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LATERAL STABILITY AND CONTROL DERIVATIVES OF A JET FIGHTER AIRPLANE EXTRACTED FROM FLIGHT TEST DATA BY UTILIZING MAXIMUM LIKELIHOOD ESTIMATION

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16. Abstract

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SUMMARY

A method of parameter extraction for stability and control derivatives of aircraft from flight test data, implementing maximum likelihood estimation, has been developed and successfully applied to actual lateral flight test data from a modern sophisticated jet fighter. This application demonstrates the important role played by the analyst in combining engineering judgment and estimator statistics to yield meaningful results. During the analysis, the problems of uniqueness of the extracted set of parameters and of longitudinal coupling effects were encountered and resolved. The results for all flight runs are presented in tabular form and as time history comparisons between the estimated states and the actual flight test data.

INTRODUCTION

A method of parameter extraction for stability and control derivatives of aircraft from flight test data has been developed at the Langley Research Center (ref. 1). This method, utilizing maximum likelihood estimation, has been applied to actual longitudinal flight test data from a modern sophisticated jet fighter airplane (ref. 2) to establish the merits of the estimation technique and its computer implementation.

In the present study, the application of the method to actual lateral flight test data from the same airplane has also been used to establish the merits of the estimation technique and its computer implementation by extracting, from the flight data, a set of stability and control derivatives that are well defined in terms of their standard deviations. During the analysis, the problems of uniqueness of the extracted set of parameters and of longitudinal coupling effects were encountered and resolved. The results presented demonstrate that the technique provides sufficient information to identify the uniqueness problem, if one exists, in terms of parameter correlations. The results also demonstrate that sufficient excitation of the aircraft will yield a unique set of derivatives and that incomplete modeling will be indicated by a poor fit of the data.

The flight test runs utilized in this study are lateral responses generated by rudder and/or aileron deflections in the neighborhood of $\pm 10^{\circ}$ and $\pm 20^{\circ}$, respectively. The changes in angle of sideslip, roll angle, rolling velocity, yawing velocity, and lateral acceleration are typically $\pm 10^{\circ}$, $\pm 30^{\circ}$, and $\pm 40^{\circ}$ per second, $\pm 15^{\circ}$ per second, and ± 0.2g, respectively. The parameters extracted were the standard linear body-axis lateral stability and control derivatives, with additional nonlinear derivatives dependent upon angle of attack. These nonlinear derivatives were found to be necessary due to strong longitudinal motion present in some of the flight test runs.

SYMBOLS

Measurements and calculations were made in the U.S. Customary Units. They are presented herein in the International System of Units (SI) with the equivalent values in the U.S. Customary Units given parenthetically.

$a_{Y,cg}$ lateral acceleration at center of gravity, g units

$a_{Y,I}$ lateral acceleration at accelerometer location, g units

b wing span, meters (ft)

$\bar{c}$ mean aerodynamic chord, meters (ft)

C_l rolling-moment coefficient

C_{l_p} damping-in-roll derivative, $\dfrac{\partial C_l}{\partial\left(\frac{pb}{2V}\right)}$, per radian

$C_{l_r} = \dfrac{\partial C_l}{\partial\left(\frac{rb}{2V}\right)}$ per radian

C_{l_β} effective-dihedral derivative, $\dfrac{\partial C_l}{\partial \beta}$, per radian

$C_{l_{\delta_a}} = \dfrac{\partial C_l}{\partial \delta_a}$ per radian

$C_{l_{\delta_r}} = \dfrac{\partial C_l}{\partial \delta_r}$ per radian

$(C_l)_{\beta_T,\delta_{r,T},\delta_{a,T}}$ rolling-moment coefficient at $\beta = \beta_T$, $\delta_r = \delta_{r,T}$, $\delta_a = \delta_{a,T}$

$(C_{l_r})_{\alpha_T}$ C_{l_r} at $\alpha = \alpha_T$, per radian

$(C_{l_\beta})_{\alpha_T}$ C_{l_β} at $\alpha = \alpha_T$, per radian

C_n yawing-moment coefficient

$C_{n_p} = \dfrac{\partial C_n}{\partial\left(\frac{pb}{2V}\right)}$ per radian

$C_{n_r} = \dfrac{\partial C_n}{\partial\left(\frac{rb}{2V}\right)}$ per radian

C_{n_β} static directional-stability derivative, $\dfrac{\partial C_n}{\partial \beta}$, per radian

$C_{n_{\delta_a}} = \dfrac{\partial C_n}{\partial \delta_a}$ per radian

$C_{n_{\delta_r}} = \dfrac{\partial C_n}{\partial \delta_r}$ per radian

$(C_n)_{\beta_T,\delta_{r,T},\delta_{a,T}}$ yawing-moment coefficient at $\beta = \beta_T$, $\delta_r = \delta_{r,T}$, $\delta_a = \delta_{a,T}$

$(C_{n_p})_{\alpha_T}$ C_{n_p} at $\alpha = \alpha_T$, per radian

$(C_{n_{\delta_a}})_{\alpha_T}$ $C_{n_{\delta_a}}$ at $\alpha = \alpha_T$, per radian

C_Y side-force coefficient

$C_{Y_p} = \dfrac{\partial C_Y}{\partial\left(\frac{pb}{2V}\right)}$ per radian

$C_{Y_r} = \dfrac{\partial C_Y}{\partial\left(\frac{rb}{2V}\right)}$ per radian

$C_{Y_{\beta}} = \dfrac{\partial C_Y}{\partial \beta}$ per radian

$C_{Y_{\delta_a}} = \dfrac{\partial C_Y}{\partial \delta_a}$ per radian

$C_{Y_{\delta_r}} = \dfrac{\partial C_Y}{\partial \delta_r}$ per radian

$(C_Y)_{\beta_T,\delta_{r,T},\delta_{a,T}}$ side-force coefficient at $\beta = \beta_T$, $\delta_r = \delta_{r,T}$, $\delta_a = \delta_{a,T}$

$(C_{Y_p})_{\alpha_T}$ C_{Y_p} at $\alpha = \alpha_T$

g acceleration due to gravity, meters/second^2 (ft/sec^2)

I_X aircraft moment of inertia about the body X-axis, kilogram-meters^2 (slug-ft^2)

I_{XZ} product of inertia of aircraft referred to body X- and Z-axis, kilogram-meters^2 (slug-ft^2)

I_Y aircraft moment of inertia about the body Y-axis, kilogram-meters^2 (slug-ft^2)

I_Z aircraft moment of inertia about the body Z-axis, kilogram-meters^2 (slug-ft^2)

$K_{C_{l_r}}$ slope of linear variation of C_{l_r} with α, per radian^2

$K_{C_{l_\beta}}$ slope of linear variation of C_{l_β} with α, per radian^2

$K_{C_{n_p}}$ slope of linear variation of C_{n_p} with α, per radian^2

$K_{C_{n_{\delta_a}}}$ slope of linear variation of $C_{n_{\delta_a}}$ with α, per radian^2

$K_{C_{Y_p}}$ slope of linear variation of C_{Y_p} with α, per radian^2

m mass of fueled airplane, kilograms (slugs)

p rolling angular velocity, radians/second

q pitching angular velocity, radians/second

r yawing angular velocity, radians/second

S wing area, meters^2 (ft^2)

u velocity along longitudinal body axis, meters/second (ft/sec)

V true airspeed, meters/second (ft/sec)

v velocity along lateral body axis, meters/second (ft/sec)

w velocity along vertical body axis, meters/second (ft/sec)

X_y accelerometer offset coordinate from center of gravity along longitudinal body axis, meters (ft)

Y_y accelerometer offset coordinate from center of gravity along lateral body axis, meters (ft)

Z_y accelerometer offset coordinate from center of gravity along vertical body axis, meters (ft)

α angle of attack, radians

α_T trim angle of attack, radians

β sideslip angle, radians

β_T trim sideslip angle, radians

δ_a aileron deflection angle (positive when right aileron is deflected down), radians

$\delta_{a,T}$ aileron deflection angle at trim, radians

δ_r rudder deflection angle (positive when trailing edge is deflected to the right), radians

$\delta_{r,T}$	rudder deflection angle at trim, radians
Γ	fit error
γ,λ	arbitrary parameters
θ	pitch angle, radians
ϕ	roll angle, radians
ρ	mass density of air, kilograms/meter^3 (slugs/ft^3)

A dot over a variable indicates the time derivative of that variable.

FLIGHT TESTS

The flight test data were provided by the U.S. Naval Air Test Center at Patuxent River, Maryland. The flight tests were conducted by Navy test pilots as part of an investigation with a McDonnell Douglas F-4 airplane. Five different lateral response runs were made: three during one flight test of the airplane and two during a second flight test. The first three runs were made at an altitude of approximately 6096 m (20 000 ft) at Mach numbers of about 0.6, 0.7, and 0.8, respectively. Control inputs for these runs were rudder only, rudder and aileron, and rudder only, respectively. The other two runs were made at an altitude of approximately 11 277.6 m (37 000 ft) at Mach numbers of about 0.9 and 0.8, respectively. Control inputs for these runs were rudder only and rudder and aileron, respectively. The stability augmentation system (SAS) was deactivated in order to provide full response for all the test runs.

For each of the test runs, the airplane was trimmed by the pilot at the desired altitude and Mach number and held for a short period. Then the control input or inputs were applied. No attempt was made to null any longitudinal motions. Roll and pitch angles as well as Mach number, pressure altitude, rudder deflection, aileron deflections, and calibrated airspeed were recorded every tenth of a second. True airspeed was determined from figure 1 of reference 3 using Mach number, pressure altitude, and temperature from flight tests and resolved through angle-of-sideslip measurements to yield lateral velocity.

Lateral displacement of the control stick in the F-4 airplane produces a combination of aileron and spoiler deflections. The aileron deflection is limited from 0° to 30° downward and from 0° to 1° upward. The spoiler being located on the upper surface of the wing has no downward deflection and is limited to upward deflections between 0° and 43°. In the flight records only the aileron deflections were recorded. Aileron-deflection data were used in the following manner to yield a single control input, which reflects a spoiler effect. The assumption was made that a negative reading for either the right or left aileron was the indication of an aileron input. It was further assumed that the spoiler effect on the opposite side of the negative aileron deflection was equivalent to a positive aileron deflection of the same magnitude. Hence, by doubling the magnitude of the negative aileron deflection and applying the sign convention of a right aileron to the magnitude, a single right aileron input, which is effectively the total aileron input, could be used in the equations. It should be noted that the aileron coefficients $\left(C_{Y_{\delta_a}},\ C_{l_{\delta_a}},\text{ and } C_{n_{\delta_a}}\right)$ extracted by this program reflect the effect of both aileron and spoiler. Since these control surfaces are physically linked, it is impossible to uniquely determine the coefficient of each aileron and spoiler without additional information.

Instrumentation consisted of rate gyros located slightly forward and at foot level of the pilot for measuring pitching, rolling, and yawing velocities; accelerometers located in the left wheel well for measuring lateral and normal accelerations; and vanes on a nose boom for measuring angle of attack and angle of sideslip. (See fig. 1.) No documentation was available from the Navy as to the accuracy of the instrumentation, although the method of parameter extraction (ref. 1) typically yielded the following signal-to-noise amplitude ratios (the noise amplitude was the 2-sigma level):

Lateral velocity	18 decibels
Rolling velocity	24 decibels
Yawing velocity	20 decibels
Roll angle	22 decibels
Lateral acceleration	8 decibels

AIRCRAFT MATHEMATICAL MODEL

The equations of motion used by the computer program (ref. 1) were modified continually during the analysis. However, three basic models evolved. The first model consisted of mainly lateral motion, the second model contained longitudinal coupling, and the third model contained longitudinal coupling and nonlinear lateral derivatives. The nonlinear derivatives $K_{C_{Y_p}}$, $K_{C_{l_\beta}}$, $K_{C_{l_r}}$, $K_{C_{n_p}}$, and $K_{C_{n_{\delta_a}}}$ permit variations with angle of attack in those particular derivatives that exhibit such dependence in wind-tunnel results (ref. 4). The three models can be obtained from the following equations:

$$\dot{v} = g \cos\theta \sin\phi + pw - ru + \frac{1}{2}\frac{\rho V^2 S}{m}\left\{\left(C_Y\right)_{\beta_T,\delta_{r,T},\delta_{a,T}} + C_{Y_\beta}\left(\beta - \beta_T\right) + C_{Y_r}\frac{rb}{2V} + \left[\left(C_{Y_p}\right)_{\alpha_T} + K_{C_{Y_p}}\left(\alpha - \alpha_T\right)\right]\frac{pb}{2V} + C_{Y_{\delta_r}}\left(\delta_r - \delta_{r,T}\right) + C_{Y_{\delta_a}}\left(\delta_a - \delta_{a,T}\right)\right\}$$

$$\dot{p} = \frac{I_{XZ}}{I_X}(\dot{r} + pq) + \left(\frac{I_Y - I_Z}{I_X}\right)qr + \frac{1}{2}\frac{\rho V^2 Sb}{I_X}\Bigg\{(C_l)_{\beta_T,\delta_{r,T},\delta_{a,T}}$$

$$+ \left[\left(C_{l_\beta}\right)_{\alpha_T} + K_{C_{l_\beta}}\left(\alpha - \alpha_T\right)\right](\beta - \beta_T) + \left[\left(C_{l_r}\right)_{\alpha_T} + K_{C_{l_r}}\left(\alpha - \alpha_T\right)\right]\frac{rb}{2V}$$

$$+ C_{l_p}\frac{pb}{2V} + C_{l_{\delta_r}}\left(\delta_r - \delta_{r,T}\right) + C_{l_{\delta_a}}\left(\delta_a - \delta_{a,T}\right)\Bigg\}$$

$$\dot{r} = pq\left(\frac{I_X - I_Y}{I_Z}\right) + \frac{I_{XZ}}{I_Z}(\dot{p} - qr) + \frac{1}{2}\frac{\rho V^2 Sb}{I_Z}\Bigg\{(C_n)_{\beta_T,\delta_{r,T},\delta_{a,T}} + C_{n_\beta}\left(\beta - \beta_T\right)$$

$$+ \left[\left(C_{n_p}\right)_{\alpha_T} + K_{C_{n_p}}\left(\alpha - \alpha_T\right)\right]\frac{pb}{2V} + C_{n_r}\frac{rb}{2V} + C_{n_{\delta_r}}\left(\delta_r - \delta_{r,T}\right)$$

$$+ \left[\left(C_{n_{\delta_a}}\right)_{\alpha_T} + K_{C_{n_{\delta_a}}}\left(\alpha - \alpha_T\right)\right]\left(\delta_a - \delta_{a,T}\right)\Bigg\}$$

$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi)$$

$$\beta = \sin^{-1}\left(\frac{v}{V}\right)$$

$$V = \sqrt{u^2 + v^2 + w^2}$$

$$\alpha = \tan^{-1}\left(\frac{w}{u}\right)$$

$$a_{Y,cg} = \frac{1}{g}(\dot{v} + ru - pw - g\cos\theta\sin\phi)$$

$$a_{Y,I} = a_{Y,cg} + \frac{1}{g}\left[(pq + \dot{r})X_y - \left(p^2 + r^2\right)Y_y + (qr - \dot{p})Z_y\right]$$

The first model, mainly lateral motion, can be obtained from the basic equations by requiring all longitudinal variables (u, w, θ, α, and q) to be constants and the nonlinear derivatives $K_{C_{Y_p}}$, $K_{C_{l_\beta}}$, $K_{C_{l_r}}$, $K_{C_{n_p}}$, and $K_{C_{n_{\delta_a}}}$ to be zero. The second model, containing longitudinal coupling, can be obtained by using the longitudinal flight data as inputs to the equations in the same manner as rudder and aileron deflections are used. The third model, containing both coupling and nonlinear derivatives, is obtained from the second model by not restricting the nonlinear derivatives to zero.

The nonlinear derivatives $K_{C_{Y_p}}$, $K_{C_{l_\beta}}$, $K_{C_{l_r}}$, $K_{C_{n_p}}$, and $K_{C_{n_{\delta_a}}}$, as well as the longitudinal coupling terms, were discovered to be necessary in order to fit the flight test data, as is demonstrated in the next section. Also demonstrated in the next section is the problem of uniqueness mentioned in references 1 and 2.

RESULTS

The conditions for the five flight test runs are listed in table I. The analysis of these runs involved two major problems: uniqueness and longitudinal effects. The results are presented in a manner to illustrate how each problem was encountered and then resolved.

