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# COMPARATIVE IN-FLIGHT THRUST MEASUREMENTS OF THE SERT II ION THRUSTER

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# COMPARATIVE IN-FLIGHT THRUST MEASUREMENTS OF THE SERT II ION THRUSTER

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## SUMMARY

A description and an analysis of the three methods used to measure the thrust of the SERT II (space electric rocket test) ion thrusters in flight are given. The first is the use of the MESA (miniature electrostatic accelerometer) to measure the  $2 \mu g$ 's resulting from the radial component of the thrust. The second is the use of telemetered beam electrical parameters to calculate thrust. The third is the measurement of orbit change resulting from the thrust. The three methods are shown to yield values of thrust which agree with each other within the error bands of the methods. It is concluded that the thrust was measured to within  $\pm 1$  percent by the MESA, to within  $\pm 2.2$  percent by beam electrical parameters, and to within about  $\pm 5$  percent by orbit changing.

## INTRODUCTION

The successful operation of the SERT II (space electric rocket test) spacecraft (ref. 1), developed and built at the NASA Lewis Research Center, represents the first successful life test of an ion thruster in space. The primary goal of the SERT II mission is to achieve at least six months of ion thruster operation in a space environment. A secondary goal is to measure the thrust to an accuracy commensurate with the requirements of postulated, deep-space, ion-propelled vehicles.

The purpose of this report is to describe the means available on SERT II for measuring thrust, to compare them, and to present the available results. The thrust of an ion thruster very similar to that on SERT II was measured on the SERT I spacecraft to within an error of about 5 percent (ref. 2). The goal of SERT II is to reduce substantially this error. It will be shown in this report that an error of about  $\pm 1$  percent of the 28-millinewton (6.3-mlb) thrust was achieved.

Many of the contemplated uses of electric propulsion are for deep-space missions.

The primary advantage obtained by using this type of propulsion is the large specific impulse that it delivers. This large impulse, though, is delivered in the form of low thrust for a long time in contrast with other propulsion means that deliver large thrust for a short time. As a result, accurate navigation during such an electrically propelled mission requires that the thrust be known precisely.

The thrust of an ion thruster can be most easily, but not necessarily most accurately, determined by measuring the ion beam current and the potential through which the ions are accelerated. These electrical measurements can be readily made in the thruster power supply with little error. Calculation of thrust from these measurements, however, requires corrections for such things as lack of beam collimation and doubly and triply charged ions. The uncertainty in these corrections may result in an error in thrust larger than desired.

Making a direct measurement of thrust on an ion-propelled vehicle is difficult because of the inevitably low thrust-to-weight ratio or acceleration which results. One can measure the change of the flight path caused by the thrust, but this change is so gradual that very long averaging times are required. The direct approach, measuring vehicle acceleration with an accelerometer, is better if a sufficiently sensitive instrument exists. The radial component of thrust-induced acceleration on SERT II is about  $2 \mu\text{g}'\text{s}$ , which must be measured to the order of 1 percent error. This requires an instrument with an error band no greater than a few tens of  $\text{ng}'\text{s}$ . The MESA (miniature electrostatic accelerometer) (ref. 3) theoretically meets these requirements and was flown on SERT II.

This report discusses the three thrust measurement methods on SERT II and presents and compares the results obtained from them. These three methods are (1) MESA measured acceleration, (2) beam electrical parameters, and (3) flight path or orbit change.

## SPACECRAFT AND ORBIT

The SERT II spacecraft was launched on February 3, 1970, into a nearly Sun-synchronous, nearly polar, circular orbit of about 1000 kilometers altitude. An artist's conception of the spacecraft in orbit is shown in figure 1. It consists of an expended Agena stage permanently affixed to the spacecraft support and experiments section. A large solar array, attached as shown, supplies electrical power for the ion thruster, for housekeeping, and for the other experiments.

The spacecraft is stabilized in orbit such that the Agena long axis always lies along the Earth radius line and the solar panels and thrusters always lie in the orbit plane. The Earth-oriented stabilization is achieved by means of gravity-gradient torque coupled with control-moment gyros. The desired solar panel-to-Sun orientation is achieved by

