877E-01   -0.6817912E 00
  -0.1000005E 00   0.7943273E 00    0.2232903E-01    0.3184050E 00    0.9148633E-01   -0.8389352E 00
  -0.5000055E-01   0.8912456E 00    0.5119736E-01    0.3671822E 00    0.8157938E-01   -0.9955440E 00
  -0.5960464E-06   0.9999986E 00    0.5817470E-01    0.4401041E 00    0.6905323E-01   -0.1146428E 01
   0.4999964E-01   0.1122015E 01    0.5561341E-01    0.1642747E 01    0.5564755E-01   -0.1287329E 01
   0.9999883E-01   0.1258922E 01    0.4940035E-01   -0.1324313E 01    0.4361203E-01   -0.1382295E 01
   0.1499939E 00   0.1412545E 01    0.4268676E-01   -0.1849147E 00    0.3406117E-01   -0.1462824E 01
   0.1999959E 00   0.1584890E 01    0.3604855E-01   -0.8457776E-01    0.2439702E-01   -0.1529036E 01
   0.2499985E 00   0.1778274E 01    0.3125713E-01   -0.4994909E-01    0.1755113E-01   -0.1581805E 01
   0.2999786E 00   0.1995256E 01    0.2762273E-01   -0.3403790E-01    0.1231687E-01   -0.1623405E 01
   0.3499796E 00   0.2238715E 01    0.2522124E-01   -0.2549130E-01    0.8432274E-02   -0.1656794E 01
   0.3999796E 00   0.2511831E 01    0.2332577E-01   -0.2000673E-01    0.5677734E-02   -0.1684965E 01
   0.4499985E 00   0.2818472E 01    0.2316662E-01   -0.1623429E-01    0.3654178E-02   -0.1710446E 01
   0.4999985E 00   0.3162267E 01    0.2373732E-01   -0.1335124E-01    0.2301262E-02   -0.1735168E 01
   0.5499985E 00   0.3548122E 01    0.2329773E-01   -0.1102365E-01    0.1396423E-02   -0.1760442E 01
   0.5999985E 00   0.3981061E 01    0.2371323E-01   -0.9092376E-02    0.3064780E-03   -0.1787080E 01
   0.6499981E 00   0.4466516E 01    0.2421639E-01   -0.7473506E-02    0.4321840E-03   -0.1815533E 01
   0.6999958E 00   0.5011852E 01    0.2473949E-01   -0.6115835E-02    0.2024846E-03   -0.1845967E 01
   0.7499958E 00   0.5623392E 01    0.2524078E-01   -0.4982553E-02    0.6667712E-04   -0.1878407E 01
   0.7999986E 00   0.6309554E 01    0.2559737E-01   -0.4042260E-02   -0.9189613E-05   -0.1912766E 01
   0.8499979E 00   0.7079423E 01    0.2609927E-01   -0.3267629E-02   -0.4830590E-04   -0.1948906E 01
   0.8999781E 00   0.7943247E 01    0.2644430E-01   -0.2632722E-02   -0.6477151E-04   -0.1986667E 01
   0.9499982E 00   0.8912474E 01    0.2674571E-01   -0.2115626E-02   -0.6860323E-04   -0.2025876E 01
   0.9999985E 00   0.9999965E 01    0.2657738E-01   -0.1696406E-02   -0.6562588E-04   -0.2066379E 01
```


LOG(SENS(RE(H))) **********
LOG(SENS(IM(H))) ++++++++++
LOG(SENS(ABS(H))) 0000000000
LOG(FREQ)
-2.00E 00
-1.60E 00
-1.20E 00
-8.00E-01
-4.00E-01
0.00E 00
4.00E-01
8.00E-01
1.20E 00
1.60E 00
2.00E 00
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
-5.9605E-07
2.5000E-01
5.0000E-01
7.5000E-01
1.0000E 00


LOG(FREQ)
LOG(SENS(PHI(H)))
-2.00E 00
-1.60E 00
-1.20E 00
-8.00E-01
-4.00E-01
0.00E 00
4.00E-01
8.00E-01
1.20E 00
1.60E 00
2.00E 00
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
-5.9605E-07
2.5000E-01
5.0000E-01
7.5000E-01
1.0000E 00

```
SENSITIVITIES OF ZEROES AND POLES OF TRANSFER FUNCTION

ZERO        REAL             IMAG                                  SENSITIVITY
                                                    REAL                IMAG
  1     -0.1000000E 01    0.0000000E 00       0.9175604E-06      0.0000000E 00

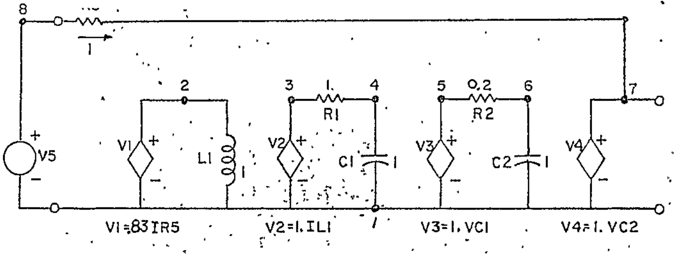
POLE        REAL             IMAG                                  SENSITIVITY
                                                    REAL                IMAG
  1     -0.1796793E 01    0.2407153E 01       0.1949336E 00     -0.2548719E 01
  2     -0.1796793E 01   -0.2407153E 01       0.1949336E 00      0.2548719E 01
  3     -0.1000002E 01   -0.4838796E-11      -0.5381470E 00      0.4601166E-07
  4     -0.1141542E 02    0.8030199E-09       0.1148264E 01     -0.5103200E-05
```

7/14/69

NASAP-PROBLEM UNCOMPENSATED PLANT

~~NONE~~ HERTZ
RAMP RESPONSE

```
V1 1 2 0.83 IR3
L1 2 1 1.H
V2 1 3 1.0 IL1
R1 3 4 1.
C1 4 1 1F
V3 1 5 1.0 VC1
R2 5 6 0.2
C2 6 1 1F
V4 1 7 1.0 VC2
R3 8 7 1.0
V5 1 8 1.0
OUTPUT
VV4/VV5/V1
FREQ 0.0 1.0 0.05
TIME 25.
EXECUTE
```



```
 TRANSFER FUNCTION   VV4/VV5/V1                                              NUMBER OF LOOPS PER ORDER

                                                                                     1=     4
                                                                                     2=     1
                  (                )
                  ( 1.00E 00       )
H(S)= 4.150E 00*-----------------------------------------------------
                  (                                  2              3 )
                  (  4.15E 00    +5.00E 00 S   +6.00E 00 S   +1.00E 00 S   )

  ZERO OF TRANSFER FUNCTION

              NONE


  POLE OF TRANSFER FUNCTION

  POLE    REAL PART    IMAG. PART

     1 -0.40461E 00  0.79736E 00
     2 -0.40461E 00 -0.79736E 00
     3 -0.51906E 01  0.62528E-12
```

```
RAMP    RESPONSE FUNCTION

F(T) =

                                                   (-0.4046E 00  J 0.7974E 00 ) T
        ( 0.5991E 00  J 0.3017E 00 ) E

                                                   (-0.4046E 00  J-0.7974E 00 ) T
        ( 0.5971E 00  J-0.3017E 00 ) E

                                                   (-0.5191E 01  J 0.6253E-12 ) T
        ( 0.6542E-02  J-0.1134E-12 ) E

                                                   ( 0.0000E 00  J 0.1000E-02 ) T
        (-0.6024E 00  J-0.5000E 03 ) E

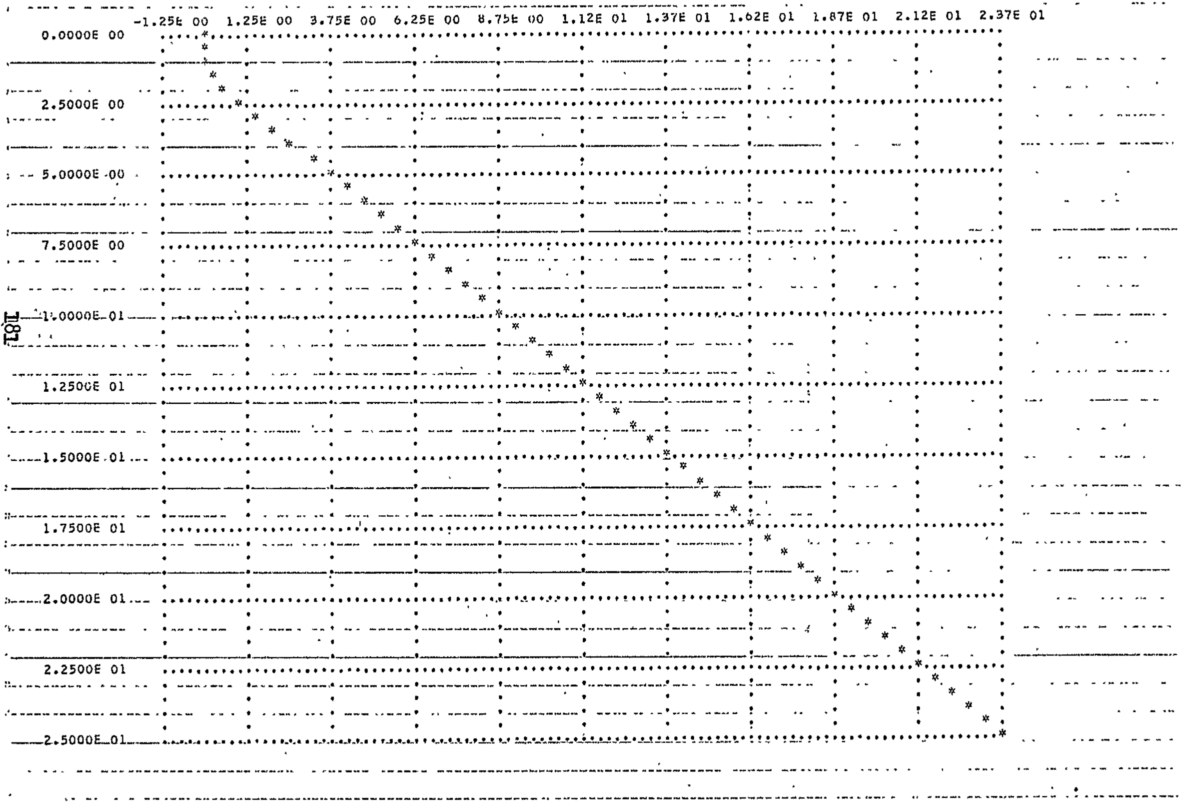
                                                   ( 0.0000E 00  J-0.1000E-02 ) T
        (-0.6024E 00  J 0.5000E 03 ) E
```

A69380

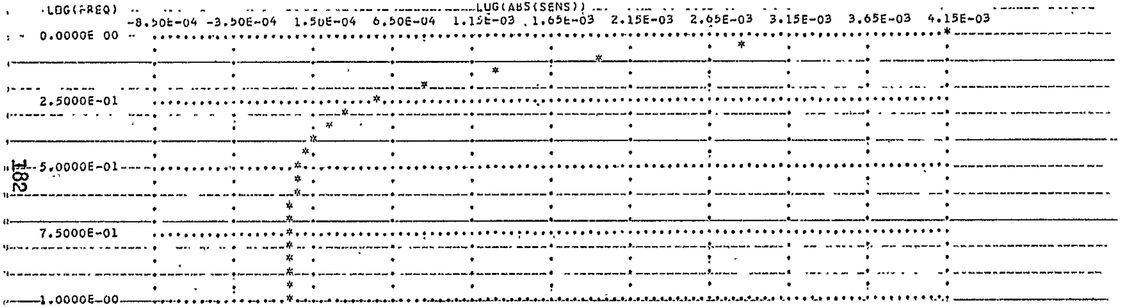
```
RAMP    RESPONSE

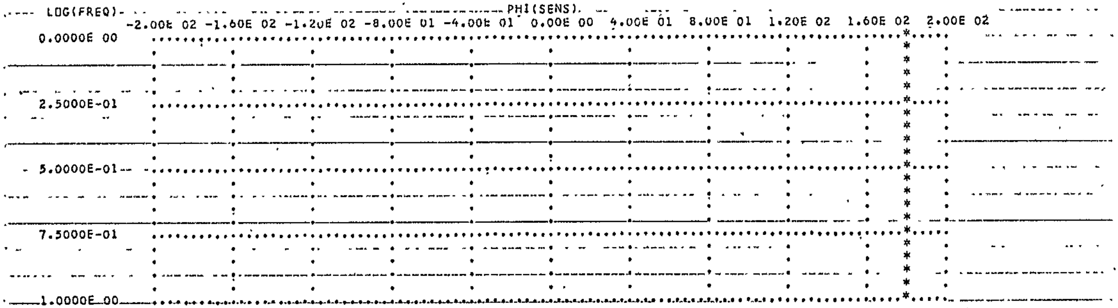
TIME                      VV4/VV5
 0.0000E 00           -0.18358231E-04
 0.5000E 00            0.63253045E-02
 0.1000E 01            0.65632403E-01
 0.1500E 01            0.22813344E 00
 0.2000E 01            0.51379776E 00
 0.2500E 01            0.91627377E 00
 0.3000E 01            0.14124537E 01
 0.3500E 01            0.19717751E 01
 0.4000E 01            0.25636339E 01
 0.4500E 01            0.31623487E 01
 0.5000E 01            0.37496891E 01
 0.5500E 01            0.43153048E 01
 0.6000E 01            0.48558273E 01
 0.6500E 01            0.53730450E 01
 0.7000E 01            0.58719320E 01
 0.7500E 01            0.63587666E 01
 0.8000E 01            0.68396826E 01
 0.8500E 01            0.73197384E 01
 0.9000E 01            0.78024302E 01
 0.9500E 01            0.82896910E 01
 0.1000E 02            0.87820425E 01
 0.1050E 02            0.92790327E 01
 0.1100E 02            0.97795935E 01
 0.1150E 02            0.10282449E 02
 0.1200E 02            0.10786363E 02
 0.1250E 02            0.11290349E 02
 0.1300E 02            0.11793727E 02
 0.1350E 02            0.12296152E 02
 0.1400E 02            0.12797535E 02
 0.1450E 02            0.13297997E 02
 0.1500E 02            0.13797764E 02
 0.1550E 02            0.14297088E 02
 0.1600E 02            0.14796228E 02
 0.1650E 02            0.15295361E 02
 0.1700E 02            0.15794614E 02
 0.1750E 02            0.16294052E 02
 0.1800E 02            0.16793701E 02
 0.1850E 02            0.17293533E 02
 0.1900E 02            0.17793468E 02
 0.1950E 02            0.18293533E 02
 0.2000E 02            0.18793594E 02
 0.2050E 02            0.19293655E 02
 0.2100E 02            0.19793671E 02
 0.2150E 02            0.20293640E 02
 0.2200E 02            0.20793579E 02
 0.2250E 02            0.21293457E 02
 0.2300E 02            0.21793320E 02
 0.2350E 02            0.22293152E 02
 0.2400E 02            0.22792984E 02
 0.2450E 02            0.23292801E 02
 0.2500E 02            0.23792618E 02
```

RAMP RESPONSE



```
      LOG(FREQ)          FREQ           ABS(SENS(H))    PHI(SENS(H))   LOG(ABS(SENS(H)))
   0.0000000E 00    0.1000000E 01    0.1009591E 01    0.1794954E 03    0.4145510E-02
   0.4999995E-01    0.1122018E 01    0.1006592E 01    0.1795993E 03    0.2853386E-02
   0.9999990E-01    0.1258925E 01    0.1004478E 01    0.1796878E 03    0.1940213E-02
   0.1499799E 00    0.1412537E 01    0.1003008E 01    0.1797606E 03    0.1304348E-02
   0.1999998E 00    0.1584892E 01    0.1001999E 01    0.1798189E 03    0.8672450E-03
   0.2499998E 00    0.1778278E 01    0.1001318E 01    0.1798646E 03    0.5720134E-03
   0.2999997E 00    0.1995261E 01    0.1000863E 01    0.1798998E 03    0.3746671E-03
   0.3499797E 00    0.2238719E 01    0.1000560E 01    0.1799265E 03    0.2430529E-03
   0.3999996E 00    0.2511885E 01    0.1000362E 01    0.1799463E 03    0.1573582E-03
   0.4499996E 00    0.2818380E 01    0.1000243E 01    0.1799611E 03    0.1010470E-03
   0.4999995E 00    0.3162274E 01    0.1000149E 01    0.1799720E 03    0.6460656E-04
   0.5499995E 00    0.3548130E 01    0.1000095E 01    0.1799798E 03    0.4141557E-04
   0.5999994E 00    0.3981067E 01    0.1000061E 01    0.1799856E 03    0.2650642E-04
   0.6499994E 00    0.4466829E 01    0.1000038E 01    0.1799897E 03    0.1656670E-04
   0.6999993E 00    0.5011865E 01    0.1000024E 01    0.1799927E 03    0.1035426E-04
   0.7499993E 00    0.5623405E 01    0.1000015E 01    0.1799948E 03    0.6626757E-05
   0.7999992E 00    0.6309563E 01    0.1000009E 01    0.1799963E 03    0.3727562E-05
   0.8499992E 00    0.7079445E 01    0.1000006E 01    0.1799974E 03    0.2485045E-05
   0.8999991E 00    0.7943267E 01    0.1000004E 01    0.1799981E 03    0.1656698E-05
   0.9499991E 00    0.8912491E 01    0.1000001E 01    0.1799987E 03    0.4141752E-06
   0.9999990E 00    0.9999978E 01    0.1000000E 01    0.1799991E 03    0.0000000E 00
```





```
   SENSITIVITIES OF ZEROES AND POLES OF TRANSFER FUNCTION

        ZERO         REAL            IMAG                        SENSITIVITY
                                                          REAL              IMAG
          1      0.0000000E 00   0.0000000E 00      0.9900000E 36   0.0000000E 00

        POLE         REAL            IMAG                        SENSITIVITY
                                                          REAL              IMAG
          1     -0.4046128E 00   0.7973596E 00     -0.4830204E 00   0.3556389E 00
          2     -0.4046128E 00  -0.7973596E 00     -0.4830204E 00  -0.3556389E 00
          3     -0.5190775E 01   0.6252776E-12     -0.3395869E-01  -0.2135725E-13
```

$-.0339(-5.19) = +.175$

$$\therefore \; S_{x}^{p_2} = -.175 = \frac{-x\,B(p_2)}{P'(p_2)}$$

where $P(p) = A(p) + x\,B(p)$

$$= \frac{-RSJ2 \cdot K_0}{(RDEN)'}$$

$(-.48+j.36)(-.40+j.80)$

$= +(.48)(.40) - (.36)(.80) + j\left[(-.40)(.36) + (-.48)(.80)\right]$

$= .192 - .288 - j\left[.144 + .384\right]$

$= -.096 - j.528$

$$\therefore \; S_{X}^{p_2} = .096 + j.528$$