Uniqueness Problem

Before presenting the uniqueness problem as encountered in this study, it would be well to describe the problem and the means of detecting its presence. The problem itself can be best described as follows: Given a set of parameters that minimize the fit error between measured and computed variables, does another set of parameters exist that will yield the same fit error? If the answer is yes, a uniqueness problem exists. Detection of the problem is facilitated by the use of the covariance matrix provided by the maximum likelihood estimation technique. Minor manipulation of this matrix, as described in reference 1, yields pairwise parameter correlation coefficients which estimate the degree of linear dependence between two parameters. Figure 2 illustrates the existence of a uniqueness problem due to linear correlation between arbitrary parameters λ and γ. Values of λ and γ that lie on the line of dependence yield the same fit error. However, it should be emphasized that two parameters may exhibit high correlation without indicating a uniqueness problem. Thus, it is necessary for the analyst to test any parameters with significant correlation coefficients to determine whether a uniqueness problem is present. The test is simply to determine whether the fit error changes as the parameters vary along the line of dependence. The procedure for carrying out the test is to assign to one of the correlated parameters several values in the range of interest and then extract the other parameter's values; this determines the line of dependence. In figure 2, the fit error Γ does not change as λ and γ vary along the line of dependence. Thus, a uniqueness problem is present. If the fit error did change, both parameters would be identified by the estimation technique at the point of minimum fit error and no uniqueness problem would exist, although the parameters would still be correlated. In this hypothetical illustration, the correlation between λ and γ is perfectly linear and will cause divergence of the estimation technique when an attempt is made to extract both parameters. However, in the use of real data, the presence of noise usually prevents perfect linear correlation, and thus divergence.

Figure 3 presents the model responses generated by the estimates of the stability derivatives of test run 1 and the respective flight test data, using the first model with all longitudinal variables fixed as constants (average values obtained from the flight data for each variable). (Note that symbols in figure 3 and subsequent machine plots presenting model responses and respective flight test data are not the standard symbols defined in the Symbols section.) Table II presents the estimates of the derivatives obtained, and table III presents a form of the covariance matrix for these estimates. Diagonal elements of this matrix are the standard deviations of the estimates, and the off-diagonal terms are correlation coefficients. As denoted by the asterisks of table III, C_{Y_β}, C_{Y_p}, and C_{Y_r}; C_{l_β}, C_{l_p}, and C_{l_r}; C_{n_β}, C_{n_p}, and C_{n_r}; and C_{l_β} and C_{n_β} all have significant correlation. Investigation of these parameters revealed the existence of a uniqueness problem.

A major cause of uniqueness problems is generally admitted to be insufficient excitation of the aircraft (for example, ref. 5). Test run 1 had rudder deflections only. Test run 2 contained both rudder and aileron deflections, and the model responses generated by the derivative estimates for this test run and the respective flight test data are shown in figure 4. Again the longitudinal variables were fixed as constants during the extraction process. Table IV presents the estimates of the derivatives obtained, and table V presents the modified covariance matrix. As pointed out in section 7.8.3 of reference 5, the likelihood of obtaining a unique set of derivatives is increased when both a rudder input and an aileron input are used to excite the airframe, as is evidenced by the lack of correlation exhibited in table V.

Longitudinal Coupling Effects

Examination of figure 4 (test run 2 responses) reveals poor fits for all the lateral variables; these poor fits indicate a possibly incomplete model. The longitudinal data for test run 2 are presented in figure 5 and indicate a substantial amount of longitudinal motion. Use of the longitudinal data as input, together with the modeling of angle of attack dependence of some of the derivatives, resulted in the extraction of a new set of derivatives for test run 2. Figure 6 presents the model responses generated by this set of derivatives and table VI contains the derivatives and their standard deviations. No significant correlation was present and, thus, a unique set of derivatives has been extracted. It should be noted that a lack of confidence exists for all the C_Y derivatives with the exception of C_{Y_β}, due to the large standard deviations of the estimates, as is the case with some of the nonlinear derivatives.

Solution of the Uniqueness Problem

The uniqueness problem of test run 1 was resolved by fixing the values of the nonlinear derivatives and $\left(C_{Y_p}\right)_{\alpha_T}$, C_{l_p}, and C_{n_r} at the values obtained in test run 2 (the wind-tunnel results presented in ref. 4 show these derivatives to be fairly insensitive to Mach number variations in this flight regime) and extracting the remaining derivatives. This same procedure was used to solve the uniqueness problem of test run 3, which also had a rudder-only input. The model responses generated by the final estimates of the derivatives for test run 1 and test run 3 are shown with the respective flight data in figures 7 and 8, respectively. The values of the derivatives and their standard deviations are presented in table VII for test run 1 and table VIII for test run 3.

Test run 4 had essentially a rudder-only input, whereas test run 5 had both rudder and aileron inputs. Again, the results of test run 5 were used to solve the uniqueness problem of test run 4. The model responses generated by the final derivative estimates of test run 4 and the respective flight data are shown in figure 9, and the estimates with the standard deviations are presented in table IX. The results of test run 5 are presented in figure 10 and table X.

The total results of the analysis are summarized in figures 11 to 13, which illustrate the variation of the extracted derivatives with Mach number, altitude, and angle of attack. The results shown in figure 13 are presented with the intercept values located at the trim angle of attack (symbol location) and the slope of the lines determined by the nonlinear derivatives.

CONCLUDING REMARKS

It is believed that the importance of the analyst, exercising engineering judgment tempered with estimator statistics, has been aptly demonstrated by the results of this study in recognizing and resolving the problems of uniqueness and longitudinal coupling effects. Thus, the extraction technique and its computer implementation have been shown to provide the means for identifying both modeling and uniqueness problems and to yield a unique set of derivatives from actual lateral flight test data, provided the flight data contain sufficient information.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., August 4, 1972.

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TABLE I.- FLIGHT TEST CONDITIONS

Test run	Altitude		Mach number	Center of gravity, % $\bar{c}$	Input
	m	ft			
1	6 096	20 000	0.6	32.19	Rudder
2	6 096	20 000	.7	31.85	Rudder and aileron
3	6 096	20 000	.8	31.49	Rudder
4	11 277.6	37 000	.9	29.18	Rudder
5	11 277.6	37 000	.8	29.11	Rudder and aileron

TABLE II.- DERIVATIVE ESTIMATES OF TEST RUN 1 OBTAINED WITH MODEL 1

C_{Y_β}	-0.392
C_{Y_p}	1.97
C_{Y_r}	3.75
$C_{Y_{\delta_r}}$	-0.0487
C_{l_β}	-0.0938
C_{l_p}	-0.355
C_{l_r}	-0.230
$C_{l_{\delta_r}}$	0.00221
C_{n_β}	0.120
C_{n_p}	0.162
C_{n_r}	-0.0664
$C_{n_{\delta_r}}$	0.0462

TABLE III.- COVARIANCE ESTIMATES OF TEST RUN 1 OBTAINED WITH MODEL 1

	C_{Y_β}	C_{Y_p}	C_{Y_r}	$C_{Y_{\delta_r}}$	C_{l_β}	C_{l_p}	C_{l_r}	$C_{l_{\delta_r}}$	C_{n_β}	C_{n_p}	C_{n_r}	$C_{n_{\delta_r}}$
C_{Y_β}	0.0916	0.95*	0.91*	-0.59	0.06	0.01	0.07	-0.11	-0.03	0.06	-0.05	0.32
C_{Y_p}	0.95*	0.669	0.95*	-0.56	0.11	0.08	0.11	-0.15	-0.15	-0.06	-0.13	0.42
C_{Y_r}	0.91*	0.95*	1.839	-0.63	0.06	0.05	0.08	-0.15	-0.04	-0.0002	-0.07	0.37
$C_{Y_{\delta_r}}$	-0.59	-0.56	-0.63	0.0334	0.08	0.15	0.12	0.05	-0.26	-0.35	-0.30	-0.03
C_{l_β}	0.06	0.11	0.06	0.08	0.00342	0.92*	0.90*	-0.40	-0.91*	-0.82	-0.85	0.54
C_{l_p}	0.01	0.08	0.05	0.15	0.92*	0.02	0.89*	-0.56	-0.76	-0.75	-0.74	0.52
C_{l_r}	0.07	0.11	0.08	0.12	0.90*	0.89*	0.0613	-0.59	-0.56	0.58	-0.65	0.54
$C_{l_{\delta_r}}$	-0.11	-0.15	-0.15	0.05	-0.40	-0.56	-0.59	0.00138	0.13	0.12	0.11	-0.46
C_{n_β}	-0.03	-0.15	-0.04	-0.26	-0.91*	-0.76	-0.56	0.13	0.00179	0.96*	0.94*	-0.62
C_{n_p}	0.06	-0.06	-0.0002	-0.35	-0.82	-0.75	-0.58	0.12	0.96*	0.0129	0.96*	-0.59
C_{n_r}	-0.05	-0.13	-0.7	-0.30	-0.85	-0.74	-0.65	0.11	0.94*	0.96*	0.0362	-0.69
$C_{n_{\delta_r}}$	0.32	0.42	0.37	0.03	0.54	0.52	0.54	-0.46	-0.62	-0.59	-0.69	0.000632

*Significant correlation.

TABLE IV.- DERIVATIVE ESTIMATES OF TEST RUN 2 OBTAINED WITH MODEL 1

C_{Y_β}	-0.303
C_{Y_p}	0.858
C_{Y_r}	2.18
$C_{Y_{\delta_r}}$	-0.00341
$C_{Y_{\delta_a}}$	0.0252
C_{l_β}	-0.0452
C_{l_p}	-0.126
C_{l_r}	0.277
$C_{l_{\delta_r}}$	-0.0121
$C_{l_{\delta_a}}$	-0.118
C_{n_β}	0.105
C_{n_p}	0.0670
C_{n_r}	-0.266
$C_{n_{\delta_r}}$	0.0519
$C_{n_{\delta_a}}$	0.00710

TABLE V.- COVARIANCE ESTIMATES OF TEST RUN 2 OBTAINED WITH MODEL 1

	C_{Y_β}	C_{Y_p}	C_{Y_r}	$C_{Y_{\delta_r}}$	$C_{Y_{\delta_a}}$	C_{l_β}	C_{l_p}	C_{l_r}	$C_{l_{\delta_r}}$	$C_{l_{\delta_a}}$	C_{n_β}	C_{n_p}	C_{n_r}	$C_{n_{\delta_r}}$	$C_{n_{\delta_a}}$
C_{Y_β}	0.304	0.73	0.64	-0.16	0.43	-0.15	-0.22	-0.01	-0.01	-0.09	0.09	0.11	0.06	0.03	-0.9
C_{Y_p}	0.73	0.317	0.76	-0.17	0.36	-0.05	-0.20	-0.11	0.05	-0.002	0.07	0.09	0.07	0.05	-0.12
C_{Y_r}	0.64	0.76	0.686	-0.38	0.31	-0.13	-0.18	-0.16	-0.01	0.01	0.11	0.10	0.09	0.06	-0.11
$C_{Y_{\delta_r}}$	-0.16	-0.17	-0.38	0.0190	-0.08	0.01	0.07	0.12	-0.09	-0.04	-0.06	-0.08	-0.09	0.01	0.06
$C_{Y_{\delta_a}}$	0.43	0.36	0.31	-0.08	0.0174	-0.08	-0.20	-0.09	0.04	-0.15	0.21	0.22	0.23	-0.04	0.07
C_{l_β}	-0.15	-0.05	-0.13	0.01	-0.08	0.00145	0.50	0.07	0.60	0.45	-0.21	-0.28	-0.36	0.05	-0.11
C_{l_p}	-0.22	-0.20	-0.18	0.07	-0.20	0.50	0.00824	0.41	-0.06	0.45	-0.67	-0.69	-0.68	-0.15	-0.28
C_{l_r}	-0.01	-0.11	-0.16	0.12	-0.09	0.07	0.41	0.0251	-0.31	-0.15	-0.37	-0.42	-0.37	0.02	-0.22
$C_{l_{\delta_r}}$	-0.01	0.05	-0.01	-0.09	0.04	0.60	-0.06	-0.31	0.000821	-0.002	-0.03	-0.14	-0.13	-0.28	-0.03
$C_{l_{\delta_a}}$	-0.09	-0.002	0.01	-0.04	-0.15	0.45	0.45	-0.15	-0.002	0.000436	-0.17	-0.09	-0.15	-0.03	-0.44
C_{n_β}	0.09	0.07	0.11	-0.06	0.21	-0.21	-0.67	-0.37	-0.03	-0.17	0.00123	0.74	0.71	0.12	0.48
C_{n_p}	0.11	0.09	0.10	-0.08	0.22	-0.28	-0.69	-0.42	-0.14	-0.09	0.74	0.0157	0.75	0.18	0.40
C_{n_r}	0.06	0.07	0.09	-0.09	0.23	-0.36	-0.68	-0.37	-0.13	-0.15	0.71	0.75	0.0292	-0.05	0.38
$C_{n_{\delta_r}}$	0.03	0.05	0.06	0.01	-0.04	-0.05	-0.15	0.02	-0.28	-0.03	0.12	0.18	-0.05	0.000764	0.08
$C_{n_{\delta_a}}$	-0.09	-0.12	-0.11	0.06	0.07	-0.11	-0.28	-0.22	-0.03	-0.44	0.48	0.40	0.38	0.08	0.000573

TABLE VI.- FINAL DERIVATIVE ESTIMATES OF TEST RUN 2 OBTAINED WITH MODEL 3 FOR $\alpha_T = 0^\circ$

	Estimate	Standard deviation
C_{Y_β}	-0.341	0.0142
$\left(C_{Y_p}\right)_{\alpha_T}$	0.086	0.271
$K_{C_{Y_p}}$	2.3	5.60
C_{Y_r}	1.27	0.301
$C_{Y_{\delta_r}}$	-0.035	0.0163
$C_{Y_{\delta_a}}$	0.036	0.0180
$\left(C_{l_\beta}\right)_{\alpha_T}$	-0.0382	0.000996
$K_{C_{l_\beta}}$	-0.73	0.0524
C_{l_p}	-0.363	0.00574
$\left(C_{l_r}\right)_{\alpha_T}$	-0.085	0.0175
$K_{C_{l_r}}$	0.60	0.681
$C_{l_{\delta_r}}$	-0.0163	0.000632
$C_{l_{\delta_a}}$	-0.0350	0.000429
C_{n_β}	0.101	0.000791
$\left(C_{n_p}\right)_{\alpha_T}$	0.0990	0.00801
$K_{C_{n_p}}$	-0.90	0.152
C_{n_r}	-0.229	0.0164
$C_{n_{\delta_r}}$	0.0527	0.000617
$\left(C_{n_{\delta_a}}\right)_{\alpha_T}$	-0.00074	0.000614
$K_{C_{n_{\delta_a}}}$	0.04	0.0139

TABLE VII.- FINAL DERIVATIVE ESTIMATES OF TEST RUN 1 OBTAINED WITH MODEL 3 FOR $\alpha_T = 0^o$

	Estimate	Standard deviation
C_{Y_β}	-0.295	0.0238
$(C_{Y_p})_{\alpha_T}$	0.086	*
$K_{C_{Y_p}}$	2.3	*
C_{Y_r}	1.6	0.513
$C_{Y_{\delta_r}}$	-0.0240	0.0232
$(C_{l_\beta})_{\alpha_T}$	-0.0388	0.000879
$K_{C_{l_\beta}}$	-0.73	*
C_{l_p}	-0.363	*
$(C_{l_r})_{\alpha_T}$	-0.132	0.0206
$K_{C_{l_r}}$	0.60	*
$C_{l_{\delta_r}}$	-0.0160	0.00132
C_{n_β}	0.0870	0.000393
$(C_{n_p})_{\alpha_T}$	0.0823	0.00291
$K_{C_{n_p}}$	-0.90	*
C_{n_r}	-0.228	*
$C_{n_{\delta_r}}$	0.0510	0.000521

*Fixed from test run 2.

TABLE VIII.- FINAL DERIVATIVE ESTIMATES OF TEST RUN 3 OBTAINED WITH MODEL 3 FOR $\alpha_T = 0^o$

	Estimate	Standard deviation
C_{Y_β}	-0.293	0.0138
$(C_{Y_p})_{\alpha_T}$	0.086	*
$K_{C_{Y_p}}$	2.3	*
C_{Y_r}	1.20	1.09
$C_{Y_{\delta_r}}$	-0.0513	0.0164
$(C_{l_\beta})_{\alpha_T}$	-0.0486	0.000643
$K_{C_{l_\beta}}$	-0.73	*
C_{l_p}	-0.363	*
$(C_{l_r})_{\alpha_T}$	-0.0318	0.0143
$K_{C_{l_r}}$	0.60	*
$C_{l_{\delta_r}}$	-0.0164	0.000648
C_{n_β}	0.0987	0.000491
$(C_{n_p})_{\alpha_T}$	0.0974	0.00423
$K_{C_{n_p}}$	-0.90	*
C_{n_r}	-0.228	*
$C_{n_{\delta_r}}$	0.0483	0.000618

*Fixed from test run 2.