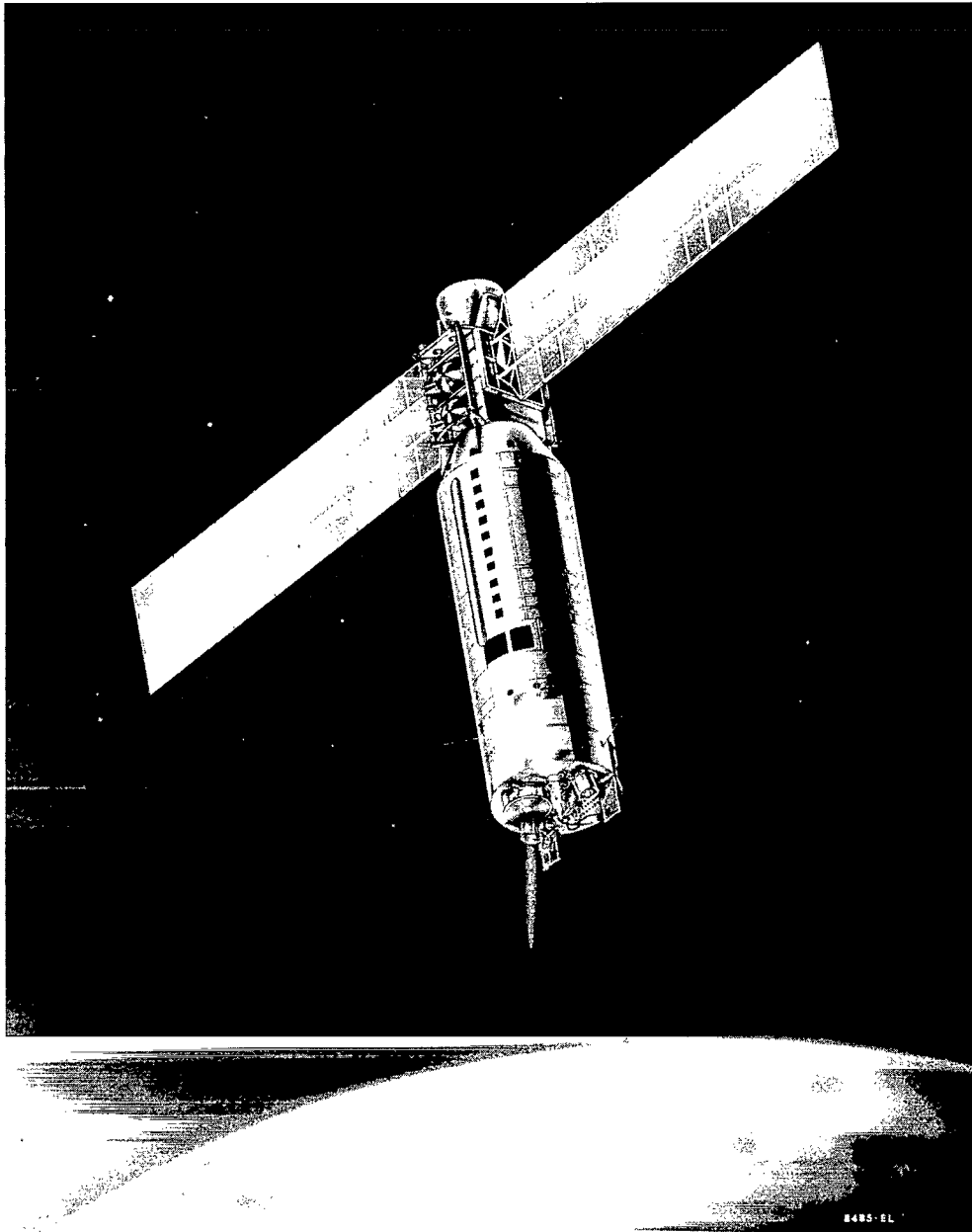


Figure 1. - SERT-II spacecraft in orbit (artist's conception).

inclining the orbit such that it precesses at the rate required (ref. 1).

There are two ion thrusters aboard, only one of which operates at any time. Each thrust vector is aligned by means of gimbals to pass through the spacecraft center of mass and form a  $10^{\circ}$  angle with the spacecraft long or yaw axis. Thus, one component of thrust, which is directed along the orbit radius, is proportional to the cosine of  $10^{\circ}$ , and another component, which is directed along the orbit tangent, is proportional to the sine of  $10^{\circ}$ . The MESA accelerometer is oriented with its sensitive axis parallel to the yaw axis so that it senses the radial component of thrust. The tangential component causes the orbit altitude to change with time and is the basis for the orbit raising thrust measurement method.

It should be mentioned at this point that a fourth means of measuring thrust exists aboard SERT II. This consists of purposely gimbaling the thruster so that the thrust vector does not pass through the center of mass. The resulting torque will be balanced by the torque from the gravity gradient at some attitude offset angle. Measurement of this offset yields a measure of thrust. An error analysis of this method shows that a measurement with no less than  $\pm 10$  percent error would be achieved primarily because of angle uncertainties. Therefore, it will not be treated further in this report.

## THRUST MEASUREMENT METHODS

### MESA Accelerometer

The electrostatic accelerometer is a single-axis instrument designed for measurement of extremely low accelerations in a low-gravity environment. State-of-the-art techniques in machining are required to produce a successful instrument based on the principle of electrostatic force. A detailed description of the basic MESA structure and operating principles is found in references 3 and 4.

Electrostatic forces are used for both the support of the acceleration sensing element (the proof mass) and the measurement of external acceleration. The proof mass is suspended orthogonal to the accelerometer sensitive axis by means of ac voltages through series tuned LC circuits. The magnitude of this voltage is selected to support the proof mass against the expected level of cross-axis acceleration forces. The lower the cross-axis g environment, the lower the required cross-axis voltages. It is advantageous to reduce the support voltages as low as practical, commensurate with the expected environment, to reduce the instrument null bias. This null bias represents the instrument output with zero input acceleration. In general, the null bias term is proportional to the cross-axis support forces. The cross-axis suspension capability for the SERT II MESA was  $100 \mu g$ 's.



The MESA uses an electrostatic force rebalance method in the sensitive axis to measure acceleration. The frequency of voltage pulses required to maintain the proof mass at its balance point is proportional to the external acceleration. The voltage pulse amplitude, width, and maximum frequency determine the full-scale sensitive-axis acceleration capability of the instrument. For SERT II, the MESA full scale was set for  $100 \mu\text{g}$ 's, even though only about  $2 \mu\text{g}$ 's were to be measured. This full scale was chosen as the minimum that would allow accurate calibration on Earth. Unlike most instruments, the MESA measurement error is a percentage of reading, plus null uncertainty, rather than the usual percentage of full scale, plus null uncertainty. Thus, in space, 2 percent of full scale can be measured with the same accuracy as 100 percent of full scale except for the null uncertainty (ref. 3). The internal data conditioning provides 100-second averages of the measured frequency. Because the MESA output is based on frequency averaging and because of large internal damping, the MESA is a steady-state instrument.

