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**EFFECT OF RADIOMETRIC ERRORS
ON ACCURACY OF TEMPERATURE-PROFILE
MEASUREMENT BY SPECTRAL SCANNING
USING ABSORPTION-EMISSION PYROMETRY**

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16. Abstract <p>The spectral-scanning method may be used to determine the temperature profile of a jet- or rocket-engine exhaust stream by measurements of gas radiation and transmittance, at two or more wavelengths. A single, fixed line of sight is used, using immobile radiators outside of the gas stream, and there is no interference with the flow. At least two sets of measurements are made, each set consisting of the conventional three radiometric measurements of absorption-emission pyrometry, but each set is taken over a different spectral interval that gives different weight to the radiation from a different portion of the optical path. Thereby, discrimination is obtained with respect to location along the path. A given radiometric error causes an error in computed temperatures. The ratio between temperature error and radiometric error depends on profile shape, path length, temperature level, and strength of line absorption, and the absorption coefficient and its temperature dependency. These influence the choice of wavelengths, for any given gas. Conditions for minimum temperature error are derived. Numerical results are presented for a two-wavelength measurement on a family of profiles that may be expected in a practical case of hydrogen-oxygen combustion. Under favorable conditions, the fractional error in temperature approximates the fractional error in radiant-flux measurement.</p>					
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EFFECT OF RADIOMETRIC ERRORS ON ACCURACY OF TEMPERATURE-
PROFILE MEASUREMENT BY SPECTRAL SCANNING USING
ABSORPTION-EMISSION PYROMETRY

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SUMMARY

The spectral-scanning method may be used to determine the temperature profile of a jet- or rocket-engine exhaust stream by measurements of gas radiation and transmittance at two or more wavelengths. A single, fixed line of sight is used, using immobile radiators and radiometers outside of the gas stream, and there is no interference with the flow. In the spectral-scanning method, the wavelengths of measurement are chosen to give greater weight to the radiation from particular portions of the optical path. Then, if a prior assumption is made that the temperature profile can be characterized by j independent parameters, these parameters can be determined from measurements at j wavelengths. A given radiometric error causes an error in computed temperatures. The ratio between temperature error and radiometric error depends on profile shape, path length, temperature level, the strength of line absorption, and the absorption coefficient and its temperature dependency. These influence the choice of wavelengths for any given gas.

In the error analysis the fluxes are computed for an assumed profile shape. Differentiation then relates the radiometric error to the temperature error in terms of the more important experimental parameters of wavelength, absorption coefficient, and its temperature coefficient. Conditions for minimum temperature error are derived. Numerical results are presented for a two-wavelength measurement on a family of profiles that may be expected in a practical case of hydrogen-oxygen combustion. Under favorable conditions, the fractional error in temperature approximates the fractional error in radiant-flux measurement.

INTRODUCTION

The spectral scanning technique in the absorption-emission method of gas pyrometry (ref. 1) provides a means of determining gas temperature along a path through the gas by using radiation sources and receivers located only at the ends of the path. In the case of a jet- or rocket-engine exhaust stream, the instrumentation may therefore be outside the stream.

Any one measurement of radiant flux at the receiver represents the integral of radiant flux from every element of the path length, weighted by the transmission factor of the path between that element and the receiver. Both the flux and the transmission factor are themselves functions of gas temperature. Any one measurement of the numerical value of this complicated integral cannot provide the local temperature distribution. In the spectral scanning technique, two or more measurements are made, each measure (i. e. , each integral) itself weighted by a function that favors radiation from a narrow spectral interval, the interval being different for each measurement. The spectral intervals are chosen to give greater weight to the radiation from particular portions of the optical path. Then, if the additional prior assumption is made that the temperature distribution can be characterized by j independent parameters, this distribution can be derived, in principle, from j separate and independent measurements. One must also know the identity of the gas, the local gas pressure, and the dependence of absorption coefficient on temperature, wavelength, and pressure. From dimensional considerations, one of the j independent parameters must be a temperature. The other parameters may be nondimensional quantities, such as temperature ratios or distance ratios, that define the shape of the temperature-distribution curve.

Each of the "independent measurements" mentioned actually consists of the usual set of three flux measurements used in absorption-emission pyrometry: flux from the gas alone, flux from a reference source alone, and flux from the reference source after it has passed through the gas.

The accuracy with which one can determine the temperature distribution will depend on the judicious choice of the weighting functions for the individual integrals. Thus, the choice of the spectral interval that each measurement favors must maximize the accuracy of temperature measurement. This report addresses itself to the problem of spectral-interval selection, for the case $j = 2$, for profiles that might be encountered in turbojet, ramjet, or rocket engines using hydrogen as the principal fuel and oxygen as the principal oxidant.

The problem definition also requires an exact definition of what is meant by "accuracy." The measure of accuracy that will be used in this report is the error in each of the j parameters of the temperature profile.

The temperature distribution will be assumed to be represented by a curve charac-

terized by two parameters - a temperature and a nondimensional variable that defines the shape of the curve. The fractional error in temperature due to a given fractional error in flux measurement will then be computed for various combinations of wavelength and absorption coefficient, in order to identify the combination that yields the smallest fractional error in temperature. In order to provide specific numerical results, the example of infrared radiation from water, as developed in hydrogen-oxygen or hydrogen-air combustion, will be treated in some detail.

SYMBOLS

A	spectrometer constant, eq. (4)
b	average line spacing
c_2	Planck's second radiation constant
E	error ratio
F, f	factors in error formulas, eqs. (22) and (23)
G	integral of slit function
g	slit function
H	temperature coefficient of absorption coefficient
k	absorption coefficient
k_0	absorption coefficient at reference pressure P_0
L	path length
m	number of zones
N_λ	spectral radiance
\bar{N}_λ	zone-averaged spectral radiance
n	profile shape parameter
P	partial pressure of emitting gas
P_z	partial pressure of gas z
T	temperature
\bar{T}	zone-averaged temperature
T_m	temperature parameter
u	optical depth
V, v	factors in error formulas, eqs. (20), (24), and (25)

x	distance along path
y	function of x/L
β	fine-structure coefficient
γ	line half-width
ϵ	absorption coefficient error ratio
η	exponent in formula for optical depth
λ	wavelength
$\Delta\lambda$	effective slit width
ξ	variable of integration
σ_1 to σ_7	summations
τ	transmission factor
φ	flux

Subscripts:

a, b	set of measurements
av	average
L	value at $x = L$
0	reference

Superscripts:

c	calculated
m	measured

ANALYSIS

Method of Treatment

In absorption-emission pyrometry, three flux measurements φ_1 , φ_2 , and φ_3 are made. These represent, respectively, the fluxes from the gas column alone, from the source alone, and from the source after passing through the gas column. It is convenient for computational purposes also to introduce an intermediate variable, the overall transmission factor $\tau_L = \varphi_3/\varphi_2$.

