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COMPUTER PROGRAM FOR SOLVING COMPRESSIBLE NONSIMILAR-BOUNDARY-LAYER EQUATIONS FOR LAMINAR, TRANSITIONAL, OR TURBULENT FLOWS OF A PERFECT GAS

by Joseph M. Price and Julius E. Harris

Langley Research Center
Hampton, Va. 23365

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Joseph M. Price and Julius E. Harris

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16. Abstract

A computer program is described which solves the compressible laminar, transitional, or turbulent boundary-layer equations for planar or axisymmetric flows. Three-point implicit difference relations are used to reduce the momentum and energy equations to finite-difference form. These equations are solved simultaneously without iteration. Turbulent flow is treated by the inclusion of either a two-layer eddy-viscosity model or a mixing-length formulation. The eddy conductivity is related to the eddy viscosity through a static turbulent Prandtl number which may be an arbitrary function of the distance from the wall boundary. The transitional boundary layer is treated by the inclusion of an intermittency function which modifies the fully turbulent model. The laminar-boundary-layer equations are recovered when the intermittency is zero, and the fully turbulent equations are solved when the intermittency is unity. This program was used to obtain the solutions presented in NASA TR R-368.

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COMPUTER PROGRAM FOR SOLVING COMPRESSIBLE NONSIMILAR-BOUNDARY-LAYER EQUATIONS FOR LAMINAR, TRANSITIONAL, OR TURBULENT FLOWS OF A PERFECT GAS

By Joseph M. Price and Julius E. Harris
Langley Research Center

SUMMARY

A computer program is described which solves the compressible laminar, transitional, or turbulent boundary-layer equations for planar or axisymmetric flows by an implicit finite-difference procedure. The program was used to obtain the solutions presented in NASA TR R-368. Turbulent flow is treated by the inclusion of either a two-layer eddy-viscosity model or a mixing-length formulation. The eddy conductivity is related to the eddy viscosity by the static turbulent Prandtl number which may be an arbitrary function of the distance from the wall boundary. The transitional boundary layer is treated by introducing an intermittency function which modifies the fully turbulent model. The intermittency function describes the probability distribution of turbulent spots and ranges from zero for laminar flow to unity for a fully turbulent flow.

INTRODUCTION

A number of finite-difference methods are currently available for computing the development of compressible turbulent boundary layers. (See, for example, refs. 1 to 8.) The numerical methods used to solve the governing equations in these references are generally different, in particular those given in references 7 and 8; however, the results are similar when common eddy-viscosity formulations are used. References 1 to 7 use implicit or Crank Nicolson type differences to reduce the governing differential equations to finite-difference form, whereas explicit-type differences are used in reference 8. A coupled solution technique is used in reference 7 (see also ref. 9) which requires no iteration procedure. However, the methods used in references 1 to 6 require iteration since the momentum and energy equations are uncoupled. Another difference, of course, among the various methods is in the formulation of the eddy-viscosity and turbulent Prandtl number formulations used to model the turbulent flux terms appearing in the mean flow equations.

This report describes a computer program developed to solve the compressible nonsimilar-boundary-layer equations for laminar, transitional, or turbulent flows of a perfect gas. The program was used to obtain the results reported in references 7 and 10. A coupled, implicit finite-difference procedure similar to that used for laminar flows in references 9 and 11 is used to solve the momentum and energy equations without iteration. The program will solve problems for two-dimensional or axisymmetric flow geometry for the flow of a perfect gas. Currently, power-law and perfect-air viscosity (Sutherland's) relations are included in the code; however, any perfect gas can be treated by inserting the correct viscosity temperature relations. Transverse curvature effects are included with the option of being neglected if desired. The equations are solved so that nonsimilar terms may also be neglected if desired.

Options are provided for either a two-layer eddy-viscosity model or an arbitrary mixing-length formulation. The turbulent Prandtl number may be either a constant or a function of distance normal to the wall boundary. The transitional region of the flow is modeled through an intermittency distribution which modifies the fully turbulent eddy-viscosity models. Transition location and extent must be specified either from experimental data or correlation relations. The laminar equations are recovered by setting the intermittency to zero.

SYMBOLS

Values are given in both SI and U.S. Customary Units. The measurements and calculations were made in U.S. Customary Units.

A	damping function, $26\nu/u_\tau$
$A^+ = Au_\tau/\nu$	
$A1_n, B1_n, C1_n, D1_n,$ $E1_n, F1_n, G1_n$	coefficients in difference equation (43a) and defined by equations (B3) to (B9)
$A2_n, B2_n, C2_n, D2_n,$ $E2_n, F2_n, G2_n$	coefficients in difference equation (43b) and defined by equations (B10) to (B16)
C_f	skin-friction coefficient, $\dfrac{\tau_w}{\frac{1}{2}\rho u^2}$
C_{m1}, C'_{m1}	defined in equations (B45) and (B46), respectively

c_p	specific heat at constant pressure
$\bar{E}_{m1},\hat{E}_{m1}$	defined in equations (B36) and (B37), respectively
$\hat{E}_Y$	defined in equation (B39)
F	velocity ratio, u/u_e
F_{m1}	defined in equation (B29)
F_{m2}	defined in equation (B32)
F_Y	defined in equation (B40)
G,H	typical variables in the boundary layer (see appendix A)
$H_1,H_2,H_3,\ldots,H_{12}$	coefficients defined by equations (B17) to (B28)
h	heat-transfer coefficient
i	index used in grid-point notation (see eq. (41))
j	flow index; $j = 0$ in planar flow, $j = 1$ in axisymmetric flow
k	grid-point spacing parameter (see eq. (41))
k_l	thermal conductivity
k_T	eddy conductivity (see eq. (15))
k_1	constant in eddy-viscosity model (see eq. (6))
k_2	constant in eddy-viscosity model (see eq. (7))
k_3	see equation (8)
k_4	constant in intermittency function (see eq. (10))
L	reference length

L_{m1},L'_{m1}	defined in equations (B34) and (B35), respectively
l	defined in equation (30)
$\bar{l}$	mixing length (see eq. (13))
M	Mach number
m	grid-point index in X-direction (see fig. 2)
N	number of grid points at each x-station (see fig. 2)
N_{Pr}	Prandtl number, $c_p\mu/k_l$
$N_{Pr,t}$	static turbulent Prandtl number (see eqs. (16) and (17))
N_{St}	Stanton number, $h/(c_p\rho u)$
n	grid-point index in Y-direction (see fig. 2)
$P^{(1)},P^{(2)},P^{(3)}$	defined in equations (48)
p	pressure
$Q^{(1)},Q^{(2)},Q^{(3)}$	defined in equations (48)
q	heat-transfer rate
R,Z	body axes system with origin at the stagnation point, where Z is positive downstream and R is positive radially outward (see fig. 1)
R_e	unit Reynolds number, u_e/ν_e
$R_{e,x}$	Reynolds number based on x, $u_e x/\nu_e$
$R_{e,x_{t,i}}$	Reynolds number at transition, $u_e x_{t,i}/\nu_e$
R_{e,δ^*}	Reynolds number based on displacement thickness, $u_e\delta^*/\nu_e$

$R_{e,\theta}$	Reynolds number based on momentum thickness, $u_e\theta/\nu_e$
R_g	gas constant (see eq. (19))
r	radial body coordinate measured normal to Z-axis (see fig. 1)
r_o	body radius (see fig. 1)
S	Sutherland's viscosity constant, 110.3° K (198.6° R)
T	static temperature
T_Y	defined in equation (B41)
T_{aw}	adiabatic wall temperature
T_{m1},T_{m2}	defined in equations (B30) and (B33), respectively
t	transverse curvature term (see eq. (23))
u	velocity component in X-direction (fig. 1)
u^+	law of wall coordinate, u/u_τ
u_τ	friction velocity, $\sqrt{\tau_w/\rho}$
V	transformed normal-velocity component (see eq. (26))
V_{m1}	defined in equation (B31)
v	velocity component in Y-direction
$\tilde{v}$	velocity component, $v + \frac{\rho' v'}{\rho}$
X,Y	orthogonal boundary-layer coordinate system with origin at stagnation point, where X lies along the body surface and is positive downstream and Y is normal to the body surface and positive outward (see fig. 1)
$X_1,X_2,\ldots,X_5$	functions of grid-point distribution (see eqs. (A4) to (A8))

x	boundary-layer coordinate along X-axis (see fig. 1)
$x_{t,f}$	end of transition (see fig. 1)
$x_{t,i}$	beginning of transition (see fig. 1)
$Y_1,Y_2,\ldots,Y_6$	functions of grid-point distributions (see eqs. (A12) to (A17))
y	boundary-layer coordinate along Y-axis (see fig. 1)
y^+	law of wall coordinate, yu_τ/ν
y_m	match point for two-layer eddy-viscosity model
z	axial body coordinate (see fig. 1)
α	defined in equation (30)
β	defined in equation (30)
Γ	streamwise intermittency distribution (see eq. (38))
γ	ratio of specific heats
$\bar{\gamma}$	transverse intermittency distribution (see eq. (10))
Δ^*	defined in equation (48g)
$\Delta x,\Delta y$	grid-point spacing, physical plane
Δx_t	transition extent, $x_{t,f} - x_{t,i}$
$\Delta\xi,\Delta\eta$	grid-point spacing, transformed plane (see fig. 2)
δ	boundary-layer thickness
δ^*	displacement thickness
δ^*_{inc}	incompressible displacement thickness, $\int_0^\infty (1 - F)\,dy$

$\delta_w^+ = \delta u_{\tau,w}/\nu_w$

ϵ	eddy viscosity, $-\rho \frac{\overline{u'v'}}{\partial u/\partial y}$
$\bar{\epsilon}$	eddy-viscosity function (see eq. (4))
$\bar{\epsilon}_{av}$	defined in equation (B36b)
$\tilde{\epsilon}$	eddy-viscosity function (see eq. (5))
η	transformed normal boundary-layer coordinate (see fig. 2)
Θ	static-temperature ratio, T/T_e
θ	momentum thickness (see fig. 1)
θ_s	shock-wave angle (see fig. 1)
λ	defined in equation (40)
μ	molecular viscosity
ν	kinematic viscosity
$\bar{\nu}$	average kinematic viscosity
ξ	transformed streamwise boundary-layer coordinate (see fig. 2)
$\bar{\xi}$	defined in equation (39)
ρ	density
σ	exponent in power-law viscosity relation (see eq. (21))
τ	shear stress
ϕ	local surface angle (see fig. 1)
χ	vorticity Reynolds number, $\frac{y^2}{\nu} \frac{\partial u}{\partial y}$

χ_{max}	maximum local value of χ
ψ	stream function

$$\omega = \left(\frac{\rho_r u_r L}{\mu_r}\right)^{-1/2}$$

Subscripts:

e	based on boundary-layer edge conditions
i	inner region of turbulent layer
m	mesh point in ξ-direction (see fig. 2)
n	mesh point in η-direction (see fig. 2)
o	outer region of turbulent layer
r	reference quantity
s	shock
t	total condition
w	wall value
∞	free stream

Superscript:

j	flow index; $j = 0$ in planar flow, $j = 1$ in axisymmetric flow

A prime on a symbol denotes a fluctuating component.

A bar over a symbol denotes the time average value.

A coordinate used as a subscript denotes the partial differential with respect to the coordinate. (See eqs. (A1).)

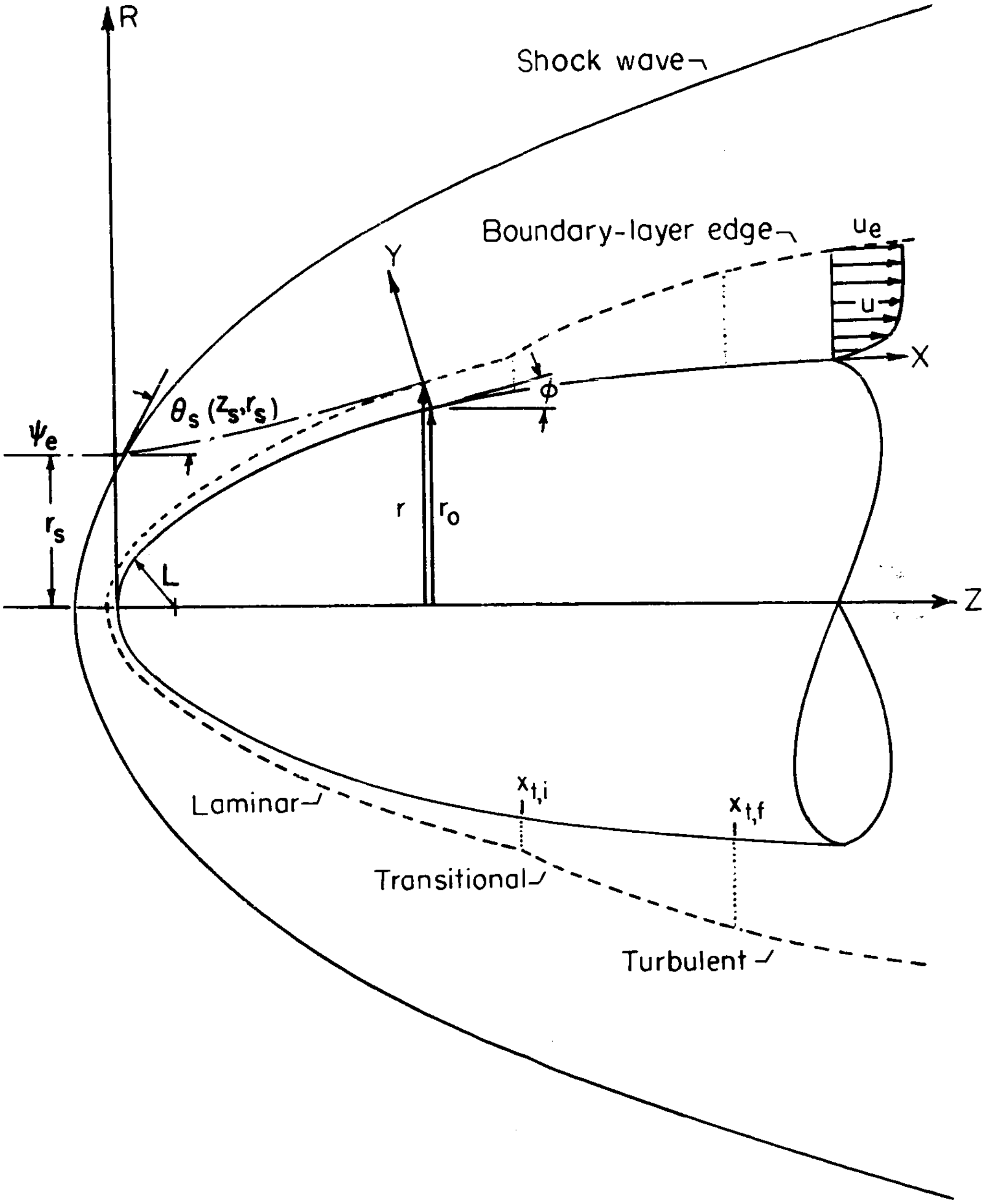


Figure 1.- Coordinate system and notation.

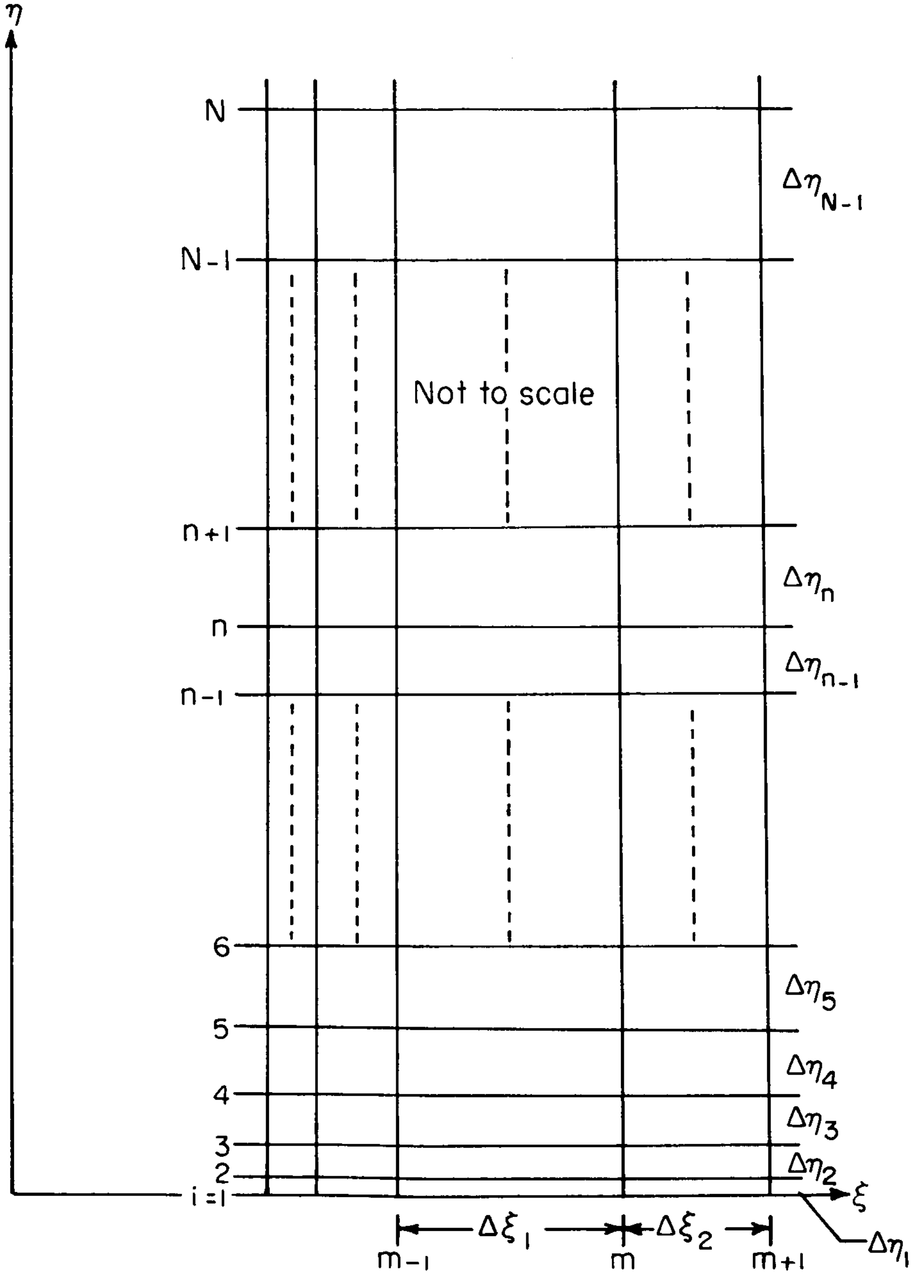


Figure 2.- Finite-difference grid model.

PROBLEM DESCRIPTION

This section presents the governing equations for compressible laminar, transitional, or turbulent boundary-layer flows together with the required boundary conditions. The eddy viscosity, eddy conductivity, transition location and extent, and transitional-flow-structure models are presented and briefly discussed; however, the reader interested in a detailed discussion of these models is referred to references 7 and 10.

Basic Partial Differential Equations

Governing equations.- The partial differential equations describing laminar, transitional, or turbulent compressible boundary-layer flows over planar or axisymmetric geometries are as follows (see refs. 7 and 12):

Continuity

$$\frac{\partial}{\partial x}\left(r^j \rho u\right) + \frac{\partial}{\partial y}\left(r^j \rho \tilde{v}\right) = 0 \tag{1}$$

Momentum

$$\rho\left(u\frac{\partial u}{\partial x} + \tilde{v}\frac{\partial u}{\partial y}\right) = -\frac{dp}{dx} + \frac{1}{r^j}\frac{\partial}{\partial y}\left(r^j \bar{\epsilon}\frac{\partial u}{\partial y}\right) \tag{2}$$

Energy

$$\rho\left[u\frac{\partial}{\partial x}\left(c_p T\right) + \tilde{v}\frac{\partial}{\partial y}\left(c_p T\right)\right] = u\frac{dp}{dx} + \bar{\epsilon}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{r^j}\frac{\partial}{\partial y}\left[r^j \tilde{\epsilon}\frac{\partial}{\partial y}\left(c_p T\right)\right] \tag{3}$$

where the conventional overbar notation for time mean-average variables has been dropped for brevity. The eddy-viscosity parameters $\bar{\epsilon}$ and $\tilde{\epsilon}$ are defined, respectively, as follows:

$$\bar{\epsilon} = \mu\left(1 + \frac{\epsilon}{\mu}\Gamma\right) \tag{4}$$

and

$$\tilde{\epsilon} = \frac{\mu}{N_{Pr}}\left(1 + \frac{\epsilon}{\mu}\frac{N_{Pr}}{N_{Pr,t}}\Gamma\right) \tag{5}$$

The intermittency-distribution parameter Γ is discussed in a subsequent section. (See eq. (38).)

Eddy viscosity.- Options are provided in the coded program for selecting either a two-layer eddy-viscosity model (KODVIS = 1) or a straight-forward mixing-length model (KODVIS = 2).

Two-layer model.- The equations describing the two-layer model are as follows (see ref. 7):

$$\left(\frac{\epsilon}{\mu}\right)_i = \frac{\rho}{\mu}\left(k_1 y D\right)^2 \left|\frac{\partial u}{\partial y}\right| \qquad \left(0 \leqq y \leqq y_m\right) \tag{6}$$

$$\left(\frac{\epsilon}{\mu}\right)_o = \frac{\rho}{\mu} k_2 u_e \delta^*_{inc} \bar{\gamma} \qquad \left(y_m < y\right) \tag{7}$$

where

$$D = 1 - \exp\left\{-\left[\sqrt{\frac{\nu_w}{\bar{\nu}}}\left(1 + k_3\right) - k_3\right]\frac{y}{A}\right\} \tag{8}$$

$$\delta^*_{inc} = \int_0^\infty \left(1 - \frac{u}{u_e}\right) dy \tag{9}$$

and

$$\bar{\gamma} = \frac{1 - \operatorname{erf}\left[5\left(\frac{y}{\delta} - k_4\right)\right]}{2} \tag{10}$$

The boundary-layer thickness δ appearing in equation (10) is defined as the distance normal to the wall boundary where $u/u_e = 0.995$. The empirical constants k_1, k_2, k_3, and k_4 are assigned values of 0.4, 0.0168, 0.0, and 0.78, respectively. Note that for $k_3 = -1.0$, the kinematic-viscosity term is removed from equation (8) (XT5 = -1.0). The empirical constants k_1 and k_2 can easily be treated as functions of some correlation parameter, say δ_w^+, in order to account properly for low Reynolds number effects in hypersonic flows. (See ref. 13 for discussion of low Reynolds number effects.) The location of the boundary separating the two layers y_m is determined from the continuity of eddy viscosity; that is, where

$$\left(\frac{\epsilon}{\mu}\right)_i = \left(\frac{\epsilon}{\mu}\right)_o \tag{11}$$

Mixing-length model.- A mixing-length formulation is provided (KODVIS = 2) for those interested in utilizing experimental mixing-length distributions. (See ref. 13, for example.) The eddy-viscosity distribution across the boundary layer can be written as follows:

$$\frac{\epsilon}{\mu} = \frac{\rho}{\mu} \bar{l}^2 \left|\frac{\partial u}{\partial y}\right| \tag{12}$$

where the mixing length $\bar{l}$ may be written as

$$\frac{\bar{l}}{\delta} = D\bar{\gamma}f\left(\frac{y}{\delta}\right) \tag{13}$$

Currently, the simplest possible formulation is provided in the digital code for $f\left(\frac{y}{\delta}\right)$ as follows:

$$f\left(\frac{y}{\delta}\right) = \begin{cases} 0.4\left(\frac{y}{\delta}\right) & \left(\frac{y}{\delta} \leqq 0.2\right) \\ 0.08 & \left(\frac{y}{\delta} > 0.2\right) \end{cases} \tag{14}$$

However, it should be noted that any functional variation can be utilized in the program.

Eddy conductivity and static turbulent Prandtl number.- The eddy conductivity defined as

$$k_T = -c_p\rho \frac{\overline{v'T'}}{\partial T/\partial y} \tag{15}$$

is modeled as a static turbulent Prandtl number $N_{Pr,t}$ as follows:

$$N_{Pr,t} = \frac{c_p\epsilon}{k_T} \tag{16}$$

where

$$N_{Pr,t} = \frac{\overline{u'v'}}{\overline{v'T'}}\left(\frac{\partial T/\partial y}{\partial u/\partial y}\right) \tag{17}$$

Any desired functional relation for $N_{Pr,t} = f\left(\frac{y}{\delta}\right)$ may be utilized in the digital code; three options are available. These options are (1) a constant, say $N_{Pr,t} = 0.95$, (2) an arbitrary distribution $N_{Pr,t} = f\left(\frac{y}{\delta}\right)$ supplied in tabular form, and (3) the Rotta distribution (see ref. 14) as follows:

$$N_{Pr,t} = 0.45\left[2 - \left(\frac{y}{\delta}\right)^2\right] \tag{18}$$

The system of equations is closed by the addition of the perfect-gas laws and a viscosity-temperature relation. The perfect-gas law is expressed as

$$P = \rho R_g T \tag{19}$$

Currently, the digital code is written to include the Sutherland viscosity-temperature relation for air (IGAS = 1)

$$\frac{\mu}{\mu_r} = \left(\frac{T}{T_r}\right)^{3/2}\left(\frac{T_r + S}{T + S}\right) \tag{20}$$

as well as the power-law expression

$$\frac{\mu}{\mu_r} = \left(\frac{T}{T_r}\right)^{\sigma} \tag{21}$$

where $\sigma = 0.647$ for helium (IGAS = 2).

Transformed plane.- The system of governing equations is singular at $x = 0$. The Probstein-Elliott (ref. 15) and Levy-Lees (ref. 16) transformation is used to remove this singularity as well as to reduce the growth of the boundary layer as the solution proceeds downstream. This transformation can be written as follows:

$$\xi(x) = \int_0^x \rho_e u_e \mu_e r_o^{2j}\, dx \tag{22a}$$

$$\eta(x,y) = \frac{\rho_e u_e r_o^j}{\sqrt{2\xi}} \int_0^y t^j \left(\frac{\rho}{\rho_e}\right) dy \tag{22b}$$

where the parameter t appearing in equation (22b) is the transverse curvature term, defined as

$$t = 1 + \frac{r}{r_o} \tag{23a}$$

or, in terms of the y-coordinate, as

$$t = 1 + \frac{y}{r_o} \cos \phi \tag{23b}$$

The relation between derivatives in the physical (x,y) and transformed (ξ,η) coordinate system is as follows:

$$\left(\frac{\partial}{\partial x}\right)_y = \rho_e u_e \mu_e r_o^{2j} \left(\frac{\partial}{\partial \xi}\right)_\eta + \left(\frac{\partial \eta}{\partial x}\right)\left(\frac{\partial}{\partial \eta}\right)_\xi \tag{24a}$$

$$\left(\frac{\partial}{\partial y}\right)_x = \frac{\rho_e u_e r_o^j t^j}{\sqrt{2\xi}} \left(\frac{\rho}{\rho_e}\right)\left(\frac{\partial}{\partial \eta}\right)_\xi \tag{24b}$$

Two new parameters F and Θ are introduced and defined as

$$\left.\begin{aligned} F &= \frac{u}{u_e} \\ \Theta &= \frac{T}{T_e} \end{aligned}\right\} \tag{25}$$

as well as a transformed normal velocity

$$V = \frac{2\xi}{\rho_e u_e \mu_e r_o^{2j}}\left[F\left(\frac{\partial \eta}{\partial x}\right) + \frac{\rho \tilde{v} r_o^j t^j}{\sqrt{2\xi}}\right] \tag{26}$$

The governing equations in the transformed plane can then be expressed as follows:

<u>Continuity</u>

$$\frac{\partial V}{\partial \eta} + 2\xi \frac{\partial F}{\partial \xi} + F = 0 \tag{27}$$

<u>Momentum</u>

$$2\xi F \frac{\partial F}{\partial \xi} + V \frac{\partial F}{\partial \eta} - \frac{\partial}{\partial \eta}\left(t^{2j} l \bar{\epsilon} \frac{\partial F}{\partial \eta}\right) + \beta\left(F^2 - \Theta\right) = 0 \tag{28}$$

<u>Energy</u>

$$2\xi F \frac{\partial \Theta}{\partial \xi} + V \frac{\partial \Theta}{\partial \eta} - \frac{\partial}{\partial \eta}\left(t^{2j} \frac{l}{N_{Pr}} \tilde{\epsilon} \frac{\partial \Theta}{\partial \eta}\right) - \alpha l t^{2j} \bar{\epsilon} \left(\frac{\partial F}{\partial \eta}\right)^2 = 0 \tag{29}$$

where

$$\left.\begin{aligned} l &= \frac{\rho \mu}{(\rho \mu)_e} \\ \alpha &= \frac{u_e^2}{c_p T_e} \\ \beta &= \frac{2\xi}{u_e}\left(\frac{du_e}{d\xi}\right) \end{aligned}\right\} \tag{30}$$

By using the viscosity relations (eqs. (20) and (21)) and the equation of state (eq. (19)), the parameter l can be written as follows:

$$l = \sqrt{\Theta}\left(\frac{1 + \bar{S}}{\Theta + \bar{S}}\right) \qquad \text{(Air only)} \tag{31a}$$

$$l = (\Theta)^{\sigma-1.0} \qquad \text{(Power-law viscosity)} \tag{31b}$$

where $\bar{S} = S/T_e$.

The transverse-curvature term can be written in terms of the transformed variables as

$$t = \pm\left(1 + \frac{2\sqrt{2\xi}\cos\phi}{\rho_e u_e}\int_0^\eta \frac{\rho_e}{\rho}\,d\eta\right)^{1/2} \tag{32}$$

The physical coordinate normal to the wall is obtained from the inverse transformation; namely,

$$y = \frac{r_o}{\cos\phi}\left[-1 \pm \left(\frac{1 + 2\sqrt{2\xi}\cos\phi}{\rho_e u_e r_o^{2j}}\int_0^\eta \Theta\,d\eta\right)^{1/2}\right] \tag{33}$$

The positive sign is used in equations (32) and (33) for axisymmetric flow over bodies of revolution (SIGN = 1.0), and the negative sign is used for flow inside axisymmetric ducts (SIGN = -1.0).

The boundary conditions in the transformed plane are as follows:

Wall boundary

$$\left.\begin{aligned} F(\xi,0) &= 0 \\ V(\xi,0) &= V_w(\xi) \\ \Theta(\xi,0) &= \Theta_w(\xi) \end{aligned}\right\} \tag{34a}$$

or

$$\left(\frac{\partial\Theta}{\partial\eta}\right)_{\xi,0} = \left(\frac{\partial\Theta}{\partial\eta}\right)_w$$

Edge conditions

$$\left.\begin{aligned} F(\xi,\eta_e) &= 1 \\ \Theta(\xi,\eta_e) &= 1 \end{aligned}\right\} \tag{34b}$$

The boundary condition at the wall for the transformed V component can be related to the physical plane as (see ref. 7)

$$V_w = \frac{\sqrt{2\xi}}{\mu_e r_o^{2j}} \left(\frac{\rho_w v_w}{\rho_e u_e}\right) \tag{35}$$

Transition location.- Many parameters influence the location of transition. These parameters are discussed in an extensive review presented in reference 17. (See also ref. 7.)

It is not currently possible to predict with assurance the location of transition for a general geometry; however, for some particular classes of flow such as those caused by sharp cones, empirical correlations are available which can be used with confidence providing it is realized that a probable range of transition locations is being predicted and not an exact fixed point. (See ref. 7.) In the present digital code either the transition location (SST) or the stability index (SMXTR) must be specified; however, any correlation relation may be directly incorporated into the program if desired.

Transition extent.- The assumption of a universal intermittency distribution implies that the transition-zone length (transition extent) can be expressed as a function of the transition Reynolds number, $u_e x_{t,i}/\nu_e$. In reference 18 it is shown, for the transition data considered, that the data are represented on the average by the equation

$$R_{e,\Delta x_t} = 5\left(R_{e,x_{t,i}}\right)^{0.8} \tag{36}$$

where $R_{e,\Delta x_t} = \frac{u_e}{\nu_e}\left(x_{t,f} - x_{t,i}\right)$. The location of the end of transition, $x_{t,f}$ can then be obtained directly from equation (36) as follows:

$$x_{t,f} = x_{t,i} + 5R_e^{-1}\left(R_{e,x_{t,i}}\right)^{0.8} \tag{37}$$

where R_e is the local unit Reynolds number, u_e/ν_e.

In the present digital code, due to the lack of general correlations for the extent of transition, this quantity $\left(x_{t,f} - x_{t,i}\right)$ can be specified in one of two ways: (1) from equation (37) (KTCOD = 1), or (2) from the specification of $x_{t,f}/x_{t,i}$ obtained from experimental data (TLNGTH, KTCOD = 2). It should be noted that the digital code can be modified to include any desired correlation or equation in place of equation (37).

Intermittency distribution.- The parameter Γ appearing in equations (4) and (5) represents the streamwise intermittency distribution which models the turbulent spot distribution in the transitional region; the parameter Γ is a function of the x-coordinate only and is defined as follows (see ref. 18):

$$\Gamma(\bar{\xi}) = 1 - \exp\left(-0.412\bar{\xi}^2\right) \tag{38}$$

where

$$\bar{\xi} = \frac{x - x_{t,i}}{\lambda} \tag{39}$$

and

$$\lambda = (x)_{\Gamma=\frac{3}{4}} - (x)_{\Gamma=\frac{1}{4}} \tag{40}$$

It should be noted that $\Gamma = 0$ for laminar flow, $\Gamma = 1$ for fully turbulent flow, and Γ ranges from 0 to 1 through the transitional-flow region. Equations (1) to (3) reduce to the classical laminar boundary-layer equations when Γ is set to zero. (See ref. 19.)