TABLE IX.- FINAL DERIVATIVE ESTIMATES OF TEST RUN 4 OBTAINED WITH MODEL 3 FOR $\alpha_T = 0^o$

	Estimate	Standard deviation
C_{Y_β}	-0.269	0.0164
$\left(C_{Y_p}\right)_{\alpha_T}$	0.090	*
$K_{C_{Y_p}}$	2.3	*
C_{Y_r}	1.03	0.578
$C_{Y_{\delta_r}}$	-0.0767	0.0155
$C_{Y_{\delta_a}}$	0.0602	*
$\left(C_{l_\beta}\right)_{\alpha_T}$	-0.0549	0.000979
$K_{C_{l_\beta}}$	-0.367	*
C_{l_p}	-0.282	*
$\left(C_{l_r}\right)_{\alpha_T}$	0.170	0.0244
$K_{C_{l_r}}$	0.80	*
$C_{l_{\delta_r}}$	-0.00363	0.000978
$C_{l_{\delta_a}}$	-0.0244	*
C_{n_β}	0.107	0.000533
$\left(C_{n_p}\right)_{\alpha_T}$	0.125	0.00369
$K_{C_{n_p}}$	-0.90	*
C_{n_r}	-0.158	*
$C_{n_{\delta_r}}$	0.0542	0.000506
$\left(C_{n_{\delta_a}}\right)_{\alpha_T}$	-0.00529	*
$K_{C_{n_{\delta_a}}}$	0.04	*

*Fixed from test run 5.

TABLE X.- FINAL DERIVATIVE ESTIMATES OF TEST RUN 5 OBTAINED WITH MODEL 3 FOR $\alpha_T = 0^o$

	Estimate	Standard deviation
C_{Y_β}	-0.383	0.0434
$(C_{Y_p})_{\alpha_T}$	0.090	0.181
$K_{C_{Y_p}}$	2.3	*
C_{Y_r}	1.01	1.21
$C_{Y_{\delta_r}}$	-0.0473	0.0342
$C_{Y_{\delta_a}}$	0.0602	0.00931
$(C_{l_\beta})_{\alpha_T}$	-0.0577	0.00752
$K_{C_{l_\beta}}$	-0.367	0.0875
C_{l_p}	-0.282	0.00531
$(C_{l_r})_{\alpha_T}$	0.0362	0.0410
$K_{C_{l_r}}$	0.80	0.824
$C_{l_{\delta_r}}$	-0.00360	0.00145
$C_{l_{\delta_a}}$	-0.0244	0.000682
C_{n_β}	0.0965	0.00107
$(C_{n_p})_{\alpha_T}$	0.0987	0.00526
$K_{C_{n_p}}$	-0.90	0.493
C_{n_r}	-0.158	0.0351
$C_{n_{\delta_r}}$	0.0525	0.000907
$(C_{n_{\delta_a}})_{\alpha_T}$	-0.00529	0.000342
$K_{C_{n_{\delta_a}}}$	0.04	*

*Fixed from test run 2.

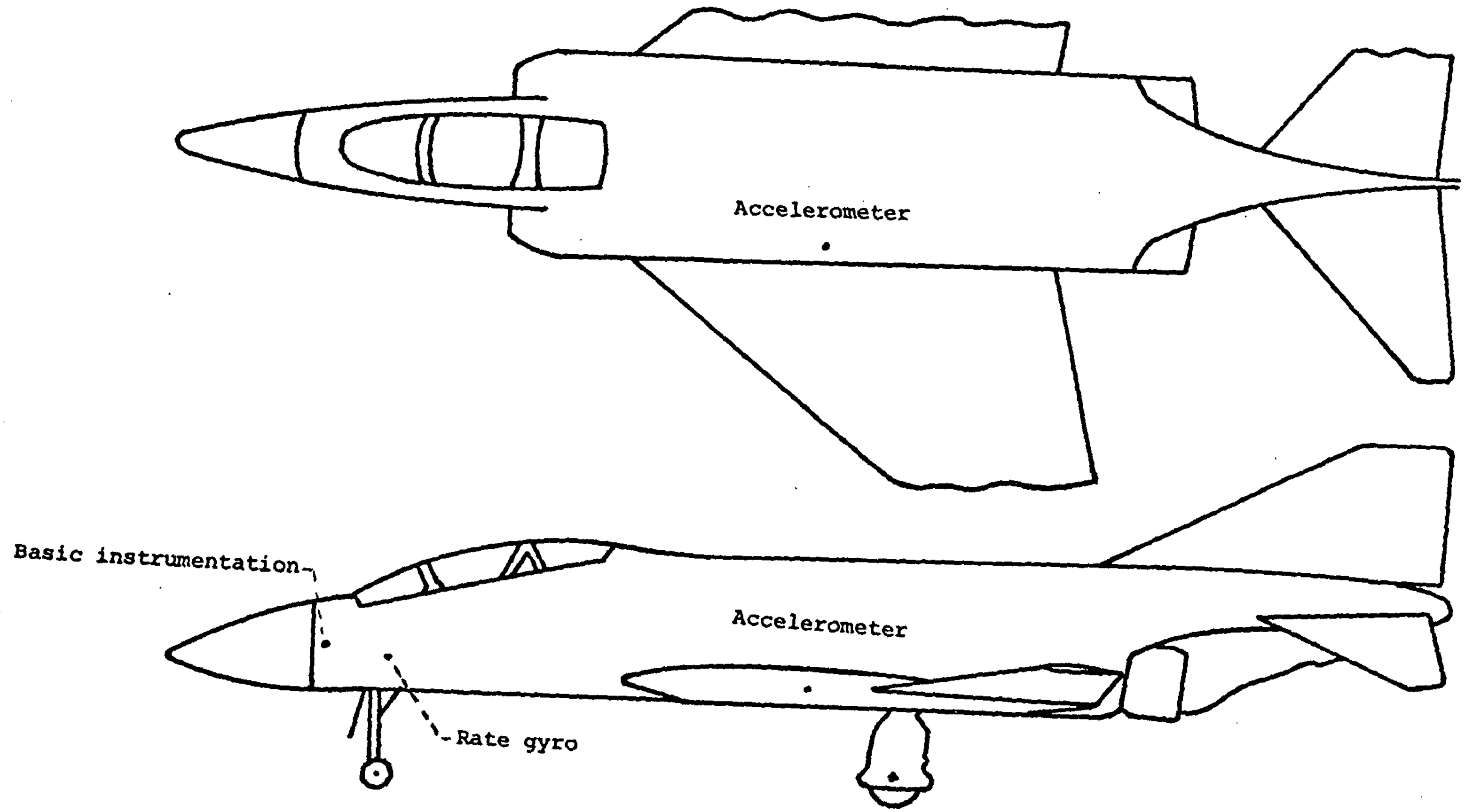


Figure 1.- Instrument locations.

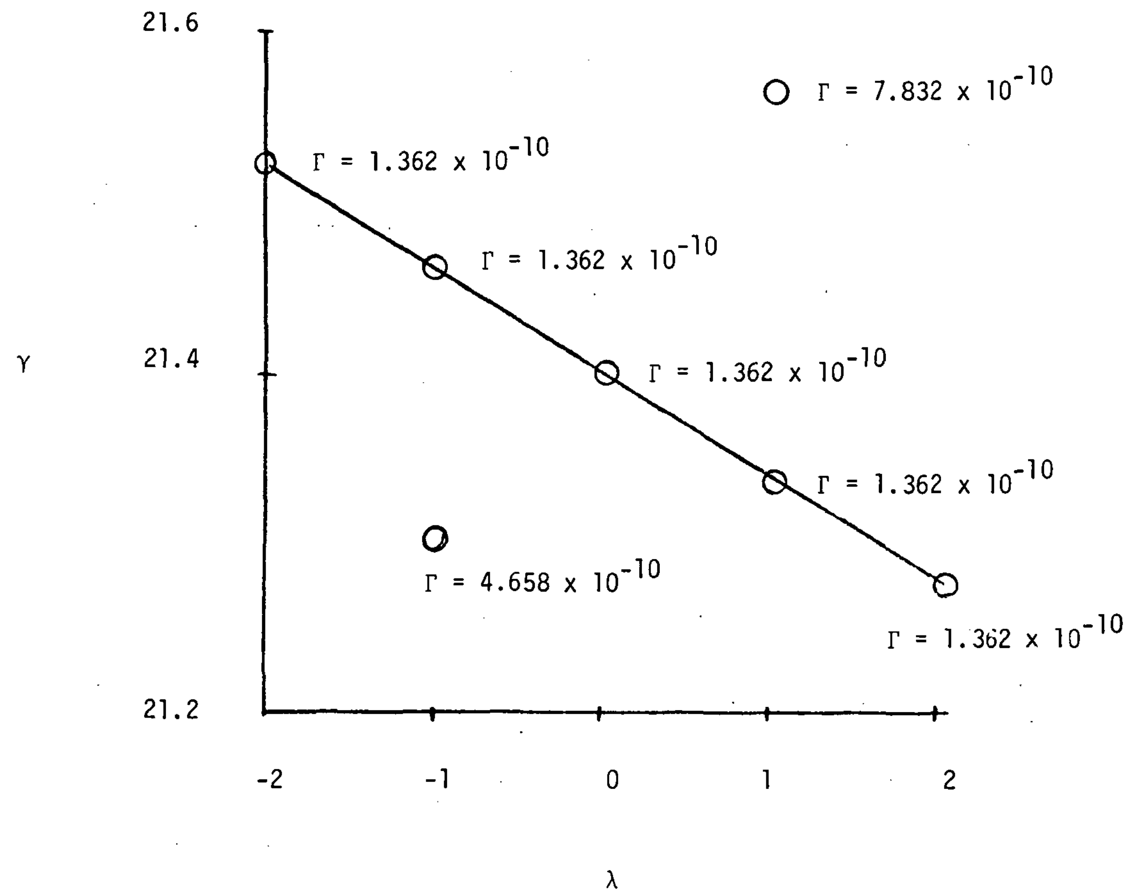
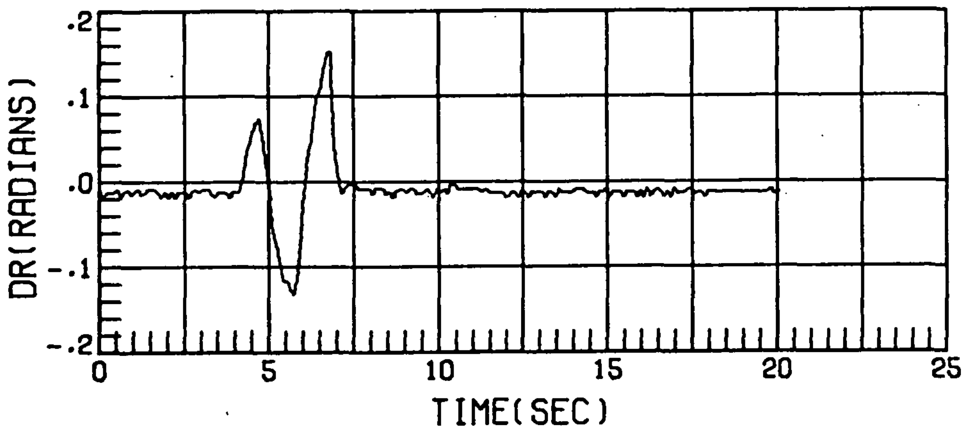


Figure 2.- Hypothetical illustration of correlation of two parameters.

- flight data



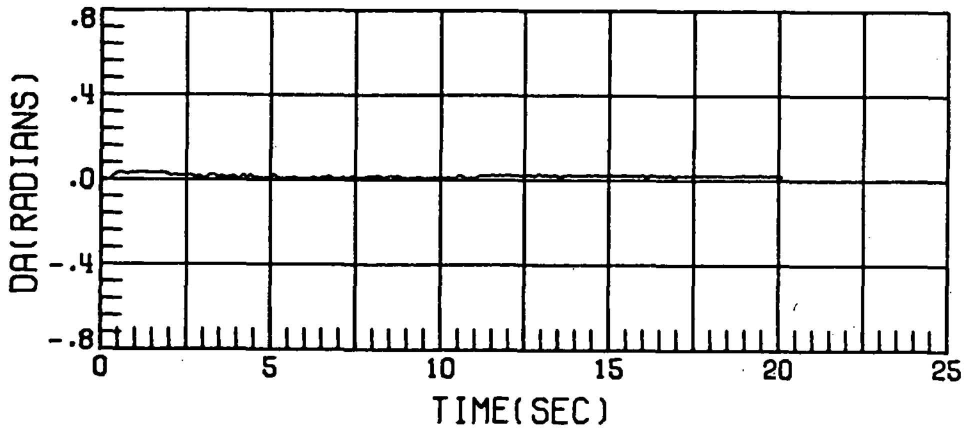


Figure 3.- Model responses generated by derivative estimates of test run 1 with longitudinal variables constant.

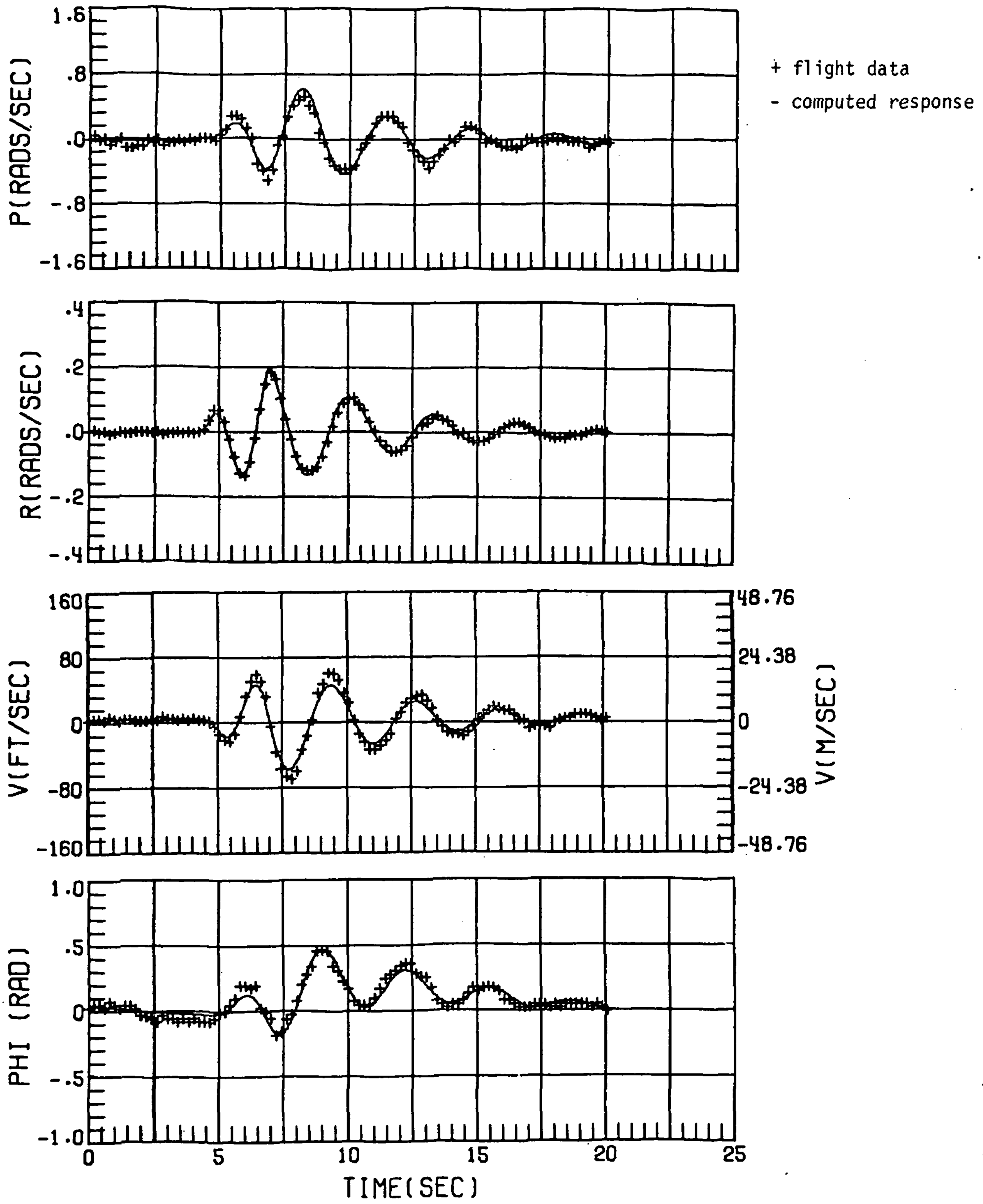


Figure 3.- Continued.

