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PERFORMANCE EVALUATION OF A CLASS OF SYSTEMATIC, RATE $\left(\frac{M-1}{M}\right)$, CONVOLUTIONAL CODES

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ABSTRACT

This report describes the implementation and performance evaluation of a class of Rate $\left(\frac{M - 1}{M}\right)$, systematic, convolutional codes being decoded with a simple majority logic decoder. The encoding logic appends one parity bit for each PCM telemetry word (typically 7 to 10 bits for NASA applications). It is shown that over the critical range of received PCM telemetry signal-to-noise ratios, this coding procedure produces a net coding gain of from 1.5 to 2.5 db relative to an equal power transmission of uncoded PCM telemetry. Being a low-redundancy systematic code, it is also possible to process this data without convolutional decoding if one is willing to incur a small rate loss penalty of about 0.5 db. The report suggest that this class of code be considered for NASA missions where a moderate increase in system gain is desirable.

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PERFORMANCE EVALUATION OF A CLASS OF SYSTEMATIC, RATE $\left(\frac{M-1}{M}\right)$, CONVOLUTIONAL CODES

INTRODUCTION

Convolutional coding is finding increased acceptance as a means of improving spacecraft telemetry performance. The earlier uses of convolutional codes in NASA deep space projects could be considered virtually a matter of necessity since a brute force solution by an increase in power, system gain and/or bandwidth was not technically feasible. In these cases the increase in system performance via the coding gain was therefore reluctantly accepted. More recently convolution coding is being implemented on other NASA satellites such as RAE-B and IMP-I/J where the deliberate choice of convolutional coding was not taken out of a sense of desperation but was made because it represented the most cost-effective solution and the problem. Traditionally, the communications designer has had to juggle the basic resources of power, system gain, and bandwidth into a system design with acceptable performance characteristics. Often the designer is faced with a dilemma that the resources that are convenient to allocate do not match the requirements and that to add the next increment of resources sufficient for required performance is unduly expensive. Convolutional codes are a viable tool that may permit a stretching of the resource to meet these performance requirements in a cost effective manner.

Convolutional codes and decoding devices run the gamut from extremely complex and operationally cumbersome techniques with high coding gain to simple and easily implemented techniques with low to moderate coding gain. Most of the information theory literature treats the former case and very little attention has been given to the simpler techniques which offer more modest coding gain. This report discusses one class of code which do offer simple encoding and decoding implementation and operability. Although the coding gain is modest, typically only 1.5 to 2.5 db, this class of code has many potential applications in NASA projects. To put the average achievable coding gain of 1.9 db into proper perspective it should be noted that the equivalent improvement could be achieved by:

(1) Increasing the spacecraft transmitting power by 55% or

(2) Replacing an 85 foot antenna dish with a comparably equipped 106 foot dish.

Certainly the latter alternative is very unpleasant to contemplate and the former item may be equally impractical under some conditions. Thus one should not sneer at improvements in the order of 1.9 db, especially if one is coming out slightly on the short side in the circuit margin calculations.

All convolutional codes used by NASA to date have been rate 1/M binary codes in which a binary signalling alphabet is used and M bits are transmitted for every information bit. By far the most common type of convolutional code is the rate 1/2 code. The code is systematic if one of the sub-bit stream imbedded in the overall encoded bit stream represents the information bits in an undistorted form. Otherwise the code is nonsystematic. Figure 1A, B show a simple example of a rate 1/2 systematic and nonsystematic encoder respectively.

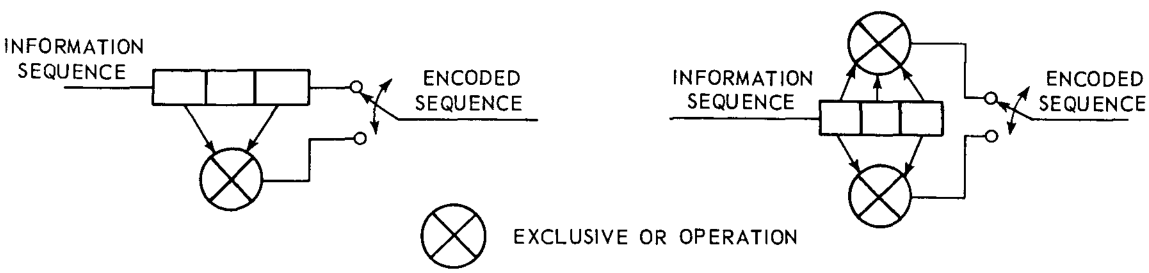


Figure 1A. Systematic Encoder

Figure 1B. Non-Systematic Encoder

With most convolutional codes, the encoding technique is relatively straight forward and easy to implement. This is fortunate because, for many NASA applications, it is the encoding equipment which must reside in the spacecraft. A much greater variety of decoding techniques and equipment exist ranging from the very simple to the extremely complex. In general you get what you pay for with the operational complexity going up rapidly with each db of improvement in coding gain. Figure 2 illustrates a very simple majority logic decoder for the systematic encoding algorithm shown in Figure 1A*.

In this report a class of systematic, Rate $\left(\frac{M-1}{M}\right)$, convolutional codes will be described. In these codes one parity bit is generated for each telemetry word. Since a telemetry word typically contains between seven and ten bits for most NASA missions, the reduction in signal energy per information bit is minimized. The performance evaluation contained in the latter portion of this report will compare convolutional coding performance versus uncoded performance on the basis of equal transmitted signal power. Therefore since more

*With all the techniques described in this report it is assumed that bit and character synchronization have been achieved prior to the convolutional decoding process. This can be done by conventional frame synchronization techniques.