```
SENSITIVITY ANALYSIS **

        LOG(FREQ)          FREQ          ABS(SENS(H))     PHI(SENS(H))    LOG(ABS(SENS(H)))
     -0.9999996E 00    0.1000001E 00    0.1002636E 01    0.1798199E 03    0.1143275E-02
     -0.9499997E 00    0.1122019E 00    0.1003289E 01    0.1797897E 03    0.1426147E-02
     -0.8999997E 00    0.1258926E 00    0.1004095E 01    0.1797527E 03    0.1774837E-02
     -0.8499998E 00    0.1412538E 00    0.1005084E 01    0.1797070E 03    0.2202375E-02
     -0.7999998E 00    0.1584894E 00    0.1006290E 01    0.1796500E 03    0.2723344E-02
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     -0.6500000E 00    0.2238721E 00    0.1011587E 01    0.1793740E 03    0.5003300E-02
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     -0.5500001E 00    0.2818382E 00    0.1016841E 01    0.1790480E 03    0.7253017E-02
     -0.5000001E 00    0.3162277E 00    0.1020034E 01    0.1788203E 03    0.8614574E-02
     -0.4500002E 00    0.3548133E 00    0.1023593E 01    0.1785374E 03    0.1012324E-01
     -0.4000002E 00    0.3981070E 00    0.1027440E 01    0.1781895E 03    0.1175650E-01
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     -0.2500004E 00    0.5623409E 00    0.1039975E 01    0.1766624E 03    0.1702297E-01
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      0.7499985E 00    0.5623393E 01    0.1046923E 01    0.1290575E 03    0.1991460E-01
      0.7999986E 00    0.6309554E 01    0.1057462E 01    0.1230376E 03    0.2426466E-01
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      0.1299997E 01    0.1995250E 02    0.2216687E 00   -0.4379515E 02   -0.6542953E 00
      0.1349998E 01    0.2238708E 02    0.1455541E 00   -0.5080815E 02   -0.8369633E 00
      0.1399998E 01    0.2511873E 02    0.9729779E-01   -0.5626796E 02   -0.1011896E 01
      0.1449997E 01    0.2818362E 02    0.6592953E-01   -0.6068202E 02   -0.1180920E 01
      0.1499997E 01    0.3162257E 02    0.4513260E-01   -0.6434824E 02   -0.1345509E 01
      0.1549997E 01    0.3548112E 02    0.3113472E-01   -0.6744966E 02   -0.1506754E 01
      0.1599998E 01    0.3981049E 02    0.2160375E-01   -0.7010991E 02   -0.1665470E 01
      0.1649997E 01    0.4466803E 02    0.1515705E-01   -0.7241505E 02   -0.1822259E 01
      0.1699997E 01    0.5011838E 02    0.1053014E-01   -0.7442323E 02   -0.1977565E 01
      0.1749997E 01    0.5623376E 02    0.7383704E-02   -0.7618335E 02   -0.2131725E 01
      0.1799997E 01    0.6309535E 02    0.5188081E-02   -0.7773132E 02   -0.2284992E 01
      0.1849997E 01    0.7079402E 02    0.3651155E-02   -0.7909932E 02   -0.2437569E 01
      0.1899997E 01    0.7943224E 02    0.2572703E-02   -0.8031140E 02   -0.2589610E 01
      0.1949997E 01    0.8912447E 02    0.1814584E-02   -0.8138557E 02   -0.2741222E 01
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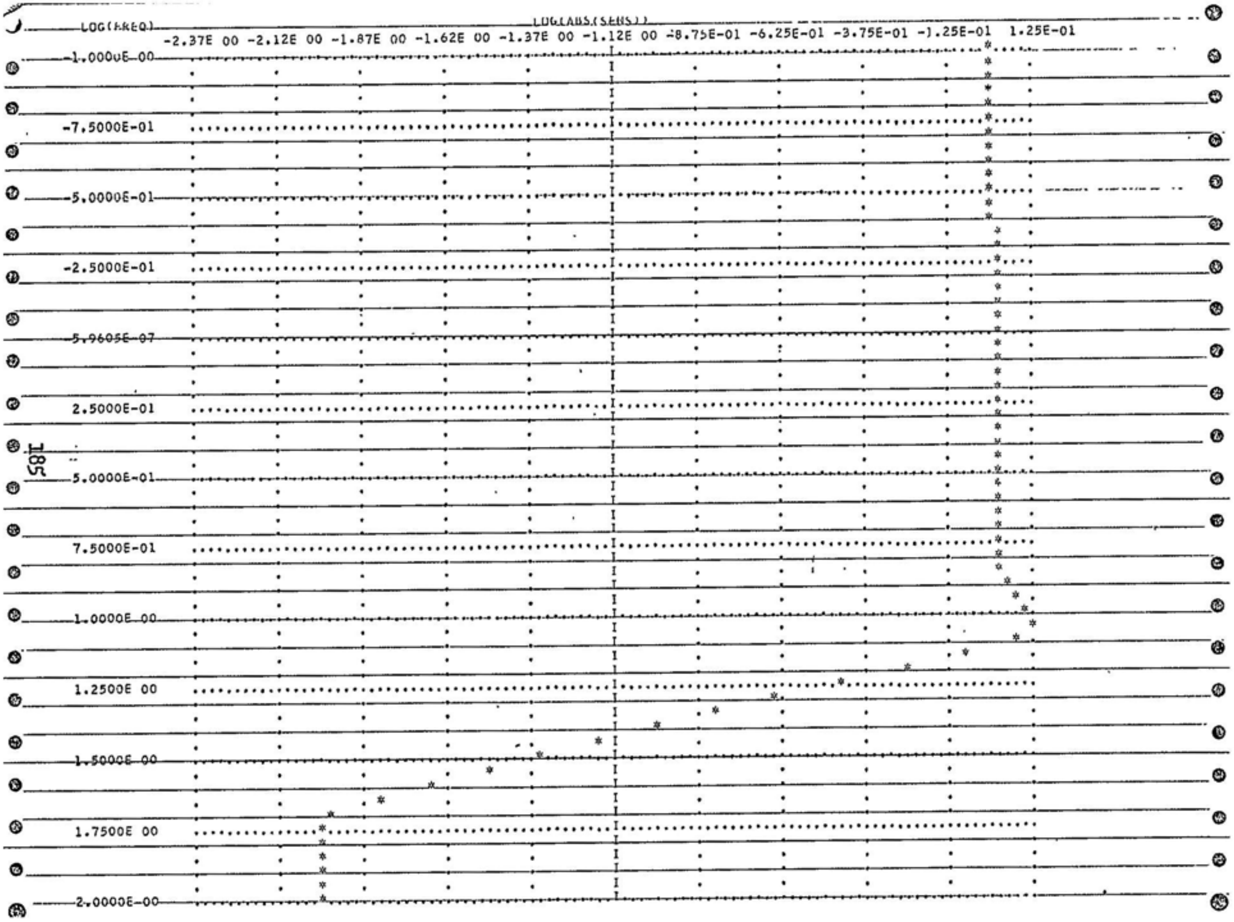
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-1.87E 00
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-8.75E-01
-6.25E-01
-3.75E-01
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1.25E-01
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
5.9605E-07
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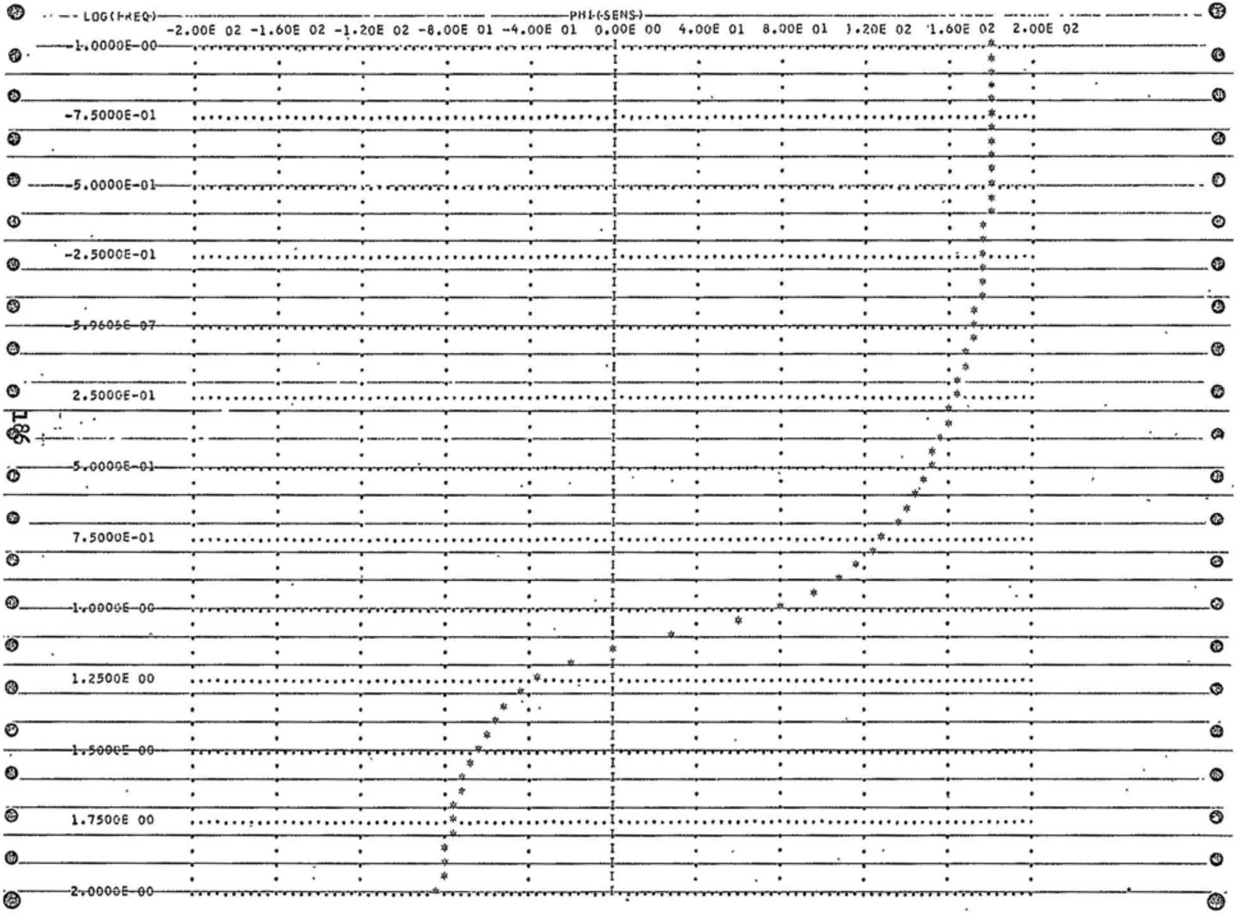
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PHI-(SENS)
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-1.60E 02
-1.20E 02
-8.00E 01
-4.00E 01
0.00E 00
4.00E 01
8.00E 01
1.20E 02
1.60E 02
2.00E 02
-1.0000E-00
-7.5000E-01
-5.0000E-01
-2.5000E-01
-5.9605E-07
2.5000E-01
5.0000E-01
7.5000E-01
1.0000E-00
1.2500E 00
1.5000E-00
1.7500E 00
2.0000E-00

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-0.9499997E 00  0.1122019E 00  -0.9991519E 00  -0.1006567E 01  -0.1003284E 01  0.1602272E-02
-0.8999997E 00  0.1258926E 00  -0.9994902E 00  -0.1008173E 01  -0.1004086E 01  0.1862695E-02
-0.8499998E 00  0.1412538E 00  -0.9998618E 00  -0.1010143E 01  -0.1005072E 01  0.2187954E-02
-0.7999998E 00  0.1584894E 00  -0.1000246E 01  -0.1012545E 01  -0.1006272E 01  0.2598320E-02
-0.7499999E 00  0.1778280E 00  -0.1000606E 01  -0.1015450E 01  -0.1007725E 01  0.3120501E-02
-0.6999999E 00  0.1995263E 00  -0.1000870E 01  -0.1018930E 01  -0.1009465E 01  0.3789346E-02
-0.6500000E 00  0.2238721E 00  -0.1000928E 01  -0.1023054E 01  -0.1011527E 01  0.4649825E-02
-0.6000000E 00  0.2511886E 00  -0.1000598E 01  -0.1027870E 01  -0.1013935E 01  0.5759098E-02
-0.5500001E 00  0.2818382E 00  -0.9996098E 00  -0.1033403E 01  -0.1016702E 01  0.7188428E-02
-0.5000001E 00  0.3162277E 00  -0.9975623E 00  -0.1039637E 01  -0.1019818E 01  0.9024482E-02
-0.4500002E 00  0.3548133E 00  -0.9948893E 00  -0.1046501E 01  -0.1023251E 01  0.1136992E-01
-0.4000002E 00  0.3981070E 00  -0.9877963E 00  -0.1053856E 01  -0.1026928E 01  0.1434250E-01
-0.3500003E 00  0.4466833E 00  -0.9782071E 00  -0.1061491E 01  -0.1030746E 01  0.1807306E-01
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-0.2500004E 00  0.5623409E 00  -0.9422776E 00  -0.1076424E 01  -0.1038212E 01  0.2837518E-01
-0.2000004E 00  0.6309568E 00  -0.9114126E 00  -0.1083002E 01  -0.1041501E 01  0.3524147E-01
-0.1500005E 00  0.7079450E 00  -0.8675125E 00  -0.1088455E 01  -0.1044228E 01  0.4345018E-01
-0.1000005E 00  0.7943273E 00  -0.8045043E 00  -0.1092379E 01  -0.1046189E 01  0.5315230E-01
-0.5000055E-01  0.8912498E 00  -0.7178245E 00  -0.1094366E 01  -0.1047183E 01  0.6450325E-01
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 0.9999986E-01  0.1258922E 01  -0.1300538E 00  -0.1084620E 01  -0.1042310E 01  0.1121919E 00
 0.1499990E 00  0.1412535E 01   0.3256893E 00  -0.1074612E 01  -0.1037306E 01  0.1299878E 00
 0.1999992E 00  0.1584890E 01   0.1165605E 01  -0.1060303E 01  -0.1030151E 01  0.1524829E 00
 0.2499985E 00  0.1778273E 01   0.3141147E 01  -0.1040985E 01  -0.1020493E 01  0.1779776E 00
 0.2999986E 00  0.1995256E 01   0.1274615E 02  -0.1015816E 01  -0.1007908E 01  0.2068093E 00
 0.3499988E 00  0.2238715E 01  -0.1797743E 02  -0.9838155E 00  -0.9919078E 00  0.2393481E 00
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 0.4499983E 00  0.2818372E 01  -0.5004240E 01  -0.8946664E 00  -0.9473332E 00  0.3171780E 00
 0.4999985E 00  0.3162267E 01  -0.4102077E 01  -0.8348843E 00  -0.9174420E 00  0.3633568E 00
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 0.5999988E 00  0.3981061E 01  -0.3245922E 01  -0.6775975E 00  -0.8387986E 00  0.4727807E 00
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 0.8499979E 00  0.7079423E 01  -0.2256956E 01   0.5313903E-01  -0.4734304E 00  0.9011121E 00
 0.8999981E 00  0.7943247E 01  -0.2042824E 01   0.3268102E 00  -0.3365945E 00  0.1050014E 01
 0.9499983E 00  0.8912474E 01  -0.1715013E 01   0.7281504E 00  -0.1359243E 00  0.1250772E 01
 0.9999985E 00  0.9999965E 01  -0.1116174E 01   0.1374160E 01   0.1870804E 00  0.1537979E 01
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 0.1199998E 01  0.1584886E 02   0.4879668E 00   0.2024986E 01   0.5124903E 00  0.1520636E 01
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 0.1599998E 01  0.3981049E 02   0.6940279E-02   0.1014702E 01   0.7349968E-02  0.1007489E 01
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 0.1699997E 01  0.5011838E 02   0.2725046E-02   0.1005658E 01   0.2827644E-02  0.1002865E 01
 0.1749997E 01  0.5623376E 02   0.1712024E-02   0.1003515E 01   0.1763344E-02  0.1001769E 01
 0.1799997E 01  0.6309535E 02   0.1076775E-02   0.1002202E 01   0.1102448E-02  0.1001109E 01
 0.1849997E 01  0.7079402E 02   0.6776147E-03   0.1001378E 01   0.6904602E-03  0.1000692E 01
 0.1899997E 01  0.7943224E 02   0.4265395E-03   0.1000850E 01   0.4329681E-03  0.1000420E 01
 0.1949997E 01  0.8912447E 02   0.2685790E-03   0.1000518E 01   0.2717972E-03  0.1000248E 01
 0.1999997E 01  0.9999934E 02   0.1690964E-03   0.1000353E 01   0.1707077E-03  0.1000182E 01
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   LOG(FREQ)          FREQ         LOG(SENS(RE(H)))  LOG(SENS(IM(H)))  LOG(SENS(ABS(H)))  LOG(SENS(PHI(H)))
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-0.9499997E 00   0.1122019E 00   -0.3684876E-03    0.2842688E-02    0.1423670E-02    -0.2795262E 01
-0.8999997E 00   0.1258926E 00   -0.2214591E-03    0.3535057E-02    0.1771124E-02    -0.2729857E 01
-0.8499998E 00   0.1412538E 00   -0.6003369E-04    0.4382975E-02    0.2197018E-02    -0.2659961E 01
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-0.7499999E 00   0.1778280E 00    0.2629217E-03    0.6658338E-02    0.3341928E-02    -0.2505774E 01
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-0.5500001E 00   0.2818382E 00   -0.1604826E-03    0.1426988E-01    0.7193547E-02    -0.2143366E 01
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-0.3500003E 00   0.4466833E 00   -0.9569179E-02    0.2591632E-01    0.1315144E-01    -0.1742968E 01
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-0.5000055E-01   0.8912498E 00   -0.1439817E 00    0.3916262E-01    0.2002259E-01    -0.1190417E 01
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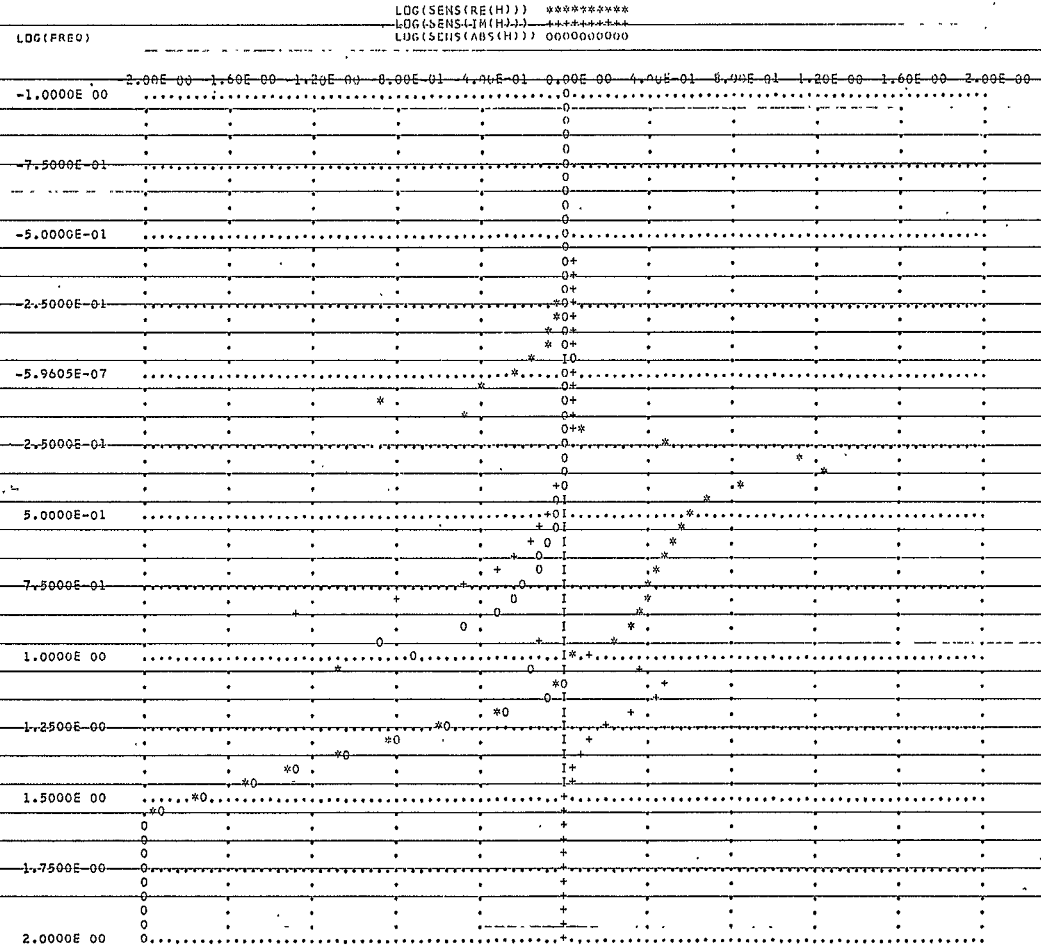
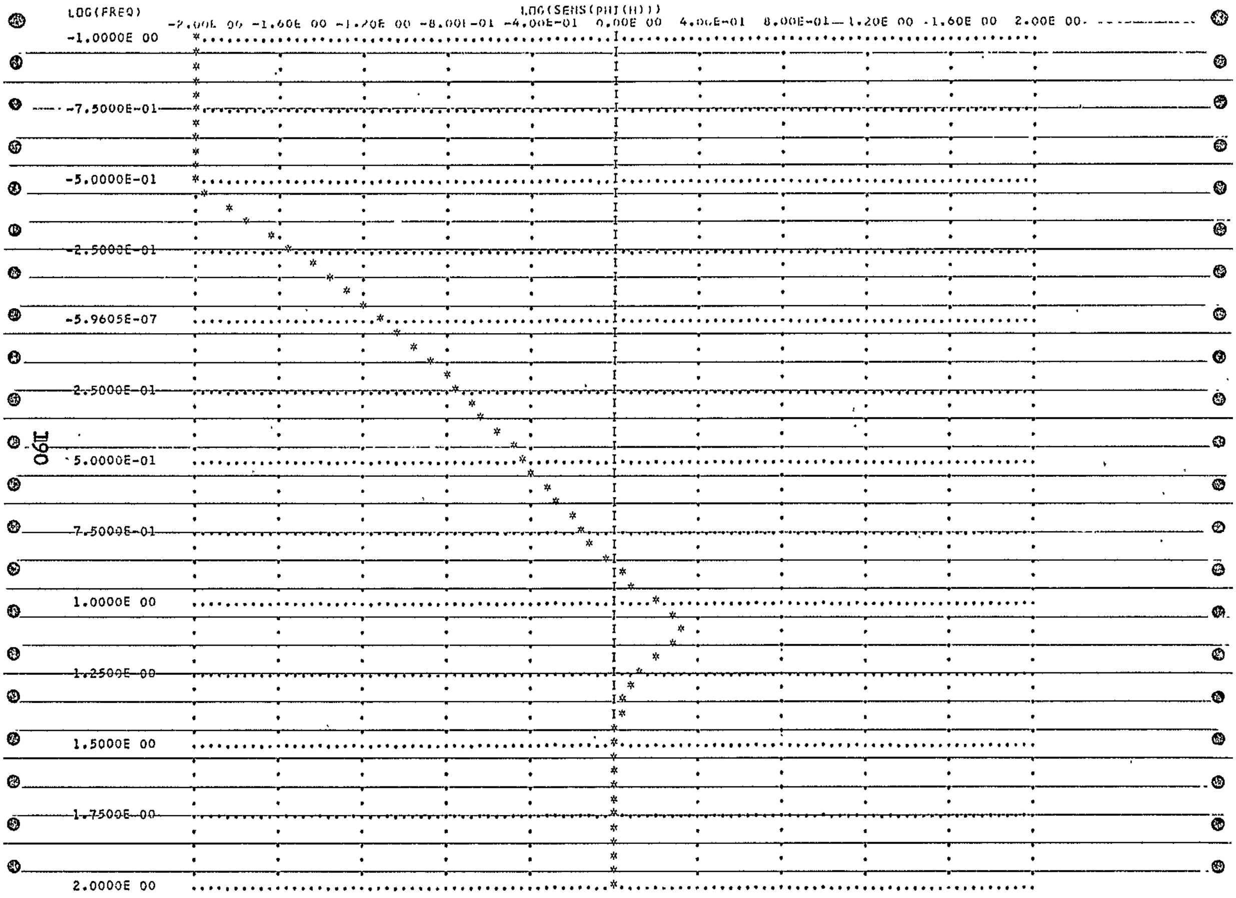
 0.3499988E 00   0.2238715E 01    0.1254726E 01   -0.7086340E-02   -0.3528704E-02    -0.6209695E 00
 0.3999990E 00   0.2511881E 01    0.6514544E 00   -0.2509601E-01   -0.1236676E-01    -0.5590988E 00
 0.4499983E 00   0.2818372E 01    0.6993377E 00   -0.4833883E-01   -0.2349723E-01    -0.4906966E 00
 0.4999985E 00   0.3162267E 01    0.6130037E 00   -0.7837367E-01   -0.3742135E-01    -0.4396663E 00
 0.5499986E 00   0.3548122E 01    0.5549371E 00   -0.1174498E 00   -0.5476660E-01    -0.3819197E 00
 0.5999988E 00   0.3981061E 01    0.5113377E 00   -0.1690282E 00   -0.7634223E-01    -0.3253402E 00
 0.6499981E 00   0.4466816E 01    0.4756963E 00   -0.2389607E 00   -0.1032481E 00    -0.2697343E 00
 0.6999983E 00   0.5011852E 01    0.4444343E 00   -0.3386427E 00   -0.1371180E 00    -0.2147442E 00
 0.7499985E 00   0.5623393E 01    0.4152318E 00   -0.4957593E 00   -0.1806765E 00    -0.1597232E 00
 0.7999986E 00   0.6309554E 01    0.3860010E 00   -0.8152141E 00   -0.2391886E 00    -0.1035349E 00
 0.8499979E 00   0.7079423E 01    0.3535231E 00   -0.1274586E 01   -0.3247438E 00    -0.4425835E-01
 0.8999981E 00   0.7943247E 01    0.3102309E 00   -0.4857039E 00   -0.4728927E 00     0.2119490E-01
 0.9499983E 00   0.8912474E 01    0.2342672E 00   -0.1377789E 00   -0.8667027E 00     0.9717792E-01
 0.9999985E 00   0.9999965E 01    0.4773170E-01    0.1380372E 00   -0.7279714E 00     0.1849504E 00
 0.1049997E 01   0.1122012E 02   -0.1071560E 01    0.3707036E 00   -0.1713285E 00     0.2792162E 00
 0.1099998E 01   0.1258918E 02   -0.5879128E-01    0.4881124E 00    0.1638329E-01     0.3206300E 00
 0.1149998E 01   0.1412530E 02   -0.6668890E-01    0.4334727E 00   -0.6668752E-01     0.2684587E 00
 0.1199998E 01   0.1584886E 02   -0.3116096E 00    0.3064271E 00   -0.2903143E 00     0.1820251E 00
 0.1249997E 01   0.1778268E 02   -0.5924771E 00    0.1958979E 00   -0.5451609E 00     0.1121846E 00
 0.1299997E 01   0.1995250E 02   -0.8547555E 00    0.1205763E 00   -0.7958671E 00     0.4697202E-01
 0.1349998E 01   0.2238708E 02   -0.1095142E 01    0.7333595E-01   -0.1036302E 01     0.3975111E-01
 0.1399998E 01   0.2511872E 02   -0.1320404E 01    0.4456249E-01   -0.1267361E 01     0.2370343E-01
 0.1449997E 01   0.2818362E 02   -0.1536381E 01    0.2717089E-01   -0.1491028E 01     0.1424543E-01
 0.1499997E 01   0.3162257E 02   -0.1746797E 01    0.1664566E-01   -0.1709121E 01     0.8632034E-02
 0.1549997E 01   0.3548112E 02   -0.1953779E 01    0.1024825E-01   -0.1922995E 01     0.5270984E-02
 0.1599998E 01   0.3981049E 02   -0.2158623E 01    0.6338447E-02   -0.2133714E 01     0.3240400E-02
 0.1649997E 01   0.4466803E 02   -0.2362116E 01    0.3930081E-02   -0.2342081E 01     0.1997523E-02
 0.1699997E 01   0.5011838E 02   -0.2564626E 01    0.2450377E-02   -0.2548574E 01     0.1242404E-02
 0.1749997E 01   0.5623376E 02   -0.2766489E 01    0.1523974E-02   -0.2753662E 01     0.7676166E-03
 0.1799997E 01   0.6309535E 02   -0.2967875E 01    0.9552797E-03   -0.2957641E 01     0.4814190E-03
 0.1849997E 01   0.7079402E 02   -0.3169017E 01    0.5980714E-03   -0.3160860E 01     0.3005872E-03
 0.1899997E 01   0.7943224E 02   -0.3370044E 01    0.3688736E-03   -0.3363544E 01     0.1821989E-03
 0.1949997E 01   0.8912447E 02   -0.3570978E 01    0.2248390E-03   -0.3565754E 01     0.1076722E-03
 0.1999997E 01   0.9999934E 02   -0.3771865E 01    0.1532178E-03   -0.3767746E 01     0.7910030E-04
```

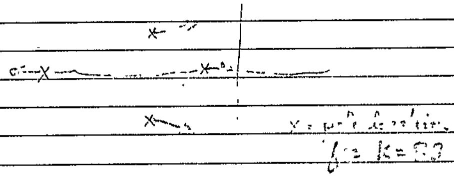

LOG(SENS(RE(H))) **********
LOG(SENS(IM(H))) ++++++++++
LOG(SENS(ABS(H))) 0000000000
LOG(FREQ)
-2.00E 00
-1.60E 00
-1.20E 00
-8.00E-01
-4.00E-01
0.00E 00
4.00E-01
8.00E-01
1.20E 00
1.60E 00
2.00E 00
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
-5.9605E-07
2.5000E-01
5.0000E-01
7.5000E-01
1.0000E 00
1.2500E 00
1.5000E 00
1.7500E 00
2.0000E 00


LOG(FREQ)
LOG(SENS(PHI(H)))
-2.00E 00 -1.60E 00 -1.20E 00 -8.00E-01 -4.00E-01 0.00E 00 4.00E-01 8.00E-01 1.20E 00 1.60E 00 2.00E 00
-1.0000E 00
-7.5000E-01
-5.0000E-01
-2.5000E-01
-5.9605E-07
2.5000E-01
5.0000E-01
7.5000E-01
1.0000E 00
1.2500E 00
1.5000E 00
1.7500E 00
2.0000E 00

