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# RING STRUCTURE OF A NEUTRAL GAS CLOUD STUDIED IN A ONE-DIMENSIONAL EXPANSION INTO SPACE

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# RING STRUCTURE OF A NEUTRAL GAS CLOUD STUDIED IN A ONE-DIMENSIONAL EXPANSION INTO SPACE

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## SUMMARY

In the present analysis the following assumption is made: When a gas cloud of uncharged particles expands into vacuum, the whole cloud does not change from continuum to free molecular flow at the same time; instead, some regions of the cloud make the transition sooner than others. An attempt is made to choose the simplest possible initial conditions, namely, a uniform distribution with instantaneous release, and then try to find some effect, within the context of ordinary gas dynamics, that tends toward the ring structure which has been observed in barium cloud experiments, admitting only the necessity to consider how the cloud goes from continuum to free molecular flow. The velocity distributions for the two kinds of flow, when superposed, are shown to yield a velocity distribution for the whole cloud that exhibits ring structure. The treatment is in one dimension.

## INTRODUCTION

Neutral clouds fall into two main classes: those used for atmospheric research and those that occur in space experiments. The latter class, where the cloud expands into vacuum, is the one of interest in the present paper. The expansion of an initially uniform distribution of gas into vacuum has been studied theoretically, and the solution where the cloud is assumed to be a continuum throughout is known in one dimension. In one dimension the problem, although difficult, is analytically tractable; however, in three dimensions numerical integration must be used. The results of numerical integration for various initial conditions in three dimensions are given in reference 1, wherein both continuum and particle approaches were used. These results do not differ greatly for the two approaches.

The experimental results are derived from space experiments where clouds are actually injected into the magnetosphere (refs. 2 and 3). Neutral clouds, made visible by photoexcitation, show a ring or shell-like structure (fig. 1); that is, the outer portion of the cloud forms a ring of brightness. A densitometer trace of a photographic image of a neutral cloud is given in figure V-3 of reference 3.

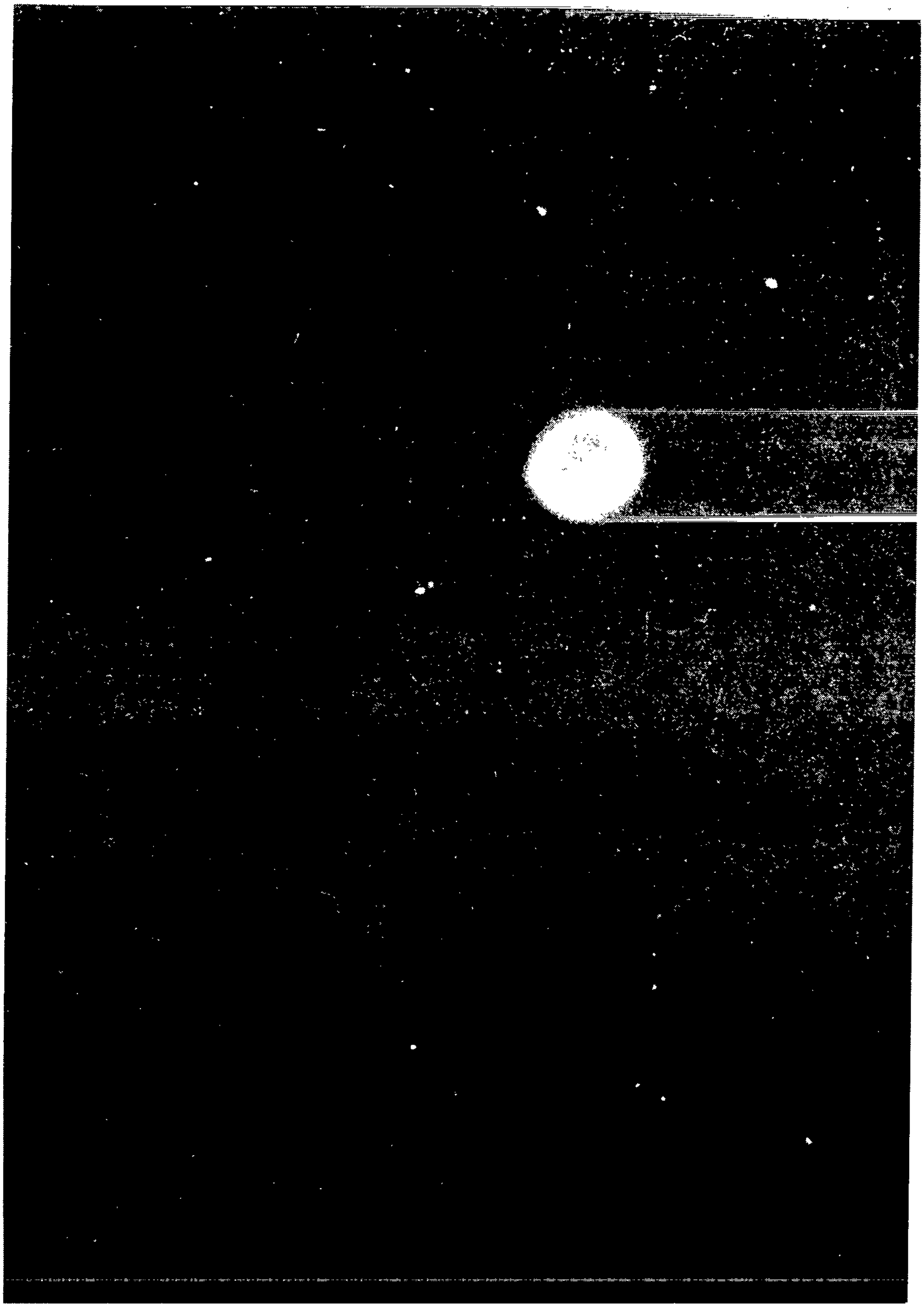

Figure 1.- Ring structure in barium cloud released at 930-km altitude.

The observed ring structure exhibited by a neutral cloud released in the almost perfect vacuum existing at very high altitudes implies a higher particle density near the radius of the ring. Since this structure is persistent, the particles within the bright ring clearly have very similar velocities. Thus, the phenomenon indicates that an unexpectedly large portion of the gas expands with a certain preferential velocity. This bunching of velocity is extremely pronounced. Thus, calculations made from optical observations of a release of barium at an altitude of almost 930 kilometers indicate that the radially outward velocity of the shell was of the order of 1.2 km/sec (ref. 3) and the rate of increase of the shell width (thermal spread) was of the order of 0.15 km/sec.