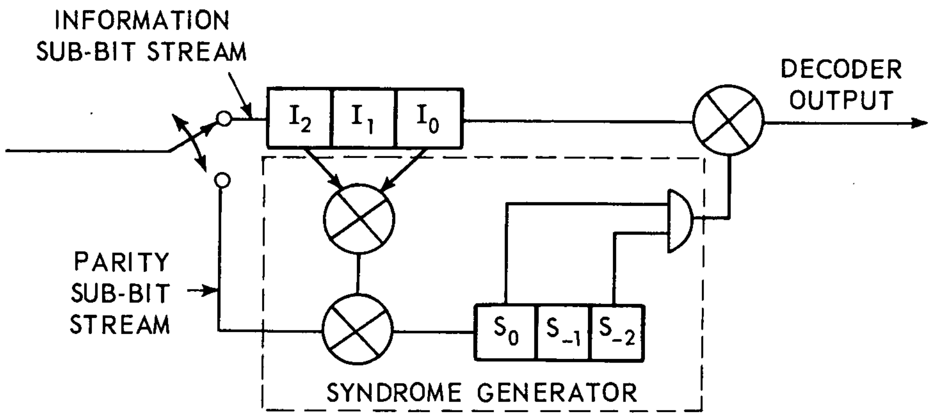


Figure 2. Majority Logic Decoder for a Rate 1/2 Systematic Code

bits are transmitted in the coded case, the signal energy per bit and signal-to-noise ratio must necessarily decrease relative to the uncoded case. This loss is referred to as the rate loss and is tabulated below for word lengths from 2 to 12.

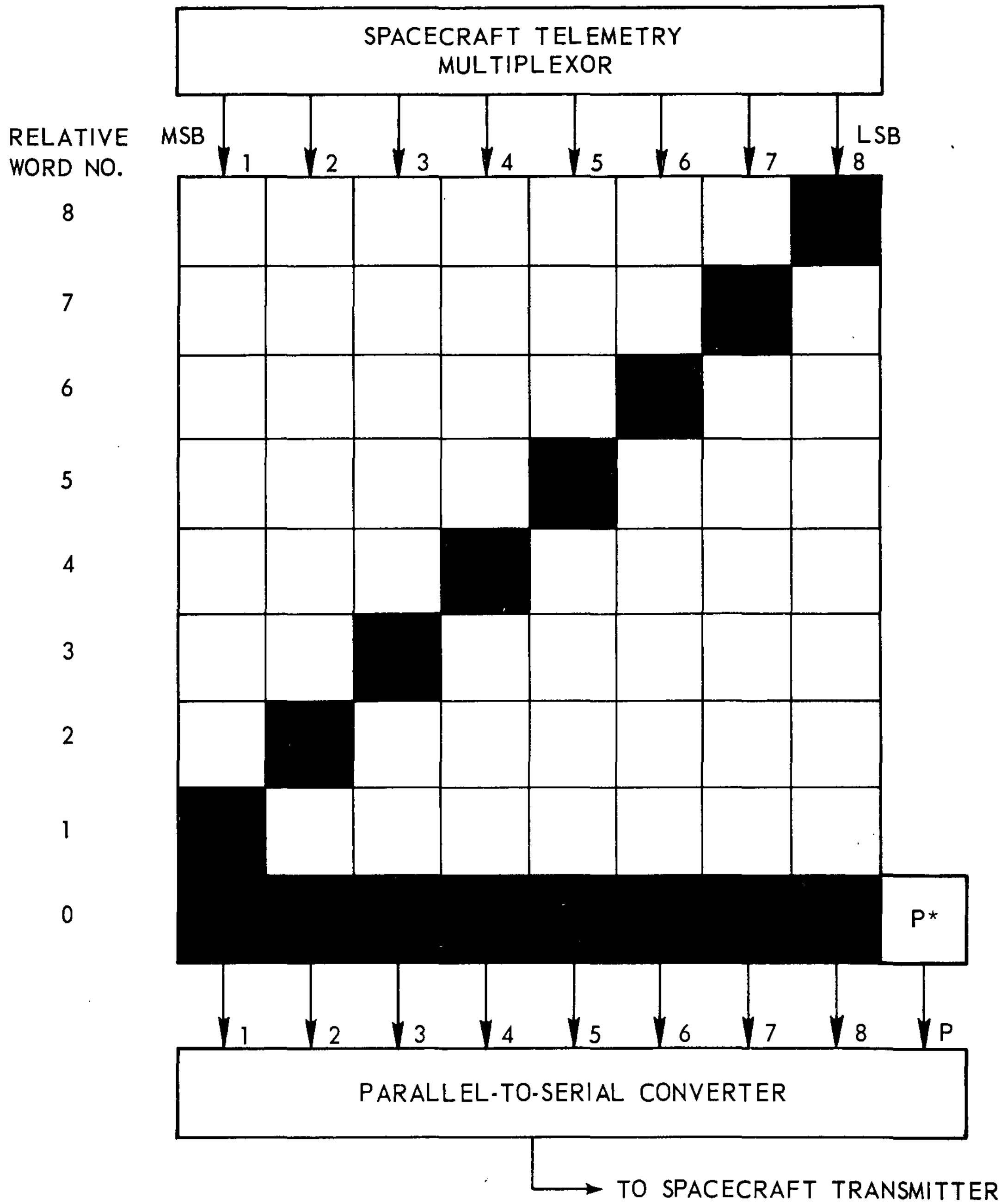
Word Length (bits)	Code Rate	Rate Loss (db)
2	1/2	3.01
3	2/3	1.76
4	3/4	1.24
5	4/5	0.96
6	5/6	0.79
7	6/7	0.67
8	7/8	0.58
9	8/9	0.51
10	9/10	0.46
11	10/11	0.41
12	11/12	0.38

The rate 1/2 encoding and decoding scheme illustrated in Figures 1A and 2 respectively is not a cost effective means of improving telemetry performance. The reason for this is that the scheme requires the transmission of one parity bit for every information bit; therefore, the transmitted energy per bit is reduced by 50% from an uncoded transmission. The simple coding scheme shown in this illustration does not possess sufficient gain to overcome this initial 3 db handicap.

