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**PRESSURE WAVE PROPAGATION THROUGH  
ANNULAR AND MIST FLOWS**

by Robert E. Henry  
Lewis Research Center  
Cleveland, Ohio

TECHNICAL PAPER proposed for presentation at  
Sixty-third Annual Meeting of the American  
Institute of Chemical Engineers  
Chicago, Illinois, November 29-December 3, 1970

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by Robert E. Henry

National Aeronautics and Space Administration  
Lewis Research Center  
Cleveland, Ohio

ABSTRACT

One-dimensional models are formulated to describe pressure wave propagation through one- and two-component annular and mist flow patterns. The solutions include the effect of inertial interphase momentum in these configurations. In addition, the nature and importance of interphase mass transfer is discussed for one-component systems. The models exhibit good agreement with the available data.

## INTRODUCTION

The phenomenon of pressure wave propagation in one- and two-component annular, annular-dispersed, and mist flow patterns has been the subject of several experimental investigations.<sup>(1-6)</sup> The common object of these studies was to obtain pertinent information regarding the critical flow of such mixtures. The detailed results have shown conclusively that the propagation velocity and the critical flow rate cannot be directly related as they are for single phase systems. The data also show that homogeneous equilibrium models, such as that proposed by Steltz,<sup>(7)</sup> are in error by as much as 200%.

Fauske<sup>(8)</sup> has shown that a realistic model for bubbly flows must include the rate of interphase momentum transfer. Henry<sup>(9)</sup> presented a formulation which correlated the interphase momentum transfer in terms of the virtual mass associated with the discrete gaseous bubbles. This correlation exhibits good agreement with the available bubbly flow data in one- and two-component mixtures. In contrast, Henry et al.<sup>(10)</sup> developed a propagation model for stratified and annular flows which assumes negligible rates for interphase heat, mass, and momentum transfer. The proposed solution is in excellent agreement with the two-component stratified data. The model was also compared to high quality, steam-water mist flows and agreed to within 10% of the available data, however, it did not predict the dependency upon the mixture quality which the experimental data exhibited.

The one-dimensional analyses in this paper consider first the role of virtual mass in the interphase momentum transport. This concept is analyzed for both droplet and wavy-annular flows. Secondly, the inter-

phase mass transport rates for one-component mixtures are considered for rarefaction and compression waves. Based on geometrical and interface conditions, approximations are made to the thermodynamic equilibrium relations to describe such processes.

### ANALYSIS

The propagation of a pressure wave through a stationary gas-liquid mixture is a transient phenomenon when viewed in an Eulerian reference frame. As shown in Fig. 1, if the propagation is viewed from a Lagrangian reference system moving with the wave, the wave front becomes stationary, and the previously stationary mixture will become a homogeneous mixture ( $k = 1$ ) which is approaching the wave front with the propagation velocity  $a_{TP}$ . As the amplitude of the wave becomes small, the gas and liquid velocities behind the wave become small such that  $u_g \approx u_l \approx a_{TP}$ . This approximation enables one to express the one-dimensional continuity and momentum equations for the moving reference frame as:

$$a_{TP} \frac{d}{dZ} [\alpha \rho_g + (1 - \alpha) \rho_l] + \alpha \rho_g \frac{du_g}{dZ} + (1 - \alpha) \rho_l \frac{du_l}{dZ} = 0 \quad (1)$$

and

$$\frac{dP}{dZ} + a_{TP}^2 \frac{d}{dZ} [\alpha \rho_g + (1 - \alpha) \rho_l] + 2a_{TP} \left[ \alpha \rho_g \frac{du_g}{dZ} + (1 - \alpha) \rho_l \frac{du_l}{dZ} \right] = 0 \quad (2)$$

It is assumed that  $\alpha$ ,  $\rho_g$ , and  $\rho_l$  are of the form  $f[P(Z)]$  such that

$$\frac{df}{dZ} = \frac{df}{dP} \frac{dP}{dZ}$$

then Eqs. (1) and (2) can be combined to give a general expression for the propagation velocity

$$a_{TP}^2 = \left\{ \frac{d}{dP} \left[ \alpha \rho_g + (1 - \alpha) \rho_l \right] \right\}^{-1} \quad (3)$$

The void fraction is given by

$$\alpha = \frac{x \rho_l}{(1 - x) k \rho_g + x \rho_l} \quad (4)$$

which can be differentiated to give

$$\frac{d\alpha}{dP} = \frac{\alpha(1 - \alpha)}{\rho_l} \frac{d\rho_l}{dP} + \frac{\alpha(1 - \alpha)}{x(1 - x)} \frac{dx}{dP} - \alpha(1 - \alpha) \frac{dk}{dP} - \frac{\alpha(1 - \alpha)}{\rho_g} \frac{d\rho_g}{dP} \quad (5)$$

where the magnitude of the velocity ratio has been set equal to unity.

Substituting Eq. (5) into Eq. (3) gives:

$$a_{TP}^2 = \left\{ \left[ \alpha^2 + \alpha(1 - \alpha) \frac{\rho_l}{\rho_g} \right] \frac{d\rho_g}{dP} + \left[ (1 - \alpha)^2 + \alpha(1 - \alpha) \frac{\rho_g}{\rho_l} \right] \frac{d\rho_l}{dP} + (\rho_g - \rho_l) \frac{\alpha(1 - \alpha)}{x(1 - x)} \frac{dx}{dP} - \alpha(1 - \alpha)(\rho_g - \rho_l) \frac{dk}{dP} \right\}^{-1} \quad (6)$$

For the void fractions commensurate with annular, annular-dispersed, and mist flow patterns, the compressibility of the liquid phase can be neglected  $[(d\rho_l/dP) = 0]$  and for the systems considered herein, the density of the gaseous phase is negligible compared to that of the liquid.

#### TWO-COMPONENT FLOWS

In two-component systems such as air-water mixtures, the inter-phase mass transport is zero  $[(dx/dP) = 0]$ . This fact and the above approximations simplify Eq. (6) to:

$$a_{TP}^2 = \left\{ \left[ \alpha^2 + \alpha(1 - \alpha) \frac{\rho_l}{\rho_g} \right] \frac{d\rho_g}{dP} + \alpha(1 - \alpha) \rho_l \frac{dk}{dP} \right\}^{-1} \quad (7)$$

In Ref. 10, Henry et al. presented a model for stratified and annular flows. As will be subsequently shown, this model, which assumed negligible rates of interphase heat, mass, and momentum transfer, is applicable to stratified and annular flows with smooth interfaces. This solution will be outlined below since the derivation is pertinent to all the models presented herein.