When the initial conditions are uniform, neither continuum nor free molecular flow solutions predict the ring structure (ref. 1). The ring structure is attributed in the present paper to the manner of transition from continuum to free molecular flow. It seems implausible to assume that such transition occurs instantaneously throughout the cloud. Instead, transition is assumed to occur gradually (starting when the average mean free path becomes appropriately large compared to some nominal cloud dimension). At any instant, the local mean free path is expected to vary considerably from the center to the outer reaches of the cloud, and free molecular flow is expected to prevail in the outer reaches before it occurs in the center. The relative sizes of the two regimes will not necessarily remain fixed. The demarcation sphere or transition interface is expected to grow with the cloud in some way, depending on the criterion for transition from continuum to free molecular flow.

Although contrary to actuality, for simplicity no intermediate state between continuum and collisionless flow is assumed to exist in the present study. Of importance is the fact that once the particles pass into the free molecular flow region they will be unaccelerated, although there is a possibility of their being peaked around a certain velocity.

Although when a given mass element of gas makes the transition it also begins to spread, this does not mean that ring structure tends to wash out with time, because the whole cloud is spreading at the same time. Although ring structure is concerned with the space distribution of particles, the velocity distribution is intimately related to the space distribution when a sufficiently long time is allowed to elapse after that time at which the particles may be considered to have become noninteracting. For example, if given a one-dimensional container of noninteracting gas particles, then the velocity distribution converges to the space distribution with the velocity scale labeled $x/t$ and with the distribution function scale labeled $nt/n_0 l$ where $n$ is the particle density within the cloud, $n_0$ is the initial particle density, and $l$ is the location of the wall. The convergence involves the ratio of the distance traveled by a large fraction of the particles to the original container size. Thus, if $x$ is the distance traveled by a particle in time $t$ at velocity $w$, from a container of size $d$, then all particles that had velocity $w$ will be within $x = wt \pm d$; but the uncertainty in position $d$ becomes negligible compared to the distance traveled $x$ as $t \to \infty$. In the present paper, these considerations will be implicit when velocity distributions are discussed as if they were space distributions.

For simplicity, one-dimensional flow only is considered. Caution should be used in interpreting the results as applicable to three-dimensional flow. However, use of the present cloud model in two- and three-dimensional flow analyses could shed additional light on its efficacy.

Reference 4 attempts to explain ring structure; however, an error in the analysis (eqs. (4), (11), and (16)) completely invalidates the conclusions. The error arises (in calculating density) from neglecting to allow the $v^2$ in the volume element

$$4\pi r^2\, dr = 4\pi t^3 v^2\, dv$$

to cancel the term $v^2$ in the three-dimensional speed distribution $v^2 f(v)$ where $f$ is the familiar bell-shape. Obviously, this error would introduce considerable ring structure because $v^2 f(v)$ has a peak displaced from $v = 0$, whereas $f(v)$ peaks at $v = 0$. In the foregoing equation $v$ is the velocity of the gas and $r$ is the radius vector, as used in reference 4.

## SYMBOLS

| | |
|---|---|
| $C$ | dimensionless form of $c$ |
| $C_c$ | dimensionless critical sound speed |
| $c$ | speed of sound |
| $f$ | velocity distribution function |
| $l$ | location of wall |
| $N$ | density-velocity distribution function |
| $n$ | particle density within cloud |
| $Q$ | dimensionless constant |
| $R, r$ | constants six and one, respectively, for $\gamma = 5/3$ |

| | |
|---|---|
| T | dimensionless form of t |
| t | time coordinate |
| U | dimensionless form of u |
| u | flow velocity |
| V | dimensionless form of v |
| v | velocity of transition interface |
| W | dimensionless form of w |
| w | particle velocity |
| X | dimensionless form of x |
| x | space coordinate |
| $\gamma$ | ratio of specific heats, 5/3 |
| $\Lambda$ | ratio of mean free path to its initial value |
| $\lambda$ | mean free path |
| $\theta$ | thermodynamic state function equal to $(3c)^2$ for $\gamma = 5/3$ |
| $\tau$ | integration variable for time |

Subscript:

| | |
|---|---|
| o | initial condition |

Superscript:

| | |
|---|---|
| * | time at which reflected wave catches transition interface |

Mathematical notation:

:: is proportional to

O order of

## PRELIMINARY REMARKS

When a given volume element of gas passes through the transition interface, the particle acceleration drops to zero and no more collisions occur, but the particles do not move with the same velocity as the gas element did before transition – only the center of gravity of the gas element does. The situation is therefore very complicated microscopically even in free molecular flow. Note, however, that the velocity distribution function for a volume element of gas which has passed through the transition interface is no longer a function of time nor position, but only of velocity. In particular, the velocity distribution function does not spread. The space distribution function for that element spreads linearly, as is also true for the total amount of gas that has passed through the transition interface at any given time; however, the velocity distribution function depends only on velocity. This invariance of the velocity distribution function is very useful in studying ring structure, because when a sufficiently long time has elapsed after the cloud has made the transition to free molecular flow, the velocity distribution function converts to the space distribution by simply relabeling the axes $f \rightarrow nt/n_0 l$ and $w \rightarrow x/t$.

### Equivalence of Continuum and Free Molecular Flow at Large Values of Time

Equations (18;23) and (18;24) of reference 5 show that as $t \rightarrow \infty$, the density depends only on velocity and time, not on the spatial coordinate, by assuming that the velocity is the space coordinate divided by the time. Thus, the actual asymptotic expressions found in reference 5 to hold for large values of time are

$$u = \frac{x}{t}$$

$$n :: \frac{1}{t}\left[1 - \left(\frac{u}{\theta_0}\right)^2\right]^2$$

Since $u$ and $n$ depend only on $t$ and $x/t$, these equations are indicative of free molecular flow. Therefore it is not surprising that a criterion can be chosen (as is actually done in this paper) for transition that would mean that some of the expanding gas never makes the transition, because for large values of time the continuum equations show that continuum and free molecular flow are functionally similar.

Apparently, if a peak can be demonstrated in the velocity distribution function, then an indication of ring structure is obtained because the velocity distribution function is closely related to the space distribution function for sufficiently long times. This statement applies not only to the free molecular flow regime but to the continuum flow regime, in view of the equations just discussed.