MESA calibration. - The only presently practical method of on-Earth calibration of the MESA uses accurate small-angle deflection and measurement to produce low-magnitude input accelerations derived from the Earth gravity vector.

Figure 2 illustrates the MESA dividing-head calibration method. With the sensitive

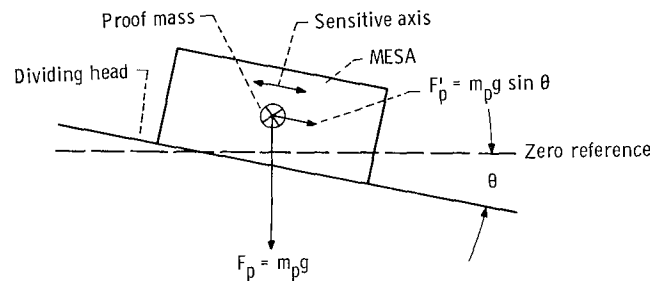


Figure 2. - MESA calibration method.

axis perpendicular to the Earth  $g$ -vector, the mounting surface is varied to obtain an instrument null. The angle  $\theta$  is then varied to provide input accelerations of  $g \sin \theta$  or  $g\theta$  for the small full-scale angles of 20 arcseconds required for the SERT II MESA. Use of autocollimator angle measuring techniques for small angles results in an angle measurement accurate to within  $\pm 0.05$  arcsecond (unpublished data of Z. H. Milburn of Bell Aerosystems), with a corresponding calibration error of  $\pm 0.25$  percent.

Of course, on-Earth calibration requires a 1-g cross-axis suspension capability for the MESA and results in the accompanying increase in null bias due to cross-coupling. Measurement of null bias under these conditions resulted in a value of

50  $\mu\text{g}'\text{s}$ . Theoretically, the null bias scales down with the reduction of cross-axis voltages. The expected null bias for the SERT II level of 100- $\mu\text{g}$  suspension is therefore 0.005  $\mu\text{g}$ .

SERT II accelerations. - Figure 3 shows the SERT II spacecraft configuration details pertinent to the MESA. The MESA is located at a distance  $d$  of  $2.39 \pm 0.015$  meters from

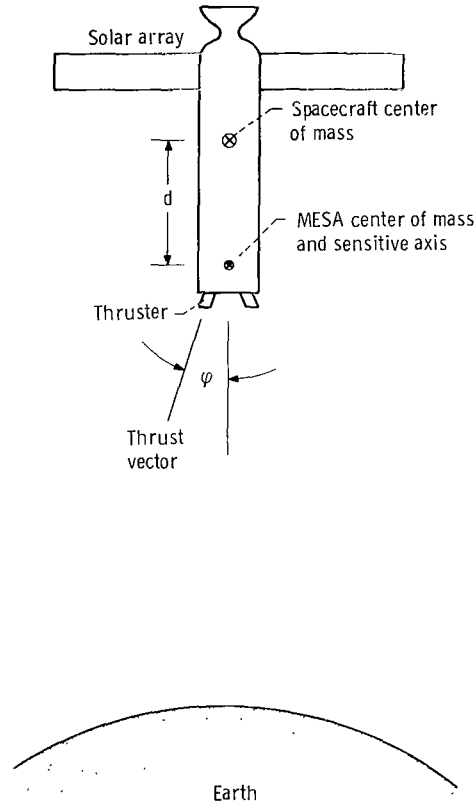


Figure 3. - SERT II acceleration measurement configuration.

the vehicle center of mass with the sensitive axis aligned with the spacecraft yaw axis. The angle  $\varphi$  between the thrust vector and the MESA sensitive axis is  $10^\circ$ . Equation (1) is the expected sensitive-axis acceleration at the MESA location consisting of the thrust-produced term,  $F \cos \varphi$  divided by the spacecraft mass, and two simplified orbital components:

$$a = \left( \frac{F \cos \varphi}{m_s} + d\omega^2 + \frac{2d\mu}{r^3} \right) \frac{1}{g} \quad (1)$$

where

- a MESA sensitive-axis acceleration, g's
- F thrust, N
- $\varphi$  angle between MESA sensitive axis and thrust vector, deg
- $m_s$  spacecraft mass, kg
- d distance from spacecraft center of mass to that of MESA proof mass, m
- $\omega$  orbital rate, rad/sec
- $\mu$  universal gravitational constant times mass of Earth,  $m^3/\text{sec}^2$
- r distance from center of Earth to spacecraft center of mass,  $7.4 \times 10^6$  m
- g free-fall standard acceleration ( $9.807 \text{ m}/\text{sec}^2$ )

(Symbols are defined in the appendix.)

Since the spacecraft is gravity gradient stabilized, it has a pitch rate of one revolution per orbit. The MESA thus revolves about the spacecraft center of mass at a once per orbit rate and senses a centripetal acceleration  $d\omega^2$ . The third term in equation (1) is the gravity-gradient component which results from the MESA being closer to the Earth than is the spacecraft center of mass.

The orbital accelerations for SERT II were calculated to be  $0.73 \mu\text{g}$ . Other sensitive-axis accelerations due to vehicle attitude and accelerometer misalignment contribute terms two orders of magnitude less than these orbital accelerations and are considered insignificant.

The orbital acceleration reading of the MESA prior to thruster turnon can be considered a secondary calibration source for the MESA, although it is impossible to separate null bias from scale factor, since only one datum point is available. However, if it is assumed that the MESA scale factor does not change with cross-axis voltage changes, this orbital acceleration measurement can be used to determine the null bias scaling of the MESA with respect to the 1-g calibration value.