The formulation is one-dimensional, however, in a system such as an air-water mixture, the mismatch between the acoustical impedances of the phases is considerable and one must be aware of the two-dimensional aspects of the propagation. Davies<sup>(11)</sup> states that, in annular flow, each phase will transmit a pressure disturbance at its own propagation velocity. Such behavior is shown in Fig. 2(a), and it is immediately apparent that this situation produces transverse pressure discontinuities which cannot exist. The relaxation of these discontinuities produces a transverse motion similar to that experienced by pressure waves propagating through a liquid contained in a flexible pipe.<sup>(12)</sup> It is proposed that these transverse forces skew the initially planar pulse into a configuration like that shown in Fig. 2(b). When the depth of the liquid layer is much smaller than the length over which the pulse travels, the wave can be considered of constant shape and to travel with a single velocity. It is this velocity that the following derivations are concerned with, and all the approximations made must be considered in light of this two-dimensional flow pattern.

Since the liquid is essentially incompressible, any temperature change due to the passage of a wave front is negligible. Therefore, any interphase heat transfer is dependent upon the gas behavior. Because of the large thermal capacity of the liquid phase ( $\rho_l c$ ), the interphase heat transfer is

governed by transient conduction through the gas to the essentially constant temperature liquid. For the large gas volumes commensurate with typical stratified and annular flows, the thermal response of the gas can be neglected such that

$$\frac{d\rho_g}{dP} = \frac{\rho_g}{\gamma P} \quad (8)$$

By definition

$$k = \frac{u_g}{u_l} \quad (9)$$

such that

$$\frac{dk}{dP} \approx \frac{1}{a_{TP}} \left( \frac{du_g}{dP} - \frac{du_l}{dP} \right) \quad (10)$$

For smooth stratified and annular systems, where each phase is continuous, the momentum equations for the separate phases can be expressed as

$$\frac{dP}{dZ} + \rho_g a_{TP} \frac{du_g}{dZ} = 0 \quad (11)$$

and

$$\frac{dP}{dZ} + \rho_l a_{TP} \frac{du_l}{dZ} = 0 \quad (12)$$

Combining Eqs. (10), (11), and (12) gives

$$\frac{dk}{dP} \approx - \frac{1}{a_{TP}^2 \rho_g} \quad (13)$$

Equations (8) and (13) can be substituted into Eq. (7) to give:

$$\frac{a_{TP}^2}{a_g^2} = \frac{1 + \alpha(1 - \alpha) \frac{\rho_l}{\rho_g}}{\alpha^2 + \alpha(1 - \alpha) \frac{\rho_l}{\rho_g}} \quad (14)$$

The above ratio is essentially equal to unity for the mixtures of interest here ( $\alpha \geq 0.50$ ). The prediction of Eq. (14) will hereafter be referred to as the "smooth annular" model.

This separated flow model was used as an approximate solution for mist or droplet flows in Ref. 9. However, in such flows, the liquid phase is discrete and one must include the virtual mass of the discrete phase in the continuous medium.<sup>(9)</sup> If it is assumed that the droplet is spherical, the momentum equation for a given drop can be written as:

$$-\frac{dP}{dZ} = \rho_l u_l \frac{du_l}{dZ} - \frac{\rho_g}{2} \left( u_g \frac{du_g}{dZ} - u_l \frac{du_l}{dZ} \right) \quad (15)$$

If, in addition, it is assumed that the behavior of a given droplet is characteristic of all droplets in the flow field, Eq. (15) can be combined with the system momentum equation (Eq. (2)) and substituted into Eq. (10) to give:

$$\frac{dk}{dP} = - \frac{1}{\rho_g a_{TP}^2} \left( \frac{2\alpha}{1 + \alpha + \alpha \frac{\rho_g}{\rho_l}} \right) \quad (16)$$

which for low pressures ( $\rho_g \ll \rho_l$ ) can be approximated by



$$\frac{dk}{dP} = - \frac{2\alpha}{a_{TP}^2 \rho_g (1 + \alpha)} \quad (17)$$

The derivative  $d\rho_g/dP$  is dependent upon the rate of interphase heat transfer. Here, as in the smooth annular model, it is assumed that, as a first approximation, the rate of heat conduction through the gas phase is negligible.

The above information can be substituted into Eq. (7) to obtain an expression for pressure wave propagation in two-component droplet or mist flow configurations.

$$\frac{a_{TP}}{a_g} = \left[ \frac{1 + \frac{2\alpha^2(1-\alpha)\rho_l}{(1+\alpha)\rho_g}}{\alpha^2 + \alpha(1-\alpha)\frac{\rho_l}{\rho_g}} \right]^{1/2} \quad (18)$$

For  $\alpha > 0.50$ , Eq. (18) is closely approximated by

$$\frac{a_{TP}}{a_g} = \sqrt{\frac{2\alpha}{1+\alpha}} \quad (19)$$

Most annular flows are characterized by a wavy interface. These surface waves also experience a virtual mass effect when there is relative acceleration with respect to the gaseous core. The effect of surface waves will be investigated by analyzing a case where the wave amplitude is large compared to the minimum film thickness such as the configuration shown in Fig. 3. It is assumed that such a geometry can be approximated by hemicylindrical filaments resting on the surface. For a cylin-

dricul rod in cross flow, the coefficient of virtual mass is equal to unity.<sup>(13)</sup>

Therefore, the momentum equation for a given filament can be written as:

$$-\frac{dP}{dZ} = \rho_l u_l \frac{du_l}{dZ} - \rho_g \left( u_g \frac{du_g}{dZ} - u_l \frac{du_l}{dZ} \right) \quad (20)$$

Following the procedure used for droplet flows, one can derive the following expression for the propagation velocity in the approximate flow configuration shown in Fig. 3.

$$\frac{a_{TP}}{a_g} = \sqrt{\alpha} \quad (21)$$

This formulation will be referred to as the wavy-annular model.

#### COMPARISON WITH TWO-COMPONENT DATA

Since each of the above models assumes that all the liquid is in the geometrical configuration considered in that model, the three solutions should bracket the results in the high void fraction region ( $\alpha > 0.50$ ).

Figure 4 compares the smooth annular model to the experimental data for quiescent stratified (or smooth interface) systems as reported in Ref. 10. (All two-component data presented herein has been corrected for 100% relative humidity.) The agreement for these idealized conditions is excellent.

Evans et al.<sup>(1)</sup> and Hamilton<sup>(2)</sup> reported experimental results for flows which are generally in the annular-dispersed or mist flow regimes. These data are shown in Fig. 5 and are in general agreement with the proposed models. Except at very low air velocities, films are relatively thin which minimizes the wave amplitude. Therefore, one would expect better agreement with the droplet model than with the wavy-annular approach. The data and flow regime observations of Ref. 1 substantiate

such reasoning. The flow regime for the data point at  $\alpha = 0.57$  should be approaching annular-slug transition and thus should contain large surface waves. Therefore, this particular point should be, and is, more closely approximated by the wavy annular model.