### Overall Procedure

The procedure is to calculate the velocity distribution at some appropriate time for the whole flow (continuum and free molecular), and then to assume that at that time the particles become free molecular, thereby implying that at some indefinitely large, later value of time the calculated velocity distribution converts to space distribution as previously discussed. The time chosen to make the calculation must be long enough so that the results either become reinforced or do not change with further increases in time. Then, for some much larger but unspecified value of time, the resultant velocity distributions, with any peaks that show up, are interpreted as space distributions with ring structure.

## FOUNDATION MATERIAL

The problem of the one-dimensional continuum expansion into a vacuum is discussed in reference 5, in which an initially uniform gas is contained in a tube between a diaphragm

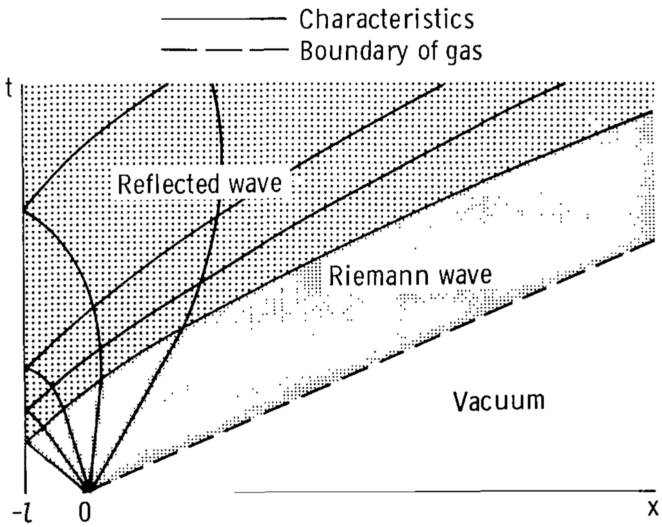


Figure 2.- Riemann and reflected wave represented schematically in x-t plane.

at $x = 0$ and a wall at $x = -l$, where $x$ is the distance along the tube. The diaphragm at station $x = 0$ is ruptured at time $t = 0$, and the gas expands in the positive x-direction. The flow, as shown in figure 2, divides itself into two regions: (1) The Riemann wave which is the flow ahead of the first reflected characteristic that propagates from the wall at $x = -l$, and (2) the reflected wave that constitutes the rest of the flow. Here the language of reference 5 is used where wave means a region of the expansion flow rather than a characteristic. The solution to this problem (ref. 6) was first given by Riemann.

## Riemann Wave

The velocity $u$ and the speed of sound $c$ at position $x$ and at time $t$ are related by the equation

$$u - c = \frac{x}{t} \tag{1a}$$

$$u = \frac{2}{\gamma - 1}(c_o - c) \tag{1b}$$

where $c_o$ is the speed of sound prior to diaphragm rupture, and $\gamma$ is the ratio of specific heats.

Now for convenience, make equations (1) dimensionless. Let

$$\left.\begin{aligned} X &= \frac{x}{l} \\ U &= \frac{u}{c_o} \\ C &= \frac{c}{c_o} \\ T &= \frac{c_o t}{l} \end{aligned}\right\} \tag{2}$$

where $T$ is seen to be the time in units of the time it takes for a sound wave to go from $x = -l$ to 0 in the initial distribution.

By introducing these dimensionless quantities into equations (1) and adopting the value $\gamma = 5/3$

$$U - C = \frac{X}{T} \tag{3a}$$

$$U = 3(1 - C) \tag{3b}$$

which show that the limit velocity (C = 0) is 3, which is higher than the corresponding value for steady flow by a factor $\sqrt{3}$; for $\gamma = 7/5$, the factor is $\sqrt{5}$.

By solving for U and C

$$U = \frac{3}{4}\left(1 + \frac{X}{T}\right) \tag{4a}$$

$$C = \frac{1}{4}\left(3 - \frac{X}{T}\right) \tag{4b}$$

which show that at any time T, the quantities U and C are linear in X in the Riemann wave.

The partial differential equation of which equations (1) are solutions is hyperbolic. Therefore characteristics, which will be curves in the space-time plane, exist. The two families of characteristics (ref. 6, p. 93) are classed as forward or backward facing according to whether the flow crosses from positive to negative face or negative to positive face. (Positive face is in the direction of increasing X.) In the present problem, the characteristics that move away from the position X = 0 at T = 0 are backward facing, and only these are of concern in the Riemann wave.

In the space-time plane, the backward-facing characteristics are a fan of straight lines originating at X = 0, T = 0. The mathematical importance of the backward-facing characteristics is contained in the general statement

$$U - 3C = \text{Constant on curve } \frac{dX}{dT} = U - C \tag{5}$$

If equations (4) are used in the differential equation (5), the backward-facing characteristics are

$$\frac{X}{T} = \text{Constant} \tag{6}$$

or straight lines through the origin. Thus, for the Riemann wave, both U and C are constant on characteristics.

Now $\lambda :: n^{-1}$; furthermore, for isentropic flow

$$\frac{n}{n_o} = \left(\frac{c}{c_o}\right)^{\frac{2}{\gamma-1}} \tag{7a}$$

Denoting by $\Lambda$ the ratio of mean free paths $\lambda/\lambda_o$

$$\Lambda = \frac{\lambda}{\lambda_o} = C^{-3} \tag{7b}$$

for the special case of $\gamma = 5/3$. Thus, within the Riemann wave, the mean free path is constant along characteristics.

## Reflected Wave

The following analytical expressions for the reflected wave are derived in reference 5:

$$t = \frac{lR}{2r!} \frac{\partial^r}{\partial \theta^r} \frac{\left[\left(\sqrt{\theta} + u\right)^2 - \theta_o\right]^r}{\sqrt{\theta}} \tag{8a}$$

$$x = ut - \frac{l}{(r - 1)!} \frac{\partial^{r-1}}{\partial \theta^{r-1}} \frac{\left(\sqrt{\theta} + u\right)\left[\left(\sqrt{\theta} + u\right)^2 - \theta_o\right]^{r-1}}{\sqrt{\theta}} \tag{8b}$$

where $r$ is an index related to $\gamma$ by the expression

$$\gamma = \frac{2r + 3}{2r + 1}$$

and

$$R = 2(2r + 1)$$

$$\theta = \left(\frac{2c}{\gamma - 1}\right)^2$$

For

$$\gamma = 5/3$$

$$r = 1$$

and

$$R = 6$$

$$\theta = (3c)^2$$

By making these substitutions

$$t = \frac{l}{3c^2}\left\{(3c + u) - \frac{1}{6c}\left[(3c + u)^2 - (3c_o)^2\right]\right\} \tag{9a}$$

$$x = ut - \frac{l(3c + u)}{3c} \tag{9b}$$

By using equations (2) to make these dimensionless

$$T = \frac{1}{3C^2}\left\{(3C + U) - \frac{1}{6C}\left[(3C + U)^2 - 9\right]\right\} \tag{10a}$$

$$X = UT - \frac{3C + U}{3C} \tag{10b}$$

Equations (10) represent a complete solution to the problem of the expansion into vacuum for the part of the flow in the reflected wave; equations (10) taken with equations (3) or (4) completely solve the problem of the expansion into vacuum for continuum flow.