In the spectral-scanning method, several sets of such measurements are made at different wavelengths and perhaps at different effective slit widths. The number of sets

must be not less than the number of independent parameters that characterize the temperature distribution along the gas column. When the number of sets exceeds the number of parameters, the redundancy helps to reduce the statistical inaccuracy of the results. In this report, the temperature distribution over the path $0 \leq x \leq L$ will be assumed to be characterized by two parameters: a temperature T_m and a nondimensional variable n that characterizes the shape of the temperature profile. Thus,

$$T = T(x, T_m, n) \quad (1)$$

It will be assumed that the two sets of flux measurements are made, assigned subscripts a and b , respectively, and that for each set one may select a mean wavelength λ and an effective slit width $\Delta\lambda$. The formal error analysis problem is to determine the error in the deduced values of T_m and n due to errors in the six flux measurements. The inverse experiment-design problem is to determine values of λ_a , λ_b , $\Delta\lambda_a$, and $\Delta\lambda_b$ that will lead to low values of the errors $|\delta T_m / T_m|$ and $|\delta n / n|$ when all six flux measurements have the same fractional error $|\delta\phi / \phi|$. The solution of the problem requires assumption of a physical model of the mechanisms of spectral emission and absorption at each of the two spectrometer settings. Specifically, in the present treatment, it requires assumptions of the magnitude and temperature coefficient of the absorption coefficient.

A convenient approach to the problem is to compute the fluxes for an assumed temperature distribution so that the ϕ_a 's and ϕ_b 's are expressed as functions of T_m and n . Differentiation then relates the error in each ϕ to errors in T_m and n through formulas of the general form

$$d\phi = \left(\frac{\partial \phi}{\partial T_m} \right) dT_m + \left(\frac{\partial \phi}{\partial n} \right) dn \quad (2)$$

Solution of the simultaneous relations then yields the inverse expression of dT_m and dn in terms of the errors in the individual ϕ 's. The algebraic expressions are then converted to a statistical summation of errors by summing the squares of the individual, independent error terms. There will be six such terms, corresponding to the six flux measurements. Finally, the individual values of fractional error in flux measurements are set equal to each other to establish the error ratios

$$\left. \begin{aligned} E(T_m) &\equiv \left| \frac{\delta T_m / T_m}{\delta \varphi / \varphi} \right| \\ E(n) &\equiv \left| \frac{\delta n / n}{\delta \varphi / \varphi} \right| \end{aligned} \right\} \quad (3)$$

General Equation for Gas Radiation

The flux from a nonisothermal gas column of length L , measured by a receiver located at $x = 0$, is

$$\varphi_1 = A \int_0^\infty g(\lambda) \int_0^L N_\lambda(\lambda, x) k(\lambda, x) \exp \left[- \int_0^x k(\lambda, \xi) d\xi \right] dx d\lambda \quad (4)$$

where A is a constant of the spectrometer, equal to the product of spectrometer transmission factor, entrance-slit area, and dispersing-element area (or collimating-element area if there is one between the slit and the dispersor), divided by the square of the distance between slit and dispersor or collimating element.

The spectral radiance N_λ is given adequately for error analysis by Wien's approximation to Planck's equation. The exponential is the gas transmission factor $\tau(\lambda, x)$. Plass (ref. 2) shows that the product $k(\lambda, x)\tau(\lambda, x)$ may be written as $-\partial\tau(\lambda, x)/\partial x$.

The spectrometer slit function $g(\lambda)$ is appreciably different from zero over only a narrow span of wavelength. We define the effective slit width $\Delta\lambda$ as representing this span, and represent the integral by G , so that

$$G \equiv \int_0^\infty g(\lambda) d\lambda = \int_{\Delta\lambda} g(\lambda) d\lambda \quad (5)$$

Equation (4) may be integrated over the wavelength by use of two approximations: (1) when $\Delta\lambda$ is small, N_λ may be considered independent of λ over the span $\Delta\lambda$, and (2) when a spectral band model is available that allows one to effectively integrate $\tau(\lambda, x)$ over the span $\Delta\lambda$, $\tau(\lambda, x)$ may be replaced by

$$\tau(x) = \frac{\int_{\Delta\lambda} g(\lambda)\tau(\lambda, x)d\lambda}{\int_{\Delta\lambda} g(\lambda)d\lambda} \quad (6)$$

If the band model is to be valid, $\Delta\lambda$ must include enough spectral lines to produce an average τ that is not sensitive to a further increase in $\Delta\lambda$. On the other hand, $\Delta\lambda$ must be small enough so that it lies within the band and so that it does not exceed the limits within which conditions (1) and (2) are reasonably valid.

With these approximations, equation (4) becomes

$$\varphi_1 = AG \int_0^L N_\lambda(x) \cdot \left[-\frac{d\tau(x)}{dx} \right] dx \quad (7)$$

In the absorption-emission method, where measurements are made of φ_1 and of $\tau(L) = \varphi_3/\varphi_2 \equiv \tau_L$, all three fluxes must be measured with the same value of G .

Equation (7) cannot be integrated analytically with most temperature profiles found in practice, but adequate numerical integration can be performed with any smooth profile. The path is divided into m isothermal zones, so that equation (7) becomes

$$\varphi_1 = AG \sum_{i=1}^m \bar{N}_i(\tau_i - \tau_{i+1}) \quad (8)$$

where \bar{N}_i is the arithmetic average of $N_{\lambda,i}$ and $N_{\lambda,i+1}$ and $(\tau_i - \tau_{i+1})$ is computed, using a band model, as described in the next section and in an example given later on.