The experimental data of Garrard<sup>(3)</sup> is shown in Fig. 6(a). The void fractions reported in Ref. 3 are determined from film thickness measurements using wall mounted conductance probes. Such measurements are insensitive to liquid entrained in the gaseous core. On the basis of the measurements reported by Hewitt,<sup>(14)</sup> it seems likely that the vertically upward flows described in Ref. 3 would experience some entrainment. To afford the reader a basis of comparison, the total liquid flow areas were estimated with the Lockhart-Martinelli<sup>(15)</sup> void fraction correlation. The data is replotted as a function of the calculated void fraction in Fig. 6(b). For the thin wall films reported by Garrard, one would expect a relatively smooth annulus with entrainment in the core, thus, the data should be bracketed by the smooth annular and droplet models, which is indeed the case in Fig. 6(b). Since the annulus contributes no momentum transfer, the mist flow model and the data can be compared on the basis of the core void fraction. The void fraction in the dispersed core can be estimated by subtracting the liquid film area from the total liquid area predicted by the Lockhart-Martinelli correlation. This comparison is given in Fig. 6(c) and it is seen that the experimental data are in good agreement with the mist flow prediction.

In summary, propagation models have been presented for two-component smooth annular, wavy-annular, and mist flow patterns. The models emphasize the effect of the flow configuration on the rate of inter-

phase momentum transport, and all exhibit good agreement with experimental data from corresponding flow regimes.

### ONE-COMPONENT FLOWS

The various momentum considerations discussed in the two-component section will be assumed valid for equivalent one-component flow patterns.

A one-component mixture introduces an additional degree of freedom to the propagation phenomenon; namely, interphase mass transfer. As discussed in Ref. 9, the experimental data for bubbly flows indicates that the mass transfer rate in the wave front is negligible. However, this conclusion may not be applicable to other flow regimes.

Considerable effort has been directed toward analyzing pressure wave propagation in reacting gas mixtures,<sup>(16, 17)</sup> which is a problem closely allied with those considered herein. The major conclusion of these studies is that for any finite reaction rate, the wave front propagates in a frozen (no mass transfer,  $dx/dP = 0$ ) manner. This particular conclusion is substantiated by some very ingenious and detailed shock wave data taken in a supersonic nozzle.<sup>(18)</sup> Another important conclusion of Refs. 16 and 17 is that the amplitude of the wave front decays rapidly and the bulk of the wave travels at some velocity less than the frozen speed. It is shown in Ref. 17 that for times

$$t \gg \frac{2\lambda}{\frac{a_F^2}{a_E^2} - 1} \quad (22)$$

the bulk of the wave travels at a velocity close to that given by an equilibrium mass transfer formulation. Therefore, an experimental investigation which claims to have measured frontal velocities may actually be

measuring bulk wave velocities.

When the frontal velocity is considered in light of the decaying amplitude and the two-dimensional aspects discussed above and illustrated in Fig. 2, it seems reasonable that the bulk velocity is the practical quantity to deal with when describing the overall one-dimensional behavior of a liquid-vapor system. Thus, the following solutions are intended to represent the bulk wave behavior.

As shown in Eq. (22), to determine the effects of mass transfer one must have a knowledge of the reaction (mass transfer) rate. To obtain such information for a two-phase system requires an understanding of the governing processes and the interfacial surface area. The latter requirement demands a detailed picture of the flow configuration which is the general unknown in two-phase flow. Therefore, one cannot present quantitative models, but by considering the nature of the processes and the general flow pattern, approximate solutions can be generated which give new insight into the overall one-dimensional behavior of one-component, two-phase systems.

The nature of interphase mass transfer differs considerably between compression and rarefaction waves. A compression wave propagating through a mixture initially in equilibrium produces superheated vapor and subcooled liquid which are each individually stable states. To relax to equilibrium, the liquid must first be heated by the vapor which is a process essentially controlled by conduction in the vapor. Thus, the one-component compression wave case is very similar to two-component flows. In contrast, rarefaction waves generate superheated liquid and supersaturated vapor which are both metastable states and each may relax

independently of the other. Such a relaxation for the vapor requires condensation which in turn requires a nucleation site. The liquid temperature is greater than that of the vapor, hence, the liquid phase cannot serve as the site. As illustrated by the condensation shocks witnessed in the flow of saturated vapor through converging-diverging nozzles, vapor can be considerably supersaturated before any spontaneous relaxation to equilibrium is initiated.<sup>(19)</sup> Therefore, it will be assumed that in the wave front, no condensation exists and the vapor behaves isentropically as if it were superheated steam ( $\gamma \approx 1.3$ ). The superheated liquid vaporizes as it relaxes to a stable equilibrium condition. Since there is a definite waiting time involved in the growth of vapor bubbles, it is assumed no bubbles are formed within the liquid phase. Thus, the vaporization occurs at the interface. Vaporization at the liquid interface yields a surface temperature equal to the local saturation value which remains essentially constant regardless of the amount vaporized. The response of the liquid phase and the mass transfer is thus governed by the conduction response of the liquid volume with the imposed constant temperature boundary condition.

As outlined above, the interphase mass transfer rate is governed by the transient conduction response of the vapor and the liquid for compression and rarefaction waves, respectively. For identical geometries, the transient conduction response of water is more than two orders of magnitude faster than that of steam.<sup>(20)</sup> For typical annular and mist flows, the vapor volume fraction is one order of magnitude greater than that of the liquid. Hence, the transient response of the liquid is 1000 times faster than that of the vapor. Based on these crude estimates, it is assumed that the transient conduction response of the vapor is negligible ( $dx/dP = 0$ ).

Hence, the solutions for compression waves propagating through one-component annular, wavy-annular, and mist flows are identical to the two-component solutions presented above. (One possible exception will be discussed later.) However, the mass transfer response in rarefaction waves requires further consideration.

As was discussed above, the rate at which the mass transfer occurs is dependent on the nature of the process and the area available for transfer has been equated to the rate at which liquid conducts heat to the interface or

$$q = -K_l A_s \left. \frac{dT_l}{dy} \right)_s \quad (23)$$

where  $A_s$  is the interfacial surface area per pound mass of liquid and  $dT_l/dy)_s$  is the temperature gradient at the surface. For heat transfer purposes two geometries are considered herein - thick films (smooth and wave-annular flows) and small droplets (mist flows). The surface area for mist flows can be one to three orders of magnitude greater than that for the annular flows. Hence, if the interphase mass transfer is appreciable, the effect will be most noticeable in the mist flow regime. It is assumed, as a first approximation, that the mass transfer rate for films is negligible because of their comparatively small surface area and thick geometry, but that the rate of small droplets can be approximated by the equilibrium rate for vaporizing liquid. Therefore the rarefaction wave models for smooth annular and wavy-annular flows are identical to the two-component solutions. For mist flows, the total system and vapor entropies are assumed constant such that

$$ds_0 = d[(1-x)s_f + xs_g] = 0 \quad (24)$$

and the mass transfer rate can be expressed by

$$\frac{dx}{dP} = - \frac{(1-x)}{s_{fg}} \frac{ds_f}{dP} \quad (25)$$

Hence, the mist flow solution for rarefaction waves can be expressed as

$$a_{TP} = \left\{ \frac{1 + \frac{2\alpha^2(1-\alpha)\rho_f}{(1+\alpha)\rho_g}}{\left[ \alpha^2 + \alpha(1-\alpha) \frac{\rho_f}{\rho_g} \right] a_g^2 + \rho_f \frac{\alpha(1-\alpha)}{xs_{fg}} \frac{ds_f}{dP}} \right\}^{1/2} \quad (26)$$

In summary, considerations of the interphase mass transfer characteristics has lead to the following approximate solutions for the bulk wave velocity of pressure disturbances propagating through one-component mixtures.