The expected ion thrust for SERT II was about 28 millinewtons (6.3 mlb). For  $\varphi = 10^0$  and  $m_s = 1434$  kilograms (3162 lb), the thrust-produced acceleration was calculated to be  $1.9 \mu\text{g}'\text{s}$ .

## Beam Electrical Parameters

A measurement of thrust can be made by using

$$F = I_B \sqrt{2 \frac{m}{q}} \sqrt{V_{net}} \quad (2)$$

where

$I_B$  ion beam current, A

$m/q$  mass-to-charge ratio of singly ionized propellant molecule, kg/C

$V_{net}$  net accelerating potential, V

In this equation,  $I_B$  and  $V_{net}$  are measured values telemetered from the spacecraft. This equation is the result of a one-dimensional analysis of the ion thruster (ref. 5). (Additional treatment of the thrust equation is presented in ref. 6.)

The one-dimensional equation is used to determine an ideal thrust for the SERT II thruster. Corrections for significant real effects, such as doubly ionized propellant atoms, ion beam divergence, and uncertainty in the thrust vector direction, are treated in the section ERRORS.

## Orbit Changing

Another method of obtaining the thrust is to measure the change in orbit radius over a period of time. The equation governing this measurement is obtained by solving the equations of motion and integrating over the averaging time:

$$\frac{dr}{dt} = \frac{2F \sin \varphi}{m_s \sqrt{\mu}} r^{3/2} \quad (3)$$

(see ref. 7)

$$F = \frac{m_s \mu^{1/2}}{t \sin \varphi} \left( r_0^{-1/2} - r^{-1/2} \right) \quad (4)$$

where

$t$  time, sec

$r_0$  orbit radius at  $t = 0$  (about  $7.4 \times 10^6$  m)

With the expected thrust level of 28 millinewtons (6.3 mlb), the orbit will increase in radius by about 570 meters per day. Measuring the rate of orbit radius change thus yields a value of thrust.

## ERRORS

### MESA Accelerometer

The engine thrust determined from the MESA acceleration measurements is given by

$$F = \frac{m_s}{\cos \varphi} (a_{\text{on}} - a_{\text{off}}) \quad (5)$$

$$F = \frac{m_s}{\cos \varphi} [(a_T + a_o + a_{\text{NB}}) - (a_o + a_{\text{NB}})] \quad (6)$$

$$F = \frac{m_s}{\cos \varphi} a_T \quad (7)$$

where

- $a_{\text{on}}$  acceleration with thruster on, g's
- $a_{\text{off}}$  acceleration with thruster off, g's
- $a_T$  acceleration due to thruster, g's
- $a_o$  orbital acceleration, g's
- $a_{\text{NB}}$  null bias, g's

Therefore, the thrust measurement (eq. (6)) is independent of orbital acceleration and null bias (if null bias is constant).

The error in  $F$ ,  $dF$ , is a function of errors in the mass of the spacecraft, errors in determination of the angle  $\varphi$ , and accelerometer errors. These errors are discussed and root sum squared to provide a total error in the measurement of  $F$ .

$$\left(\frac{dF}{F}\right)_{m_s} = \frac{dm_s}{m_s}$$

$$\left(\frac{dF}{F}\right)_{\varphi} = \tan \varphi d\varphi$$

$$\left(\frac{dF}{F}\right)_a = \frac{da}{a}$$

$$\frac{dm_s}{m_s} = \pm 0.22 \text{ percent (ref. 1)}$$

where  $\varphi$  is known to within  $\pm 0.5^\circ$  or  $\pm 0.0087$  radian. Therefore, for  $\varphi = 10^\circ$ ,  $\tan \varphi d\varphi = \pm 0.15$  percent. The error  $da/a$  is composed of several possible errors:

- (1) Scale factor calibration error
- (2) Scale factor stability and repeatability
- (3) Null bias drift
- (4) Readout system error
- (5) Basic instrument linearity

As mentioned in the section MESA calibration, the final on-Earth scale factor calibration is considered known to within  $\pm 0.25$  percent. However, the measured scale factor was found to vary by as much as 0.8 percent between calibration tests following the vibration, shock, and thermal-vacuum qualification tests. (It is possible that other calibration errors, such as test base stability, contribute to this variation rather than actual physical changes in the MESA.) If it is assumed that no added change in scale factor occurs with a  $10^{-4}$ -g cross-axis suspension, the final in-orbit scale factor is known to  $\pm \sqrt{(0.25)^2 + (0.8)^2} = \pm 0.84$  percent.

The null bias drift was studied by using the average drift in orbital acceleration data prior to thruster turn-on and was determined to be less than 0.5 percent over a short (500 sec) time period.

The readout system error is a maximum of  $\pm 0.2$  percent for a 100-second reading and decreases proportionately with time averaging over longer periods.

The basic instrument linearity is  $\pm 0.1$  percent of reading (ref. 4).

Root-sum-square totaling is used to combine the independent percentage errors in  $a$ :

$$\frac{da}{a} = \pm \sqrt{(0.84)^2 + (0.2)^2 + (0.1)^2 + (0.5)^2}$$

$$\frac{da}{a} = \pm 1.00 \text{ percent}$$

Therefore, the root-sum-square error for thrust  $F$  is determined by

$$\frac{dF}{F} = \pm \sqrt{(0.22)^2 + (0.16)^2 + (1.00)^2}$$

$$\frac{dF}{F} = \pm 1.03 \text{ percent}$$

Hereinafter, the error in thrust as determined by the MESA will be considered to be  $\pm 1$  percent.