Numerical Solution of the Governing Equations

The system of governing equations (eqs. (19) to (21) and (27) to 29)) is parabolic and, therefore, can be numerically integrated in a step-by-step procedure in the streamwise direction. In order to cast the equations into a form in which the step-by-step procedure can be efficiently used, the derivatives with respect to ξ and η are replaced by finite-difference quotients. The method of linearization and solution used in the analysis closely parallels that of references 9 and 11.

Finite-difference mesh model.- It has been shown for laminar boundary layers that equally spaced grid points can be used in the normal coordinate direction. (See refs. 9 and 11.) However, for transitional and turbulent boundary layers the use of equally spaced grid points is not practical because the fine-mesh size required to obtain accurate results near the wall boundary is inefficient for the entire boundary layer. The grid-point spacing in the η-direction used in the program is such that the ratio of any two successive strips is a constant; that is, the successive $\Delta\eta_i$-coordinates form a geometric progression. In constructing the difference quotients, the sketch of the grid-point distribution presented in figure 2 is useful for reference. The dependent variables F, Θ, and V are assumed known at each of the N grid points along the $m-1$ and m stations, but are unknown at station $m+1$. The $\Delta\xi_1$ and $\Delta\xi_2$ values, not specified to be equal, are obtained from the specified x values $\left(x_{m-1}, x_m, x_{m+1}\right)$ and from equation (22a). The relationship between the $\Delta\eta_i$-coordinates for the chosen grid-point spacing is given by the following equation (see ref. 1):

$$\Delta\eta_i = (k)^{i-1}\Delta\eta_1 \qquad (i = 1,2,3,\ldots,N) \qquad (41)$$

where k is the ratio of any two successive steps (XK), $\Delta\eta_1$ is the spacing between the second grid point and the wall (note that the first grid point is at the wall boundary), and N denotes the total number of grid points across the chosen η strip. The total thickness of the η strip can then be expressed as follows:

$$\eta_N = \Delta\eta_i\left[\frac{1 - (k)^{N-1}}{1 - k}\right] \qquad (k \neq 1) \qquad (42)$$

The selection of the optimum k and N values for a specified η_N-coordinate depends upon the particular problem under consideration. The main objective in the selection is to obtain the minimum number of grid points with which a convergent solution may be obtained and thereby minimize the computer-processing time for each test case. The laminar boundary layer presents no problem since a k value of unity is acceptable; however, for transitional or turbulent layers, the value of k will be a number slightly greater than unity, say between 1.02 and 1.04. If transitional or turbulent flow occurs in a given problem, the laminar portion of the boundary layer is calculated with the value of k used for the turbulent region; that is, for a given problem, k is invariant.

Difference equations.- Three-point implicit difference relations (see appendix A) are used to reduce the transformed momentum and energy equations (eqs. (28) and (29), respectively) to finite-difference form. The difference quotients produce linear difference equations when substituted into the momentum and energy equations provided truncation terms of the order $\Delta\xi_{m-1}\,\Delta\xi_m$ and $\Delta\eta_{n-1}\,\Delta\eta_n$ are neglected. (It should be noted that the truncation term for $\partial^2F/\partial\eta^2$ is of the order $\Delta\eta_{n-1} - \Delta\eta_n$.) The resulting difference equations may be written as follows:

$$A1_nF_{m+1,n-1} + B1_nF_{m+1,n} + C1_nF_{m+1,n+1} + D1_n\Theta_{m+1,n-1}$$
$$+ E1_n\Theta_{m+1,n} + F1_n\Theta_{m+1,n+1} = G1_n \qquad (43a)$$

$$A2_nF_{m+1,n-1} + B2_nF_{m+1,n} + C2_nF_{m+1,n+1} + D2_n\Theta_{m+1,n-1}$$
$$+ E2_n\Theta_{m+1,n} + F2_n\Theta_{m+1,n+1} = G2_n \qquad (43b)$$

The coefficients $A1_n, B1_n, \ldots, G1_n$ and $A2_n, B2_n, \ldots, G2_n$ (see appendix B) are functions of known quantities at stations m and m-1. It is important to note that equations (43) are coupled through the dependent variables F and Θ; however, the dependent variable V does not appear explicitly as an unknown at station $m+1$. The variable V is uncoupled from the system because of the particular way that the nonlinear terms $V\frac{\partial F}{\partial \eta}$ and $V\frac{\partial \Theta}{\partial \eta}$ (see eqs. (28) and (29), respectively) are linearized. (See eq. (A23).)

Solution of difference equations.- The system of difference equations (eqs. (43)) represents a set of exactly $2(N - 1)$ linear algebraic equations for $2(N - 1)$ unknowns. The proper boundary conditions to be used with the difference equations are specified in equations (34). The $2(N - 1)$ linear algebraic equations may be written in tridiagonal matrix form; consequently, an efficient algorithm (Gaussian elimination) is available for simultaneous solution.

The simultaneous or coupled-solution technique is presented in appendix B of reference 9; however, because of differences between the present work and that presented in reference 9, the solution technique is discussed here in some detail.

Because of the special form of equations (43), the following relations exist (see ref. 20):

$$F_{m+1,n-1} = P^{(1)}_{m+1,n-1} + P^{(2)}_{m+1,n-1}F_{m+1,n} + P^{(3)}_{m+1,n-1}\Theta_{m+1,n} \tag{44a}$$

$$\Theta_{m+1,n-1} = Q^{(1)}_{m+1,n-1} + Q^{(2)}_{m+1,n-1}F_{m+1,n} + Q^{(3)}_{m+1,n-1}\Theta_{m+1,n} \tag{44b}$$

Next, equations (44) are substituted into equations (43) to obtain the following relations:

$$\begin{aligned} B1^*_{m+1,n}F_{m+1,n} + E1^*_{m+1,n}\Theta_{m+1,n} = G1^*_{m+1,n} &- C1_{m+1,n}F_{m+1,n+1} \\ &- F1_{m+1,n}\Theta_{m+1,n+1} \end{aligned} \tag{45a}$$

$$\begin{aligned} B2^*_{m+1,n}F_{m+1,n} + E2^*_{m+1,n}\Theta_{m+1,n} = G2^*_{m+1,n} &- C2_{m+1,n}F_{m+1,n+1} \\ &- F1_{m+1,n}\Theta_{m+1,n+1} \end{aligned} \tag{45b}$$

where

$$B1^*_{m+1,n} = B1_{m+1,n} + A1_{m+1,n}P^{(2)}_{m+1,n-1} + D1_{m+1,n}Q^{(2)}_{m+1,n-1} \tag{46a}$$

$$E1^*_{m+1,n} = E1_{m+1,n} + A1_{m+1,n}P^{(3)}_{m+1,n-1} + D1_{m+1,n}Q^{(3)}_{m+1,n-1} \tag{46b}$$

$$G1^*_{m+1,n} = G1_{m+1,n} - A1_{m+1,n}P^{(1)}_{m+1,n-1} - D1_{m+1,n}Q^{(1)}_{m+1,n-1} \qquad (46c)$$

$$B2^*_{m+1,n} = B2_{m+1,n} + A2_{m+1,n}P^{(2)}_{m+1,n-1} + D2_{m+1,n}Q^{(2)}_{m+1,n-1} \qquad (46d)$$

$$E2^*_{m+1,n} = E2_{m+1,n} + A2_{m+1,n}P^{(3)}_{m+1,n-1} + D2_{m+1,n}Q^{(3)}_{m+1,n-1} \qquad (46e)$$

and

$$G2^*_{m+1,n} = G2_{m+1,n} - A2_{m+1,n}P^{(1)}_{m+1,n-1} - D2_{m+1,n}Q^{(1)}_{m+1,n-1} \qquad (46f)$$

The unknown values of F and Θ at station $m+1,n$ are obtained from equations (45) as follows:

$$F_{m+1,n} = P^{(1)}_{m+1,n} + P^{(2)}_{m+1,n}F_{m+1,n+1} + P^{(3)}_{m+1,n}\Theta_{m+1,n+1} \qquad (47a)$$

$$\Theta_{m+1,n} = Q^{(1)}_{m+1,n} + Q^{(2)}_{m+1,n}F_{m+1,n+1} + Q^{(3)}_{m+1,n}\Theta_{m+1,n+1} \qquad (47b)$$

where

$$P^{(1)}_{m+1,n} = \left(E2^*_{m+1,n}G1^*_{m+1,n} - E1^*_{m+1,n}G2^*_{m+1,n}\right)\Delta^*_{m+1,n} \qquad (48a)$$

$$P^{(2)}_{m+1,n} = \left(E1^*_{m+1,n}C2_{m+1,n} - E2^*_{m+1,n}C1_{m+1,n}\right)\Delta^*_{m+1,n} \qquad (48b)$$

$$P^{(3)}_{m+1,n} = \left(E1^*_{m+1,n}F2_{m+1,n} - E2^*_{m+1,n}F1_{m+1,n}\right)\Delta^*_{m+1,n} \qquad (48c)$$

$$Q^{(1)}_{m+1,n} = \left(B1^*_{m+1,n}G2^*_{m+1,n} - B2^*_{m+1,n}G1^*_{m+1,n}\right)\Delta^*_{m+1,n} \qquad (48d)$$

$$Q^{(2)}_{m+1,n} = \left(B2^*_{m+1,n}C1_{m+1,n} - B1^*_{m+1,n}C2_{m+1,n}\right)\Delta^*_{m+1,n} \qquad (48e)$$

$$Q^{(3)}_{m+1,n} = \left(B2^*_{m+1,n}F1_{m+1,n} - B1^*_{m+1,n}F2_{m+1,n}\right)\Delta^*_{m+1,n} \qquad (48f)$$

and

$$\Delta^*_{m+1,n} = \frac{1}{\left(B1^*_{m+1,n}E2^*_{m+1,n} - B2^*_{m+1,n}E1^*_{m+1,n}\right)} \qquad (48g)$$

Next, equations (44) are rewritten as follows (where $n = n+1$)

$$F_{m+1,n} = P^{(1)}_{m+1,n} + P^{(2)}_{m+1,n} F_{m+1,n+1} + P^{(3)}_{m+1,n} \Theta_{m+1,n+1} \tag{49a}$$

$$\Theta_{m+1,n} = Q^{(1)}_{m+1,n} + Q^{(2)}_{m+1,n} F_{m+1,n+1} + Q^{(3)}_{m+1,n} \Theta_{m+1,n+1} \tag{49b}$$

The "no-slip" boundary condition $\left(F_{m+1,1} = 0\right)$ is applied at the wall boundary to obtain the values of $P^{(i)}_{m+1,1}$ where $i = 1,2,3$; that is,

$$P^{(1)}_{m+1,1} = P^{(2)}_{m+1,1} = P^{(3)}_{m+1,1} = 0 \tag{50}$$

The thermal condition at the wall boundary may be specified in one of two ways: (1) specified wall-temperature distribution, or (2) specified heat-transfer distribution. For a specified wall-temperature distribution it can be seen directly from equation (49b) that

$$\left.\begin{aligned} Q^{(1)}_{m+1,1} &= \Theta_{m+1,1} \\ Q^{(2)}_{m+1,1} &= Q^{(3)}_{m+1,1} = 0 \end{aligned}\right\} \tag{51}$$

The case in which a heat-transfer distribution is specified presents a more difficult problem; however, this class of flows is often of interest. (For example, consider adiabatic flows.)

The heat transfer at the wall boundary can be written in the transformed plane as follows (see ref. 7):

$$q_{m+1,1} = \frac{-\mu_r u_r^2}{\omega L} \left(\frac{\rho_e u_e T_c \mu_e r_o^j}{N_{Pr}\sqrt{2\xi}}\right)_{m+1,N} l_{m+1,1} \left(\frac{\partial \Theta}{\partial \eta}\right)_{m+1,1} \tag{52}$$

Then, for a specified value of $q_{m+1,1}$, the gradient of Θ can be obtained directly as follows:

$$\left(\frac{\partial \Theta}{\partial \eta}\right)_{m+1,1} = -q_{m+1,1} \left(\frac{\omega L}{\mu_r u_r^2}\right) \left(\frac{N_{Pr}\sqrt{2\xi}}{\rho_e u_e T_e \mu_e r_o^j}\right)_{m+1,N} \left(\frac{1}{l}\right)_{m+1,1} \tag{53}$$

For the grid-point spacing used in the analysis (geometric progression, see eq. (41)), the gradient of Θ evaluated at the wall, by using a 3-point relation, is as follows:

$$\left(\frac{\partial\Theta}{\partial\eta}\right)_{m+1,1} = \frac{\left[1 - (1 + k)^2\right]\Theta_{m+1,1} + (1 + k)^2\Theta_{m+1,2} - \Theta_{m+1,3}}{k(1 + k)\,\Delta\eta_1} \tag{54}$$

Equations (53) and (54) then yield the following expression for $\Theta_{m+1,1}$:

$$\Theta_{m+1,1} = \frac{k(1 + k)\,\Delta\eta_1}{1 - (1 + k)^2}\left(\frac{\partial\Theta}{\partial\eta}\right)_{m+1,1} - \frac{(1 + k)^2}{1 - (1 + k)^2}\Theta_{m+1,2} + \frac{1}{1 - (1 + k)^2}\Theta_{m+1,3} \tag{55}$$

where $\left(\frac{\partial\Theta}{\partial\eta}\right)_{m+1,1}$ is evaluated from equation (53). Equations (43) are next written at the $m+1,2$ point to obtain two equations in terms of $F_{m+1,n}$ and $\Theta_{m+1,n}$ where $n = 1,2,3$. (Note that $F_{m+1,1} = 0$.) The quantity $F_{m+1,3}$ is next eliminated from these two equations to obtain one equation in terms of $F_{m+1,2}$ and $\Theta_{m+1,n}$ where $n = 1,2,3$. The quantity $\Theta_{m+1,3}$ is next eliminated through use of equation (55) to obtain the relation

$$\Theta_{m+1,1} = \bar{Q}^{(1)}_{m+1,1} + \bar{Q}^{(2)}_{m+1,1}F_{m+1,2} + \bar{Q}^{(3)}_{m+1,1}\Theta_{m+1,2} \tag{56}$$

where

$$\bar{Q}^{(1)}_{m+1,1} = \frac{\left[(C2)(G1) - (C1)(G2)\right]_{m+1,2} + \left[(C2)(F1) - (C1)(F2)\right]_{m+1,2}\left[k(1 + k)\,\Delta\eta_1\right]\left(\frac{\partial\Theta}{\partial\eta}\right)_{m+1,1}}{\Delta_{m+1,2}} \tag{57a}$$

$$\bar{Q}^{(2)}_{m+1,1} = \frac{\left[(C1)(B2) - (C2)(B1)\right]_{m+1,2}}{\Delta_{m+1,2}} \tag{57b}$$

$$\bar{Q}^{(3)}_{m+1,1} = \frac{\left[(C1)(E2) - (C2)(E1)\right]_{m+1,2} + \left[(C1)(F2) - (C2)(F1)\right]_{m+1,2}(1 + k)^2}{\Delta_{m+1,2}} \tag{57c}$$

and

$$\Delta_{m+1,2} = \left\{\left[(C2)(D1) - (C1)(D2)\right] + \left[(C2)(F1) - (C1)(F2)\right]\left[1 - (1 + k)^2\right]\right\}_{m+1,2} \tag{57d}$$

By comparing equations (49b) and (56) it is observed that

$$Q^{(i)}_{m+1,1} = \overline{Q}^{(i)}_{m+1,1} \qquad (i = 1,2,3) \qquad (58)$$

which completes the desired boundary condition for the case of a specified heat-transfer distribution along the wall boundary. The temperature at the wall is obtained directly from equation (55) once $\Theta_{m+1,2}$ and $\Theta_{m+1,3}$ are known.

The quantities $P^{(i)}_{m+1,n}$ and $Q^{(i)}_{m+1,n}$ where $i = 1,2,3$ (see eqs. (49)) must first be determined across the boundary layer at the $m+1$ station where $n = 1,2,\ldots,N$. These quantities are calculated by the following procedure:

(1) Perform the following steps at the first grid point away from the wall ($n = 2$):

(a) Calculate $A1_2,B1_2,\ldots,G1_2$ from equations (B3) to (B9).

(b) Calculate $A2_2,B2_2,\ldots,G2_2$ from equations (B10) to (B16).

(c) By using the results from steps (a) and (b) and the boundary conditions (eqs. (50) and (51) or (57)), calculate $B1_2^*$, $B2_2^*$, $E1_2^*$, $E2_2^*$, $G1_2^*$, and $G2_2^*$ from equations (46).

(d) By using the results from steps (a) to (c), calculate $P_2^{(i)}$ and $Q_2^{(i)}$ where $i = 1,2,3$ from equations (48).

(2) The procedure outlined in step (1) is now repeated at grid point with $n = 3$ by using the results obtained at $n = 2$. This procedure is repeated until the entire boundary layer is traversed ($n = N$) and all values of $P^{(i)}_{m+1,n}$ and $Q^{(i)}_{m+1,n}$ are determined where $i = 1,2,3$ and $n = 2,3,4,\ldots,N$.

(3) By knowing the values of $P^{(i)}_{m+1,n}$ and $Q^{(i)}_{m+1,n}$ where $i = 1,2,3$ and $n = 2,3,4,\ldots,N$, the values of $F_{m+1,n}$ and $\Theta_{m+1,n}$ where $n = N-1,N-2,\ldots,2$ are calculated from equations (47). It should be noted that $F_{m+1,N}$ and $\Theta_{m+1,N}$ are specified edge boundary conditions (eqs. (34b)). The wall-boundary values of F and Θ are obtained from equations (34a) or equation (55) for the case of a specified wall-boundary heat-transfer distribution. Before the computations can proceed downstream, the transformed velocity $V_{m+1,n}$ must be determined across the boundary layer where $n = 2,3,\ldots,N$. This requires the solution of the continuity equation. (See eq. (27).)

Solution of continuity equation. - The continuity equation (eq. (27)) is solved numerically for the $N - 1$ unknown values of V at station $m+1$. Equation (27) is integrated once to yield the following relation for $V_{m+1,n}$:

$$V_{m+1,n} = V_{m+1,1} - \int_0^{\eta_n} \left(2\xi \frac{\partial F}{\partial \xi} + F\right)_{m+1} d\eta \tag{59}$$

where $V_{m+1,1}$ represents the boundary condition at the wall V_w. (See eq. (35).) The integral appearing in equation (59) is numerically integrated across the η-strip to obtain the $N - 1$ values of V. In the present program the trapezoidal rule of integration is used.

Initial profiles.- Initial profiles for starting the finite-difference scheme are required at two x-stations since three-point differences are utilized. The initial profiles at the stagnation point or line for blunt bodies, or near $x = 0$ for sharp-tipped bodies, are obtained by numerically solving the similar boundary-layer equations. (See eqs. (B47) to (B49).) The equations are solved by a fourth-order Runge-Kutta scheme with a Newton iteration method to modify the initial estimates of the gradients of F and Θ evaluated at the wall boundary. The $N - 1$ values of F, Θ, and V obtained at the equally spaced $N - 1$ grid points are numerically redistributed to $N - 1$ grid points whose spacing is determined from equations (41) and (42) if a variable spacing is required. (As noted previously, variable spacing is required if transitional or turbulent flow occurs.) The second initial profile located at station m is assumed identical to the one located at station $m-1$. Any errors that might be incurred because of this assumption are minimized by using an extremely small value of $\Delta\xi$; that is, an initial step size in the physical plane on the order of $\Delta x = 1 \times 10^{-5}$ is used. The solution at the unknown station $m+1$ is then obtained by the finite-difference method. Extremely small, equally spaced $\Delta\xi$-steps are used in the region of the initial profile. The step size is increased after errors due to the starting procedure have approached zero, that is, after 10 to 15 steps in $\Delta\xi$.

Evaluation of wall derivatives.- The shear stress and heat transfer at the wall are directly proportional to the gradient of F and Θ evaluated at the wall, respectively. By using G to represent a general quantity, where $G_{m+1,1}$ is the value of G evaluated at the wall, the four-point difference scheme used to evaluate derivatives at the wall is given as

$$\left(\frac{\partial G}{\partial \eta}\right)_{m+1,1} = Y_7 G_{m+1,1} + Y_8 G_{m+1,2} + Y_9 G_{m+1,3} + Y_{10} G_{m+1,4} \tag{60}$$

where the coefficients $Y_7, \ldots, Y_{10}$ are defined by the following relations:

$$Y_7 = -\frac{\left(1 + k + k^2\right)^2 \left[k(1 + k) - 1\right] + (1 + k)}{(1 + k)\left(1 + k + k^2\right)k^3 \Delta\eta_1} \tag{61a}$$

$$Y_8 = \frac{\left(1 + k + k^2\right)}{k^2 \Delta\eta_1} \tag{61b}$$

$$Y_9 = -\frac{\left(1 + k + k^2\right)}{(1 + k)k^3 \Delta\eta_1} \tag{61c}$$

and

$$Y_{10} = \frac{1}{\left(1 + k + k^2\right)k^3 \Delta\eta_1} \tag{61d}$$

For the case of equally spaced grid points in the η-direction (k = 1), equations (61) become

$$Y_7 = -\frac{11}{6 \Delta\eta} \tag{62a}$$

$$Y_8 = \frac{18}{6 \Delta\eta} \tag{62b}$$

$$Y_9 = -\frac{9}{6 \Delta\eta} \tag{62c}$$

$$Y_{10} = \frac{2}{6 \Delta\eta} \tag{62d}$$

and equation (60) reduces to the familiar four-point relation; that is,

$$\left(\frac{\partial G}{\partial \eta}\right)_{m+1,1} = -\frac{1}{6 \Delta\eta}\left(11G_{m+1,1} - 18G_{m+1,2} + 9G_{m+1,3} - 2G_{m+1,4}\right) \tag{63}$$

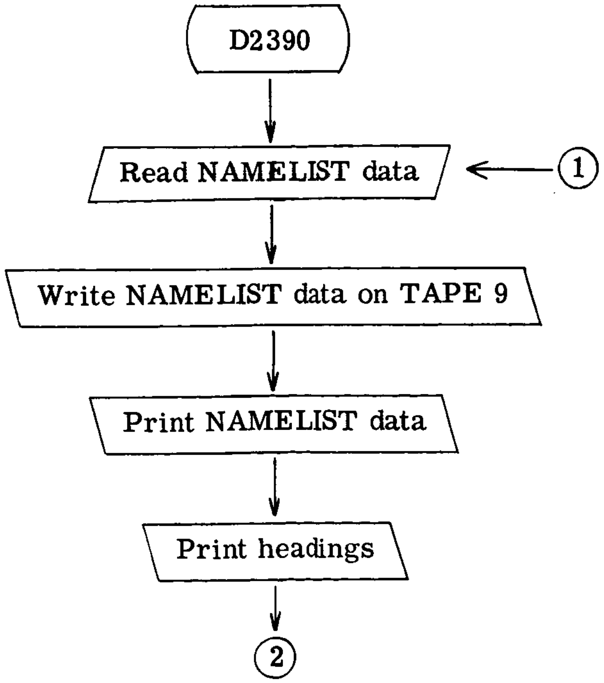
PROGRAM DESCRIPTION

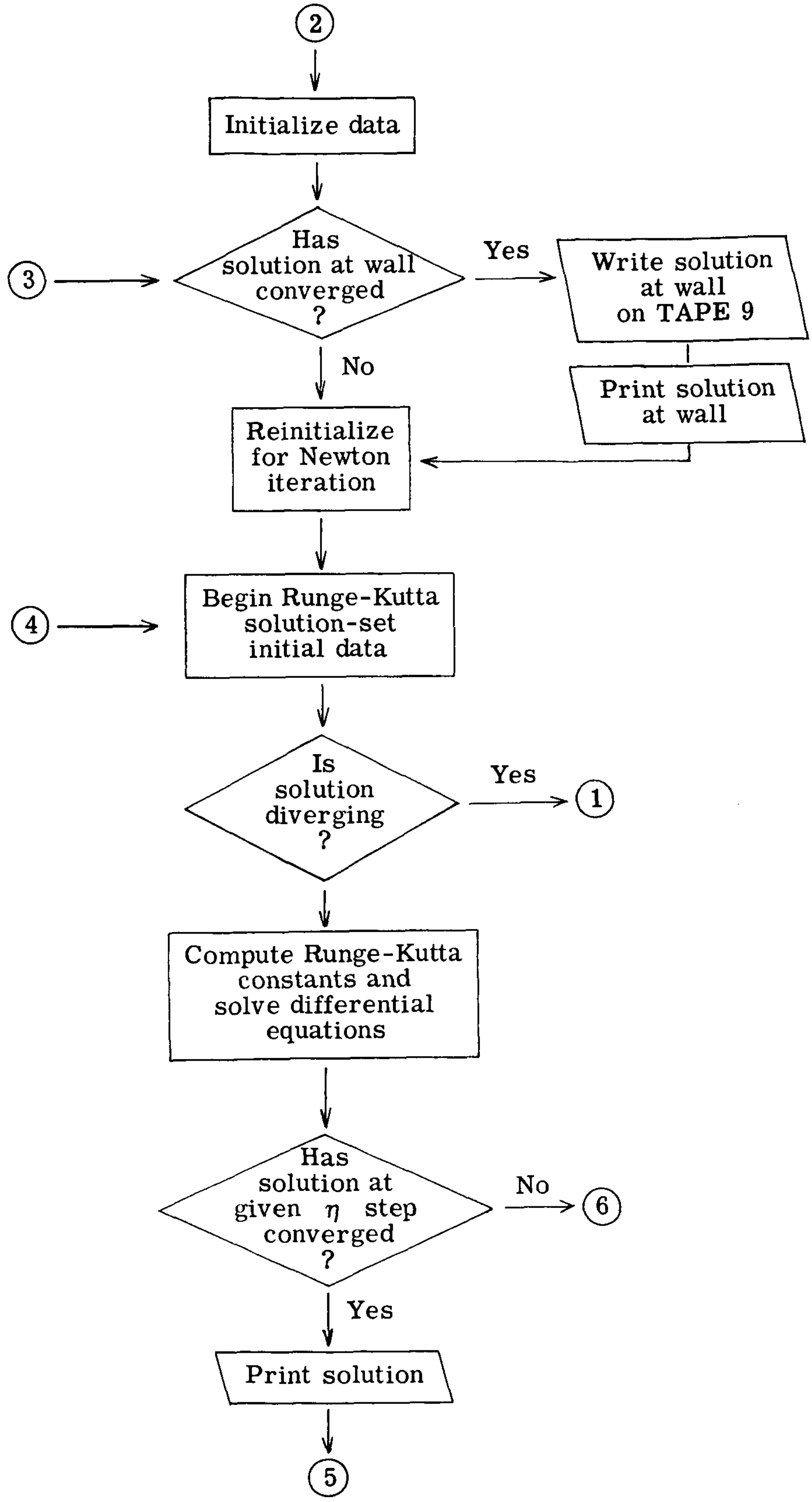
General Discussion

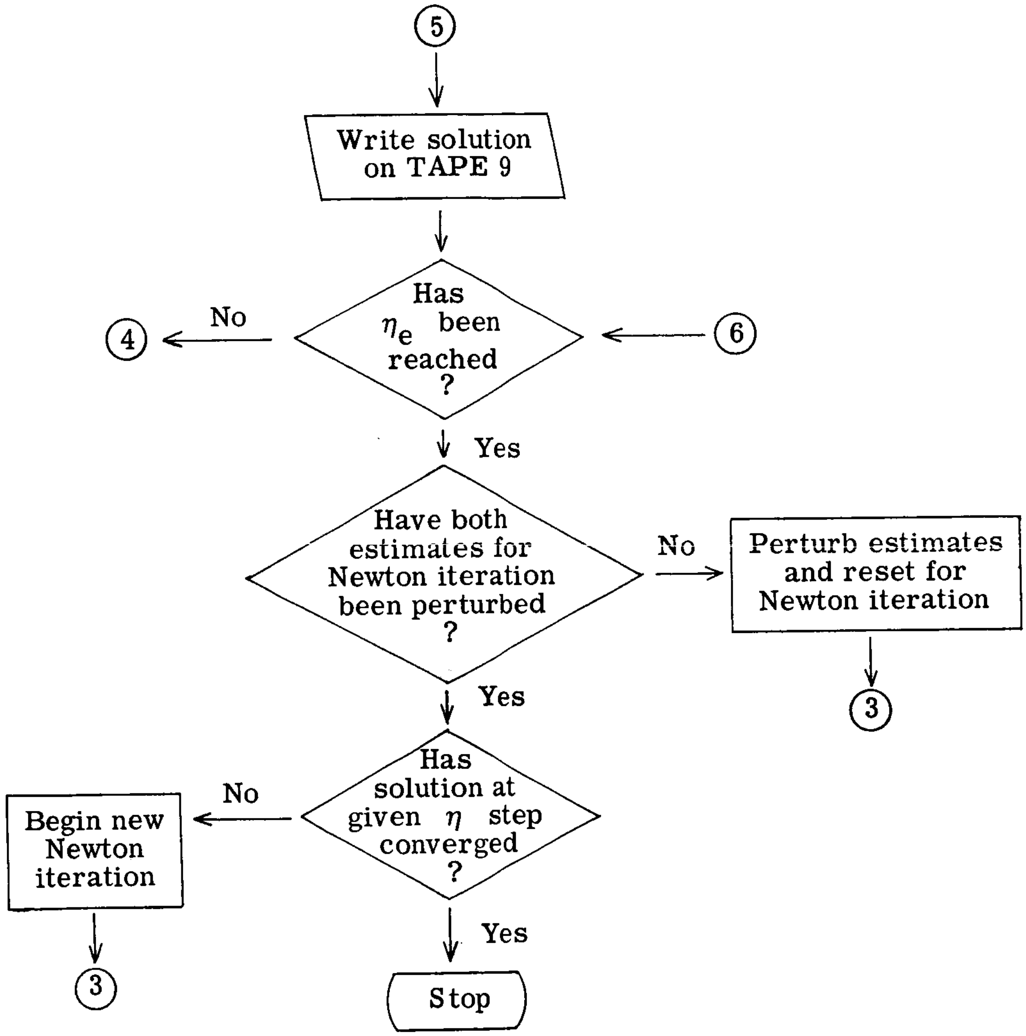
The program, written in FORTRAN IV for the CDC 6000 series computers, consists of three main programs, D2390, D23901, and D2401. Program D2390 computes the initial similarity solution to equations (B47) to (B49) with the points equally spaced in the η-direction. Program D23901 takes this solution and redistributes the values of η geometrically in subroutine GEOM, then interpolates to provide the solution over the geometrically spaced points. Program D2401 takes these data as a starting profile, reads other input, and computes initial conditions. Steps are then taken down the body to solve the momentum and energy equations in finite-difference form, and the continuity equation is numerically integrated. Various boundary-layer parameters such as boundary-layer thickness, displacement thickness, momentum thickness, and skin-friction and heat-transfer coefficients are then calculated and the output is printed. Program D2401 uses the following subroutines: TURBLNT calculates the eddy viscosity, its derivatives, and the intermittency distributions required for the solution of transitional and turbulent flows; VARENT reads the variable-entropy input in tabular form, computes dr_S/dz, and prints the input and the derivatives; TABLE reads the body-geometry input, nondimensionalizes it, and, if necessary, distributes the values according to specified steps and computes derivatives; SETUP determines from input where profiles and wall values are to be printed; function INTEGT integrates by using the trapezoidal rule; INUNIT converts data, if necessary, to the U.S. Customary System of Units for computation and back to the International System for printout; FTLUP performs a second-order interpolation to find intermediate values from a tabular array (see appendix C).