(1) The interphase mass transfer in compression waves is governed by the conduction response of the vapor which is a comparatively slow process. Thus, it was assumed the mass transfer rate was negligible and the solutions for the three flow patterns considered are the same as the two-component models.

(2) In rarefaction waves the liquid conduction response controls the mass transfer. This is a much faster process, however, it was assumed the small surface area and thick geometry of the annular flows severely restricted this response and the mass transfer rate was negligible. Hence, these two models are also the same as their two-component counterparts. However, for mist flows, the large interfacial area and minute geometries were considered conducive to large mass transfer rates. An equilibrium liquid vaporization solution was assumed for these flows.



## COMPARISON WITH ONE-COMPONENT DATA

The steam-water stratified data of Grolmes and Fauske<sup>(21)</sup> is compared to the smooth interface, no mass transfer model in Fig. 7. It is readily apparent that the compression waves exhibit excellent agreement with the analytical prediction. The rarefaction waves also are in good agreement with the model, however, there is a small but consistent difference between the two. This small discrepancy is probably the result of some surface evaporation but since the conduction resistance of the liquid film is appreciable, the liquid mass transfer rate is considerably less than the equilibrium value and, for all practical purposes, negligible as was assumed.

White and D'Arcy<sup>(22)</sup> measured the propagation velocities of rarefaction waves in steam-water annular flow systems. The mixer was designed to generate annular flows in a horizontal test section. No information is given about the shape of the interface in these flows, however, the smooth and wavy-annular models should bracket the data. As is illustrated in Fig. 8 this is generally the case. (The void fractions were calculated by the authors using a correlation given in Ref. 23.)

Figure 9 compares the experimental results for compression and rarefaction waves propagating through a single phase medium (air) and a 50 percent quality steam-water mixture in mist flow. These data, taken by DeJong and Firey,<sup>(6)</sup> clearly illustrate the difference in the propagation phenomenon between the two media. The one-component mist flow models developed herein show excellent agreement with the measured velocities of small amplitude waves.

Figure 10 compares the models and the experimental data of England et al.<sup>(5)</sup> for rarefaction waves propagating through steam-water mist flows. (Since the single phase propagation velocity is slightly dependent on the air content in the steam, which was not specified,  $a_g$  was taken to be the average value measured in slightly superheated steam.) Again there is generally good agreement between the model and the data. As stated in Ref. 5, at the lower qualities ( $x \approx 0.20$ ), some liquid was forming on the wall, thus, decreasing the rate of interphase mass transfer. For such flow regimes, the data should be bracketed by the mist and annular predictions which is indeed the case. In fact, since the wall film contributes no mass transfer, one would expect that propagation velocities for annular-dispersed flows could be calculated based on the mixture quality in the dispersed core. This is substantiated in Fig. 11 which exhibits the experimental results of Collingham and Firey<sup>(4)</sup> for rarefaction waves propagating through annular-dispersed steam-water mixtures. The data are reported as functions of both gross and core qualities. As seen in Fig. 11(a), the data are bracketed by the mist and annular models as a function of gross quality, and, as shown in Fig. 11(b), the mist flow model affords a good prediction when based on the quality of the dispersed core.

In Figs. 9(b), 10, and 11, the mist flow velocities for rarefaction waves are slightly less than the analytical predictions. As was discussed above, the bulk wave behavior is preceded by a low amplitude wave front which travels at the frozen velocity. The small interphase velocity differences set up behind this front could cause some viscous momentum transfer in the bulk behavior which would produce such lower velocities.

To summarize, considerations of the interphase mass transfer processes lead to the conclusion that such phenomenon are negligible in all cases except for rarefaction waves propagating through mist flows. A model which assumed negligible vapor condensation and equilibrium liquid vaporization was proposed for mist flows.

One final point to be mentioned here is that the models and data discussed above are characteristic of small pulses traveling through a mixture which is initially in equilibrium. Therefore, they are representative of neither sound waves nor critical flow and should not be construed as "sonic" or "critical" velocities. As discussed in Ref. 9 for bubbly mixtures and in Ref. 24 for mist flows, the velocity of sound in two-phase media is frequency dependent because of the relaxation times associated with the interphase transport of heat, mass, and momentum. It has also been shown in Refs. 25 and 26 that the velocities associated with one-component critical flows are characterized by considerable nonequilibrium in the mass transfer process, and such flows cannot be treated by assuming negligible interphase mass transfer.

#### COMMENTS ON THE DATA OF SEMENOV AND KOSTERIN

Other than the annular flow results of White and D'Arcy, the only one-component data reported for the void fraction range 0.50 to 0.90 are the steam-water compression wave results of Semenov and Kosterin.<sup>(27)</sup> In light of the other experimental studies enumerated above, this investigation requires special attention. The experimental results, which were obtained in vertical-up flow, are shown in Fig. 12. For comparison, the air-water data given in Ref. 27 for a horizontal channel are also given. It is obvious that the air-water data does not agree with the results of Refs.

1-3, which raises questions regarding what flow configuration the data characterizes. No flow regime observations are given, but it is possible that some type of slug flow existed and the propagation was of the inertial type described in Refs. 10 and 28. For most of the void fraction range reported, this could also be true of one-component flows. However, for void fractions in the range from 0.90 to 1.0, a dispersed-annular configuration seems more likely than a slug pattern. Therefore, are the results characteristic of compression waves propagating through a one-component dispersed flow?

As was discussed in the previous section, the vapor controlled mass transfer for compression waves is generally so small that it can be neglected. However, Eq. (22) shows the bulk wave behavior is also dependent upon the ratio of the frozen and equilibrium velocities. In the void fraction range investigated, this difference is quite large. Thus, perhaps the data of Ref. 27 satisfy Eq. (22) and, thus, are representative of the bulk of the wave traveling at the equilibrium velocity. The limiting mass transfer curves are shown in Fig. 12 (only virtual mass momentum transfer is considered) and the equilibrium mass transfer solutions describe the general character of the data.