## Riemann-Wave—Reflected-Wave Interface

The Riemann-wave—reflected-wave interface is the first characteristic reflected from the wall after the diaphragm is ruptured. In one-dimensional flow it is a plane normal to the flow that is reflected downstream (after having traveled to the wall from the diaphragm). In X-T space it is a curve like the Mach waves of supersonic aerodynamics.

On the interface separating the two flow regimes, equations (3) and (10) are simultaneously satisfied; if equations (3) and (10a) are used, the following relations are established:

$$T = \frac{1}{C^2} \tag{11a}$$

$$X = 3T - 4T^{1/2} \tag{11b}$$

These relations are used to establish limits for certain required integrals.

## PRESENT ANALYSIS

### Continuum Flow Region

If given a quantity of gas between two neighboring sections $dx$ apart in the tube such that the speed of sound is $c(x)$, and the flow velocity is $u(x)$, and denoting particle velocity by $w$, the velocity distribution function assumes the form

$$f(w)\, dw = \left(\frac{\gamma}{2\pi}\right)^{1/2} \frac{1}{c(x)} \exp\left\{-\frac{\gamma}{2}\left[\frac{w - u(x)}{c(x)}\right]^2\right\} dw \tag{12}$$

Within the continuum regime, if $n(x)$ denotes density of particles, then the total number of particles between sections $x_1$ and $x_2$ having velocity $w$ within the range $dw$ is given by

$$N(w)\, dw = \left(\frac{\gamma}{2\pi}\right)^{1/2} dw \int_{x_1}^{x_2} \frac{n(x)}{c(x)} \exp\left\{-\frac{\gamma}{2}\left[\frac{w - u(x)}{c(x)}\right]^2\right\} dx \tag{13}$$

By using equations (7) and introducing dimensionless parameters as defined by equations (2)

$$N(W)\, dW = n_o l\left(\frac{\gamma}{2\pi}\right)^{1/2} dW \int_{X_1}^{X_2} C^2 \exp\left[-\frac{\gamma}{2}\left(\frac{W - U}{C}\right)^2\right] dX \tag{14}$$

where

$$W = \frac{w}{c_o}$$

## Free Molecular Flow Region

Now, it is desired to find the velocity distribution function for that part of the expanding gas that has passed through the interface at which the transition to free molecular flow takes place up to time $t$. At time $\tau$, the volume rate of flow for particles which are passing through the interface at speeds between $w$ and $w + dw$ is equal to $(w - v) \times$ (Unit cross section), where $v$ is the velocity of the interface. Furthermore, the particle density at time $\tau$, at the interface, with speeds between $w$ and $w + dw$, is $nf$, obtained by multiplying the particle density by the velocity distribution function.

The velocity distribution function will be that of the continuum gas at the interface, the position of which is known at any time

$$f(w,\tau) = \left(\frac{\gamma}{2\pi}\right)^{1/2} \frac{1}{c(\tau)} \exp\left\{-\frac{\gamma}{2}\left[\frac{w - u(\tau)}{c(\tau)}\right]^2\right\} \tag{15}$$

Hence, the number of particles having velocity in the range $w$ to $w + dw$ which have passed through the interface at time $t$ is given by

$$N(w)\, dw = \left(\frac{\gamma}{2\pi}\right)^{1/2} dw \int_0^t \left[w - v(\tau)\right]\frac{n(\tau)}{c(\tau)} \exp\left\{-\frac{\gamma}{2}\left[\frac{w - u(\tau)}{c(\tau)}\right]^2\right\} d\tau \tag{16}$$

where $c(\tau)$, $u(\tau)$, $v(\tau)$, and $n(\tau)$ are velocity of sound, flow velocity, interface velocity, and particle density at location of interface at time $\tau$, respectively.

Nondimensionalizing as before

$$N(w) = n_0 l\left(\frac{\gamma}{2\pi}\right)^{1/2} \int_0^T (W - V)C^2 \exp\left[-\frac{\gamma}{2}\left(\frac{W - U}{C}\right)^2\right] dT \tag{17}$$

## Transition Criterion

The transition between the continuum region and the free molecular region was assumed to be sharp on a moving boundary between these flow regions. The criterion that will be used to define the transition interface in this paper is

$$U = Q \tag{18}$$

where $Q$ is a constant. This equation, with the flow equations for either the Riemann or the reflected wave, serves to identify a section (which will be called the "transition interface") that moves in the tube such that it occupies the position where the streaming velocity has the given value $Q$.

It is now possible to see how the criterion $U = Q$ is effective in building a peak in the velocity distribution function. Set equal to unity, in equation (17), the constant before the integral and the $C$ and the $\gamma/2$ inside. By using dimensionless variables, the incremental velocity distribution function for a time element $\Delta T$ is

$$\Delta N(W) = (W - V)\exp\left[(W - Q)^2\right]\Delta T \tag{19}$$

in which the criterion $U = Q$ has been used. Figure 3 shows $\Delta N(W)$. The displaced Maxwellian $\exp\left[(W - Q)^2\right]$ of the gas element undergoing transition is weighted by the $W - V$ such that a peak occurs for $W$ somewhat greater than $Q$ and a valley for $W$ somewhat less than $V$. The implications of the criterion with respect to the notion of mean free path as compared to the size of the phenomenon are brought out in a subsequent section entitled "DISCUSSION."

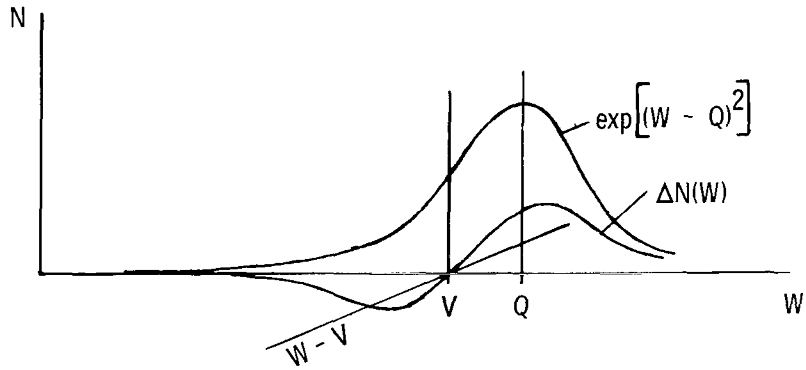


Figure 3.- Illustration of manner in which equation (19) builds a peak.