Gas Transmission Function

The band-model formulation for a nonisothermal gas is based on equation (5) of reference 3. This formulation has been justified and applied in reference 4, and summarized in reference 5. Other band models can be found in the literature (ref. 6). The transmission factor τ is a function of optical depth u given by

$$\tau = \exp \left\{ - \frac{u}{\left[1 + \left(\frac{\pi u}{2\beta_{av}} \right)^2 \right]^{1/2}} \right\} \quad (9)$$

$$u = \int_0^x k \cdot \left(\frac{\beta}{\beta_{av}} \right)^\eta dx \quad (10)$$

$$\beta_{av} = \frac{\left[\int_0^L \left(\frac{\beta P}{T} \right) dx \right]}{\int_0^L \left(\frac{P}{T} \right) dx} \quad (11)$$

where k is the absorption coefficient at temperature T and partial pressure P , and β is the fine-structure coefficient. Both k and β are functions of T and hence of location x along the optical path $0 \leq x \leq L$. The value of β may be computed as

$$\beta = \frac{2\pi\gamma}{b} \quad (12)$$

where γ is the line half-width and b is average line spacing. The quantities k , γ , and b must be known. The quantity β_{av} is a mass-weighted average of β over the entire path. An empirically derived specification of the exponent η in terms of the overall optical depth u_L is (ref. 5)

$$\left. \begin{aligned} \eta &= 0 && \text{for } u_L < \beta_{av} \\ \eta &= 1 && \text{for } u_L > 10\beta_{av} \\ \eta &= \frac{\left[\left(\frac{u_L}{\beta_{av}} \right) - 1 \right]}{9} && \text{for } 1 \leq \left(\frac{u_L}{\beta_{av}} \right) \leq 10 \end{aligned} \right\} \quad (13)$$

Although these equations are used directly for profile determination, approximations are necessary to permit presentation of the results of an error analysis in sufficiently simple form. The number of independent variables must be limited to those with the greatest effect on accuracy. The variables we choose are the overall transmission factor $\tau(L) \equiv \tau_L$ (in preference to the absorption coefficient), the temperature coefficient H of the absorption coefficient where $H \equiv (T/k) \cdot (\partial k/\partial T)$, and the wavelength λ . The wavelength is important because of its strong influence on the spectral radiance, through Planck's law. The transmission factor is chosen in preference to the absorption coefficient because the former includes the effect of path length. A convenient and adequate approximation of H is computed by using the linearization

$$k_0(T) = k_0(T_m) \left(\frac{T_m}{T} \right) \left[1 + H \left(\frac{T - T_m}{T_m} \right) \right] \quad (14)$$

where k_0 is the absorption coefficient at standard reference pressure P_0 , and taking $T = T_m - 2(T_m - T_{av})$, where T_{av} is a temperature that is representative of the major flux-contributing portion of the path and T_m is a temperature arbitrarily chosen to be somewhat higher than T_{av} . When equation (14) leads to a negative value of $k_0(T)$, $k_0(T)$ is to be taken as zero. A reasonable criterion for choosing T_{av} is

$$T_{av} \approx \left(\frac{1}{L} \right) \int_0^L T(x) dx \quad (15)$$

To calculate $\tau(x)$, $k_0(T_m)$ is first determined, and then equations (14), (10), and (9) are used, in that order.

The value of $k_0(T_m)$ may be obtained from a direct tabulation of this quantity, but this method is desirable only when τ_L is close to unity (a more exact statement of this criterion will be given later on). Ordinarily, greater accuracy is achieved by computing $k_0(T_m)$ from the measurement of τ_L . The procedure is as follows:

Given $T(x)$ from an assumed temperature profile, $P(T)$ from the gas composition and partial pressure, and β from a direct tabulation or from values of γ and b , find β_{av} by equation (11). From the measured $\tau_L \equiv \tau(L)$, solve equation (9) for u_L and then estimate η from equation (13). Assuming a value of H , equation (14) is used to express $k_0(T)$ in terms of $k_0(T_m)$ in equation (10). Integrating equation (10) over the entire path length L then yields

$$k_0(T_m) = \frac{u_L P_0}{T_m \int_0^L \left(\frac{P}{T}\right) \left[1 + H\left(\frac{T - T_m}{T_m}\right)\right] \left(\frac{\beta}{\beta_{av}}\right)^\eta dx} \quad (16)$$

Error Analysis

Error analysis is effected by differentiation of equation (7) and then replacing differentials of significant quantities by the errors in those quantities. Differentiation of equation (7) requires determination of the differentials dN_λ and $d(d\tau/dx)$. The expression for dN_λ , obtained by differentiating Wien's equation, is

$$dN_\lambda = \left(\frac{c_2 N_\lambda}{\lambda T^2}\right) dT \quad (17)$$

where c_2 is Planck's second radiation constant.

The quantity $d\tau/dx \equiv \tau'$ may be represented as a function of T , k_0 , and β if tabulated values of k_0 are used to determine $\tau(x)$. Then, one can write

$$d\tau' = \left(\frac{\partial \tau'}{\partial T}\right) dT + \left(\frac{\partial \tau'}{\partial k_0}\right) dk_0 + \left(\frac{\partial \tau'}{\partial \beta}\right) d\beta \quad (18a)$$

On the other hand, if the measured τ_L is used to establish $\tau(x)$, one can write

$$d\tau' = \left(\frac{\partial \tau'}{\partial T}\right) dT + \left(\frac{\partial \tau'}{\partial \tau_L}\right) d\tau_L + \left(\frac{\partial \tau'}{\partial \beta}\right) d\beta \quad (18b)$$

We shall treat this latter form in detail, and merely summarize, later, the result of using equation (18a).

The last term on the right side of equations (18a) and (18b) will be neglected, for the purposes of error analysis, because it is of smaller magnitude than the other two terms.

We note in passing that

$$\frac{d\tau_L}{\tau_L} = \left(\frac{d\varphi_3}{\varphi_3}\right) - \left(\frac{d\varphi_2}{\varphi_2}\right) \quad (19)$$

but continue to work with τ_L for conciseness.

When equation (7) and its differential are combined with equations (17) and (18) and when dT is represented in terms of T_m and n through the differential of equation (1), the result may be written as

$$dn - f \frac{dT_m}{T_m} = V \equiv v_1 \frac{d\varphi_1}{\varphi_1} - v_2 d\tau_L \quad (20)$$

where f , v_1 , and v_2 depend on the temperature profile.