From the above discussion, it is apparent that a more definitive study of this regime is needed. Such a study should include not only rarefaction and compression wave data but also detailed flow regime observations.

#### SUMMARY AND CONCLUSIONS

One-dimensional models are developed for the propagation of pressure waves through one- and two-component mixtures in smooth annular,

wavy-annular, and mist flow configurations. The two-component models describe the nature of the interphase momentum transfer resulting from the virtual mass of the liquid configuration. In addition to momentum transfer, the one-component solutions incorporate assumptions based on the nature of the mass transfer processes in rarefaction and compression waves as well as the flow configuration. The various solutions presented exhibit generally good agreement with data characteristic of corresponding flow patterns.

The following points can be concluded from this investigation.

(1) If the phases are not configured in a smooth interface manner, the virtual mass of the liquid phase must be considered.

(2) The vapor controlled mass transfer process for compression waves is considerably slower than the liquid controlled rarefaction process, and thus, the interphase mass transfer is more significant in rarefaction waves.

(3) The interphase mass transfer is also strongly dependent upon the interfacial area which is determined by the flow pattern. Thus the interphase mass transfer is more significant in dispersed flows than in annular configurations.

#### NOMENCLATURE

A	area
a	propagation velocity
c	specific heat
f	function
K	thermal conductivity
k	velocity ratio, $u_g/u_l$

P	pressure
q	heat transferred per unit time per unit mass
s	entropy
T	temperature
t	time
u	velocity
x	quality
y	transverse length
Z	axial length
$\alpha$	void fraction
$\gamma$	isentropic exponent
$\lambda$	mass transfer relaxation time
$\rho$	density

## Subscripts:

E	equilibrium
F	frozen, $dx/dP = 0$
f	saturated liquid phase
fg	difference between liquid
g	gaseous phase
l	subcooled liquid phase
o	stagnation
P	constant pressure
s	surface area
TP	two-phase

## REFERENCES

1. Evans, R. G., S. W. Gouse, Jr., and A. E. Bergles, Massachusetts Institute of Technology Rept. No. DSR 74629-2 (1968).
2. Hamilton, L. J., Ph.D. Thesis, University of California, Berkeley (1968).
3. Garrard, Jr., C. W., Ph.D. Thesis, Texas A & M University (1968).
4. Collingham, R. E., and J. C. Firey, "Velocity of Sound Measurements in Wet Steam," I & EC Process Design and Dev., 2, No. 3, p. 197 (1963).
5. England, W. G., J. C. Firey, and O. E. Trapp, "Additional Velocity of Sound Measurements in Wet Steam," I & EC Process Design and Dev., 5, No. 2, p. 198 (1966).
6. DeJong, V. J. and J. C. Firey, "Effect of Slip and Phase Change on Sound Velocity in Steam-Water Mixtures and the Relation to Critical Flow," I & EC Process Design and Dev., 7, No. 3, p. 454 (1968).
7. Steltz, W. B., "The Critical and Two-Phase Flow of Steam," Jr. Eng. for Power, 83, No. 2, p. 145 (1961).
8. Fauske, H. K., Paper presented at the EURATOM and Technische Hogeschool te Eindhoven, Symposium on Two-Phase Flow Dynamics, Eindhoven, Netherlands (1967).
9. R. E. Henry, "Pressure Wave Propagation in Two-Phase Mixtures," to be published in Chem. Eng. Prog. Sym. Ser. Heat Transfer - Minneapolis, Vol. 66 (1966).
10. Rhodes, E. and D. S. Scott, eds., Cocurrent Gas-Liquid Flow, p. 1-18, Plenum Press, New York (1969).

11. Davies, A. L., Atomic Energy Establishment, Winfrith Report - No. M452 (1965).
12. Streeter, V. L., Fluid Mechanics, " 2nd Ed. McGraw Hill, New York (1958).
13. Milne-Thompson, L. M., "Theoretical Hydrodynamics, " 4th Edition, The Macmillan Co., New York (1960).
14. Hewitt, G. F., Atomic Energy Research Establishment Rept. No. R-4683 (1964).
15. Lockhart, R. W. and R. C. Martinelli, "Proposed Correlation of Data for Isothermal Two-Phase, Two-Component Flow in Pipes, " Chemical Engineering Progress, 45, No. 1, p. 39 (1949).
16. Roseman, J. and V. Agosta, Polytechnic Institute of Brooklyn, Propulsion Research Laboratory Report No. PRL-61-12 (1961).
17. Springer, R. W. and S. Gratch, American Society of Mechanical Engineers Paper No. 67-FE-2 (1967).
18. Combustion Institute, Eighth Symposium on Combustion, International, p. 348, Williams and Wilkins, Maryland (1962).
19. Hill, P. G., "Condensation of Water Vapour During Supersonic Expansion in Nozzles, " Jr. of Fluid Mechanics, 25, No. 3, p. 593 (1966).
20. Kreith, F., "Principles of Heat Transfer, " International Textbook, Scranton (1958).
21. Grolmes, M. A. and H. K. Fauske, "Propagation Characteristics of Compression and Rarefaction Pressure Pulses in One-Component Vapor-Liquid Mixtures, " Nuclear Engineering and Design, 11, p. 137 (1969).



22. White, R. F. and D. F. D'Arcy, Paper No. 5 presented at Sym. of Fluid Mech. and Measurements in Two-Phase Flow Systems, Univ. of Leeds, England (1969).
23. Collier, J. G., Atomic Energy of Canada Ltd. Rept. No. 1631 (1962).
24. Deich, M. E., G. A. Fillippov, E. V. Stekol'shchikov, and Anisimova, M. P., "Experimental Study of the Velocity of Sound in Wet Steam," Thermal Engineering, 14, No. 4, p. 59 (1967).
25. Henry, R. E., Argonne National Laboratory Rept. No. 7430 (1968).
26. Henry, R. E., Discussion of "A Pressure Pulse Model for Two-Phase Critical Flow and Sonic Velocity," Trans. ASME, J. Heat Transfer, Series C, 91, No. 3, p. 382 (1969).
27. Semenov, N. I. and S. I. Kosterin, "Results of Studying the Speed of Sound in Moving Gas-Liquid Systems," Thermal Engineering, 11, No. 6, p. 59 (1964).
28. Bowles, R. E. and F. M. Manion, U.S. Army Aviation Material Laboratories, Report No. 65-77 (1966).