Descriptions, Flow Charts, and Listings of the Main Programs and Subprograms

Main program D2390.- The main program D2390 performs the locally similar solution to the continuity, momentum, and energy equations ((B47), (B48), and (B49), respectively), by using a fourth-order Runge-Kutta technique with Newton's iteration method. The flow diagram of main program D2390 is as follows:




2
Initialize data
3
Has solution at wall converged ?
Yes
Write solution at wall on TAPE 9
Print solution at wall
No
Reinitialize for Newton iteration
4
Begin Runge-Kutta solution-set initial data
Is solution diverging ?
Yes
1
Compute Runge-Kutta constants and solve differential equations
Has solution at given η step converged ?
No
6
Yes
Print solution
5


5
Write solution on TAPE 9
Has η_e been reached ?
No
4
6
Yes
Have both estimates for Newton iteration been perturbed ?
No
Perturb estimates and reset for Newton iteration
3
Yes
Has solution at given η step converged ?
No
Begin new Newton iteration
3
Yes
Stop

The program listing for the main program D2390 is as follows:

```
      PROGRAM D2390(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE9)            A   1     100000
                                                                                      200000
*        RUNGE-KUTTA METHOD FOR INITIAL VALUE PROBLEM                       A   2     300000
*        INVOLVING FIRST ORDER DIFFERENTIAL EQUATIONS                       A   3     400000
                                                                                      500000
      DIMENSION YO(6), YN(6), Q(6), V(6), C(6), F(6), D(6), W(6)            A   4     600000
      NAMELIST /NAM1/ HPR,XEND,H,PR,XKK,BETA,ALPHA,XO,YO,IEDGE,XK,IGAS,V   A   5     700000
     1ISCON,VISPOW,KODUNIT                                                  A   6     800000
                                                                                      900000
*        INITIALIZE DATA                                                    A   7    1000000
                                                                                     1100000
      DATA HPR,XEND,H,PR,XKK,BETA,ALPHA,XO,N/.1,10.,.01,.7,4*0.,6/          A   8    1200000
      DATA XK,IGAS/1.,1/,KODUNIT/0/,VISCON/0.0/,VISPOW/0.0/                 A  13    1300000
      IGAS=1                                                                A  10    1400000
    1 CONTINUE                                                              A  11    1500000
      DO 2 I=1,6                                                            A  12    1600000
    2 YO(I)=0.                                                              A  13    1700000
                                                                                     1800000
*        INPUT DATA                                                         A  14    1900000
                                                                                     2000000
      READ (5,NAM1)                                                         A  15    2100000
      IF (ENDFILE 5) 3,4                                                    A  16    2200000
    3 STOP 6                                                                A  17    2300000
    4 CONTINUE                                                              A  18    2400000
      IEDGE=XEND/HPR+1.5                                                    A  19    2500000
      WRITE (6,NAM1)                                                        A  20    2600000
      WRITE (9,NAM1)                                                        A  21    2700000
      WRITE (6,31)                                                          A  22    2800000
      WRITE (6,32)                                                          A  23    2900000
      YW31=0.0                                                              A  24    3000000
      YW33=100.0                                                            A  25    3100000
      TFACT=100.0                                                           A  26    3200000
      NDELT=-1                                                              A  27    3300000
    5 IF (ABS(TFACT)-.000001) 6,7,7                                         A  28    3400000
                                                                                     3500000
*        WRITE SOLUTION AT WALL                                             A  29    3600000
                                                                                     3700000
    6 WRITE (6,34) XO,YO(1),YO(2),YO(3),YO(4),YO(5),YO(6)                   A  30    3800000
      WRITE (9,34) XO,YO(1),YO(2),YO(3),YO(4),YO(5),YO(6)                   A  31    3900000
    7 YW1=YO(1)                                                             A  32    4000000
      YW2=YO(2)                                                             A  33    4100000
      YW3=YO(3)                                                             A  34    4200000
      YW4=YO(4)                                                             A  35    4300000
      YW5=YO(5)                                                             A  36    4400000
      YW6=YO(6)                                                             A  37    4500000
      XO1=XO                                                                A  38    4600000
      HO1=H                                                                 A  39    4700000
      XPR=HPR                                                               A  40    4800000
                                                                                     4900000
```

```
*          BEGIN RUNGE-KUTTA                                           A  41    5000000
                                                                                5100000
      X=X0                                                             A  42    5200000
      DO 8 I=1,N                                                       A  43    5300000
      YN(I)=Y0(I)                                                      A  44    5400000
    8 Q(I)=0.0                                                         A  45    5500000
    9 U=X                                                              A  46    5600000
      DO 10 I=1,N                                                      A  47    5700000
   10 V(I)=YN(I)                                                       A  48    5800000
      I1=1                                                             A  49    5900000
      GO TO 29                                                         A  50    6000000
                                                                                6100000
*          CCMPUTE RUNGE-KUTTA CONSTANTS                               A  51    6200000
*             COMPUTE K1                                               A  52    6300000
                                                                                6400000
   11 DO 12 I=1,N                                                      A  53    6500000
      C(I)=H*F(I)                                                      A  54    6600000
      D(I)=.5*(C(I)-2.0*Q(I))                                          A  55    6700000
      W(I)=YN(I)+D(I)                                                  A  56    6800000
      Q(I)=Q(I)+3.0*D(I)-.5*C(I)                                       A  57    6900000
   12 V(I)=W(I)                                                        A  58    7000000
      U=X+.5*H                                                         A  59    7100000
      I1=2                                                             A  60    7200000
      GO TO 29                                                         A  61    7300000
                                                                                7400000
*             CCMPUTE K2                                               A  62    7500000
                                                                                7600000
   13 DO 14 I=1,N                                                      A  63    7700000
      C(I)=H*F(I)                                                      A  64    7800000
      D(I)=.29289325*(C(I)-Q(I))                                       A  65    7900000
      W(I)=W(I)+D(I)                                                   A  66    8000000
      Q(I)=Q(I)+3.0*D(I)-.29289325*C(I)                                A  67    8100000
   14 V(I)=W(I)                                                        A  68    8200000
      I1=3                                                             A  69    8300000
      GO TO 29                                                         A  70    8400000
                                                                                8500000
*             COMPUTE K3                                               A  71    8600000
                                                                                8700000
   15 DO 16 I=1,N                                                      A  72    8800000
      C(I)=H*F(I)                                                      A  73    8900000
      D(I)=1.7071067*(C(I)-Q(I))                                       A  74    9000000
      W(I)=W(I)+D(I)                                                   A  75    9100000
      Q(I)=Q(I)+3.0*D(I)-1.7071067*C(I)                                A  76    9200000
   16 V(I)=W(I)                                                        A  77    9300000
      U=X+H                                                            A  78    9400000
      I1=4                                                             A  79    9500000
      GO TO 29                                                         A  80    9600000
                                                                                9700000
```

```
*         COMPUTE K4                                                      A  81    9800000
                                                                                   9900000
   17  DO 18 I=1,N                                                        A  82   10000000
       C(I)=H*F(I)                                                        A  83   10100000
       D(I)=.16666667*(C(I)-2.0*Q(I))                                     A  84   10200000
       YN(I)=W(I)+D(I)                                                    A  85   10300000
   18  Q(I)=Q(I)+3.0*D(I)-.5*C(I)                                         A  86   10400000
       X=X+H                                                              A  87   10500000
       IF (X-XPR+.000001) 9,19,19                                         A  88   10600000
   19  CONTINUE                                                           A  89   10700000
       IF (ABS(TFACT)-.000001) 20,21,21                                   A  90   10800000
                                                                                  10900000
*        OUTPUT ONTO TAPE AND PRINT                                       A  91   11000000
                                                                                  11100000
   20  WRITE (6,34) X,YN(1),YN(2),YN(3),YN(4),YN(5),YN(6)                 A  92   11200000
       WRITE (9,34) X,YN(1),YN(2),YN(3),YN(4),YN(5),YN(6)                 A  93   11300000
                                                                                  11400000
*        RESET VALUES FOR NEXT NEWTONS ITERATION                          A  94   11500000
                                                                                  11600000
   21  IF (X-XEND) 28,22,22                                               A  95   11700000
   22  CONTINUE                                                           A  96   11800000
       IF (NDELT) 23,26,27                                                A  97   11900000
   23  IF (ABS(TFACT)-0.000001) 24,25,25                                  A  98   12000000
   24  CONTINUE                                                           A  99   12100000
       STOP                                                               A 100   12200000
   25  YW31=YW3                                                           A 101   12300000
       YW51=YW5                                                           A 102   12400000
       YI21=YN(2)                                                         A 103   12500000
       YI41=YN(4)                                                         A 104   12600000
       YO(3)=YW31+0.0001                                                  A 105   12700000
       YO(1)=YW1                                                          A 106   12800000
       YO(2)=YW2                                                          A 107   12900000
       YO(4)=YW4                                                          A 108   13000000
       YO(5)=YW5                                                          A 109   13100000
       YO(6)=YW6                                                          A 110   13200000
       XO=XO1                                                             A 111   13300000
       H=HO1                                                              A 112   13400000
       NDELT=0                                                            A 113   13500000
       GO TO 5                                                            A 114   13600000
   26  YI22=YN(2)                                                         A 115   13700000
       YI42=YN(4)                                                         A 116   13800000
       YO(5)=YW51+0.0001                                                  A 117   13900000
       YO(1)=YW1                                                          A 118   14000000
       YO(2)=YW2                                                          A 119   14100000
       YO(3)=YW31                                                         A 120   14200000
       YO(4)=YW4                                                          A 121   14300000
       YO(6)=YW6                                                          A 122   14400000
       XO=XO1                                                             A 123   14500000
       H=HO1                                                              A 124   14600000
       NDELT=+1                                                           A 125   14700000
       GO TO 5                                                            A 126   14800000
                                                                                  14900000
```

```
*        COMPUTE DETERMINANT

  27  DENOM=((YI22-YI21)*(YN(4)-YI41))/(0.0001**2.0)-((YN(2)-YI21)*(YI42
     1-YI41))/(0.0001**2.0)
      YO(3)=YW31+((YN(4)-YI41)*(1.0-YI21)-(YN(2)-YI21)*(1.0-YI41))/(DENO
     1M*0.0001)
      YO(5)=YW51+((YI22-YI21)*(1.0-YI41)-(YI42-YI41)*(1.0-YI21))/(DENOM*
     10.0001)
      YO(1)=YW1
      YO(2)=YW2
      YO(4)=YW4
      YO(6)=YW6
      YW33=YO(3)
      TFACT=YW33-YW31
      XO=XO1
      H=HO1
      NDELT=-1
      GO TO 5
  28  XPR=XPR+HPR
      GO TO 9
  29  CONTINUE
      IF (V(4).GE.0.) GO TO 30
      WRITE (6,33)
      GO TO 1
  30  CONTINUE
      IF (IGAS.EQ.1) VISC=SQRT(V(4))*(1.0+XKK)/(V(4)+XKK)
      IF (IGAS.EQ.2) VISC=V(4)**(VISPOW-1.)

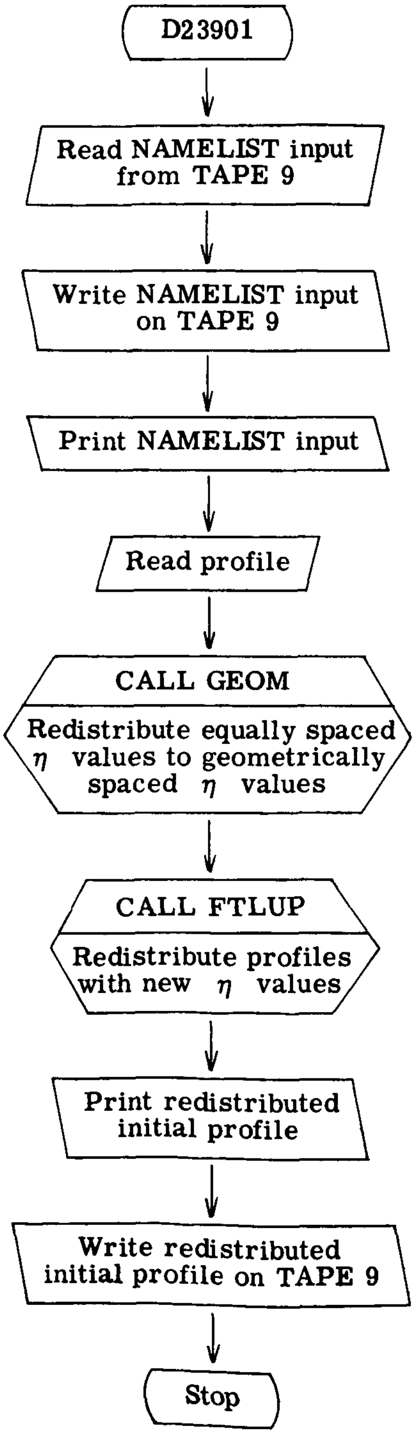
*        SOLVE DIFFERENTIAL EQUATIONS

      F(1)=V(2)
      F(2)=V(3)/VISC
      F(3)=-V(1)*V(3)/VISC-(V(4)-V(2)*V(2))*BETA
      F(4)=V(5)/VISC
      F(5)=-PR*V(1)*V(5)/VISC-PR*ALPHA*V(3)*V(3)/VISC
      F(6)=V(4)-V(2)
      GO TO (11,13,15,17), I1

*

  31  FORMAT (1H1)
  32  FORMAT (6X,3HETA9X,5HYO(1)8X,5HYO(2)8X,5HYO(3)8X,5HYO(4)8X,5HYO(5)
     18X,5HYO(6))
  33  FORMAT (//95H SOLUTION DID NOT CONVERGE.  AN ATTEMPT WILL BE MADE
     1TO READ NEW ESTIMATES FOR YO(3) AND YO(5).)
  34  FORMAT (7E13.6)
      END
```

Main program D23901.- Main program D23901 takes the initial profile data from the program D2390 solution and redistributes the equally spaced η values geometrically according to input-distribution constant XK and then interpolates to redistribute the profile to correspond with the new η values. The flow diagram of the main program D23901 is as follows:



The program listing for the main program D23901 is as follows:

```
      PROGRAM D23901(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE9)

*        THIS PROGRAM READS THE ARRAYS WRITTEN BY D2390 AND CHANGES
*        THE DATA FROM EQUALLY SPACED POINTS TO A GEOMETRIC PROGRESSION

      DIMENSION A(301), AA(301), B(301), BB(301), C(301), CC(301), D(301
     1), DD(301), E(301), EE(301), F(301), FF(301), G(301), GG(301)
      DIMENSION YO(6)
      NAMELIST /NAM1/ HPR,XEND,H,PR,XKK,BETA,ALPHA,XO,YO,IEDGE,XK,IGAS,V
     1ISCON,VISPOW,KODUNIT

*       READ INITIAL VALUE INPUT FROM D2390

      REWIND 9
      READ (9,NAM1)
      IF (ENDFILE 9) 1,2
    1 STOP 7
    2 CONTINUE
      READ (9,6) (A(N),B(N),C(N),D(N),E(N),F(N),G(N),N=1,IEDGE)
      IF (ENDFILE 9) 3,4
    3 STOP 10
    4 CONTINUE

*       REDISTRIBUTE ETAS FROM D2390

      CALL GEOM (XK,XEND,IEDGE,AA)

*       REDISTRIBUTE INITIAL PROFILES WITH NEW ETAS

      DO 5 I=1,IEDGE
      CALL FTLUP (AA(I),BB(I),2,IEDGE,A,B)
      CALL FTLUP (AA(I),CC(I),2,IEDGE,A,C)
      CALL FTLUP (AA(I),DD(I),2,IEDGE,A,D)
      CALL FTLUP (AA(I),EE(I),2,IEDGE,A,E)
      CALL FTLUP (AA(I),FF(I),2,IEDGE,A,F)
    5 CALL FTLUP (AA(I),GG(I),2,IEDGE,A,G)
      REWIND 9
      WRITE (9,NAM1)

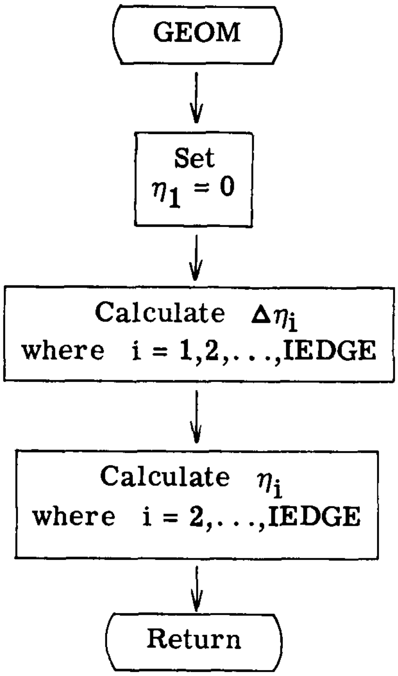
*       WRITE NEW INITIAL PROFILE

      WRITE (9,6) (AA(N),BB(N),CC(N),DD(N),EE(N),FF(N),GG(N),N=1,IEDGE)
      WRITE (6,NAM1)
      WRITE (6,7)
      WRITE (6,8)
      WRITE (6,6) (AA(N),BB(N),CC(N),DD(N),EE(N),FF(N),GG(N),N=1,IEDGE)
      STOP

*

    6 FORMAT (7E13.6)
    7 FORMAT (1H1)
    8 FORMAT (6X,3HETA9X,5HYO(1)8X,5HYO(2)8X,5HYO(3)8X,5HYO(4)8X,5HYO(5)
     18X,5HYO(6))
      END
```

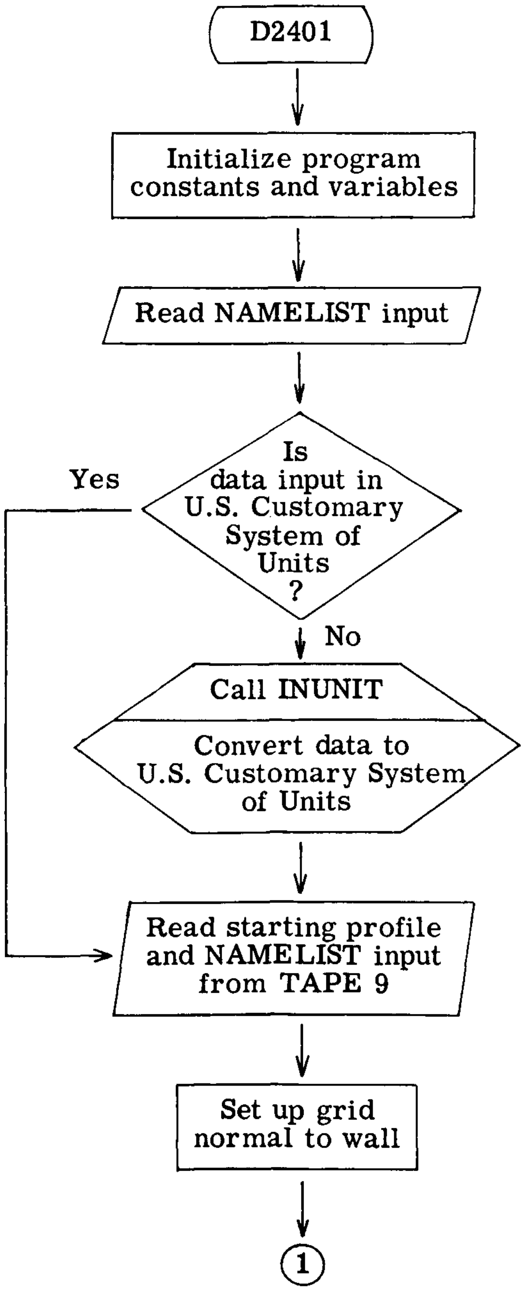
Subroutine GEOM.- Subroutine GEOM redistributes the equally spaced η values to geometrically spaced values according to input-distribution constant XK. The flow diagram for subroutine GEOM is as follows:

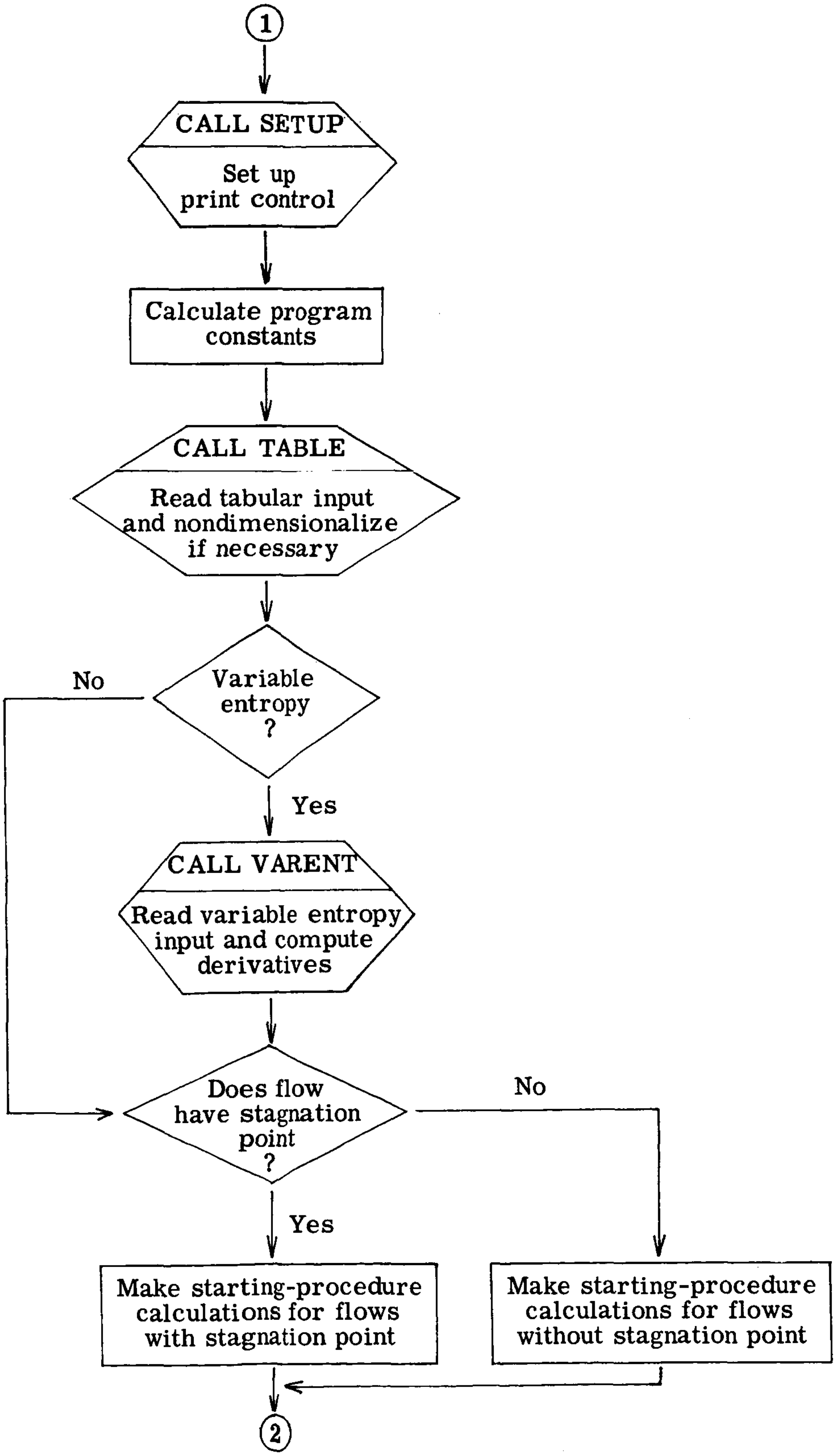


The program listing for subroutine GEOM is as follows:

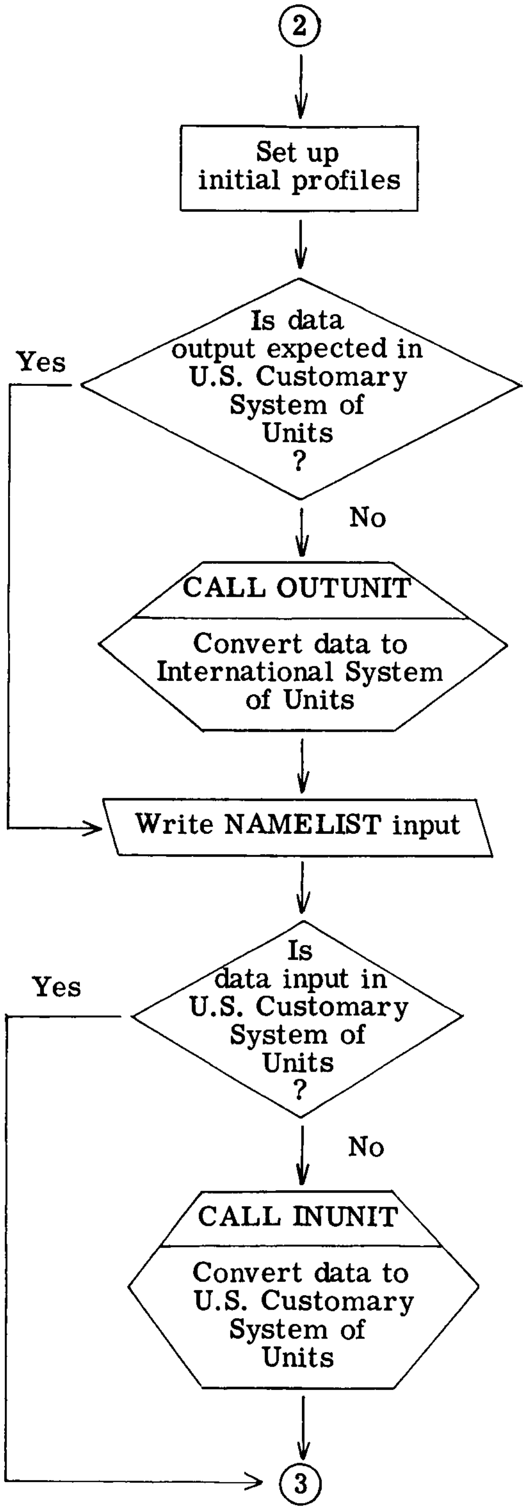
```
      SUBROUTINE GEOM (K,ETAEDGE,IEDGE,AA)                       B   1    5600000
      REAL K                                                     B   2    5700000
      DIMENSION DETA(301), AA(301)                               B   3    5800000
      AA(1)=0.                                                   B   4    5900000
      IF (K.EQ.1.) GO TO 1                                       B   5    6000000
      DETA(1)=((1-K)/(1-K**(IEDGE-1)))*ETAEDGE                   B   6    6100000
      GO TO 2                                                    B   7    6200000
1     DETA(1)=ETAEDGE/(IEDGE-1)                                  B   8    6300000
2     AA(2)=DETA(1)                                              B   9    6400000
      DETA(2)=K*DETA(1)                                          B  10    6500000
      DO 3 N=3,IEDGE                                             B  11    6600000
      DETA(N)=(K**(N-1))*DETA(1)                                 B  12    6700000
3     AA(N)=AA(N-1)+DETA(N-1)                                    B  13    6800000
      RETURN                                                     B  14    6900000
      END                                                        B  15-   7000000
```

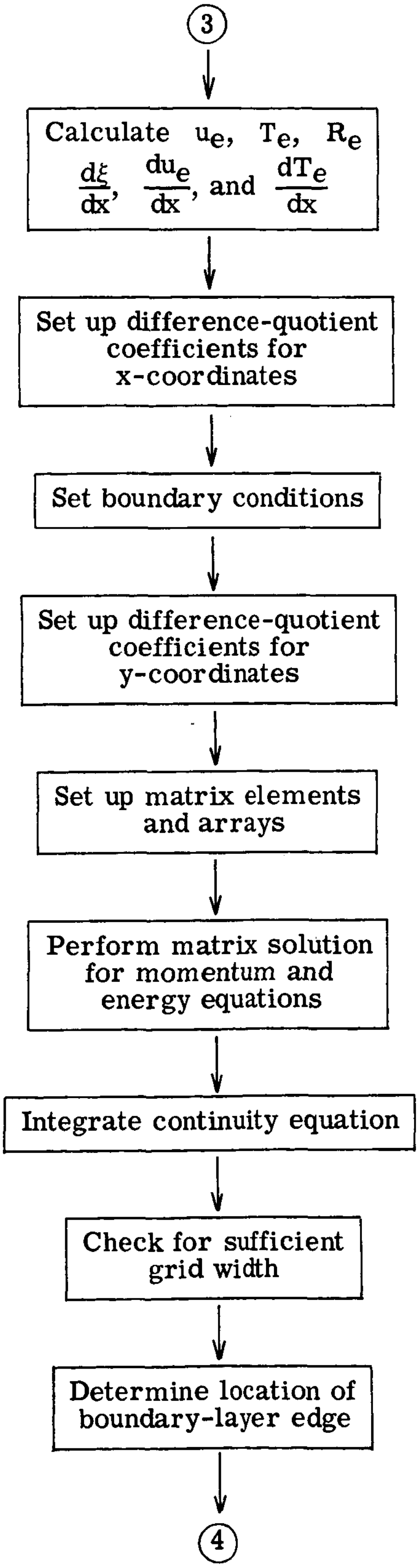
Main program D2401.- The main program D2401 controls the finite-difference solution of the boundary-layer equations. It reads the initial profile data (which may come from D2390 or D23901) and other input, computes initial conditions, solves the momentum and energy equations in finite-difference form, numerically integrates the continuity equation, calculates the boundary-layer thickness, displacement thickness, momentum thickness, and skin-friction and heat-transfer coefficients, and prints the output. The flow diagram of the main program D2401 is as follows:

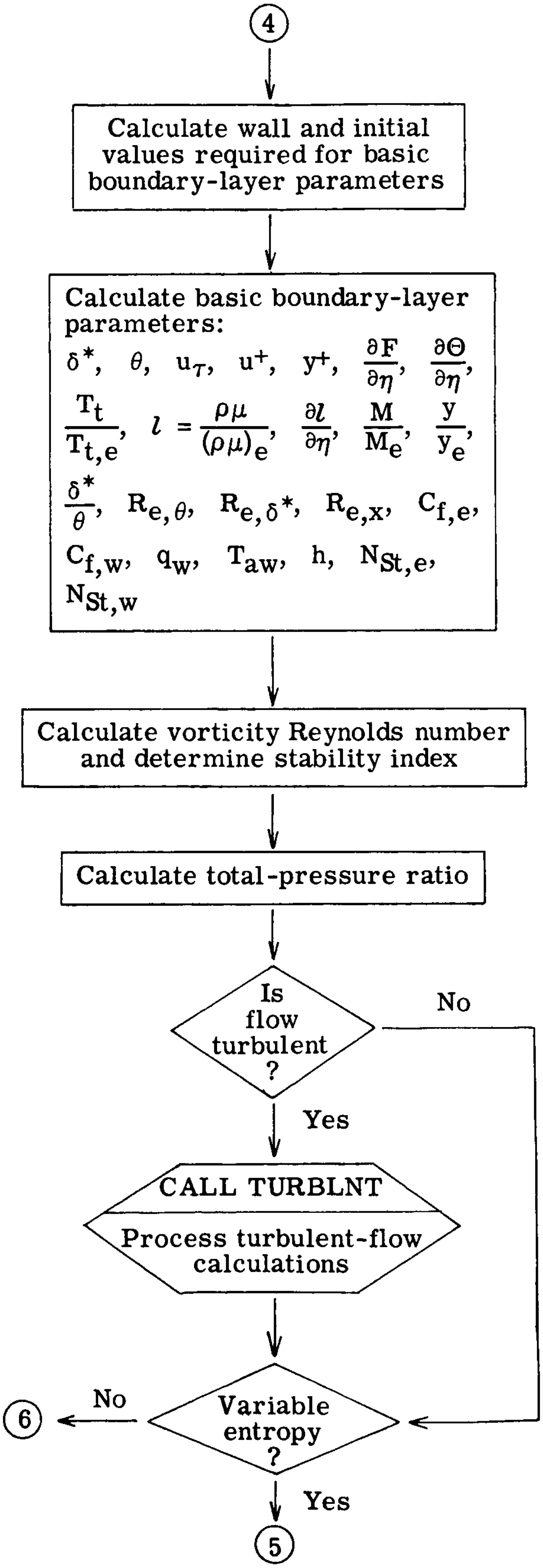


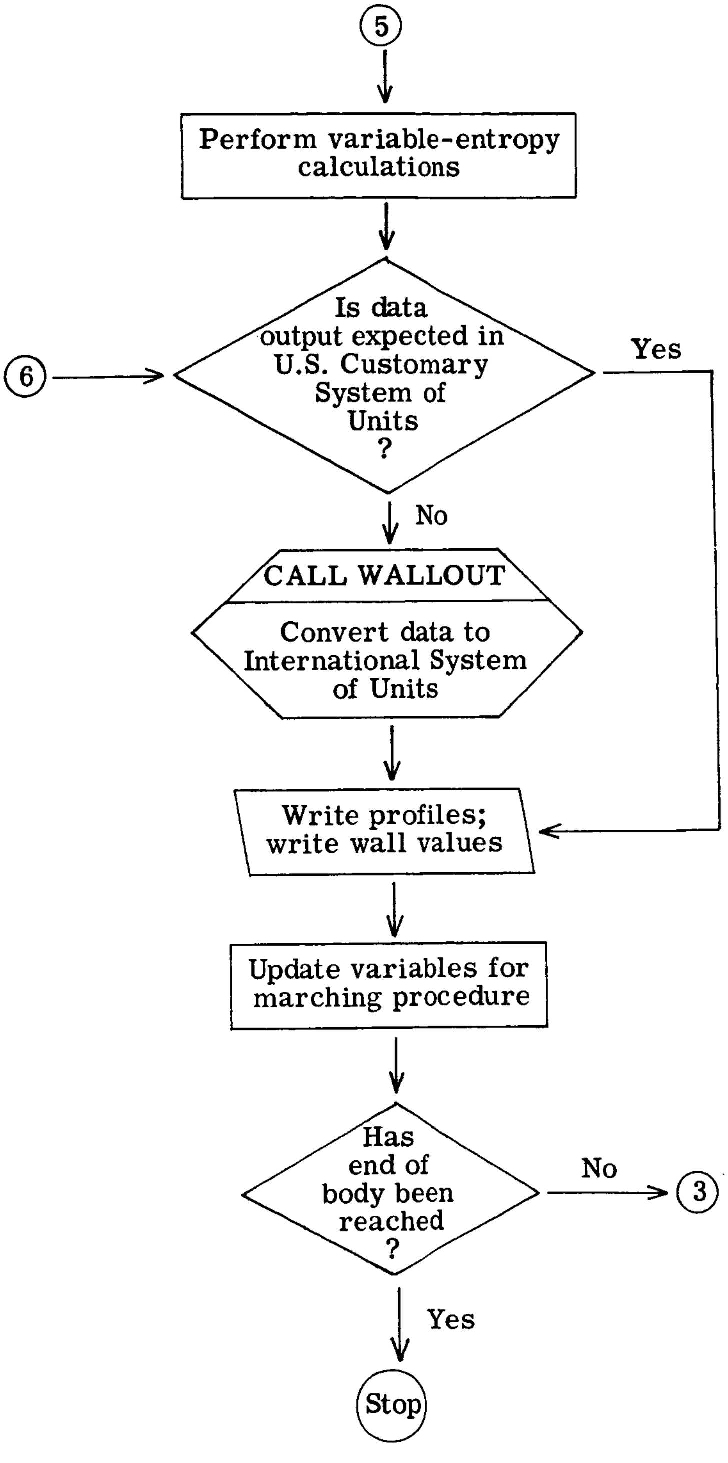


1
CALL SETUP
Set up print control
Calculate program constants
CALL TABLE
Read tabular input and nondimensionalize if necessary
Variable entropy ?
No
Yes
CALL VARENT
Read variable entropy input and compute derivatives
Does flow have stagnation point ?
No
Yes
Make starting-procedure calculations for flows with stagnation point
Make starting-procedure calculations for flows without stagnation point
2