When two sets of measurements are made, identified by respective subscripts a and b , there result two equations of the form of equation (20) representing the differentials in the six flux measurements (through eq. (19)) due to differentials dT_m and dn . Simultaneous solution of the two equations yields

$$dn = \frac{f_a V_b - f_b V_a}{f_a - f_b} \quad (21a)$$

$$\frac{dT_m}{T_m} = \frac{V_b - V_a}{f_a - f_b} \quad (21b)$$

If all six flux measurements are now assigned equal fractional error, the corresponding errors in T_m and n are given by

$$E^2(T_m) = \frac{F_b^2 + F_a^2}{(f_a - f_b)^2} \quad (22)$$

$$E^2(n) = \left(\frac{1}{n^2}\right) \frac{f_a^2 F_b^2 + f_b^2 F_a^2}{(f_a - f_b)^2} \quad (23)$$

where

$$F^2 = v_1^2 + 2v_2^2 \tau_L^2 \quad (24)$$

with subscript a or b assigned to F , τ_L , f , v_1 , and v_2 . The error ratios are defined by equation (3). These represent the sensitivity of T_m and n to errors in flux measurement. The sensitivity is greatest when $f_a \approx f_b$; hence this difference should be held large, and the F -values should be as small as possible.

Alternative determination of absorption. - In those situations when τ_L is near unity, so that the gas is quite transparent, F becomes undesirably large. In that case a smaller value of F may be obtained by estimating τ_L by a computation based on intrinsic gas properties rather than by obtaining it from experimental flux measurements φ_2 and φ_3 . This alternative approach is possible if a tabulation of k_0 is available and if a good a priori estimate of the temperature profile can be made. (This estimate might be the profile first determined by the three experimental flux measurements.) Let $\epsilon = (dk_0/k_0)/(d\varphi/\varphi)$ be the ratio of the error of the tabulated k_0 to the error of flux measurement. Then equation (24) can be replaced by

$$F^2 = v_1^2 + v_2^2 \tau_L^2 (\epsilon \ln \tau_L)^2 \quad (25)$$

Accuracy is thereby improved whenever $(\epsilon \ln \tau_L)^2 < 2$.

EXAMPLE

Assumptions and Data for Computation of f and F

To clarify what magnitudes of error ratios might be achieved in a practical application and what control might be exercised on these error ratios, numerical calculations have been made for a specific family of temperature profiles and for the infrared radiation from H_2O .

The profiles will be assumed to be given by

$$\frac{T_m - T}{T_m - T_L} = y^{1/n} \quad (26)$$

where

$$y = \left| \left(\frac{2x}{L} \right) - 1 \right| \quad (27)$$

and

$$T_L = 300\text{K}, \quad 1000 \leq T_m \leq 3000\text{K}, \quad 0 < n \leq 1/2$$

These profiles are shown in figure 1. They resemble the transverse velocity and temperature distributions often found in ducts with axisymmetric flow.

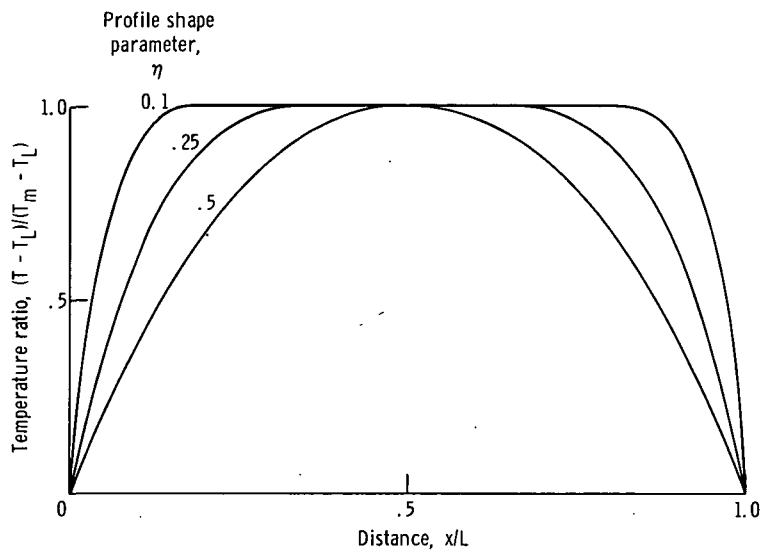


Figure 1. - Assumed temperature profiles characterized by two parameters.

For this profile, dT in equation (17) becomes

$$dT = (1 - y^{1/n})dT_m + (T_m - T_L)(y^{1/n} \ln y) \left(\frac{1}{n^2} \right) dn \quad (28)$$

and the values of f , v_1 , and v_2 in equation (20) become

$$\begin{aligned}
f &= -T_m \frac{\frac{\sigma_2 c_2}{\lambda(T_m - T_L)} + \sigma_3}{\sigma_1} \\
v_1 &= \frac{\sigma_4}{\sigma_1} \\
v_2 &= \frac{\sigma_5}{\sigma_1} \\
\sigma_1 &= \left(\frac{\sigma_6 c_2}{\lambda n^2} \right) + \sigma_7 \\
\sigma_2 &= \sum \bar{N}_i \Delta \tau_i \frac{(\bar{T}_i - T_L)}{\bar{T}_i^2} \\
\sigma_3 &= \sum \left(\frac{\bar{N}_i}{\delta T_m} \right) \left[(\Delta \tau_i)_{T_m + \delta T_m} - (\Delta \tau_i)_{T_m} \right] \\
\sigma_4 &= \sum \bar{N}_i \Delta \tau_i \\
\sigma_5 &= \sum \left(\frac{\bar{N}_i}{\delta \tau_L} \right) \left[(\Delta \tau_i)_{\tau_L + \delta \tau_L} - (\Delta \tau_i)_{\tau_L} \right] \\
\sigma_6 &= \sum \bar{N}_i \Delta \tau_i (\ln y_i) \frac{(T_m - \bar{T}_i)}{\bar{T}_i^2} \\
\sigma_7 &= \sum \left(\frac{\bar{N}_i}{\delta n} \right) \left[(\Delta \tau_i)_{n + \delta n} - (\Delta \tau_i)_n \right] \\
\bar{T}_i &= \frac{(T_i + T_{i+1})}{2} \\
\Delta \tau_i &= \tau_i - \tau_{i+1}
\end{aligned} \tag{29}$$

where all summations are for $i = 1$ to $i = m$. To compute σ_3 , τ_L and n are held constant; to compute σ_5 , T_m and n are held constant; to compute σ_7 , τ_L and T_m are held constant.