```
SENSITIVITIES OF ZEROES AND POLES OF TRANSFER FUNCTION

   ZERO         REAL             IMAG                              SENSITIVITY
                                                       REAL           IMAG
     1     0.1000000E-29     0.0000000E 00        0.0000000E 00   0.0000000E 00
     2    -0.8108258E-01     0.0000000E 00        0.0000000E 00   0.0000000E 00
     3    -0.6876126E 01     0.1258056E 02        0.0000000E 00   0.0000000E 00
     4    -0.6876126E 01    -0.1258056E 02        0.0000000E 00   0.0000000E 00

   POLE         REAL             IMAG                              SENSITIVITY
                                                       REAL           IMAG
     1    -0.5386047E 00     0.0000000E 00        0.4560962E-01   0.0000000E 00
     2    -0.2880999E 01     0.1215740E 02        0.3735610E 01   0.1442457E 01
     3    -0.2880999E 01    -0.1215740E 02        0.3735610E 01  -0.1442457E 01
     4    -0.7532733E 01     0.0000000E 00       -0.7516842E 01   0.0000000E 00
```



```
NASAP PROBLE  RANGER ATTITUDE CONTROL

NONE
STEP RESPONSE

V1 1 2 1. I..
R1 2 3 -1.
R2 3 4 1.
R3 4 1 1.
C1 4 1   F
V2 1 5 6. I..
L1 5 1 1.
V3 1 6  .0.67 IL1
L2 6 1 1.
V4 1 7 1. I
R4 8 7 1.
V5 1 8 1.
OUTPUT
VV4/VV5/.2
FREQ 0.  1.  ...
TIME 5..
EXECUTE


TRANSFER FUNCTION  VV4/VV5/.2


                       (                                     )
                       (  5.00E-01     +1.00E 00 S  )
   H(S)= 0.04...
                       (                                    2 )
                       (  4.00E-02     +8.00E-02 S   +1.00E 00 S   )


ZERO OF TRANSFER FUNCTION

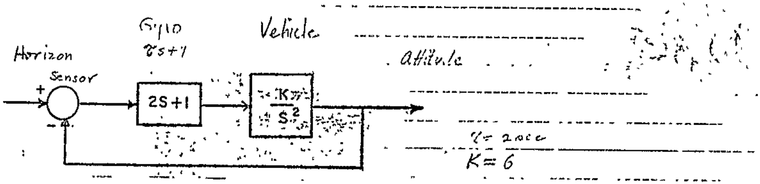
ZERO   REAL PART    IMAG. PART

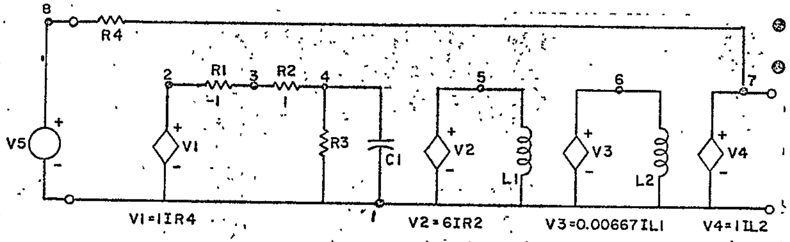
  1 -0.50000E 00  0.00000E 00


POLE OF TRANSFER FUNCTION

POLE   REAL PART    IMAG. PART

  1 -0.4002 E-01  0.19616E 00
  2 -0.4002 E-01 -0.19616E  )
```





```
      STEP    RESPONSE FUNCTION

      F(T) =

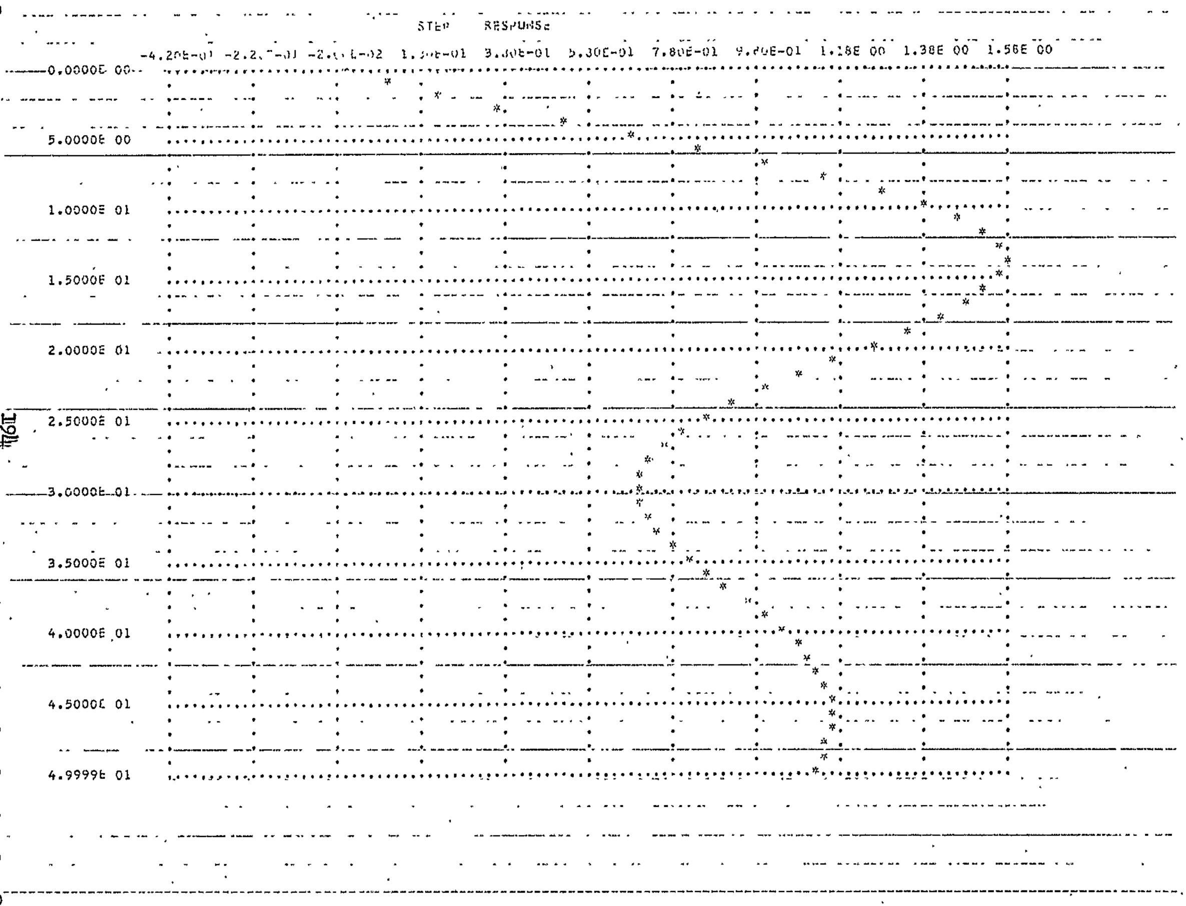
                                                 (-0.4002E-01   J 0.1960E 00 ) T
            (-0.5000E 00   J-0.1021E 00 ) E

                                                 (-0.4002E-01   J-0.1960E 00 ) T
            (-0.5000E 00   J 0.1021E 00 ) E

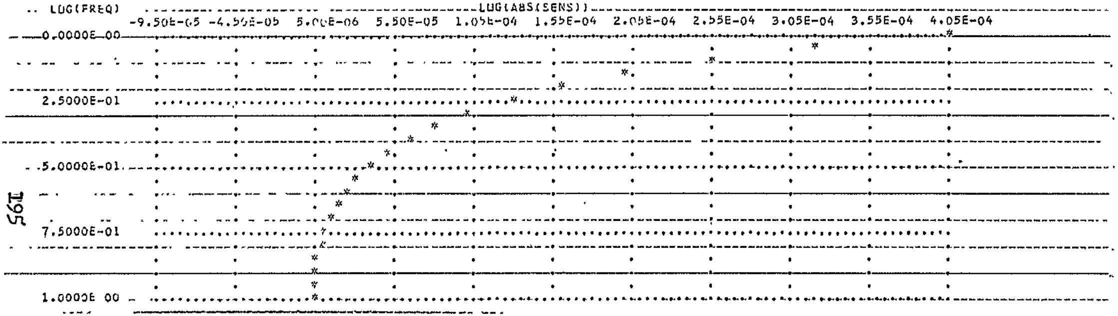
                                                 ( 0.0000E 00   J 0.0000E 00 ) T
            (-0.1000E-01   J 0.0000E 00 ) E
```

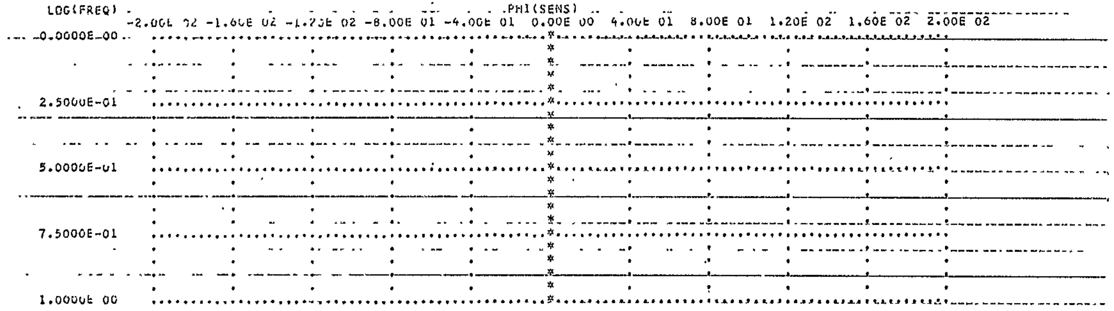
```
      STEP    RESPONSE

      TIME                      /V4/VV5
       0.0000E 00              0.95367432E-06
       0.1000E 01              0.75429010E-01
       0.2000E 01              0.21494611E 00
       0.3000E 01              0.36253381E 00
       0.4000E 01              0.51051136E 00
       0.5000E 01              0.68383224E 00
       0.6000E 01              0.84574419E 00
       0.7000E 01              0.10020437E 01
       0.8000E 01              0.11462402E 01
       0.9000E 01              0.12737398E 01
       0.1000E 02              0.13209385E 01
       0.1100E 02              0.14652491E 01
       0.1200E 02              0.15253162E 01
       0.1300E 02              0.15505898E 01
       0.1400E 02              0.15716610E 01
       0.1500E 02              0.15599680E 01
       0.1600E 02              0.15277090E 01
       0.1700E 02              0.14777040E 01
       0.1800E 02              0.14132347E 01
       0.1900E 02              0.13378849E 01
       0.2000E 02              0.12553797E 01
       0.2100E 02              0.11694307E 01
       0.2200E 02              0.10835991E 01
       0.2300E 02              0.10011554E 01
       0.2400E 02              0.92502826E 00
       0.2500E 02              0.85762044E 00
       0.2600E 02              0.80085526E 00
       0.2700E 02              0.75508769E 00
       0.2800E 02              0.72411668E 00
       0.2900E 02              0.70317802E 00
       0.3000E 02              0.69399702E 00
       0.3100E 02              0.70482910E 00
       0.3200E 02              0.72152626E 00
       0.3300E 02              0.74761463E 00
       0.3400E 02              0.78136921E 00
       0.3500E 02              0.82190807E 00
       0.3600E 02              0.86427345E 00
       0.3700E 02              0.90950012E 00
       0.3800E 02              0.95371978E 00
       0.3900E 02              0.99419646E 00
       0.4000E 02              0.10393974E 01
       0.4100E 02              0.10740356E 01
       0.4200E 02              0.11040945E 01
       0.4300E 02              0.11276535E 01
       0.4400E 02              0.11448723E 01
       0.4500E 02              0.11556446E 01
       0.4600E 02              0.11584845E 01
       0.4700E 02              0.11555953E 01
       0.4800E 02              0.11469469E 01
       0.4900E 02              0.11333330E 01
       0.5000E 02              0.11156654E 01
```


STEP RESPONSE
-4.20E-01
-2.2.E-01
-2.E-02
1.30E-01
3.30E-01
5.30E-01
7.80E-01
9.80E-01
1.18E 00
1.38E 00
1.58E 00
0.0000E 00
5.0000E 00
1.0000E 01
1.5000E 01
2.0000E 01
2.5000E 01
3.0000E 01
3.5000E 01
4.0000E 01
4.5000E 01
4.9999E 01

```
   LOG(FREQ)          FREQ            ABS(SENS(H))     PHI(SENS(H))    LOG(ABS(SENS(H)))
-0.0000000E 00    0.1000000E 01    0.1000934E 01    0.7305771E 00    0.4052085E-03
 0.4999995E-01    0.1122018E 01    0.1000741E 01    0.6509995E 00    0.3216951E-03
 0.9999997E-01    0.1258925E 01    0.1000587E 01    0.5801130E 00    0.2550571E-03
 0.1499999E 00    0.1412537E 01    0.1000466E 01    0.5169612E 00    0.2024346E-03
 0.1999998E 00    0.1584892E 01    0.1000370E 01    0.4606965E 00    0.1606703E-03
 0.2499999E 00    0.1778279E 01    0.1000294E 01    0.4105638E 00    0.1275472E-03
 0.2999997E 00    0.1995261E 01    0.1000234E 01    0.3658924E 00    0.1014611E-03
 0.3499997E 00    0.2238719E 01    0.1000186E 01    0.3260856E 00    0.8075670E-04
 0.3999996E 00    0.2511885E 01    0.1000147E 01    0.2906126E 00    0.6377832E-04
 0.4499996E 00    0.2818380E 01    0.1000116E 01    0.2590013E 00    0.5052646E-04
 0.4999995E 00    0.3162274E 01    0.1000093E 01    0.2308295E 00    0.4017315E-04
 0.5499995E 00    0.3548130E 01    0.1000072E 01    0.2057230E 00    0.3147619E-04
 0.5999994E 00    0.3981067E 01    0.1000058E 01    0.1833479E 00    0.2526396E-04
 0.6499994E 00    0.4466829E 01    0.1000046E 01    0.1634070E 00    0.1987995E-04
 0.6999993E 00    0.5011865E 01    0.1000035E 01    0.1456352E 00    0.1532421E-04
 0.7499993E 00    0.5623405E 01    0.1000029E 01    0.1297964E 00    0.1242508E-04
 0.7999992E 00    0.6309563E 01    0.1000023E 01    0.1156805E 00    0.9940097E-05
 0.8499992E 00    0.7079445E 01    0.1000017E 01    0.1030998E 00    0.7455089E-05
 0.8999991E 00    0.7943267E 01    0.1000014E 01    0.9188730E-01    0.6212587E-05
 0.9499991E 00    0.8912491E 01    0.1000011E 01    0.8189446E-01    0.4970077E-05
 0.9999990E 00    0.9999978E 01    0.1000009E 01    0.7298833E-01    0.3727562E-05
```





```
   SENSITIVITY ANALYSIS

            LOG(FREQ)          FREQ          SENS(RE(H))     SENS(IM(H))     SENS(ABS(H))    SENS(PHI(H))
        0.0000000E 00    0.1000000E 01    0.8096766E 00    0.1001704E 01    0.1000853E 01    -0.7794116E-02
        0.4999995E-01    0.1122018E 01    0.8096220E 00    0.1001353E 01    0.1000677E 01    -0.6974593E-02
        0.9999990E-01    0.1258925E 01    0.8095778E 00    0.1001074E 01    0.1000537E 01    -0.6238859E-02
        0.1499999E 00    0.1412537E 01    0.8095437E 00    0.1000853E 01    0.1000426E 01    -0.5578786E-02
        0.1999998E 00    0.1584892F 01    0.8095166E 00    0.1000677E 01    0.1000339E 01    -0.4986938E-02
        0.2499998E 00    0.1778278E 01    0.8094950E 00    0.1000538E 01    0.1000269E 01    -0.4456572E-02
        0.2999997E 00    0.1995261E 01    0.8094779E 00    0.1000427E 01    0.1000214E 01    -0.3981538E-02
        0.3499997E 00    0.2238719E 01    0.8094643E 00    0.1000340E 01    0.1000170E 01    -0.3556268E-02
        0.3999996E 00    0.2511885E 01    0.8094532E 00    0.1000269E 01    0.1000134E 01    -0.3175717E-02
        0.4499996E 00    0.2818380E 01    0.8094442E 00    0.1000214E 01    0.1000107E 01    -0.2835326E-02
        0.4999995E 00    0.3162274E 01    0.8094374E 00    0.1000170E 01    0.1000085E 01    -0.2530954E-02
        0.5499995E 00    0.3548130E 01    0.8094318E 00    0.1000134E 01    0.1000067E 01    -0.2258887E-02
        0.5999994E 00    0.3981067E 01    0.8094273E 00    0.1000107E 01    0.1000053E 01    -0.2015772E-02
        0.6499994E 00    0.4466829E 01    0.8094236E 00    0.1000084E 01    0.1000042E 01    -0.1798581E-02
        0.6999993E 00    0.5011865F 01    0.8094202E 00    0.1000066E 01    0.1000032E 01    -0.1604602E-02
        0.7499993E 00    0.5623405E 01    0.8094192E 00    0.1000053E 01    0.1000027E 01    -0.1431390E-02
        0.7999992E 00    0.6309563F 01    0.8094174E 00    0.1000042E 01    0.1000021E 01    -0.1276756E-02
        0.8499992E 00    0.7079445E 01    0.8094158E 00    0.1000032E 01    0.1000016E 01    -0.1138728E-02
        0.8999991E 00    0.7943267E 01    0.8094155E 00    0.1000027E 01    0.1000013E 01    -0.1015543E-02
        0.9499991E 00    0.8912491E 01    0.8094145E 00    0.1000021E 01    0.1000010E 01    -0.9056237E-03
        0.9999990E 00    0.9999978E 01    0.8094139E 00    0.1000016E 01    0.1000009E 01    -0.8075521E-03
```

```
          LOG(FREQ)          FREQ      LOG(SENS(RE(H))) LOG(SENS(IM(H))) LOG(SENS(ABS(H))) LOG(SENS(PHI(H)))
        0.0000000E 00    0.1000000E 01    0.9168923E-01   -0.7340078E-03   -0.3692873E-03    0.2108232E 01
        0.4999995E-01    0.1122012E 01    0.9171836E-01   -0.5873174E-03   -0.2935510E-03    0.2156479E 01
        0.9999990E-01    0.1258925E 01    0.9174126E-01   -0.4665251E-03   -0.2331182E-03    0.2204894E 01
        0.1499999E 00    0.1412537F 01    0.9175950E-01   -0.3705258E-03   -0.1855110E-03    0.2253459E 01
        0.1999998E 00    0.1584892E 01    0.9177482E-01   -0.2943790E-03   -0.1470073E-03    0.2302161E 01
        0.2499998E 00    0.1778278E 01    0.9178674E-01   -0.2331182E-03   -0.1159536E-03    0.2350997E 01
        0.2999997E 00    0.1995261E 01    0.9179610E-01   -0.1355110E-03   -0.9276539E-04    0.2399944E 01
        0.3499997E 00    0.2238719E 01    0.9180319E-01   -0.1474214E-03   -0.7330286E-04    0.2449001E 01
        0.3999996E 00    0.2511885E 01    0.9180925E-01   -0.1171958E-03   -0.5880892E-04    0.2498152E 01
        0.4499996E 00    0.2818380E 01    0.9181397E-01   -0.9276539E-04   -0.4597103E-04    0.2547393F 01
        0.4999995E 00    0.3162274E 01    0.9181786E-01   -0.7330266E-04   -0.3644591E-04    0.2596712E 01
        0.5499995E 00    0.3548130E 01    0.9182161E-01   -0.5880892E-04   -0.2940545E-04    0.2646095E 01
        0.5999994E 00    0.3981067E 01    0.9182417E-01   -0.4679929E-04   -0.2319321E-04    0.2695548E 01
        0.6499994E 00    0.4466829E 01    0.9182578E-01   -0.3686004E-04   -0.1822333E-04    0.2745061E 01
        0.6999993E 00    0.5011865E 01    0.9182751E-01   -0.2899130E-04   -0.1408173E-04    0.2794624E 01
        0.7499993E 00    0.5623405E 01    0.9182882E-01   -0.2279904E-04   -0.1076842E-04    0.2844234E 01
        0.7999992E 00    0.6309563E 01    0.9183055E-01   -0.1822333E-04   -0.8697596E-05    0.2893879E 01
        0.8499992E 00    0.7079445F 01    0.9183174E-01   -0.1449589E-04   -0.7040920E-05    0.2943563E 01
        0.8999991E 00    0.7943267E 01    0.9183264E-01   -0.1118259E-04   -0.4970077E-05    0.2993285E 01
        0.9499991E 00    0.8912491E 01    0.9183282E-01   -0.9111759E-05   -0.4555905E-05    0.3043035E 01
        0.9999990E 00    0.9999978E 01    0.9183419E-01   -0.7455089E-05   -0.3313391E-05    0.3092807E 01
```

```
 SENSITIVITIES OF ZEROES AND POLES OF TRANSFER FUNCTION

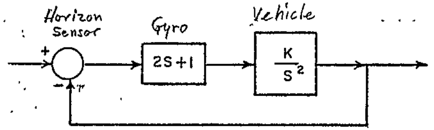
        ZERO       REAL             IMAG                            SENSITIVITY
                                                       REAL             IMAG
          1     -0.5000000E 00   0.0000000E 00    0.0000000E 00   0.0000000E 00

        POLE       REAL             IMAG                            SENSITIVITY
                                                       REAL             IMAG
          1     -0.4001990E-01   0.1960058E 00   -0.4001988E-01   0.9391737E-01
          2     -0.4001990E-01  -0.1960058E 00   -0.4001988E-01  -0.9391737E-01
```

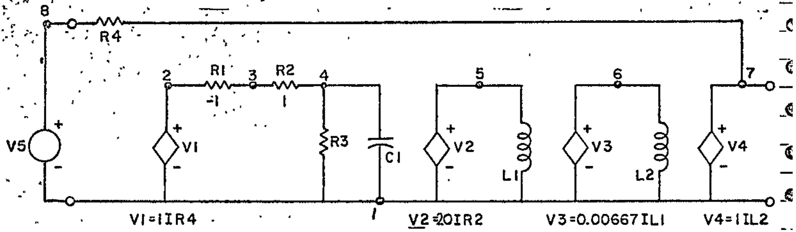
```
NASAP PROBLEM RANGER ATTITUDE CONTROL