2
Set up initial profiles
Is data output expected in U.S. Customary System of Units ?
Yes
No
CALL OUTUNIT
Convert data to International System of Units
Write NAMELIST input
Is data input in U.S. Customary System of Units ?
Yes
No
CALL INUNIT
Convert data to U.S. Customary System of Units
3

③
Calculate u_e, T_e, R_e
$\frac{d\xi}{dx}$, $\frac{du_e}{dx}$, and $\frac{dT_e}{dx}$
Set up difference-quotient coefficients for x-coordinates
Set boundary conditions
Set up difference-quotient coefficients for y-coordinates
Set up matrix elements and arrays
Perform matrix solution for momentum and energy equations
Integrate continuity equation
Check for sufficient grid width
Determine location of boundary-layer edge
④


④
Calculate wall and initial values required for basic boundary-layer parameters
Calculate basic boundary-layer parameters:
δ^*, θ, u_τ, u^+, y^+, $\frac{\partial F}{\partial \eta}$, $\frac{\partial \Theta}{\partial \eta}$,
$\frac{T_t}{T_{t,e}}$, $l = \frac{\rho\mu}{(\rho\mu)_e}$, $\frac{\partial l}{\partial \eta}$, $\frac{M}{M_e}$, $\frac{y}{y_e}$,
$\frac{\delta^*}{\theta}$, $R_{e,\theta}$, R_{e,δ^*}, $R_{e,x}$, $C_{f,e}$,
$C_{f,w}$, q_w, T_{aw}, h, $N_{St,e}$,
$N_{St,w}$
Calculate vorticity Reynolds number and determine stability index
Calculate total-pressure ratio
Is flow turbulent ?
No
Yes
CALL TURBLNT
Process turbulent-flow calculations
Variable entropy ?
No
⑥
Yes
⑤


5
Perform variable-entropy calculations
6
Is data output expected in U.S. Customary System of Units ?
Yes
No
CALL WALLOUT
Convert data to International System of Units
Write profiles; write wall values
Update variables for marching procedure
Has end of body been reached ?
No
3
Yes
Stop

The program listing for the main program D2401 is as follows:

```
      PROGRAM D2401(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE21,TAPE9,T
     1APE4)
      DIMENSION FN(JK), FO(JK), FP(JK), TN(JK), TO(JK), TP(JK), VN(JK),
     1VO(JK), VP(JK), EP(JK), EPP(JK), FZ(JK), TZ(JK), PTOPT(JK), STAB2(
     2JK), MOME(JK), XN(JK), Y(JK), DY(JK), XK1(JK), XK2(JK), XK3(JK), X
     3L1(JK), XL2(JK), XL3(JK), XLM11(JK), XLPM11(JK), RATON(JK), RATOO(
     4JK), RATOP(JK), TTOTT(JK), CROCCO(JK), UOUPL(JK), TCORD(JK), UDEF(
     5JK), NONDEL(JK), EETM(JK), EEPM(JK), UFE(JL), PROVAL(JN), PRNTVAL(
     6JM)
      DIMENSION VARA(JK), VARB(JK), VARC(JK), VARD(JK), VARE(JK)
      DIMENSION YO(6)
      DIMENSION RRS(100), ZZS(100), DRSDZS(100)
      DIMENSION PRTAR(100), GLAR(100)
      COMMON /TRBULNT/ S,KSTR,TLNGTH,CORP,TRFACT,TBAR,XLBAR,DISINC,XT1,X
     1T2,XT3,XT4,XT5,RE,UE,XNUE,J,RMI,EPS,JPOINT,IEDGE,WW1,WW2,WW3,WW4,W
     2W5,N,ETAEDG,KODVIS,A,XBE,X,PR,CONSTNT,KODPRT,PRT,PRTAR,GLAR,NUMB1
      COMMON /UNIT/ VISCON,PT1,TT1,WAVE,R,SU,CONE,DS,SST,RT1,P1,T1,R1,U1
     1,AA1,TREF,VISREF,PESTAR,TESTAR,RESTAR,UESTAR,MUESTAR,YESTAR,THETA,
     2TAUD,QSD,HD,UPLUS,DISP,PE,Z,TW,QW,RVWALD,PROINC,PRNTINC
      INTEGER ETAEDG,W
      REAL MOME,NUE,NUW,KWD,KED,INTGRL,INTEGT,INTEGL,NONDEL,MUESTAR
      EXTERNAL INTEGT
      NAMELIST /NAM1/ HPR,XEND,H,PR,XKK,BETA,ALPHA,XO,YO,IEDGE,XK,IGAS,V
     1ISCON,VISPOW,KODUNIT
      NAMELIST /NAM2/ XMA,PT1,TT1,WAVE,XY1,XY2,XY3,G,R,SU,PR,PRT,IBODY,J
     1,W,FT,KODE,KODWAL,IENTRO,CONE,IEND1,A,DS,DY,KODVIS,SST,SMXTR,TLNGT
     2H,CORP,CONSTNT,XT1,XT2,XT3,XT4,XT5,PROINC,PRNTINC,IPRO,PROVAL,IPRN
     3T,PRNTVAL,NAUXPRO,BLNGTH,NPUTYPE,KODPRT,NUMB1,PRTAR,GLAR,KTCOD
      NAMELIST /NAM/ P10,T10,G,REY,RT1,P1,T1,R1,U1,AA1,TREF,VISREF,R10
      NAMELIST /DBG/ S,Y1,Y2,Y3,Y4,Y5,Y6,Z1,Z2,Z3,Z4,Z5,X,OY,OZ,ITRO,XK1
     1,XK2,XK3,XL1,XL2,XL3,FN,FO,FP,TN,TO,TP,VN,VO,VP,EP,EPP,XLM11,XLPM1
     21,RATOP

*        INITIALIZE DATA TO STANDARD INPUT

      DATA WAVE/90./,G/1.4/,R/1716./,SU/198.6/,PR/.72/,PRT/.9/,W/0/,FT/1
     1.0/,KODE/0/,KODWAL/1/,IENTRO/1/,A/1./,DS/.01/,KODVIS/1/,SST/1.E8/,
     2SMXTR/1.E8/,TLNGTH/2./,CORP/.412/,CONSTNT/0./,XT1/.4/,XT2/26./,XT3
     3/.0168/,XT4/.78/,XT5/0./,PROINC/1./,PRNTINC/.1/,NAUXPRO/0/,BLNGTH/
     40./,NPUTYPE/1/,IPRO/0/,IPRNT/0/
      DATA THATS/0./,RS/0./,P20/0./,H1,H2,H3,H4,H5,H6,H7,H8,H9,H10,H11/1
     11*0./,NSUBLY/3/,XPX/.5/,SIGN/1./,SMXN/0./,SMXO/0./,SMXP/0./,ST2MAX
     2/1000001/,TRFACT/0./,KSTR/0/,ITMAX/3/,NOIT/0/,DPEDS/0./,Z/0./,KODP
     3RT/1/,KTCOD/2/,KTCD/0/,CONE/0.0/
      REWIND 9
      DO 1 I=1,JN
 1    PROVAL(I)=0.
```

```
      DO 2 I=1,JM
    2 PRNTVAL(I)=0.
      DO 3 I=1,JL
    3 UFE(I)=0.
      DO 4 I=1,JK
      EPPM(I)=STAB2(I)=Y(I)=XN(I)=VN(I)=VO(I)=VP(I)=EPP(I)=XK1(I)=XK2(I)
     1=XK3(I)=XL1(I)=XL2(I)=XL3(I)=FZ(I)=TZ(I)=XLPM11(I)=0.0
      RATOP(I)=RATOO(I)=RATON(I)=TN(I)=TO(I)=TP(I)=FN(I)=FO(I)=FP(I)=EP(
     1I)=XLM11(I)=1.0
    4 CONTINUE
*
*          READ NAMELIST INPUT
*
      READ (9,NAM1)
      IF (ENDFILE 9) 5,6
    5 STOP 2
    6 CONTINUE
      WRITE (6,NAM1)
      READ (5,NAM2)
      IF (ENDFILE 5) 7,8
    7 STOP 3
    8 CONTINUE
      IF (KODUNIT.EQ.1) CALL INUNIT (PROVAL,PRNTVAL,JM,JN)
      SI=DS
*
*          READ STARTING PROFILE
*
      READ (9,104) Y(1),VN(1),FN(1),DUM,TN(1),X51,DUM
      READ (9,104) (Y(N),VN(N),FN(N),DUM,TN(N),DUM,X6N,N=2,IEDGE)
      IF (ENDFILE 9) 9,10
    9 STOP 4
   10 CONTINUE
*
*         SET UP GRID NORMAL TO WALL
*
      W1=XK
      W2=1+W1
      W3=1+W1+W1*W1
      WW1=W3*W3*(W1*W2-1.)+W2
      WW2=W1*W2*W3*W3
      WW3=W3*W3
      WW4=1.+W1
      WW5=W1*W1*W1*W2*W3
      IF (XK.EQ.1.) GO TO 11
      DY(1)=((1.-XK)/(1.-XK**(IEDGE-1)))*XEND
      GO TO 12
   11 DY(1)=XEND/(IEDGE-1)
```

```
   12 CONTINUE                                                          A  87     9500000
      FETM(1)=FETM(2)=1./PR                                             A  88     9600000
      DY(2)=XK*DY(1)                                                    A  89     9700000
      DO 13 N=3,JK                                                      A  90     9800000
      DY(N)=(XK**(N-1))*DY(1)                                           A  91     9900000
      FETM(N)=1./PR                                                     A  92    10000000
   13 CONTINUE                                                          A  93    10100000
                                                                                 10200000
*        SET UP PRINT CONTROL                                           A  94    10300000
                                                                                 10400000
      ALL=DS*IEND1                                                      A  95    10500000
      IF (BLNGTH.NE.0.) ALL=BLNGTH                                      A  96    10600000
      CALL SETUP (PRDINC,PRDVAL,ALL,JN,IPRD)                            A  97    10700000
      CALL SETUP (PRNTINC,PRNTVAL,ALL,JM,IPRNT)                         A  98    10800000
      WRITE (6,101)                                                     A  99    10900000
      WRITE (6,102)                                                     A 100    11000000
                                                                                 11100000
*        WRITE INITIAL PROFILES                                         A 101    11200000
                                                                                 11300000
      WRITE (6,103) (Y(N),VN(N),FN(N),TN(N),X51,X6N,N=1,IEDGE)          A 102    11400000
                                                                                 11500000
*        PROGRAM CONSTANTS                                              A 103    11600000
                                                                                 11700000
   14 CONTINUE                                                          A 104    11800000
      XMAC=1.+.5*(G-1.)*XMA**2                                          A 105    11900000
      RT1=PT1/(R*TT1)                                                   A 106    12000000
      P1=PT1/(XMAC)**(G/(G-1.))                                         A 107    12100000
      R1=RT1/(XMAC)**(1./(G-1.))                                        A 108    12200000
      T1=TT1/XMAC                                                       A 109    12300000
      AA1=SQRT(G*P1/R1)                                                 A 110    12400000
      U1=XMA*AA1                                                        A 111    12500000
      TREF=U1**2/((G/(G-1.))*R)                                         A 112    12600000
      IF (IGAS.EQ.2) GO TO 15                                           A 113    12700000
      VIS1=(2.27E-8)*(T1**1.5)/(T1+SU)                                  A 114    12800000
      VISREF=(2.27E-8)*(TREF**1.5)/(TREF+SU)                            A 115    12900000
      GO TO 16                                                          A 116    13000000
   15 VIS1=VISCON*(T1**VISPOW)                                          A 117    13100000
      VISREF=VISCON*(TREF**VISPOW)                                      A 118    13200000
   16 CONTINUE                                                          A 119    13300000
      REY=R1*U1*A/VIS1                                                  A 120    13400000
      REYREF=REY*VIS1/VISREF                                            A 121    13500000
      XCONE=.0174533*CONE                                               A 122    13600000
      EPS=1./SQRT(REYREF)                                               A 123    13700000
      XWAVE=1.74533E-02*WAVE                                            A 124    13800000
      ABC=(XMA*SIN(XWAVE))**2                                           A 125    13900000
      P10=PT1/(R1*U1*U1)                                                A 126    14000000
      IF (XWAVE.GT..0000001.OR.XWAVE.LT.-.0000001) P10=(1./(G*XMA*XMA))* A 127    14100000
```

```
     1((XMAC*ABC*(G+1.))/(ABC*(G-1.)+2.))**(G/(G-1.))*((G+1.)/(2.*G*ABC-  A 128   14200000
     2(G-1.)))**(1./(G-1.))                                                A 129   14300000
      T10=0.5+1.0/(XMA*XMA*(G-1.0))                                        A 130   14400000
      R10=G*P10/(T10*(G-1.0))                                              A 131   14500000
      TC=SU/(T1*XMAC)                                                      A 132   14600000
      REL=SQRT(PR)                                                         A 133   14700000
      RET=PR**0.33333                                                      A 134   14800000
                                                                                   14900000
*         READ TABULAR DATA                                                A 135   15000000
                                                                                   15100000
      IF (NOIT.GT.0) GO TO 21                                              A 136   15200000
      NABC=NPUTYPE+1                                                       A 137   15300000
      GO TO (21,17,18), NABC                                               A 138   15400000
   17 CALL TABLE (IEND1,DS,R1,U1,A,TREF,KODWAL,VISREF,KODUNIT)             A 139   15500000
      GO TO 19                                                             A 140   15600000
   18 CALL TABLE1 (IEND1,DS,R1,U1,A,TREF,KODWAL,VISREF,KODUNIT)            A 141   15700000
   19 GO TO (21,20), IENTRO                                                A 142   15800000
   20 CALL VARENT (RRS,ZZS,DRSDZS,NNN)                                     A 143   15900000
   21 CONTINUE                                                             A 144   16000000
      GO TO (22,24), IBODY                                                 A 145   16100000
                                                                                   16200000
*       STARTING PROCEDURE FOR FLOWS WITH STAGNATION POINT                 A 146   16300000
                                                                                   16400000
   22 CONTINUE                                                             A 147   16500000
      IF (IGAS.EQ.1) VIS10=(T10**1.5)*(1.0+T10*TC)/(T10+T10*TC)            A 148   16600000
      IF (IGAS.EQ.2) VIS10=T10**VISPOW                                     A 149   16700000
      P1PT2=P1/PT1                                                         A 150   16800000
      IF (XMA.LE.1.) GO TO 23                                              A 151   16900000
      P1PT2=((((G+1.)*XMA**2)/2.)**(-G/(G-1.)))*((G+1.)/((2.*G*XMA**2)-(   A 152   17000000
     1G-1.)))**(-1./(G-1.))                                                A 153   17100000
   23 CONTINUE                                                             A 154   17200000
      DUEDS=SQRT(2.*((G-1.)/G)*T10*(1.-P1PT2))                             A 155   17300000
      QUAN1=DUEDS                                                          A 156   17400000
      BELA=R10*VIS10*DUEDS                                                 A 157   17500000
      RMI=SI                                                               A 158   17600000
      DX1DS=BELA*(SI-DS)**(2*J+1)                                          A 159   17700000
      DXDS=BELA*SI**(2*J+1)                                                A 160   17800000
      DX1=(BELA*SI**(2*J+2))/(2*J+2)                                       A 161   17900000
      X=DX1                                                                A 162   18000000
      QS=-R10*QUAN1*SI*VIS10*T10*RMI*X51/(PR*SQRT(2.*X))                   A 163   18100000
      DISP=SQRT(2.*X)*X6N/(R10*QUAN1*SI*RMI)                               A 164   18200000
      GO TO 27                                                             A 165   18300000
                                                                                   18400000
*       STARTING PROCEDURE FOR FLOWS WITHOUT STAGNATION POINT              A 166   18500000
                                                                                   18600000
   24 CONTINUE                                                             A 167   18700000
      RE=XY1                                                               A 168   18800000
```

```
      TESTR=XY2*T1
      IF (J.EQ.0) GO TO 25
      PE=((G-1.0)/G)*RE*(TESTR/TREF)
      UE=XY3*SQRT(G*R*TESTR)/U1
      IF (NOIT.GT.0) UE=UEE(M)
      IF (IGAS.EQ.1) XNUE=((TESTR/TREF)**1.5)*((TREF+SU)/(TESTR+SU))
      IF (IGAS.EQ.2) XNUE=(TESTR/TREF)**VISPOW
      RMI=SI*SIN(XCONE)
      BELEX=RE*UE*XNUE*(SIN(XCONE)**(2*J))
      IF (J.NE.0.AND.CONE.EQ.0.) BELEX=RE*UE*XNUE
      DX1DS=BELEX*((SI-DS)**(2*J))
      GO TO 26
   25 CONTINUE
      PE=P1/(R1*U1*U1)
      TE=T1/TREF
      UE=1.
      IF (NOIT.GT.0) UE=UEE(M)
      IF (IGAS.EQ.1) XNUE=(TE**1.5)*((1.+SU/TREF)/(TE+SU/TREF))
      IF (IGAS.EQ.2) XNUE=(T1/TREF)**.647
      RMI=1.
      BELEX=RE*UE*XNUE
      DX1DS=BELEX
   26 DXDS=BELEX*SI**(2*J)
      DX1=(BELEX*(SI**(2*J+1)))/(2*J+1)
      X=DX1
      QS=DISP=0.
   27 CONTINUE

*       SET UP INITIAL PROFILES

      DO 28 N=1,IEDGE
      VN(N)=-VN(N)
      FO(N)=FN(N)
      TO(N)=TN(N)
   28 VO(N)=VN(N)
      IF (KODUNIT.EQ.1) CALL OUTUNIT (PROVAL,PRNTVAL,JM,JN)
      WRITE (6,NAM2)
      WRITE (6,NAM)
      IF (KODUNIT.EQ.1) CALL INUNIT (PROVAL,PRNTVAL,JM,JN)
      ZS=0.
      XLMIN=1.
      XLMIC=XLMIN

*       BEGIN MARCHING PROCEDURE ALONG SURFACE

      SMI=0.
      S=DS
```

```
      DO 93 M=2,IEND1                                                  A 212   23600000
      SM2=SM1                                                          A 213   23700000
      SM1=S                                                            A 214   23800000
      READ (4) S,PE,RMI,TW,Z,DPEDS,RVWALD,DRDZ,QW                      A 215   23900000
      IF (KODWAL.NE.1) CW=QW*VISREF*U1*U1/(778.26*A)                   A 216   24000000
      PHI=ATAN(DRDZ)                                                   A 217   24100000
      COSTH=COS(PHI)                                                   A 218   24200000
      PP=DPEDS                                                         A 219   24300000
      UE=SQRT(2.0*T10*(1.0-(PE/P10)**((G-1.0)/G)))                     A 220   24400000
      IF (NOIT.GT.0) UE=UEE(M)                                         A 221   24500000
      TE=T10-0.5*UE*UE                                                 A 222   24600000
      RE=G*PE/((G-1.0)*TE)                                             A 223   24700000
      IF (IGAS.EQ.1) XNUE=(TE**1.5)*(1.0+T10*TC)/(TE+T10*TC)           A 224   24800000
      IF (IGAS.EQ.2) XNUE=TE**VISPOW                                   A 225   24900000
      DX2DS=DX1DS                                                      A 226   25000000
      DX1DS=DXDS                                                       A 227   25100000
      DXDS=RE*UE*XNUE*(RMI**(2*J))                                     A 228   25200000
                                                                               25300000
*       CHECK FOR CHANGE IN STEP INCREMENT                             A 229   25400000
                                                                               25500000
      CKK=(S-SM1)/(SM1-SM2)                                            A 230   25600000
      IF (CKK.LT..999999999) GO TO 29                                  A 231   25700000
      IF (CKK.GT.1.0000001) GO TO 29                                   A 232   25800000
      DX2=(S-SM1)*(5.*DXDS+8.*DX1DS-DX2DS)/12.                         A 233   25900000
      GO TO 30                                                         A 234   26000000
   29 DX2=(S-SM1)*(DX1DS+DXDS)/2.                                      A 235   26100000
   30 CONTINUE                                                         A 236   26200000
      X=X+DX2                                                          A 237   26300000
      DY=(2.*X)**(2.*XPX)                                              A 238   26400000
      DZ=(2.*X)**(XPX)                                                 A 239   26500000
      DUEDX=-PP/(RE*UE*DXDS)                                           A 240   26600000
      DTEDX=-UE*DUEDX                                                  A 241   26700000
      XAL=UE*UE/TE                                                     A 242   26800000
      XBE=DY*DUEDX/UE                                                  A 243   26900000
      TR=T10*TC/TE                                                     A 244   27000000
                                                                               27100000
*       SET UP DIFFERENCE-QUOTIENT COEFFICIENTS FOR X-COORDINATE       A 245   27200000
                                                                               27300000
      Z1=2.*((DX1+2.*DX2)/(DX1+DX2))                                   A 246   27400000
      Z2=2.*(DX1+DX2)/DX1                                              A 247   27500000
      Z3=2.*((DX2*DX2)/(DX1*(DX1+DX2)))                                A 248   27600000
      Z4=(DX1+DX2)/DX1                                                 A 249   27700000
      Z5=DX2/DX1                                                       A 250   27800000
                                                                               27900000
*                            SET BOUNDARY CONDITIONS                   A 251   28000000
                                                                               28100000
      XK1(1)=0.0                                                       A 252   28200000
```

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      XK2(1)=0.0                                                        A 253   28300000
      XK3(1)=0.0                                                        A 254   28400000
      XL2(1)=0.0                                                        A 255   28500000
      XL3(1)=0.0                                                        A 256   28600000
      IF (KODWAL.EQ.1) GO TO 31                                         A 257   28700000
      XLM1P=Z4*XLM1O-Z5*XLM1N                                           A 258   28800000
      TZ(1)=-QW*778.26*(EPS*A/(VISREF*U1**2))*(PR*OZ/(RE*UE*TE*XNUE*RMI*A 259   28900000
     1*J))*(1.0/XLM1P)                                                  A 260   29000000
      GO TO 32                                                          A 261   29100000
   31 CONTINUE                                                          A 262   29200000
      XL1(1)=TW/TE                                                      A 263   29300000
      TP(1)=XL1(1)                                                      A 264   29400000
   32 CONTINUE                                                          A 265   29500000
      FP(1)=0.0                                                         A 266   29600000
      N=IEDGE+1                                                         A 267   29700000
      KON=N+1                                                           A 268   29800000
      FAA=OZ/(RE*UE*(RMI**J))                                           A 269   29900000
      FAB=2.*EPS*W*FAA*COSTH/(RMI**J)                                   A 270   30000000
      FAC=2.*EPS*OZ*COSTH/(RE*UE*RMI*RMI)                               A 271   30100000
      FAD=RMI/(EPS*COSTH)                                               A 272   30200000
      ITRO=0                                                            A 273   30300000
                                                                                30400000
*         SET UP DIFFERENCE COEFFICIENTS FOR Y COORDINATE              A 274   30500000
                                                                                30600000
      DY1=DY(1)                                                         A 275   30700000
      DO 39 N=2,IEDGE                                                   A 276   30800000
      DY2=XK*DY1                                                        A 277   30900000
      Y1=2./((DY1+DY2)*DY2)                                             A 278   31000000
      Y2=2./(DY1*DY2)                                                   A 279   31100000
      Y3=Y1*(DY2/DY1)                                                   A 280   31200000
      Y4=Y1*(DY1/2.)                                                    A 281   31300000
      Y5=(DY1-DY2)/(DY1*DY2)                                            A 282   31400000
      Y6=DY2/(DY1*(DY1+DY2))                                            A 283   31500000
                                                                                31600000
*        SET UP MATRIX ELEMENTS                                         A 284   31700000
                                                                                31800000
      IF (ITRO.EQ.0) GO TO 33                                           A 285   31900000
      TM1=TP(N)                                                         A 286   32000000
      RATO=RATOP(N)                                                     A 287   32100000
      DRATO=FAB*TM1                                                     A 288   32200000
      RATO2J=RATO**(2.*J)                                               A 289   32300000
      XLM1=XLM11(N)                                                     A 290   32400000
      XLPM1=XLPM11(N)                                                   A 291   32500000
      TY=TZ(N)                                                          A 292   32600000
      FY=FZ(N)                                                          A 293   32700000
      FM1=FP(N)                                                         A 294   32800000
      EM1=EP(N)                                                         A 295   32900000
```