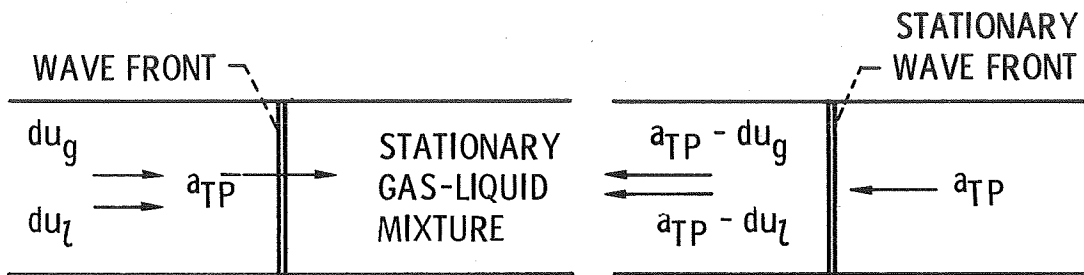
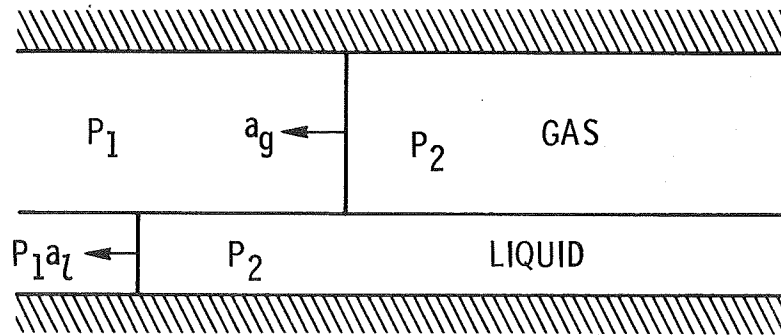
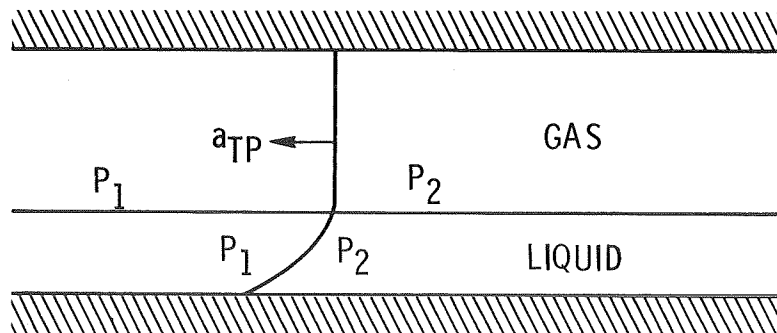


Figure 1. - One-dimensional wave propagation velocity model.



(a) PRESSURE WAVE PROPAGATING SEPARATELY IN EACH PHASE.



(B) PROPOSED WAVE FORM RESULTING FROM TWO-DIMENSIONAL ASPECTS OF THE SYSTEM.

Figure 2

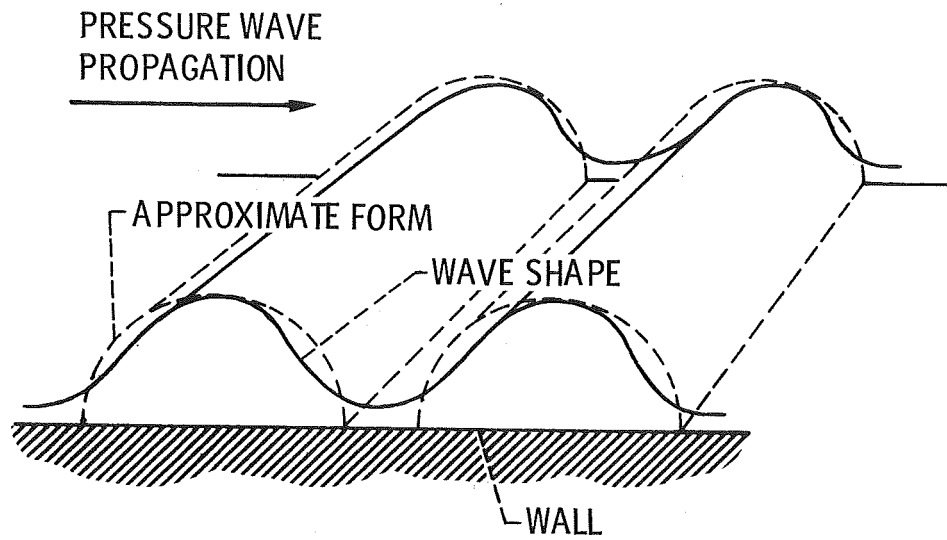


Figure 3. - Approximate wave form employed to investigate momentum transfer in wavy-annular flow.

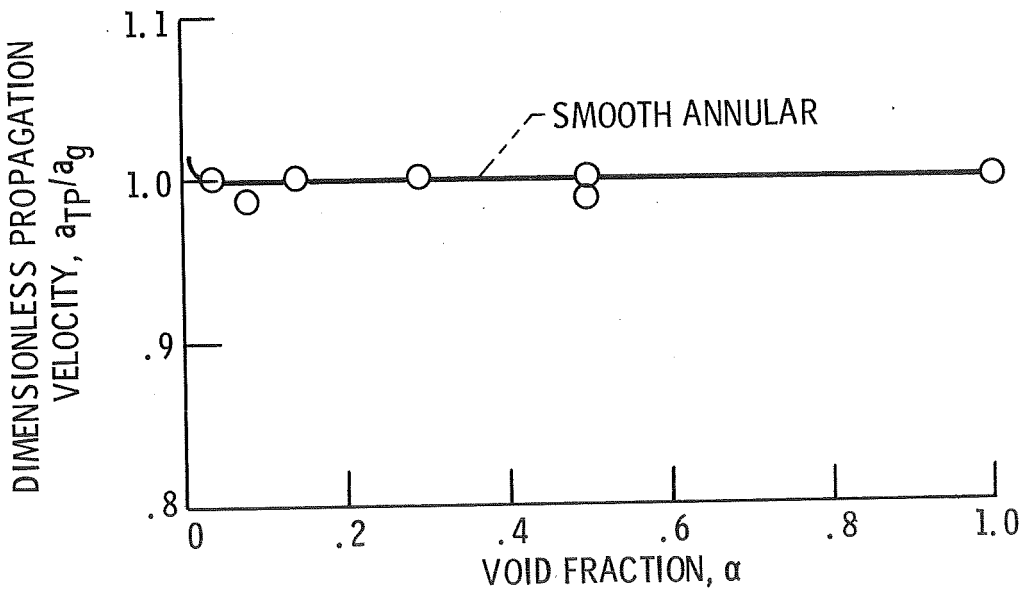


Figure 4. - Comparison of the smooth annular model with the stratified rarefaction wave data of reference 10.