NONE
STEP RESPONSE
```

K=20



```
V1 1 2 1. IR4
R1 2 3  -1.
R2 3 4 1.
R3 4 1 1.
C1 4 1  2F
V2 1 5 20 IR2
L1 5 1 1H
V3 1 6  0.00667  IL1
L2 6 1 1H
V4 1 7 1. IL2
R4 8 7 1.
V5 1 8 1.
OUTPUT
VV4/VV5/V2
FREQ 0.0 1.0 0.05
TIME 50.
EXECUTE
```



```
TRANSFER FUNCTION   VV4/VV5/V2


                        (                                   )
                        (   5.00E-01      +1.00E 00 S       )
H(S)= 2.668E-01*----------------------------------------------
                        (                               2 )
                        (   1.33E-01      +2.67E-01 S     +1.00E 00 S   )

                                                  NUMBER OF LOOPS PER ORDER

                                                      1=     5
                                                      2=     3

  ZERO OF TRANSFER FUNCTION

  ZERO    REAL PART    IMAG. PART

    1 -0.50000E 00   0.00000E 00


  POLE OF TRANSFER FUNCTION

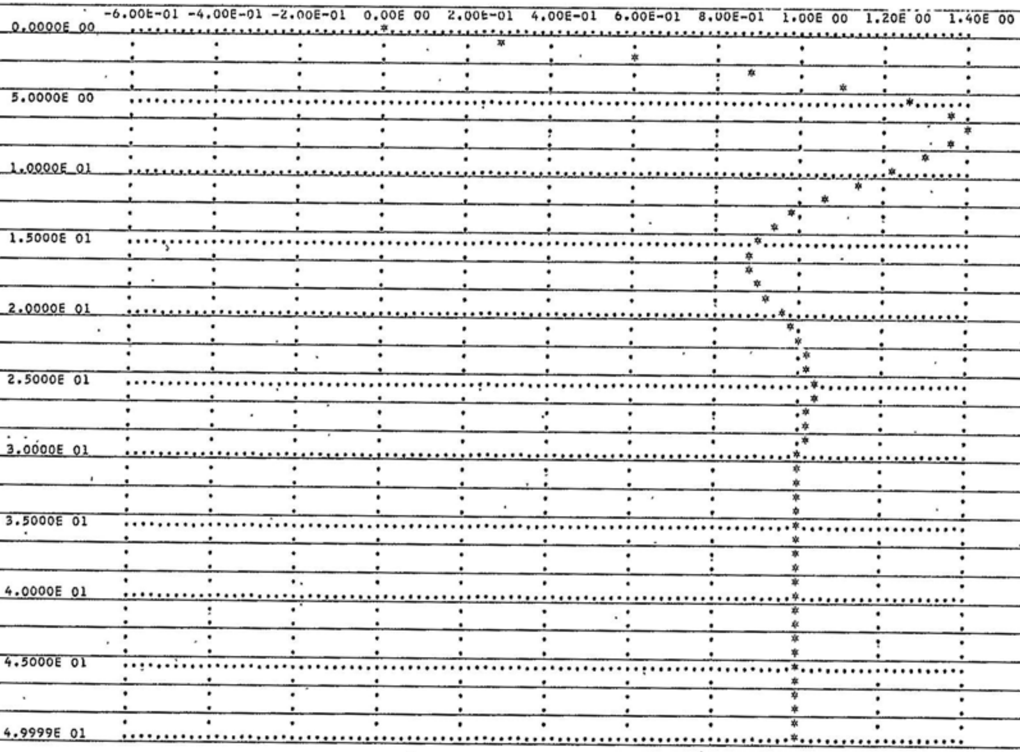
  POLE    REAL PART    IMAG. PART

    1 -0.13340E 00   0.34001E 00
    2 -0.13340E 00  -0.34001E 00
```

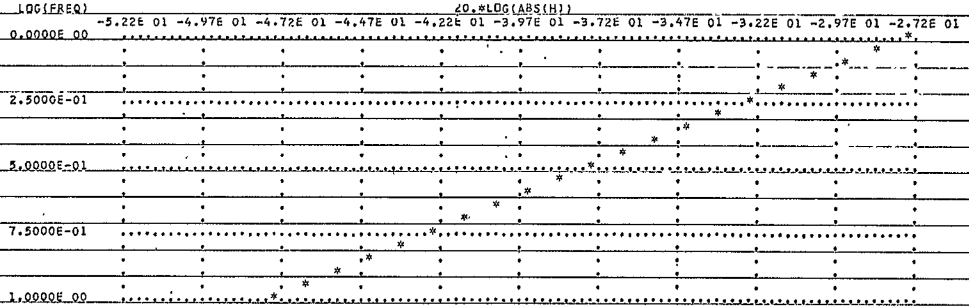
```
  STEP    RESPONSE FUNCTION                                        STEP    RESPONSE

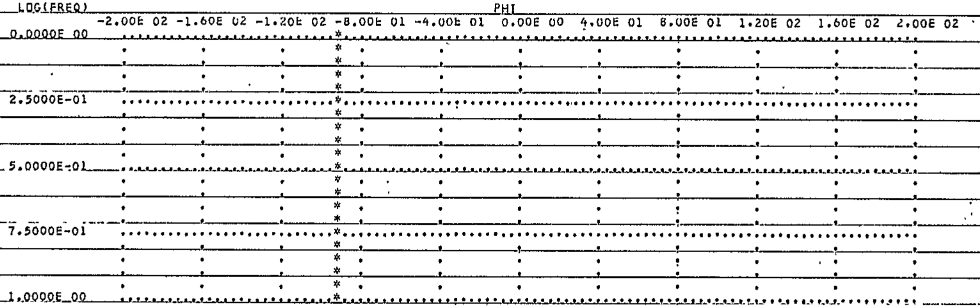
  F(T) *                                                           TIME               VV4/VV5
                                                                    0.0000E 00        -0.59604645E-07
                                 (-0.1334E 00  J 0.3400E 00 ) T     0.1000E 01         0.28948647E 00
        (-0.5000E 00  J-0.1962E 00 ) E                              0.2000E 01         0.59345466E 00
                                                                    0.3000E 01         0.87331635E 00
                                 (-0.1334E 00  J-0.3400E 00 ) T     0.4000E 01         0.11023102E 01
        (-0.5000E 00  J 0.1962E 00 ) E                              0.5000E 01         0.12658359E 01
                                                                    0.6000E 01         0.13602829E 01
                                 ( 0.0000E 00  J 0.0000E 00 ) T     0.7000E 01         0.13908958E 01
        ( 0.1000E 01  J 0.0000E 00 ) E                              0.8000E 01         0.13690777E 01
                                                                    0.9000E 01         0.13096333E 01
                                                                    0.1000E 02         0.12282543E 01
                                                                    0.1100E 02         0.11395025E 01
                                                                    0.1200E 02         0.10553808E 01
                                                                    0.1300E 02         0.98454595E 00
                                                                    0.1400E 02         0.93208754E 00
                                                                    0.1500E 02         0.89977700E 00
                                                                    0.1600E 02         0.88663769E 00
                                                                    0.1700E 02         0.88970166E 00
                                                                    0.1800E 02         0.90481949E 00
                                                                    0.1900E 02         0.92741799E 00
                                                                    0.2000E 02         0.95312858E 00
                                                                    0.2100E 02         0.97824574E 00
                                                                    0.2200E 02         0.10000000E 01
                                                                    0.2300E 02         0.10166597E 01
                                                                    0.2400E 02         0.10274897E 01
                                                                    0.2500E 02         0.10325994E 01
                                                                    0.2600E 02         0.10327387E 01
                                                                    0.2700E 02         0.10290546E 01
                                                                    0.2800E 02         0.10228682E 01
                                                                    0.2900E 02         0.10154827E 01
                                                                    0.3000E 02         0.10080338E 01
                                                                    0.3100E 02         0.10013990E 01
                                                                    0.3200E 02         0.99615616E 00
                                                                    0.3300E 02         0.99258560E 00
                                                                    0.3400E 02         0.99070960E 00
                                                                    0.3500E 02         0.99034858E 00
                                                                    0.3600E 02         0.99118954E 00
                                                                    0.3700E 02         0.99285364E 00
                                                                    0.3800E 02         0.99495542E 00
                                                                    0.3900E 02         0.99714911E 00
                                                                    0.4000E 02         0.99915910E 00
                                                                    0.4100E 02         0.10007954E 01
                                                                    0.4200E 02         0.10019560E 01
                                                                    0.4300E 02         0.10026188E 01
                                                                    0.4400E 02         0.10028229E 01
                                                                    0.4500E 02         0.10026522E 01
                                                                    0.4600E 02         0.10022144E 01
                                                                    0.4700E 02         0.10016222E 01
                                                                    0.4800E 02         0.10009813E 01
                                                                    0.4900E 02         0.10003757E 01
                                                                    0.5000E 02         0.99986970E 00
```

STEP RESPONSE



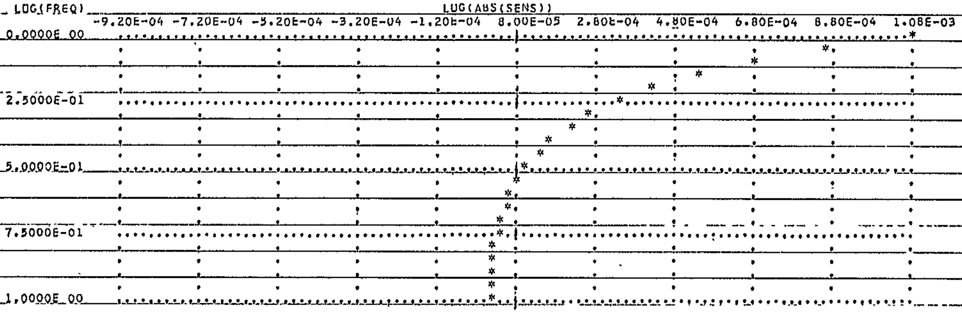
```
     LOG(FREQ)          FREQ          20.*LOG(ABS(H))      PHI(H)           ABS(H)         LOG(ABS(H))
   0.0000000E 00     0.1000000E 01    -0.2739093E 02    -0.9211026E 02    0.4270237E-01    -0.1369547E 01
   0.4999995E-01     0.1122018E 01    -0.2840099E 02    -0.9188379E 02    0.3801452E-01    -0.1420050E 01
   0.9999990E-01     0.1258925E 01    -0.2940898E 02    -0.9168105E 02    0.3384930E-01    -0.1470449E 01
   0.1499999E 00     0.1412537E 01    -0.3041531E 02    -0.9149976E 02    0.3014620E-01    -0.1520766E 01
   0.1999998E 00     0.1584892E 01    -0.3142038E 02    -0.9133772E 02    0.2685222E-01    -0.1571019E 01
   0.2499998E 00     0.1778278E 01    -0.3242439E 02    -0.9119301E 02    0.2392103E-01    -0.1621220E 01
   0.2999997E 00     0.1995761E 01    -0.3342758E 02    -0.9106381E 02    0.2131181E-01    -0.1671379E 01
   0.3499997E 00     0.2238719E 01    -0.3443010E 02    -0.9094853E 02    0.1898864E-01    -0.1721505E 01
   0.3999996E 00     0.2511885E 01    -0.3543213E 02    -0.9084564E 02    0.1691972E-01    -0.1771606E 01
   0.4499996E 00     0.2818380E 01    -0.3643369E 02    -0.9075386E 02    0.1507698E-01    -0.1821685E 01
   0.4999995E 00     0.3162274E 01    -0.3743497E 02    -0.9067201E 02    0.1343540E-01    -0.1871749E 01
   0.5499995E 00     0.3548130E 01    -0.3843597E 02    -0.9059903E 02    0.1197292E-01    -0.1921799E 01
   0.5999994E 00     0.3981067E 01    -0.3943677E 02    -0.9053398E 02    0.1066990E-01    -0.1971839E 01
   0.6499994E 00     0.4466829E 01    -0.4043741E 02    -0.9047597E 02    0.9508859E-02    -0.2021871E 01
   0.6999993E 00     0.5011865E 01    -0.4143793E 02    -0.9042422E 02    0.8474287E-02    -0.2071897E 01
   0.7499993E 00     0.5623405E 01    -0.4243831E 02    -0.9037811E 02    0.7552367E-02    -0.2121916E 01
   0.7999992E 00     0.6309563E 01    -0.4343864E 02    -0.9033701E 02    0.6730810E-02    -0.2171932E 01
   0.8499992E 00     0.7079445E 01    -0.4443889E 02    -0.9030043E 02    0.5998664E-02    -0.2221945E 01
   0.8999991E 00     0.7943267E 01    -0.4543909E 02    -0.9026775E 02    0.5346190E-02    -0.2271955E 01
   0.9499991E 00     0.8912491E 01    -0.4643925E 02    -0.9023869E 02    0.4764710E-02    -0.2321963E 01
   0.9999990E 00     0.9999978E 01    -0.4743938E 02    -0.9021272E 02    0.4246488E-02    -0.2371969E 01
```

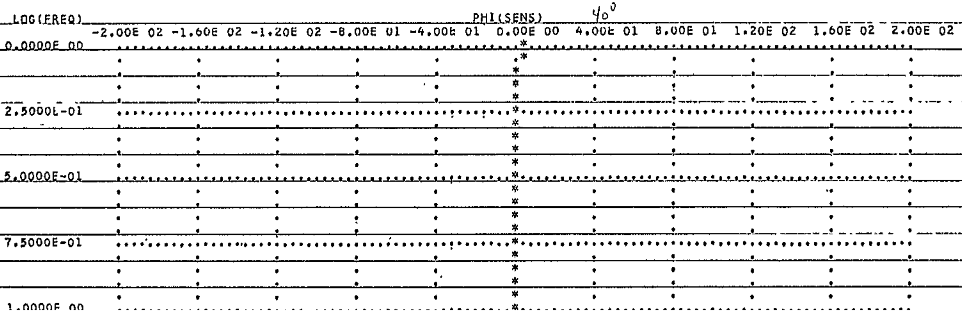




```
SENSITIVITY ANALYSIS **

      LOG(FREQ)         FRLQ            ABS(SENS(H))     PHI(SENS(H))    LOG(ABS(SENS(H)))
    0.0000000E 00   0.1000000E 01   0.1002481E 01   0.2439694E 01   0.1075936E-02
    0.4999995E-01   0.1122018E 01   0.1001969E 01   0.2173139E 01   0.8544312E-03
    0.9999990E-01   0.1258925E 01   0.1001563E 01   0.1935930E 01   0.6783034E-03
    0.1499999E 00   0.1412537E 01   0.1001242E 01   0.1724776E 01   0.5389217E-03
    0.1999998E 00   0.1584892E 01   0.1000986E 01   0.1536767E 01   0.4280463E-03
    0.2499998E 00   0.1778278E 01   0.1000783E 01   0.1369333E 01   0.3399048E-03
    0.2999997E 00   0.1995261E 01   0.1000621E 01   0.1220200E 01   0.2695443E-03
    0.3499997E 00   0.2238719E 01   0.1000493E 01   0.1087347E 01   0.2140759E-03
    0.3999996E 00   0.2511885E 01   0.1000391E 01   0.9689891E 00   0.1697787E-03
    0.4499996E 00   0.2818380E 01   0.1000312E 01   0.8635350E 00   0.1354142E-03
    0.4999995E 00   0.3162274E 01   0.1000247E 01   0.7695715E 00   0.1072582E-03
    0.5499995E 00   0.3548130E 01   0.1000196E 01   0.6858408E 00   0.8531177E-04
    0.5999994E 00   0.3981067E 01   0.1000154E 01   0.6112304E 00   0.6709123E-04
    0.6499994E 00   0.4466829E 01   0.1000124E 01   0.5447392E 00   0.5383947E-04
    0.6999993E 00   0.5011865E 01   0.1000097E 01   0.4854859E 00   0.4224383E-04
    0.7499993E 00   0.5623405E 01   0.1000077E 01   0.4326795E 00   0.3354691E-04
    0.7999992E 00   0.6309563E 01   0.1000061E 01   0.3856190E 00   0.2650642E-04
    0.8499992E 00   0.7079445E 01   0.1000049E 01   0.3436785E 00   0.2112243E-04
    0.8999991E 00   0.7943267E 01   0.1000038E 01   0.3063000E 00   0.1656670E-04
    0.9499991E 00   0.8912491E 01   0.1000030E 01   0.2729879E 00   0.1283924E-04
    0.9999990E 00   0.9999978E 01   0.1000024E 01   0.2432992E 00   0.1035426E-04
```





```
 SENSITIVITIES OF ZERDES AND POLES OF TRANSFER FUNCTION

      ZERD          REAL             IMAG                             SENSITIVITY
                                                        REAL              IMAG
       1      -0.4999998E 00    0.0000000E 00    -0.4468126E-06    0.0000000E 00

      POLE          REAL             IMAG                             SENSITIVITY
                                                        REAL              IMAG
       1      -0.1333996E 00    0.3400062E 00    -0.1333997E 00    0.1438336E 00
       2      -0.1333996E 00   -0.3400062E 00    -0.1333997E 00   -0.1438336E 00
```

```
NASAP PROBLEM RANGER ATTITUDE CONTROL

NONE
STEP RESPONSE

V1 1 2 1. IR4
R1 2 3  -1.
R2 3 4 1.
R3 4 1 1.
C1 4 1  2F
V2 1 5 75 IR2
L1 5 1 1H
V3 1 6  0.00667  IL1
L2 6 1 1H
V4 1 7 1. IL2
R4 8 7 1.
V5 1 8 1.
OUTPUT
VV4/VV5/V2
FREQ 0.0 1.0 0.05
TIME 50.
EXECUTE
```

K=75

```
TRANSFER FUNCTION   VV4/VV5/V2                                                    NUMBER OF LOOPS PER ORDER

                                                                                    1=    5
                                                                                    2=    3
                  (                                )
                  (  5.00E-01     +1.00E 00 S      )
H(S)= 1.000E 00*------------------------------------------------------
                  (                                        2 )
                  (  5.00E-01     +1.00E 00 S    +1.00E 00 S   )


 ZERO OF TRANSFER FUNCTION

 ZERO    REAL PART    IMAG. PART

   1 -0.50000E 00  0.00000E 00

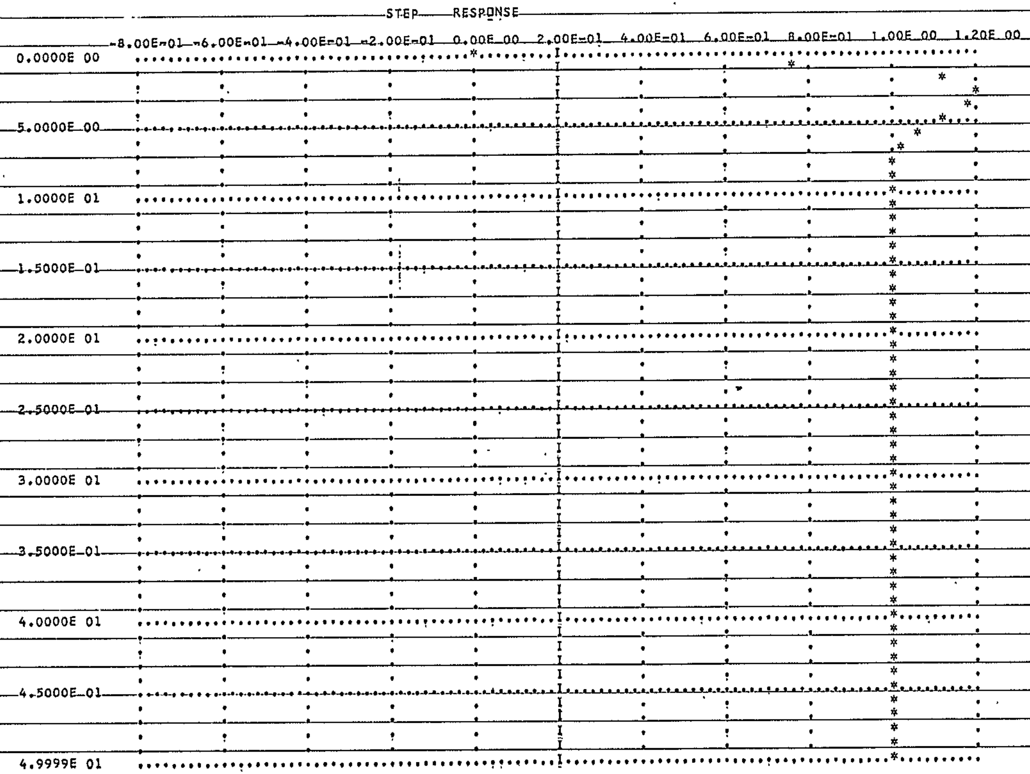

 POLE OF TRANSFER FUNCTION

 POLE    REAL PART    IMAG. PART

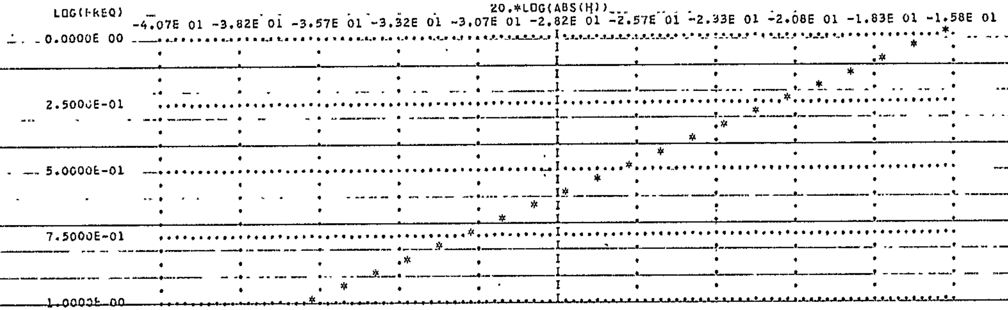
   1 -0.50025E 00  0.50000E 00
   2 -0.50025E 00 -0.50000E 00
```

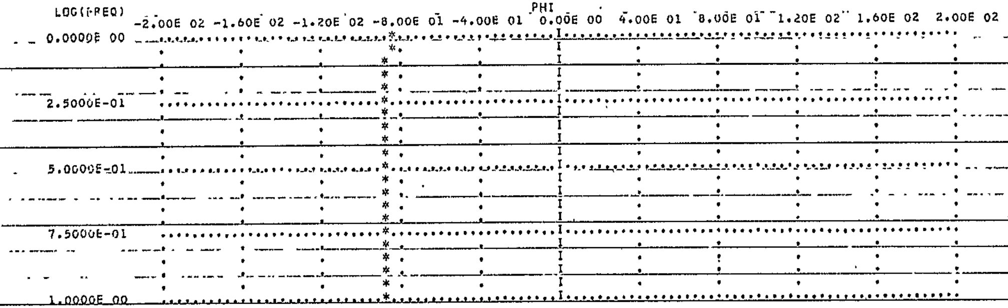
```
  STEP    RESPONSE-FUNCTION                                                     STEP    RESPONSE