```
      EPM1=EPP(N)                                              A 296    33000000
      GO TO 37                                                 A 297    33100000
   33 CONTINUE                                                 A 298    33200000
      TM1=Z4*TO(N)-Z5*TN(N)                                    A 299    33300000
      IF (TM1.GT.0.) GO TO 34                                  A 300    33400000
      TM1=(TO(N)+TN(N))/2.                                     A 301    33500000
      WRITE (6,100) S,M,N                                      A 302    33600000
   34 CONTINUE                                                 A 303    33700000
      RATO=Z4*RATOO(N)-Z5*RATON(N)                             A 304    33800000
      DRATO=FAB*TM1                                            A 305    33900000
      RATO2J=RATO**(2*J)                                       A 306    34000000
      IF (IGAS.EQ.2) GO TO 35                                  A 307    34100000
      XLM1=((1.+TR)*SQRT(TM1)/(TM1+TR))                        A 308    34200000
      XLPM1=XLM1*(TR-TM1)/(2.*TM1*(TM1+TR))                    A 309    34300000
      GO TO 36                                                 A 310    34400000
   35 XLM1=TM1**(VISPOW-1.)                                    A 311    34500000
      XLPM1=(VISPOW-1.)*(TM1**(VISPOW-2.))                     A 312    34600000
   36 CONTINUE                                                 A 313    34700000
      FY=(FO(N+1)-FO(N-1))/(DY(N-1)+DY(N))                     A 314    34800000
      TY=(TO(N+1)-TO(N-1))/(DY(N-1)+DY(N))                     A 315    34900000
      FM1=Z4*FO(N)-Z5*FN(N)                                    A 316    35000000
      EM1=EP(N)                                                A 317    35100000
      EPM1=EPP(N)                                              A 318    35200000
   37 CONTINUE                                                 A 319    35300000
      VM1=Z4*VO(N)-Z5*VN(N)                                    A 320    35400000
      TM2=Z2*TO(N)-Z3*TN(N)                                    A 321    35500000
      FM2=Z2*FO(N)-Z3*FN(N)                                    A 322    35600000
      ETM1=EETM(N)                                             A 323    35700000
      ETPM1=EEPM(N)                                            A 324    35800000
      H1=DY*FM1*FT/(2.*DX2)                                    A 325    35900000
      H2=VM1-XLM1*((RATO2J*EPM1)+(EM1*DRATO))                  A 326    36000000
      H3=-RATO2J*XLM1*EM1                                      A 327    36100000
      H4=H3*XLPM1/XLM1                                         A 328    36200000
      H5=XBE*FM1                                               A 329    36300000
      H6=-XBE                                                  A 330    36400000
      H7=VM1-XLM1*((RATO2J*ETPM1)+(ETM1*DRATO))                A 331    36500000
      H8=-XAL*RATO2J*XLM1*EM1                                  A 332    36600000
      H9=-RATO2J*XLPM1*ETM1                                    A 333    36700000
      H10=H9*XLM1/XLPM1                                        A 334    36800000
      H11=H2+H4*TY                                             A 335    36900000
      H12=H7+2.*H9*TY                                          A 336    37000000
      A1=-Y6*H11+Y3*H3                                         A 337    37100000
      B1=-Y5*H11-Y2*H3+H5+Z1*H1                                A 338    37200000
      C1=Y4*H11+Y1*H3                                          A 339    37300000
      D1=-Y6*H4*FY                                             A 340    37400000
      E1=(D1*Y5/Y6)+H6                                         A 341    37500000
      F1=-D1*Y4/Y6                                             A 342    37600000
```

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      G1=H1*FM2+H4*TY*FY                                                 A 343   37700000
      A2=-2.*H8*FY*Y6                                                    A 344   37800000
      B2=A2*Y5/Y6                                                        A 345   37900000
      C2=-A2*Y4/Y6                                                       A 346   38000000
      D2=-Y6*H12+H10*Y3                                                  A 347   38100000
      E2=-Y5*H12-H10*Y2+H1*Z1                                            A 348   38200000
      F2=Y4*H12+H10*Y1                                                   A 349   38300000
      G2=H1*TM2+H8*FY*FY+H9*TY*TY                                        A 350   38400000
      IF (KODWAL.EQ.1) GO TO 38                                          A 351   38500000
      IF (N.GT.2) GO TO 38                                               A 352   38600000
      DID=(C2*D1-C1*D2)-((C2*F1-C1*F2)*(((1.+XK)**2)-1.))                A 353   38700000
      XL1(1)=((C2*G1-C1*G2)+(C2*F1-C1*F2)*(XK*(1+XK)*DY(1))*TZ(1))/DID   A 354   38800000
      XL2(1)=-(C2*B1-C1*B2)/DID                                          A 355   38900000
      XL3(1)=-((C2*E1-C1*E2)+((C2*F1-C1*F2)*((1.+XK)**2)))/DID           A 356   39000000
   38 CONTINUE                                                           A 357   39100000
                                                                                 39200000
*        SET UP MATRIX ARRAYS                                            A 358   39300000
                                                                                 39400000
      B1S=B1+A1*XK2(N-1)+D1*XL2(N-1)                                     A 359   39500000
      B2S=B2+A2*XK2(N-1)+D2*XL2(N-1)                                     A 360   39600000
      E1S=E1+A1*XK3(N-1)+D1*XL3(N-1)                                     A 361   39700000
      E2S=E2+A2*XK3(N-1)+D2*XL3(N-1)                                     A 362   39800000
      G1S=G1-A1*XK1(N-1)-D1*XL1(N-1)                                     A 363   39900000
      G2S=G2-A2*XK1(N-1)-D2*XL1(N-1)                                     A 364   40000000
      D=1.0/(B1S*E2S-E1S*B2S)                                            A 365   40100000
      XK1(N)=D*(G1S*E2S-G2S*E1S)                                         A 366   40200000
      XK2(N)=D*(E1S*C2-C1*E2S)                                           A 367   40300000
      XK3(N)=D*(E1S*F2-F1*E2S)                                           A 368   40400000
      XL1(N)=D*(B1S*G2S-B2S*G1S)                                         A 369   40500000
      XL2(N)=D*(C1*B2S-B1S*C2)                                           A 370   40600000
      XL3(N)=D*(F1*B2S-B1S*F2)                                           A 371   40700000
      DY1=DY2                                                            A 372   40800000
   39 CONTINUE                                                           A 373   40900000
      N=IEDGE+1                                                          A 374   41000000
      NN=N-1                                                             A 375   41100000
      KON=NN                                                             A 376   41200000
                                                                                 41300000
*         MATRIX SOLUTION FOR MOMENTUM AND ENERGY EQUATIONS              A 377   41400000
                                                                                 41500000
      DO 40 N=2,NN                                                       A 378   41600000
      FP(KON)=XK1(KON)+XK2(KON)*FP(KON+1)+XK3(KON)*TP(KON+1)             A 379   41700000
      TP(KON)=XL1(KON)+XL2(KON)*FP(KON+1)+XL3(KON)*TP(KON+1)             A 380   41800000
   40 KON=KON-1                                                          A 381   41900000
      IF (KODWAL.EQ.1) GO TO 41                                          A 382   42000000
      TP(1)=(XK*(1+XK)*DY(1)*XLM1P-(1+XK)**2*TP(2)+TP(3))/(1-(1+XK)**2)  A 383   42100000
      TW=TP(1)*TE                                                        A 384   42200000
      RFCTOR=(TP(1)-1.)/((TT1/(TE*TREF))-1.)                             A 385   42300000
```

```
   41 CONTINUE
      TWOTT1=TP(1)*TE*TREF/TT1

*       INTEGRATE CONTINUITY EQUATION

      VP(1)=(RVWALD*FAA)/(R1*U1*EPS*XNUE)
      B0=0.0
      DO 42 N=2,NN
      B=DY*((XPX*FP(N)/X)+FT*((Z1*FP(N)-Z2*FO(N)+Z3*FN(N))/(2.*DX2)))
      VP(N)=VP(N-1)-0.5*(B+B0)*DY(N-1)
   42 B0=B

*      CHECK FOR SUFFICIENT GRID WIDTH

      IF (M-20) 48,48,43
   43 IF (ABS(FP(IEDGE-15)-FP(IEDGE-16))-.0001) 44,44,45
   44 IF (ABS(TP(IEDGE-15)-TP(IEDGE-16))-.0001) 48,48,45
   45 IF (XK.EQ.1.) GO TO 46
      WRITE (6,105)
      STOP
   46 IF (TRFACT.LT.1.) GO TO 47
      WRITE (6,106)
      STOP
   47 IF (IEDGE.LE.301) GO TO 49
      WRITE (6,107)
      STOP
   48 IF (XK.NE.1..OR.TRFACT.LT.1.) GO TO 50
      WRITE (6,108)
      STOP
   49 IEDGE=IEDGE+1
   50 CONTINUE

*      DETERMINE LOCATION OF BOUNDARY LAYER EDGE

      NCOUNT=0
      DO 51 I=1,IEDGE
      NCOUNT=NCOUNT+1
      IF (FP(I).GE..9999) GO TO 52
   51 CONTINUE
      WRITE (6,109)
      WRITE (6,DBG)
      CALL DUMP
      STOP
   52 ETAEDG=NCOUNT

*      CALCULATE WALL AND INITIAL VALUES REQUIRED
*     FOR BASIC BOUNDARY LAYER PARAMETERS
```

```
      IF (IGAS.EQ.2) GO TO 53
      XLM11(1)=((1.+TE)*SQRT(TP(1))/(TP(1)+TR))
      XLM1P=XLM11(1)
      XLPM11(1)=XLM11(1)*(TR-TP(1))/(2.*TP(1)*(TP(1)+TR))
      GO TO 54
   53 XLM11(1)=TP(1)**(VISPOW-1.)
      XLM1P=XLM11(1)
      XLPM11(1)=(VISPOW-1.)*(TP(1)**(VISPOW-2.))
   54 CONTINUE
      MOMF(1)=FP(1)/SQRT(TP(1))
      IF (KODWAL.NE.1) GO TO 55
      TZ(1)=(-WW1*TP(1)+WW2*TP(2)-WW3*TP(3)+WW4*TP(4))/(WW5*DY(1))
   55 CONTINUE
      FZ(1)=(-WW1*FP(1)+WW2*FP(2)-WW3*FP(3)+WW4*FP(4))/(WW5*DY(1))
      XMF1=2.0+XAL
      XMF2=TP(1)*TE/T10
      XMF3=1.0-XMF2
      TTOTT(1)=(2.0*TP(1)+XAL*FP(1)*FP(1))/XMF1
      CROCCO(1)=(TTOTT(1)-XMF2)/XMF3
      UOUPL(1)=0.0
      TCORD(1)=0.0
      UDEF(1)=0.0
      NONDFL(1)=0.0
      XMF4=EPS*XNUE*UE*UE*(RMI**J)*XLM11(1)*FZ(1)/OZ
      CO=0.0
      DISP=0.0
      THETA=0.0
      DISINC=0.0
      TPK=0.0
      KON=IEDGE+2
*       CALCULATE BASIC BOUNDARY LAYER PARAMETERS
      DO 60 N=2,KON
      TPK=TPK+0.5*(TP(N-1)+TP(N))*DY(N-1)
      RATOP(N)=SIGN*SQRT(1.0+FAB*TPK)
      XN(N)=FAD*(-1.0+SIGN*SQRT(1.0+FAC*TPK))
      DISP=DISP+.5*((TP(N-1)-FP(N-1))+(TP(N)-FP(N)))*DY(N-1)
      C=1.0/(RATOP(N)**J)
      C=C*(FP(N)*(1.-FP(N)))
      THETA=THETA+0.5*(CO+C)*DY(N-1)
      CO=C
      IF (TRFACT.EQ.0.0) GO TO 56
      UPLUS=SQRT(XMF4*TP(N))
      UOUPL(N)=UE*FP(N)/UPLUS
      TCORD(N)=(XN(N)*UPLUS*RE)/(EPS*XNUE*((TP(N)*XLM11(N))**2))
```

```
      UDEF(N)=UE*(1.-FP(N))/UPLUS                                         A 470     51800000
      SUBTST=ABS(UDUPL(N)-TCORD(N))/((UDUPL(N)+TCORD(N))/2.)              A 471     51900000
      IF (SUBTST.GE.0.06) GO TO 56                                        A 472     52000000
      NSUBLY=N                                                            A 473     52100000
   56 CONTINUE                                                            A 474     52200000
      DISINC=DISINC+0.5*((TP(N-1)*(1.0-FP(N-1))/RATOP(N-1))+(TP(N)*(1.0-  A 475     52300000
     1FP(N))/RATOP(N)))*DY(N-1)                                           A 476     52400000
      FZ(N)=(FP(N+1)-FP(N-1))/(DY(N-1)+DY(N))                             A 477     52500000
      TZ(N)=(TP(N+1)-TP(N-1))/(DY(N-1)+DY(N))                             A 478     52600000
      TTOTT(N)=(2.0*TP(N)+XAL*FP(N)*FP(N))/XMF1                           A 479     52700000
      CROCCO(N)=(TTOTT(N)-XMF2)/XMF3                                      A 480     52800000
      IF (TP(N).GE.0.) GO TO 57                                           A 481     52900000
      WRITE (6,110) N,S,N,TP(N)                                           A 482     53000000
      WRITE (6,DBG)                                                       A 483     53100000
      STOP 2                                                              A 484     53200000
   57 CONTINUE                                                            A 485     53300000
      IF (IGAS.EQ.2) GO TO 58                                             A 486     53400000
      XLM11(N)=((1.+TR)*SQRT(TP(N))/(TP(N)+TR))                           A 487     53500000
      XLPM11(N)=XLM11(N)*(TR-TP(N))/(2.*TP(N)*(TP(N)+TR))                 A 488     53600000
      GO TO 59                                                            A 489     53700000
   58 XLM11(N)=TP(N)**(VISPOW-1.)                                         A 490     53800000
      XLPM11(N)=(VISPOW-1.)*(TP(N)**(VISPOW-2.))                          A 491     53900000
   59 CONTINUE                                                            A 492     54000000
      MOME(N)=FP(N)/SQRT(TP(N))                                           A 493     54100000
   60 CONTINUE                                                            A 494     54200000
      DISINC=DISINC*EPS*FAA                                               A 495     54300000
      THETA=THETA*EPS*FAA*A                                               A 496     54400000
      DISP=DISP*EPS*FAA*A                                                 A 497     54500000
      DISP=ABS(DISP)                                                      A 498     54600000
      XNDEN=XN(ETAEDG)                                                    A 499     54700000
      DO 61 N=2,KON                                                       A 500     54800000
   61 NONDEL(N)=XN(N)/XNDEN                                               A 501     54900000
      IF (J.EQ.1) GO TO 62                                                A 502     55000000
      GO TO 63                                                            A 503     55100000
   62 DISP=RMI*(-1.+SQRT(1.+(2.*DISP/RMI)))                               A 504     55200000
   63 CONTINUE                                                            A 505     55300000
      THADIS=DISP/THETA                                                   A 506     55400000
      REDELT=(RE*UE*DISP/XNUE)*REYREF                                     A 507     55500000
      RETHET=REDELT/THADIS                                                A 508     55600000
      RES=(RE*UE*S/XNUE)*REYREF                                           A 509     55700000
      XMAE=UE/SQRT(TE*(G-1.))                                             A 510     55800000
      TAUD=VISREF*U1*(XLM11(1)*RE*XNUE*UE*UE*(RMI**J)*FZ(1)/DZ)/(EPS*A)   A 511     55900000
      CFE=TAUD/(.5*R1*U1*U1*RE*UE*UE)                                     A 512     56000000
      CFW=CFE*TW/TE                                                       A 513     56100000
      IF (KODWAL.NE.1) GO TO 64                                           A 514     56200000
      QS=-XLM11(1)*RE*UE*TE*XNUE*(RMI**J)*TZ(1)/(PR*DZ*EPS)               A 515     56300000
      QSD=QS*VISREF*U1*U1/(778.26*A)                                      A 516     56400000
```

```
      GO TO 65                                                          A 517   56500000
   64 QSD=QW                                                            A 518   56600000
   65 CONTINUE                                                          A 519   56700000
      TWD=TREF*TW                                                       A 520   56800000
      TAWD=(RFL+(RFT-RFL)*TRFACT)*(TT1-TE*TREF)+TE*TREF                 A 521   56900000
      HD=QSD/(TWD-TAWD)                                                 A 522   57000000
      CHE=778.26*HD*(G-1.)/(G*R*R1*U1*RE*UE)                            A 523   57100000
      CHW=CHE*TW/TE                                                     A 524   57200000
      HSD=HD*A*S                                                        A 525   57300000
      KED=R*VISREF*XNUE*G/(778.26*PR*(G-1.))                            A 526   57400000
      KWD=KED*XLM11(1)*TP(1)                                            A 527   57500000
      NUE=HSD/KED                                                       A 528   57600000
      NUW=HSD/KWD                                                       A 529   57700000
      TBAR=TP(1)                                                        A 530   57800000
      XLBAR=XLM11(1)                                                    A 531   57900000
      IF (TRFACT.EQ.0.0) GO TO 67                                       A 532   58000000
      SUM1=0.                                                           A 533   58100000
      SUM2=SUM1                                                         A 534   58200000
      DO 66 N=1,NSUBLY                                                  A 535   58300000
      SUM1=SUM1+TP(N)                                                   A 536   58400000
   66 SUM2=SUM2+XLM11(N)                                                A 537   58500000
      TBAR=SUM1/NSUBLY                                                  A 538   58600000
      XLBAR=SUM2/NSUBLY                                                 A 539   58700000
   67 CONTINUE                                                          A 540   58800000
                                                                                58900000
*              TOTAL PRESSURE RATIO, (PT2)BL/(PT2)REF                   A 541   59000000
                                                                                59100000
      AZ1=XMA*XMA*(G+1.)/2.                                             A 542   59200000
      AZ2=(G+1.)/(2.*G*XMA*XMA-G+1.)                                    A 543   59300000
      PTREF=PT1                                                         A 544   59400000
      IF (XMA.LE.1.) GO TO 68                                           A 545   59500000
      PTREF=(AZ1**(G/(G-1.))*(AZ2**(1./(G-1.))))/(G*XMA*XMA)            A 546   59600000
   68 CONTINUE                                                          A 547   59700000
      DO 71 I=1,IEDGE                                                   A 548   59800000
      ZEB=(MOME(I)*XMAE)**2                                             A 549   59900000
      IF (ZEB-1.) 69,69,70                                              A 550   60000000
   69 PT2=PE*((1.+(G-1.)*ZEB/2.)**(G/(G-1.)))                           A 551   60100000
      GO TO 71                                                          A 552   60200000
   70 PT2=PE*((((G+1.)*ZEB/2.)**(G/(G-1.)))*(((G+1.)/(2.*G*ZEB-G+1.))**(A 553   60300000
     11./(G-1.))))                                                      A 554   60400000
   71 PTOPT(I)=PT2/PTREF                                                A 555   60500000
                                                                                60600000
*         CALCULATE VORTICITY REYNOLDS NUMBER                           A 556   60700000
*         AND DETERMINE STABILITY INDEX                                 A 557   60800000
                                                                                60900000
      IF (TRFACT.GT.0.0.AND.TRFACT.LT.0.9999) GO TO 74                  A 558   61000000
      DO 72 I=1,ETAEDG                                                  A 559   61100000
```

```
      INTGRL=INTEGT(TP,I,DY)
      TERMB=((OZ/((RMI**J)*XNUE))*(INTGRL**2))/EPS
   72 STAB2(I)=TERMB*FZ(I)/(XLM11(I)*TP(I)**3)
      ST2MAX=STAB2(1)
      DO 73 IX=2,IEDGE
      IF (ST2MAX.GT.STAB2(IX)) GO TO 73
      ST2MAX=STAB2(IX)
   73 CONTINUE
      SMXP=ST2MAX
   74 CONTINUE
      DSMXO=(SMXP-SMXN)/(2.*DS)
      SMXN=SMXO
      SMXO=SMXP
      IF (KTCOD.EQ.2.OR.KTCD.EQ.1) GO TO 75
      KTCD=1
      RRRR=RE*UE*R1*U1/(XNUE*VISREF)
      TLNGTH=1.+(5.*RRRR**(-1)*(RRRR*A*SST)**.8)/SST
   75 CONTINUE
*
      TURBULENT PARAMETER CALCULATIONS

      IF (SMXTR.LE.ST2MAX) CALL TURBLNT (TP,XLM11,FZ,XN,RATOP,DY,EP,FP,E
     1PP,EETM,EEPM,VARA,VARB,VARC,VARD,VARE,JK)
      IF (SST.LE.S*A) CALL TURBLNT (TP,XLM11,FZ,XN,RATOP,DY,EP,FP,EPP,EE
     1TM,EEPM,VARA,VARB,VARC,VARD,VARE,JK)

*     END TURBULENT PARAMETER CALCULATIONS
*        VARIABLE ENTROPY CALCULATIONS

      GO TO (77,76), IENTRO
   76 INTEGL=INTEGT(FP,ETAEDG,DY)
      XXJ=J
      RS=((J+1)*EPS*OZ*INTEGL)**((2.-XXJ)/2.)
      CALL FTLUP (RS,ZS,2,NNN,RRS,ZZS)
      CALL FTLUP (RS,TANTHTS,2,NNN,RRS,DRSDZS)
      THATS=ATAN(TANTHTS)
      PTTWO=(G+1.)*XMA**2*SIN(THATS)**2
      PTTWO=PTTWO/((G-1.)*(XMA)**2*SIN(THATS)**2+2.)
      PTTWO=PTTWO**(G/(G-1.))*PT1
      PTTWO=PTTWO*((G+1.)/(2.*G*XMA**2*SIN(THATS)**2-(G-1.)))**(1./(G-1.
     1))
      P20=PTTWO/(R1*U1*U1)
      UEE(M)=SQRT(2.*T10*(1.-(PE/P20)**((G-1.)/G)))
      NN=IEDGE+2
   77 CONTINUE
      IF (IENTRO.EQ.1) P20=P10
```

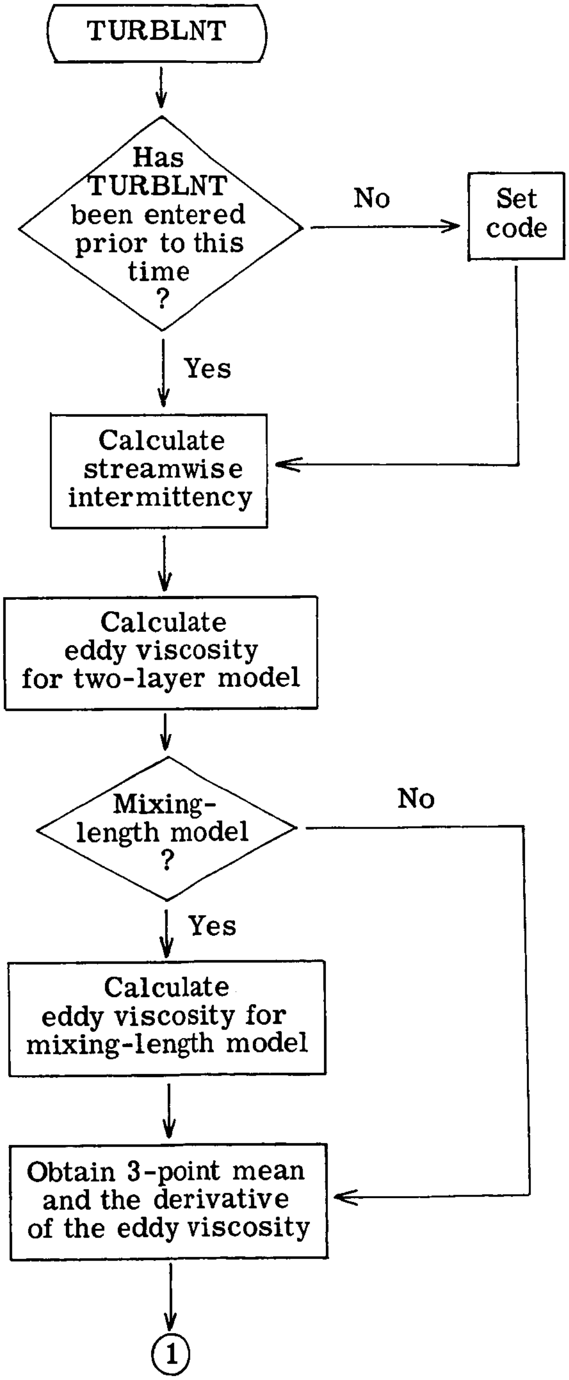
```
*        PRINT PROFILES AND WALL VALUES

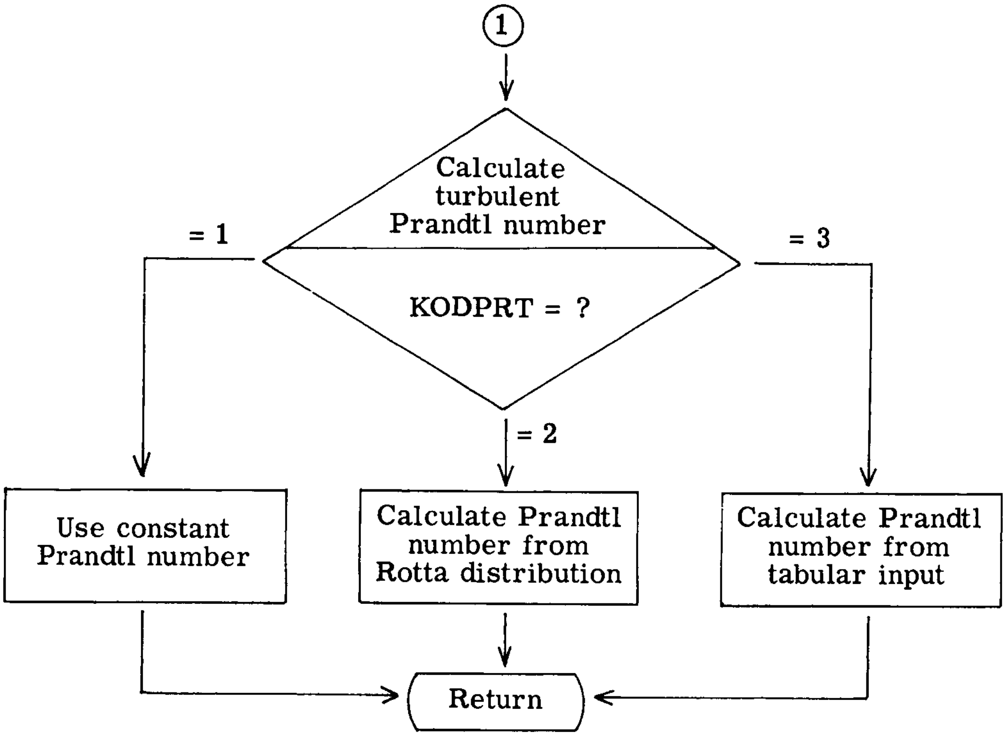
      S=S*A
      DO 78 NUMBR=1,JN
      IF (S.GT.PROVAL(NUMBR)-.000001.AND.S.LT.PROVAL(NUMBR)+.000001) GO
     1TO 79
   78 CONTINUE
      GO TO 88
   79 CONTINUE
      WRITE (6,111) S
      IEDGEX=ETAEDG+10
      IF (NAUXPRO.NE.1) GO TO 81
      WRITE (6,112)
      DO 80 I=1,IEDGEX
   80 WRITE (6,113) (Y(I),NONDEL(I),VP(I),FZ(I),TZ(I),VARA(I),VARB(I),VA
     1RC(I),VARD(I),EP(I),VARE(I))
   81 CONTINUE
      IF (KODE.EQ.1) GO TO 83
      IF (TRFACT.GT.0.9999) GO TO 86
      IF (TRFACT.GT.0.0.AND.TRFACT.LT.0.9999) GO TO 83
      WRITE (6,114)
      DO 82 I=1,IEDGEX
   82 WRITE (6,115) Y(I),NONDEL(I),FP(I),TP(I),TTOTT(I),CROCCO(I),PTOPT(
     1I),MCME(I),FZ(I),TZ(I),STAB2(I),XLM11(I)
      GO TO 88
   83 WRITE (6,114)
      DO 84 I=1,IEDGEX
   84 WRITE (6,115) Y(I),NONDEL(I),FP(I),TP(I),TTOTT(I),CROCCO(I),PTOPT(
     1I),MCME(I),FZ(I),TZ(I),STAB2(I),XLM11(I)
      WRITE (6,116)
      DO 85 I=1,IEDGEX
   85 WRITE (6,115) Y(I),NONDEL(I),FP(I),TP(I),TTOTT(I),CROCCO(I),PTOPT(
     1I),MCME(I),TCORD(I),UOUPL(I),UDEF(I),EP(I)
      GO TO 88
   86 WRITE (6,116)
      DO 87 I=1,IEDGEX
   87 WRITE (6,115) Y(I),NONDEL(I),FP(I),TP(I),TTOTT(I),CROCCO(I),PTOPT(
     1I),MCME(I),TCORD(I),UOUPL(I),UDEF(I),EP(I)
   88 CONTINUE
      DO 89 NUMBER=1,JM
      IF (S.GT.PRNTVAL(NUMBER)-.000001.AND.S.LT.PRNTVAL(NUMBER)+.000001)
     1 GO TO 90
   89 CONTINUE
      GO TO 91
   90 CONTINUE
      PESTAR=PE*R1*U1*U1
      TESTAR=TE*TREF
```

```
      RESTAR=RE*R1
      UESTAR=UE*U1
      MUESTAR=XNUE*VISREF
      YESTAR=XNDEN*EPS*A
      IF (KODUNIT.EQ.1) CALL WALLOUT (PROVAL,PRNTVAL,JM,JN)
      WRITE (6,117) S,RETHET,PP,CFW,ZS,YESTAR,X,RES,DTEDX,QSD,RS,UPLUS,R
     1MI,PESTAR,DUEDX,HD,ITRO,PTREF,Z,TESTAR,DISP,CHE,TWOTT1,JPOINT,XBE,
     2RESTAR,THETA,CHW,RFCTOR,P20
      WRITE (6,118) TRFACT,UESTAR,THADIS,NUE,ST2MAX,EPS,RVWALD,XMAE,TAUD
     1,NUW,DSMXD,REDELT,MUESTAR,CFE,THATS,XNDEN
      IF (KODUNIT.EQ.1) S=S*3.280839895
   91 CONTINUE
*
*        UPDATE VARIABLES FOR MARCHING PROCEDURE
*
      DO 92 N=1,NN
      FN(N)=FO(N)
      FO(N)=FP(N)
      TN(N)=TO(N)
      TO(N)=TP(N)
      VN(N)=VO(N)
      VO(N)=VP(N)
      RATON(N)=RATOO(N)
      RATOO(N)=RATOP(N)
      XLM1N=XLM1O
      XLM1O=XLM1P
   92 CONTINUE
   93 DX1=DX2
      IF (IENTRO.EQ.1) STOP 100
      IF (NOIT.GE.ITMAX) STOP 77
      NOIT=NOIT+1
      REWIND 4
      REWIND 9
      READ (9,NAM1)
      IF (ENDFILE 9) 94,95
   94 STOP 12
   95 CONTINUE
      READ (9,104) Y(1),VN(1),FN(1),DUM,TN(1),X51,DUM
      IF (ENDFILE 9) 96,97
   96 STOP 13
   97 CONTINUE
      READ (9,104) (Y(N),VN(N),FN(N),DUM,TN(N),DUM,X6N,N=2,IEDGE)
      IF (ENDFILE 9) 98,99
   98 STOP 14
   99 CONTINUE
      GO TO 14
```

```
*                                                                          A 692    75300000
                                                                                    75400000
 100 FORMAT (/64H NEGATIVE TM1 REPLACED WITH ARITHMETIC MEAN OF TO AND     A 693    75500000
    1TN AT S = ,F10.4,5H M = ,I3,5H N = ,I3/)                              A 694    75600000
 101 FORMAT (1H1)                                                          A 695    75700000
 102 FORMAT (6X,3HETA9X,5HYD(1)8X,5HYD(2)8X,5HYD(4)8X,5HYD(5)8X,5HYD(6)    A 696    75800000
    1)                                                                     A 697    75900000
 103 FORMAT (6E13.6)                                                       A 698    76000000
 104 FORMAT (7F13.6)                                                       A 699    76100000
 105 FORMAT (22H  GRID WIDTH TOO SMALL/35H  CANNOT ADD MORE STEPS WITH     A 700    76200000
    1XK*1.0)                                                               A 701    76300000
 106 FORMAT (22H  GRID WIDTH TOO SMALL/24HMAKE XK>1.0 AND RESUBMIT)        A 702    76400000
 107 FORMAT (22H  GRID WIDTH TOO SMALL/78HMAXIMUM LIMIT (301) FOR IEDGE    A 703    76500000
    1 HAS BEEN EXCEED.   CHANGE XK AND/OR XEND AND HPR)                    A 704    76600000
 108 FORMAT (53H     XK=1.0  -   XK MUST BE > 1.0 FOR TURBULENT FLOWS)     A 705    76700000
 109 FORMAT (1X,26HFP(I) NEVER EQ OR GT .9999)                             A 706    76800000
 110 FORMAT (/23H YOU DID IT AGAIN - TP(,I3,13H) IS NEGATIVE/3H S=,F10.    A 707    76900000
    14,4H TP(,I3,2H)=,F10.5/)                                              A 708    77000000
 111 FORMAT (//1X,2HX=F14.4,9H  PROFILE/)                                  A 709    77100000
 112 FORMAT (130H        ETA          Y/YE            V    GRAD(U/UE)  GRAD A 710    77200000
    1(T/TE)         FC1          DAMP           EP1            EP2           EP A 711    77300000
    2     MIXDEL/)                                                         A 712    77400000
 113 FORMAT (11E12.3)                                                      A 713    77500000
 114 FORMAT (//4X,3HETA,8X,4HY/YE,7X,4HU/UE,7X,4HT/TE,6X,6HTT/TTE,5X,6H    A 714    77600000
    1CROCCO,5X,6HPT/PTR,6X,4HM/ME,8X,2HFZ,9X,2HTZ,6X,7HVORTREY,6X,5HXLM    A 715    77700000
    211/)                                                                  A 716    77800000
 115 FORMAT (12E11.3)                                                      A 717    77900000
 116 FORMAT (//4X,3HETA,8X,4HY/YE,7X,4HU/UE,7X,4HT/TE,6X,6HTT/TTE,5X,6H    A 718    78000000
    1CROCCO,5X,6HPT/PTR,6X,4HM/ME,7X,5HYPLUS,6X,5HUPLUS,6X,4HUDEF,6X,6H    A 719    78100000
    2VISEFF/)                                                              A 720    78200000
 117 FORMAT (/2X,7HX =     ,E12.5,2X,7HRETHET=,F12.5,2X,7HDPEDX =,F12.5,   A 721    78300000
    12X,7HCFW =   ,E12.5,2X,7HZSHK = ,E12.5,2X,7HYE =    ,E12.5,/2X,7HXI   A 722    78400000
    2=    ,E12.5,2X,7HREX =  ,E12.5,2X,7HDTEDX =,E12.5,2X,7HQSD =  ,E12.   A 723    78500000
    35,2X,7HRSHK = ,E12.5,2X,7HUTAU = ,E12.5,/2X,7HRAD =   ,E12.5,2X,7HP   A 724    78600000
    4E =    ,E12.5,2X,7HDUEDX =,E12.5,2X,7HHD =    ,E12.5,2X,7HITRO = ,I1  A 725    78700000
    52,2X,7HPTR =   ,E12.5,/2X,7HZ =     ,E12.5,2X,7HTE =    ,E12.5,2X,7HD A 726    78800000
    6LTAST=,E12.5,2X,7HNSTE = ,E12.5,2X,7HTW/TT =,E12.5,2X,7HYMP =   ,I1   A 727    78900000
    72,/2X,7HBETA = ,F12.5,2X,7HRE =    ,E12.5,2X,7HTHETA =,E12.5,2X,7HN   A 728    79000000
    8STW = ,E12.5,2X,7HRFTRUE=,E12.5,2X,7HP20 =   ,E12.5)                  A 729    79100000
 118 FORMAT (2X,7HTRFCT =,E12.5,2X,7HUE =    ,E12.5,2X,7HD/T =   ,E12.5,2  A 730    79200000
    1X,7HNUE =   ,E12.5,2X,7HROUSE =,E12.5,2X,7HOMEGA =,E12.5,/2X,7HRVWA   A 731    79300000
    2LD=,E12.5,2X,7HME =    ,E12.5,2X,7HTAUD = ,E12.5,2X,7HNUW =   ,E12.5  A 732    79400000
    3,2X,7HDSMXO =,E12.5/2X,7HREDELT=,E12.5,2X,7HMUE =   ,E12.5,2X,7HCFE   A 733    79500000
    4 =   ,E12.5,2X,7HSWANG =,E12.5,2X,7HXD =     ,E12.5)                  A 734    79600000
     END                                                                   A 735-   79700000
```

Subroutine TURBLNT.- Subroutine TURBLNT calculates the eddy viscosity, its derivatives, and the intermittency distributions required for the solution of transitional and turbulent flows. The flow diagram for subroutine TURBLNT is as follows:




1
Calculate turbulent Prandtl number
KODPRT = ?
= 1
= 2
= 3
Use constant Prandtl number
Calculate Prandtl number from Rotta distribution
Calculate Prandtl number from tabular input
Return

The program listing for subroutine TURBLNT is as follows:

```
      SUBROUTINE TURBLNT (TP,XLM11,FZ,XN,RATOP,DY,EP,FP,EPP,EETM,EEPM,VA
     1RA,VARB,VARC,VARD,VARE,JK)
      DIMENSION VARA(JK), VARB(JK), VARC(JK), VARD(JK), VARE(JK)
      DIMENSION TP(JK), XLM11(JK), FZ(JK), XN(JK), RATOP(JK), DY(JK)
      DIMENSION EP(JK), FP(JK), EPP(JK)
      DIMENSION EETM(JK), EEPM(JK)
      DIMENSION PRTAR(100), GLAR(100)
      INTEGER ETAEDG
      COMMON /TRBULNT/ S,KSTR,TLNGTH,CORP,TRFACT,TBAR,XLBAR,DISINC,XT1,X
     1T2,XT3,XT4,XT5,RE,UE,XNUE,J,RMI,EPS,JPOINT,IEDGE,WW1,WW2,WW3,WW4,W
     2W5,N,ETAEDG,KODVIS,A,XBE,X,PR,CONSTNT,KODPRT,PRT,PRTAR,GLAR,NUMB1
      IF (KSTR.EQ.0) GO TO 1
      GO TO 2
 1    KSTR=1
      STR=S
      XLAMDA=STR*(TLNGTH-1.)/(SQRT((ALOG(50.))/CORP))
 2    CONTINUE

*          CALCULATE STREAMWISE INTERMITANCY

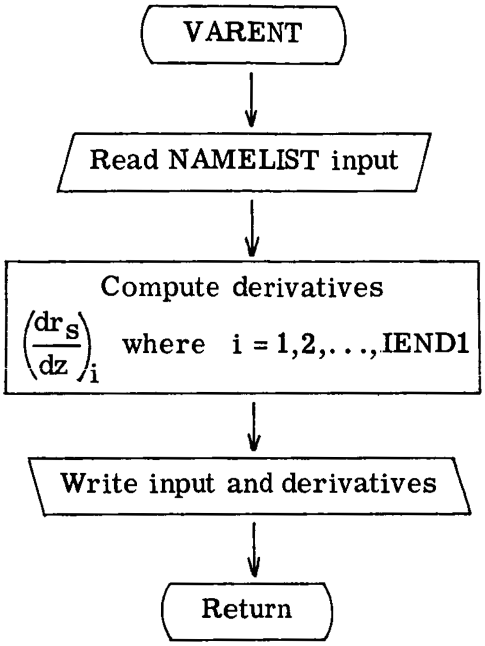
      TRFACT=1.-EXP(-1.*CORP*(((S-STR)/XLAMDA)**2))

*          CALCULATE EDDY-VISCOSITY
*                2 LAYER MODEL

      IFC=0
      DO 9 N=2,IEDGE
      YINTER=.5*(1.-ERF(5.*((XN(N)/XN(ETAEDG))-XT4)))
      IF (IFC.EQ.1) GO TO 3
      XMF=RE*RE*UE*UE*((RMI*RATOP(N))**J)*ABS(FZ(N))
      XMF=XMF/(((EPS*TP(N))**3)*A*A*XNUE*SQRT(2.*X)*XLM11(N))
      FC1=XMF
      FC2=(SQRT(XLM11(1)/XLBAR)*(TP(1)/TBAR))*(1.+XT5)-XT5
      FC3=EPS*A*XN(N)
      XMF=ABS((XLM11(1)*FZ(1))-CONSTNT*(RE*UE*(RMI**J)*XBE*XN(N)/SQRT(2.
     1*X)))
      XMF=XMF*RE*RE*UE*UE*(RMI**J)/(XNUE*EPS*SQRT(2.*X))
      XMF=XMF/((XLM11(N)**2)*(TP(N)**3))
      XMF=SQRT(XMF)/(XT2*A*EPS)
      FC4=XMF
      DAMP=1.-EXP(-FC2*FC3*FC4)
      IF (KODVIS.EQ.2) GO TO 4
      VARB(N)=DAMP
      XMIXL=XT1*FC3*DAMP
      EP1=1.+TRFACT*FC1*(XMIXL**2)*YINTER
      VARC(N)=EP1
 3    XMF=XT3*RE*UE*DISINC*YINTER/(((EPS*TP(N))**2)*XNUE*XLM11(N))
      FC1=RE*RE*UE*UE*((RMI*RATOP(N))**J)*ABS(FZ(N))
      FC1=FC1/(((EPS*TP(N))**3)*A*A*XNUE*SQRT(2.*X)*XLM11(N))
      VARA(N)=FC1
      EP2=1.+TRFACT*XMF
```

```
      VARD(N)=EP2                                                   F  48     97500000
      IF (IFC.EQ.1) GO TO 8                                         F  49     97600000
      IF (EP1.LE.EP2.AND.IFC.EQ.0) GO TO 7                          F  50     97700000
      IFC=1                                                         F  51     97800000
      JPCINT=N                                                      F  52     97900000
      GO TO 8                                                       F  53     98000000
                                                                              98100000
*              MIXING LENGTH MODEL                                  F  54     98200000
                                                                              98300000
    4 XMIXTP=0.2*XN(ETAEDG)                                         F  55     98400000
      IF (XN(N).GT.XMIXTP) GO TO 5                                  F  56     98500000
      XMIXL=0.4*EPS*A*XN(N)                                         F  57     98600000
      GO TO 6                                                       F  58     98700000
    5 XMIXL=0.08*EPS*A*XN(ETAEDG)                                   F  59     98800000
    6 EP(N)=1.+TRFACT*FC1*((XMIXL*DAMP)**2)*YINTER                  F  60     98900000
      GO TO 9                                                       F  61     99000000
    7 EP(N)=EP1                                                     F  62     99100000
      GO TO 9                                                       F  63     99200000
    8 EP(N)=EP2                                                     F  64     99300000
    9 CONTINUE                                                      F  65     99400000
                                                                              99500000
*          OBTAIN THE THREE POINT MEAN AND                          F  66     99600000
*          THE DERIVATIVE OF EDDY-VISCOSITY                         F  67     99700000
                                                                              99800000
      DO 10 N=1,IEDGE                                               F  68     99900000
      EP(N)=(EP(N)+EP(N+1)+EP(N+2))/3.                              F  69    100000000
      IF (EP(N).GE..9999) EP(N)=1.                                  F  70    100100000
      IF (N.EQ.1.OR.N.EQ.2) GO TO 10                                F  71    100200000
      EPP(N-1)=(EP(N)-EP(N-2))/(DY(N-1)+DY(N-2))                    F  72    100300000
   10 CONTINUE                                                      F  73    100400000
                                                                             100500000
*         CALCULATE TURBULENT PRANDTL NUMBER                        F  74    100600000
                                                                             100700000
      DO 14 N=2,IEDGE                                               F  75    100800000
      GO TO (13,11,12), KODPRT                                      F  76    100900000
   11 PRT=0.95-0.45*((XN(N)/XN(ETAEDG))**2)                         F  77    101000000
      GO TO 13                                                      F  78    101100000
   12 GL=XN(N)/XN(ETAEDG)                                           F  79    101200000
      CALL FTLUP (GL,PRT,1,NUMB1,GLAR,PRTAR)                        F  80    101300000
   13 EETM(N)=(PRT+(EP(N)-1.)*PR)/(PR*PRT)                          F  81    101400000
      EETM(N)=(PRT+(EP(N)-1.)*PR)/(PR*PRT)                          F  82    101500000
   14 CONTINUE                                                      F  83    101600000
      DO 15 N=2,IEDGE                                               F  84    101700000
   15 EEPM(N)=(EETM(N+1)-EETM(N-1))/(DY(N)+DY(N-1))                 F  85    101800000
      EEPM(1)=(-WW1*EETM(1)+WW2*EETM(2)-WW3*EETM(3)+WW4*EETM(4))    F  86    101900000
      EEPM(1)=EEPM(1)/(WW5*DY(1))                                   F  87    102000000
      XFFX=EPS*XN(ETAEDG)                                           F  88    102100000
      DO 16 N=2,IEDGE                                               F  89    102200000
      IF (EP(N).LT.1.0) EP(N)=1.0                                   F  90    102300000
      IF (VARA(N).EQ.0.0) VARA(N)=1.0                               F  91    102400000
   16 VARE(N)=SQRT((EP(N)-1.)/VARA(N))/XFFX                         F  92    102500000
      RETURN                                                        F  93    102600000
      END                                                           F  94-   102700000
```