The water radiation will be assumed to lie within the 1.9-, 2.7-, and 6.3-micrometer bands. Other assumed limits will be overall transmission factor, $0.1 < \tau_L < 1$; and temperature coefficient of k , $-3 \leq H \leq 2$. For the profiles of equation (26), equation (15) leads to values of T_{av} that lie between 70 and 100 percent of T_m . Therefore, a safe rule of thumb for T_{av} is $T_{av} \approx 0.85T_m$. Table I gives values of k_0 and H for wavelengths that are of particular interest, because their use can lead to relatively small values of the error ratios. The values in table I are computed from the tabulations by Ludwig (ref. 7) of values of $k_0 T_m / T_0$ at 1 bar pressure. The value of ϵ for the data is on the order of 10.

TABLE I. - ABSORPTION COEFFICIENT $k_0 T_m / T_0$ AND ITS TEMPERATURE COEFFICIENT H FOR WATER AT 100 KILONEWTONS PER SQUARE METER (1 BAR)

Wave-length, λ , μm	Temperature parameter, T_m , K					
	1000		2000		3000	
	$k_0 T_m / T_0$ (a)	H (b)	$k_0 T_m / T_0$ (a)	H (b)	$k_0 T_m / T_0$ (a)	H (b)
6.061	0.237	-2.04	0.171	-0.47	0.170	-0.02
5.970	.294	-1.80	.210	-.53	.202	-.12
5.882	.434	-.69	.268	-1.23	.173	-1.51
5.797	.528	-.62	.300	-1.48	.191	-1.71
2.703	.673	-.13	.268	-2.45	.150	-2.36
2.685	.566	-1.17	.194	-2.60	.113	-2.15
2.597	.277	-2.77	.161	-.57	.154	-.14
2.500	.186	.69	.159	-.50	.161	.04
2.484	.113	.75	.124	.0	.132	.18
2.424	.012	1.41	.035	1.22	.060	1.22
2.094	.001	2.02	.019	2.78	.040	1.61
2.020	.010	1.99	.049	1.42	.048	-.09
1.961	.053	1.56	.048	-.64	.040	-.60
1.810	.045	.56	.045	-.11	.044	-.10

^aThis quantity is tabulated rather than k_0 to facilitate linear interpolation between columns. $T_0 = 273$ K; $k_0 = k_0(T_m)$.

^bEvaluated at $T_{av} = 0.85 T_m$.

The values of γ and b are also computed from reference 7. Using values there tabulated, one obtains

$$\gamma = 0.09 \left(\frac{1}{P_0} \right) \left(\frac{T_0}{T} \right)^{1/2} (P_{H_2O} + P_{N_2}) + 0.44 \left(\frac{T_0}{T} \right) \left(\frac{P_{H_2O}}{P_0} \right) \quad (30)$$

The value of b (designated d in ref. 7) can be approximated for H_2O by making it independent of wavelength and with a temperature dependence given by

$$b = b_0 \left(\frac{T_0}{T} \right)^{2.5} \quad (31)$$

where

$$b_0 = 0.5 \quad \text{at } T_0 = 1500 \text{ K}$$

For a given amount of fuel and oxidant, the more important partial pressures P_{H_2O} and P_{N_2} can be determined (ref. 8) to allow computation of γ and, thereby, of β . The approximations make $\beta(T)$ the same at all wavelengths in the range of λ .

The error-ratio parameters f (eq. 29)) and F (eqs. (24) and (29)) are listed in tables II and III for a particular wavelength in two of the water bands; interpolation may be used for any other wavelength. The quantity f is affected to a lesser degree by τ_L than it is by λ or H . On the other hand, parameter F is more strongly affected by τ_L than it is by λ or H .

To find values of f and F by use of tables II and III, τ_L must be estimated from the optical depth u_L . Figure 2 shows the relation between τ_L and the optical depth u_L of an isothermal gas column, in accordance with equation (9). The optical depth u_L of a nonisothermal gas column is an integral of the local value of k over the path L . For the profile of equation (26), this integral is approximated by

$$u_L = k_0(T_m) \left(\frac{LP}{P_0} \right) \left[1 + 2n(1 - H) \left(\frac{T_m - T_L}{T_m} \right)^2 \right] \quad (32)$$

for $H \leq T_m/(T_m - T_L)$. Because a larger value of H would give a negative value of k

TABLE II. - VALUES OF f FOR SELECTED VALUES OF OTHER PARAMETERS

[Linear interpolation may be used to obtain intermediate values. Use λ^{-1} as argument for linear interpolation between wavelengths.]