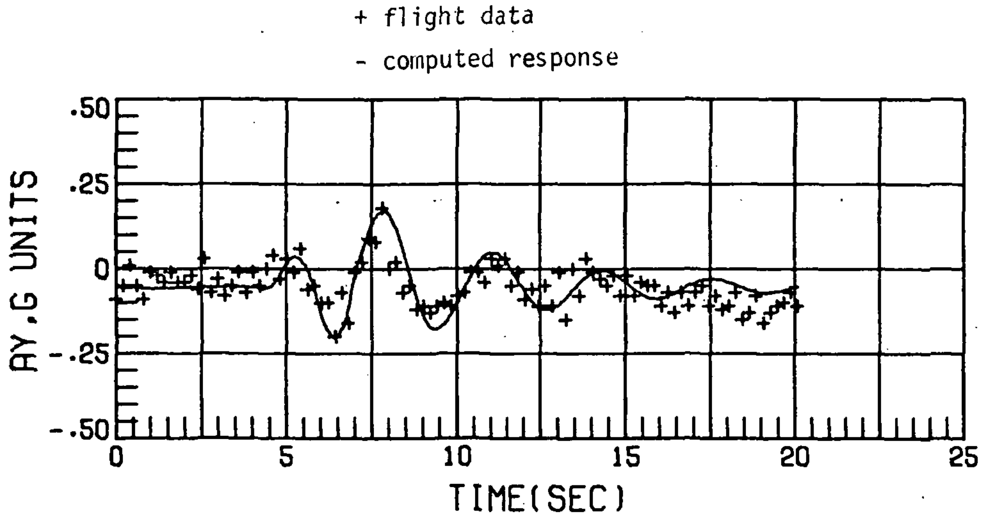
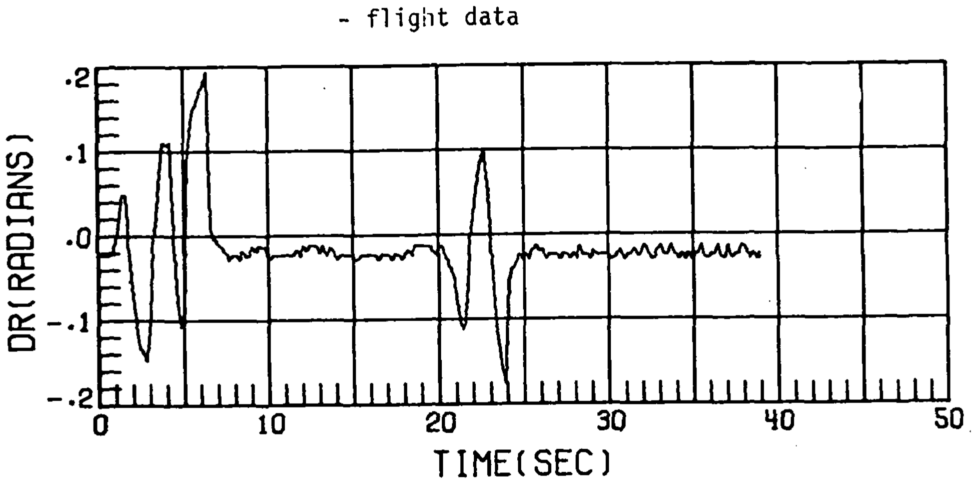


Figure 3.- Concluded.



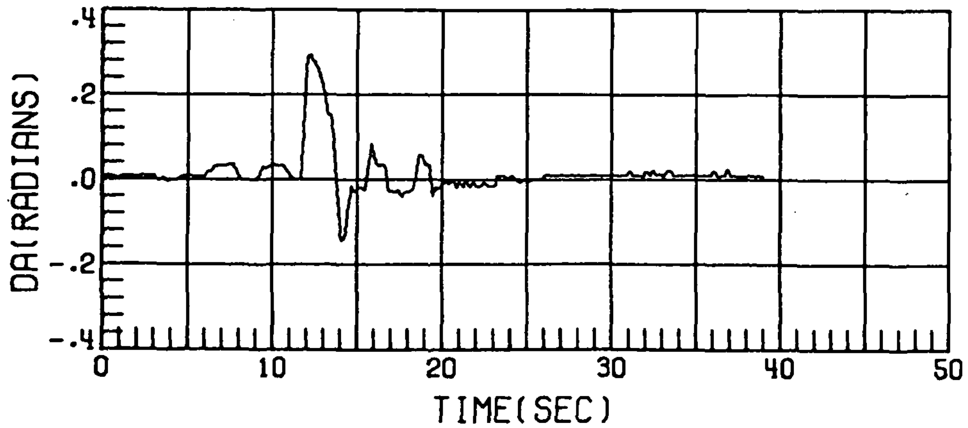


Figure 4.- Model responses generated by derivative estimates of test run 2 with longitudinal variables constant.

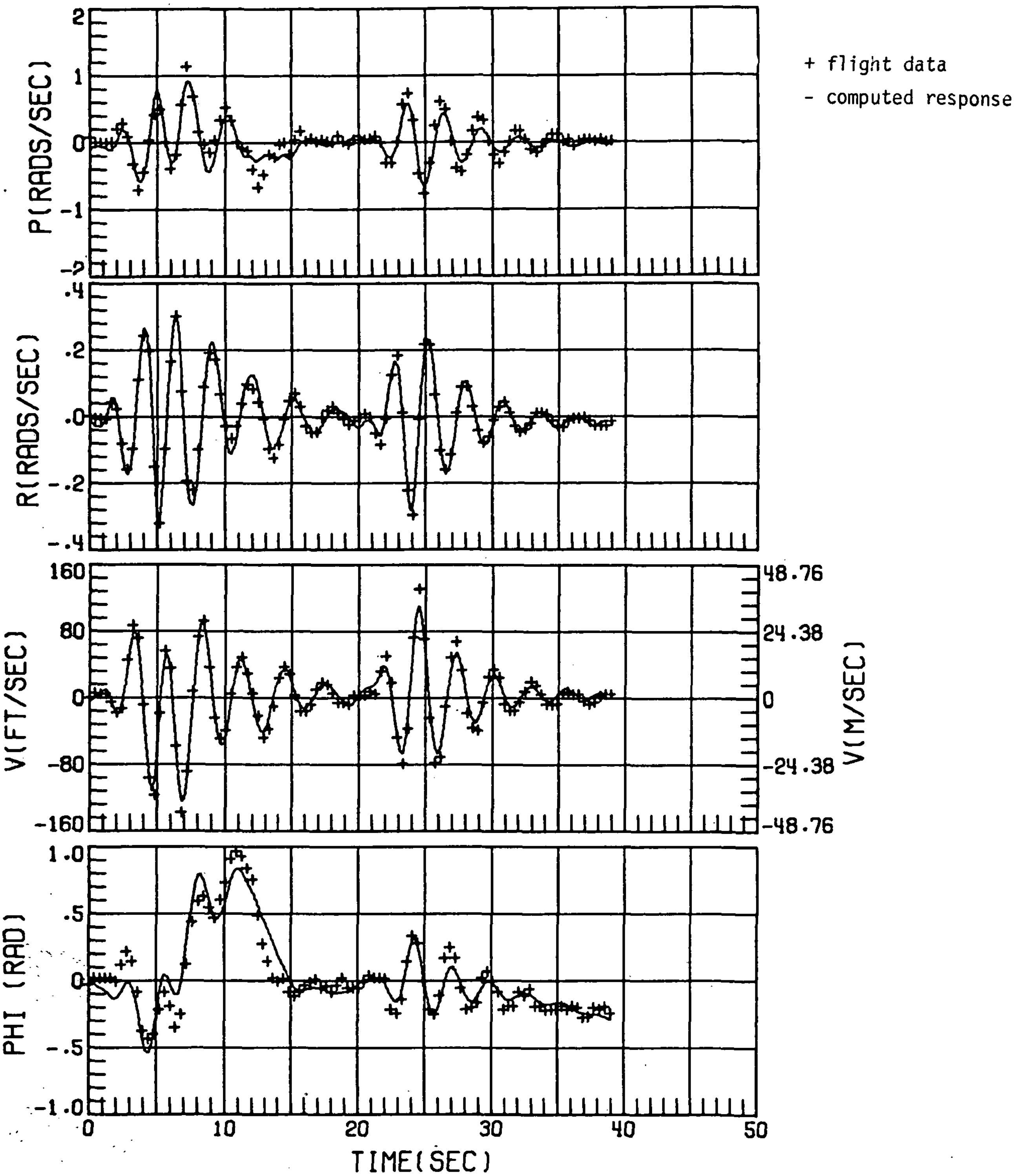


Figure 4.- Continued.

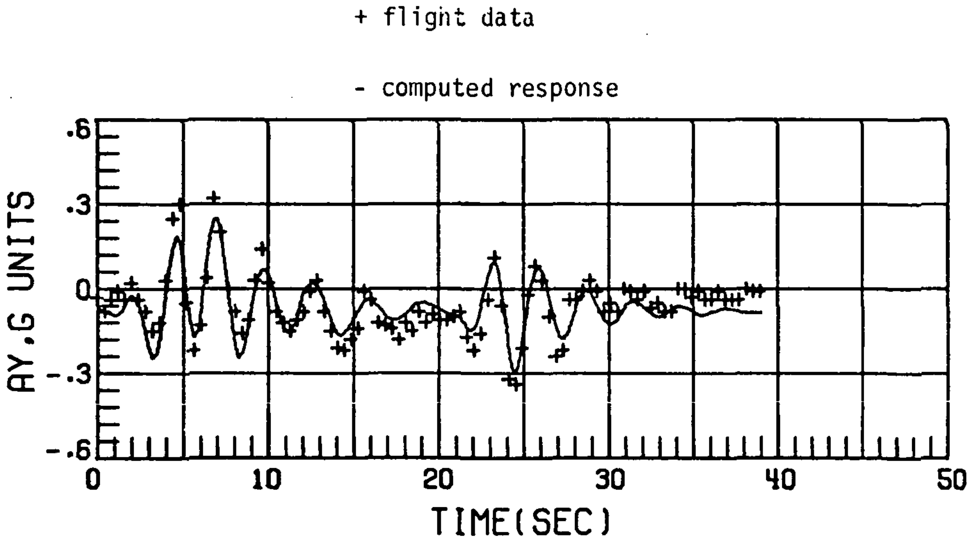


Figure 4.- Concluded.

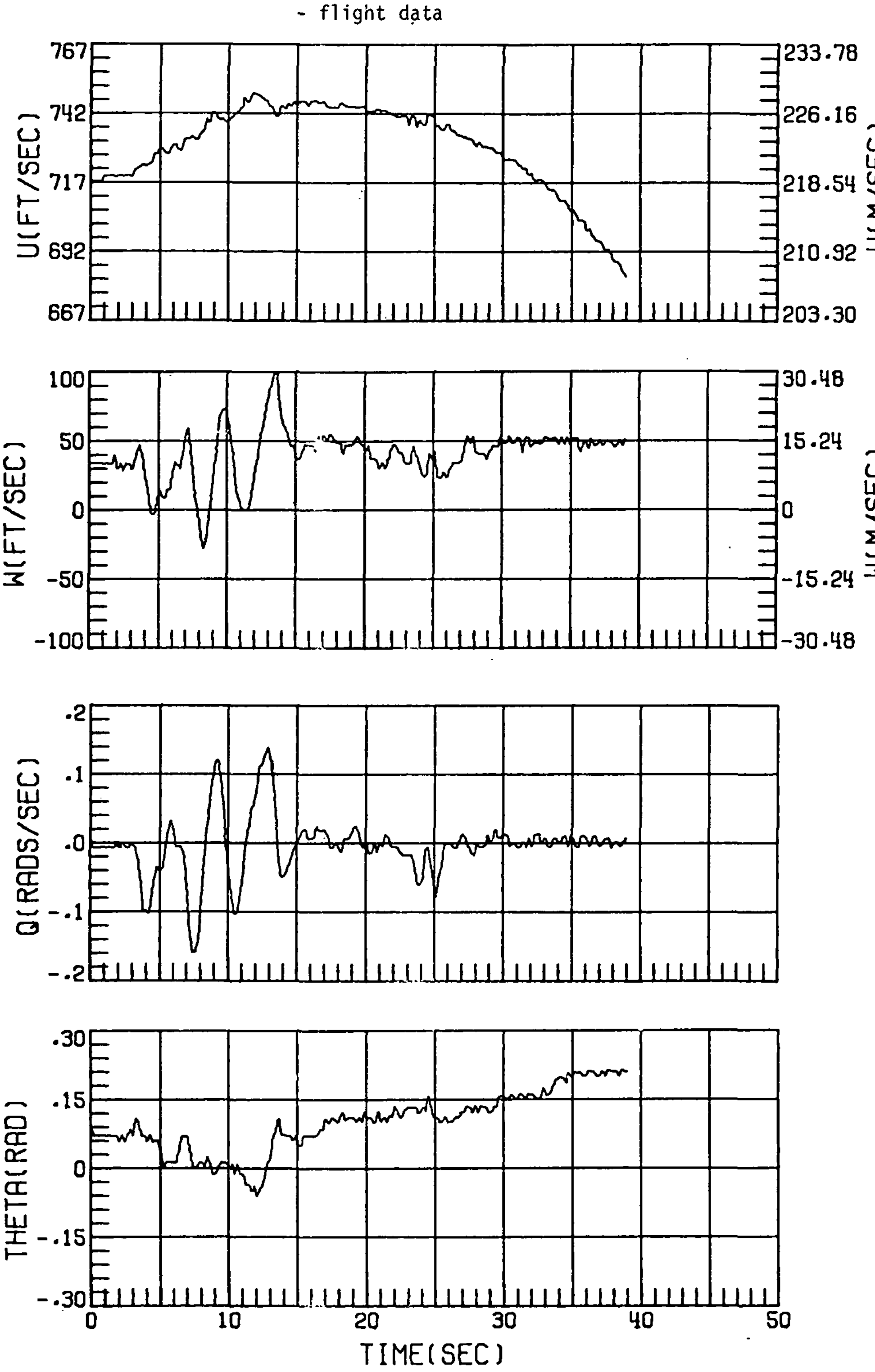


Figure 5.- Longitudinal data of test run 2.

- flight data

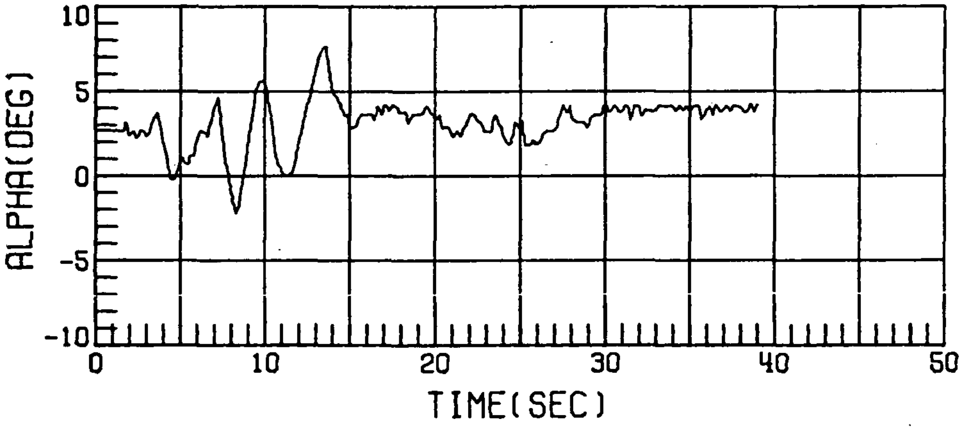


Figure 5.- Concluded.