SYSTEMATIC, RATE $\left(\frac{M - 1}{M}\right)$, CONVOLUTIONAL CODES

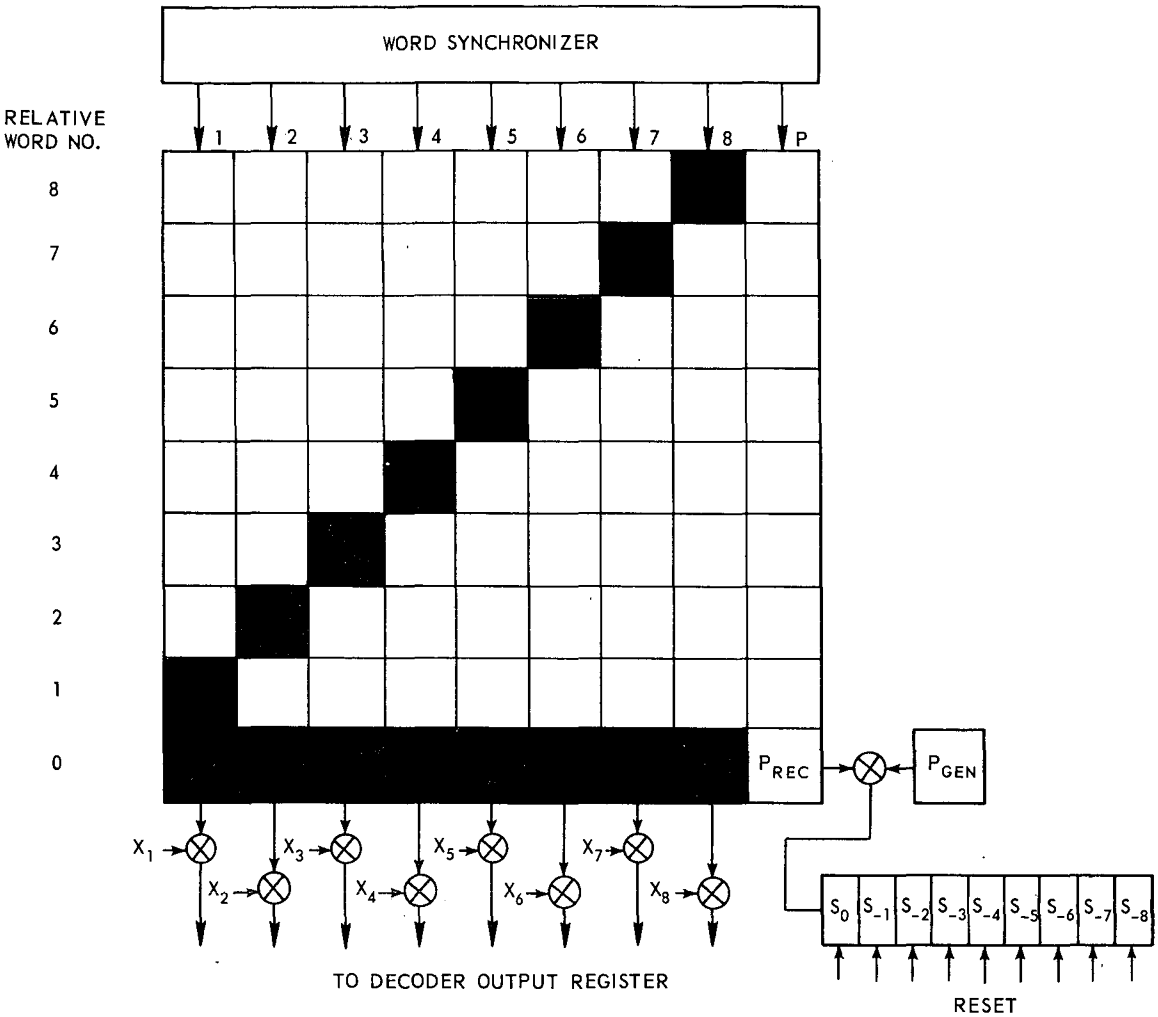
Figures 3A and 3B illustrate the encoder and decoder respectively of a Rate 8/9 convolutional code. In Figure 3A an 8-bit word is received from the spacecraft telemetry multiplexor and each bit is shifted into the corresponding entrance stage of eight shift registers. Each shift register has 9 stages. The telemetry word is shifted down one stage each bit time, and after nine such shifts it is dumped into a parallel to serial converter and is then modulated on the telemetry link. Note also that as each individual bit is being shifted down the shift register it will pass through two stages which are shaded in Figure 3A. Each word time a parity bit is generated which is the Modulo 2 addition (Exclusive OR) of the contents of all 16 stages which are shaded in this diagram. This parity bit is transmitted as the 9th bit of each telemetry word.