  F(T) =                                                                        TIME                     VV4/VV5
                                                                                  0.0000E 00            -0.23841858E-06
                                                 (-0.5002E 00  J 0.5000E 00 ) T   0.1000E 01             0.75870979E 00
                   (-0.5000E 00  J-0.5002E 00 ) E                                 0.2000E 01             0.11108913E 01
                                                                                  0.3000E 01             0.12067432E 01
                                                 (-0.5002E 00  J-0.5000E 00 ) T   0.4000E 01             0.11792612E 01
                   (-0.5000E 00  J 0.5002E 00 ) E                                 0.5000E 01             0.11147690E 01
                                                                                  0.6000E 01             0.10562334E 01
                                                 ( 0.0000E 00  J 0.0000E 00 ) T   0.7000E 01             0.10176487E 01
                   ( 0.1000E 01  J 0.0000E 00 ) E                                 0.8000E 01             0.99810702E 00
                                                                                  0.9000E 01             0.99149549E 00
                                                                                  0.1000E 02             0.99164456E 00
                                                                                  0.1100E 02             0.99423426E 00
                                                                                  0.1200E 02             0.99693567E 00
                                                                                  0.1300E 02             0.99885851E 00
                                                                                  0.1400E 02             0.99991173E 00
                                                                                  0.1500E 02             0.10003252E 01
                                                                                  0.1600E 02             0.10003786E 01
                                                                                  0.1700E 02             0.10002832E 01
                                                                                  0.1800E 02             0.10001621E 01
                                                                                  0.1900E 02             0.10000677E 01
                                                                                  0.2000E 02             0.10000124E 01
                                                                                  0.2100E 02             0.99998844E 00
                                                                                  0.2200E 02             0.99998283E 00
                                                                                  0.2300E 02             0.99998581E 00
                                                                                  0.2400E 02             0.99999106E 00
                                                                                  0.2500E 02             0.99999553E 00
                                                                                  0.2600E 02             0.99999839E 00
                                                                                  0.2700E 02             0.99999976E 00
                                                                                  0.2800E 02             0.10000000E 01
                                                                                  0.2900E 02             0.10000000E 01
                                                                                  0.3000E 02             0.99999994E 00
                                                                                  0.3100E 02             0.99999970E 00
                                                                                  0.3200E 02             0.99999958E 00
                                                                                  0.3300E 02             0.99999952E 00
                                                                                  0.3400E 02             0.99999946E 00
                                                                                  0.3500E 02             0.99999946E 00
                                                                                  0.3600E 02             0.99999946E 00
                                                                                  0.3700E 02             0.99999946E 00
                                                                                  0.3800E 02             0.99999946E 00
                                                                                  0.3900E 02             0.99999952E 00
                                                                                  0.4000E 02             0.99999952E 00
                                                                                  0.4100E 02             0.99999952E 00
                                                                                  0.4200E 02             0.99999952E 00
                                                                                  0.4300E 02             0.99999952E 00
                                                                                  0.4400E 02             0.99999952E 00
                                                                                  0.4500E 02             0.99999952E 00
                                                                                  0.4600E 02             0.99999952E 00
                                                                                  0.4700E 02             0.99999952E 00
                                                                                  0.4800E 02             0.99999952E 00
                                                                                  0.4900E 02             0.99999952E 00
                                                                                  0.5000E 02             0.99999952E 00
```


STEP RESPONSE
-8.00E-01
-6.00E-01
-4.00E-01
-2.00E-01
0.00E 00
2.00E-01
4.00E-01
6.00E-01
8.00E-01
1.00E 00
1.20E 00
0.0000E 00
5.0000E 00
1.0000E 01
1.5000E 01
2.0000E 01
2.5000E 01
3.0000E 01
3.5000E 01
4.0000E 01
4.5000E 01
4.9999E 01

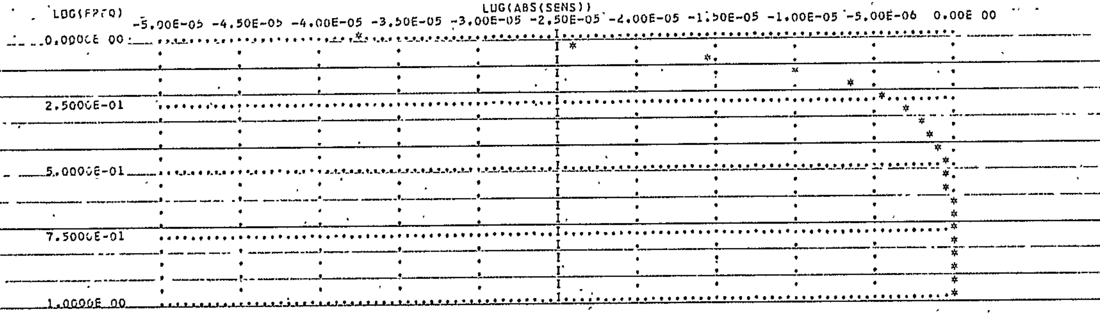
```
    LOG(FREQ)          FREQ         20.*LOG(ABS(H))      PHI(H)           ABS(H)        LOG(ABS(H))
  0.0000000E 00    0.1000000E 01    -0.1593262E 02   -0.8538820E 02   0.1597236E 00   -0.7966309E 00
  0.4999995E-01    0.1122018E 01    -0.1693796E 02   -0.8589842E 02   0.1422662E 00   -0.8468981E 00
  0.9999990E-01    0.1258925E 01    -0.1794226E 02   -0.8635066E 02   0.1267319E 00   -0.8971136E 00
  0.1499999E 00    0.1412537E 01    -0.1894571E 02   -0.8675188E 02   0.1129052E 00   -0.9472860E 00
  0.1999998E 00    0.1584892E 01    -0.1994847E 02   -0.8710826E 02   0.1005949E 00   -0.9974236E 00
  0.2499998E 00    0.1778278E 01    -0.2095065E 02   -0.8742490E 02   0.8963263E-01   -0.1047533E 01
  0.2999997E 00    0.1995261E 01    -0.2195241E 02   -0.8770654E 02   0.7986897E-01   -0.1097621E 01
  0.3499997E 00    0.2238719E 01    -0.2295383E 02   -0.8795700E 02   0.7117176E-01   -0.1147692E 01
  0.3999996E 00    0.2511885E 01    -0.2395494E 02   -0.8817999E 02   0.6342381E-01   -0.1197747E 01
  0.4499996E 00    0.2818380E 01    -0.2495580E 02   -0.8837846E 02   0.5652089E-01   -0.1247790E 01
  0.4999995E 00    0.3162274E 01    -0.2595651E 02   -0.8855515E 02   0.5037018E-01   -0.1297826E 01
  0.5499995E 00    0.3548130E 01    -0.2695706E 02   -0.8871260E 02   0.4488956E-01   -0.1347854E 01
  0.5999994E 00    0.3981067E 01    -0.2795750E 02   -0.8885277E 02   0.4000582E-01   -0.1397876E 01
  0.6499994E 00    0.4466829E 01    -0.2895787E 02   -0.8897774E 02   0.3565376E-01   -0.1447894E 01
  0.6999993E 00    0.5011865E 01    -0.2995816E 02   -0.8908896E 02   0.3177541E-01   -0.1497909E 01
  0.7499993E 00    0.5623405E 01    -0.3095837E 02   -0.8918810E 02   0.2831915E-01   -0.1547919E 01
  0.7999992E 00    0.6309563E 01    -0.3195856E 02   -0.8927644E 02   0.2523895E-01   -0.1597928E 01
  0.8499992E 00    0.7079445E 01    -0.3295869E 02   -0.8935516E 02   0.2249387E-01   -0.1647935E 01
  0.8999991E 00    0.7943267E 01    -0.3395882E 02   -0.8942526E 02   0.2004742E-01   -0.1697941E 01
  0.9499991E 00    0.8912491E 01    -0.3495889E 02   -0.8948782E 02   0.1786710E-01   -0.1747945E 01
  0.9999990E 00    0.9999978E 01    -0.3595897E 02   -0.8954355E 02   0.1592393E-01   -0.1797949E 01
```

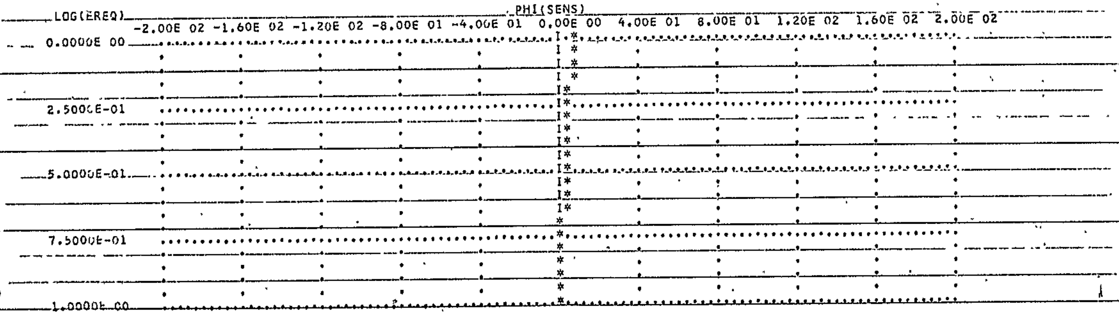




```
SENSITIVITY-ANALYSIS **

    LOG(FREQ)          FREQ           ABS(SENS(H))      PHI(SENS(H))    LOG(ABS(SENS(H)))
  0.0000000E 00    0.1000000E 01    0.9999137E 00    0.9161649E 01    -0.3748448E-04
  0.4999995E-01    0.1122018E 01    0.9999444E 00    0.8158376E 01    -0.2412638E-04
  0.9999990E-01    0.1258925E 01    0.9999644E 00    0.7266206E 01    -0.1548008E-04
  0.1499999E 00    0.1412537E 01    0.9999767E 00    0.6472507E 01    -0.1012153E-04
  0.1999998E 00    0.1584892E 01    0.9999847E 00    0.5766136E 01    -0.6652744E-05
  0.2499998E 00    0.1778278E 01    0.9999900E 00    0.5137315E 01    -0.4348864E-05
  0.2999997E 00    0.1995261E 01    0.9999933E 00    0.4577395E 01    -0.2899237E-05
  0.3499997E 00    0.2238719E 01    0.9999956E 00    0.4078719E 01    -0.1889679E-05
  0.3999996E 00    0.2511885E 01    0.9999970E 00    0.3634531E 01    -0.1320186E-05
  0.4499996E 00    0.2818380E 01    0.9999980E 00    0.3238844E 01    -0.8801237E-06
  0.4999995E 00    0.3162274E 01    0.9999986E 00    0.2886308E 01    -0.5953776E-06
  0.5499995E 00    0.3548130E 01    0.9999989E 00    0.2572201E 01    -0.4659474E-06
  0.5999994E 00    0.3981067E 01    0.9999990E 00    0.2292319E 01    -0.4400614E-06
  0.6499994E 00    0.4466829E 01    0.1000000E 01    0.2042920E 01     0.0000000E 00
  0.6999793E 00    0.5011865E 01    0.1000000E 01    0.1820675E 01     0.0000000E 00
  0.7499993E 00    0.5623405E 01    0.9999999E 00    0.1622622E 01    -0.2588597E-07
  0.7999992E 00    0.6309563E 01    0.9999999E 00    0.1446123E 01    -0.2588597E-07
  0.8499992E 00    0.7079445E 01    0.9999998E 00    0.1288829E 01    -0.1035439E-06
  0.8999991E 00    0.7943267E 01    0.1000000E 01    0.1148652E 01     0.0000000E 00
  0.9499991E 00    0.8912491E 01    0.1000000E 01    0.1023722E 01     0.0000000E 00
  0.9999990E 00    0.9999978E 01    0.9999999E 00    0.9123837E 00    -0.5177193E-07
```





```
 SENSITIVITIES OF ZEROES AND POLES OF TRANSFER FUNCTION


      ZERO          REAL             IMAG                                     SENSITIVITY
                                                            REAL                  IMAG
        1      -0.4999999E 00    0.0000000E 00       -0.1191500E-06      0.0000000E 00


      POLE          REAL             IMAG                                     SENSITIVITY
                                                            REAL                  IMAG
        1      -0.5002484E 00    0.5000001E 00       -0.5002484E 00     -0.2489090E-03
        2      -0.5002484E 00   -0.5000001E 00       -0.5002484E 00      0.2489090E-03
```

CHAPTER VII

SPECIAL CONTROL SYSTEM EXAMPLES

In the previous chapters of this manual we have used a variety of control circuit problems to illustrate various facets of computer aided design with NASAP. The examples were basic to control theory with explicit or implicit connection to aerospace applications. In this final chapter we have selected a few control system examples that emphasize the capabilities and also the limitations of the present version of NASAP as modified at the Moore School. From these examples it will be apparent that, in general, NASAP can be an effective aid to the control system engineer dealing with moderately complex linear control systems.

VIIA EXAMPLE INVOLVING TIME DOMAIN APPROXIMATION

Approximation of system response is a long standing problem related to control system analysis and synthesis. It is often necessary to approximate a system before suitable compensation can be applied. Eisenberg [EI 1] has described a technique for approximately identifying high order system responses based on certain characteristic responses to a unit step input. Many feedback systems exhibit time domain response to a unit step input that includes an overshoot followed by variations which subsequently settle to a steady state value. The approximate system response is generated by a closed loop transfer function whose open loop transfer function contains a transport lag and a first order lag. This technique only requires knowledge of the first peak overshoot, the time to peak and the settling time of the unknown system response.

We represent the unknown response by that of a unity-feedback control system with G(s) representing the plant. The general form of G(s) is assumed to be

$$G(s) = \frac{K_p e^{-sTd}}{\tau s + 1} \tag{7.1}$$

This particular approximation is quite convenient and yields the system transfer function

$$\frac{C}{R}(s) = \frac{G(s)}{1 + G(s)} = \frac{\frac{K_p}{\tau} e^{-sTd}}{s + \frac{1}{\tau} + \frac{K_p}{\tau} e^{-sTd}} \tag{7.2}$$

To facilitate further consideration of this expression, we normalize it with respect to T_d. Defining

$$\bar{s} = T_d s = \sigma T_d + j\omega T_d = \bar{\sigma} + j\bar{\omega}$$

we can write

$$\frac{C}{R}(\bar{s}) = \frac{\alpha e^{-\bar{s}}}{\bar{s} + \beta + \alpha e^{-\bar{s}}} \tag{7.3}$$

where

$$\alpha = \frac{K_p T_d}{\tau}, \ \beta = \frac{T_d}{\tau}, \ \alpha = K_p \beta$$

The problem of system approximation is now reduced to that of determining the parameters α and β such that the denominator of (7.3) has a pair of clearly dominant complex conjugate roots which will exhibit a second-order response. In particular it is desired that the responses shown in Fig. 7.1 be similar, for the set of conditions

$$M_h \cong M_s = M, \qquad T_{ph} \cong T_{ps} = T, \qquad T_{sh} \cong T_{ss} = T_s \tag{7.4}$$

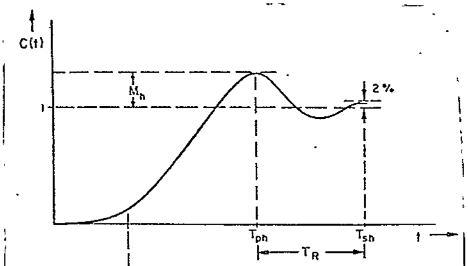
where the values of M_h, T_{ph}, and T_{sh} are measured from the unknown system step response.

One way of accomplishing the selection of α and β is to use the generalized curves found in [EI 1]. An alternative is to find a suitable electric circuit model for G(s) in (7.1) and then use NASAP. The new feature here is the exponential, e^{-sTd}. We can obtain a rational function representation for this exponential by a Padé approximation. Specifically we use the biquadratic

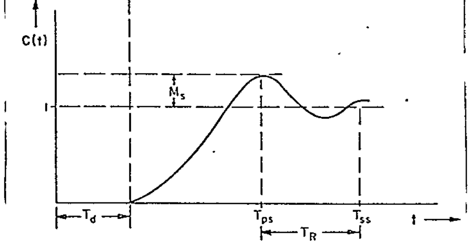
$$\frac{s^2 - as + b}{s^2 + as + b}$$

In Chapter III, we showed how to obtain a ladder network for this with negative as well as positive elements. The input impedance of the network is the desired circuit model.

To illustrate this approximation technique we again consider the seventh order system with an open loop transfer function



(a) Closed-Loop Response of an Unknown System



(b) Second-Order System Response Delayed in Time

Fig. 7.1 System Approximation by Time Domain Response (Subscripts "h" and "s" refer to higher order and second order system respectively)

$$G(s) = \frac{1}{(6s + 1)(2s + 1)^3(s + 1)^3} \tag{7.5}$$

The step response for this in a unity feedback system was given in Chapter V as the "Eisenberg" problem. Consideration of the actual closed-loop unit step response of this system, the lower response curve shown in Fig. 7.2 from [EI 1], yields the three data point $M_h = 31.7\%$, $T_{ph} = 20s$, and $T_{sh} = 55.4s^2$.

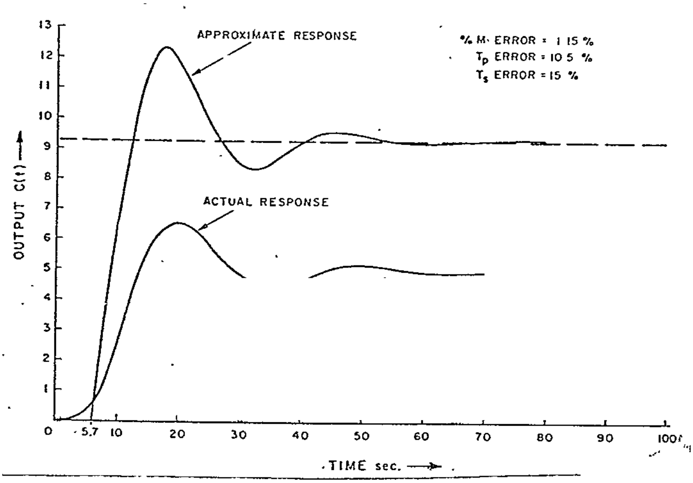


Fig. 7.2 Comparison of System Time Responses.

To obtain the approximate system parameters we must fall back on the well known relationships for a second-order system

$$M_s = e^{\left(\frac{\pi\zeta}{\sqrt{1 - \zeta^2}}\right)} \tag{7.6}$$

$$T_{ps} = \frac{\pi}{\omega_n\sqrt{1 - \zeta^2}} \tag{7.7}$$

$$T_{ss} = \frac{4}{\zeta\omega_s} \quad \text{(for a 2\% deviation of the response envelope from the steady-state value)} \tag{7.8}$$

The determination of ζ and ω_n and ultimately location of the dominant roots involves use of (7.4). Since $M_h \cong M_s$, where M_h is a measured quantity, we have the numerical value M_s. Hence the value of ζ can either be computed from (7.6) as

$$\zeta = \frac{|\ln M|}{\sqrt{(\ln M)^2 + \pi^2}} \tag{7.9}$$

or it can be obtained from a universal curve of M versus ζ. Next to determine ω_n. Note from Fig. 7.1 a and b and (7.4) that

$$T_R = T_{sh} - T_{ph} = T_{ss} - T_{ps} \tag{7.10}$$

Substituting the required values and solving for ω_n gives

$$\omega_n = \frac{4}{T_R \zeta} - \frac{\pi}{T_R \sqrt{1 - \zeta^2}} \tag{7.11}$$

Since ζ is known ω_n can be expressed in terms of measured quantities as

$$\omega_n = \frac{\sqrt{(\ln M)^2 + \pi^2}}{T_R} \left(\frac{4}{|\ln M|} - 1 \right) \tag{7.12}$$

Next we find T_d as a function of the measured parameters. From Fig. 7.1a and b

$$T_d = T_{ph} - T_{ps} \tag{7.13}$$

where T_{ph} is a measured quantity and T_{ps} can be obtained from (7.7). Substituting (7.7) into (7.13) gives

$$T_d = \frac{4T_{ph} - |\ln M| T_{sh}}{4 - |\ln M|} \tag{7.14}$$

so that the values of ζ, ω_n, and T_d are completely specified from the key characteristics of the unknown system response. Thus the values of the approximate system parameters are found to be $T_d = 5.7$s, $\tau = 89.7$s, and

$K_p = 13.1$. These give an approximate open-loop transfer function of

$$G(s) = \frac{13.1e^{-5.7s}}{89.7s + 1} \tag{7.15}$$

The closed-loop response corresponding to this G(s) obtained in the reference is shown in the upper response curve in Fig. 7.2. The percentage differences between the indicated characteristics of the two responses are also shown. It should be pointed out that altough the responses match quite closely with respect to M, T_p, and T_s the steady-state levels will not match. This mismatch is unimportant because the steady-state levels can always be matched exactly by simply adding gain outside the feedback loop. Also the gain of the unknown system response is easily computed from the measurable steady-state level, ($c_{ss}(t)$), via the relationship

$$K = \frac{c_{ss}(t)}{1 - c_{ss}(t)} \tag{7.16}$$

The NASAP print out and step response for (7.15) is shown in Figs. 7.3 and 7.4.

It should be noted for aerospace control applications a distinct advantage of the technique is that it is not necessary to open any feedback loops to effect the approximation. Thus, the system response can be approximated without interrupting normal operation. This is in contrast to some methods of system approximation, this technique is not hampered by the rare cases where the open-loop response is oscillatory. There are other methods available for system approximation that can be used on a closed-loop basis but these methods usually require that the system gain be increased until critical cycling is attained (oscillations). For many practical applications, however, it is not advisable to bring a control system to the verge of instability.