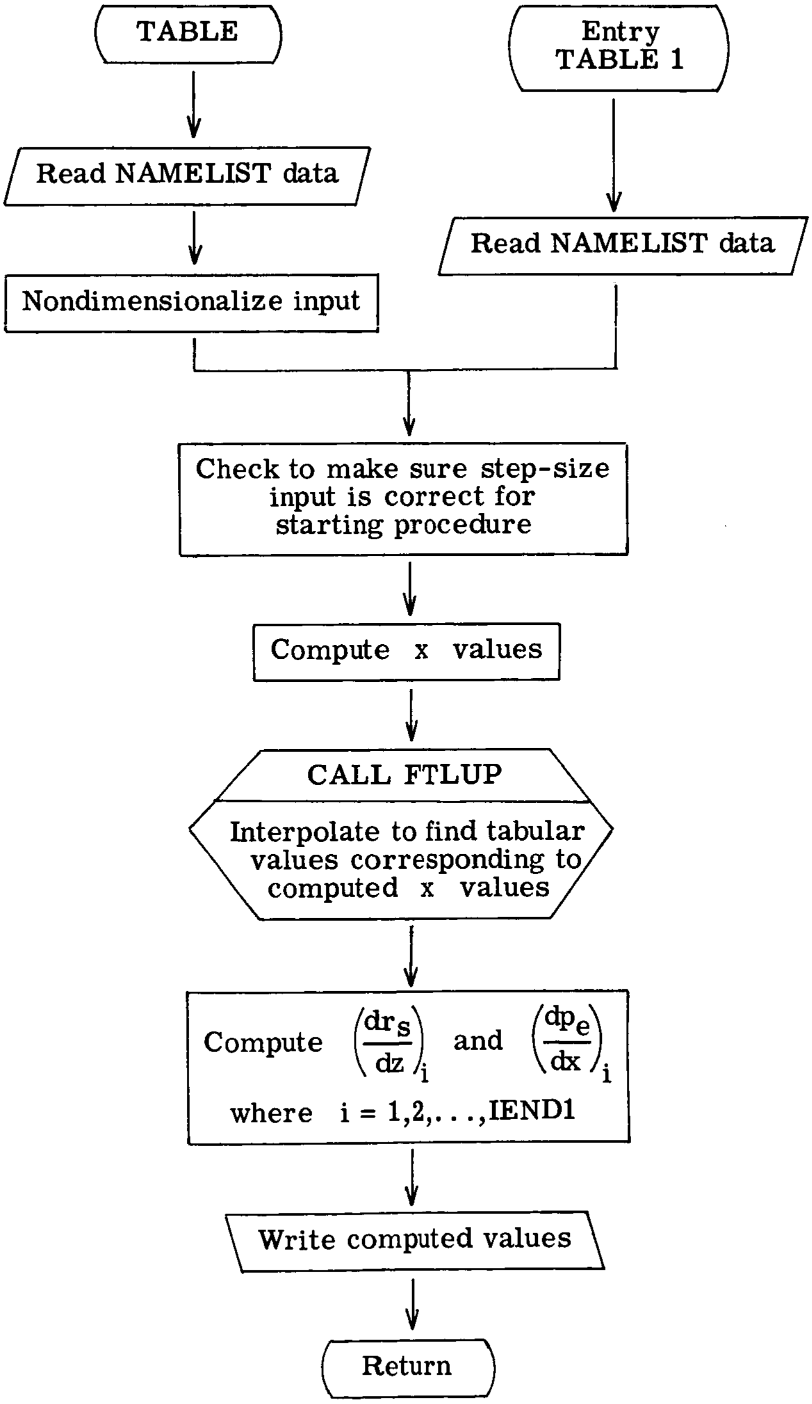
Subroutine VARENT.- Subroutine VARENT reads the variable-entropy input in tabular form, computes dr_s/dz, and then writes the input and the derivatives. The flow diagram for subroutine VARENT is as follows:



The program listing for subroutine VARENT is as follows:

```
      SUBROUTINE VARENT (RRS,ZZS,DRSDZS,NUMBER)                         E   1   91100000
      DIMENSION RRS(100), ZZS(100), DRSDZS(100)                         E   2   91200000
      NAMELIST /NAM4/ NUMBER,RRS,ZZS,DRSDZS                             E   3   91300000
      READ (5,NAM4)                                                     E   4   91400000
      NUMM1=NUMBER-1                                                    E   5   91500000
      DO 1 I=2,NUMM1                                                    E   6   91600000
    1 DRSDZS(I)=(RRS(I+1)-RRS(I-1))/(ZZS(I+1)-ZZS(I-1))                 E   7   91700000
      DRSDZS(1)=(RRS(2)-RRS(1))/(ZZS(2)-ZZS(1))                         E   8   91800000
      DRSDZS(NUMBER)=(RRS(NUMBER)-RRS(NUMBER-1))/(ZZS(NUMBER)-ZZS(NUMBER E   9   91900000
     1-1))                                                              E  10   92000000
      WRITE (6,NAM4)                                                    E  11   92100000
      RETURN                                                            E  12   92200000
      END                                                               E  13-  92300000
```

Subroutine TABLE.- Subroutine TABLE reads tabular input for body geometry, nondimensionalizes the input if necessary, distributes the values according to specified steps, and computes the derivatives. The flow diagram for subroutine TABLE is as follows:



The program listing for subroutine TABLE is as follows:

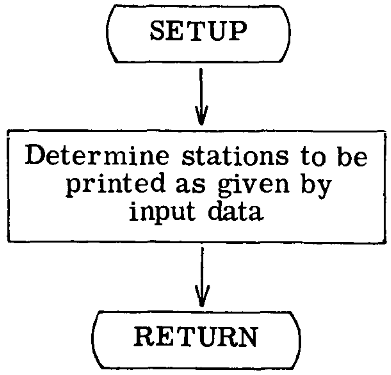
```
      SUBROUTINE TABLE (IEND1,DS,R1,U1,A,TREF,KODWAL,VISREF,KODUNIT)
      DIMENSION PE(100), Z(100), RMI(100), TW(100), S(100), RVWALD(100),
     1 QW(100), PD(1000), ZED(1000), RMIDD(1000), SS(1000)
      NAMELIST /NAM3/ NUMBER,L,PE,Z,RMI,TW,S,RVWALD,SS,QW
      DATA SS(2)/0./,L/1/
      DATA (RVWALD(I),I=1,100)/100*0./
      DATA (RMI(I),I=1,100)/100*0./
      DATA C1/.0208854346/,C2/1.8/,C5/3.280839895/,C15/.0063658804/,C16/
     1.0000881/
      READ (5,NAM3)
      IF (ENDFILE 5) 1,2
    1 STOP 5
    2 WRITE (6,NAM3)
      IF (KODUNIT.NE.1) GO TO 6

*         CONVERT $NAM3  INPUT DATA TO U.S.STANDARD UNITS

      DO 3 I=1,IEND1
    3 SS(I)=SS(I)*C5
      DO 5 I=1,NUMBER
      S(I)=S(I)*C5
      Z(I)=Z(I)*C5
      RMI(I)=RMI(I)*C5
      PE(I)=PE(I)*C1
      RVWALD(I)=RVWALD(I)*C15
      IF (KODWAL.NE.1) GO TO 4
      TW(I)=TW(I)*C2
      GO TO 5
    4 QW(I)=QW(I)*C16
    5 CONTINUE
    6 DO 8 I=1,NUMBER
      PE(I)=PE(I)/(R1*U1*U1)
      S(I)=S(I)/A
      RMI(I)=RMI(I)/A
      IF (KODWAL.NE.1) GO TO 7
      TW(I)=TW(I)/TREF
      GO TO 8
    7 QW(I)=QW(I)*778.26*A/(VISREF*U1*U1)
    8 Z(I)=Z(I)/A
      GO TO 10
      ENTRY TABLE1
      READ (5,NAM3)
      WRITE (6,NAM3)
      IF (ENDFILE 5) 9,10
    9 STOP 14
   10 CONTINUE
      IF (SS(1).GT.DS+.000001.OR.SS(1).LT.DS-.000001) GO TO 11
```

```
      IF (SS(2).GT.DS+.000001.OR.SS(2).LT.DS-.000001) GO TO 11            D  46    86500000
      IF (SS(3).GT.DS+.000001.OR.SS(3).LT.DS-.000001) GO TO 11            D  47    86600000
      GO TO 12                                                            D  48    86700000
   11 WRITE (6,20) SS(1),SS(2),SS(3),DS                                   D  49    86800000
      STOP 77                                                             D  50    86900000
   12 CONTINUE                                                            D  51    87000000
      TEMP=0.                                                             D  52    87100000
      DO 13 I=1,IEND1                                                     D  53    87200000
      SS(I)=TEMP+SS(I)                                                    D  54    87300000
   13 TEMP=SS(I)                                                          D  55    87400000
      DO 14 I=1,IEND1                                                     D  56    87500000
      SD=DS*I                                                             D  57    87600000
      IF (SS(2).NE.0.) SD=SS(I)                                           D  58    87700000
      CALL FTLUP (SD,ZED(I),L,NUMBER,S,Z)                                 D  59    87800000
      CALL FTLUP (SD,RMIDD(I),L,NUMBER,S,RMI)                             D  60    87900000
   14 CALL FTLUP (SD,PD(I),L,NUMBER,S,PE)                                 D  61    88000000
      TWODS=2.*DS                                                         D  62    88100000
      DO 19 I=2,IEND1                                                     D  63    88200000
      SD=DS*I                                                             D  64    88300000
      IF (SS(2).NE.0.) SD=SS(I)                                           D  65    88400000
      IF (KCDWAL.EQ.1) GO TO 15                                           D  66    88500000
      CALL FTLUP (SD,QWD,L,NUMBER,S,QW)                                   D  67    88600000
      GO TO 16                                                            D  68    88700000
   15 CONTINUE                                                            D  69    88800000
      CALL FTLUP (SD,TWD,L,NUMBER,S,TW)                                   D  70    88900000
   16 CONTINUE                                                            D  71    89000000
      CALL FTLUP (SD,RVWALDD,L,NUMBER,S,RVWALD)                           D  72    89100000
      IF (I.EQ.IEND1) GO TO 17                                            D  73    89200000
      DRDZ=(RMIDD(I+1)-RMIDD(I-1))/(ZED(I+1)-ZED(I-1))                    D  74    89300000
      IF (SS(2).NE.0.) TWODS=SS(I+1)-SS(I-1)                              D  75    89400000
      DPEDSD=(PD(I+1)-PD(I-1))/TWODS                                      D  76    89500000
      GO TO 18                                                            D  77    89600000
   17 IF (SS(2).NE.0.) DDS=SS(I)-SS(I-1)                                  D  78    89700000
      DPEDSD=(PD(I)-PD(I-1))/DDS                                          D  79    89800000
      DRDZ=(RMIDD(I)-RMIDD(I-1))/(ZED(I)-ZED(I-1))                        D  80    89900000
   18 WRITE (4) SD,PD(I),RMIDD(I),TWD,ZED(I),DPEDSD,RVWALDD,DRDZ,QWD      D  81    90000000
   19 CONTINUE                                                            D  82    90100000
      REWIND 4                                                            D  83    90200000
      RETURN                                                              D  84    90300000
                                                                                   90400000
*                                                                         D  85    90500000
                                                                                   90600000
   20 FORMAT (1X,7HSS(1) =,F9.4,/1X,7HSS(2) =,F9.4,/1X,7HSS(3) =,F9.4,/1  D  86    90700000
     1X,7HDS    =,F9.4,/1X,53HTHESE VALUES MUST BE EQUAL FOR THE STARTIN  D  87    90800000
     2G PROCEDURE)                                                        D  88    90900000
      END                                                                 D  89-   91000000
```

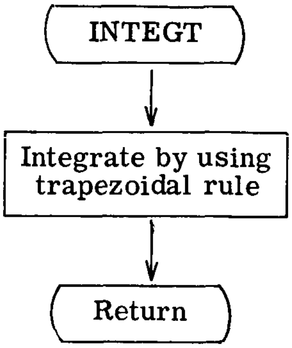
Subroutine SETUP.- Subroutine SETUP determines from the input where profiles and wall values are to be printed. The flow diagram for subroutine SETUP is as follows:



The program listing for subroutine SETUP is as follows:

```
      SUBROUTINE SETUP (A,B,C,J,K)                                  C  1   80700000
      DIMENSION B(J)                                                C  2   80800000
      IF (A.EQ.0) RETURN                                            C  3   80900000
      KPLUS2=K+2                                                    C  4   81000000
      B(K+1)=A                                                      C  5   81100000
      DO 1 I=KPLUS2,J                                               C  6   81200000
      B(I)=B(I-1)+A                                                 C  7   81300000
      IF (B(I).GE.C) RETURN                                         C  8   81400000
    1 CONTINUE                                                      C  9   81500000
      RETURN                                                        C 10   81600000
      END                                                           C 11-  81700000
```

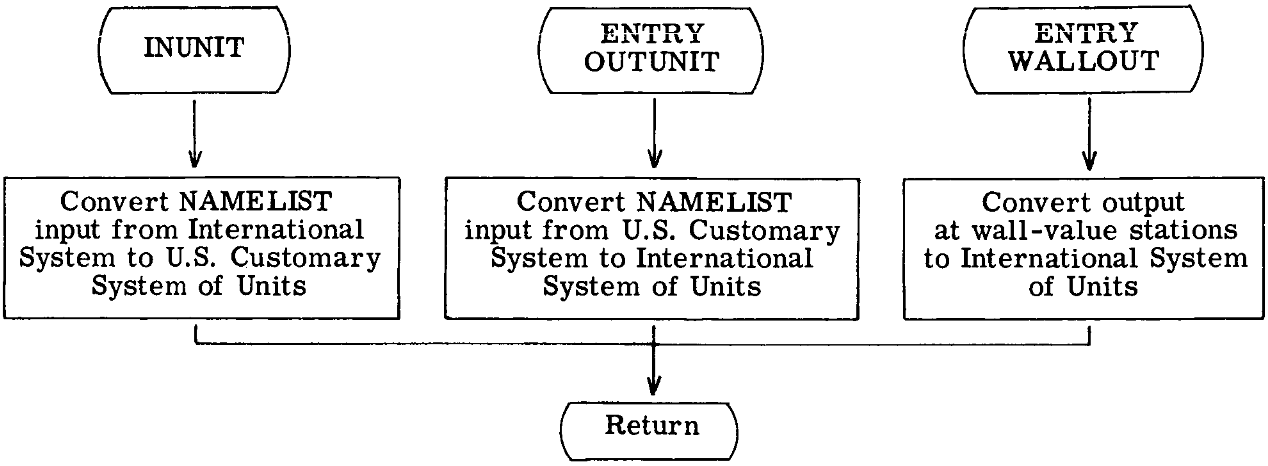
Function INTEGT.- Function subroutine INTEGT integrates by using the trapezoidal rule. The flow diagram for function INTEGT is as follows:



The program listing for function INTEGT is as follows:

```
      FUNCTION INTEGT (YY,NOPTS,DX)                                B  1   79800000
      DIMENSION YY(201), DX(201)                                   B  2   79900000
      REAL INTEGT                                                  B  3   80000000
      INTEGT=0.                                                    B  4   80100000
      IF (NOPTS.LT.2) GO TO 2                                      B  5   80200000
      DO 1 N=2,NOPTS                                               B  6   80300000
  1   INTEGT=INTEGT+(DX(N-1)/2.)*(YY(N-1)+YY(N))                   B  7   80400000
  2   RETURN                                                       B  8   80500000
      END                                                          B  9-  80600000
```

Subroutine INUNIT.- Subroutine INUNIT converts International System dimensional-input data to the U.S. Customary System of Units for calculations in the program. The subroutine then converts the data back to the International System before output. The flow diagram for subroutine INUNIT is as follows:



The program listing for subroutine INUNIT is as follows:

```
      SUBROUTINE INUNIT (PROVAL,PRNTVAL,JM,JN)                          G   1  102800000
      COMMON /TRBULNT/ S,KSTR,TLNGTH,CORP,TRFACT,TBAR,XLBAR,DISINC,XT1,X G   2  102900000
     1T2,XT3,XT4,XT5,PE,UE,XNUE,J,RMI,EPS,JPOINT,IEDGE,WW1,WW2,WW3,WW4,W G   3  103000000
     2W5,N,ETAEDG,KODVIS,A,XBE,X,PR,CONSTNT,KODPRT                      G   4  103100000
      COMMON /UNIT/ VISCON,PT1,TT1,WAVE,R,SU,CONE,DS,SST,RT1,P1,T1,R1,U1 G   5  103200000
     1,AA1,TREF,VISREF,PESTAR,TESTAR,PESTAR,UESTAR,MUESTAR,YESTAR,THETA, G   6  103300000
     2TAUD,QSD,HD,UPLUS,DISP,PE,Z,TW,QW,RVWALD,PROINC,PRNTINC           G   7  103400000
      DIMENSION PROVAL(JN), PRNTVAL(JM)                                 G   8  103500000
      REAL MUESTAR                                                      G   9  103600000
      DATA RT1,P1,T1,R1,U1,AA1,TREF,VISREF/8*1.0/,UPLUS/0.0/            G  10  103700000
      DATA CC1/.0208854346/,CC2/1.8/,CC3/57.2957810375/,CC4/10.76391041/ G  11  103800000
     1,CC5/3.280839895/,CC6/.0019403196/,CC7/47.880258/,CC8/.55555555/,C G  12  103900000
     2C9/.017453292/,CC10/.09290304/,CC11/.3048/,CC12/515.379/,CC13/1134 G  13  104000000
     38.93/,CC14/20428.0758/                                            G  14  104100000
                                                                               104200000
*         CONVERT $NAM2 AND $NAM VALUES TO U.S. STANDARD UNITS           G  15  104300000
                                                                               104400000
      PROINC=PROINC*CC5                                                 G  16  104500000
      PRNTINC=PRNTINC*CC5                                               G  17  104600000
      DO 1 I=1,JN                                                       G  18  104700000
    1 PROVAL(I)=PROVAL(I)*CC5                                           G  19  104800000
      DO 2 I=1,JM                                                       G  20  104900000
    2 PRNTVAL(I)=PRNTVAL(I)*CC5                                         G  21  105000000
      A=A*CC5                                                           G  22  105100000
      U1=U1*CC5                                                         G  23  105200000
      AA1=AA1*CC5                                                       G  24  105300000
      DS=DS*CC5                                                         G  25  105400000
      SST=SST*CC5                                                       G  26  105500000
      VISCON=VISCON*CC1                                                 G  27  105600000
      PT1=PT1*CC1                                                       G  28  105700000
      P1=P1*CC1                                                         G  29  105800000
      VISREF=VISREF*CC1                                                 G  30  105900000
      TREF=TREF*CC2                                                     G  31  106000000
      T1=T1*CC2                                                         G  32  106100000
      SU=SU*CC2                                                         G  33  106200000
      TT1=TT1*CC2                                                       G  34  106300000
      CONE=CONE*CC3                                                     G  35  106400000
      WAVE=WAVE*CC3                                                     G  36  106500000
      R=R*CC4                                                           G  37  106600000
      RT1=RT1*CC6                                                       G  38  106700000
      R1=R1*CC6                                                         G  39  106800000
      GO TO 5                                                           G  40  106900000
                                                                               107000000
*         CONVERT $NAM2 AND $NAM VALUES TO INTERNATIONAL STANDARD UNITS  G  41  107100000
                                                                               107200000
      ENTRY OUTUNIT                                                     G  42  107300000
      PROINC=PROINC*CC11                                                G  43  107400000
```

```
      PRNTINC=PRNTINC*CC11                                                   G  44  107500000
      DO 3 I=1,JN                                                            G  45  107600000
    3 PROVAL(I)=PROVAL(I)*CC11                                               G  46  107700000
      DO 4 I=1,JM                                                            G  47  107800000
    4 PRNTVAL(I)=PRNTVAL(I)*CC11                                             G  48  107900000
      A=A*CC11                                                               G  49  108000000
      DS=DS*CC11                                                             G  50  108100000
      SST=SST*CC11                                                           G  51  108200000
      U1=U1*CC11                                                             G  52  108300000
      AA1=AA1*CC11                                                           G  53  108400000
      VISCON=VISCON*CC7                                                      G  54  108500000
      PT1=PT1*CC7                                                            G  55  108600000
      P1=P1*CC7                                                              G  56  108700000
      VISREF=VISREF*CC7                                                      G  57  108800000
      TREF=TREF*CC8                                                          G  58  108900000
      T1=T1*CC8                                                              G  59  109000000
      TT1=TT1*CC8                                                            G  60  109100000
      SU=SU*CC8                                                              G  61  109200000
      WAVE=WAVE*CC9                                                          G  62  109300000
      CONF=CONF*CC9                                                          G  63  109400000
      R=R*CC10                                                               G  64  109500000
      RT1=RT1*CC12                                                           G  65  109600000
      R1=R1*CC12                                                             G  66  109700000
      GO TO 5                                                                G  67  109800000
                                                                                    109900000
*         CONVERT WALL VALUES TO  INTERNATIONAL STANDARD UNITS               G  68  110000000
                                                                                    110100000
      ENTRY WALLOUT                                                          G  69  110200000
      PESTAR=PESTAR*CC7                                                      G  70  110300000
      MUESTAR=MUESTAR*CC7                                                    G  71  110400000
      TAUD=TAUD*CC7                                                          G  72  110500000
      TESTAR=TESTAR*CC8                                                      G  73  110600000
      S=S*CC11                                                               G  74  110700000
      UPLUS=UPLUS*CC11                                                       G  75  110800000
      DISP=DISP*CC11                                                         G  76  110900000
      UESTAR=UESTAR*CC11                                                     G  77  111000000
      YESTAR=YESTAR*CC11                                                     G  78  111100000
      THETA=THETA*CC11                                                       G  79  111200000
      RESTAR=RESTAR*CC12                                                     G  80  111300000
      QSD=QSD*CC13                                                           G  81  111400000
      HD=HD*CC14                                                             G  82  111500000
    5 RETURN                                                                 G  83  111600000
      END                                                                    G  84- 111700000
```

USAGE

The programs are run on the Control Data 6000 Series computer under the SCOPE 3.0 operating system. The CPU time required for running all three programs is approximately 0.003 second per mesh point.

Array Dimensions

Program D2401 uses the variable-dimension capability of the preprocessor installed at the Langley Research Center to enable the user to use a minimum amount of storage for each case. If this capability is not available at the user's installation, the dimension statements at the beginning of program D2401 should be modified by inserting the following numbers in place of their equivalent designations:

JK	maximum number of steps in η-direction plus 10
JL	= 1, if case is considering only constant entropy maximum number of steps in X-direction, if case is considering variable entropy
JM	maximum number of wall-value stations to be printed
JN	maximum number of profiles to be printed

FORTRAN statements setting these values should also be inserted immediately following the NAMELIST statements in program D2401.

Intermediate Data Storage

The output for the initial solution found in program D2390 is written on TAPE 9 as well as on the output file. Program D23901 will then read the data from TAPE 9, redistribute the points geometrically, and write the redistributed solution in place of the original distribution on TAPE 9 as well as on the output file. Program D2401 will then read the redistributed solution from TAPE 9. If redistributing the points is unnecessary, executing program D23901 may be eliminated and the solution from D2390 will be read directly by program D2401. Generally, TAPE 9 will be a disk file to be used only for a current run. However, if many cases are to be run with the same initial solution, a physical tape can be requested so that D2390 and D23901 need not be rerun for each case. When using TAPE 9 as either a disk file or a tape file, it is automatically rewound at the beginning of D23901 and again at the beginning of D2401.

Input Description

Input for all programs is standard CDC NAMELIST. Program D2390 reads input listed under $NAM1 and copies these input data as well as the output data onto TAPE 9.

Program D23901 then reads these data from TAPE 9 as input. No other input is required for D23901. Program D2401 requires the input from TAPE 9 (written by either D2390 or D23901) and the data found listed under $NAM2. Subroutine TABLE (in program D2401) requires the data listed under $NAM3. If the case being considered is using variable entropy, then subroutine VARENT (in program D2401) requires the data listed under $NAM4.

Dimensional input and output may be in either the International System of Units (KODUNIT = 1) or the U.S. Customary System of Units (KODUNIT = 0). The following listing of input and output data gives the units in the International System, followed in parentheses by the units in the U.S. Customary System. Where no units are given, the data are nondimensional.

The $NAM1 input data for program D2390 are given as follows:

HPR	η, increment for which values will be printed and stored on TAPE 9 (Default = 0.1)
XEND	η_N (see fig. 2) (Default = 10.0)
H	Runge-Kutta integration increment (Default = 0.01)
PR	N_{Pr} (Default = 0.7)
XKK	S/T_e (Default = 0.0)
BETA	β (Default = 0.0)
ALPHA	α (Default = 0.0)
XO	η_0 (Default = 0.0)
YO(1)	V_w (Default = 0.0)
YO(2)	F_w (Default = 0.0)
YO(3)	$\left(\frac{\partial F}{\partial \eta}\right)_w$ (Default = 0.0)
YO(4)	Θ_w (Default = 0.0)
YO(5)	$\left(\frac{\partial \Theta}{\partial \eta}\right)_w$ (Default = 0.0)

YO(6)	$\left(\frac{\partial^2\Theta}{\partial\eta^2}\right)_w$ (Default = 0.0)
XK	k (If k = 1, then program D23901 does not change data from D2390.) (Default = 1.0)
IGAS	= 1 for air (Sutherland's viscosity constant) = 2 for power-law viscosity relation (Default = 1)
VISCON	constant in power-law viscosity relation, newton-sec/m^2 (lb-sec/ft^2)
VISPOW	σ, exponent in power-law viscosity relation
KODUNIT	= 0 if all dimensional input and output are in the U.S. Customary System of Units = 1 if all dimensional input and output are in the International System of Units (Default = 0)

The $NAM2 input data for program D2401 are given as follows:

XMA	M_∞
PT1	$p_{t,\infty}$, newton/m^2 (lb/ft^2)
TT1	$T_{t,\infty}$, $^\circ$K ($^\circ$R)
WAVE	shock-wave angle at tip of sharp body or stagnation point of blunt body (Default = 90.0), radians (degrees)
XY1	$\left(\frac{p_e}{p_\infty}\right)_{x=0}$
XY2	$\left(\frac{T_e}{T_\infty}\right)_{x=0}$
XY3	$\left(M_e\right)_{x=0}$
G	γ (Default = 1.4)
R	R_g, gas constant (Default = 1716.0), m^2/sec^2-$^\circ$K (ft^2/sec^2-$^\circ$R)

SU	S (Default = 198.6), $^{\circ}$K ($^{\circ}$R)
PR	N_{Pr} (Default = 0.72)
PRT	$N_{Pr,t}$ (Default = 0.9)
IBODY	= 1 for stagnation-point flows = 2 otherwise
J	j
W	= 0 if transverse curvature is neglected = 1 if transverse curvature is included (Default = 0)
FT	= 1.0 for nonsimilar solution = 0.0 for similar solution (Default = 1.0)
KODE	= 1 if both laminar and turbulent profile values are defined for diagnostic reasons after flow is fully turbulent = 0 otherwise (Default = 0)
KODWAL	= 1 for specified temperature distribution = 2 for specified heat-transfer distribution (Default = 1)
IENTRO	= 1 for constant entropy = 2 for variable entropy (Default = 1)
CONE	cone semiapex angle (Default = 0.0), radians (degrees)
IEND1	number of steps in X-direction
A	reference length (Default = 1.0), meters (feet)
DS	initial step length in X-direction (Default = 0.01), meters (feet)
KODVIS	= 1 for two-layer eddy-viscosity model = 2 for mixing-length model (Default = 1)
SST	x-location at which transition occurs (Default = 1.0E08), meters (feet)

SMXTR	critical vorticity Reynolds number (Default = 1.0E08)
TLNGTH	$x_{t,f}/x_{t,i}$ (Default = 2.0)
CORP	coefficient in equation (38) (Default = 0.412)
CONSTNT	transition model (Default = 0.0)
XT1	k_1 (See eq. (6).) (Default = 0.4)
XT2	A^+ (Default = 26.0)
XT3	k_2 (See eq. (7).) (Default = 0.0168)
XT4	k_4 (See eq. (10).) (Default = 0.78)
XT5	k_3 (See eq. (8).) (Default = 0.0)
PROINC	incremental x value for which profile printouts will be made = 0 if only certain specified profile printouts will be made (Default = 1.0), meters (feet)
PRNTINC	incremental x value for which wall-value printouts will be made = 0 if only certain specified wall-value stations printouts are desired (Default = 0.1), meters (feet)
IPRO	number of specified profile printouts desired (other than those determined by PROINC) (Default = 0)
PROVAL	array of IPRO specific x values for which profile printouts are desired meters (feet)
IPRNT	number of specified wall-value printouts desired (other than those determined by PRNTINC) (Default = 0)
PRNTVAL	array of IPRNT specific x values for which printouts are desired, meters (feet)

NAUXPRO	= 1 if auxiliary profile printouts are desired (see output description) ≠ 1 otherwise (Default = 0)
BLNGTH	= 0 if using constant step size in X-direction length of body if using variable step size in X-direction (Default = 0.0), meters (feet)
NPUTYPE	= 1 for dimensional input = 2 for nondimensional input (Default = 1)
KODPRT	= 1 for constant $N_{Pr,t}$ = 2 for Rotta distribution = 3 for tabular $N_{Pr,t} = f(y/\delta)$ (Default = 1)
NUMB1	number of values read into PRTAR and GLAR arrays if KODPRT = 3
PRTAR	turbulent Prandtl number array, used only if KODPRT = 3 (NUMB1 values)
GLAR	y/δ array corresponding to PRTAR, used only if KODPRT = 3 (NUMB1 values)
KTCOD	= 1 if transition extent is calculated from equation (37) = 2 if transition extent is read in as TLNGTH (Default = 2)

The $NAM3 input data for program D2401 are given as follows:

NUMBER	number of values read into $NAM3 tables (Maximum = 100)
L	order of interpolation to be used for $NAM3 table (Default = 1)
PE	pressure-distribution array (NUMBER values), newton/m^2 (lb/ft^2)
Z	axial-coordinate array (NUMBER values), meters (feet)
RMI	body radial-coordinate array (NUMBER values) (Default = 1.0), meters (feet)
TW	wall temperature-distribution array (NUMBER values), $^{\circ}$K ($^{\circ}$R)

QW	wall heat-transfer-distribution array (NUMBER values), watts/m^2 (Btu/ft^2-sec)
RVWALD	mass flux at wall array, V_w (NUMBER values) (Default = 0.0), newton-sec/m^3 (lb-sec/ft^3)
S	x-station array corresponding to above table inputs (NUMBER values), meters (feet)
SS	array of incremental values between adjacent x stations for computation (Maximum = 1000), meters (feet) The first three values for SS must equal DS for the starting procedure; that is, SS(1) = SS(2) = SS(3) = DS.