At the decoder end (shown in Figure 3B) the telemetry data is bit-, word-, and frame-synchronized and the 9-bit parallel output of the word synchronizer is input to 9 nine-stage shift registers. A parity bit (P_{GEN}) is generated from the information bits in exactly the same manner as in the encoder. If no errors occur in transmission, the P_{GEN} bit will be identical to the received parity bit (P_{REC}) at each word time. If $P_{GEN} \oplus P_{REC} = 1$ then the received and generated parity bits differ and this is symptomatic of transmission error(s). To identify the location of the error the output of the parity compare circuit ($P_{GEN} \oplus P_{REC}$) is gated into a syndrome shift register. If only one error has occurred within the data currently stored in the shift registers, and if this is an information bit, then, when the bit in error reaches the bottom stage of the shift register, S_0 will be 1 and one and only one of the other syndrome stages S_{-1} through S_{-8} will also be set to 1. This syndrome stage (S_{-1} to S_{-8}) which is set uniquely identifies the information bit in error and permits the flipping of the appropriate bit to correct the error. This decoder cannot correct all combination of errors but it will be shown that, over a broad range of signal-to-noise ratios, a net coding gain is achievable. Figures 3A and 3B can be easily extrapolated for any other word length.



*PARITY BIT (P) IS THE MODULO 2 ADDITION OF ALL SHADED BITS

Figure 3A. Systematic Rate 8/9 Convolutional Encoder



P_{GEN} = LOCALLY GENERATED PARITY REPRESENTING MODULO 2 ADDITION OF ALL RECEIVED BITS IN SHADED STAGES OF SHIFT REGISTERS.

$$X_i = S_0 \cdot S_{-i} \cdot \prod_{\substack{j=1 \\ j \neq i}}^{8} \bar{S}_{-j} \qquad \text{RESET} = \sum_{i=1}^{8} X_i$$

Figure 3B. Systematic Rate 8/9 Convolutional Decoder

EVALUATION OF CODING PERFORMANCE

The probability of an error out of the telemetry bit synchronizer/detector is expressed as:

$$\epsilon = \frac{1}{2} \operatorname{erfc} \left(\sqrt{(SNR) \cdot (RATE)} \right) \quad (1)$$

where

ϵ = pre-correction bit error rate

SNR = Equivalent signal-to-noise ratio for uncoded PCM telemetry expressed as a linear ratio (rather than a logarithmic ratio as in Decibels)

RATE = Coding rate (RATE = 1 for uncoded PCM and (M - 1)/M for coded PCM telemetry)

$$\operatorname{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$$

In the case of convolutionally encoded telemetry, the post-correction bit error rate, E, can be expressed as

$$E = P_{uc} + P_{fc} \quad (3)$$

where

P_{uc} = probability or an uncorrectable error, and

P_{fc} = probability of false correction.

To evaluate P_{uc} and P_{fc} it is necessary to understand the combination of conditions which give rise to both phenomena. This analysis is described in Appendix A. From Equations A-1 and A-2 we have

$$P_{uc} = 2 - (1 - \epsilon)^{\frac{1}{2}(M^2 + 3M - 2)} - (1 - E)^{(M-1)^2} \quad (4)$$

$$P_{fc} = M^2 \epsilon (1 - \epsilon)^{M-1} \left[\frac{\epsilon}{2} \cdot (1 - \epsilon)^{\frac{M}{2} - 1} + E \cdot (1 - E)^{M-2} \right] \quad (5)$$

The post-correction bit error rate, E, may be found by substituting (4) and (5) into (3) and solving for E by iterations.

A FORTRAN program was written to evaluate the performance of this class of code. The program listing is shown in Appendix B and the resulting computer printout is contained in Appendix C.

Figure 4 is a graph of the effective bit error rate versus S/N ratio for uncoded PCM telemetry and for convolutionally coded PCM telemetry at rates 2/3, 3/4, 4/5, 6/7, 8/9, and 10/11. Note that the convolution coding performance crosses the uncoded performance curve between 5.0 db and 6.4 db. At signal-to-noise ratios above this cross-over range, the convolutional codes give superior performance. This cross-over range corresponds to an uncoded bit error rate between 1.5×10^{-3} and 6×10^{-3} and is too high for normal telemetry processing.

Figure 5 illustrates the coding gain (or loss) in decibels for convolutionally encoded PCM versus the uncoded case. This coding gain includes the previously mentioned rate loss effect. For most telemetry applications, the maximum acceptable bit error rate is 10^{-4} which corresponds to an uncoded S/N ratio of 8.4 db. Above 12 db the bit error rate is so low even for uncoded telemetry that any further improvement via coding is generally unwarranted. A Rate 8/9 convolutional code will provide coding gains of 1.5 db and 2.3 db at signal-to-noise ratios corresponding to uncoded telemetry reception at 8.4 db and 12 db respectively. The average coding gain over this critical range is 1.9 db.