## Motion of Transition Interface

Riemann wave.- The result of substituting equation (18) into equations (3) is

$$\frac{X}{T} = \frac{4}{3}Q - 1 \tag{20a}$$

$$C = 1 - \frac{Q}{3} \tag{20b}$$

Equation (20a) shows that the transition section in the Riemann wave moves at the constant speed

$$V = \frac{dX}{dT} = \frac{4}{3}Q - 1 \tag{21}$$

and its path in the X-T plane is a straight line through the origin, identical to a backward-facing characteristic. Equation (20b) shows that the speed of sound at the transition interface is constant.

Equations (20a) and (11b) can be used to find where the transition interface meets the first reflected characteristic. Thus

$$T^* = \left(1 - \frac{1}{3}Q\right)^{-2} \tag{22}$$

Reflected wave.- By substituting equation (18) into equations (10) the following parametric equations for the transition interface path in X-T space are obtained

$$T = \frac{3(3C + Q)}{(3C)^2} - \frac{3}{2}\,\frac{(3C + Q)^2 - 9}{(3C)^3} \tag{23a}$$

$$X = QT - \frac{3C + Q}{3C} \tag{23b}$$

Equation (23a) may be rearranged as

$$(3C)^3 - \frac{3}{2T}(3C)^2 + \frac{3}{2T}\left(Q^2 - 9\right) = 0 \tag{24}$$

which is cubic in 3C with Q and T as parameters. Therefore, for any T and preselected value of Q, a value for C can be determined. (The appropriate root is defined by the need to be equal to that derived previously for the Riemann wave at the bounding point between the two flow regimes.)

The velocity V can then be obtained from the parametric path representations of equations (23) as follows:

$$V = \frac{dX/dC}{dT/dC} = Q - \frac{\frac{d}{dC}\left(\frac{3C + Q}{3C}\right)}{dT/dC}$$

$$V = Q\left[1 - \frac{2C^2}{3C^2 - \left(Q^2 - 9\right)}\right] \tag{25}$$

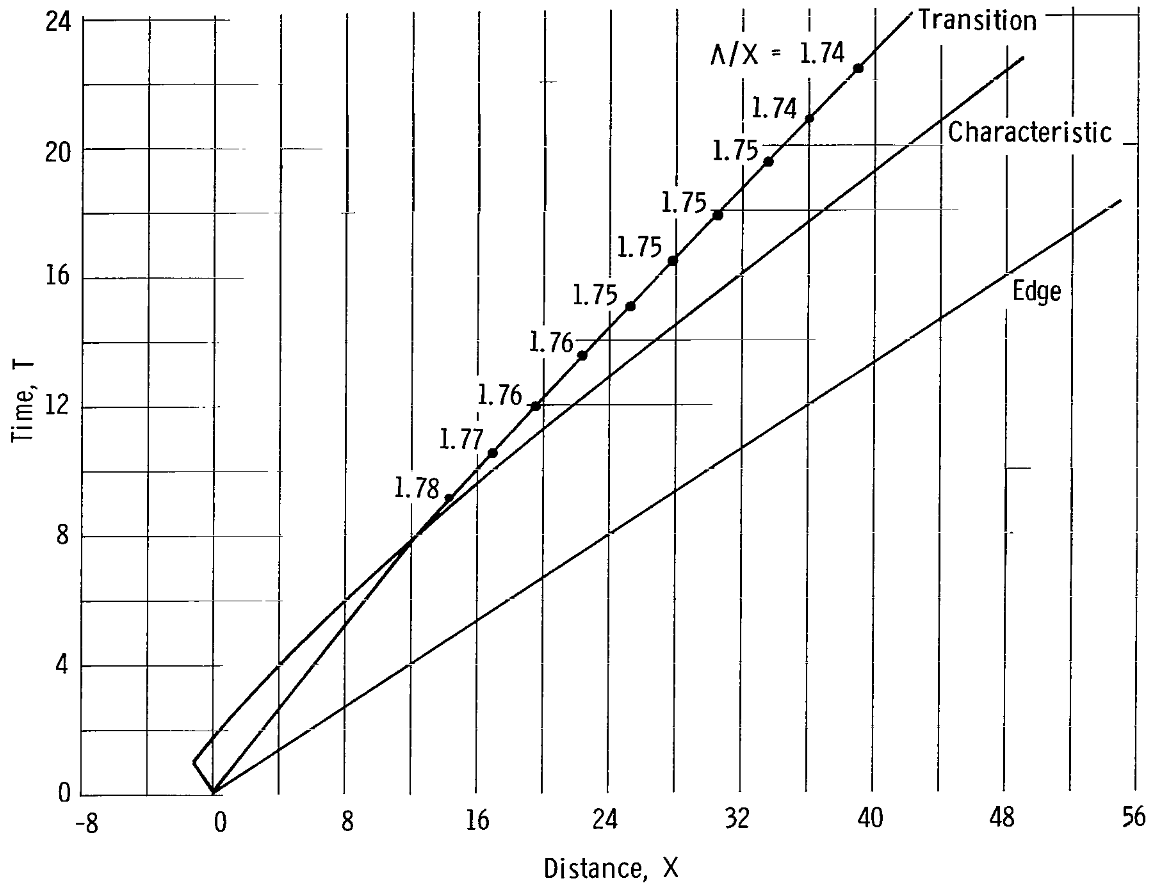


Figure 4.- Path of transition section, first reflected characteristic, and leading edge in X-T plane. Q = 1.9.

The path of the transition section in the X-T plane is shown in figure 4 along with that for the first reflected characteristic for a value of Q = 1.9. The leading edge of the expansion is a line through the origin with a slope of 1/3. (The limit velocity is 3.) Note that a considerable portion of the flow does not make the transition.

## Equations for Computation of Velocity Distribution Function

The integration paths in equations (14) and (17) are shown in figure 5.