### Beam Electrical Parameters

Measurement accuracy. - The determination of thrust using equation (2) for the SERT II thruster is made by substituting flight data as received. Equation (2) becomes

$$F = (I_5 - I_6 + \Delta I_6) \sqrt{2 \frac{m}{q}} \sqrt{V_5 + V_4 - V_{SP}} \quad (8)$$

where (see refs. 1 and 6)

- $I_5$  screen current, A
- $I_6$  accelerator current, A
- $\Delta I_6$  accelerator current due to neutralizer ions (ref. 8), A
- $V_5$  positive high potential, V
- $V_4$  anode potential, V
- $V_{SP}$  space probe potential, V

The relation of these parameters is shown in figure 4, which is a plot of the potential seen by an ion on its way out of the thruster.

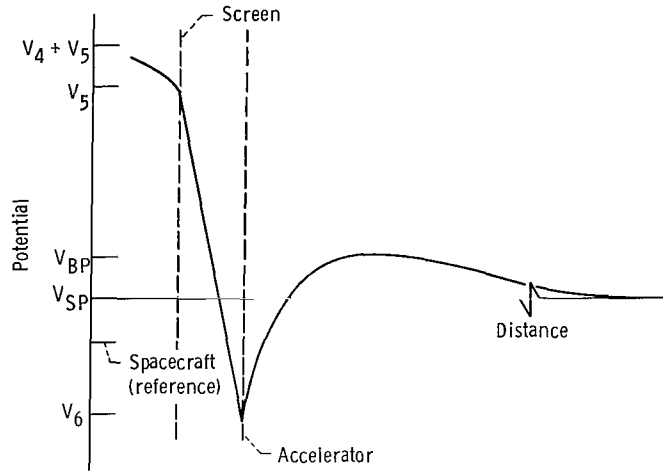


Figure 4. - Thruster potentials.

The thrust determined by using equation (8) is subject to errors in the measured quantities. An error analysis of this method yields the following errors:

$$\left(\frac{dF}{F}\right)_{I_5} = \frac{dI_5}{I_5 - I_6 + \Delta I_6} \quad (9a)$$

$$\left(\frac{dF}{F}\right)_{I_6} = \frac{dI_6}{I_5 - I_6 + \Delta I_6} \quad (9b)$$

$$\left(\frac{dF}{F}\right)_{\Delta I_6} = \frac{d\Delta I_6}{I_5 - I_6 + \Delta I_6} \quad (9c)$$

$$\left(\frac{dF}{F}\right)_{V_5} = \frac{1}{2} \frac{dV_5}{V_5 + V_4 - V_{SP}} \quad (9d)$$

$$\left(\frac{dF}{F}\right)_{V_4} = \frac{1}{2} \frac{dV_4}{V_5 + V_4 - V_{SP}} \quad (9e)$$

$$\left(\frac{dF}{F}\right)_{V_{SP}} = \frac{1}{2} \frac{dV_{SP}}{V_5 + V_4 - V_{SP}} \quad (9f)$$

The root-sum-square error in the SERT II analog flight data is (including the quantizing



error of the digital telemetry system) 1.59 percent of the full-scale value of the parameter measured. This was applied to the parameters for each thruster. It was determined that the contributions of the terms in equations (9b), (9c), (9e), and (9f) to the error are negligible.

The root-sum-square errors in thrust, as determined by equation (8), for each SERT II thruster at each nominal operating condition during the SERT II mission are given in table I. Variations in error and thrust with time arise from two sources:  $V_5$  decreases as the solar array degrades;  $V_4$  decreases as the thruster wears with operating time.

TABLE I. - ROOT-SUM-SQUARE ERROR IN THRUST AS DETERMINED BY EQUATION (8)

Nominal thrust level, percent	Thruster 1	Thruster 2
	Root-sum-square error, percent	
30	5.5	5.7
80	2.7	2.6
<sup>a</sup> 100	2.2	2.2
<sup>b</sup> 100	2.3	---
<sup>c</sup> 100	2.3	---

<sup>a</sup>10 hr from thruster beam turnon.

<sup>b</sup>100 hr from thruster beam turnon.

<sup>c</sup>4400 hr from thruster beam turnon (estimated from projected operation).

Nonideal effects. - Effects of thruster construction and operation are now considered. Not all the propellant in the SERT II thruster is ionized; approximately 15 percent is expelled as neutral particles (ref. 9). The net momentum contribution of this efflux to the total thrust is less than one part in  $10^5$ . This contribution will be neglected.

Some of the propellant in a mercury electron-bombardment ion thruster is doubly or triply ionized. Studies of the ionization process to determine the extent of these phenomena have been undertaken (refs. 10 and 11). The net result of these studies is that, for nominal SERT II thruster operating conditions, a reduction in thrust of 0 to 2.5 percent can be applied. This report will assume a 0-percent reduction.

The problem of unwanted thrust vector deflection in mercury electron-bombardment ion thrusters due to electrode (grid system) misalignments has been treated in reference 12. The analysis used a digital computer program and was applied to the SERT II thruster configuration. Misalignment types were considered as translational, rotational, and skew.

The limits of the fabrication and assembly tolerances of the SERT II thruster can be applied to the results of the analyses in reference 12. Thrust vector deflection due to translational misalignment can be  $1.46^\circ$  or less. Skew misalignment can produce a thrust vector deflection of  $0.04^\circ$  or less. (Additional, and potentially the most unpredictable and damaging, skew misalignment can result from thermal-mechanical warping of the electrodes during thruster operation. However, the bulk of experience with SERT II thrusters leads to the conclusion that, for the SERT II thruster configuration, thermal-mechanical warping occurs primarily in the screen grid, and this warping is radially near-symmetrical. Any skew contribution to thrust vector misalignment from this source is deemed negligible.) Rotational misalignment produces an azimuthal component of thrust that results in a couple about the longitudinal axis of the thruster. SERT II spacecraft attitude data lead to the conclusion that rotational misalignment is negligible.