```
STEP      RESPONSE FUNCTION

F(T) =
                                    (-0.8220E-01  J 0.2209E 00 ) T
         (-0.1513E 00  J 0.3452E 00 ) E
                                    (-0.8220E-01  J-0.2209E 00 ) T
         (-0.1513E 00  J-0.3452E 00 ) E
                                    (-0.1045E 01  J 0.0000E 00 ) T
         (-0.1973E 00  J 0.1710E-07 ) E
                                    ( 0.0000E 00  J 0.0000E 00 ) T
         ( 0.4998E 00  J-0.1577E-07 ) E
```

Steady state Error $C_{ss} = 0.500$

```
STEP      RESPONSE

TIME              VV5/VV1
 0.0000E 00        0.23641856E-06
 0.1000E 01        0.19278191E-01
 0.2000E 01       -0.70372620E-02
 0.3000E 01       -0.27167184E-01
 0.4000E 01       -0.25492847E-01
 0.5000E 01       -0.29194455E-03
 0.6000E 01        0.45559863E-01
 0.7000E 01        0.10725963E 00
 0.8000E 01        0.17949197E 00
 0.9000E 01        0.25706337E 00
 0.1000E 02        0.33573571E 00
 0.1100E 02        0.40794364E 00
 0.1200E 02        0.47790766E 00
 0.1300E 02        0.53568649E 00
 0.1400E 02        0.58467635E 00
 0.1500E 02        0.62106609E 00
 0.1600E 02        0.64575958E 00
 0.1700E 02        0.65927076E 00
 0.1800E 02        0.66260619E 00
 0.1900E 02        0.65713841E 00
 0.2000E 02        0.64448053E 00
 0.2100E 02        0.62636787E 00
 0.2200E 02        0.60455073E 00
 0.2300E 02        0.58070276E 00
 0.2400E 02        0.55634767E 00
 0.2500E 02        0.53280771E 00
 0.2600E 02        0.51114539E 00
 0.2700E 02        0.49219918E 00
 0.2800E 02        0.47651201E 00
 0.2900E 02        0.46439201E 00
 0.3000E 02        0.45591635E 00
 0.3100E 02        0.45096457E 00
 0.3200E 02        0.44925499E 00
 0.3300E 02        0.45038351E 00
 0.3400E 02        0.45386177E 00
 0.3500E 02        0.45915681E 00
 0.3600E 02        0.46572310E 00
 0.3700E 02        0.47303331E 00
 0.3800E 02        0.48060185E 00
 0.3900E 02        0.48800279E 00
 0.4000E 02        0.49488711E 00
 0.4100E 02        0.50097942E 00
 0.4200E 02        0.50609010E 00
 0.4300E 02        0.51010745E 00
 0.4400E 02        0.51299738E 00
 0.4500E 02        0.51476945E 00
 0.4600E 02        0.51551604E 00
 0.4700E 02        0.51535028E 00
 0.4800E 02        0.51441896E 00
 0.4900E 02        0.51288563E 00
 0.5000E 02        0.51091963E 00
```

Peak ← 32.4% overshoot

Fig 7.4a

```
NASAP ETRANSLAG APPROXIMATION

NONE
STEP RESPONSE
                                          NUMBER OF LOOPS PER ORDER
V1 1 2 1
R8 2 3 1                                     1=     8
V2 1 4 1 IR8                                 2=    19
R1 4 5 89.7                                  3=    13
C1 5 1 1F                                    4=     1
I3 1 6 13.1    VC1
R4 6 1 10K
R2 6 7 1
C2 7 1 -0.475F
R3 7 8 -2
C3 8 1 1.425F
R5 8 1 2
V4 1 3 1 VR4
V5 1 9 0.538 VR4
OUTPUT
VV5/VV1
TIME 50
EXECUTE


TRANSFER FUNCTION    VV5/VV1


                    (                                       2 )
                    (  3.69E-01      -1.05E 00 S   +1.00E 00 S    )
H(S)= 7.856E-02*-------------------------------------------------------------
                    (                                       2                3 )
                    (  5.81E-02      +2.27E-01 S   +1.21E 00 S   +1.00E 00 S   )


ZERO OF TRANSFER FUNCTION

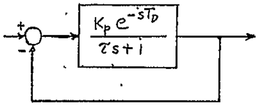
ZERO   REAL PART   IMAG. PART

   1  0.5262E 00 -0.30387E 00
   2  0.5262E 00  0.30387E 00


POLE OF TRANSFER FUNCTION

POLE   REAL PART   IMAG. PART

   1 -0.82196E-01  0.22087E 00
   2 -0.82196E-01 -0.22087E 00
   3 -0.10452E 01  0.00000E 00
```



$\tau = 89.7$
$T_D = 5.7$
$K_p = 13.1$

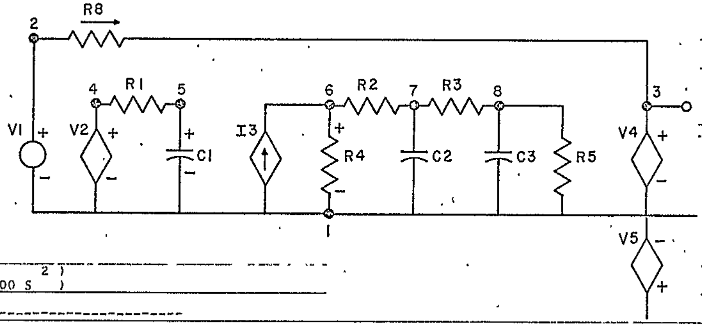
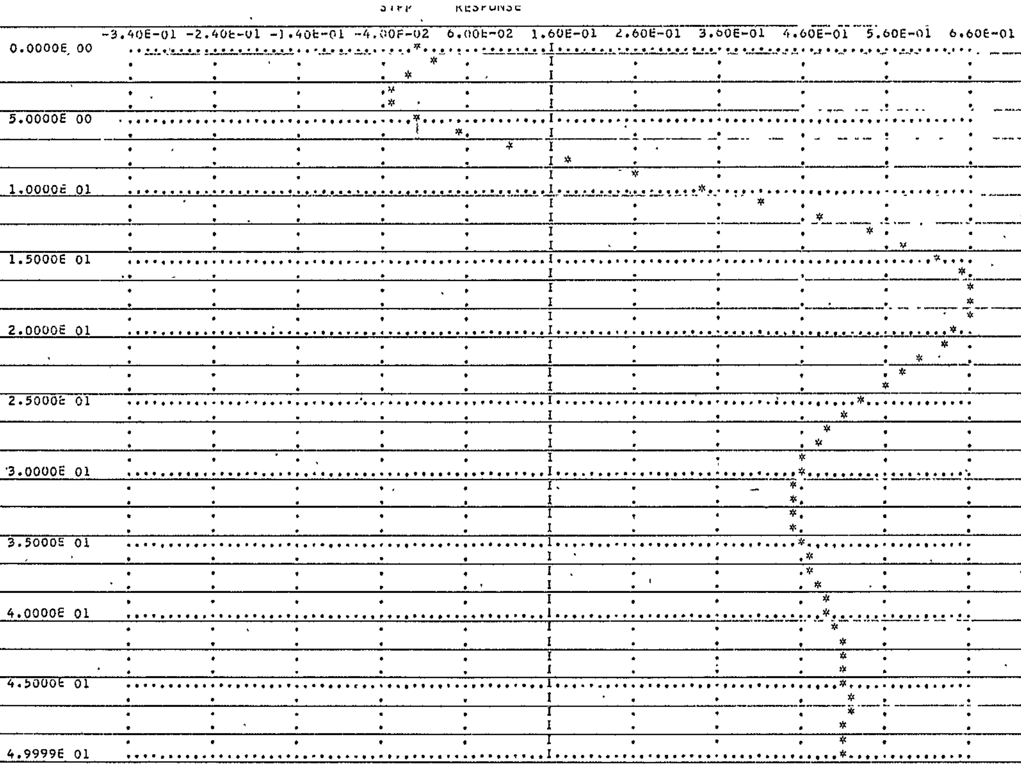


Fig 7.3



STEP RESPONSE
-3.40E-01
-2.40E-01
-1.40E-01
-4.00E-02
6.00E-02
1.60E-01
2.60E-01
3.60E-01
4.60E-01
5.60E-01
6.60E-01
0.0000E 00
5.0000E 00
1.0000E 01
1.5000E 01
2.0000E 01
2.5000E 01
3.0000E 01
3.5000E 01
4.0000E 01
4.5000E 01
4.9999E 01

Fig 7.4b

VIIB EXAMPLE OF A CONTROL SYSTEM WITH TRANSPORT LAG

As an extension of the concepts introduced in the previous example, we consider a given control system that has a plant with transport lag along with three simple lags shown in Fig. 7.5. Lupfer and Oglesby [LU 1] discussed such a problem wherein the object was to find a proportional-integral controller. They treated this problem with the aid of an analog computer. An alternative solution was presented by Eisenberg [EI 2] using the parameter plane approach to develop a graphical technique.

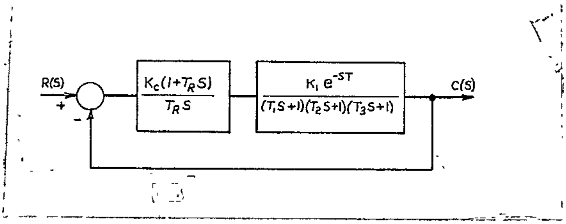


Fig. 7.5 Control system with transport lag
Reactor process from [LU 1]

To use NASAP we must obtain a circuit model for the control system with transport lag. From the previous approximation problem we know how to handle e^{-sT_D} by using a Padé approximation that yields a rational function realized by a ladder network. For the controller of this system we require a different type of model. We can write

$$\frac{K_c(T_r s + 1)}{T_r s} = K_c\left(1 + \frac{1}{T_r s}\right) \qquad (7.17)$$

Now we use the model shown in Fig. 7.6 where V7 and I2 are dependent on the error signal and I1 is dependent on the current through L1. The first three elements model the $\frac{1}{T_r s}$ term while the fourth element accounts for the unity term. The rest of the model poses no new problems.

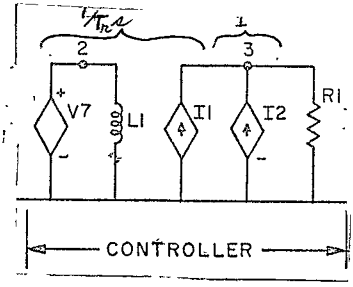


Fig. 7.6 Controller Circuit Model

To be specific then we are given the transfer function of the plant

$$G(s) = \frac{K_1 e^{-sT}}{(T_1 s + 1)(T_2 s + 1)(T_3 s + 1)} \tag{7.18}$$

where $K_1 = 1.0$

$T_1 = 13.1$ min

$T_2 = 11.1$ min

$T_3 = 0.5$ min

$T = 9.5$ min

To obtain the unit step response we must determine the controller gain K_c and the reset time T_r. From [LU 1] we have the experimental results obtained with an analog computer. This is shown in Fig. 7.7 where the key response characteristics are labeled and the controller parameters used by Lupfer and Oglesby are indicated.

Following Eisenberg [EI 2] we can restate the design problem as:

> Choose values of controller gain constant K_c and reset time T_r to yield an output time domain response with

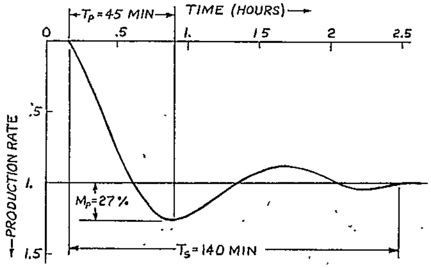


Fig. 7.7 Reactor process; response to unit step input, $K_c = 1.0$ and $T_r = 20.8$ min.

the characteristics of Fig. 7.7. Specifically, it is desired that (1) the peak overshoot $M_p = 27\%$, (2) the time to reach a peak $T_p = 45$ min (where the output response begins after the initial $T = 9.5$ min delay), and (3) the time for the response to reach 2% of the steady-state value $T_s = 140$ min.

At this point we do not have to reformulate the system transfer function as in [EI 2] but simply obtain the NASAP model shown in Fig. 7.8.

Finally we obtain the unit step response using NASAP. Compare this output Fig. 7.9 with that given in Fig. 7.7. This ability to handle transport lag considerably broadens the range of aerospace control problems that can be assisted by the use of NASAP. To permit detailed comparisons Fig. 7.9a is based on K_c=1 and T_r= 20.8 min. while Fig. 7.9b is a rerun of this example using the values K_c=0.9 and T_r=21.9 min. taken from [EI 2].

NSM-NASAP
4/14/70
43.0270 SEC.

using { Kc=1
Tr = 20.8

```
NASAP LUPFER-UGLESBY CONTROL PROCESS

NONE
STEP RESPONSE
                                                NUMBER OF LOOPS PER ORDER

V1 1 10 1                                            1=     12
R6 10 11 1                                           2=     40
V7 1 2 1 IR6                                         3=     47
L1 2 1 1248H                                         4=     26
I2 1 3 1 IR6                                         5=      9
I1 1 3 1 IL1                                         6=      1
R1 3 1 1
V2 1 4 1 VR1
R2 4 5 786
C2 5 1 1F
V3 1 6 1 VC2
R3 6 7 666
C3 7 1 1F
V4 1 8 1 VC3
R4 8 9 30
C4 9 1 1F
I5 1 12 1 VC4
R7 12 1 10K
R5 12 13 1
C5 13 1 -47.5F
R9 13 14 -2
C9 14 1 142.5F
R8 14 1 2
V6 1 11 1 VR7
OUTPUT
VTMEV9000
EXECUTE
```

VV6/VV1
TIME=9000

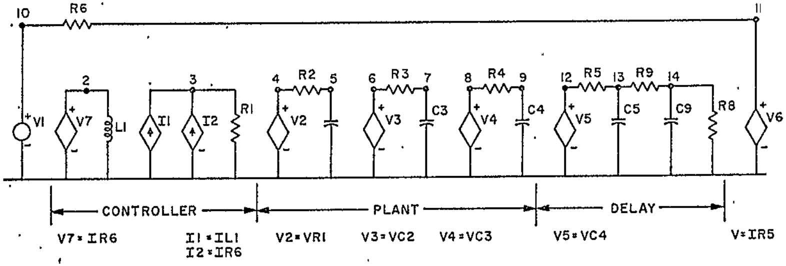


Fig. 7.8 NASAP Model for Fig. 7.5

```
TRANSFER FUNCTION   VV6/VV1

                 (                                             2           3 )
                 (  2.96E-08    +2.85E-05 S   -9.73E-03 S    +1.00E 00 S     )
H(S)= 6.367E-08*-----------------------------------------------------------------------------------------------------
                 (                                             2           3           4           5           6 )
                 (  1.88E-15    +4.17E-12 S   +3.54E-09 S    +2.45E-06 S  +5.11E-04 S  +4.66E-02 S  +1.00E 00 S  )

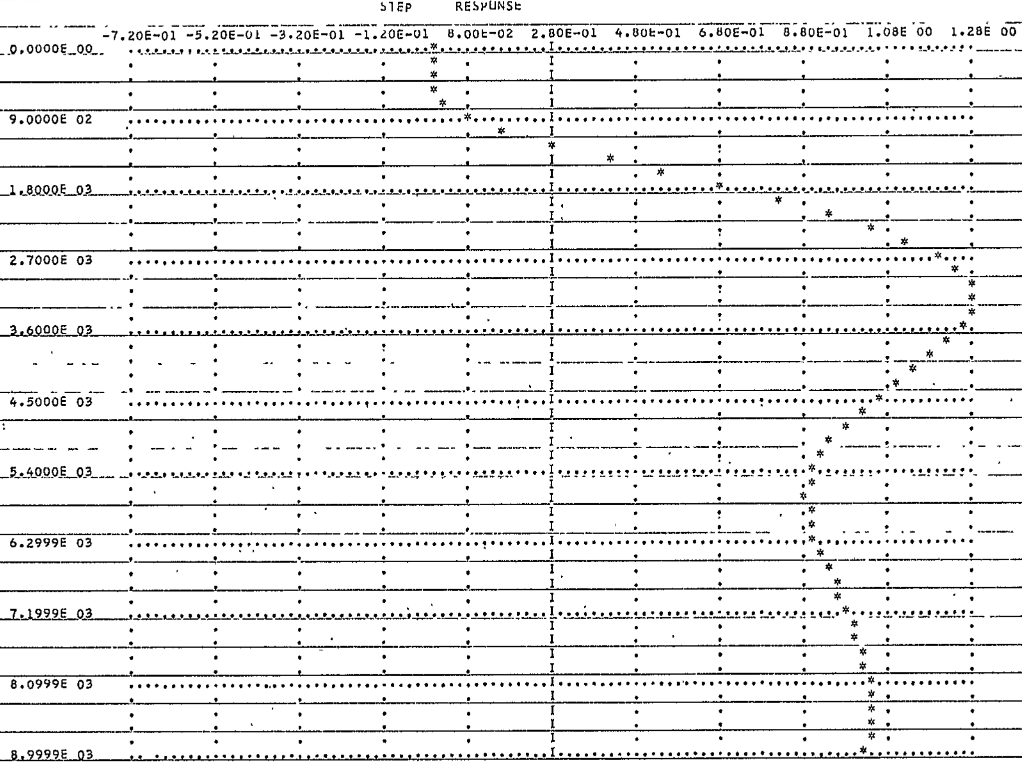
   ZERO OF TRANSFER FUNCTION                POLE OF TRANSFEER FUNCTION

   ZERO   REAL PART    IMAG. PART           POLE   REAL PART    IMAG. PART

     1  0.52632E-02  0.30387E-02              1 -0.58469E-02  0.40666E-02
     2  0.52632E-02 -0.30387E-02              2 -0.58469E-02 -0.40666E-02
     3 -0.80128E-03  0.00000E 00              3 -0.68800E-03  0.00000E 00
                                              4 -0.40133E-03 -0.12055E-02
                                              5 -0.40133E-03  0.12055E-02
                                              6 -0.33447E-01  0.00000E 00
```

```
     STEP      RE PONSE FUNCTION                                                        STEP      RESPONSE

     F(T) =                                                                             TIME                     VV6/VV1
                                                                                        0.0000E 00              -0.89406967E-06
                                   (-0.5847E-02   J 0.4067E-02 ) T                      0.1800E 03               0.50302744E-02
           (-0.1059E 00   J 0.6185E-01 ) E                                              0.3600E 03              -0.27541518E-02
                                                                                        0.5400E 03              -0.79850554E-02
                                   (-0.5847E-02   J-0.4067E-02 ) T                      0.7200E 03               0.15006900E-01
           (-0.1059E 00   J-0.6185E-01 ) E                                              0.9000E 03               0.73874056E-01
                                                                                        0.1080E 04               0.16472322E 00
                                   (-0.6880E-03   J 0.0000E 00 ) T                      0.1260E 04               0.27963656E 00
           (-0.2157E 00   J-0.3780E-07 ) E                                              0.1440E 04               0.41006690E 00
                                                                                        0.1620E 04               0.54791850E 00
                                   (-0.4013E-03   J-0.1206E-02 ) T                      0.1800E 04               0.68588519E 00
           (-0.2846E 00   J-0.5080E 00 ) E                                              0.1980E 04               0.81763464E 00
                                                                                        0.2160E 04               0.93795317E 00
                                   (-0.4013E-03   J 0.1206E-02 ) T                      0.2340E 04               0.10428429E 01
           (-0.2846E 00   J 0.5080E 00 ) E                                              0.2520E 04               0.11295567E 01
                                                                                        0.2700E 04               0.11965675E 01
                                   (-0.3345E-01   J 0.0000E 00 ) T                      0.2880E 04               0.12434797E 01
           (-0.3361E-02   J 0.2240E-12 ) E                                              0.3060E 04               0.12709026E 01
                                                                                        0.3240E 04               0.12802896E 01   <- peak  28% overshoot
                                   ( 0.0000E 00   J 0.0000E 00 ) T                      0.3420E 04               0.12737522E 01
           ( 0.1000E 01   J 0.3203E-06 ) E                                              0.3600E 04               0.12538738E 01
                                                                                        0.3780E 04               0.12235241E 01
                                                                                        0.3960E 04               0.11856804E 01
                                                                                        0.4140E 04               0.11432724E 01
                                                                                        0.4320E 04               0.10990458E 01
                                                                                        0.4500E 04               0.10554581E 01
                                                                                        0.4680E 04               0.10145941E 01
                                                                                        0.4860E 04               0.97812319E 00
                                                                                        0.5040E 04               0.94726765E 00
                                                                                        0.5220E 04               0.92280972E 00
                                                                                        0.5400E 04               0.90511072E 00
                                                                                        0.5580E 04               0.89415151E 00
                                                                                        0.5760E 04               0.88958508E 00
                                                                                        0.5940E 04               0.89079767E 00
                                                                                        0.6120E 04               0.89697522E 00
                                                                                        0.6300E 04               0.90716964E 00
                                                                                        0.6480F 04               0.92036349E 00
                                                                                        0.6660E 04               0.93552864E 00
                                                                                        0.6840E 04               0.95167798E 00
                                                                                        0.7020E 04               0.96790797E 00
                                                                                        0.7200E 04               0.98343128E 00
                                                                                        0.7380E 04               0.99759942E 00
                                                                                        0.7560E 04               0.10099154E 01
                                                                                        0.7740E 04               0.10200396E 01
                                                                                        0.7920E 04               0.10277815E 01
                                                                                        0.8100E 04               0.10330915E 01
                                                                                        0.8280E 04               0.10360460E 01
                                                                                        0.8460E 04               0.10368204E 01
                                                                                        0.8640E 04               0.10356770E 01
                                                                                        0.8820E 04               0.10329304E 01
                                                                                        0.9000E 04               0.10289335E 01
```


STEP RESPONSE
-7.20E-01
-5.20E-01
-3.20E-01
-1.20E-01
8.00E-02
2.80E-01
4.80E-01
6.80E-01
8.80E-01
1.08E 00
1.28E 00
0.0000E 00
9.0000E 02
1.8000E 03
2.7000E 03
3.6000E 03
4.5000E 03
5.4000E 03
6.2999E 03
7.1999E 03
8.0999E 03
8.9999E 03

ASM-NASAP
4/20/70
39.7947 SEC.

REPEATED using EISENBERG Values [EI 2]

$K_c = 0.9$

$T_R = 21.9$

```
NASAP LUPFER-OGLESBY CONTROL PROCESS

NONE
STEP RESPONSE

V1 1 10 1
R6 10 11 1
V7 1 2 1 IR6
L1 2 1 1314H
I2 1 3 1 IR6
I1 1 3 1 IL1
R1 3 1 0.9
V2 1 4 1 VR1
R2 4 5 766
C2 5 1 1F
V3 1 6 1 VC2                      NUMBER OF LOOPS PER ORDER
R3 6 7 666
C3 7 1 1F                            1=     11
V4 1 8 1 VC3                         2=     30
R4 8 9 30                            3=     26
C4 9 1 1F                            4=     13
V5 1 12 1 VC4                        5=      3
R5 12 13 1
C5 13 1  47.5F
P9 13 14 -2
C9 14 1 -142.5F
R8 14 1 2
V6 1 11 1 IR5
OUTPUT
VV6/VV1
TIME 9000
EXECUTE


TRANSFER FUNCTION   VV6/VV1

                (                                               2                 3 )
                (  2.81E-08     +2.89E-05 S    -9.77E-03 S    +1.00E 00 S    )

H(S)= 5.731E-08*----------------------------------------------------------------------------------------------------------------
                (                                               2                 3                 4                 5               6 )
                (  1.61E-15     +4.01E-12 S    +3.60E-09 S    +2.45E-06 S    +5.11E-04 S    +4.66E-02 S    +1.00E 00 S    )

                                              POLE OF TRANSFER FUNCTION

 ZERO OF TRANSFER FUNCTION                    POLE    REAL PART     IMAG. PART

 ZERO    REAL PART    IMAG. PART
                                                1 -0.58147E-02  0.39870E-02
                                                2 -0.58147E-02 -0.39870E-02
    1  0.52622E-02  0.30387E-02                 3 -0.47543E-03 -0.11591E-02
    2  0.52622E-02 -0.30387E-02                 4 -0.47543E-03  0.11591E-02
    3 -0.76104E-03  0.00000E 00                 5 -0.61755E-03  0.00000E 00
                                                6 -0.13446E-01  0.00000E 00
```

Fig 7.9b

```
STEP     RESPONSE FUNCTION

F(T) =
                                   (-0.5815E-02  J 0.3987E-02 ) T
           (-0.1012E 00  J 0.5839E-01 ) E
                                   (-0.5815E-02  J-0.3987E-02 ) T
           (-0.1012E 00  J-0.5839E-01 ) E
                                   (-0.4754E-03  J-0.1159E-02 ) T
           (-0.2453E 00  J-0.5323E 00 ) E
                                   (-0.4754E-03  J 0.1159E-02 ) T
           (-0.2453E 00  J 0.5323E 00 ) E
                                   (-0.6175E-03  J 0.0000E 00 ) T
           (-0.3038E 00  J-0.1050E-06 ) E
                                   (-0.3344E-01  J 0.0000E 00 ) T
           (-0.3035E-02  J 0.7430E-10 ) E
                                   ( 0.0000E 00  J 0.0000E 00 ) T
           ( 0.1000E 01  J 0.6701E-06 ) E
```

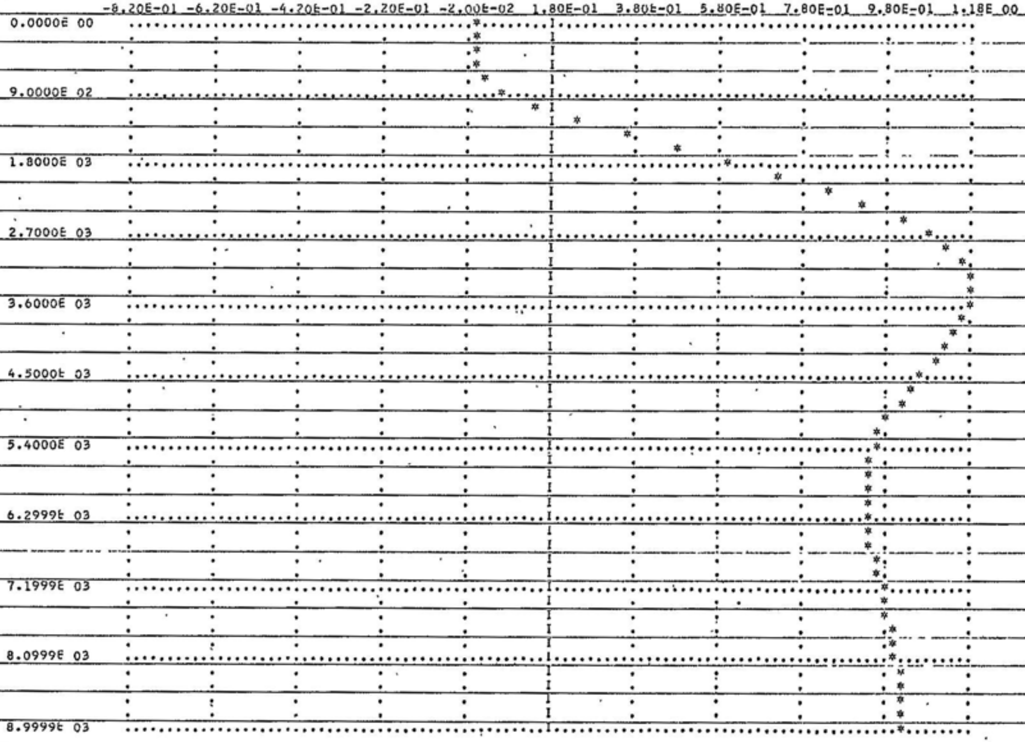
```
STEP     RESPONSE