Without transition

$$N(W,T)\,dW = n_o l\left(\frac{\gamma}{2\pi}\right)^{1/2} dW\left\{\int_{-1}^{X_2} C^2 \exp\left[-\frac{\gamma}{2}\left(\frac{W-U}{C}\right)^2\right]dX + \int_{X_2}^{3T} C^2 \exp\left[-\frac{\gamma}{2}\left(\frac{W-U}{C}\right)^2\right]dX\right\} \tag{26}$$

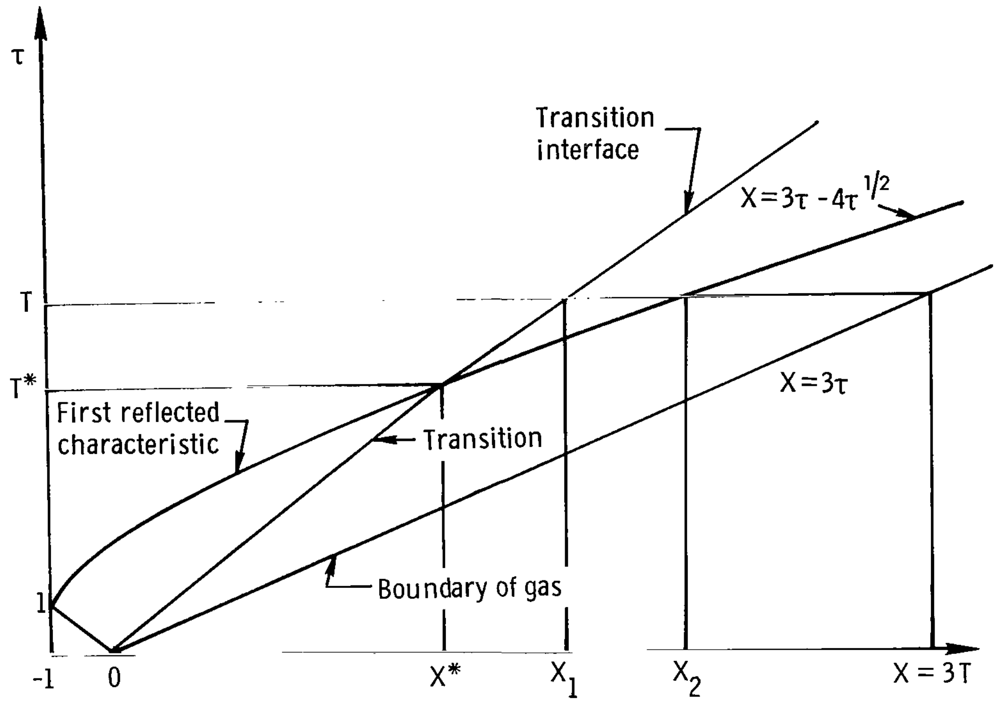


Figure 5.- Paths of integration for equations (14) and (17).

In the first integral, for any $X$ at a given $T$, the quantities $U$ and $C$ are given by equations (10); $X_2$ is given by equation (11b). In the second integral, equations (4) are used.

With transition

$$N(W,T)\,dW = n_0 l\left(\frac{\gamma}{2\pi}\right)^{1/2} dW\left\{\int_{-1}^{X_1} C^2 \exp\left[-\frac{\gamma}{2}\left(\frac{W - U}{C}\right)^2\right]dX + \int_0^{T^*} (W - V)C^2 \exp\left[-\frac{\gamma}{2}\left(\frac{W - Q}{C}\right)^2\right]d\tau + \int_{T^*}^{T} (W - V)C^2 \exp\left[-\frac{\gamma}{2}\left(\frac{W - Q}{C}\right)^2\right]d\tau\right\} \tag{27}$$

In the first integral, $U$ and $C$ are again evaluated by using equations (10). In the second integral, equations (20) and (21) are used. In the third integral, equations (24) and (25) are used. The quantity $X_1$ is given by equations (24) and (23b); $T^*$ by equation (22).

Equations (26) and (27) were programed on the Control Data series 6000 computer systems. Figure 6 shows the distribution functions (which for large values of time convert to space distributions) for various values of $Q$ and for a time value of $20T^*$. Ring