Each SERT II thruster system is mounted to a gimbal system capable of moving the thrust vector plus or minus  $10^\circ$ , in two orthogonal axes, from the preflight geometric alignment through the spacecraft center of mass. The anticipated in-flight corrections to the thrust vector of an operating thruster have been unnecessary for SERT II. Nine opportunities for a change in thrust vector direction with a commanded change in thruster operating conditions occurred: one thruster 2 (backup) startup through three discrete thrust levels and two thruster 1 (primary) startups through three discrete thrust levels. (Thruster 1 was shut off during the solar eclipse of March 7, 1970.) SERT II spacecraft attitude data show that the thrust vector of each thruster is misaligned by less than  $0.63^\circ$  under all operating conditions. (In addition, results obtained in ground testing of a flight-configuration SERT II thruster have shown the thrust vector to be misaligned by less than  $0.5^\circ$ .) The correction in thrust due to thrust vector misalignment that can be applied to the thrust determined by using equation (8) is then  $\cos 0.63^\circ$ , or a reduction of 0.002 percent; this will be neglected. (As the electrodes wear during the mission, the thrust vector direction may change, gimbaling may become necessary, and the correction may become significant.)

The remaining real effect to be quantitatively considered is the divergence of the ion beam. The ion current density distribution is a function of the operating characteristics, mechanical design, and fabrication and assembly tolerances of the particular thruster under consideration.

The divergence of the ion beam and the ion current density distribution have not been determined for the SERT II thruster configuration. Experience leads to an estimate of a 1- to 3-percent reduction in the magnitude of the thrust vector, as determined by equation (8), to account for the combination of these effects (ref. 13). A 1-percent reduction will be applied in this report.

In summary, the thrust, as determined by equation (8) using SERT II thruster flight operating data, is reduced by 1 percent because of ion beam divergence and, in addition, is subject to the root-sum-square errors of table I.

### Orbit Change

As was stated previously, the thrust can be calculated from the measured change of orbit radius over a period of time. An error analysis of this method using equation (4) yields the error in thrust due to errors in the measured quantities. These individual errors are root sum squared to obtain the error in F:

$$\left(\frac{dF}{F}\right)_{m_s} = \frac{dm_s}{m_s} \tag{10a}$$

$$\left(\frac{dF}{F}\right)_t = \frac{dt}{t} \tag{10b}$$

$$\left(\frac{dF}{F}\right)_r = \frac{\sqrt{2} dr}{\Delta r} \tag{10c}$$

$$\left(\frac{dF}{F}\right)_\varphi = \text{ctn } \varphi d\varphi \tag{10d}$$

where

$$\Delta r = r - r_0, \text{ m}$$

As was stated before,

$$\frac{dm_s}{m_s} = 0.22 \text{ percent}$$

For a reasonably long averaging time (days),  $dt/t$  is completely negligible.

For  $\varphi = 10^0$  and  $d\varphi = 0.5^0$

$$\text{ctn } \varphi \text{ d}\varphi = 5 \text{ percent}$$

The equation given for error in thrust due to inaccuracy in measuring the radius (eq. (10c)) assumes that the change in radius is very small compared with the radius. This assumption is valid for the low-thrust case being considered in this report. The quantity  $\Delta r$  increases with time and can be calculated from equation (3) for SERT II:

$$\frac{\Delta r}{\Delta t} = 570 \text{ m/day}$$

So that

$$\left(\frac{dF}{F}\right)_r = \frac{\sqrt{2} \text{ dr}}{570 T}$$

where  $T$  is the time in days.

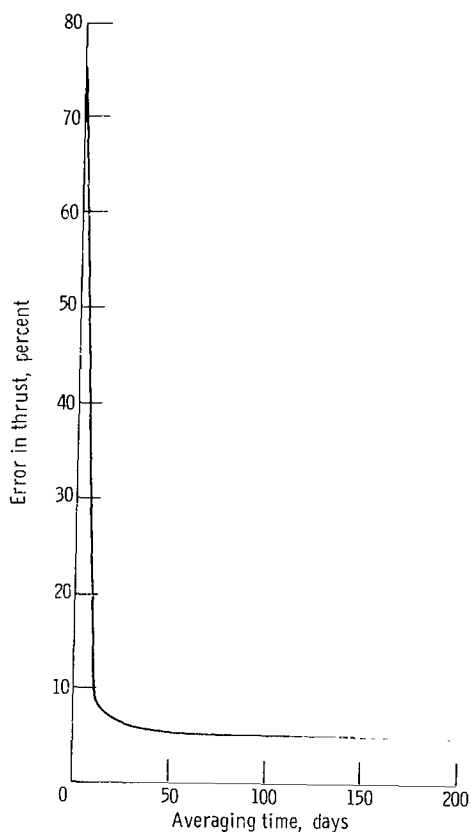


Figure 5. - Root-sum-square error in thrust as function of averaging time for orbit radius change measurements.

The stated accuracy of the radius measurement is  $\pm 300$  meters:

$$\left(\frac{dF}{F}\right)_r = \frac{\sqrt{2} \times 300}{570 T} = \left(\frac{75}{T}\right) \text{ percent}$$

A plot of the total root-sum-square error in thrust as a function of time is shown in figure 5.