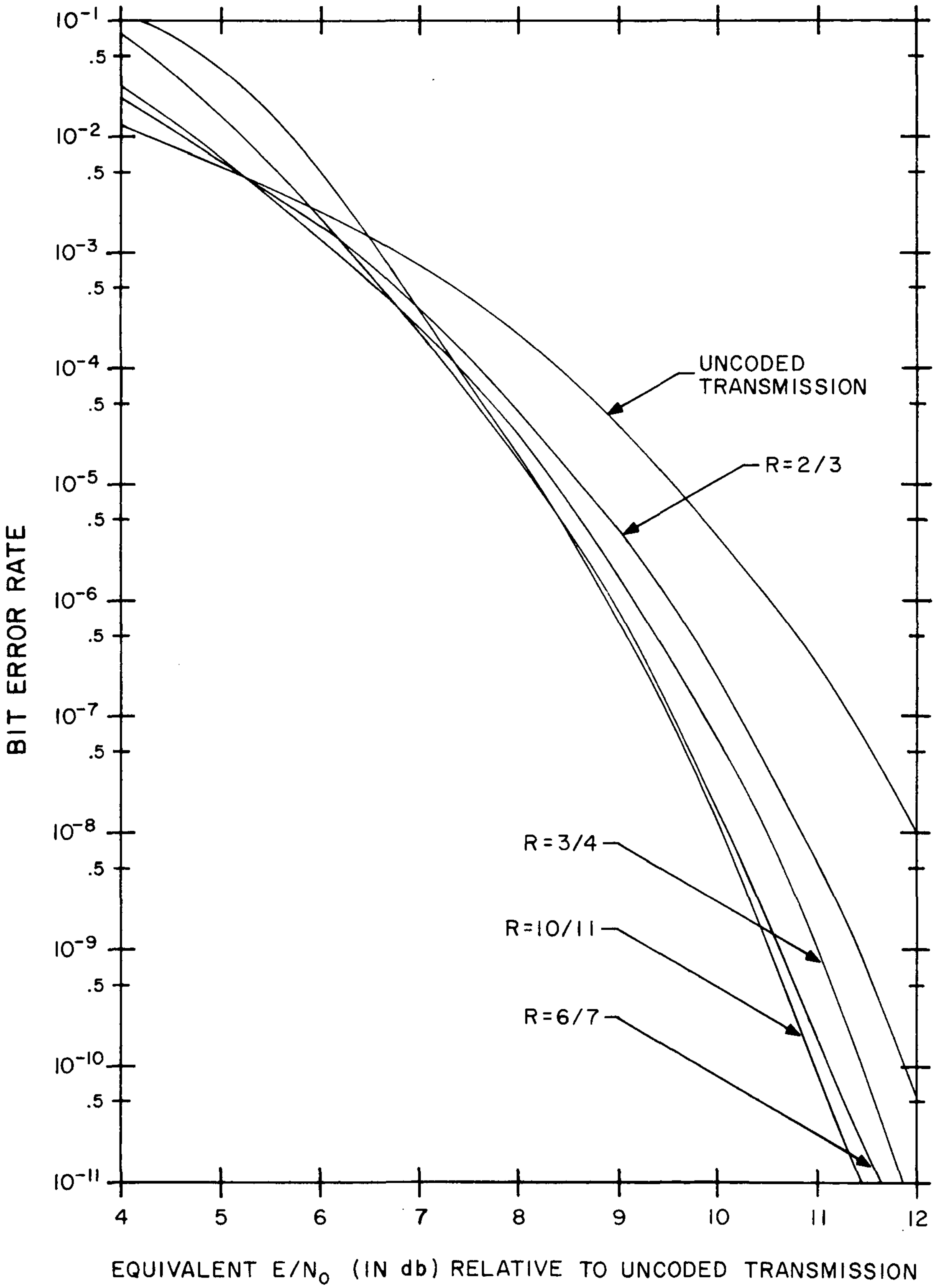


Figure 4. Comparison of Bit Error Rate Versus S/N Ratio for Coded and Uncoded Telemetry