Temperature parameter, T_m , K	Profile shape parameter, n	Temperature coefficient of absorption coefficient, H	Transmission factor at $x = L, \tau_L$							
			0.1		0.4		0.8		0.95	
			Wavelength, $\lambda, \mu m$							
			1.9	6.3	1.9	6.3	1.9	6.3	1.9	6.3
			Reciprocal wavelength, $1/\lambda$							
			0.526	0.159	0.526	0.159	0.526	0.159	0.526	0.159
			f							
1000	0.1	-2	1.5	0.7	1.9	1.0	1.5	0.6	1.6	0.7
		0	1.9	.9	2.4	1.3	2.2	1.1	2.4	1.2
		1	2.1	1.2	2.8	1.7	3.0	1.6	3.3	1.8
		2	2.5	1.6	3.5	2.5	4.5	3.1	5.0	3.5
	0.25	-2	2.4	1.1	2.7	1.4	2.0	0.9	2.1	0.9
		0	2.9	1.6	3.3	1.9	2.7	1.4	2.8	1.5
		1	3.3	1.9	3.8	2.3	3.4	2.0	3.6	2.1
		2	3.9	2.6	4.6	3.2	4.9	3.5	5.3	3.8
	0.5	-2	3.3	1.7	3.4	1.9	2.7	1.4	2.7	1.4
		0	3.9	2.2	4.1	2.5	3.4	1.9	3.3	1.9
		1	4.3	2.6	4.6	2.9	4.0	2.5	4.0	2.5
		2	5.1	3.6	5.5	4.1	5.4	4.0	5.7	4.3
2000	0.1	-2	0.6	0.1	0.6	0.2	0.6	0.2	0.6	0.2
		0	1.0	.5	1.0	.5	1.1	.5	1.2	.6
		1	1.4	.9	1.6	.9	1.9	1.1	2.0	1.1
		2	2.0	1.6	2.8	2.2	3.7	2.9	3.8	3.0
	0.25	-2	1.3	0.7	0.9	0.3	0.9	0.3	0.9	0.3
		0	1.7	1.0	1.4	.7	1.4	.7	1.4	.8
		1	2.0	1.3	2.0	1.2	2.1	1.3	2.2	1.3
		2	2.7	2.1	3.2	2.6	3.9	3.2	4.1	3.3
	0.5	-2	1.9	1.1	1.6	0.4	1.3	0.6	1.3	0.6
		0	2.4	1.5	2.3	1.2	1.9	1.1	1.8	1.1
		1	2.8	1.9	2.8	1.8	2.5	1.6	2.6	1.7
		2	3.5	2.9	4.0	3.3	4.3	3.6	4.5	3.7
3000	0.1	-2	0.1	-0.2	0.1	-0.1	0.1	-0.1	0.1	-0.1
		0	.3	0	.4	.1	.4	.1	.4	.1
		1	.6	.3	.8	.4	1.0	.5	1.0	.5
		2	1.2	1.1	2.1	1.8	2.7	2.4	2.8	2.5
	0.25	-2	0.4	-0.7	0.1	-0.3	0.1	-0.3	0.1	-0.3
		0	.8	.2	.5	.1	.6	.1	.6	.1
		1	1.1	.6	1.0	.5	1.1	.6	1.2	.6
		2	1.9	1.7	2.5	2.2	3.1	2.8	3.1	2.9
	0.5	-2	1.6	1.5	0.5	-0.1	0.5	0	0.5	0
		0	2.1	1.9	1.0	.4	.9	.4	.9	.4
		1	2.2	1.8	1.5	.8	1.5	.8	1.5	.8
		2	3.0	3.2	3.2	3.0	3.7	3.6	3.8	3.7

TABLE III. - VALUES OF $(1 - \tau_L)F$ FOR SELECTED VALUES OF OTHER PARAMETERS

[Linear interpolation may be used to obtain intermediate values.]

Temperature parameter, T_m , K	Profile shape parameter, n	Temperature coefficient of absorption coefficient, H	Transmission factor at $x = L$, τ_L							
			0.1		0.4		0.8		0.95	
			Wavelength, λ , μm							
			1.9	6.3	1.9	6.3	1.9	6.3	1.9	6.3
$(1 - \tau_L)F$										
1000	0.1	-2	0.2	0.3	0.2	0.3	0.2	0.4	0.3	0.5
		0	.2	.3	.2	.4	.3	.5	.4	.7
		1	.2	.4	.3	.5	.4	.8	.6	1.1
		2	.3	.6	.3	.8	.7	1.5	.9	2.1
	0.25	-2	0.3	0.5	0.2	0.4	0.3	0.6	0.4	0.7
		0	.3	.6	.3	.5	.4	.7	.5	.9
		1	.4	.7	.3	.6	.5	.9	.6	1.2
		2	.5	1.0	.4	1.0	.7	1.7	.9	2.3
	0.5	-2	0.4	0.8	0.3	0.6	0.5	1.0	0.5	1.1
0		.5	.8	.3	.7	.5	1.0	.6	1.2	
1		.5	.9	.4	.8	.6	1.2	.7	1.4	
2		.6	1.5	.5	1.3	.8	2.0	1.0	2.6	
2000	0.1	-2	0.2	0.4	0.1	0.2	0.2	0.4	0.3	0.4
		3	.3	.5	.2	.3	.3	.6	.4	.7
		1	.3	.7	.3	.6	.6	1.1	.7	1.3
		2	.5	1.3	.6	1.6	1.1	3.0	1.4	3.6
	0.25	-2	0.3	0.5	0.3	0.5	0.3	0.6	0.4	0.7
		0	.4	.6	.3	.6	.4	.7	.5	.9
		1	.4	.8	.4	.8	.6	1.2	.8	1.5
		2	.6	1.6	.7	1.8	1.2	3.3	1.5	4.0
	0.5	-2	0.5	0.8	0.7	1.4	0.5	1.0	0.6	1.2
0		.5	.8	.6	1.3	.5	1.0	.7	1.3	
1		.6	1.0	.6	1.3	.7	1.3	.9	1.7	
2		.8	2.3	.8	2.4	1.3	3.8	1.6	4.7	
3000	0.1	-2	0.2	0.3	0.1	0.2	0.2	0.3	0.2	0.4
		0	.2	.3	.2	.3	.3	.5	.3	.6
		1	.3	.5	.3	.5	.5	1.0	.6	1.2
		2	.5	1.7	.7	2.3	1.4	4.1	1.6	4.9
	0.25	-2	0.5	2.1	0.3	0.6	0.4	0.8	0.5	1.0
		0	.5	1.4	.3	.6	.4	.8	.5	1.0
		1	.6	1.4	.4	.8	.7	1.4	.8	1.6
		2	.9	3.1	.9	3.1	1.6	5.3	2.0	6.3
	0.5	-2	1.4	7.5	0.6	1.3	0.6	1.3	0.8	1.6
0		1.1	3.6	.5	1.1	.6	1.2	.8	1.5	
1		1.0	2.8	.6	1.2	.8	1.7	1.0	2.1	
2		1.6	6.9	1.3	5.1	2.2	8.0	2.7	9.7	

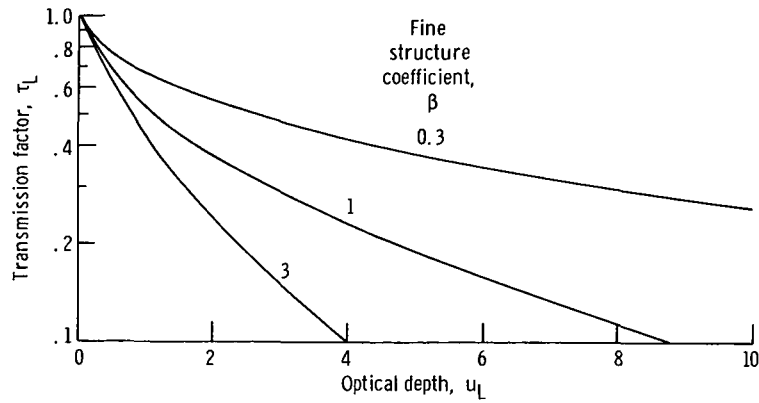


Figure 2. - Overall transmission factor of isothermal gas column.

at T_L , the quantity H should arbitrarily be assigned the limiting value $T_m/(T_m - T_L)$ if H is actually larger than this value. Pressure P of the emitting gas is a representative value over the temperature range from T_L to T_m . Representative values of P and β are their values at the temperature T_{av} used in the determination of H .