## RESULTS AND DISCUSSION

Thrust measurements for initial thruster 1 operation were obtained by the three described methods. Table II shows the comparative measurement results for each phase of the thruster 1 startup operation.

The MESA-obtained thrust data were derived from the MESA acceleration measurements using two different averaging techniques. The first technique used only MESA data that were obtained within 500 seconds before and after each step change in thrust. Figure 6 is the MESA data edited and plotted to show only these step changes. The step changes in acceleration were then summed to obtain the 80- and 100-percent thrust levels. The total acceleration change measured between the thruster off and full on conditions is  $0.725 + 0.866 + 0.328 = 1.919 \mu\text{g}'\text{s}$ .

The second averaging technique involved using all the MESA acceleration data available at the time of this writing. An average thrust-off acceleration and an average 100-

TABLE II. - SERT II COMPARATIVE THRUST  
MEASUREMENTS FOR THRUSTER 1

Method	Thrust at three nominal levels, percent						Error band, percent
	30		80		100		
	mN	mlb	mN	mlb	mN	mlb	
MESA	10.4	2.33	22.7	5.11	27.4	6.16	$\pm 1$
Electrical							
Uncorrected	11.3	2.54	23.2	5.21	28.5	6.41	$\pm 2.2$
Corrected	11.2	2.52	23.0	5.17	28.2	6.34	$\pm 2.2$
Orbit change	(a)	(a)	(a)	(a)	28.0	6.3	$\pm 5$

<sup>a</sup>Not applicable.

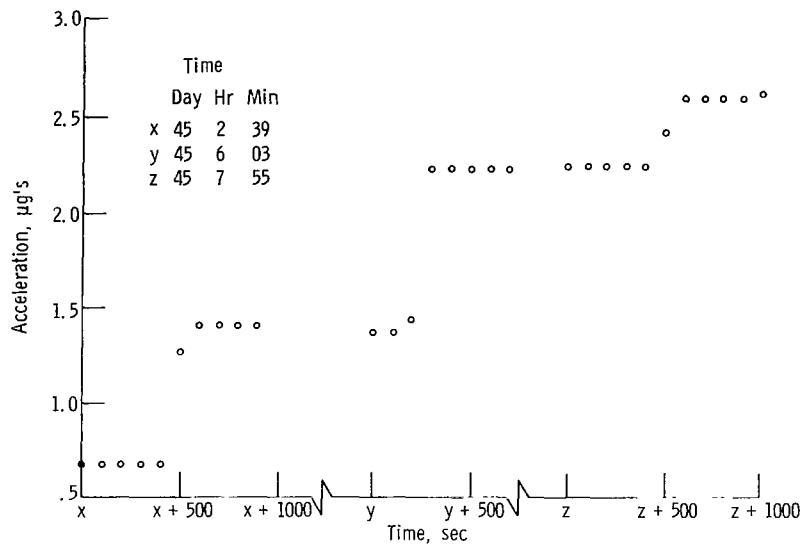


Figure 6. - MESA acceleration data during thruster turnon procedures.

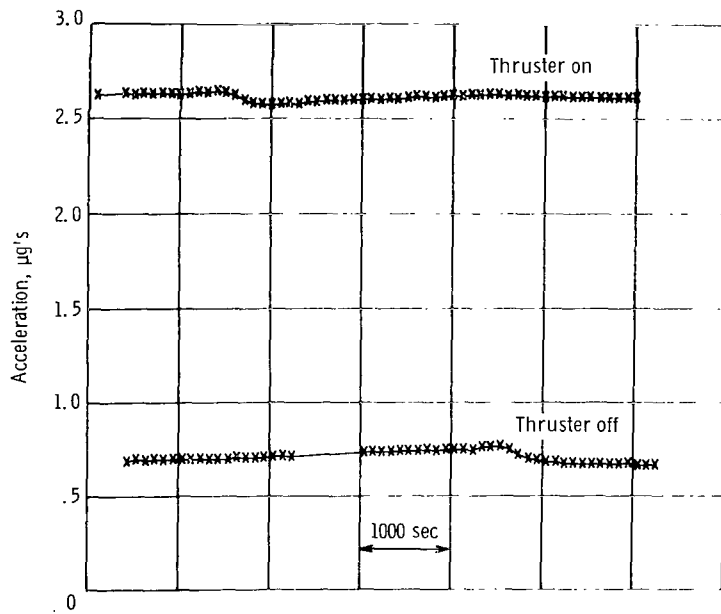


Figure 7. - MESA acceleration data.

percent thrust-on acceleration were calculated to be 0.67 and 2.60  $\mu\text{g}'\text{s}$ , respectively. Taking the difference results in an acceleration term due to thrust of 1.93  $\mu\text{g}'\text{s}$ , in close (0.5 percent) agreement with the step change data.

Figure 7 illustrates typical MESA data for two full orbits and shows the as yet unexplained orbital variation pattern that was observed in much of the MESA data, whether the thruster was operating or not. Computer studies are being made to correlate these 0.1- $\mu\text{g}$  transient peaks with the spacecraft location over the Earth. The close agreement between thrust calculations based on short-term and long-term averaged data supports the contention that these variations do not significantly affect the MESA thrust measurement accuracy.