The $NAM4 input data for program D2401 are given as follows:

NUMBER	number of values read into $NAM4 tables (Maximum = 100)
RRS	array of radial coordinates of shock wave (NUMBER values)
ZZS	array of axial coordinates of shock wave (NUMBER values)

Output Description

The output for programs D2390 and D23901 consists of printing and the intermediate data on TAPE 9 as discussed earlier in this section. In program D2390, the $NAM1 input data are printed, followed by the initial profile consisting of the following values:

Initial profile

ETA = η

YO(1) = V

YO(2) = F

YO(4) = Θ

YO(5) = $\dfrac{\partial\Theta}{\partial\eta}$

YO(6) = $\dfrac{\partial^2\Theta}{\partial\eta^2}$

In program D23901 the output is printed in a form identical to that in D2390 except that the profile is redistributed. This same output is repeated in program D2401 for convenience. Next, the $NAM3 input data are printed. If the particular case is considering variable entropy, this is followed by the $NAM4 input data.

Next, the $NAM2 input data and the $NAM constants which consist of the following values are given:

P10	$\left(\frac{p_{t,e}}{\rho_r u_r^2}\right)_{x=0}$
T10	$\frac{T_{t,\infty}}{T_r}$
G	γ, ratio of specific heats
REY	$\frac{\rho_\infty u_\infty A}{\mu_\infty}$ free-stream Reynolds number
RT1	$\rho_{t,\infty}$, kilogram/m^3 (slug/ft^3)
P1	p_∞, free-stream pressure, newton/m^2 (lb/ft^2)
T1	T_∞, free-stream temperature, $^{\circ}$K ($^{\circ}$R)
R1	ρ_∞, free-stream density, kilogram/m^3 (slug/ft^3)
U1	u_∞, free-stream velocity, m/sec (ft/sec)
AA1	a_∞, free-stream speed of sound, m/sec (ft/sec)
TREF	T_r, reference temperature, $^{\circ}$K ($^{\circ}$R)
VISREF	ρ_r, reference viscosity, newton-sec/m^2 (lb-sec/ft^2)
R10	$\frac{\rho_{t,\infty}}{\rho_\infty}$

Next, the profile values are printed according to the specifications in the input. These consist of the following:

Laminar-profile values

ETA	η
Y/YE	y/y_e
U/UE	F
T/TE	Θ
TT/TTE	$T_t/T_{t,e}$
CROCCO	$\frac{T_t - T_w}{T_{t,e} - T_w}$
PT/PTR	$p_t/p_{t,r}$, total pressure ratio
M/ME	M/M_e, Mach number ratio
FZ	$\left(\frac{\partial F}{\partial \eta}\right)_{m+1,n}$
TZ	$\left(\frac{\partial \Theta}{\partial \eta}\right)_{m+1,n}$
VORTREY	$(\chi)_{m+1,n}$, vorticity Reynolds number
XLM11	$\frac{(\rho\mu)_{m+1,n}}{(\rho\mu)_e}$

Additional values for transitional and turbulent profiles

YPLUS	$\frac{yu_\tau}{\nu}$
UPLUS	$\frac{u}{u_\tau}$
UDEF	$\frac{u_e - u}{u_\tau}$
VISEFF	$1 + \frac{\epsilon}{\mu}\Gamma$, effective viscosity parameter

Auxiliary-profile values

V	V (see eq. (27))
GRAD(U/UE)	FZ
GRAD(T/TE)	TZ
FC1	$\left(\frac{\rho}{\mu}\left\lvert\frac{\partial u}{\partial y}\right\rvert\right)_{m+1,n}$
DAMP	$\left(1 - \exp\left\{-\left[\sqrt{\frac{\nu_w}{\bar{\nu}}}\left(1 + k_3\right) - k_3\right]\frac{y}{A}\right\}\right)_{m+1,n}$
EP1	$\left[\left(\frac{\epsilon}{\mu}\right)_i\right]_{m+1,n}$ (see eq. (6))
EP2	$\left[\left(\frac{\epsilon}{\mu}\right)_o\right]_{m+1,n}$ (see eq. (7))
EP	$\left(\frac{\epsilon}{\mu}\right)_{m+1,n}$
MIXDEL	$\left(\frac{l}{\delta}\right)_{m+1,n}$

Next, the wall-value stations are printed according to the specifications in the input. These consist of the following:

X	spatial coordinate, meters (feet)
XI	ξ
RAD	r_0, body radius (see fig. 1)
Z	axial coordinate of body (see fig. 1)
BETA	β, pressure-gradient parameter (see eqs. (30))
TRFCT	Γ, intermittency distribution (see eq. (38))

RVWALD	dimensional mass flux at wall (see eq. (35))
REDELT	$\frac{\rho_e u_e \delta^*}{\mu_e}$, Reynolds number based on local displacement thickness
RETHET	$\frac{\rho_e u_e \theta}{\mu_e}$
REX	$\frac{\rho_e u_e x}{\mu_e}$, local Reynolds number
PE	$\frac{p_e}{\rho_r u_r^2}$, edge pressure, newton/m^2 (lb/ft^2)
TE	$\frac{T_e}{T_r}$, edge temperature, $^{\circ}$K ($^{\circ}$R)
RE	$\frac{\rho_e}{\rho_r}$, edge density, kilogram/m^3 (slug/ft^3)
UE	$\frac{u_e}{u_r}$, edge velocity, m/sec (ft/sec)
ME	M_e, edge Mach number
MUE	$\frac{\mu_e}{\mu_r}$, newton-sec/m^2 (lb-sec/ft^2)
DPEDX	$\frac{\partial p_e}{\partial x}$, pressure gradient
DTEDX	$\frac{\partial T_e}{\partial x}$, temperature gradient
DUEDX	$\frac{\partial u_e}{\partial x}$, velocity gradient
DLTAST	δ^*, displacement thickness, meters (feet)
THETA	θ, momentum thickness, meters (feet)
D/T	$\frac{\delta^*}{\theta}$, shape factor
TAUD	τ_w, wall shear stress, newton/m^2 (lb/ft^2)
CFE	$\frac{\tau_w}{\frac{1}{2}\rho_e u_e^2}$, skin-friction coefficient based on edge condition

CFW	$\frac{\tau_w}{\frac{1}{2}\rho_w u_e^2}$, skin-friction coefficient based on wall density
QSD	heat transfer, watt/m^2 (Btu/ft^2-sec)
HD	$\frac{q_w}{T_w - T_{aw}}$, heat-transfer coefficient, watt/m^2-^{o}K (Btu/ft^2-sec-^{o}R)
NSTE	$\frac{h}{c_p(\rho u)_e}$, Stanton number based on edge condition
NSTW	$\frac{h}{c_p \rho_w u_e}$, Stanton number based on wall condition
NUE	Nusselt number based on edge condition
NUW	Nusselt number based on wall condition
SWANG	local shock-wave angle, degrees
ZSHK	axial coordinate of shock wave
RSHK	local radius of shock wave
ITRO	number of iterations performed for variable entropy
TW/TT	$\frac{\tau_w}{T_{t,\infty}}$
RFTRUE	$\frac{T_{aw} - T_e}{T_t - T_e}$, recovery factor
ROUSE	χ_{max}
DSMXO	$\left(\frac{\partial \chi}{\partial x}\right)_m$
XD	$y\sqrt{\frac{\rho_r u_r L}{\mu_r}}$
YE	δ_e, boundary-layer thickness, meters (feet)
UTAU	$u_\tau = \sqrt{\frac{T_w}{\rho}}$, m/sec (ft/sec)

PTR	reference total pressure
YMP	y_m
P20	$\frac{\rho_{t,e}}{\rho_r u_r^2}$
OMEGA	$\left(\frac{\rho_r u_r L}{\mu_r}\right)^{-1/2}$

Sample Cases

Two sample cases are presented in order to illustrate the input and output quantities in relation to the test conditions of the particular case being considered. These cases include laminar flow over a blunt axisymmetric body and laminar, transitional, and turbulent flow over a flat plate.

Case 1.- An example of laminar flow over a blunt, axisymmetric body is given in reference 21. The body is a spherically blunted, 25° half-angle cone. The wind-tunnel test conditions are as follows:

$$M_\infty = 7.95$$

$$p_{t,\infty} = 6.31 \times 10^6 \text{ N/m}^2$$

$$T_{t,\infty} = 7.83 \times 10^2 \text{ }^\circ\text{K}$$

$$\frac{T_w}{T_{t,\infty}} = 0.38$$

The wall-boundary pressure distribution used in the numerical solution was obtained by the technique presented in reference 22. (See ref. 7 for comparison of numerical results with experimental data.) This case requires 75000_8 storage on the CDC 6000 computers. The listing of the variable-dimension data for this particular case is as follows:

JK = 110

JL = 1

JM = 120

JN = 20

The listing of the input data for case 1 is given as follows:

```
$NAM1
HPR=0.1,
XEND=10.0,
H=0.01,
PR=0.72,
XKK=0.14085,
BETA=0.5,
ALPHA=0.0,
X0=0.0,
Y0(1)=0.0,0.0,0.77,0.38,0.28,0.0,
XK=1.0,
IGAS=1,
VISCON=0.7173E-08,
VISPOW=0.647,
KODUNIT=0,
$



$NAM2
XMA=7.95,
PT1=131760.0,
TT1=1410.0,
WAVE=90.0,
G=1.4,
R=1716.0,
SU=198.6,
PR=0.72,
PRT=0.9,
IBODY=1,
J=1,
W=0,
FT=1.0,
KODE=0,
KODWAL=1,
IENTRO=1,
CONE=25.0,
IEND1=103,
A=1.0,
DS=0.0005,
KODVIS=1,
SST=0.1E+09,
SMXTR=0.1E+09,
TLNGTH=2.0,
CORP=0.412,
CONSTNT=0.0,
XT1=0.4,
XT2=26.0,
XT3=0.0168,
XT4=0.78,
XT5=0.0,
PROINC=0.1,
PRNTINC=.005,
IPRO=1,
PROVAL(1)=0.345,
IPRNT=1,
PRNTVAL(1)=0.345,
NAUXPRO=0,
BLNGTH=0.0,
NPUTYPE=1,
KODPRT=2,
KTCOO=2,
$
```

```
$NAM3
NUMBER=77,
L=1,
PE(1)=1150.4,1149.43,1146.54,1141.72,1134.98,1126.32,1115.76,1103.29,
        1088.94,1072.74,1054.72,1034.89,1013.3,989.997,965.019,938.401,
        910.389,880.855,849.844,817.414,783.869,749.276,713.361,676.169,
        638.401,599.196,559.265,517.655,492.393,475.551,456.972,439.014,
        421.517,404.537,388.006,371.935,356.335,341.208,326.529,312.31,
        298.539,285.218,272.305,259.829,247.761,236.107,231.92,231.678,
        231.161,229.78,228.924,227.986,226.962,225.869,224.718,223.511,
        222.252,220.957,219.634,218.288,216.896,211.374,207.75,205.714,
        202.654,201.503,209.729,223.925,235.348,247.107,255.308,261.082,
        264.994,264.131,262.866,262.141,262.049,
Z(1)=0.0,0.00001853,0.00006226,0.0001354,0.0002381,0.0003708,0.0005337,
       0.0007273,0.000952,0.001208,0.001497,0.001819,0.002175,0.002565,
       0.002991,0.003455,0.003957,0.004498,0.005082,0.005708,0.00638,
       0.007101,0.007873,0.008699,0.009584,0.01053,0.01155,0.01265,
       0.01401,0.01456,0.01512,0.01568,0.01624,0.0168,0.01737,0.01794,
       0.01852,0.0191,0.01968,0.02026,0.02085,0.02144,0.02204,0.02263,
       0.02324,0.02384,0.02445,0.02505,0.02606,0.02813,0.02921,0.03031,
       0.03143,0.03259,0.03378,0.03501,0.03628,0.03759,0.03895,0.04036,
       0.04183,0.04843,0.0536, 0.05718,0.06457,0.08496,0.1067,0.1268,
       0.1474,0.1699,0.1895,0.21,0.2511,0.292,0.3347,0.3799,0.4203,
PMI(1)=0.0,0.001243,0.002277,0.003356,0.004448,0.005546,0.006648,
         0.007751,0.008856,0.009962,0.01107,0.01218,0.01329,0.01439,
         0.0155,0.01661,0.01772,0.01883,0.01994,0.02105,0.02216,0.02327,
         0.02437,0.02548,0.02659,0.02769,0.0288,0.0299,0.03116,0.03164,
         0.03211,0.03257,0.03301,0.03343,0.03385,0.03425,0.03464,0.03502,
         0.03539,0.03575,0.03609,0.03643,0.03675,0.03707,0.03737,0.03766,
         0.03795,0.03823,0.0387,0.03966,0.04016,0.04068,0.0412,0.04174,
         0.0423,0.04287,0.04346,0.04407,0.0447,0.04536,0.04605,0.04913,
         0.05154,0.05321,0.05665,0.06616,0.07631,0.08568,0.09528,0.1058,
         0.1149,0.1245,0.1436,0.1627,0.1826,0.2037,0.2225,
TW(1)=77*540.0,
RVWALD(1)=77*0.0,
S(1)=0.0,0.001243,0.002278,0.00336,0.004457,0.005563,0.006676,0.007797,
       0.008924,0.01006,0.0112,0.01236,0.01352,0.0147,0.01588,0.01709,
       0.01831,0.01954,0.02079,0.02207,0.02336,0.02469,0.02604,0.02742,
       0.02883,0.03029,0.03179,0.03335,0.0352,0.03594,0.03666,0.03738,
       0.0381,0.03881,0.03951,0.04021,0.04091,0.0416,0.04228,0.04297,
       0.04365,0.04433,0.04501,0.04568,0.04636,0.04703,0.04771,0.04836,
       0.04948,0.05177,0.05295,0.05416,0.05541,0.05668,0.058,0.05935,
       0.06075,0.0622,0.0637,0.06525,0.06688,0.07415,0.07986,0.08381,
       0.09196,0.1145,0.1385,0.1606,0.1834,0.2082,0.2298,0.2524,0.2977,
       0.3429,0.39,0.44,0.4845,
SS(1)=10*.0005,50*.001,43*.01,
$
```

The sample output for case 1 is given as follows:

```
      ETA          Y0(1)          Y0(2)          Y0(3)          Y0(4)          Y0(5)          Y0(6)
0.             0.             0.             7.729421E-01   3.800000E-01   3.008175E-01   0.
1.000000E-01   2.851202E-03   5.690891E-02   7.533828E-01   4.024295E-01   3.008021E-01   3.626776E-02
2.000000E-01   1.135689E-02   1.130707E-01   7.326971E-01   4.251599E-01   3.006933E-01   6.913905E-02
3.000000E-01   2.543655E-02   1.683685E-01   7.108778E-01   4.481846E-01   3.003958E-01   9.872421E-02
4.000000E-01   4.499767E-02   2.226786E-01   6.879263E-01   4.714881E-01   2.998122E-01   1.251445E-01
5.000000E-01   6.993505E-02   2.758718E-01   6.638543E-01   4.950460E-01   2.988446E-01   1.485318E-01
6.000000E-01   1.001303E-01   3.278144E-01   6.386874E-01   5.188240E-01   2.973959E-01   1.690284E-01
7.000000E-01   1.354517E-01   3.783696E-01   6.124673E-01   5.427776E-01   2.953713E-01   1.867859E-01
8.000000E-01   1.757534E-01   4.273990E-01   5.852543E-01   5.668524E-01   2.926810E-01   2.019649E-01
9.000000E-01   2.208760E-01   4.747651E-01   5.571299E-01   5.909838E-01   2.892422E-01   2.147339E-01
1.000000E+00   2.706465E-01   5.203334E-01   5.281979E-01   6.150972E-01   2.849821E-01   2.252680E-01
1.100000E+00   3.248785E-01   5.639754E-01   4.985856E-01   6.391092E-01   2.798410E-01   2.337475E-01
     .              .              .              .              .              .              .
     .              .              .              .              .              .              .
     .              .              .              .              .              .              .
6.100000E+00   4.999201E+00   9.999991E-01   3.801683E-06   9.999914E-01   3.271047E-05   2.117261E-01
6.200000E+00   5.099201E+00   9.999994E-01   2.515940E-06   9.999941E-01   2.274064E-05   2.117255E-01
6.300000E+00   5.199201E+00   9.999996E-01   1.654964E-06   9.999960E-01   1.569610E-05   2.117251E-01
6.400000E+00   5.299201E+00   9.999998E-01   1.082035E-06   9.999973E-01   1.075607E-05   2.117248E-01
6.500000E+00   5.399201E+00   9.999998E-01   7.031642E-07   9.999982E-01   7.317933E-06   2.117246E-01
6.600000E+00   5.499201E+00   9.999999E-01   4.541827E-07   9.999988E-01   4.943066E-06   2.117244E-01
6.700000E+00   5.599201E+00   9.999999E-01   2.915789E-07   9.999992E-01   3.314953E-06   2.117243E-01
6.800000E+00   5.699201E+00   1.000000E+00   1.860478E-07   9.999995E-01   2.207148E-06   2.117243E-01
6.900000E+00   5.799201E+00   1.000000E+00   1.179846E-07   9.999997E-01   1.459011E-06   2.117242E-01
7.000000E+00   5.899201E+00   1.000000E+00   7.436122E-08   9.999998E-01   9.575439E-07   2.117242E-01
7.100000E+00   5.999201E+00   1.000000E+00   4.657739E-08   9.999999E-01   6.239244E-07   2.117242E-01
7.200000E+00   6.099201E+00   1.000000E+00   2.899328E-08   9.999999E-01   4.036252E-07   2.117242E-01
7.300000E+00   6.199201E+00   1.000000E+00   1.793488E-08   9.999999E-01   2.592374E-07   2.117242E-01
7.400000E+00   6.299201E+00   1.000000E+00   1.102462E-08   1.000000E+00   1.653066E-07   2.117242E-01


$NAM

P10     =   0.92490308449337E+00,

T10     =   0.53955539733397E+00,

G       =   0.14E+01,

REY     =   0.39738221255108E+07,

RT1     =   0.54456182115757E-01,

P1      =   0.1405651212204E+02,

T1      =   0.1033686448444E+03,

R1      =   0.79244926185209E-04,
```

```
U1       =  0.3961723846778E+04,

AA1      =  0.49833004361987E+03,

TREF     =  0.26132627103112E+04,

VISREF   =  0.10784657366036E-05,

R10      =  0.59996819820952E+01,

$END


X=          .3450  PROFILE


   ETA         Y/YE        U/UE        T/TE       TT/TTE      CROCCO      PT/PTR       M/ME         FZ          TZ        VORTREY       XLM11

0.          0.          0.          5.831E-01   3.830E-01   2.879E-15   2.296E-01   0.          3.998E-01   3.100E-01   0.          1.163E+00
1.000E-01   1.427E-02   4.021E-02   6.137E-01   4.036E-01   3.345E-02   2.307E-01   5.133E-02   4.043E-01   3.015E-01   2.286E+00   1.149E+00
2.000E-01   2.926E-02   8.086E-02   6.434E-01   4.248E-01   6.781E-02   2.339E-01   1.008E-01   4.085E-01   2.924E-01   8.525E+00   1.136E+00
3.000E-01   4.494E-02   1.219E-01   6.722E-01   4.466E-01   1.031E-01   2.390E-01   1.487E-01   4.124E-01   2.827E-01   1.801E+01   1.123E+00
4.000E-01   6.130E-02   1.633E-01   7.000E-01   4.689E-01   1.392E-01   2.460E-01   1.952E-01   4.156E-01   2.723E-01   3.022E+01   1.111E+00
5.000E-01   7.830E-02   2.050E-01   7.267E-01   4.917E-01   1.762E-01   2.548E-01   2.405E-01   4.182E-01   2.613E-01   4.480E+01   1.100E+00
6.000E-01   9.591E-02   2.470E-01   7.522E-01   5.150E-01   2.139E-01   2.654E-01   2.847E-01   4.198E-01   2.495E-01   6.144E+01   1.090E+00
7.000E-01   1.141E-01   2.890E-01   7.766E-01   5.387E-01   2.524E-01   2.780E-01   3.280E-01   4.204E-01   2.371E-01   7.990E+01   1.080E+00
8.000E-01   1.329E-01   3.310E-01   7.997E-01   5.628E-01   2.914E-01   2.924E-01   3.702E-01   4.198E-01   2.240E-01   9.994E+01   1.071E+00
    .           .           .           .           .           .           .           .           .           .           .           .
    .           .           .           .           .           .           .           .           .           .           .           .
    .           .           .           .           .           .           .           .           .           .           .           .

3.900E+00   8.352E-01   9.968E-01   9.995E-01   9.975E-01   9.959E-01   8.844E-01   9.970E-01   1.183E-02   1.453E-03   6.159E+01   1.000E+00
4.000E+00   8.587E-01   9.978E-01   9.997E-01   9.982E-01   9.972E-01   8.858E-01   9.979E-01   8.752E-03   1.152E-03   4.817E+01   1.000E+00
4.100E+00   8.823E-01   9.985E-01   9.998E-01   9.988E-01   9.981E-01   8.869E-01   9.986E-01   6.344E-03   9.015E-04   3.686E+01   1.000E+00
4.200E+00   9.058E-01   9.990E-01   9.998E-01   9.992E-01   9.988E-01   8.876E-01   9.991E-01   4.495E-03   6.941E-04   2.753E+01   1.000E+00
4.300E+00   9.294E-01   9.994E-01   9.999E-01   9.995E-01   9.992E-01   8.881E-01   9.995E-01   3.101E-03   5.238E-04   2.000E+01   1.000E+00
4.400E+00   9.529E-01   9.997E-01   9.999E-01   9.997E-01   9.996E-01   8.885E-01   9.997E-01   2.073E-03   3.856E-04   1.406E+01   1.000E+00
4.500E+00   9.765E-01   9.998E-01   1.000E+00   9.999E-01   9.998E-01   8.887E-01   9.998E-01   1.332E-03   2.749E-04   9.489E+00   1.000E+00
4.600E+00   1.000E+00   9.999E-01   1.000E+00   9.999E-01   9.999E-01   8.889E-01   9.999E-01   8.128E-04   1.878E-04   6.073E+00   1.000E+00


X =      3.45000E-01  RETHET= 2.05328E+02  DPEDX =-1.91279E-02  CFW =    1.29312E-03  ZSHK =  0.               YE =    2.49507E-03
XI =     2.24736E-03  REX =    2.43046E+05  DTEDX =-5.06340E-01  QSD =   -3.41942E+00  RSHK =  0.               UTAU =  0.
RAD =    1.63587E-01  PE =     2.64075E+02  DUEDX = 8.31957E-01  HD =     4.29204E-03  ITRO =                0  PTR =   9.24903E-01
Z =      2.93904E-01  TE =     9.26012E+02  DLTAST= 5.57693E-04  NSTE =   1.38798E-03  TW/TT = 3.82979E-01  YMP =              R
BETA =   6.14417E-03  RE =     1.66185E-04  THETA = 2.91459E-04  NSTW =   8.09398E-04  RFTRUE=         IIIII  P20 =   9.24903E-01
TRFCT = 0.            UE =     2.41116E+03  D/T =   1.91345E+00  NUE =    2.42888E+02  ROUSE = 3.62804E+02  OMEGA = 1.85343E-03
RVWALD= 0.            ME =     1.61657E+00  TAUD =  1.07121E+00  NUW =    3.58218E+02  DSMXO = 1.75918E+03
REDELT= 3.92885E+02  MUE =    5.68785E-07  CFE =   2.21749E-03  SWANG = 0.            XD =    1.34620E+00
```

Case 2.- An example of planar flow is presented in reference 23. The wind-tunnel test conditions were as follows:

$M_\infty = 2.8$

$p_{t,\infty} = 9.997 \times 10^5$ N/m^2

$T_{t,\infty} = 3.11 \times 10^2$ °K

$q_w = 0$ (Adiabatic flow)

The location of transition was not reported in reference 23; therefore, for the present calculations it was assumed that transition occurred near the sharp leading edge of the flat-plate model. This case requires $120\,000_8$ storage on the CDC 6000 computer. The listing of the variable-dimension data for this particular case is as follows:

JK = 310

JL = 1

JM = 750

JN = 40

The listing of the input data for case 2 is given as follows:

```
$NAM1
HPR=0.4,
XEND=120.0,
H=0.01,
PR=0.7,
XKK=0.911,
BETA=0.0,
ALPHA=3.135,
X0=0.0,
Y0(1)=0.0,0.0,0.452,2.43,-0.04,0.0,
XK=1.02,
IGAS=1,
VISCON=0.7173E-08,
VISPOW=0.647,
KODUNIT=0,
$
```

```
$NAM2
XMA=2.8,
PT1=21024.0,
TT1=550.0,
WAVE=0.0,
XY1=1.0,
XY2=1.0,
XY3=2.8,
G=1.4,
R=1716.0,
SU=198.6,
PR=0.7,
PRT=0.95,
IBODY=2,
J=0,
W=0,
FT=1.0,
KODE=0,
KODWAL=2,
IENTRO=1,
CONE=0.0,
IEND1=1000,
A=1.0,
DS=0.01,
KODVIS=1,
SST=0.1E+09,
SMXTR=2500.0,
TLNGTH=2.0,
CORP=0.412,
CONSTNT=0.0,
XT1=0.4,
XT2=26.0,
XT3=0.0168,
XT4=0.78,
XT5=0.0,
PROINC=2.0,
PRNTINC=0.1,
IPPO=14,
PROVAL(1)=0.1,0.15,0.2,0.25,0.3,0.35,0.4,0.45,0.02,0.05,0.08,.06,.07,1.,
IPRNT=13,
PRNTVAL(1)=2.0,4.0,0.2,0.25,0.3,0.35,0.4,0.45,0.02,0.05,0.08,.06,.07,
NAUXPRO=0,
BLNGTH=0.0,
NPUTYPE=1,
KODPRT=2,
KTCOD=1,
$
```

```
$NAM3
NUMBER=41,
L=1,
PE(1)=41*774.69861580563,
Z(1)=0.0,0.5,1.0,1.5,2.0,2.5,3.0,3.5,4.0,4.5,5.0,5.5,6.0,6.5,7.0,7.5,
8.0,8.5,9.0,9.5,10.0,10.5,11.0,11.5,12.0,12.5,13.0,13.5,14.0,14.5,15.0,
15.5,16.0,16.5,17.0,17.5,18.0,18.5,19.0,19.5,20.0,
RMI(1)=41*1.0,
TW(1)=41*520.4439,
QW(1)=41*0.0,
RVWALD(1)=41*0.0,
S(1)=0.0,0.5,1.0,1.5,2.0,2.5,3.0,3.5,4.0,4.5,5.0,5.5,6.0,6.5,7.0,7.5,
8.0,8.5,9.0,9.5,10.0,10.5,11.0,11.5,12.0,12.5,13.0,13.5,14.0,14.5,15.0,
15.5,16.0,16.5,17.0,17.5,18.0,18.5,19.0,19.5,20.0,
SS(1)=1000*.01,
$
```