Procedure for Computing Error Ratios

The steps in the computation of error ratios are

- (1) For given T_m , n , H , λ , L , and partial pressure P , obtain values of f from table I and then use equation (32) to compute the optical depth u_L .
- (2) Compute temperature $T_{av} = 0.85 T_m$.
- (3) Calculate β from equation (12) at temperature T_{av} .
- (4) Obtain τ_L from figure 2.
- (5) Obtain f and F from tables II and III, choosing the procedure (measurement or calculation) that yields the smaller value of F . For the procedure that uses calculation, use the value of F at which $\tau_L = \exp(-\sqrt{2}/\epsilon)$.
- (6) Calculate the error ratios by equations (22) and (23).

Computed Error Ratios

The error ratios $E(T_m)$ and $E(n)$ are plotted in figure 3 against the path length L for three cases: both measurements in the same band ($2.7 \mu m$); measurements in adjacent bands ($2.7, 6.3 \mu m$ and $1.9, 2.7 \mu m$); measurements in widely different bands ($1.9, 6.3 \mu m$). Values of ${}^m\tau_L$ and ${}^c\tau_L$ along the curves are listed in table IV, where ${}^m\tau_L$ is the ratio of the measured fluxes φ_2 and φ_3 and ${}^c\tau_L$ is calculated from a

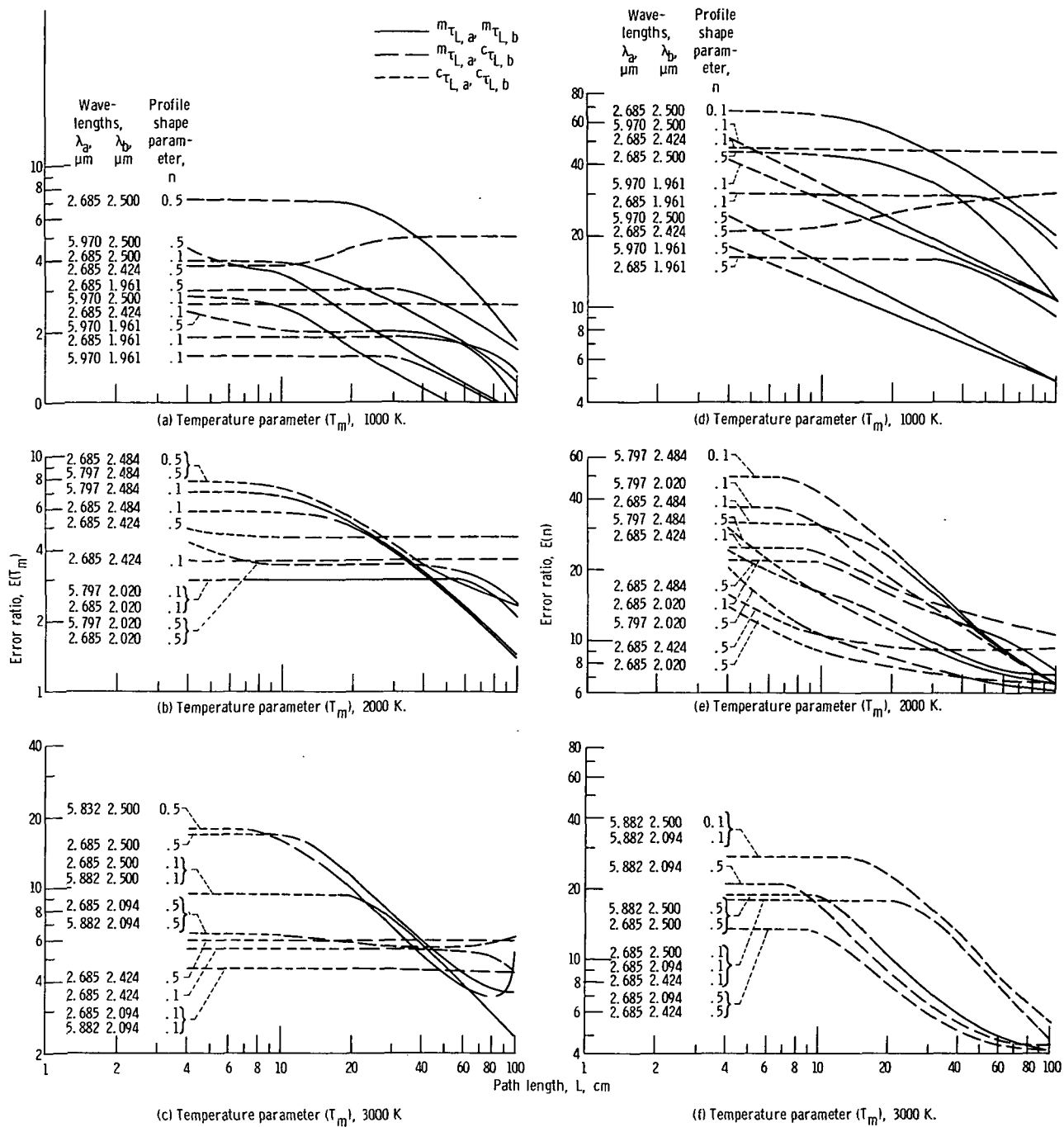


Figure 3. - Error ratios for several wavelength pairs as function of geometric path length. The choice of measured or computed values of τ_L was made so as to minimize the error ratio.