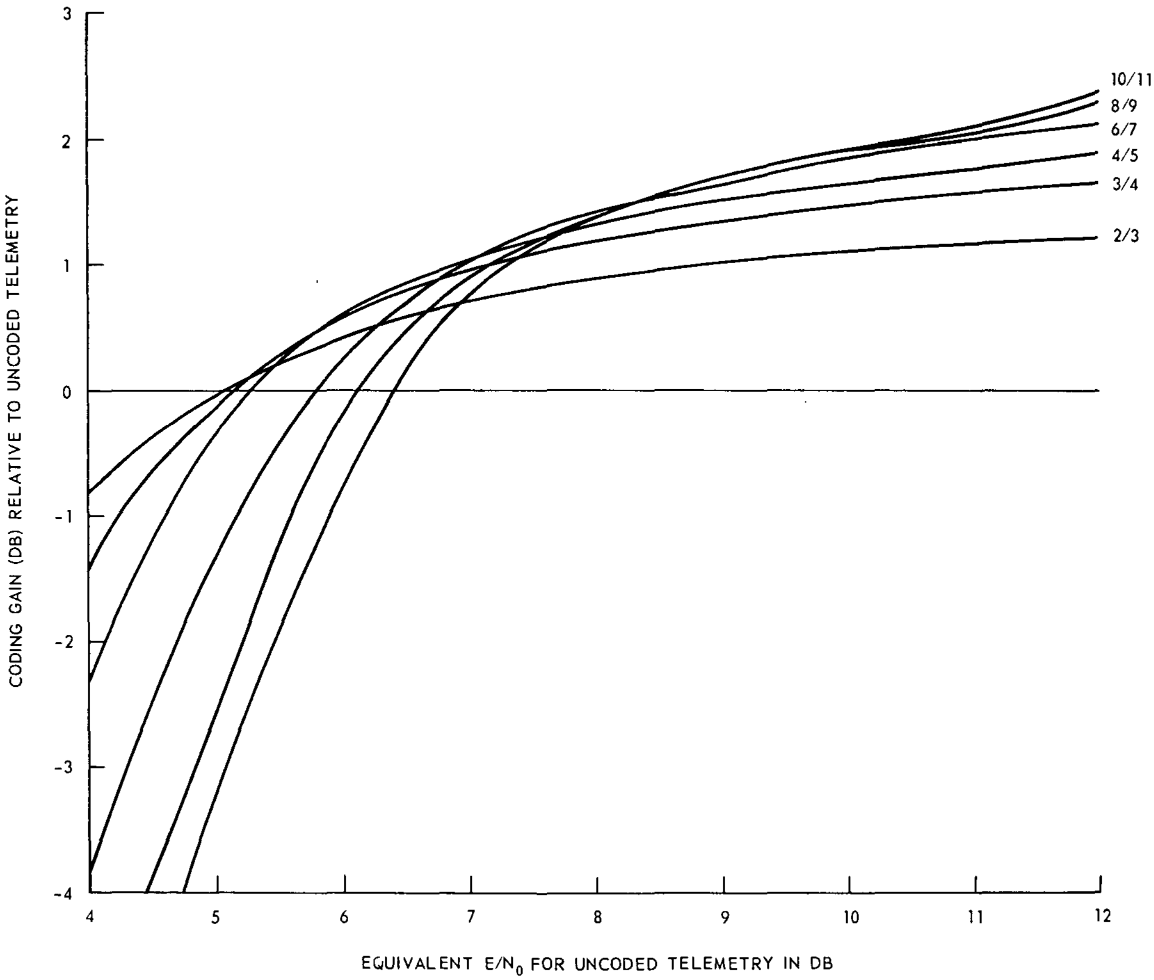


Figure 5. Coding Gain for Rate $\left(\frac{M-1}{M}\right)$ Convolutional Codes

SUMMARY

The class of systematic, Rate $\left(\frac{M - 1}{M}\right)$, convolutional codes using simple majority logic decoding possess the following features which may prove beneficial to some NASA spacecraft or ground communication application:

(1) Coding gain (including rate loss effect) between 1.5 and 2.5 db relative to uncoded PCM transmission over the critical range of signal-to-noise ratios.

(2) Low redundancy which minimizes the rate loss effect and conserves spectrum allocation.

(3) Extremely simple encoding and decoding logic which can provide a low cost reliable coding implementation.

From an information theory basis, there may appear to be little benefit in minimizing the amount of coding redundancy. It can usually be shown that with sophisticated decoding techniques, the coding gain obtainable with high redundancy codes more than compensates for the rate loss effect. Why, then, should we attempt to minimize the coding redundancy? One reason is that not all stations or mission support facilities are similarly equipped at any given point in time. Decoding equipment, even of the simple majority logic variety described in this report, may not be available at a facility for which mission support is desirable. If a nonsystematic code were used there is no hope of analyzing the data prior to decoding. If a high redundancy systematic code is used then the reduced signal-to-noise ratio due to the rate loss may make the reliability of the information sub-bit stream so low as to be unusable. For example, the rate losses for rate 1/2 and rate 1/3 codes are 3.0 db and 4.8 db respectively. However, in the case of a rate 8/9 convolutional code, the rate loss is only .5 db. Therefore, if a requirement exists to process a rate 8/9 telemetry bitstream at a facility without a convolutional decoder, the resulting bit error rate is only degraded by 0.5 db relative to an uncoded PCM transmission.

Another application may be in the area of PCM telemetry transmission of scanned image data. Here it is convenient to encode an entire scan line, which may consist of up to 100,000 bits, into one minor telemetry frame. A severe synchronization problem occurs due to the very long interval between frame sync words. A simple word parity scheme has been suggested to provide an additional aid to synchronization. The same objectives could be achieved using a systematic, Rate $\left(\frac{M - 1}{M}\right)$, convolution code with the added benefit that a coding gain relative to the word parity scheme of 2 to 3 db would be possible.

There is no intent in this report to suggest that Rate $\left(\frac{M-1}{M}\right)$ convolutional codes are a universal panacea to the various PCM telemetry design problems related to NASA mission. For many applications the S/N ratio is so good that even simple coding techniques are not warranted. On the other hand severe power/bandwidth constraints may necessitate coding gains far higher than are obtainable with this class of code. However, between these two extremes there lies a broad range of applications where a consideration of systematic Rate $\left(\frac{M-1}{M}\right)$ convolutional codes may prove cost-effective.

APPENDIX A

CALCULATION OF DECODER BIT ERROR RATE

For a $\frac{M-1}{M}$ convolutional code of a type similar to that shown in Figures 3A and 3B, the S_0 syndrome stage will be a 1 if and only if an odd number of bit errors in the transmission of the following bits:

S_0: P_0, (0, 1), (0, 2), (0, 3) . . . (0, M - 1), (1, 1), (2, 2), (3, 3) . . . (M - 1, M - 1)

In the above notation P_i refers to the parity bit transmitted with the ith word and (i, j) represent the jth information bit of the ith worth. Similarly the entire set of errors which could give rise to any non zero syndrome bits are

S_0:	P_0,	(0, 1),	(0, 2) . . .	(0, M - 1),	(1, 1),	(2, 2) . . .	(M - 1, M - 1)
S_{-1}:	P_{-1},	(- 1, 1),	(- 1, 2) . . .	(- 1, M - 1),	(0, 1),	(1, 2) . . .	(M - 2, M - 1)
S_{-2}:	P_{-2},	(- 2, 1),	(- 2, 2) . . .	(- 2, M - 1),	(- 1, 1),	(0, 2) . . .	(M - 3, M - 1)
.	.				.		.
.	.				.		.
.	.				.		.
S_{-N}:	P_{-N},	(-N, 1),	(-M, 2) . . .	(-N, M - 1),	(1-N, 1),	(2-N, 2) . . .	(M-N - 1, M - 1)
S_{1-M}:	P_{1-M},	(1-M, 1),	(1-M, 2) . . .	(1-M, M - 1),	(2-M, 1),	(3-M, 2) . . .	(0, M - 1)