TIME                VV6/VV1
 0.0000E 00        -0.95367432E-06
 0.1800E 03         0.45130849E-02
 0.3600E 03        -0.25043488E-02
 0.5400E 03        -0.71744919E-02
 0.7200E 03         0.13525188E-01
 0.9000E 03         0.66282332E-01
 0.1080E 04         0.14736784E 00
 0.1260E 04         0.24952722E 00
 0.1440E 04         0.36511564E 00
 0.1620E 04         0.48707902E 00
 0.1800E 04         0.60921687E 00
 0.1980E 04         0.72628343E 00
 0.2160E 04         0.83405477E 00
 0.2340E 04         0.92936033E 00
 0.2520E 04         0.10100660E 01
 0.2700E 04         0.10750170E 01
 0.2880E 04         0.11239424E 01
 0.3060E 04         0.11573286E 01
 0.3240E 04         0.11762857E 01
 0.3420E 04         0.11823950E 01
 0.3600E 04         0.11775694E 01
 0.3780E 04         0.11639090E 01
 0.3960E 04         0.11435785E 01
 0.4140E 04         0.11187010E 01
 0.4320E 04         0.10912590E 01
 0.4500E 04         0.10630350E 01
 0.4680E 04         0.10355549E 01
 0.4860E 04         0.10100603E 01
 0.5040E 04         0.98749757E 00
 0.5220E 04         0.96851933E 00
 0.5400E 04         0.95350277E 00
 0.5580E 04         0.94257486E 00
 0.5760E 04         0.93564713E 00
 0.5940E 04         0.93245381E 00
 0.6120E 04         0.93259305E 00
 0.6300E 04         0.93556708E 00
 0.6480E 04         0.94082105E 00
 0.6660E 04         0.94778817E 00
 0.6840E 04         0.95587045E 00
 0.7020E 04         0.96456379E 00
 0.7200E 04         0.97337794E 00
 0.7380E 04         0.98190069E 00
 0.7560E 04         0.98979634E 00
 0.7740E 04         0.99680966E 00
 0.7920E 04         0.10027647E 01
 0.8100E 04         0.10075626E 01
 0.8280E 04         0.10111713E 01
 0.8460E 04         0.10136185E 01
 0.8640E 04         0.10149803E 01
 0.8820E 04         0.10153723E 01
 0.9000E 04         0.10149336E 01
```

peak at 18.2% 57 min (3420 sec)

2% at 4770 sec.

STEP RESPONSE



VIIC. EXAMPLE SHOWING NASAP LIMITATION

We now present an active filter circuit which illustrates a flowgraph of great complexity and demonstrates some NASAP limitations. In particular the fact that it may not be sufficient that the user be judicious in the specific tree he allows the NASAP program to select. In Fig. 7.9 which is identical to Fig. 2.16 in [HU 1] is shown the 28-element NASAP circuit diagram of this Hutton problem. The input voltage V1 is fed through a low-pass tee-network to the base of a transistor which is part of a two-transistor differential amplifier. The output voltage of this differential amplifier is connected directly to the base of a simple common emitter transistor stage. The voltage at the collector of this transistor, which is also the output voltage of the circuit, is fed back through an RC twin-tee network to the base of the other transistor in the differential amplifier. Each of these three transistors is represented in Fig. 7.9b by the h-parameter equivalent circuit with $h_{12}= 0$ and with a capacitor (C_2, C_3, C_7) connected between the base and collector terminals. This capacitance is included to take into account the frequency characteristic of the transistor.

The NASAP input listing used by Hutton is reproduced as Fig. 7.10. There all 17 of the resistors are listed first and followed by the seven capacitors, in numerical sequence. With this listing, the NASAP tree selection algorithm selects as branches of the tree the elements in the following order; V1, R17, C1, C2, C3, C4, C6, C7, R4, R5, R8, R15. This particular tree generates a flowgraph possessing a total number of loops of all orders of 2,440,105 a very complicated flowgraph indeed. As noted by Hutton, 10 minutes of execution time were required on the UNIVAC 1108.

Utilizing one of the options discussed in Chapter II, a tree can be selected to yield a flowgraph with considerably fewer loops. Seven of the

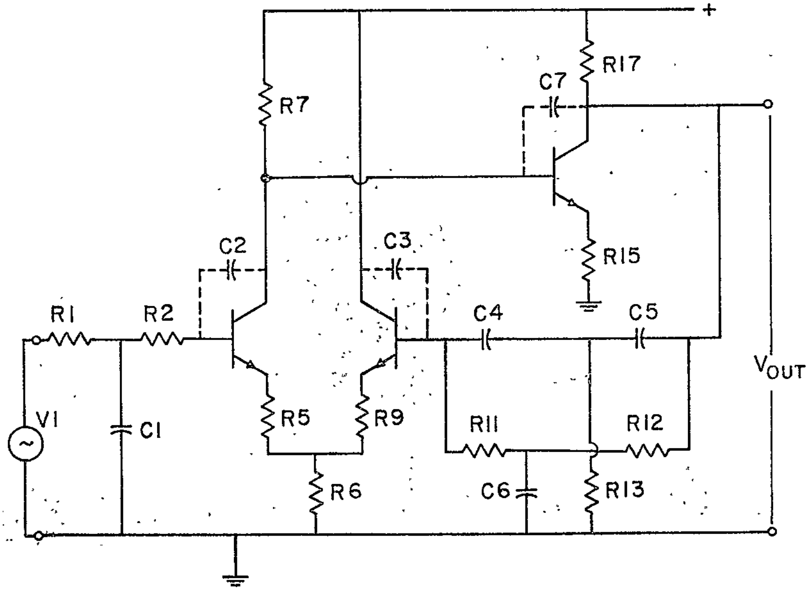


Fig. 7.9a Active Filter Circuit for Hutton Problem (Circuit 6)

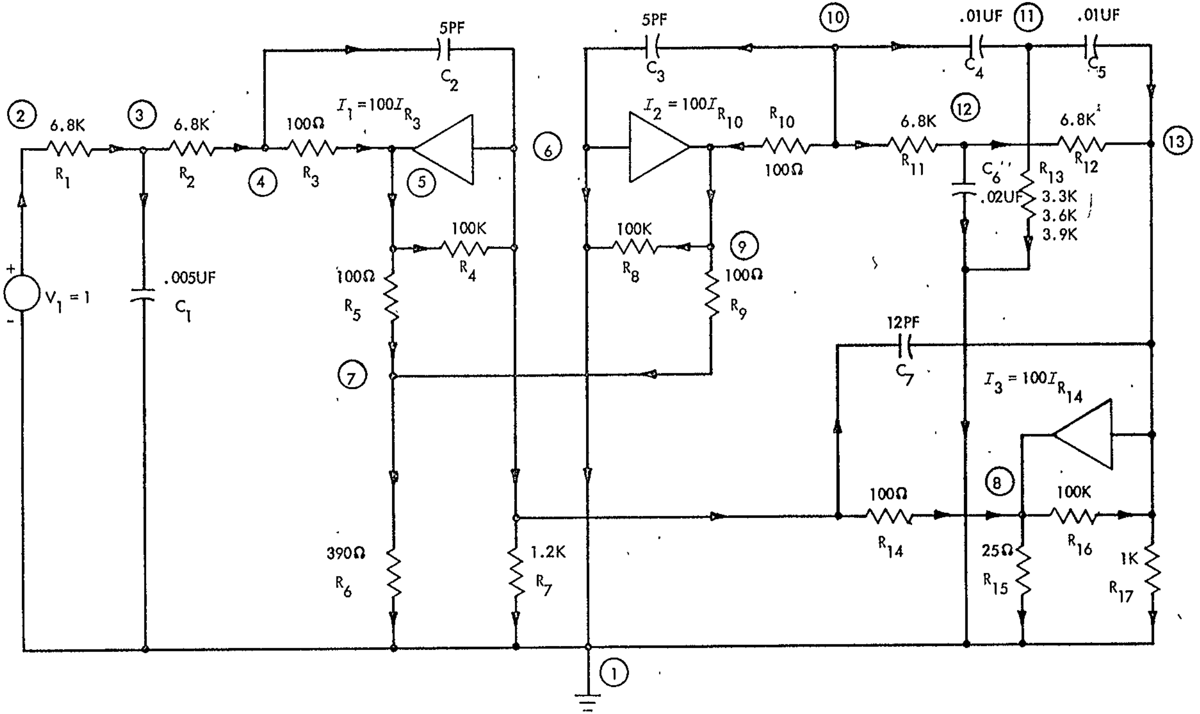


Fig. 7.9b Hutton Problem

```
NASAP                                   4/9/68

R1 2 3 6.8K
R2 3 4 6.8K                                                      NUMBER OF LOOPS PER ORDER
R3 4 5 100
R4 5 6 100K                                                         1=     696          2 2 0
R5 6 7 100                                                          2=    8969
R6 7 1 390                                                          3=   52922
R7 6 1 1.2K                                                         4=  165371
R8 9 1 100K                                                         5=  419756
R9 9 7 100                                                          6=  628827
R10 10 9 100                                                        7=  616324
R11 10 12 6.8K                                                      8=  380754
R12 12 13 5.8K                                                      9=  138488
R13 11 1 3.6K                                                      10=   26132
R14 6 8 100                                                        11=    1896
R15 8 1 25
R16 8 13 100K
R17 13 1 1K
C1 3 1 5UF
C2 4 6 .005UF
C3 10 1 .005UF
C4 10 11 10UF
C5 11 13 10UF
C6 12 1 20UF
C7 6 13 .012UF
V1 1 2 1.
I1 6 5 100 IR3
I2 1 9 100 IR10
I3 13 8 100 IR14
OUTPUT
VR17/VV1
FREQ -1 4 .1
TIME .005
EXECUTE


TRANSFER FUNCTION   VR17/VV1



                                                  2             3             4             5 )
                .16+17     - .49+16 S  - .33+15 S  - .44+13 S  - .43+07 S  + .10+01 S   )

F(S)=  .666+06*---------------------------------------------------------------------------------------------------------

                                                  2             3             4             5             6             7 )
                .11+23     + .69+21 S  - .68+20 S  - .60+19 S  - .11+18 S  - .38+13 S  + .91+07 S  + .10+01 S   )
```

Fig. 7.10 NASAP Printout for Hutton Circuit 6 elements (namely, V1, R17, C1, C3, C6, R8, R15) in the above described tree are connected to the node numbered 1 in Fig. 7.11 . However, three resistors, R6, R7, and R13 are also connected to node 1. If these three elements can be made branches of a tree which also contains the seven elements from the tree selected by Hutton, then ten of the twelve necessary tree branches will be connected to a common node. Such a tree with a definite star-like structure should yield a flowgraph with fewer loops. We shall now indicate how R6, R7, and R13 can be selected as tree branches and which branches of the original Hutton tree must be removed to make way for these resistors.

The resistor R7 forms a closed path with the original tree branches C7 and R17. Since the voltage across R17 is the specified output variable, R7 will become a tree branch only if C7 can be removed from the tree. This is easily accomplished by adding a dummy voltage source dependent on the voltage across R7. Likewise the resistor R13 forms a closed path with the original tree branches C3 and C4. Since C3 is connected to node 1, we wish to make R13 a tree branch in place of C4. A dummy voltage source dependent upon the voltage across R13 easily accomplishes this by making R13 a type 2 element instead of a type 4 element (see description of tree selection algorithm). Finally R6 can be included in the list of tree branches by making it the first resistor described in the NASAP input listing. With R6 in the tree, either R4 or R5 must be removed from the tree. The choice is easily made by noting that the co-tree element R3 will form a closed path with R4 and C2 when R4 is a tree branch. On the other hand, with R5 as a tree branch, R3 will form a closed path with R6, R7, R5 and C2. Hence R4 should remain as a tree branch. This is achieved by having R4 precede R5 in the NASAP input list.

The revised NASAP input listing that yields a tree with 10 elements connected to the same node is shown in Fig. 7.12. The controlled sources V2

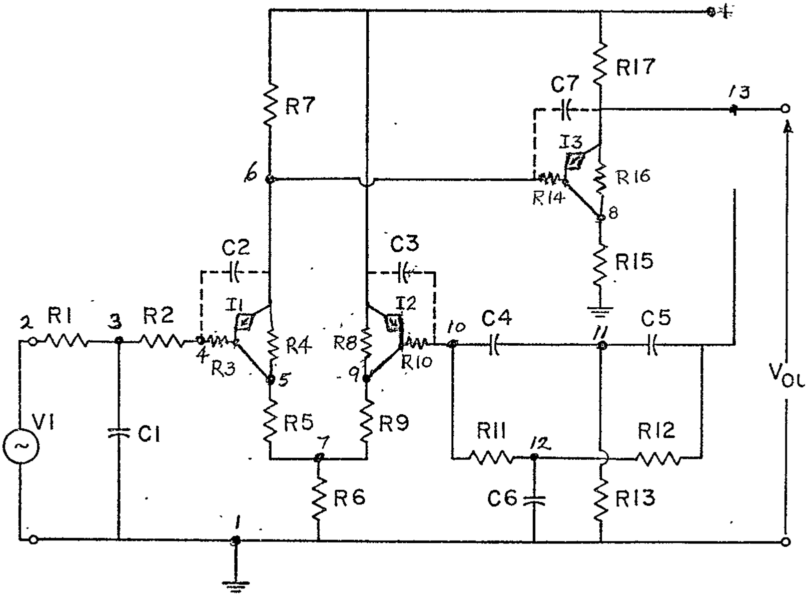


Fig. 7.11 Alternative NASAP Model for Hutton (compare Fig. 7.9)

```
NASAP PROBLEM [illegible]
NONE
NONE
V1 1 2 1.
R6 7 1 390
R15 8 1 25
R1 2 3 6.8K
R2 3 4 6.8K
R3 4 5 100
R4 5 6 100K
R5 5 7 100
R7 6 1 1.2K
R8 9 1 100K
R9 9 7 100
R10 10 9 100
R11 10 12 6.8K
R12 12 11 6.8K
R13 13 1 3.[illegible]
R14 6 8 10[illegible]
R16 8 11 100K
R17 11 1 1K
C1 3 1 5UF
C2 4 6 .005[illegible]
C3 10 1 .0[illegible]
C4 10 13 10[illegible]
C5 13 11 1[illegible]
C6 12 1 20[illegible]
C7 6 11 [illegible]
I1 6 5 100 I 3
I2 1 9 100 I 1
I3 11 8 100 I 14
V2 1 14 1.[illegible] 7
V3 1 15 1.0 11.13
OUTPUT
VR17/VV1
EXECUTE
```

ASM- NASAP
3/11/70

25 min = 1509.-2789 SEC

DUMP in MULLER

(102- 415, 515)

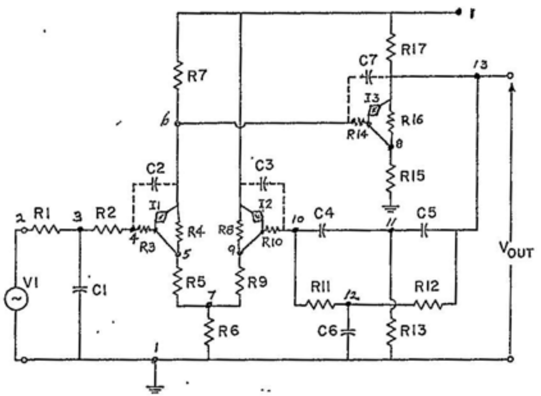


Fig. 7.12

and V3 are the dummy voltage sources necessary to include R7 and R13 in the tree selected by NASAP. With these 2 elements included, there are 30 elements in the equivalent circuit (NOTE: 30 elements are the maximum number of elements that can be used on the RCA Spectra 70 and IBM 360 machines because of their 32 bit computer words). Note in Fig. 7.12 that the flowgraph generated by this new tree contains 415, 515 loops of all orders. Thus a saving of over 2 million loops has been achieved by careful selection of the tree. The subsequent execution time was 25 minutes on the Spectra 70/46 (equivalent to approximately 3 minutes on the IBM 360/75). Note also the difference between the transfer function found by use of the original Hutton tree (Fig. 7.10) and that found by use of the tree described here (Fig. 7.12). The extra 2 million loops results in considerable error in the coefficients of the transfer function, see the discussion in [SE 1]. Note in Fig. 7.12 that only the zeros of the transfer function are given. Due to excessive floating point overflow, the MULLER (root-finding) subroutine was unable to determine the poles of the transfer function.

Through the cooperation of Prof. Alan B. Macnee the Hutton problem was run using CIRAN (a program based on state variables) on the University of Michigan IBM 360/67 computer. The impedances were scaled by 10^{-3} and frequency 10^{-6}. The circuit was found to exhibit a pair of dominant conjugate complex poles near 2.4 KHz. For a ± 10% change in the value of R13 the Q of this pole ranged from 7.5 to 20. The fact that these three analyses took only 12.13 seconds of CPU time indicates a severe limitation of NASAP for this class of problems.

In Fig. 7.13 the compensation is seen to consist of two identical RC lag network. The transfer function of the RC lag network is

$$\frac{\frac{1}{RC}}{S+\frac{1}{RC}} \quad \text{or} \quad \frac{G}{S+G}$$

The parameter G, the RC lag network pole, is chosen different from all plant poles, usually farther inside the left half plane than any pole of the desired system.

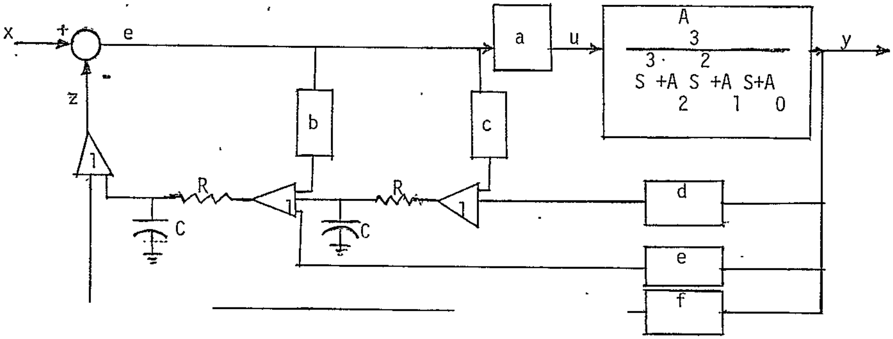


Fig. 7.13 Compensated Third Order Control System

Fig. 7.14 is equivalent to Fig. 7.13 with respect to the transfer function involved.

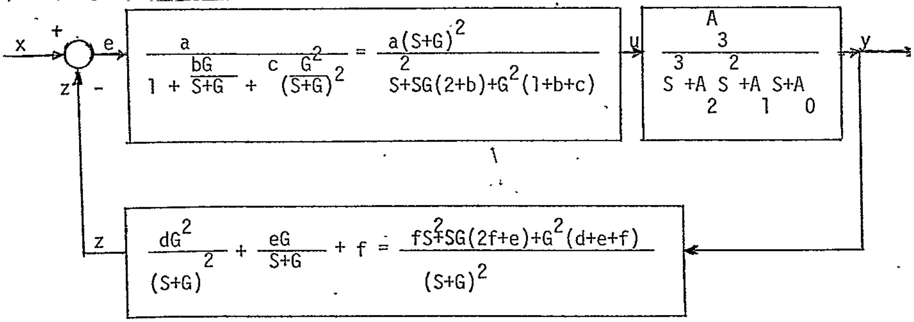


Fig. 7.14 Equivalent Feedback Control System Representation for Fig. 7.13

The closed-loop transfer function for y/x shown in Fig. 7.14 is

$$\frac{y}{x} = \frac{\dfrac{a(S+G)^2A_3}{(S^2+SG(2+b)+G^2(1+b+c))(S^3+A_2S^2+A_1S+A_0)}}{1+\dfrac{aA_3(fS^2+G(2f+e)S+G^2(d+e+f))}{(S^2+SG(2+b)+G^2(1+b+c))(S^3+A_2S^2+A_1S+A_0)}} \qquad (7.21)$$

Equation (7.20) for the desired system, when multiplied top and bottom by $(S+G)^2$ becomes

$$\frac{y}{x} = \frac{B_3(S+G)^2}{S^5+(B_2+2G)S^4+(B_1+2GB_2+G^2)S^3+(B_0+2GB_1+G^2B_2)S^2+(G^2B_1+2GB_0)S+G^2B_0} \qquad (7.22)$$

This equation can be compared termwise with (7.21) after it has been rewritte to clear fractions. Thus we find that $aA_3 = B_3$ and successively

$$b = \frac{(B_2 - A_2)}{G} \quad \text{where G is positive.}$$

$$c = \frac{(B_1 - A_1) + (G - A_2)\ (B_2 - A_2)}{G^2} \quad \text{with G is positive.}$$

$$f = \frac{(B_0 - A_0) + (B_1 - A_1)\ (2G - A_2) + (B_2 - A_2)(\ (G-A_2)^2 - A_1)}{B_3}\ , \ B_3) \neq$$

$$c = \frac{(B_1-A_1)(-3G^2 - A_1+2GA_2) + (B_2-A_2)\ (-2G(G-A_2)^2 + A_1A_2-A_0)}{GB_3}$$

and finally

$$d = \frac{(B_1-A_1)(G^3-G^2A_2+GA_1-A_0) + (B_2-A_2)(G^2A_1+A_0A_2-GA_0+G^2(G-A_2)^2-GA_1A_2)}{G^2G_3} \qquad (7.23)$$

VIID EXAMPLE INVOLVING LUENBERGER OBSERVER

For the last problem we consider an application of modern control theory based on state variables to show the versatility of NASAP. In particular we use the Luenberger observer method to implement compensation for a control system situation in which all the states of the system are not measurable. For present purposes it is sufficient to note that the Luenberger observer [LU 1] is a device which constructs an estimate of the system state vector based upon the available system inputs and outputs. Then, based upon the reconstructed state vector, simple matrix algebra manipulations can be utilized to obtain estimates of the missing states or a combination of the missing states.

Luenberger has shown that for an n-th order system with m measureable states, the order of the required observed need only contain n-m poles. Furthermore these pole locations are arbitrary as long as they are different from the eigenvalues of the system matrix.

The Luenberger observer accomplishes the desired result by adding dynamics in the feedback path of the control system.

This theory suggests a unique form for general compensation of third order systems wherein the designer can place the closed-loop poles at any desired location. This development is adapted from Newman [NE 2].

The open loop descriptions of the control system shown in Fig. 7.13 is

$$\frac{y}{u} = \frac{A_3}{S^3 + A_2 S^2 + A_1 S + A_0} \qquad (7.19)$$

For convenience the desired closed-loop system transfer function is expressed as

$$\frac{y}{x} = \frac{B_3}{S^3 + B_2 S^2 + B_1 S + B_0} \qquad (7.20)$$

These six equations define unique values for the symbols a, b, c, f, e, and d respectively.

It is worth noting [NE 2] that this method of general system compensation may be extended to higher order systems. An N-th order system will require a string of N-1 RC lag networks. Each lag network is driven by the signal e and the signal y, both passing through gain blocks as in Fig. 7.13. The z signal is formed by adding the output of the string and a signal equal to fy, as in Fig. 7.13. The a block is used in cascade with the plant as in Fig. 7.13 .

It is found easier to design compensators for plant transfer functions with numerator polynomials by reducing the coefficients in the equations to numbers, instead of trying to derive the general relationship.

For an illustrative example, we consider the open loop and desired closed loop transfer functions for the control system in Fig. 7.13 .

Open Loop System

$$\frac{y}{u} = \frac{A_3}{S^3+A_2S^2+A_1S+A_0} = \frac{10}{S(S+1)(S+10)} \tag{7.24}$$

Closed Loop System (Desired)

$$\frac{y}{x} = \frac{B_3}{S^3+B_2S^2+B_1S+B_0} = \left(\frac{5}{S+5}\right)\left(\frac{20}{S+20}\right)^2 \tag{7.25}$$

Following the procedure outlined above, we determine the constants a, b, c, d, e, f, as listed.

$$\begin{aligned} a &= 200 & b &= 2.24 \\ c &= 1.7 & d &= 4.256 \\ e &= -14.018 & f &= 10.762 \end{aligned} \tag{7.26}$$

The parameter G, was chosen as twenty (G = 20) which is different from all plant poles.

Having thus specified the Luenberger observer we need the corresponding electric circuit model for the compensated third order control system. This model is shown in Fig. 7.15. A NASAP run was made for the step response of this model. We note that the transfer function shows six critical frequencies near s = -20; two zeros and four poles. The step response shows a slight steady state error.

Execution time on the RCA Spectra 70/46 was 42.86 seconds. A comparison run was made of this third-order compensator system using CSMP (Continuous System Modeling Program) on the IBM 360/75. The CSMP step response for this control system checked very closely except that the execution time was 32 seconds (22 seconds CPU). This represents a significant cost advantage for NASAP since the 360/75 is faster by a factor of approximately 8 over the Spectra 70/46. The NASAP printout is shown in Fig. 7.16.

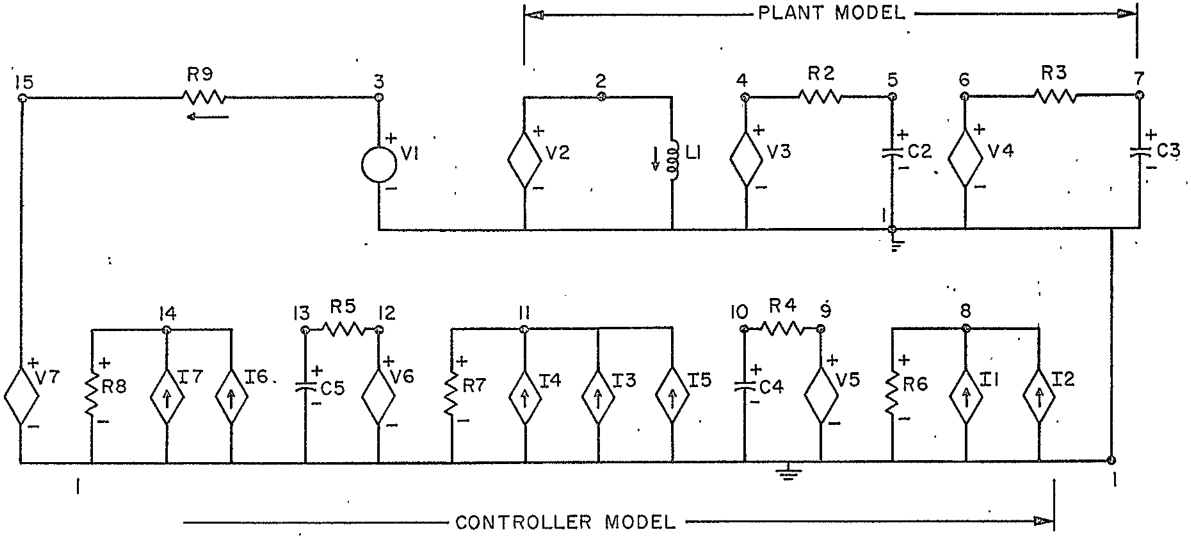
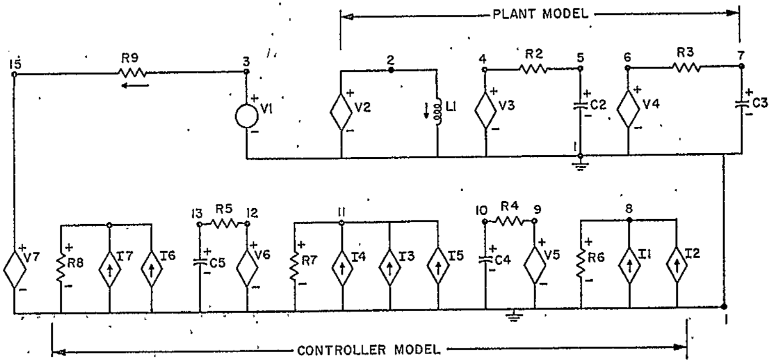


Fig. 7.15 NASAP Model for Third Order Control System in Fig. 7.13

1/19/70