TABLE IV. - TRANSMISSION FACTORS FOR CURVES

OF FIGURE 3 WITH $n = 0.5$

Temperature parameter, T_m , K	Wave-length, λ , μm	Path length, L				
		4	10	20	50	100
		Transmission factor at $x = L$				
1000	5.970	0.85	0.74	0.63	0.47	0.33
	2.685	.79	.66	.54	.37	.24
	2.500	.93	^a .87	.79	.67	.55
	2.424	^a .995	^a .988	^a .976	^a .95	^a .91
	1.961	^a .979	^a .95	^a .91	.83	.75
2000	5.797	^a 0.87	0.73	0.58	0.37	0.22
	2.685	^a .88	.76	.62	.41	.25
	2.484	^a .961	^a .91	.84	.68	.53
	2.424	^a .993	^a .982	^a .964	^a .917	.85
	2.020	^a .990	^a .975	^a .95	^a .89	.80
3000	5.882	^a 0.93	0.85	0.73	0.50	0.32
	2.685	^a .95	^a .88	.78	.57	.39
	2.500	^a .963	^a .91	.84	.67	.49
	2.424	^a .991	^a .980	^a .960	^a .91	.83
	2.094	^a .995	^a .986	^a .973	^a .94	^a .88

^aThis quantity is ${}^c\tau_L$; others are ${}^m\tau_L$.

tabulation of absorption coefficients. The error ratio $E(T_m)$ is smaller at lower temperatures because (1) $|f_a - f_b|$ is greater, and (2) F decreases with decreasing τ_L , which decreases with decreasing temperature, unless path length also decreases. For example, at a path length of 10 centimeters the lowest obtainable value of $E(T_m)$ is about 6 at 3000 K and about 2 at 1000 K. The lowest obtainable value of $E(n)$ is relatively independent of temperature.

PROCEDURE FOR SELECTING OPTIMUM WAVELENGTHS

Given a gas of path length L , pressures $P_{H_2O} \equiv P$ and P_{N_2} , wall temperature T_L , and prior estimates of T_m and n , the following steps may be used to select the pair of wavelengths λ_a, λ_b that will minimize the error ratios $E(T_m)$ and $E(n)$:

(1) Use table II to select approximate values of λ_a, λ_b , and corresponding parameters H to minimize the error ratio in accordance with the selection rule to keep

$|f_a - f_b|$ large, and in accordance with considerations of convenience such as availability of detectors, filters, or monochromators. Use step (2) to confirm that the chosen combination of H and λ exists.

(2) By use of table I, choose wavelengths λ_a, λ_b , at which the desired values of H occur. The error ratio is minimized when the longer wavelength has the more negative value of H , and the larger value of k .

(3) Compute error ratios by the steps listed under "Procedure for Computing Error Ratios."

EFFECT OF LARGE ERROR RATIO ON PROFILE COMPUTATION

In application of the spectral-scanning method, a profile is computed using flux measurements and an iterative procedure. The Newton-Raphson iteration method works well. When measured flux φ_1 and transmission factor τ_L were assumed to be in error, trial examples showed convergence of the iteration in a small number of steps when the error ratios were small, but the iteration did not always converge when the error ratios were large. Instead, a stable oscillation existed between two sets of values of T_m and n as an approximate solution. The oscillation could be eliminated (ref. 9) by use of the damped least squares method (ref. 10). However, this method was not necessary when the error ratio was small, as required for practical use of the spectral scanning method.

The stability of the iterative computational procedure depends on (a) the complexity of the assumed profile shape, that is, on the number of independent parameters assumed to characterize the profile and (b) the magnitude of the error ratios (error in parameter divided by error in flux measurement). For a given measurement error $\delta\varphi/\varphi$, the stability of the iterative solution of the simultaneous equations becomes less likely as the assumed profile becomes more complex. For a given assumed profile shape, the solution of the simultaneous equations resulting from the presumably independent flux measurements is more likely to be unstable when the measurement errors are larger.

The present error analysis, with an example of a constant-pressure gas stream, has shown that two measurements in the infrared must be made at carefully selected wavelengths to avoid excessive random error in the determination of the profile. Other calculations, which will not be detailed here, show that three measurements and an assumed three-parameter profile produced a much greater random error. In general, with both two- and three-parameter profiles, the random error is reduced and the prospects for a stable solution are increased, if measurements are made in both the infrared and the ultraviolet.

When spectral-scanning was applied to measure atmospheric temperature profiles from satellites, (refs. 11 and 12), an analysis showed that nominally independent flux

measurements may have a high degree of interdependence. In this application, there were two favorable circumstances: (1) The probable profile was known to a good approximation from accumulated statistical data, and (2) the pressure gradient along the line of sight, from satellite to Earth, was more conducive to accuracy than the constant-pressure condition assumed in this report. Nevertheless, a maximum of no more than six independent measurements was found feasible for that application. A larger number caused mathematical instability in the computations, and actually increased the random error.

CONCLUDING REMARKS

The preceding discussion has treated errors resulting from uncertainty in flux measurement. Additional sources of error lie in the uncertainty of knowledge of the characteristics of the gas itself; these include the validity of the band model and, through it, the accuracy of the band-model parameters k and β . These are affected also by uncertainties in knowledge of gas composition. On the other hand, the two wavelengths and the corresponding Planck function are known with negligible error. The use of widely separated wavelengths generally results in a desirably larger value of the quantity $|f_a - f_b|$. Such a large value of the difference is less dependent on the value of H or τ_L or on uncertainty in the band model computation of $\tau(x)$ by equation (9). Large error ratios may also lead to slow convergence of the iterative procedures ordinarily used in application of the spectral-scanning method.

The examples show that $E(T_m)$ increases as T_m increases and is quite large at short path lengths. The resulting error ratio may be excessive. It is therefore, appropriate to consider more desirable alternatives. One such alternative is to dispense with the restriction to infrared radiation and to examine the possible advantages of using ultraviolet radiation as well as infrared. At temperatures above 2000 K the pressure of OH is sufficient to permit the measurement of its radiation in the ultraviolet. It is then possible to make one temperature measurement with OH radiation and only one temperature measurement with H_2O radiation. The wavelength separation between ultraviolet and infrared is large. This wavelength separation results in a large difference of parameters $|f_a - f_b|$. The short wavelength of OH reduces the value of F as indicated by table III.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, January 17, 1972,
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