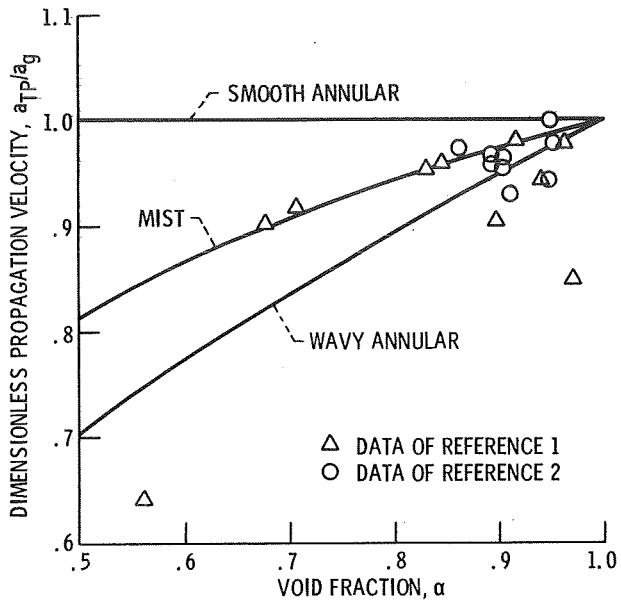


Figure 5. - Comparison of the two-component models with experimental data of references 1 and 2.

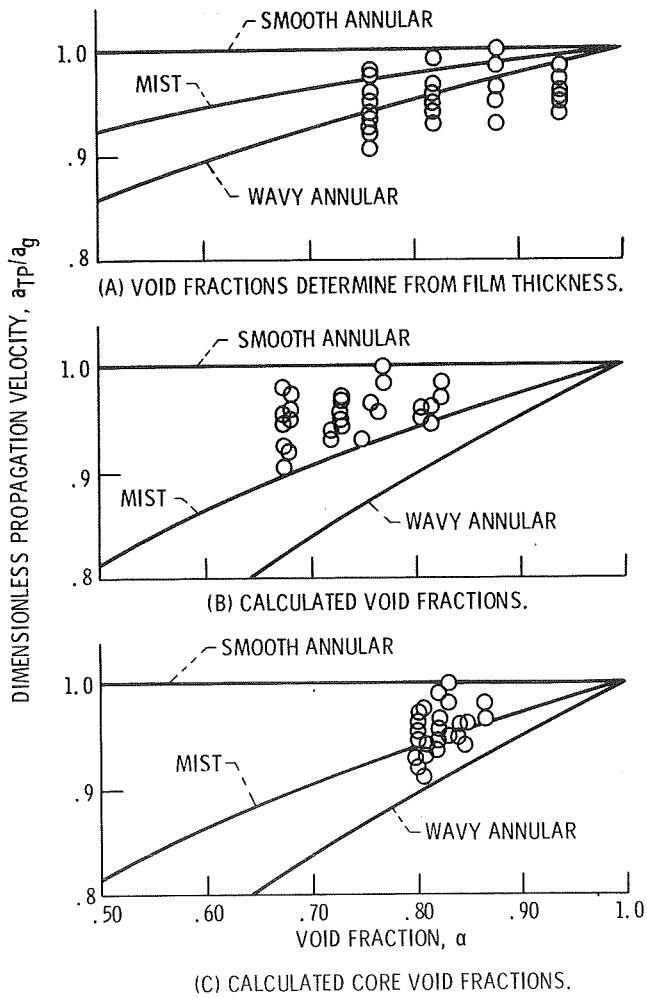


Figure 6. - Comparison of models and data of reference 3.

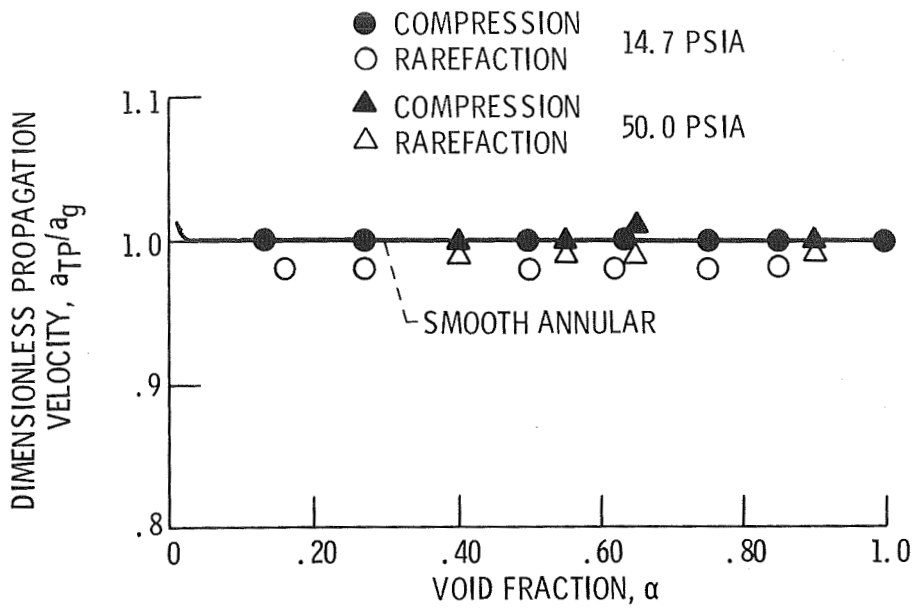


Figure 7. - Comparison of the smooth interface model and the stratified data of reference 21.

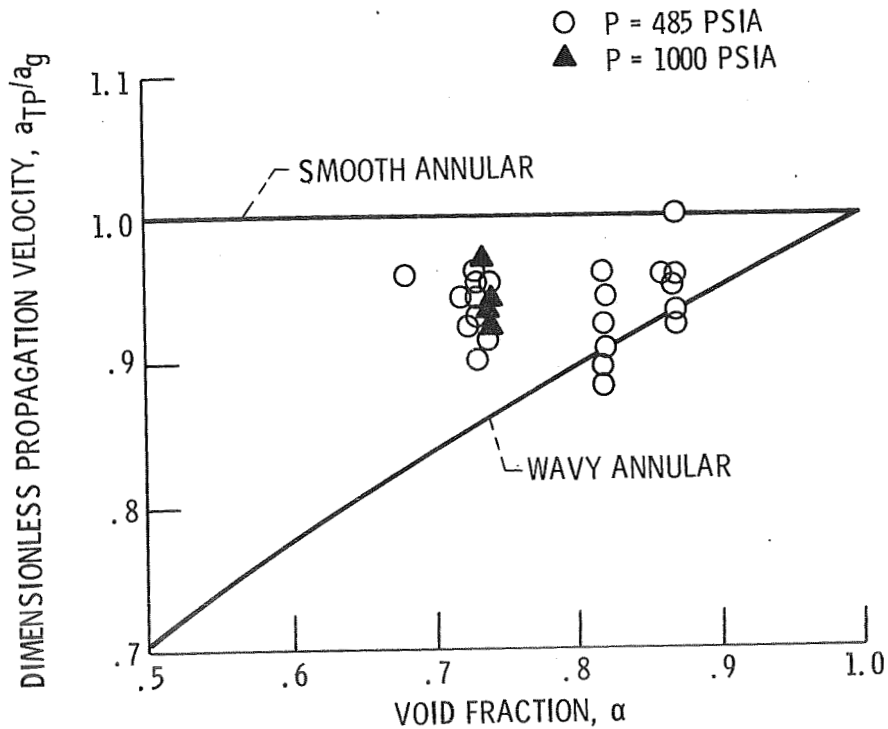
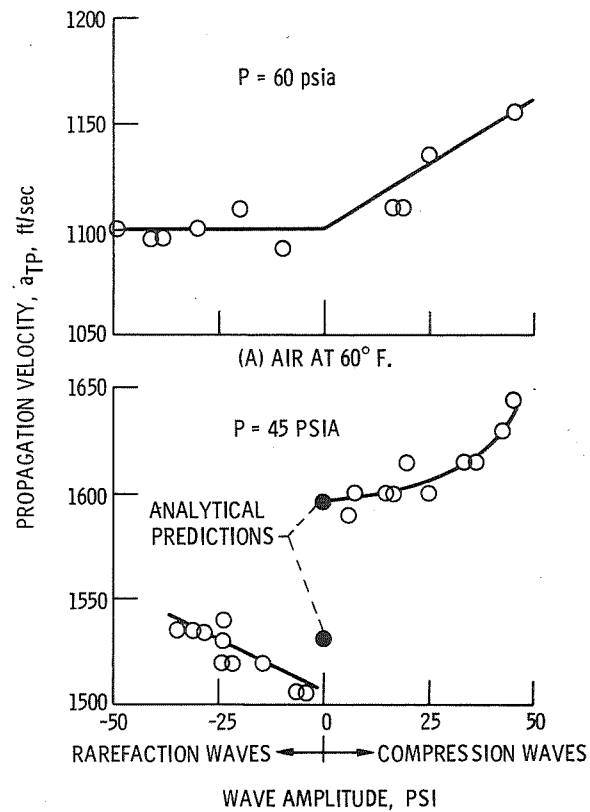