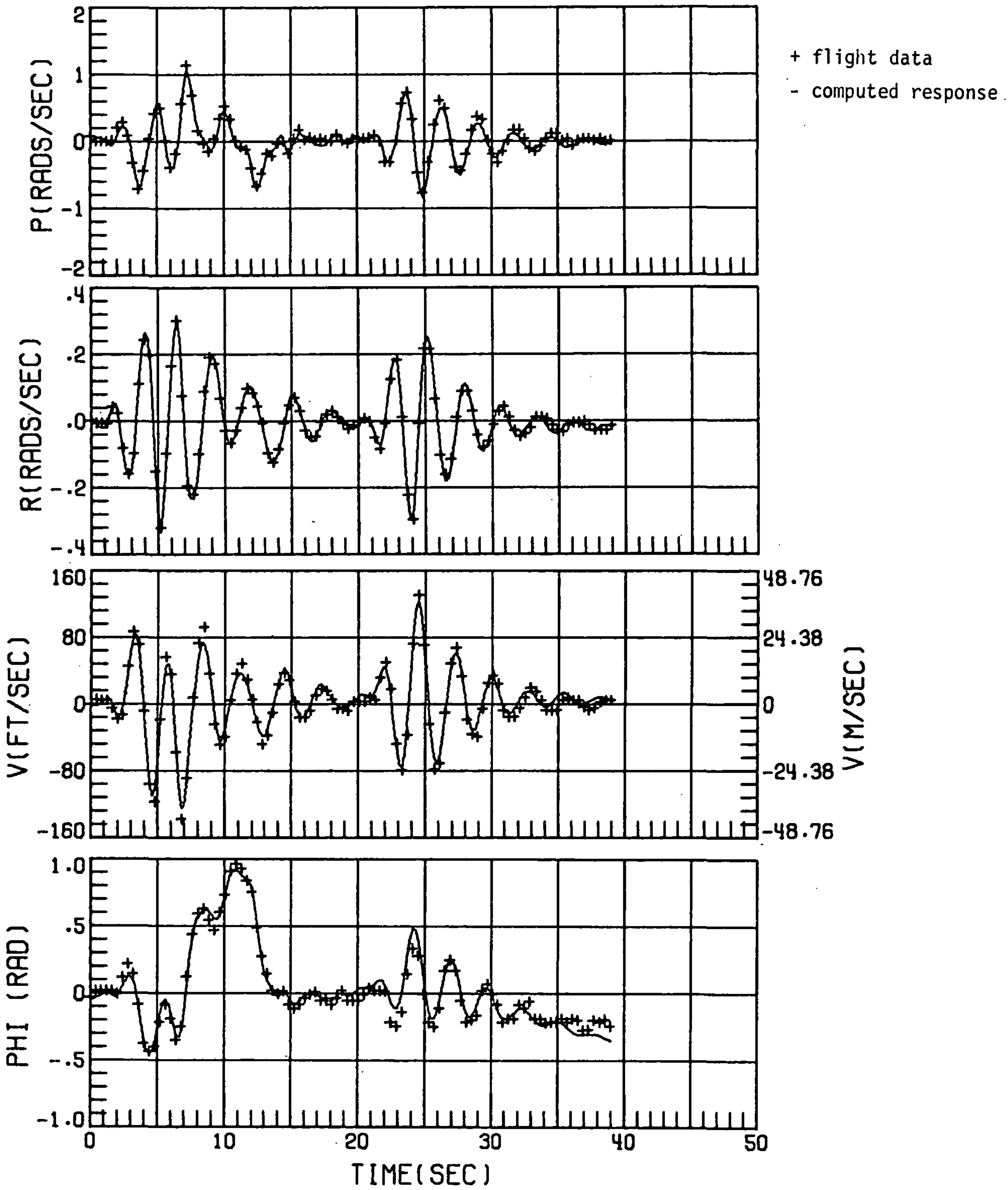


Figure 6.- Model responses generated by final derivative estimates of test run 2 with longitudinal data as input.

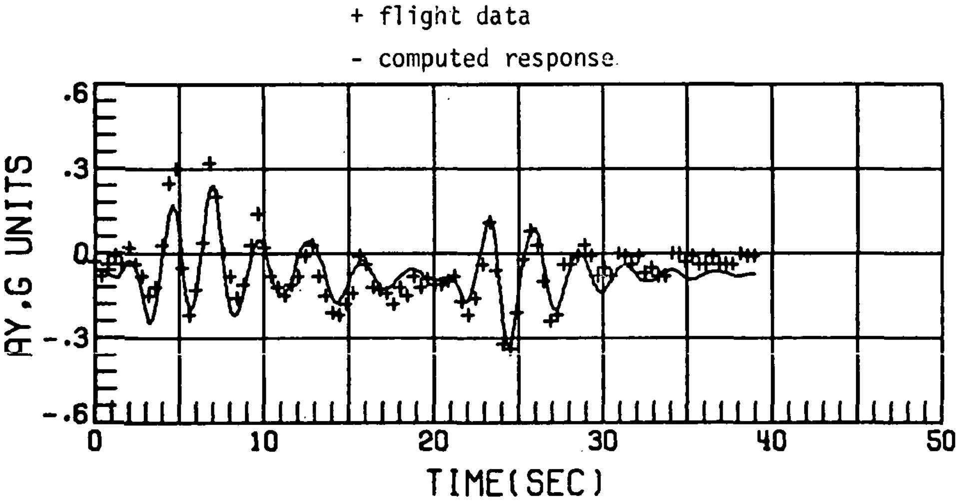


Figure 6.- Concluded.

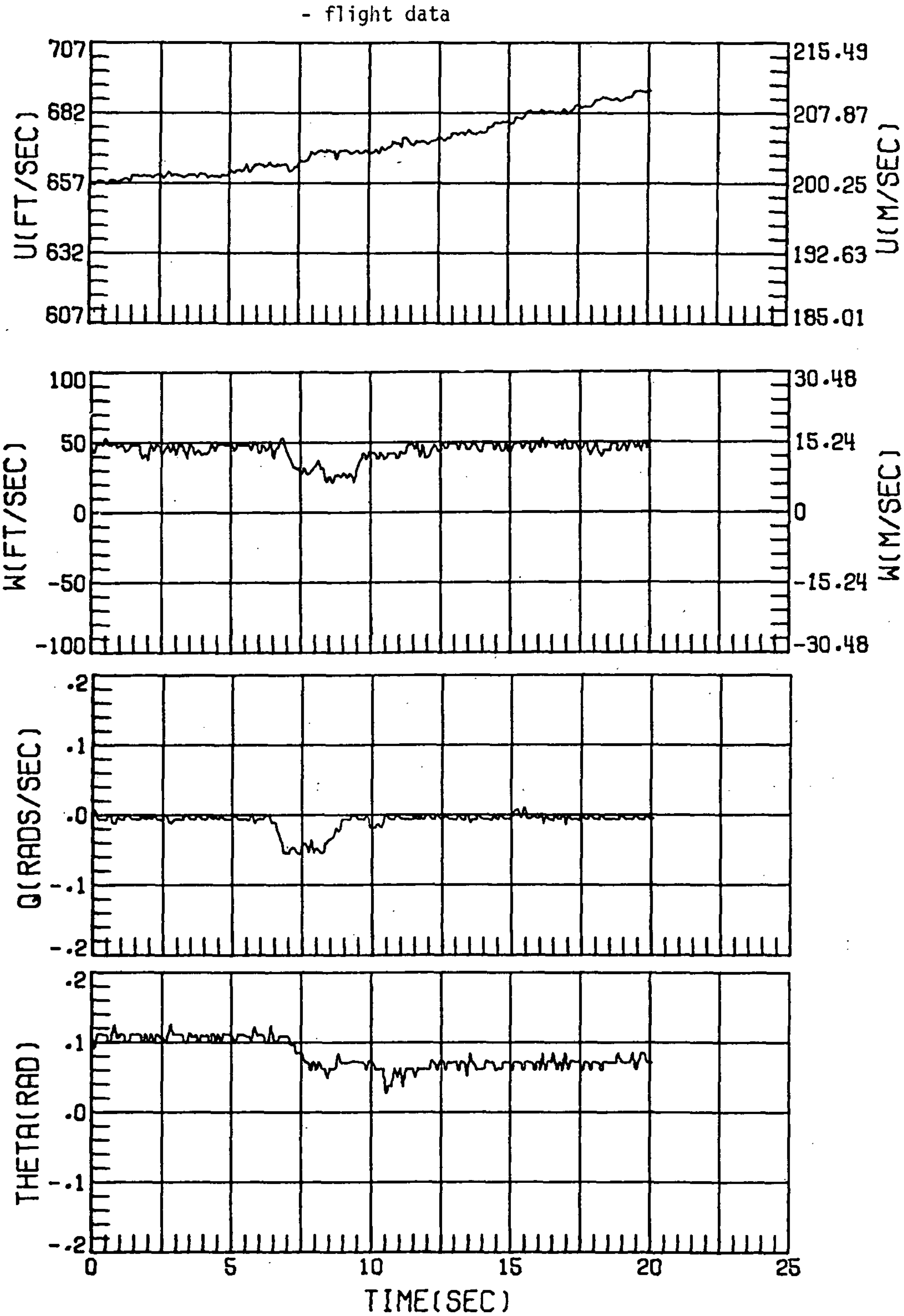
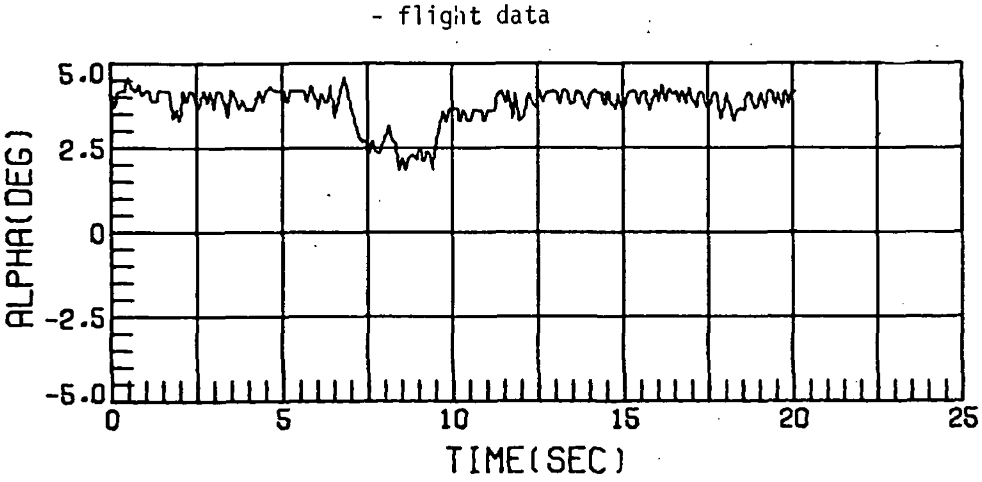


Figure 7.- Model responses generated by final derivative estimates of test run 1 with longitudinal data as input.



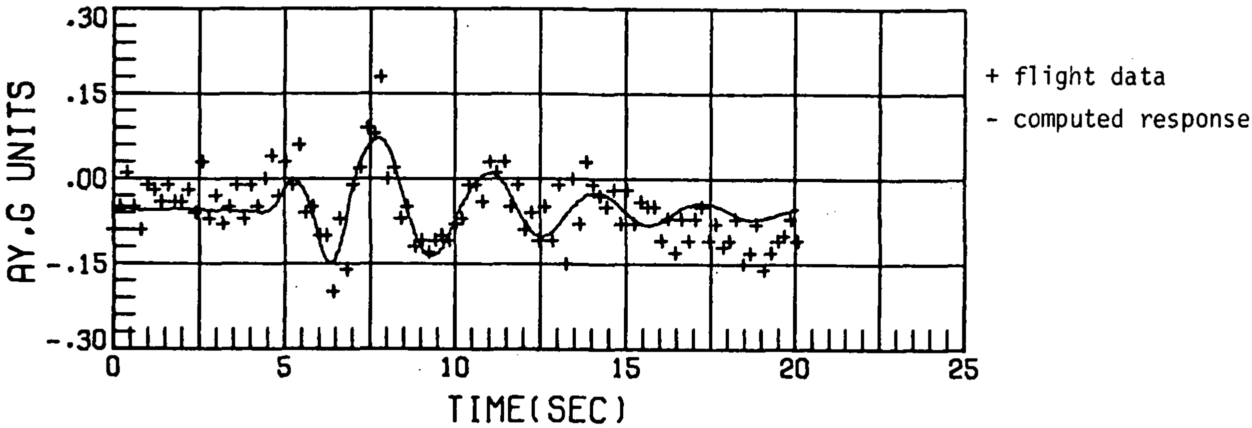


Figure 7.- Continued.

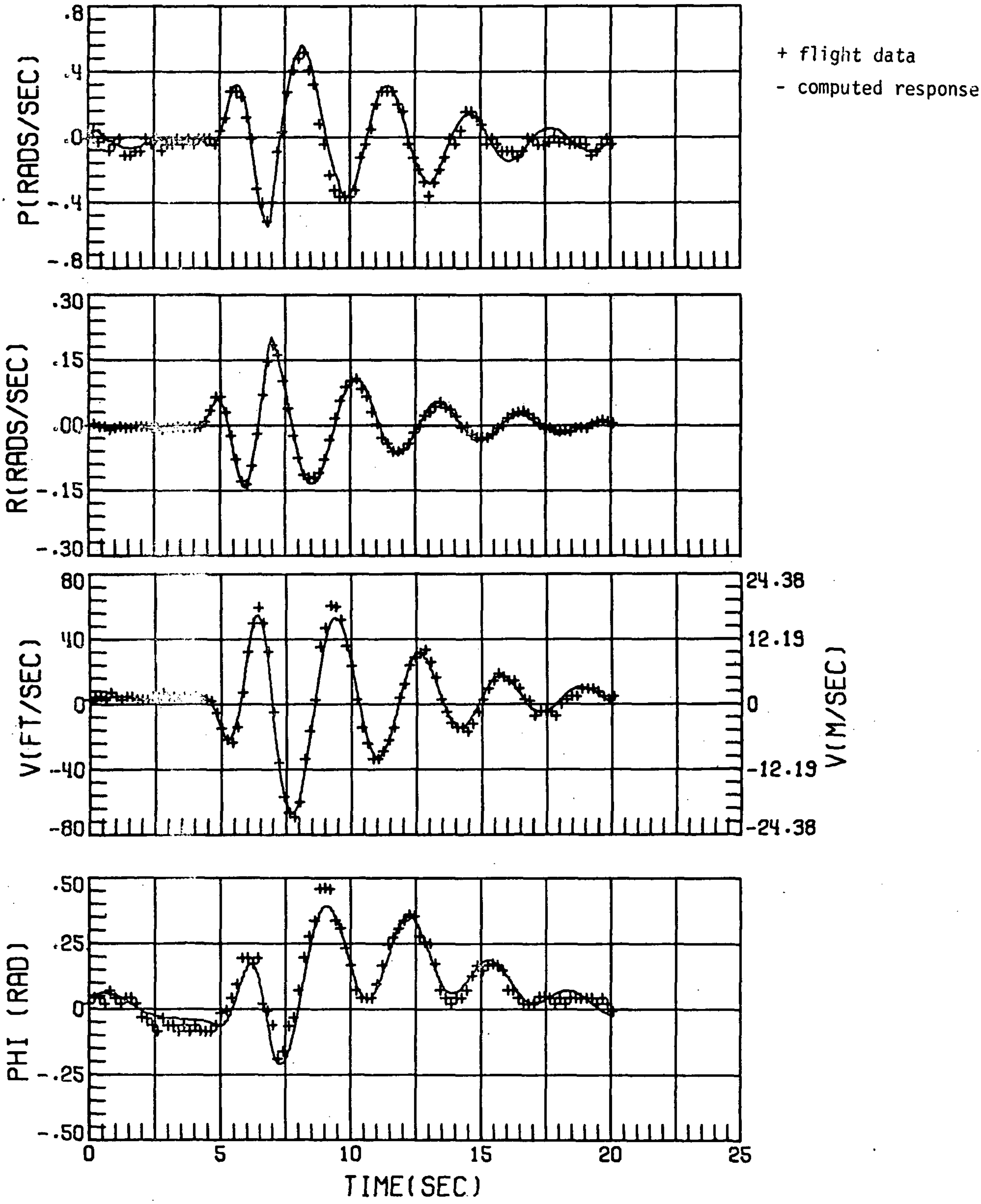
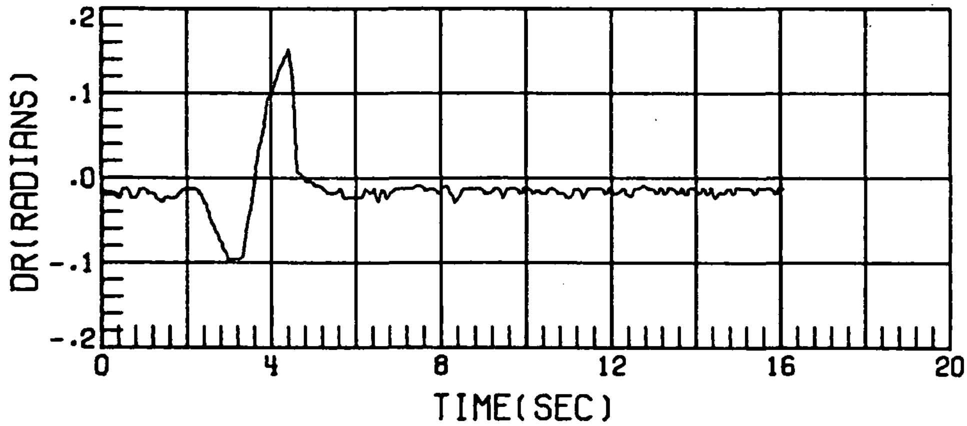


Figure 7.- Concluded.