```
NASAP  NEWMAN DESIGN PROBLEM

       NONE
       STEP RESPONSE

       V1 1 3 1.
       V2 1 2 200 IR9
       L1 2 1 1H
       V3 1 4 1 IL1
       R2 4 5 1
       C2 5 1 1F
       V4 1 6 1 VC2
       R3 6 7 0.1
       C3 7 1 1F
       I1 1 B 2.24 IR9
       I2 1 B 4.256 VC3
       R6 B 1 1
       V5 1 9 1 VR6
       P4 9 10 0.05
       C4 10 1 1F
       I3 1 11 1 VC4
       I4 1 11 1.7 IR9
       I5 1 11 -14.018 VC3
       R7 11 1 1
       V6 1 12 1 VR7
       R5 12 13 0.05
       C5 13 1 1F
       I6 1 14 1 VC5
       I7 1 14 10.762 VC3
       R8 14 1 1
       V7 1 15 1 VR8
       R9 3 15 1              NUMBER OF LOOPS PER ORDER
       OUTPUT
       VC3/VV1                       1=    10
       TIME 1.0                      2=    16
       EXECUTE                       3=    10
                                     4=     2

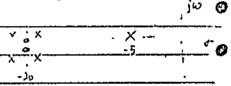
       TRANSFER FUNCTION   VC3/VV1

                    (                                   2 )
                    (  4.00E 02    +4.00E 01 S   +1.00E 00 S   )
       H(S)= 2.000E 03*------------------------------------------------------------------
                    (                                   2              3              4              5 )
                    (  8.00E 05    +3.20E 05 S   +4.40E 04 S   +2.80E 03 S   +8.50E 01 S   +1.00E 00 S   )

                                          POLE OF TRANSFER FUNCTION

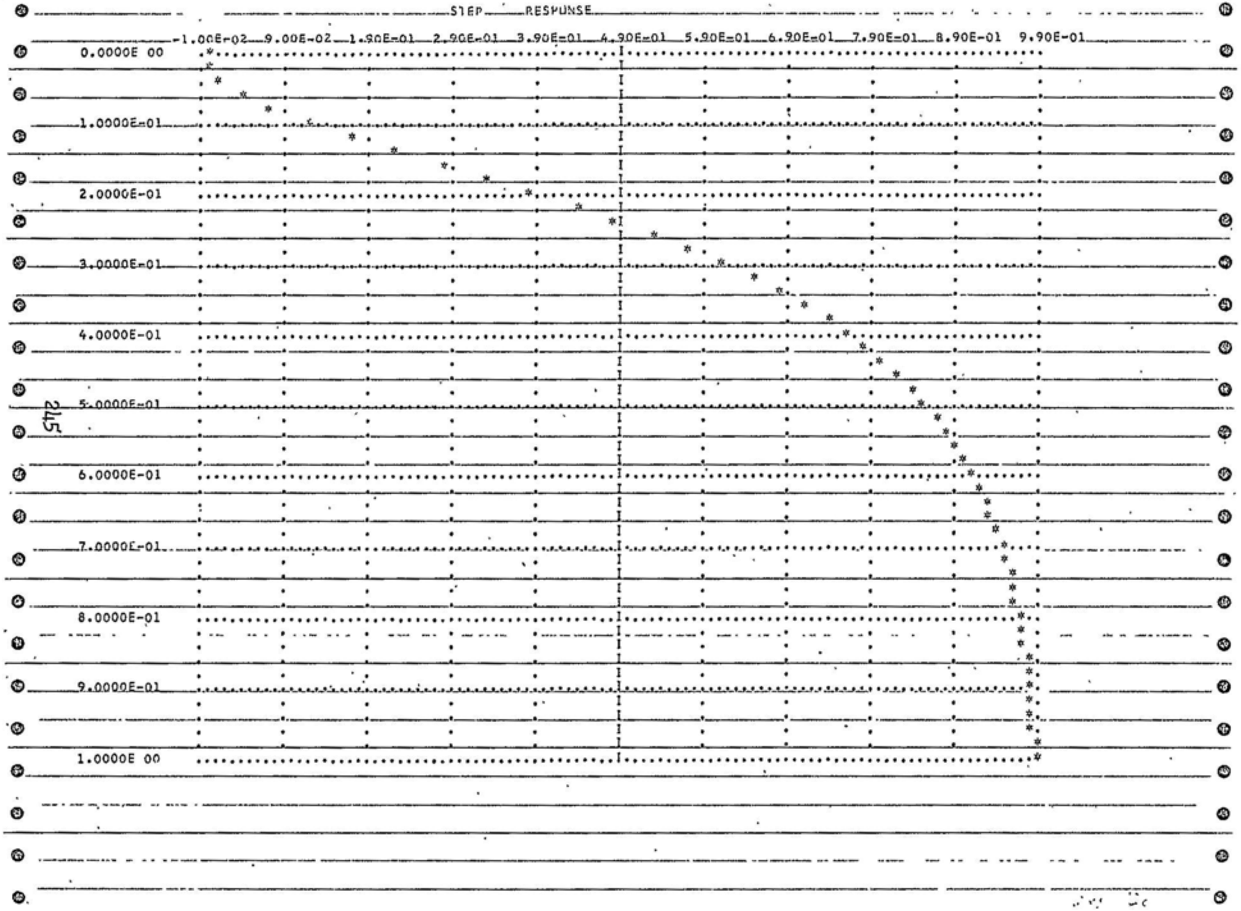
                                          POLE   REAL PART    IMAG. PART
        ZERO OF TRANSFER FUNCTION

        ZERO   REAL PART   IMAG. PART      1 -0.20682E 02 -0.69237E 00
                                           2 -0.20682E 02  0.69237E 00
                                           3 -0.19325E 02  0.66244E 00
           1 -0.20000E 02  0.16526E-01     4 -0.19325E 02 -0.66244E 00
           2 -0.20000E 02 -0.16526E-01     5 -0.50000E 01  0.00000E 00
```



```
STEP RESPONSE FUNCTION                                          STEP  RESPONSE

F(T) =                                                          TIME            VC3/VV1
                                                                0.0000E 00      0.26226044E-05
                                 (-0.2068E 02  J-0.6924E 00 ) T 0.2000E-01      0.21390915E-02
          (-0.1041E 01  J 0.1232E 01 ) E                        0.4000E-01      0.13778249E-01
                                                                0.6000E-01      0.37224972E-01
                                 (-0.2068E 02  J 0.6924E 00 ) T 0.8000E-01      0.73013723E-01
          (-0.1041E 01  J-0.1232E 01 ) E                        0.1000E 00      0.11717720E 00
                                                                0.1200E 00      0.16741727E 00
                                 (-0.1932E 02  J 0.6624E 00 ) T 0.1400E 00      0.22115964E 00
          ( 0.1430E 01  J-0.1208E 01 ) E                        0.1600E 00      0.27627476E 00
                                                                0.1800E 00      0.33111763E 00
                                 (-0.1932E 02  J-0.6624E 00 ) T 0.2000E 00      0.38449517E 00
          ( 0.1430E 01  J 0.1208E 01 ) E                        0.2200E 00      0.43559955E 00
                                                                0.2400E 00      0.48388535E 00
                                 (-0.5000E 01  J 0.0000E 00 ) T 0.2600E 00      0.52909425E 00
          (-0.1776E 01  J 0.4791E-08 ) E                        0.2800E 00      0.57109714E 00
                                                                0.3000E 00      0.60989457E 00
                                 ( 0.0000E 00  J 0.0000E 00 ) T 0.3200E 00      0.64556819E 00
          ( 0.9994E 00  J-0.1198E-08 ) E                        0.3400E 00      0.67825389E 00
                                                                0.3600E 00      0.70811856E 00
                                                                0.3800E 00      0.73534657E 00
                                                                0.4000E 00      0.76012969E 00
                                                                0.4200E 00      0.78265649E 00
                                                                0.4400E 00      0.80311209E 00
                                                                0.4600E 00      0.82167155E 00
                                                                0.4800E 00      0.83950056E 00
                                                                0.5000E 00      0.85375313E 00
                                                                0.5200E 00      0.86757113E 00
                                                                0.5400E 00      0.88008666E 00
                                                                0.5600E 00      0.89141941E 00
                                                                0.5800E 00      0.90167969E 00
                                                                0.6000E 00      0.91096765E 00
                                                                0.6200E 00      0.91937453E 00
                                                                0.6400E 00      0.92698336E 00
                                                                0.6600E 00      0.93386942E 00
                                                                0.6800E 00      0.94010109E 00
                                                                0.7000E 00      0.94574052E 00
                                                                0.7200E 00      0.95084363E 00
                                                                0.7400E 00      0.95546150E 00
                                                                0.7600E 00      0.95964009E 00
                                                                0.7800E 00      0.96342123E 00
                                                                0.8000E 00      0.96684259E 00
                                                                0.8200E 00      0.96993846E 00
                                                                0.8400E 00      0.97273967E 00
                                                                0.8600E 00      0.97527456E 00
                                                                0.8800E 00      0.97756815E 00
                                                                0.9000E 00      0.97964346E 00
                                                                0.9200E 00      0.98152131E 00
                                                                0.9400E 00      0.98322052E 00
                                                                0.9600E 00      0.98475796E 00
                                                                0.9800E 00      0.98614913E 00
                                                                0.1000E 01      0.98740792E 00
```


STEP RESPONSE
-1.00E-02
9.00E-02
1.90E-01
2.90E-01
3.90E-01
4.90E-01
5.90E-01
6.90E-01
7.90E-01
8.90E-01
9.90E-01
0.0000E 00
1.0000E-01
2.0000E-01
3.0000E-01
4.0000E-01
5.0000E-01
6.0000E-01
7.0000E-01
8.0000E-01
9.0000E-01
1.0000E 00

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APPENDIX A

STEP AND RAMP RESPONSE CAPABILITY FOR NASAP

The algorithm used by NASAP to determine the impulse response of a circuit can be easily extended to find the step and ramp response. Listed below are the modifications that are necessary to accomplish this. Note that if, for example, both the impulse and step responses are required for a particular circuit, the problem must be executed twice.

To the COMMON/FREQ/ cards of subroutines NASINP and BODE must be added:

```
,KRESP
```

The following additions must be included in subroutine NASINP. After card 1010, three cards must be added:

```
     Go to 3
824  WRITE (6, 117)                                                  1011
117  FORMAT (35H INCLUDE TYPE OF TIME RESPONSE CARD)                 1012
                                                                     1013
```

After card 1180, nine cards must be added:

```
READ (5,101) INLST                                                   1181
WRITE (6,107) INLST                                                  1182
KRESP = 1000                                                         1183
IF (ICHAR(INLST(1)).  EQ. 49. AND ICHAR(INLST(2))EQ.54) KRESP = 1    1184
IF (ICHAR(INLST(1)).  EQ. 55. AND ICHAR(INLST(2)).EQ.56)KRESP = 1    1185
IF (ICHAR(INLST(1)).  EQ. 62)                           KRESP = 2    1186
IF (ICHAR(INLST(1)).  EQ. 59  AND ICHAR(INLST(2)).EQ.41 KRESP = 3    1187
IF (KRESP. EQ.1000)   GO TO 824                                      1189
WRITE (6,116)                                                        1189A
```

The following changes must be made in INV:

```
Change card 14930
From DIMENSION F(100), T(100), COE(2)
To   DIMENSION F(100), T(100), COE(2), TYPE (2,3)
Change card 14990
From WRITE (6,50)
To   WRITE (6,50)    (TYPE(J,KRESP), J=1,2)
Change card 1500
From 50 FORMAT (26H1 IMPULSE RESPONSE FUNCTION //7H F(t)=)
To   50 FORMAT (1H1, 2A4, 18H RESPONSE FUNCTION //7H F(t)=)
Change card 15280
From WRITE (6,40) TRNS,(T(K), F(K), K=1, 51)
To   WRITE (6,40) (TYPE(J,KRESP), J=1,2), TRNS, (T(K), F(K), K=1, 51)
Change card 15290
From 40 FORMAT(17H1 IMPULSE RESPONSE // 5H TIME, 18X, 10A1/(E12,4,E23.8))
To   40 FORMAT(1H1,2A4,  9H RESPONSE // 5H TIME, 18X, 10A1/(E12.4,E23.8))
Change card 15300
From WRITE (6,41)
```

cont'd.

```
     To  WRITE (6,41) (TYPE(J,KRESP), J=1,2)
     Change card 15310
     From 41 FORMAT (1H1, 45X, 17HIMPULSE  RESPONSE, /)
     To   41 FORMAT (1H1, 45X,   2A4,  9H  RESPONSE, /)
```

The following additions must be included in subroutine AINIV.
After card 14930, two cards must be added:

```
     DATA TYPE /4HIMPU,4HLSE, 4HSTEP, 4H    ,4HRAMP, 4H             14931
     COMMON/FREQ/THI, FEQ (3), CC,Q, KRESP
```

After card 15060, twelve cards must be added:

```
     GO To (77,76,78,76), KRESP                                     15061
 76    IAD2=IAD2+1                                                  15062
     RootR (IAD2,2) = 0                                             15063
     RootI (IAD2, 2) =0                                             15064

     Go To 77                                                       15066
 78    IAD2=IAD2+1                                                  15067
     RootR(IAD2,2) = 0                                              15068
     RootI(IAD2,2) = 0.001                                          15069
     IAD2= IAD2+1                                                   15069A
     RootR (IAD2,2) = 0                                             15069B
     RootI (IAD2,2) = -0.001                                        15079C
 77      CONTINUE                                                   15069E
```

When the above modifications are incorporated in the NASAP program package, the only change from a user's point of view is that he <u>must</u> include one card, <u>immediately</u> following the NASAP PROBLEM card, on which is punched (beginning in column one) one of the four following comments:

NO RESPONSE
IMPULSE RESPONSE
STEP RESPONSE
RAMP RESPONSE

The choice of which comment is to be used depends on what type of time response, if any, is desired. In actuality, only the first two letters of each of the above comments are really necessary . In cards 1181 through 1189A of subroutine NASINP, the program reads the card after the NASAP PROBLEM card and then prints it. If the letters in columns one and two are on R and a A respectively, the time response variable KRESP is set equal to 3. If there is an N in column one and an O in column two, KRESP thenis set equal to 1. If there is an I in column 1 and an M in column two KRESP is set equal to 1. If there is an S in column one, KRESP then has a value of 2. If the program encounters none of the above letter combinations on the second input card, it will print out the error message given on card 1013 and about the job.

The variable KRESP is used in subroutine INV to select the proper elements of the two-dimensional matrix TYPE such that the headings of the printout of the time response agree with the information on the second input card. Thus, if a step response is desired(KRESP=2), the elements in the second column of TYPE will be printed. The variable KRESP is also used in INV to select the proper poles that must be included with the transfer function poles to yield the particular time response. On lines 15061, if KRESP=1(impulse response), the

cont'd

APPENDIX B

Suggested Revision that combines Subroutines SENS & SENSS of NASAP 69/I into a single shorter Subroutine SENSS

In the University of Pennsylvania monthly report for July 1969, the possibility was mentioned of eliminating subroutine SENS by using the results of the calculations of subroutine SENSS (namely S_X^H) and the four equations.

$$S_X^{|H|} = \text{Re } S_X^H$$

$$S_X^{\Phi} = \frac{1}{\Phi} \cdot \text{Im } S_X^H$$

$$S_X^{\text{Re } H} = \text{Re } S_X^H - \left(\frac{\text{ImH}}{\text{ReH}} \cdot \text{Im } S_X^H\right)$$

$$S_X^{\text{ImH}} = \text{Re } S_X^H + \left(\frac{\text{ReH}}{\text{ImH}} \cdot \text{Im } S_X^H\right)$$

where $S_X^H = \text{Re } S_X^H + j \text{ Im } S_X^H$

and $H(jw) = |H| \; e^{j\Phi} = \text{Re } H + j \text{ Im } H.$

This can be accomplished by making the following additions to subroutine SENSS. After card 17000, add the following 3 cards: cards 15370, 15380, and 15390 from SENS. After card 17030, remove cards 17040, 17050, and 17060 and add the following 6 cards:

```
     COMMON/LOOPS/DM(100), PHIH(250), LOGF(250), LGBSNS(250), ABSH(250),
   1 ABSENS(250), REH(250), IMH(250), PHISNS(250), CSENS(250), SENSRE(250),
```

card 15440 from SENS

card 15450 from SENS with DUM1(288) changed to DUM1(38) card 15490 from SENS

card 15500 from SENS with the addition of ,LGBSNS.

After card 17110, add the following 2 cards: cards 15520 and 15530 from SENS.

After card 17540, add the following 39 cards:

card 15540 from SENS

card 15550 from SENS with 1HO changed to 1H1.

card 15560 from SENS

```
    Do 802  I=1, NITR
    Z = 10.** LOGF(I)
    RE = REAL (CSENS(I))
    AIM = AIMAG (CSENS(I))
    CALL QZERO (REH,I,SENSRE,I,GLAG, $81, $82)
 81 SENSRE (I) = RE - (IMH(I)/REH(I)) * AIM
 82 CALL QZERO (IMH,I,SENSIM,I,GLAG, $83, $84)
 83 SENSIM (I) = RE + (REH(I)/IMH(I)) * AIM
 84 SENSBS (I) = RE
 97 CALL QZERO (PHIH,I,SENSFI,I,GLAG, $98, $99)
 98 SENSFI (I) = (180./PI) * (AIM/PHIH(I))
 99 WRITE (6,180) LOGF(I),Z,SENSRE(I),SENSIM(I),SENSBS(I),SENSFI(I)
180 FORMAT (5X,6(E 15.7, 2X))
802 CONTINUE
```

cards 16110 thru 16320 from SENS (22 cards)

The following 10 cards of SENSS must be slightly modified: (NOTE - etc. means that the rest of the card remains unchanged).

Change card 17120

from WRITE(6,100)

to WRITE(6,200)

card 17130

from 100 FORMAT(1H1, etc.

to 200 FORMAT(1H0, etc.

card 17390

from 96 WRITE(6,105) etc.

to 96 WRITE(6,205) etc.

card 17400

from 105 FORMAT etc.

to 205 FORMAT etc.

card 17450

from WRITE(6,102)

to WRITE(6,202)

card 17460

from 102 FORMAT etc.

to 202 FORMAT etc.

card 17470

from WRITE(6,103) etc.

to WRITE(6,203) etc.

card 17480

from 103 FORMAT etc.

to 203 FORMAT etc.

card 17520

from WRITE(6,102)

to WRITE(6,202)

card 17530

from WRITE(6,103) etc.

to WRITE(6,203) etc.

Finally in BODE remove card 13510. In SPLOT the following 2 minor modifications are needed:

On card 16670

change ARR(1000), DUMM2(1750)

to ARR(750) , DUMM2(2250)

On card 16690

change DUM1(288)

to DUM1(38)

These changes essentially delete cards 15570 thru 16100 from SENS. The calculations in this block of cards are replaced by the Do-Loop (labelled 802) given above. The remaining cards of SENS are then inserted into subroutine SENSS in the appropriate locations. On the RCA Spectra 70, the original version of SENS requires 4000 bytes of memory and the original version of SENSS requires 2540 bytes. However the suggested new version of SENSS requires 4392 bytes - thus realizing a saving of 2148 bytes. Enclosed is a listing of the new version of SENSS. Due to the use of a BCD coded program on a EBCDIC machine, the following symbols are equivalent:

\# is equivalent to =
% " " " (
< " " ")
& " " " +

Note that the printed output of the original SENSS now <u>precedes</u> the printed output of the original version of SENS.