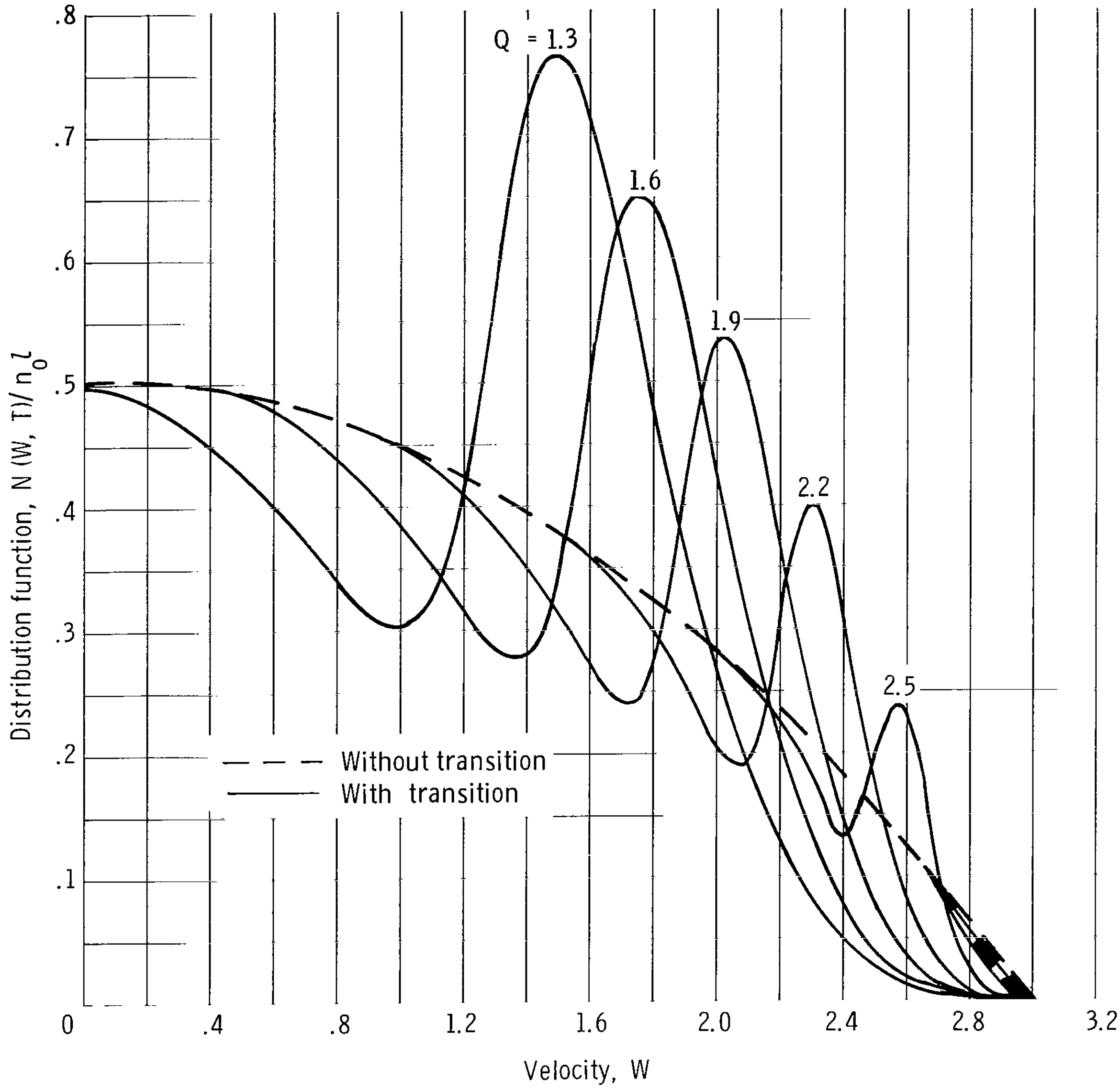


Figure 6.- Density-velocity distributions for expansion with transition compared with same without transition for $T/T^* = 20$.

structure is indeed apparent. For example, at $Q = 1.9$ and $T = 20T^*$, a peak occurs in the curve marked "with transition" that is about two times the value for the curves marked "without transition;" that is, transition appears to impose a fairly strong perturbation on the distribution curve that corresponds to an all-continuum expansion. The indication of ring structure was sufficiently strong to indicate that a more rigorous attempt (using the Boltzmann equation with a simple atomic model like a square-well potential or a Maxwellian force law) to solve the problem could be justified.

## DISCUSSION

### Actual Stagnation and Initial Conditions

The temperature of the barium charge immediately after release was $2300^{\circ}$ K (ref. 3). It is believed that the temperature of $2300^{\circ}$ K should be associated with the temperature of the gas leaving the burning charge. If a limiting situation is assumed, this gas could have a velocity as high as a Mach number of 1 with respect to the flame front advancing into the charge (ref. 6, p. 212). By neglecting the speed of the flame front as compared to the speed of the gaseous combustion products, the temperature of $2300^{\circ}$ K gives a speed of sound $C_c = 0.48$ for the gas at a Mach number of 1. The value of $C_o$ for stagnation remains to be found. Now, in steady flow

$$\frac{C_o}{C_c} = \sqrt{\frac{\gamma + 1}{2}} = 1.155$$

but for the present case equation (16) shows

$$\frac{C_o}{C_c} = \frac{\gamma + 1}{2} = \frac{4}{3}$$

where $C_c$ is sound speed at a Mach number of 1. Therefore,

$$C_o = 0.48 \times \frac{4}{3} = 0.64 \text{ km/sec}$$

and

$$Q = \frac{1.2}{0.48 \times \frac{4}{3}} = 1.875$$

where 1.2 km/sec is the expansion velocity of the cloud.

It is believed that the initial conditions are not closely linked to the original container of the burning barium charge. Instead, it would seem more appropriate to consider the burning time of approximately 0.15 sec (ref. 3, p. 16) and the expansion velocity of 1.2 km/sec to establish a size $l \approx 0.2$ km. Then by using the value for the stagnation speed of sound $c_o = 0.64$ km/sec, the unit time corresponds to $0.2/0.64 \approx 0.3$ sec.

From these numbers it is possible to see what real time a value of $N = 20$ corresponds to; for example, $Q = 1.9$. The computed value of $T$ is 148.76, and therefore the corresponding real-time value is $148.76 \times 0.3 = 45$ sec. If it took, for example, 10 times this value for the velocity distributions to convert to space distributions, the real time would be 7 to 8 minutes. This real time is greater by a factor of two than the maximum time for which the neutral cloud has been observed (ref. 3, p. 98), but if N had been chosen as 10 instead of 20, the time could have been shortened by a factor of two without greatly washing out the calculated ring structure. (See fig. 7.)