- flight data



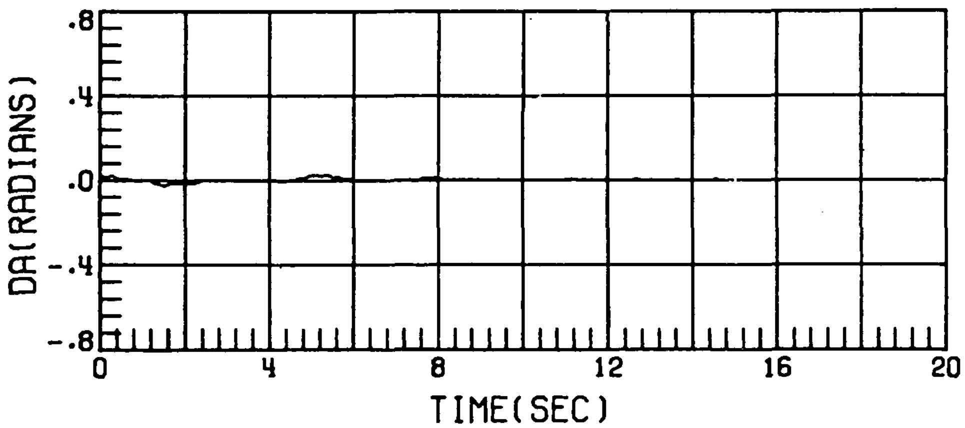


Figure 8.- Model responses generated by final derivative estimates of test run 3 with longitudinal data as input.

- flight data

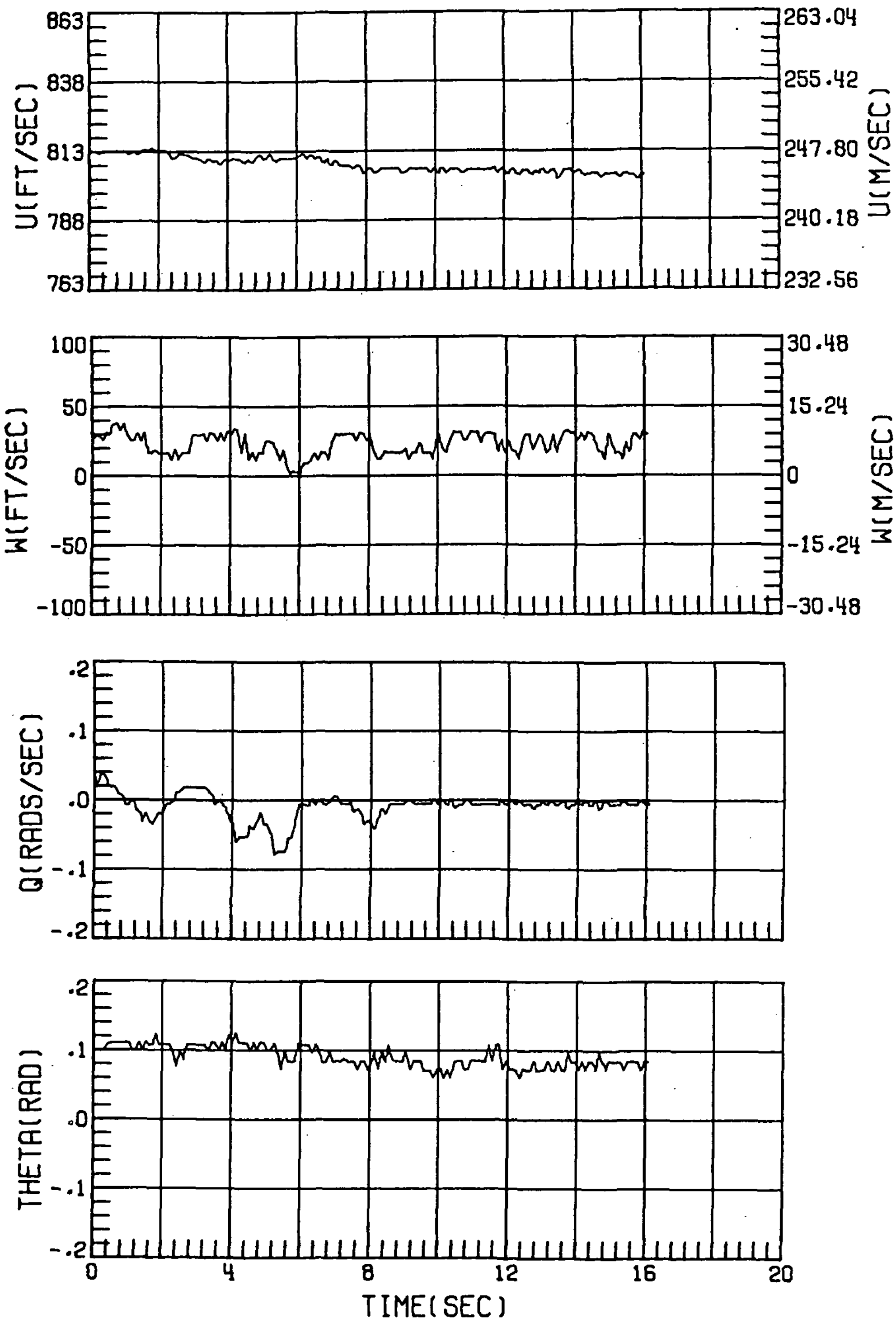
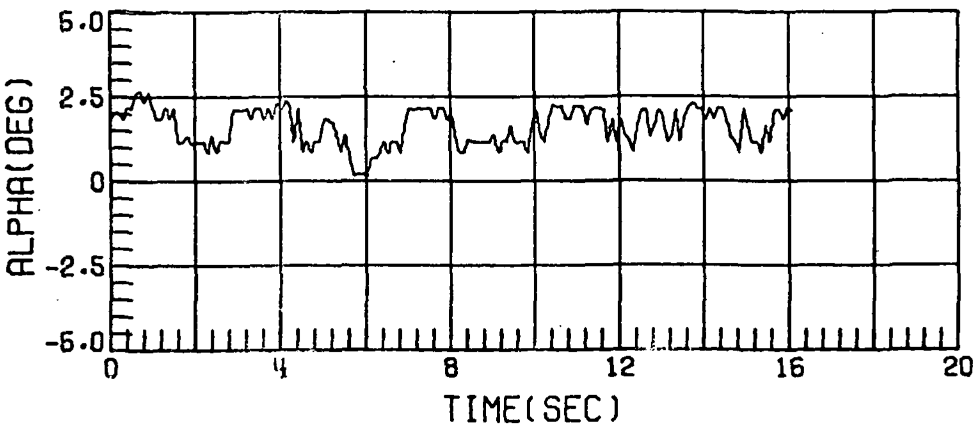
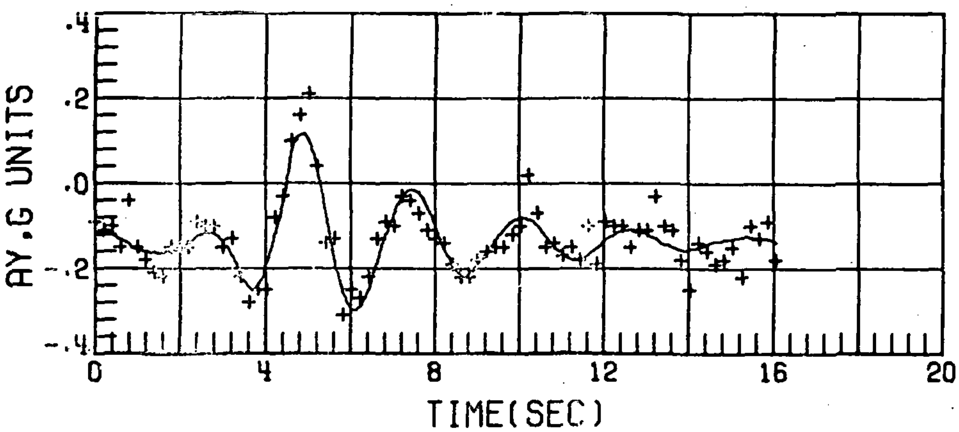


Figure 8.- Continued.

- flight data





+ flight data
- computed response

Figure 8.- Continued.

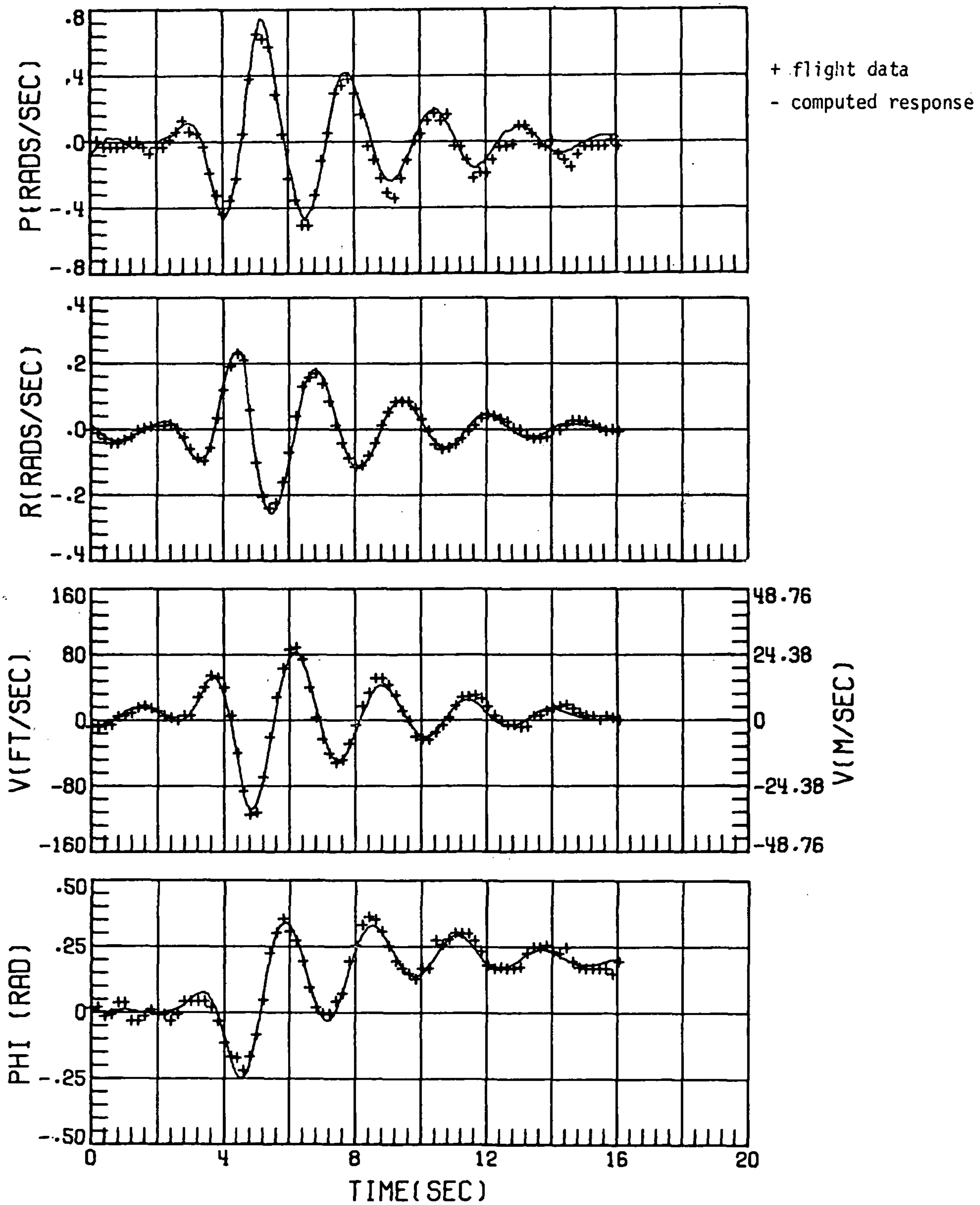
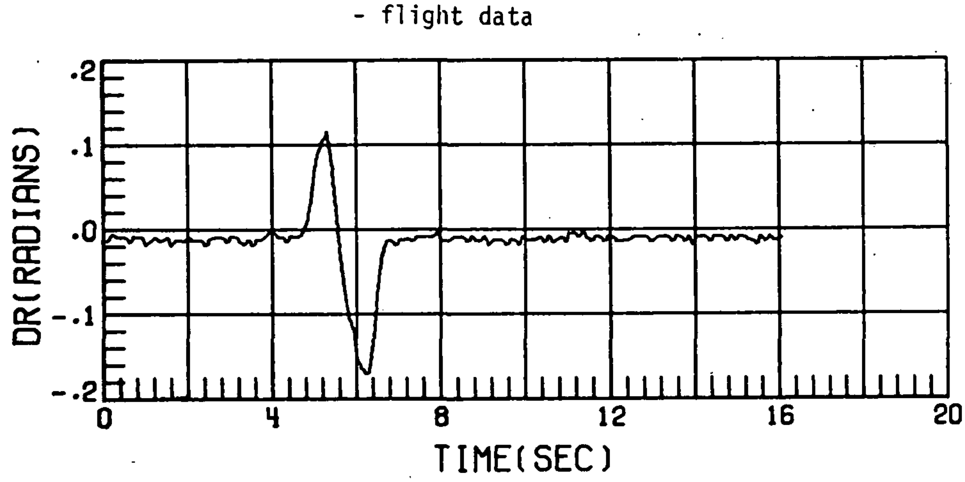


Figure 8.- Concluded.



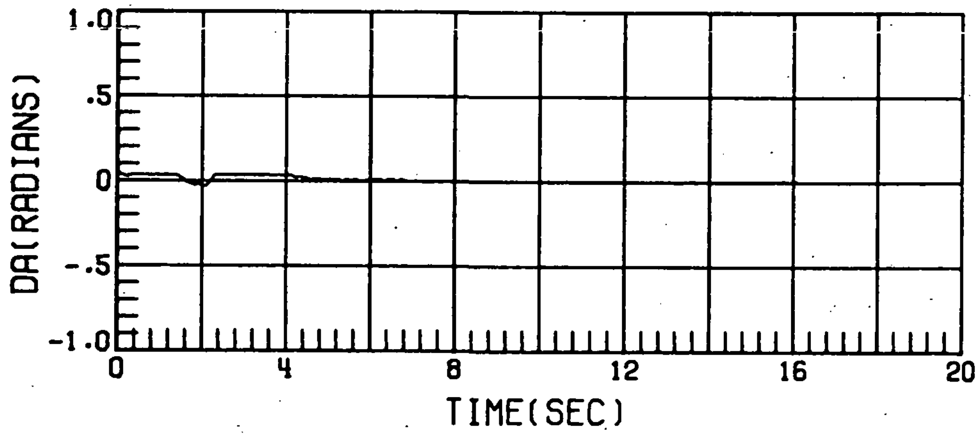


Figure 9.- Model responses generated by final derivative estimates of test run 4 with longitudinal data as input.