Information on the MESA null bias was obtained from the accelerometer readings prior to thruster turnon. As described earlier, the expected orbital acceleration for SERT II was 0.73  $\mu\text{g}$ . The actual averaged measured value was 0.67  $\mu\text{g}$ . This discrepancy can be attributed to a null bias of -0.06  $\mu\text{g}$ . The null bias was expected to scale down by  $10^4$  from the 1-g cross-axis voltage value of -50 to -0.005  $\mu\text{g}$  at the  $10^{-4}$ -g voltage setting. If this orbital acceleration discrepancy was actually due to null bias, the null bias scaling factor was only  $8.3 \times 10^2$ . The null bias of an accelerometer should either be insignificantly small or known and stable. For SERT II measurements, a null bias of 0.06  $\mu\text{g}$  is greater than 1 percent of the measured acceleration and, hence, is not negligible. Although the long-term drift of the null bias appears to be insignificant on the basis of available data, the aforementioned orbital variation of 0.1  $\mu\text{g}$  presents problems in evaluating the short-term null bias stability. On the basis of thruster-off orbital acceleration data, the total acceleration variation for periods of less than 500 seconds was generally less than  $\pm 0.01 \mu\text{g}$ . On the basis of these data, therefore, the MESA null bias was sufficiently stable to result in measurement errors of less than 0.5 percent, as was stated in the section ERRORS.

Table III lists the thrust, and expected error, determined from telemetered electrical parameters and equation (8). No corrections have been applied.

The thrust obtained by orbit-raising measurements over the period February 16 to March 21, 1970, is 28.1 millinewtons (6.3 mlb). This value was obtained from the satellite tracking group at Goddard Space Flight Center. The error band on these data is 1.4 millinewtons (0.31 mlb).

The data obtained from beam electrical parameters are more complete than those from the MESA or orbit-changing techniques. The MESA was not operating during thruster 2 operation and also ceased functioning properly about 4 days after thruster 1 came on. Thus, the MESA-derived data for thruster 1 do not apply for data after the 100-percent thrust, 10-hour data. The orbit-changing technique requires too long an averaging period and exhibits too large an error band to give a detailed table of data for all the thruster operating levels.

TABLE III. - UNCORRECTED THRUST OF SERT II ION THRUSTERS

AS DETERMINED BY EQUATION (8)

Nominal thrust level, percent	Thruster 1				Thruster 2			
	Thrust		Root-sum-square error		Thrust		Root-sum-square error	
	mN	mlb	mN	mlb	mN	mlb	mN	mlb
30	11.3	2.54	0.6	0.01	10.5	2.36	0.6	0.01
80	23.2	5.21	.6	.01	23.3	5.24	.6	.01
<sup>a</sup> 100	28.5	6.41	.6	.01	28.4	6.36	.6	.01
<sup>b</sup> 100	28.0	6.29	.6	.01	----	----	---	----
<sup>c</sup> 100	27.3	6.14	.6	.01	----	----	---	----

<sup>a</sup>10 hr from thruster beam turnon.

<sup>b</sup>1000 hr from thruster beam turnon.

<sup>c</sup>4400 hr from thruster beam turnon (estimated from projected operation).

In conclusion, it has been shown that the thrust of the SERT II thrusters has been measured to within  $\pm 1$  percent by the MESA, to within  $\pm 2.2$  percent by the beam electrical parameter method, and to within  $\pm 5$  percent by the orbit-changing method. Within these error bands, the three methods are in agreement.

Lewis Research Center,  
 National Aeronautics and Space Administration,  
 Cleveland, Ohio, June 25, 1970,  
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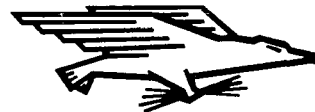


## APPENDIX - SYMBOLS

a	MESA sensitive-axis acceleration, g's	m <sub>p</sub>	MESA proof mass, kg
a <sub>NB</sub>	MESA null bias, g's	m <sub>s</sub>	mass of spacecraft (1434 kg or 3162 lb)
a <sub>o</sub>	orbital acceleration (gravity gradient plus centripetal), g's	r	distance from center of Earth to spacecraft center of mass (7.4×10 <sup>6</sup> m)
a <sub>off</sub>	total acceleration with thrusters off, g's	Δr	r - r <sub>0</sub> , m
a <sub>on</sub>	total acceleration with thrusters on, g's	r <sub>0</sub>	r at t = 0, m
a <sub>T</sub>	acceleration due to thruster, g's	T	time, day
d	distance from spacecraft center of mass to that of MESA proof mass, m	t	time, sec
F	thrust, N	V <sub>BP</sub>	spacecraft beam probe potential, V
$\left(\frac{dF}{F}\right)_X$	error in F due to error in X (typical notation used throughout)	V <sub>net</sub>	thruster net accelerating potential, V
F <sub>p</sub>	force on MESA proof mass, N	V <sub>SP</sub>	spacecraft space probe potential, V
F' <sub>p</sub>	sensitive-axis component of F <sub>p</sub> , N	V <sub>4</sub>	thruster anode potential, V
g	free-fall standard acceleration, 9.807 m/sec <sup>2</sup>	V <sub>5</sub>	thruster positive high potential, V
I <sub>B</sub>	ion beam current, A	V <sub>6</sub>	thruster negative high potential, V
I <sub>5</sub>	thruster screen current, A	θ	angular deviation from horizontal zero reference during MESA calibration, deg
I <sub>6</sub>	thruster accelerator current, A	μ	universal gravitational constant times mass of Earth, m <sup>3</sup> /sec <sup>2</sup>
ΔI <sub>6</sub>	accelerator current due to neutralizer ions (ref. 8), A	φ	angle between thrust vector and spacecraft yaw axis, deg
$\frac{m}{q}$	mass-to-charge ratio of singly ionized propellant molecule, kg/C	ω	orbital rate, rad/sec

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