Figure 8. - Comparison between the annular flow models and the one-component rarefaction wave results of reference 22.



(B) A 50 PERCENT QUALITY STEAM-WATER MIXTURE AS REPORTED IN REFERENCE 6.

Figure 9. - Compression and rarefaction wave data .

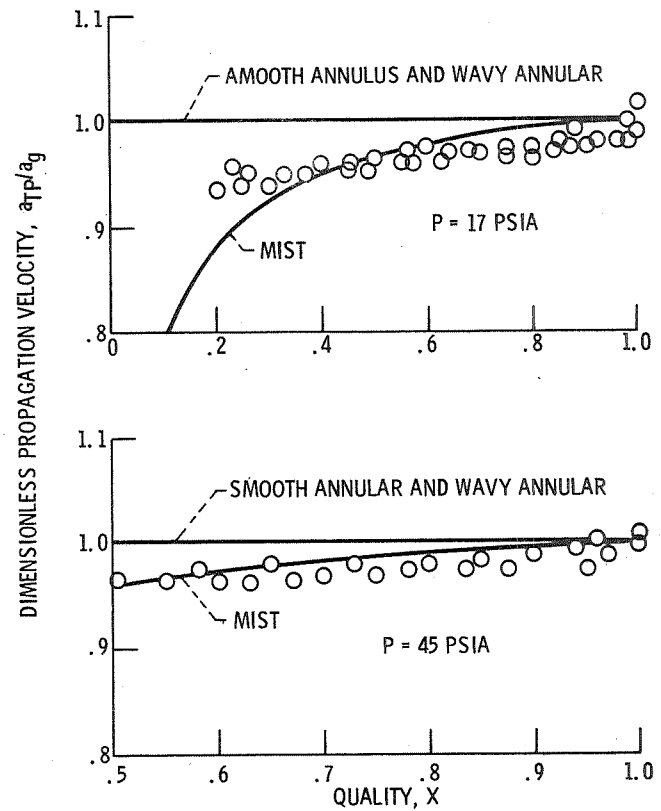


Figure 10. - Comparison of the one-component models with the rarefaction wave results of reference 5.

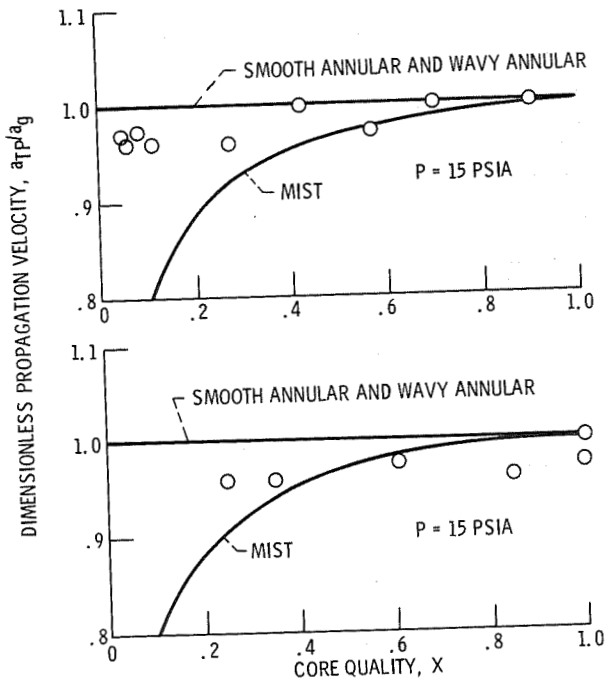


Figure 11. - Comparison of the one-component models with the rarefaction wave results of reference 4 as a function of gross and core qualities.

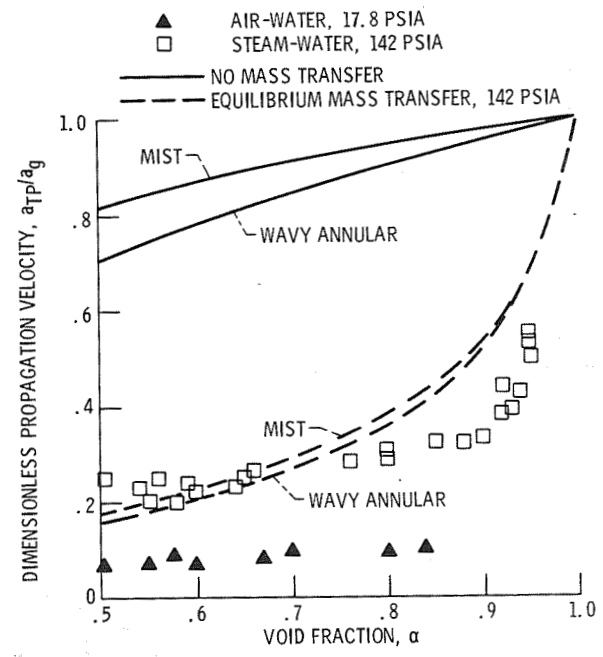


Figure 12. - One- and two-component experimental results presented in reference 27.