- flight data

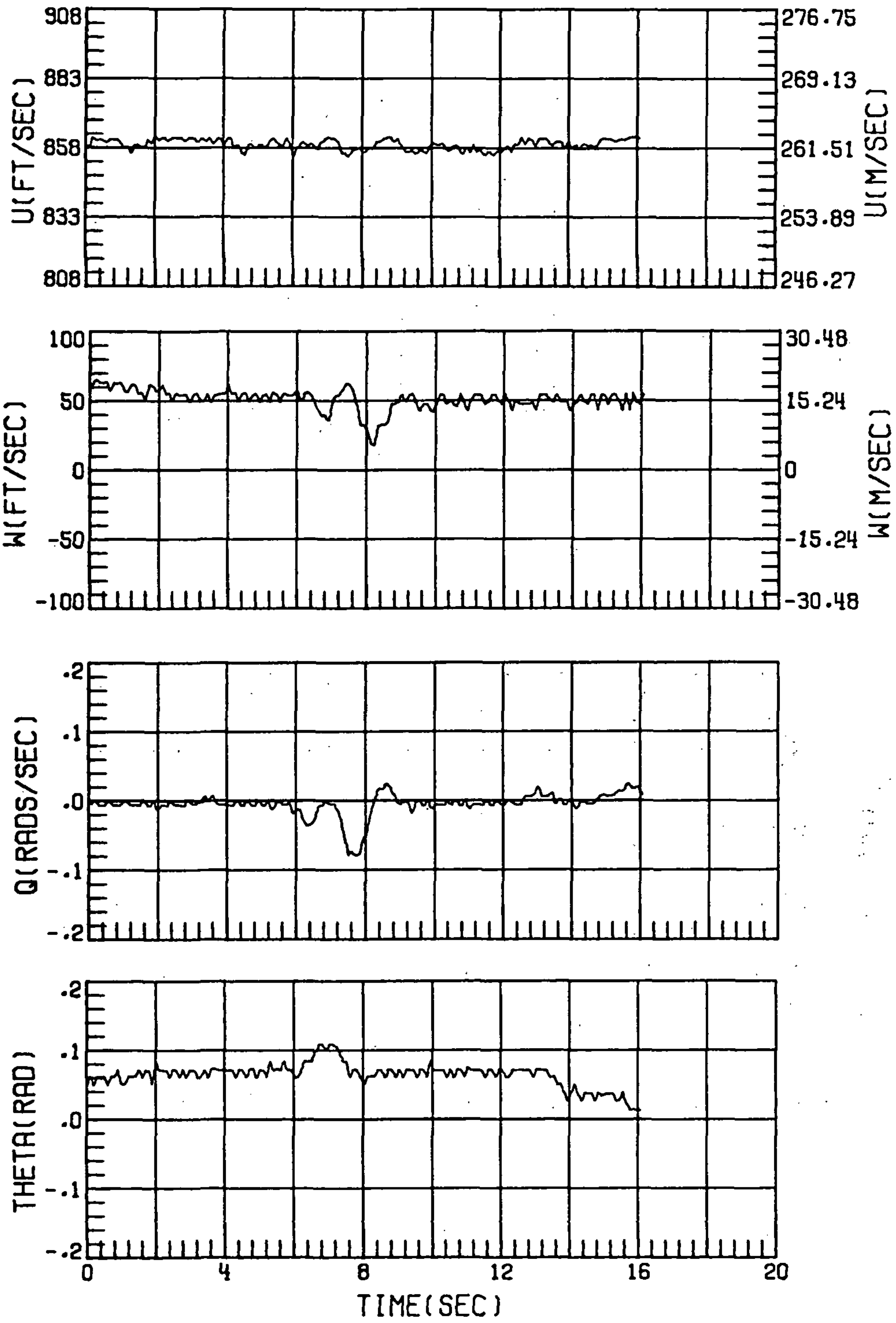
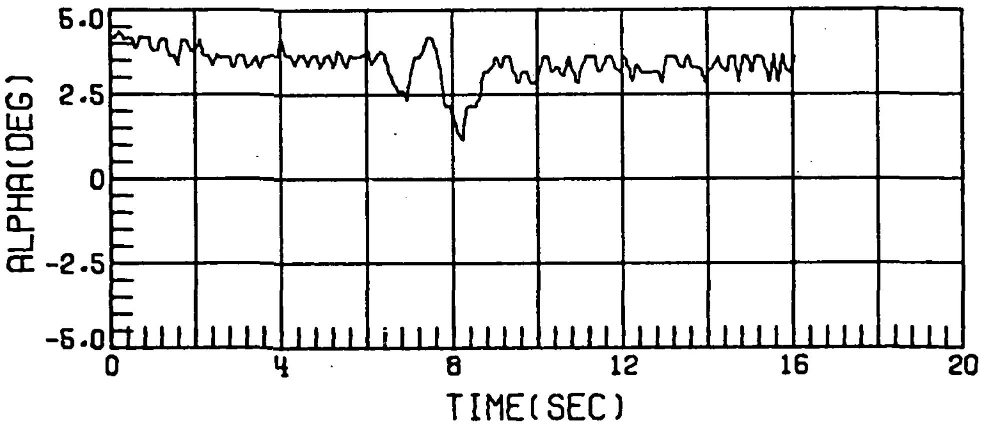


Figure 9.- Continued.

- flight data



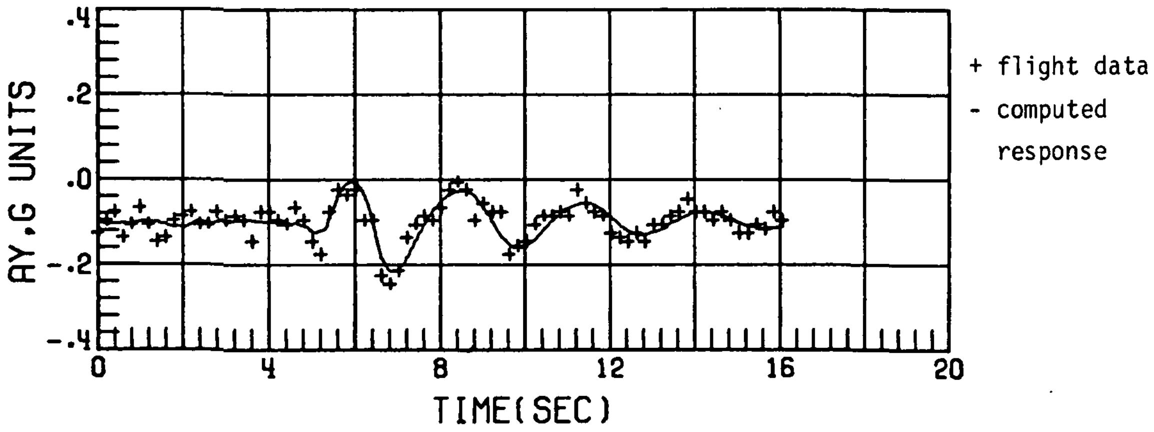


Figure 9.- Continued.

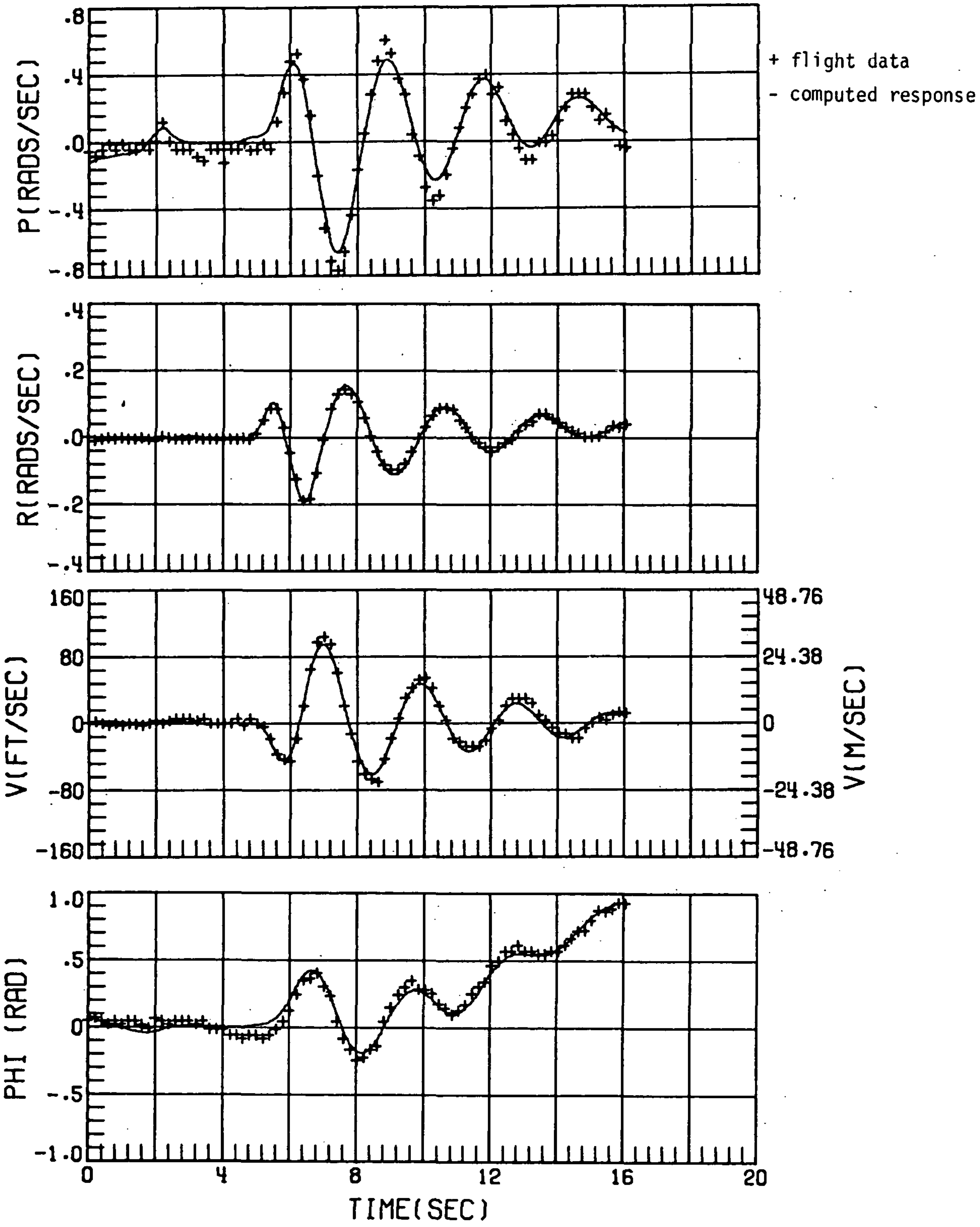
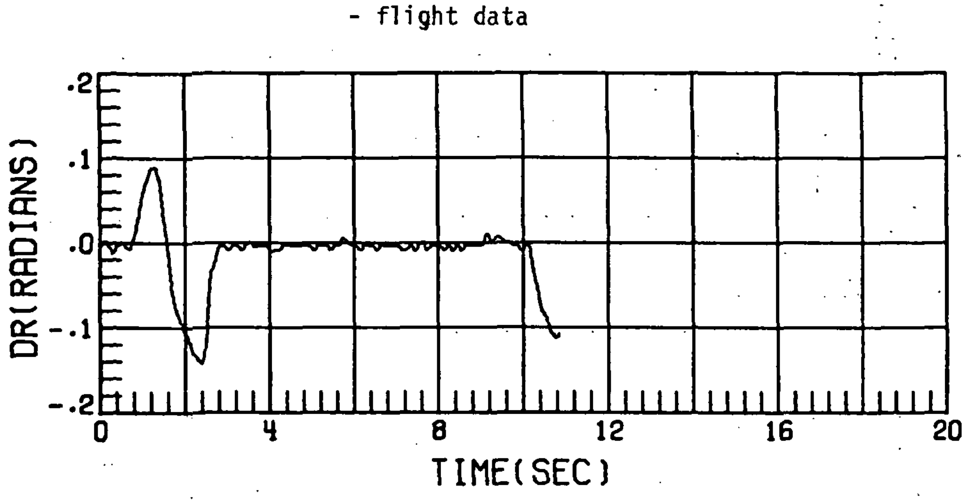


Figure 9.- Concluded.



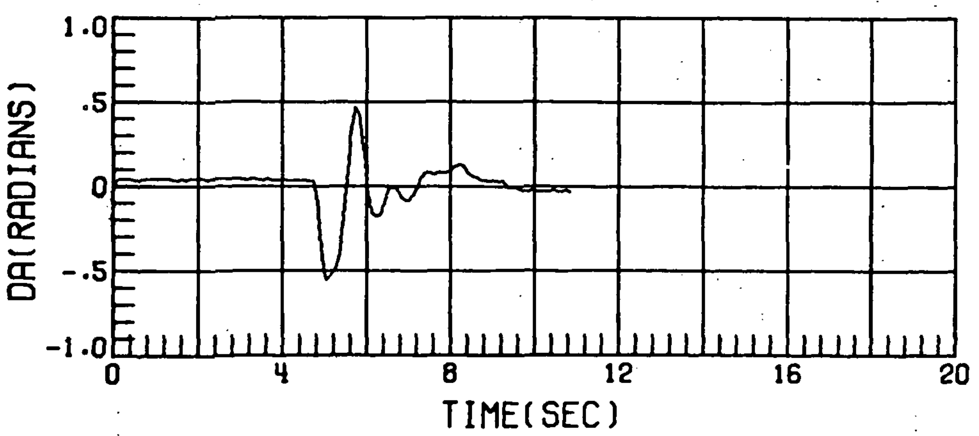


Figure 10.- Model responses generated by final derivative estimates of test run 5 with longitudinal data as input.

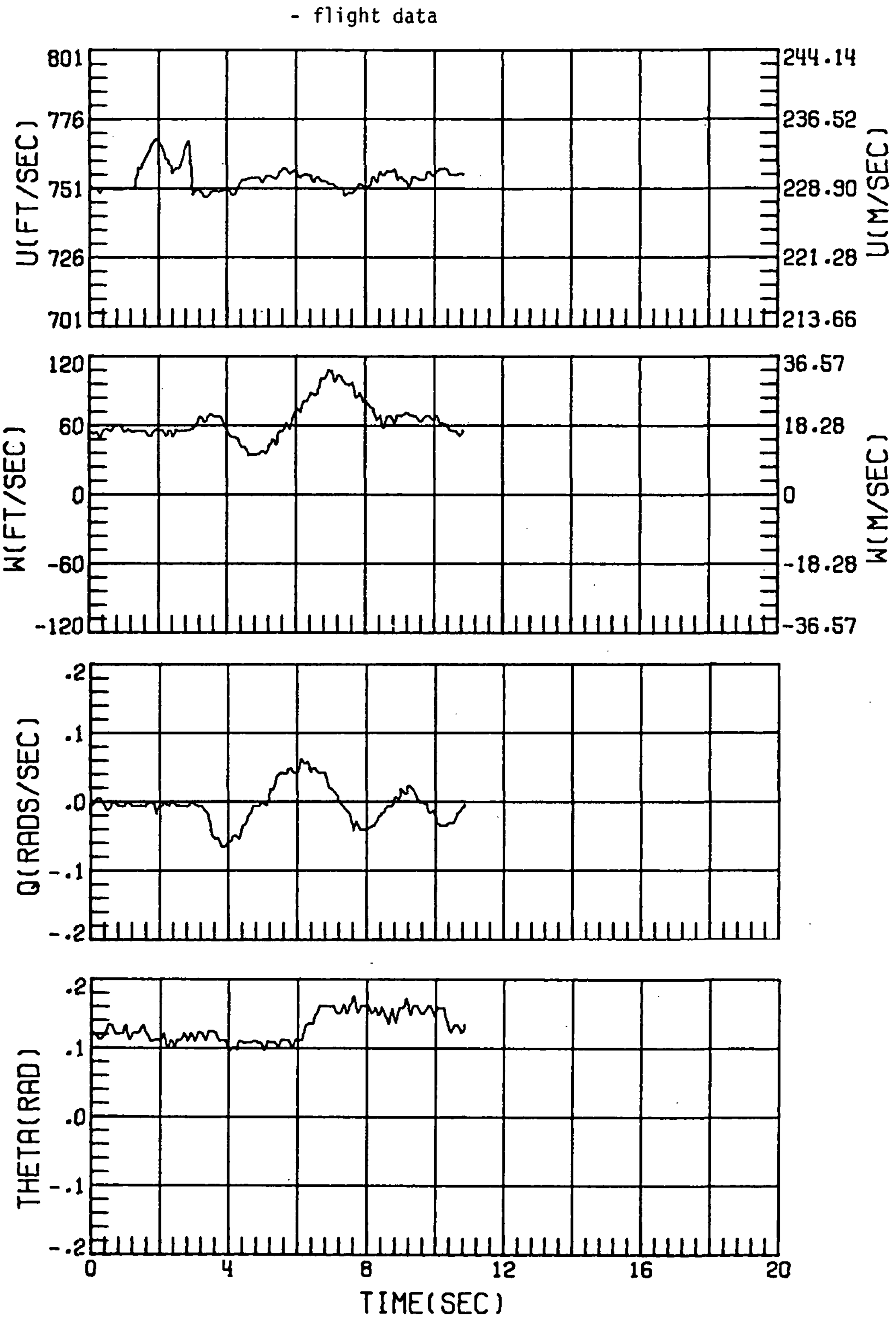
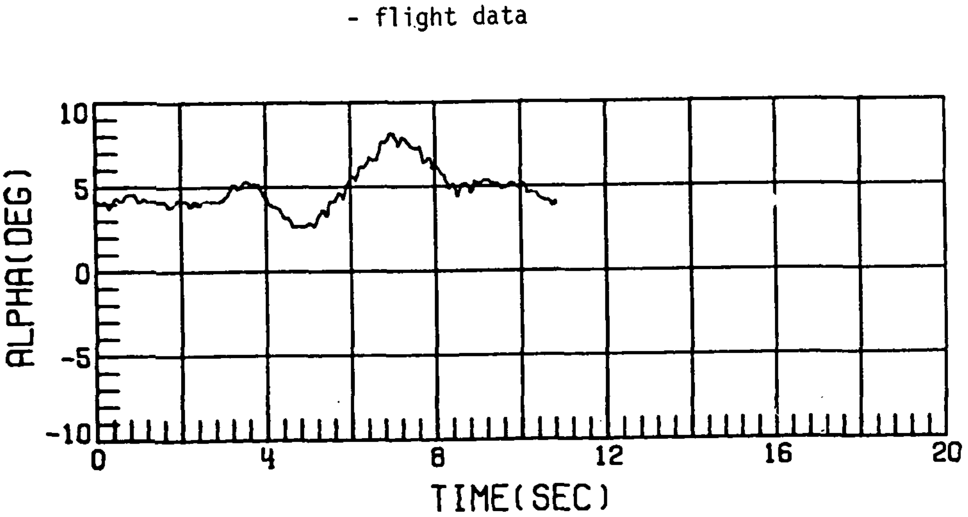


Figure 10.- Continued.