Sample outputs for case 2 for regions where the flow is laminar and turbulent are given as follows:

```
     ETA          YO(1)          YO(2)           YO(4)          YO(5)         YO(6)
0.            0.             0.             2.430000E+00-4.938715E-02  3.643092E+00
6.328538E-03  1.883040E-05   3.246450E-03   2.429598E+00-4.938715E-02  3.643092E+00
1.278365E-02  5.864116E-05   6.556230E-03   2.429166E+00-4.938715E-02  3.643092E+00
1.936786E-02  1.206843E-04   9.930542E-03   2.428702E+00-4.938715E-02  3.643092E+00
2.608375E-02  2.062704E-04   1.337061E-02   2.428206E+00-4.938715E-02  3.643092E+00
3.293397E-02  3.167714E-04   1.687767E-02   2.427676E+00-4.938715E-02  3.643092E+00
3.992118E-02  4.536230E-04   2.045301E-02   2.427109E+00-4.938715E-02  3.643092E+00
4.704814E-02  6.183275E-04   2.409790E-02   2.426504E+00-4.938715E-02  3.643092E+00
5.431765E-02  8.124567E-04   2.781366E-02   2.425860E+00-4.938715E-02  3.643092E+00
6.173254E-02  1.037655E-03   3.160162E-02   2.425174E+00-4.938715E-02  3.643092E+00
6.929573E-02  1.295641E-03   3.546315E-02   2.424445E+00-4.938715E-02  3.643092E+00
7.701018E-02  1.588214E-03   3.939963E-02   2.423670E+00-4.938715E-02  3.643092E+00

     .             .              .              .              .              .
     .             .              .              .              .              .
     .             .              .              .              .              .

5.853772E+00  4.700446E+00   9.999988E-01   1.000100E+00-4.938715E-02  3.643092E+00
5.977176E+00  4.823850E+00   9.999994E-01   1.000061E+00-4.938715E-02  3.643092E+00
6.103048E+00  4.949722E+00   9.999998E-01   1.000033E+00-4.938715E-02  3.643092E+00
6.231437E+00  5.078111E+00   9.999998E-01   1.000026E+00-4.938715E-02  3.643092E+00
6.362395E+00  5.209069E+00   9.999999E-01   1.000015E+00-4.938715E-02  3.643092E+00
6.495971E+00  5.342645E+00   1.000000E+00   1.000007E+00-4.938715E-02  3.643092E+00
6.632219E+00  5.478893E+00   1.000000E+00   1.000006E+00-4.938715E-02  3.643092E+00
6.771192E+00  5.617866E+00   1.000000E+00   1.000003E+00-4.938715E-02  3.643092E+00
6.912944E+00  5.759618E+00   1.000000E+00   1.000002E+00-4.938715E-02  3.643092E+00
7.057522E+00  5.904206E+00   1.000000E+00   1.000001E+00-4.938715E-02  3.643092E+00
7.205011E+00  6.051685E+00   1.000000E+00   1.000000E+00-4.938715E-02  3.643092E+00


$NAM

P10      =   0.24725123499179E+01,

T10      =   0.81887755102041E+00,

G        =   0.14E+01,

REY      =   0.24561099387761E+08,

RT1      =   0.22275905912269E-01,

P1       =   0.77469861580563E+03,

T1       =   0.21417445482866E+03,

R1       =   0.21078894314355E-02,
```

```
U1       =   0.20084661930434E+04,

AA1      =   0.71720935465835E+03,

TREF     =   0.67165109034267E+03,

VISREF   =   0.45404287741154E-06,

R10      =   0.10567872099961E+02,

$END



X=          .0700   PROFILE


   ETA         Y/YE          U/UE          T/TE          TT/TTE        CROCCO        PT/PTR        M/ME          FZ          TZ           VORTREY       XLM11

0.           0.            0.            2.331E+00     9.076E-01     3.843E-14     9.461E-02     0.            5.033E-01  -0.           0.            9.031E-01
6.329E-03    2.038E-03     3.185E-03     2.334E+00     9.091E-01     1.612E-02     9.462E-02     2.085E-03     5.033E-01   2.954E-01   1.771E-02     9.028E-01
1.278E-02    4.119E-03     6.434E-03     2.334E+00     9.091E-01     1.618E-02     9.462E-02     4.211E-03     5.033E-01  -7.273E-03   7.231E-02     9.028E-01
1.937E-02    6.242E-03     9.747E-03     2.334E+00     9.091E-01     1.628E-02     9.463E-02     6.380E-03     5.033E-01  -1.089E-02   1.660E-01     9.028E-01
2.608E-02    8.407E-03     1.313E-02     2.334E+00     9.091E-01     1.643E-02     9.465E-02     8.592E-03     5.033E-01  -1.462E-02   3.012E-01     9.028E-01
3.293E-02    1.061E-02     1.657E-02     2.334E+00     9.091E-01     1.664E-02     9.467E-02     1.085E-02     5.032E-01  -1.843E-02   4.803E-01     9.028E-01
3.992E-02    1.287E-02     2.009E-02     2.334E+00     9.091E-01     1.689E-02     9.470E-02     1.315E-02     5.032E-01  -2.232E-02   7.058E-01     9.028E-01
4.705E-02    1.516E-02     2.368E-02     2.334E+00     9.092E-01     1.720E-02     9.474E-02     1.550E-02     5.032E-01  -2.629E-02   9.804E-01     9.028E-01
5.432E-02    1.751E-02     2.734E-02     2.334E+00     9.092E-01     1.756E-02     9.478E-02     1.789E-02     5.032E-01  -3.033E-02   1.307E+00     9.029E-01
6.173E-02    1.990E-02     3.107E-02     2.333E+00     9.092E-01     1.799E-02     9.483E-02     2.034E-02     5.032E-01  -3.446E-02   1.689E+00     9.029E-01
6.930E-02    2.233E-02     3.487E-02     2.333E+00     9.093E-01     1.849E-02     9.488E-02     2.283E-02     5.032E-01  -3.867E-02   2.128E+00     9.029E-01
7.701E-02    2.482E-02     3.875E-02     2.333E+00     9.093E-01     1.905E-02     9.495E-02     2.537E-02     5.032E-01  -4.296E-02   2.629E+00     9.029E-01
8.488E-02    2.735E-02     4.271E-02     2.332E+00     9.094E-01     1.968E-02     9.502E-02     2.797E-02     5.031E-01  -4.734E-02   3.195E+00     9.030E-01
    .            .             .             .             .             .             .             .             .            .            .             .
    .            .             .             .             .             .             .             .             .            .            .             .
    .            .             .             .             .             .             .             .             .            .            .             .

4.549E+00    9.588E-01     9.997E-01     1.005E+00     1.002E+00     1.017E+00     9.949E-01     9.973E-01     9.665E-04  -1.294E-02   8.519E+01     9.999E-01
4.646E+00    9.722E-01     9.998E-01     1.004E+00     1.001E+00     1.013E+00     9.961E-01     9.979E-01     6.898E-04  -1.024E-02   6.273E+01     9.999E-01
4.745E+00    9.860E-01     9.999E-01     1.003E+00     1.001E+00     1.010E+00     9.970E-01     9.984E-01     4.841E-04  -8.001E-03   4.540E+01     9.999E-01
4.847E+00    1.000E+00     9.999E-01     1.002E+00     1.001E+00     1.008E+00     9.978E-01     9.988E-01     3.338E-04  -6.178E-03   3.227E+01     1.000E+00


X =      7.00000E-02  RETHET= 8.40589E+02  DPEDX = 0.            CFW =    1.14250E-03  ZSHK =  0.            YE =     5.46727E-04
XI =     2.65745E-02  REX =    1.71928E+06  DTEDX =-0.            QSD =    0.           RSHK =  0.            UTAU =  0.
RAD =    1.00000E+00  PE =     7.74699E+02  DUEDX =-0.            HD =     0.           ITRO =              0 PTR =    9.62954E-01
Z =      7.00000E-02  TE =     2.14174E+02  DLTAST= 2.68454E-04   NSTE =   0.           TW/TT  = 9.07564E-01  YMP =              R
RETA =   0.                   RE =     2.10789E-03  THETA = 3.42244E-05   NSTW =   0.           RETRUE= 8.48612E-01  P20 =    2.47251E+00
TRECT = 0.                    UE =     2.00847E+03  D/T =    7.84394E+00  NUE =    0.           ROUSE = 2.59252E+03  OMEGA = 3.27486E-04
RVWALD= 0.                    ME =     2.80000E+00  TAUD =   2.08415E+00  NUW =    0.           DSMXO = 1.99617E+04
PEDELT= 6.59353E+03  MUE =    1.72371E-07  CFE =    4.90210E-04   SWANG = 0.           XO =    1.66947E+00
```

```
X=          2.0000  PROFILE

  ETA         Y/YE        U/UE        T/TE        TT/TTE      CROCCO      PT/PTR      M/ME        YPLUS       UPLUS       UDEF        VISEFF

0.          0.          0.          2.433E+00   9.476E-01   6.780E-14   9.461E-02   0.          0.          0.          0.          1.004E+00
6.329E-03   2.884E-04   4.768E-02   2.435E+00   9.495E-01   3.668E-02   9.510E-02   3.056E-02   1.311E+00   1.171E+00   2.338E+01   1.013E+00
1.278E-02   5.822E-04   9.533E-02   2.427E+00   9.508E-01   6.082E-02   9.657E-02   6.119E-02   2.655E+00   2.344E+00   2.224E+01   1.044E+00
1.937E-02   8.807E-04   1.417E-01   2.415E+00   9.528E-01   9.826E-02   9.900E-02   9.118E-02   4.038E+00   3.493E+00   2.116E+01   1.110E+00
2.608E-02   1.183E-03   1.853E-01   2.399E+00   9.552E-01   1.442E-01   1.023E-01   1.196E-01   5.464E+00   4.583E+00   2.015E+01   1.223E+00
3.293E-02   1.490E-03   2.249E-01   2.380E+00   9.577E-01   1.927E-01   1.061E-01   1.458E-01   6.938E+00   5.584E+00   1.925E+01   1.392E+00
3.992E-02   1.800E-03   2.598E-01   2.360E+00   9.601E-01   2.387E-01   1.103E-01   1.691E-01   8.458E+00   6.480E+00   1.846E+01   1.618E+00
4.705E-02   2.113E-03   2.902E-01   2.339E+00   9.623E-01   2.797E-01   1.147E-01   1.898E-01   1.003E+01   7.270E+00   1.778E+01   1.901E+00
5.432E-02   2.430E-03   3.165E-01   2.319E+00   9.641E-01   3.149E-01   1.190E-01   2.079E-01   1.164E+01   7.964E+00   1.720E+01   2.238E+00
6.173E-02   2.751E-03   3.393E-01   2.299E+00   9.656E-01   3.444E-01   1.233E-01   2.238E-01   1.329E+01   8.572E+00   1.669E+01   2.627E+00

   .           .           .           .           .           .           .           .           .           .           .           .
   .           .           .           .           .           .           .           .           .           .           .           .
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3.709E+01   9.271E-01   9.939E-01   1.043E+00   1.009E+00   1.178E+00   9.494E-01   9.731E-01   1.205E+04   3.728E+01   2.298E-01   1.435E+02
3.784E+01   9.413E-01   9.959E-01   1.034E+00   1.008E+00   1.157E+00   9.610E-01   9.793E-01   1.239E+04   3.752E+01   1.559E-01   1.232E+02
3.860E+01   9.557E-01   9.975E-01   1.025E+00   1.007E+00   1.130E+00   9.716E-01   9.850E-01   1.273E+04   3.774E+01   9.601E-02   1.045E+02
3.938E+01   9.703E-01   9.987E-01   1.017E+00   1.005E+00   1.099E+00   9.811E-01   9.901E-01   1.307E+04   3.793E+01   5.084E-02   8.782E+01
4.017E+01   9.851E-01   9.994E-01   1.010E+00   1.003E+00   1.064E+00   9.892E-01   9.943E-01   1.341E+04   3.809E+01   2.099E-02   7.268E+01
4.098E+01   1.000E+00   1.000E+00   1.001E+00   1.000E+00   1.006E+00   9.993E-01   9.996E-01   1.380E+04   3.829E+01   1.042E-04   1.000E+00


X =       2.00000E+00  PETHET= 3.69327E+04  DPEDX = 6.66134E-14  CFW =    3.31613E-03  ZSHK  =  0.                YE =      2.15550E-02
XI =      7.59273E-01  REX =   4.91222E+07  DTEDX = 1.75466E-13  QSD =    0.           RSHK  =  0.                UTAU =    2.61030E-02
RAD =     1.00000E+00  PE =    7.74699E+02  DUEDX =-1.75466E-13  HD =     0.           ITRO  =                0  PTR =     9.62954E-01
Z =       2.00000E+00  TE =    2.14174E+02  DLTAST= 7.14388E-03  NSTE =   0.           TW/TT = 9.47602E-01  YMP =             147
BETA = -2.66454E-13    RE =    2.10789E-03  THETA = 1.50371E-03  NSTW =   0.           RFTRUE= 9.14186E-01  P20 =     2.47251E+00
TRFCT = 1.00000E+00    UE =    2.00847E+03  D/T =   4.75085E+00  NUE =    0.           ROUSE = 7.79506E+04  OMEGA =   3.27486E-04
RVWALD= 0.             ME =    2.80000E+00  TAUD =  5.79372E+00  NUW =    0.           DSMXO = 2.78534E+04
REDELT= 1.75462E+05    MUE =   1.72371E-07  CFE =   1.36273E-03  SWANG =  0.           XD =    6.58196E+01
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Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., February 11, 1972.

APPENDIX A

DIFFERENCE RELATIONS

Three-point implicit difference relations are used to reduce the transformed momentum and energy equations (eqs. (28) and (29)) to finite-difference form. It is assumed that all data are known at the solution stations m-1 and m. (See fig. 2.) Then, it is possible to obtain the unknown quantities at the grid points for the m+1 station. In the subsequent development the notations G and H are utilized to represent any typical variable.

Taylor-series expansions are first written about the unknown grid point (m+1,n) in the ξ-direction as follows:

$$G_{m,n} = G_{m+1,n} - \Delta\xi_2\left(G_\xi\right)_{m+1,n} + \frac{\Delta\xi_2^2}{2}\left(G_{\xi\xi}\right)_{m+1,n} - \frac{\Delta\xi_2^3}{6}\left(G_{\xi\xi\xi}\right)_{m+1,n} + \dots \tag{A1a}$$

and

$$G_{m-1,n} = G_{m+1,n} - \left(\Delta\xi_1 + \Delta\xi_2\right)\left(G_\xi\right)_{m+1,n} + \frac{\left(\Delta\xi_1 + \Delta\xi_2\right)^2}{2}\left(G_{\xi\xi}\right)_{m+1,n} - \frac{\left(\Delta\xi_1 + \Delta\xi_2\right)^3}{6}\left(G_{\xi\xi\xi}\right)_{m+1,n} + \dots \tag{A1b}$$

where subscript notation has been utilized to denote differentiation; that is, $G_\xi \equiv \frac{\partial G}{\partial \xi}$, and so forth.

Equations (A1a) and (A1b) can be solved to yield

$$\left(\frac{\partial G}{\partial \xi}\right)_{m+1,n} = \frac{X_1 G_{m+1,n} - X_2 G_{m,n} + X_3 G_{m-1,n}}{2\,\Delta\xi_2} + \frac{\Delta\xi_2\left(\Delta\xi_1 + \Delta\xi_2\right)}{6} G_{\xi\xi\xi} + \dots \tag{A2}$$

and

$$G_{m+1,n} = X_4 G_{m,n} - X_5 G_{m-1,n} + \frac{\Delta\xi_1\,\Delta\xi_2}{2}\left(1 + \frac{\Delta\xi_2}{\Delta\xi_1}\right) G_{\xi\xi} + \dots \tag{A3}$$

Terms of the order of $\Delta\xi_1\,\Delta\xi_2$, or smaller, are neglected. This produces truncation errors of the order of $\Delta\xi_1\,\Delta\xi_2$ instead of $\Delta\xi_2$ as in reference 9 where two-point difference relations are used. The $X_1, X_2, \ldots, X_5$ coefficients appearing in equations (A2) and (A3) are defined as follows:

$$X_1 = 2\frac{\Delta\xi_1 + 2\Delta\xi_2}{\Delta\xi_1 + \Delta\xi_2} \tag{A4}$$

$$X_2 = 2\frac{\Delta\xi_1 + \Delta\xi_2}{\Delta\xi_1} \tag{A5}$$

$$X_3 = 2\frac{\Delta\xi_1 \Delta\xi_2}{\Delta\xi_1\left(\Delta\xi_1 + \Delta\xi_2\right)} \tag{A6}$$

$$X_4 = \frac{\Delta\xi_1 + \Delta\xi_2}{\Delta\xi_1} \tag{A7}$$

and

$$X_5 = \frac{\Delta\xi_2}{\Delta\xi_1} \tag{A8}$$

Taylor-series expansions are next written about the unknown grid point (m+1,n) in the η-direction as follows:

$$G_{m+1,n+1} = G_{m+1,n} + \Delta\eta_n\left(G_\eta\right)_{m+1,n} + \frac{\Delta\eta_n^2}{2}\left(G_{\eta\eta}\right)_{m+1,n} + \frac{\Delta\eta_n^3}{6}\left(G_{\eta\eta\eta}\right)_{m+1,n} + \ldots \tag{A9a}$$

and

$$G_{m+1,n-1} = G_{m+1,n} - \Delta\eta_{n-1}\left(G_\eta\right)_{m+1,n} + \frac{\Delta\eta_{n-1}^2}{2}\left(G_{\eta\eta}\right)_{m+1,n} - \frac{\Delta\eta_{n-1}^3}{6}\left(G_{\eta\eta\eta}\right)_{m+1,n} + \ldots \tag{A9b}$$

Equations (A9a) and (A9b) can be solved to yield

$$\left(\frac{\partial^2 G}{\partial\eta^2}\right)_{m+1,n} = Y_1 G_{m+1,n+1} - Y_2 G_{m+1,n} + Y_3 G_{m+1,n-1} + \frac{\left(\Delta\eta_{n-1} - \Delta\eta_n\right)}{3} G_{\eta\eta\eta} + \ldots \tag{A10}$$

and

$$\left(\frac{\partial G}{\partial\eta}\right)_{m+1,n} = Y_4 G_{m+1,n+1} - Y_5 G_{m+1,n} - Y_6 G_{m+1,n-1} - \frac{\Delta\eta_n \Delta\eta_{n-1}}{6} G_{\eta\eta\eta} + \ldots \tag{A11}$$

The $Y_1, Y_2, \ldots, Y_6$ coefficients appearing in equations (A10) and (A11) are defined as follows:

$$Y_1 = \frac{2}{\Delta\eta_n\left(\Delta\eta_n + \Delta\eta_{n-1}\right)} \tag{A12}$$

$$Y_2 = \frac{2}{\Delta\eta_n \Delta\eta_{n-1}} \tag{A13}$$

$$Y_3 = \frac{2}{\Delta\eta_{n-1}\left(\Delta\eta_n + \Delta\eta_{n-1}\right)} \tag{A14}$$

$$Y_4 = \frac{\Delta\eta_{n-1}}{\Delta\eta_n\left(\Delta\eta_n + \Delta\eta_{n-1}\right)} \tag{A15}$$

$$Y_5 = \frac{\Delta\eta_{n-1} - \Delta\eta_n}{\Delta\eta_n \Delta\eta_{n-1}} \tag{A16}$$

and

$$Y_6 = \frac{\Delta\eta_n}{\Delta\eta_{n-1}\left(\Delta\eta_n + \Delta\eta_{n-1}\right)} \tag{A17}$$

For the case of equally spaced grid points in the ξ- and η-coordinates, equations (A4) to (A8) and (A12) to (A17) reduce to the following relations:

$$\left.\begin{aligned} X_1 &= 3 \\ X_2 &= 4 \\ X_3 &= 1 \\ X_4 &= 2 \\ X_5 &= 1 \end{aligned}\right\} \tag{A18a}$$

and

$$\left.\begin{aligned}
Y_1 &= \frac{1}{\Delta\eta^2} \\
Y_2 &= 2Y_1 \\
Y_3 &= Y_1 \\
Y_4 &= \frac{1}{2\,\Delta\eta} \\
Y_5 &= 0 \\
Y_6 &= Y_4
\end{aligned}\right\} \tag{A18b}$$

where $\Delta\xi$ and $\Delta\eta$ represent the spacing between the grid points in the ξ- and η-coordinates, respectively.

Equations (A2), (A3), (A10), and (A11) can then be written for constant grid-point spacing as follows:

$$\left(\frac{\partial G}{\partial \xi}\right)_{m+1,n} = \frac{3G_{m+1,n} - 4G_{m,n} + G_{m-1,n}}{2\,\Delta\xi} + \frac{\Delta\xi^2}{3}G_{\xi\xi\xi} + \ldots \tag{A19}$$

$$G_{m+1,n} = 2G_{m,n} - G_{m-1,n} + \Delta\xi^2 G_{\xi\xi} + \ldots \tag{A20}$$

$$\left(\frac{\partial^2 G}{\partial \eta^2}\right)_{m+1,n} = \frac{G_{m+1,n+1} - 2G_{m+1,n} + G_{m+1,n-1}}{\Delta\eta^2} - \frac{\Delta\eta^2}{12}G_{\eta\eta\eta\eta} + \ldots \tag{A21}$$

and

$$\left(\frac{\partial G}{\partial \eta}\right)_{m+1,n} = \frac{G_{m+1,n+1} - G_{m+1,n-1}}{2\,\Delta\eta} - \frac{\Delta\eta^2}{6}G_{\eta\eta\eta} + \ldots \tag{A22}$$

Equations (A19) to (A22) are recognized as the standard relations for equally spaced grid points. (See, for example, ref. 11.)

Quantities of the form $\left(G\frac{\partial H}{\partial \xi}\right)$ that appear in the governing equations must be linearized in order to obtain a system of linear difference equations. Quantities of this type are obtained from equations (A2) and (A3).

The procedure used to linearize nonlinear products such as $\left(\frac{\partial G}{\partial \eta}\right)\left(\frac{\partial H}{\partial \eta}\right)$ is the same as that used by Flügge-Lotz and Blottner (ref. 9) and is as follows:

$$\left[\left(\frac{\partial G}{\partial \eta}\right)\left(\frac{\partial H}{\partial \eta}\right)\right]_{m+1,n} = \left(\frac{\partial G}{\partial \eta}\right)_{m,n}\left(\frac{\partial H}{\partial \eta}\right)_{m+1,n} - \left(\frac{\partial G}{\partial \eta}\right)_{m,n}\left(\frac{\partial H}{\partial \eta}\right)_{m,n} + \left(\frac{\partial H}{\partial \eta}\right)_{m,n}\left(\frac{\partial G}{\partial \eta}\right)_{m+1,n} + 0\left(\Delta\xi_2\right)^2 \tag{A23}$$

where the terms $\left(\frac{\partial G}{\partial \eta}\right)_{m,n}$ and $\left(\frac{\partial H}{\partial \eta}\right)_{m,n}$ are evaluated from equation (A11), but at the known station m. By equating G to H in equation (A23), the linearized form for quantities of the type $\left(\frac{\partial G}{\partial \eta}\right)^2$ is obtained; that is,

$$\left(\frac{\partial G}{\partial \eta}\right)^2_{m+1,n} = \left(\frac{\partial G}{\partial \eta}\right)_{m,n}\left[2\left(\frac{\partial G}{\partial \eta}\right)_{m+1,n} - \left(\frac{\partial G}{\partial \eta}\right)_{m,n}\right] + 0\left(\Delta\xi_2\right)^2 \tag{A24}$$

where $\left(\frac{\partial G}{\partial \eta}\right)_{m+1,n}$ is obtained from equation (A22).

The preceding relations for the difference quotients produce linear-difference equations when substituted into the governing differential equations (eqs. (43)) for the conservation of momentum (eq. (28)) and energy (eq. (29)), respectively, since terms of the order of $(\Delta\xi)^2$ are neglected.

APPENDIX B

COEFFICIENTS FOR DIFFERENCE EQUATIONS

Equations (43) are the difference equations used to represent the partial differential equations for the conservation of momentum and energy, respectively. These equations are repeated for convenience as follows:

$$A1_nF_{m+1,n-1} + B1_nF_{m+1,n} + C1_nF_{m+1,n+1} + D1_n\Theta_{m+1,n-1}$$

$$+ E1_n\Theta_{m+1,n} + F1_n\Theta_{m+1,n+1} = G1_n \tag{B1}$$

$$A2_nF_{m+1,n-1} + B2_nF_{m+1,n} + C2_nF_{m+1,n+1} + D2_n\Theta_{m+1,n-1}$$

$$+ E2_n\Theta_{m+1,n} + F2_n\Theta_{m+1,n+1} = G2_n \tag{B2}$$

These equations are obtained from equations (28) and (29) and the difference quotients are presented in appendix A. The coefficients $A1_n$, $B1_n$, and so forth, in equations (B1) and (B2) are functions of known quantities evaluated at stations m and $m-1$. (See fig. 2.) Therefore, equations (B1) and (B2) can be solved simultaneously without iterative procedures. The coefficients $A1_n$, $B1_n$, and so forth are as follows:

$$A1_n = Y_3H_3 - Y_6H_{11} \tag{B3}$$

$$B1_n = X_1H_1 - Y_2H_3 - Y_5H_{11} + H_5 \tag{B4}$$

$$C1_n = Y_1H_3 + Y_4H_{11} \tag{B5}$$

$$D1_n = -Y_6H_4F_Y \tag{B6}$$

$$E1_n = \frac{Y_5}{Y_6}D1_n + H_6 \tag{B7}$$

$$F1_n = -\frac{Y_4}{Y_6}D1_n \tag{B8}$$

$$G1_n = H_1F_{m2} + H_4T_YF_Y \tag{B9}$$

$$A2_n = -2Y_6H_8F_Y \tag{B10}$$

$$B2_n = \frac{Y_5}{Y_6}A2_n \tag{B11}$$

$$C2_n = -\frac{Y_4}{Y_6}A2_n \tag{B12}$$

$$D2_n = Y_3H_{10} - Y_6H_{12} \tag{B13}$$

$$E2_n = X_1H_1 - Y_2H_{10} - Y_5H_{12} \tag{B14}$$

$$F2_n = Y_1H_{10} + Y_4H_{12} \tag{B15}$$

and

$$G2_n = H_1T_{m2} + H_8\left(F_Y\right)^2 + H_9\left(T_Y\right)^2 \tag{B16}$$

The coefficients $Y_1,Y_2,\ldots,Y_6$ and $X_1,\ldots,X_5$ are functions of the grid-point spacing and are defined in equations (A12) to (A17) and (A4) to (A8), respectively. The coefficients $H_1,H_2,\ldots,H_{12}$ are defined as follows:

$$H_1 = \xi_{m+1}F_{m1}\frac{(FT)}{\Delta\xi_2} \tag{B17}$$

$$H_2 = V_{m1} - L_{m1}\left(\overline{E}_{m1}C'_{m1} + \overline{E}'_{m1}C_{m1}\right) \tag{B18}$$

$$H_3 = -\overline{E}_{m1}L_{m1}C_{m1} \tag{B19}$$

$$H_4 = H_3\frac{L'_{m1}}{L_{m1}} \tag{B20}$$

$$H_5 = \beta_{m+1}F_{m1} \tag{B21}$$

$$H_6 = -\beta_{m+1} \tag{B22}$$

$$H_7 = V_{m1} - L_{m1}\left(\hat{E}_{m1}C'_{m1} + \hat{E}'_{m1}C_{m1}\right) \tag{B23}$$

$$H_8 = -\alpha_{m+1}L_{m1}\bar{E}_{m1}C_{m1} \tag{B24}$$

$$H_9 = -\hat{E}_{m1}L'_{m1}C_{m1} \tag{B25}$$

$$H_{10} = H_9 \frac{L_{m1}}{L'_{m1}} \tag{B26}$$

$$H_{11} = H_2 + H_4T_Y \tag{B27}$$

and

$$H_{12} = H_7 + 2H_9T_Y \tag{B28}$$

The undefined quantities appearing in equations (B17) to (B28) are defined as follows:

$$F_{m1} = X_4F_{m,n} - X_5F_{m-1,n} \tag{B29}$$

$$T_{m1} = X_4\Theta_{m,n} - X_5\Theta_{m-1,n} \tag{B30}$$

$$V_{m1} = X_4V_{m,n} - X_5V_{m-1,n} \tag{B31}$$

$$F_{m2} = X_2F_{m,n} - X_3F_{m-1,n} \tag{B32}$$

$$T_{m2} = X_2\Theta_{m,n} - X_3\Theta_{m-1,n} \tag{B33}$$

$$L_{m1} = \sqrt{T_{m1}}\,\frac{1 + \left(\frac{S}{T_e}\right)_{m+1}}{T_{m1} + \left(\frac{S}{T_e}\right)_{m+1}} \qquad \text{(Air only)} \tag{B34a}$$

$$L_{m1} = \left(T_{m1}\right)^{\sigma-1} \qquad \text{(Power law)} \qquad \text{(B34b)}$$

$$L'_{m1} = \frac{L_{m1}}{2T_{m1}}\left[\frac{\left(\frac{S}{T_e}\right)_{m+1} - T_{m1}}{\left(\frac{S}{T_e}\right)_{m+1} + T_{m1}}\right] \qquad \text{(Air only)} \qquad \text{(B35a)}$$

$$L'_{m1} = (\sigma - 1)\left(T_{m1}\right)^{\sigma-2} \qquad \text{(Power law)} \qquad \text{(B35b)}$$

$$\overline{E}_{m1} = \left(\overline{\epsilon}_{av}\right)_{m+1,n} \qquad \text{(B36a)}$$

where

$$\left(\overline{\epsilon}_{av}\right)_{m+1,n} = \frac{\overline{\epsilon}_{m-1,n} + \overline{\epsilon}_{m,n} + \overline{\epsilon}_{m+1,n}}{3} \qquad \text{(B36b)}$$

$$\hat{E}_{m1} = \frac{\left(\tilde{\epsilon}_{av}\right)_{m+1,n}}{\sigma} \qquad \text{(B37)}$$

$$\overline{E}_Y = Y_4\overline{\epsilon}_{m,n+1} - Y_5\overline{\epsilon}_{m,n} - Y_6\overline{\epsilon}_{m,n-1} \qquad \text{(See eq. (A11))} \qquad \text{(B38)}$$

$$\hat{E}_Y = Y_4\tilde{\epsilon}_{m,n+1} - Y_5\tilde{\epsilon}_{m,n} - Y_6\tilde{\epsilon}_{m,n-1} \qquad \text{(B39)}$$

$$F_Y = Y_4F_{m,n+1} - Y_5F_{m,n} - Y_6F_{m,n-1} \qquad \text{(B40)}$$

$$T_Y = Y_4\Theta_{m,n+1} - Y_5\Theta_{m,n} - Y_6\Theta_{m,n-1} \qquad \text{(B41)}$$

$$\beta_{m+1} = \left(\frac{2\xi}{u_e}\frac{du_e}{d\xi}\right)_{m+1} \qquad \text{(See eqs. (30))} \qquad \text{(B42)}$$

and

$$\alpha_{m+1} = \left(\frac{u_e^2}{T_e}\right)_{m+1} \qquad \text{(B43)}$$

The transverse-curvature terms are contained in the quantities C_{m1} and C'_{m1} which appear explicitly in the H_2, H_3, H_7, H_8, and H_9 coefficients. The transverse-curvature term in the transformed plane (see ref. 7) may be written as follows:

$$t^{2j} = 1 + \frac{2\omega j(W)\sqrt{2\xi}\cos\phi}{\rho_e u_e}\int_0^{\eta}\Theta\,d\eta \tag{B44}$$

where t represents the ratio r/r_o and is a known quantity for the $N-1$ grid points at station $m-1$ and m. Then, the extrapolated values at $m+1,n$ are obtained as follows where the parameter C is used to represent t^{2j}:

$$C_{m1} = X_4 C_{m,n} - X_5 C_{m-1,n} \tag{B45}$$

$$C'_{m1} = Y_4 C_{m,n+1} - Y_5 C_{m,n} - Y_6 C_{m,n-1} \tag{B46}$$

Two quantities (symbols) as of now remain undefined. These are the code symbols FT and W which appear in equations (B17) and (B44), respectively. The code symbol W appearing in equation (B44) is used either to retain or neglect the transverse-curvature terms for axisymmetric flows; that is, $W = 1$ or 0, respectively. For planar flows, the transverse-curvature term does not appear since j equals 0. The code symbol FT (flow type) appearing in equation (B17) is used either to retain or neglect the nonsimilar terms in the governing differential equations; that is, $FT = 1$ or 0, respectively. If FT is assigned a value of unity, the solution to the nonsimilar equations (eqs. (27) to (29)) is obtained. If FT is assigned a value of zero, the locally similar solution is obtained; that is, the following system of equations is solved:

Continuity

$$\frac{\partial V}{\partial \eta} + F = 0 \tag{B47}$$

Momentum

$$V\frac{\partial F}{\partial \eta} - \frac{\partial}{\partial \eta}\left(t^{2j} l \bar{\epsilon} \frac{\partial F}{\partial \eta}\right) + \beta\left(F^2 - \Theta\right) = 0 \tag{B48}$$

Energy

$$V\frac{\partial \Theta}{\partial \eta} - \frac{\partial}{\partial \eta}\left(t^{2j} l \frac{\tilde{\epsilon}}{\sigma}\frac{\partial \Theta}{\partial \eta}\right) - \alpha l t^{2j}\bar{\epsilon}\left(\frac{\partial F}{\partial \eta}\right)^2 = 0 \tag{B49}$$

The governing equations for the locally similar system are obtained from equations (27) to (29) by neglecting derivatives of the dependent variables F, Θ, and V with respect to the streamwise coordinate ξ. The capability of obtaining locally similar solutions is desirable in that for a given test case the locally similar and complete nonsimilar solutions can be obtained for the identical program inputs and numerical procedures. Consequently, the effects of the nonsimilar terms on the boundary-layer characteristics for a particular case can be determined by a direct comparison of the results obtained for solutions for $FT = 1$ and 0, respectively.

APPENDIX C

LANGLEY LIBRARY SUBROUTINE FTLUP

Language: FORTRAN

Purpose: Computes $y = F(x)$ from a table of values using first- or second-order interpolation. An option to give y a constant value for any x is also provided.

Use: CALL FTLUP(X, Y, M, N, VARI, VARD)

X	The name of the independent variable x.
Y	The name of the dependent variable $y = F(x)$.
M	The order of interpolation (an integer) M = 0 for y a constant. VARD(I) corresponds to VARI(I) for I = 1, 2, . . ., N. For M = 0 or $N \leqq 1$, $y = F(\text{VARI}(1))$ for any value of x. The program extrapolates. M = 1 or 2. First or second order if VARI is strictly increasing (not equal). M = -1 or -2. First or second order if VARI is strictly decreasing (not equal).
N	The number of points in the table (an integer).
VARI	The name of a one-dimensional array which contains the N values of the independent variable.
VARD	The name of a one-dimensional array which contains the N values of the dependent variable.

Restrictions: All the numbers must be floating point. The values of the independent variable x in the table must be strictly increasing or strictly decreasing. The following arrays must be dimensioned by the calling program as indicated: VARI(N), VARD(N).

Accuracy: A function of the order of interpolation used.

References: (a) Nielsen, Kaj L.: Methods in Numerical Analysis. The Macmillan Co., c.1956, pp. 87-91.
(b) Milne, William Edmund: Numerical Calculus. Princeton Univ. Press, c.1949, pp. 69-73.

Storage: 430_8 locations.

Error condition: If the VARI values are not in order, the subroutine will print TABLE BELOW OUT OF ORDER FOR FTLUP AT POSITION xxx TABLE IS STORED IN LOCATION xxxxxx (absolute). It then prints the contents of VARI and VARD, and STOPS the program.

Subroutine date: September 12, 1969.

REFERENCES

1. Cebeci, Tuncer; Smith, A. M. O.; and Mosinskis, G.: Calculation of Compressible Adiabatic Turbulent Boundary Layers. AIAA Paper No. 69-987, June 1969.
2. Bushnell, Dennis M.; and Beckwith, Ivan E.: Calculation of Nonequilibrium Hypersonic Turbulent Boundary Layers and Comparisons With Experimental Data. AIAA J., vol. 8, no. 8, Aug. 1970, pp. 1462-1469.
3. Hixon, Barbara A.; Beckwith, Ivan E.; and Bushnell, Dennis M.: Computer Program for Compressible Laminar or Turbulent Nonsimilar Boundary Layers. NASA TM X-2140, 1971.
4. Patankar, S. V.; and Spalding, D. B.: Heat and Mass Transfer in Boundary Layers. Morgan-Grampian (London), 1967.
5. Herring, H. James; and Mellor, George L.: A Method of Calculating Compressible Turbulent Boundary Layers. NASA CR-1144, 1968.
6. Anderson, Larry W.; and Kendal, Robert M.: A Nonsimilar Solution for Multicomponent Reacting Laminar and Turbulent Boundary Layer Flows Including Transverse Curvature. AFWL-TR-69-106, U.S. Air Force, Mar. 1970. (Available from DDC as AD 867 904.)
7. Harris, Julius E.: Numerical Solution of the Equations for Compressible Laminar, Transitional, and Turbulent Boundary Layers and Comparisons With Experimental Data. NASA TR R-368, 1971.
8. Pletcher, Richard H.: On a Finite-Difference Solution for the Constant-Property Turbulent Boundary Layer. AIAA J., vol. 7, no. 2, Feb. 1969, pp. 305-311.
9. Flügge-Lotz, I.; and Blottner, F. G.: Computation of the Compressible Laminar Boundary-Layer Flow Including Displacement-Thickness Interaction Using Finite-Difference Methods. AFOSR 2206, U.S. Air Force, Jan. 1962.
10. Harris, Julius Elmore: Numerical Solution of the Compressible Laminar, Transitional, and Turbulent Boundary Layer Equations With Comparisons to Experimental Data. Ph. D. Thesis, Virginia Polytech. Inst., May 1970.
11. Davis, R. T.; and Flügge-Lotz, I.: Laminar Compressible Flow Past Axisymmetric Blunt Bodies (Results of a Second-Order Theory). Tech. Rep. No. 143 (Grants AF-AFOSR-62-242 and AF-AFOSR-235-63), Div. Eng. Mech., Stanford Univ., Dec. 1963.
12. Van Driest, E. R.: Turbulent Boundary Layer in Compressible Fluids. J. Aeronaut. Sci., vol. 18, no. 3, Mar. 1951, pp. 145-160, 216.

13. Bushnell, Dennis M.; and Morris, Dana J.: Shear-Stress, Eddy-Viscosity, and Mixing-Length Distributions In Hypersonic Turbulent Boundary Layers. NASA TM X-2310, 1971.

14. Rotta, J. C.: Heat Transfer and Temperature Distribution in Turbulent Boundary Layers at Supersonic and Hypersonic Flow. Recent Developments in Boundary Layer Research, Pt. 1, AGARDograph 97, May 1965, pp. 35-63.

15. Probstein, Ronald F.; and Elliott, David: The Transverse Curvature Effect in Compressible Axially Symmetric Laminar Boundary-Layer Flow. J. Aeronaut. Sci., vol. 23, no. 3, Mar. 1956, pp. 208-224, 236.

16. Hayes, Wallace D.; and Probstein, Ronald F.: Hypersonic Flow Theory. Academic Press, Inc., 1959, p. 290.

17. Morkovin, Mark V.: Critical Evaluation of Transition From Laminar to Turbulent Shear Layers With Emphasis on Hypersonically Traveling Bodies. AFFDL-TR-68-149, U.S. Air Force, Mar. 1969. (Available from DDC as AD 686 178.)

18. Dhawan, S.; and Narasimha, R.: Some Properties of Boundary Layer Flow During the Transition From Laminar to Turbulent Motion. J. Fluid Mech., vol. 3, pt. 4, Jan. 1958, pp. 418-436.

19. Moore, F. K., ed.: Theory of Laminar Flows. Vol. IV of High Speed Aerodynamics and Jet Propulsion. Princeton University Press, 1964.

20. Richtmyer, Robert D.: Difference Methods for Initial-Value Problems. Interscience Publ., Inc., 1957.

21. Bushnell, Dennis M.; Jones, Robert A.; and Huffman, Jarrett K.: Heat-Transfer and Pressure Distributions on Spherically Blunted 25° Half-Angle Cone at Mach 8 and Angles of Attack up to 90°. NASA TN D-4792, 1968.

22. Lomax, Harvard; and Inouye, Mamoru: Numerical Analysis of Flow Properties About Blunt Bodies Moving at Supersonic Speeds in an Equilibrium Gas. NASA TR R-204, 1964.

23. Moore, D. R.; and Harkness, J.: Experimental Investigation of the Compressible Turbulent Boundary Layer at Very High Reynolds Numbers, M = 2.8. Rep. No. 0-71000/4R-9, LTV Res. Center, Apr. 1964.

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