In the above matrix, there are two distinct classes of potential error terms. The first class, referred to as the pre-correction class, represent those errors for which no correction is attempted (parity bits) or those information bits which have not yet reached the stage where they can be corrected. The probability that any one of these bits in the pre-correction class will be in error is simply equal to the bit error <u>rate</u> from the bit detector. The second class represent the post-correction elements and refers to those information bits which have already been decoded. There are a total of $\frac{1}{2}(M^2 + 3M)$ distinct pre-correction elements and $(M - 1)^2$ distinct post-correction elements in the above matrix.

A decoder error will be made if either:

(1) The decoder fails to correct an actual information bit error, or

(2) The decoder erroneously changes the state of a properly received information bit.

Let P_{uc} and P_{fc} be the probabilities of an uncorrectable error and a false correction error respectively.

An uncorrectable error will occur in a given information bit position if an only if:

(1) The information bit is initially received in error, and,

(2) A received error occurs in any one of the other $\frac{1}{2}(M^2 + 3M - 2)$ pre-correction elements or the $(M - 1)^2$ post correction element and virtually all multiple combinations thereof.

Thus

$$P_{uc} = \epsilon \cdot \left[2 - (1 - \epsilon)^{\frac{1}{2}(M^2+3M-2)} - (1 - E)^{(M-1)^2} \right] \qquad \text{(A-1)}$$

False correction will occur on the jth bit of word 0 if:

(1) The jth bit is initially received correctly,

(2) The S_0 and S_{-j} stages of the syndrome register are set to 1, and,

(3) All other syndrome stages are reset to zero.

From the matrix of pre- and post-correction elements it can be seen that the number of ways the above constraints can be satisfied is a function of the bit position "j" as well as M. Evaluating this probability gives:

$$P_{fc}(j) = M \cdot \epsilon \cdot (1 - \epsilon)^M \cdot \left[(M - j)\epsilon \cdot (1 - \epsilon)^{M-j-1} \right. \qquad \text{(A-2)}$$

$$\left. + (M - 1) \cdot E \cdot (1 - E)^{M-2} \right] + (M - 2)\epsilon \cdot (1 - \epsilon)^{M-2} \cdot E \cdot (1 - E)^{M-2}$$

P_{fc} will be approximated by setting j = M/2. Thus

$$P_{fc} \simeq \epsilon \cdot (1 - \epsilon)^M \left\{ \frac{M^2}{2} \epsilon \cdot (1 - \epsilon)^{\frac{M}{2} - 1} \right. \qquad \text{(A-3)}$$

$$\left. + E \cdot (1 - E)^{M-2} \cdot \left[M \cdot (M - 1) + (M - 2)(1 - \epsilon)^{-2} \right] \right\}$$

But

$$M \cdot (M - 1) + (M - 2)(1 - \epsilon)^{-2} \simeq M^2$$

for $\epsilon << 1$. Hence,

$$P_{fc} \simeq M^2 \epsilon \cdot (1 - \epsilon)^M \cdot \left[\frac{\epsilon}{2} \cdot (1 - \epsilon)^{\frac{M}{2} - 1} + E \cdot (1 - E)^{M-2} \right] \qquad \text{(A-4)}$$

The decoder error rate, E, can now be found by setting E equal to the sum of (A-1) and (A-4) and iterating until the solution for E converges.

APPENDIX B

LISTING OF FORTRAN PROGRAM FOR THE PERFORMANCE EVALUATION OF SYSTEMATIC, RATE $\left(\frac{M-1}{M}\right)$, CONVOLUTIONAL CODES

```
AUG 71)                                   OS/360  FORTRAN H

MPILER OPTIONS - NAME= MAIN,OPT=01,LINECNT=58,SIZE=0000K,
                 SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
      C     PROGRAM TO COMPUTE THE PERFORMANCE OF WEIGHT 2 RATE (M-1)/M CODES
      C
      C     JSNR =EQUIVALENT S/N RATIO OF UNCODED TRANSMISSION
      C     BERUC=UNCODED BIT ERROR RATE
      C     X    =PRE-CORRECTION BIT ERROR RATE OF RATE (M-1)/M TELEMETRY CODE
      C     E    =EFFECTIVE BIT ERROR RATE FROM DECODER
      C     PUC  =PROBABILITY THAT DECODER WILL FAIL TO CORRECT ACTUAL BIT ERROR
      C     PFC  =PROBABILITY THAT DECODER WILL INVERT PROPERLY RECEIVED BIT
      C     ADEL =CODING GAIN DUE TO (M-1)/M CODE (INCLUDES RATE LOSS EFFECT)
      C
            DIMENSION B(250)
        100 FORMAT('1',18X,'PERFORMANCE EVALUATION OF SYSTEMATIC RATE',I3,'/',
           *       I2,' CONVOLUTIONAL CODE'//' EQUIVALENT     * * * * * BIT ',
           *       'ERROR RATES * * * * * * * * *     PROBABILITY OF   PROBAB',
           *       'ILITY OF     CODING'/' UNCODED S/N'49X,'UNCORRECTABLE    ',
           *       'FALSE CORRECTION',5X,'GAIN'/' RATIO (DB.)   UNCODED    PR',
           *       'E-CORRECTION    POST-CORRECTION      ERROR (PUC)',9X'(PFC)',
           *       11X'(DB.)'//)
        102 FORMAT(I6,E17.3,E13.3,E18.3,E17.3,E18.3,F14.2/)
            RLIM=.01
            DO 5 I=1,250
            SNRT=-10.+FLOAT(I-1)/10.
            SNR=10.**(SNRT/10.)
          5 B(I)=.5*ERFC(SQRT(SNR))
            DO 1 M=2,11
            XM=M
            ML=M-1
            RATE=FLOAT(ML)/XM
            WRITE(6,100)ML,M
            DO 1 ISNR=5,13
            JSNR=ISNR-1
            SNR=10.**(FLOAT(JSNR)/10.)
            BERUC=.5*ERFC(SQRT(SNR))
            X=.5*ERFC(SQRT(RATE*SNR))
            E=0.
            I=0
          2 PUC=X*(2.-(1.-X)**((M*M+3*M-2)/2)-(1.-E)**((M-1)**2))
            PFC=XM*XM*X*(1.-X)**M*(.5*X*(1.-X)**(XM/2.-1.)+E*(1.-E)**(XM-2.))
            F=PUC+PFC
            RE=ABS(F-E)/F
            E=F
            I=I+1
            IF(RE.GT.RLIM.AND.I.LT.10)GO TO 2
            DO 6 I=1,250
            XI=I-1
            IF(E.GT.B(I))GO TO 7
          6 CONTINUE
          7 I=XI+1
            EI=XI-(E-B(I))/(B(I-1)-B(I))
            ADEL=EI/10.-10.-FLOAT(JSNR)
          1 WRITE(6,102)JSNR,BERUC,X,E,PUC,PFC,ADEL
            STOP
            END
```