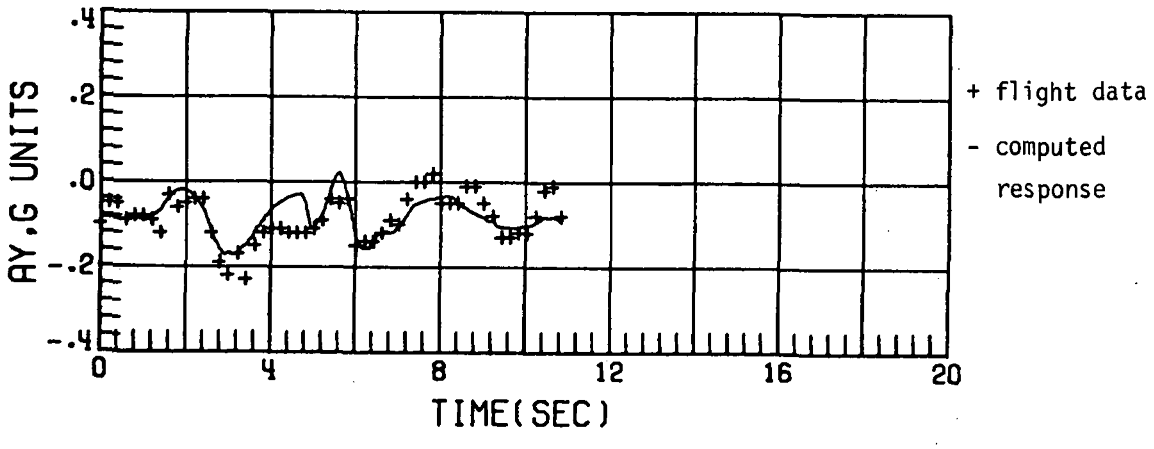


Figure 10.- Continued.

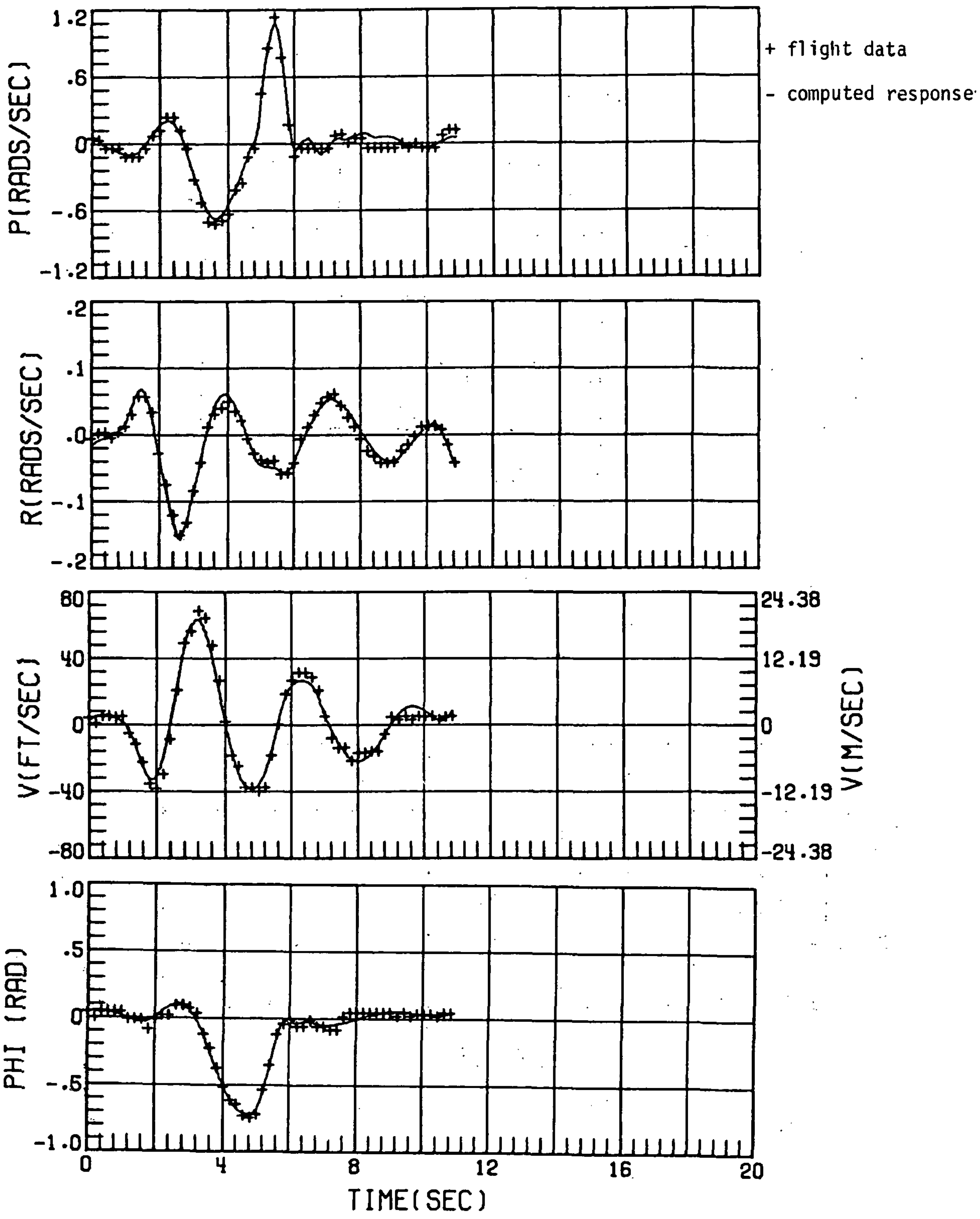


Figure 10.- Concluded.

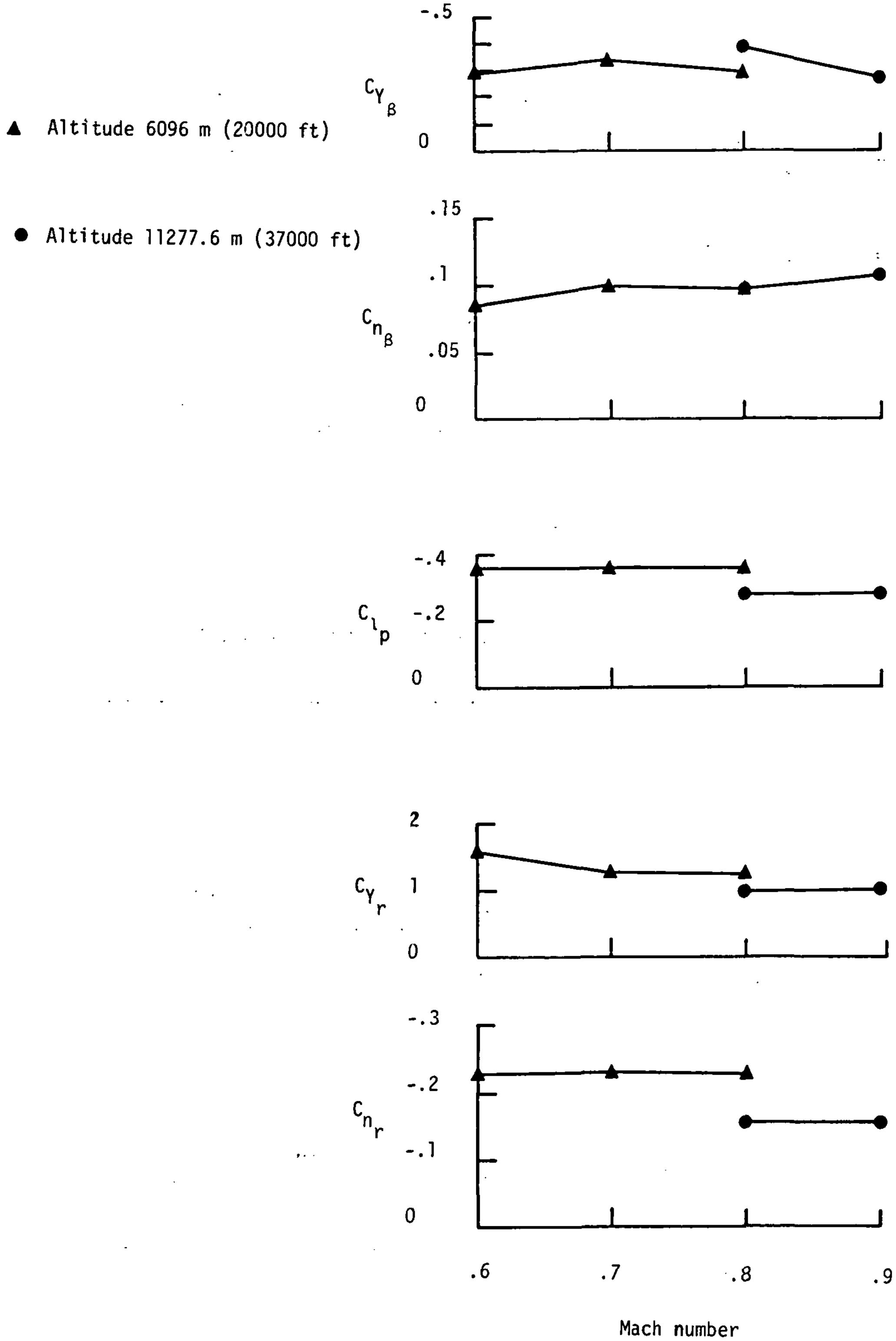


Figure 11.- Variation of stability derivatives with Mach number.

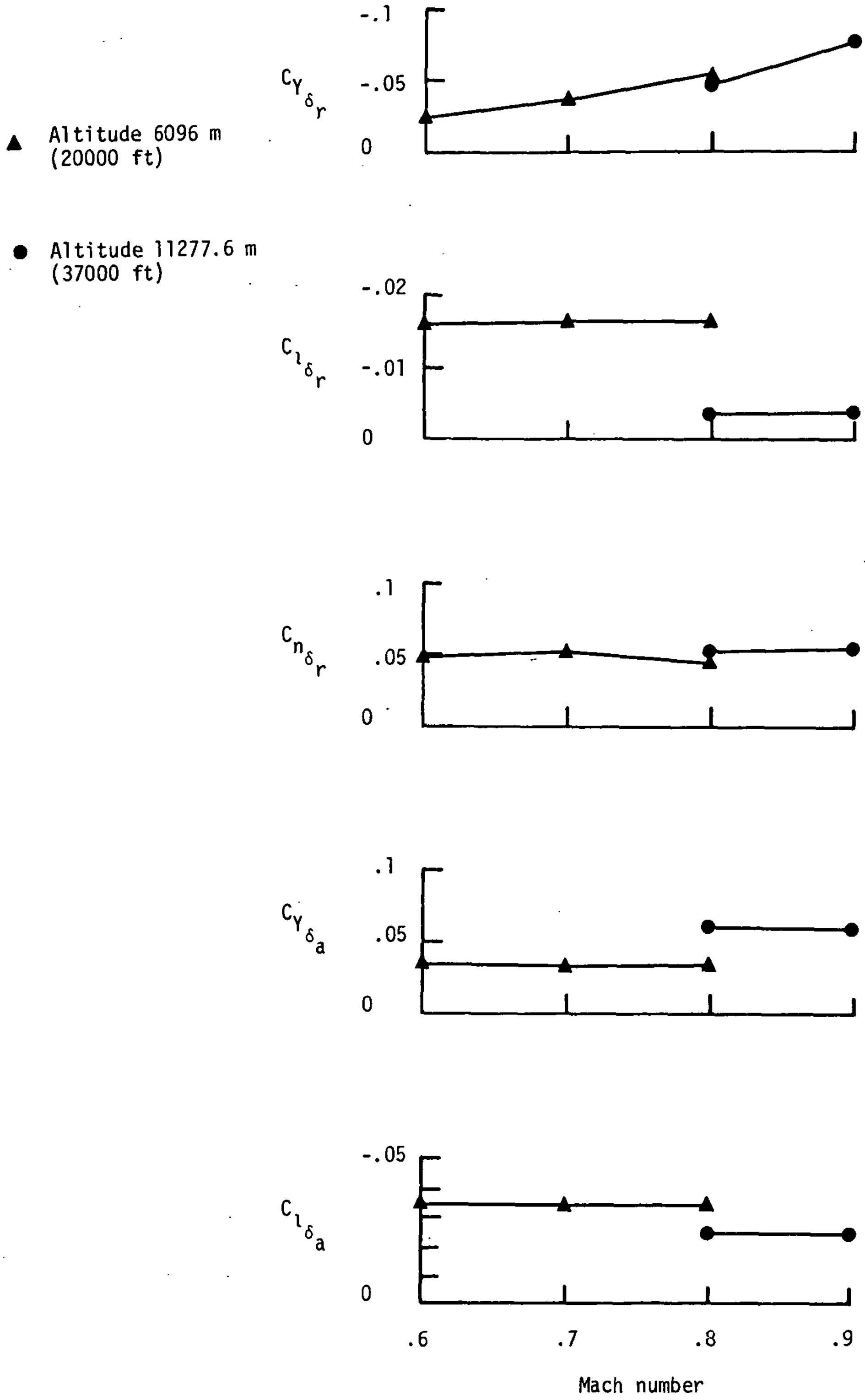


Figure 12.- Variation of control derivatives with Mach number.

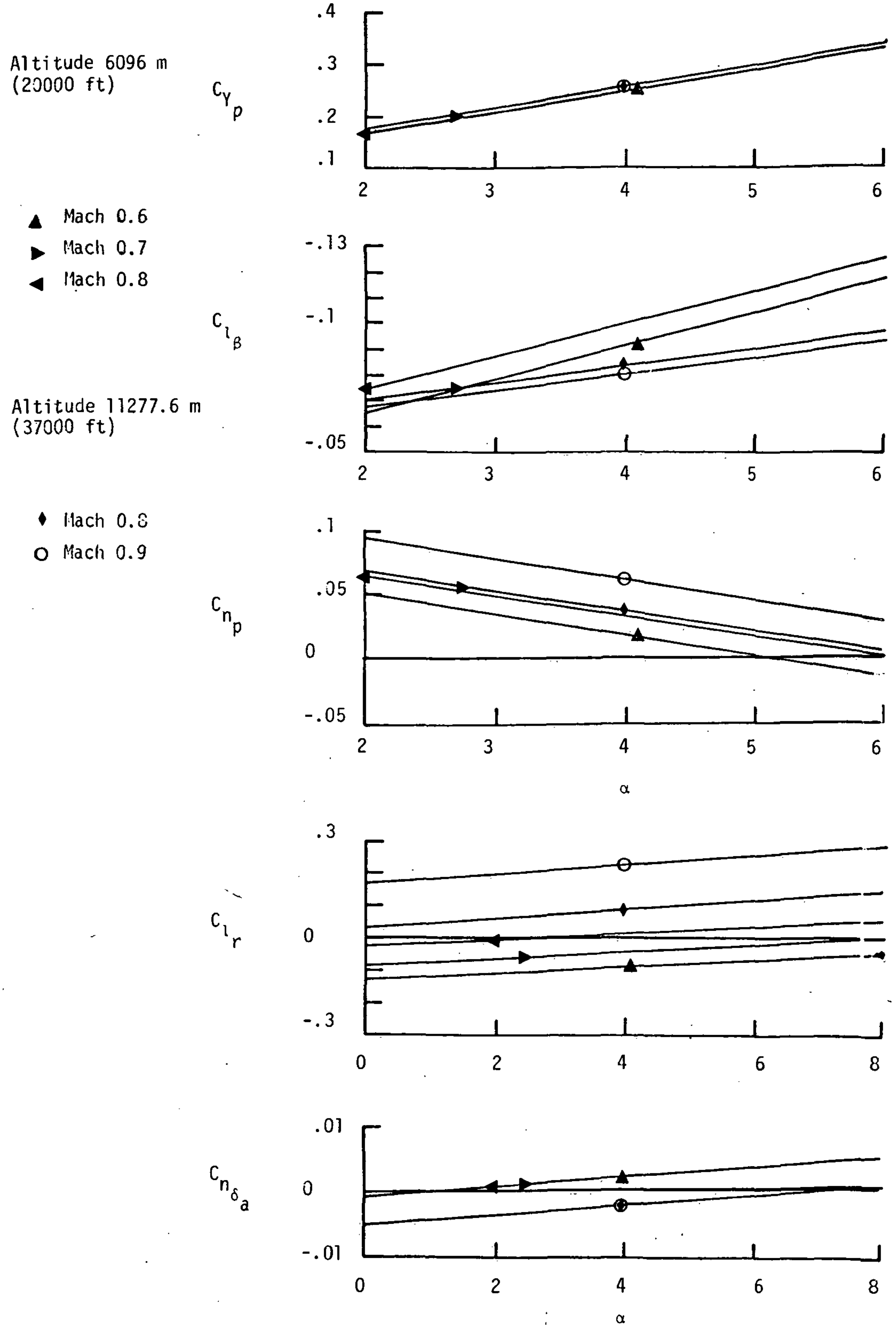


Figure 13.- Variation of parameters with angle of attack. (Symbol at trim angle of attack.)

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