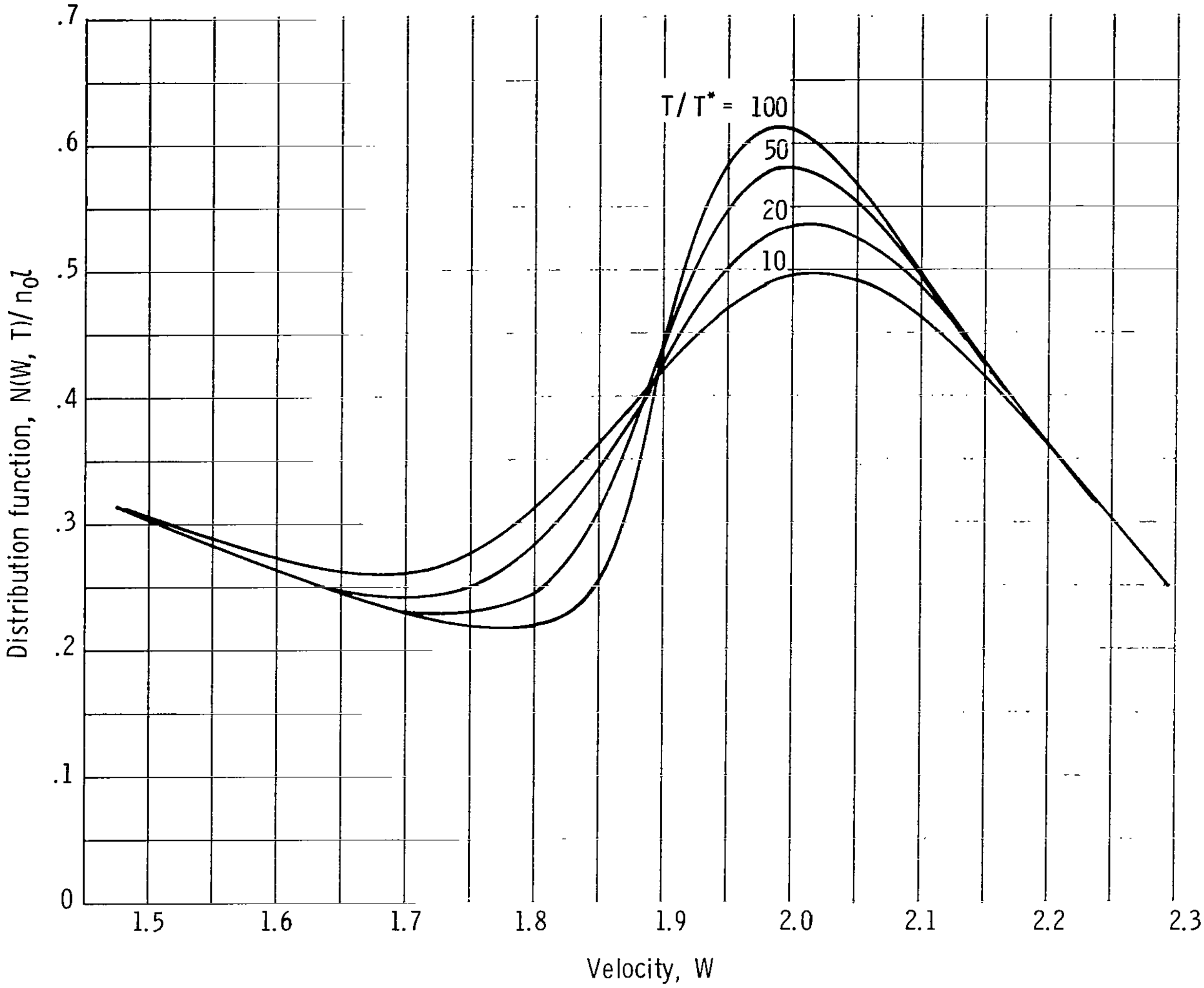


Figure 7.- Development of peak in density-velocity distribution curves for various times at $Q = 1.9$.

The initial ratio of mean free path to size is of interest. The values in reference 3, page 16, for the total number of $10^{24}$ atoms, and the collisional radius for the atoms of $3 \times 10^{-8}$ cm give, with the value given above $l = 0.2$ km

$$\frac{\lambda_o}{l} = \frac{l^2}{(\text{Total number of atoms}) \times (\text{Collision radius})^2} = 0.44$$

Thus the cloud is of low density initially on the basis of mean free path and size.

## Transition Criterion and Relation of Mean Free Path to Size

The ratio of $\Lambda$ to $X$ on the transition interface can be found from equations (23b) and (7b); thus

$$\frac{\Lambda}{X} = \frac{1}{XC^3} = \frac{1}{C^3TQ + O(C^2)} \tag{28}$$

and from equation (24)

$$C^3T = \frac{1}{18}(9 - Q^2) + O(C^2) \tag{29}$$

Hence

$$\frac{\Lambda}{X} = \frac{1}{\frac{Q(9 - Q^2)}{18} + O(C^2)}$$

At large values of time $C \rightarrow 0$. Hence

$$\frac{\Lambda}{X} = \frac{18}{Q(9 - Q^2)} \tag{30}$$

Moreover, for values of $Q$ in the range of interest, computations show that the ratio $\Lambda/X$ does not depart significantly from its asymptotic value, even for decreasing values of $X$ and $T$ down to the Riemann wave. This is seen in figure 4 where the values of $\Lambda/X$ are marked on the transition path in the X-T plane. Substitution of equations (2) and (7b) into equation (30) gives

$$\frac{\lambda}{x} = \frac{\lambda_o}{l}\frac{18}{Q(9 - Q^2)} \tag{31}$$

and for $Q = 1.9$

$$\frac{\lambda}{x} = 1.76\frac{\lambda_o}{l} \tag{32}$$

Now, if the ratio $\lambda/x$ is to be approximately unity, so that the mean free path would be approximately equal to the size of the continuum region, then

$$\frac{\lambda_o}{l} = \frac{1}{1.76} = 0.57$$

This implies that the cloud under consideration must be of low density even initially. This low initial density corresponds to the previous calculation in the section entitled "Actual Stagnation and Initial Conditions" where the ratio of mean free path to size, based on experimental data, was shown to be initially 0.44.

### Limiting Peak Height of Velocity Distribution Function

Equation (27) produces a peak in the velocity distribution function N(W) that increases with time, as may be seen from figure 7. The fact that the peaks are bounded in time can be seen from an examination of equation (27) by considering only the third integral on the right. (The first two integrals on the right do not produce any increase with time.) Use of N' to denote the integral in question, which must be evaluated on the transition interface, gives

$$N'(W) = \int_{T^*}^{T} (W - V)C^2 \exp\left[-\frac{\gamma}{2}\left(\frac{W - Q}{C}\right)^2\right] d\tau \tag{33}$$

Now, note that the limit as $T \to \infty$ is desired. First, assume that $T^*$ is large enough so that the approximation of equation (29) may be used. Next, observe that V is given exactly by equation (25). It is convenient now to change the integration variable to C, instead of T, by using equation (29) as follows

$$N'(W) = \frac{9 - Q^2}{6} \int_{C=0}^{C^*} (W - V)C^{-2} \exp\left[-\frac{\gamma}{2}\left(\frac{W - Q}{C}\right)^2\right] dC \tag{34}$$

In this expression for N'(W), all quantities are constant in the integration except C and V; V is a function of C as shown by equation (25); Q is a constant; W is arbitrary and has its interesting values, for the present purpose, a little greater than Q, since the peak is to the right of the position W = Q. Some question may arise as to whether the $C^{-2}$ in the integrand makes the integral indefinitely large when W = Q. In this case equation (25) gives

$$W - V = \frac{2C^2}{3C^2 - (Q^2 - 9)} \tag{35}$$

so that $C^{-2}$ cancels out of the integrand and no infinity exists in the integrand.

## CONCLUDING REMARKS

An analysis has been made of the one-dimensional expansion of a cloud into space. The cloud is considered to be made up of an inner continuum flow region and an outer free molecular flow region. The interplay between the two regions can produce the observed ring structure.

The transition between the continuum region and the free molecular region was assumed to be sharp on a moving boundary between these flow regions.

The criterion for transition was chosen to produce a peak in the particle density distribution.

The indication of ring structure was sufficiently strong to indicate that a more rigorous attempt (using the Boltzmann equation with a simple atomic model like a square-well potential or a Maxwellian force law) to solve the problem could be justified.

The cloud as modeled herein is of low density even initially. The fact that the ring structure only shows up well in space experiments rather than in ground experiments is not surprising.

Use of the present cloud model in two- and three-dimensional flow analyses could shed additional light on its efficacy.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., March 15, 1972.

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