APPENDIX C

TABULATION OF CODING PERFORMANCE OF SYSTEMATIC, RATE $\left(\frac{M-1}{M}\right)$, CONVOLUTIONAL CODES FOR M = 2 TO 11

PERFORMANCE EVALUATION OF SYSTEMATIC RATE 1/2 CONVOLUTIONAL CODE

EQUIVALENT UNCODED S/N RATIO (DB.)	* * * * * BIT ERROR RATES * * * * * * * * * UNCODED	PRE-CORRECTION	POST-CORRECTION	PROBABILITY OF UNCORRECTABLE ERROR (PUC)	PROBABILITY OF FALSE CORRECTION (PFC)	CODING GAIN (DB.)
4	0.125E-01	0.565E-01	0.234E-01	0.130E-01	0.104E-01	-1.04
5	0.595E-02	0.377E-01	0.971E-02	0.573E-02	0.398E-02	-0.63
6	0.239E-02	0.23[illegible]E-01	0.344E-02	0.212E-02	0.131E-02	-0.37
7	0.773E-03	0.126E-01	0.992E-03	0.634E-03	0.357E-03	-0.20
8	0.191E-03	0.6[illegible]E-02	0.221E-03	0.144E-03	0.765E-04	-0.09
9	0.336E-04	0.241E-02	0.352E-04	0.233E-04	0.119E-04	-0.02
10	0.387E-05	0.783E-03	0.368E-05	0.245E-05	0.123E-05	0.02
11	0.261E-06	0.194E-03	0.226E-06	0.150E-06	0.754E-07	0.05
12	0.5[illegible]1E-08	0.343E-04	0.703E-08	0.468E-08	0.235E-08	0.07

PERFORMANCE EVALUATION OF SYSTEMATIC RATE 2/3 CONVOLUTIONAL CODE

EQUIVALENT UNCODED S/N RATIO (DB.)	* * * * * BIT ERROR RATES * * * * * * * * * UNCODED	PRE-CORRECTION	POST-CORRECTION	PROBABILITY OF UNCORRECTABLE ERROR (PUC)	PROBABILITY OF FALSE CORRECTION (PEC)	CODING GAIN (DB.)
4	0.125E-01	0.336E-01	0.208E-01	0.107E-01	0.100E-01	-0.83
5	0.595E-02	0.200E-01	0.620E-02	0.348E-02	0.272E-02	-0.05
6	0.239E-02	0.106E-01	0.157E-02	0.935E-03	0.633E-03	0.40
7	0.773E-03	0.487E-02	0.311E-03	0.192E-03	0.118E-03	0.68
8	0.191E-03	0.186E-02	0.442E-04	0.279E-04	0.163E-04	0.86
9	0.336E-04	0.568E-03	0.406E-05	0.259E-05	0.147E-05	0.98
10	0.387E-05	0.130E-03	0.213E-06	0.136E-06	0.767E-07	1.07
11	0.261E-06	0.209E-04	0.546E-08	0.349E-08	0.197E-08	1.14
12	0.901E-08	0.214E-05	0.554E-10	0.348E-10	0.207E-10	1.19

PERFORMANCE EVALUATION OF SYSTEMATIC RATE 3/ 4 CONVOLUTIONAL CODE

EQUIVALENT UNCODED S/N RATIO (DB.)	* * * * * BIT ERROR RATES * * * * * * * * * UNCODED	PRE-CORRECTION	POST-CORRECTION	PROBABILITY OF UNCORRECTABLE ERROR (PUC)	PROBABILITY OF FALSE CORRECTION (PFC)	CODING GAIN (DB.)
4	0.125E-01	0.261E-01	0.283E-01	0.135E-01	0.148E-01	-1.41
5	0.595E-02	0.147E-01	0.638E-02	0.339E-02	0.299E-02	-0.09
6	0.239E-02	0.727E-02	0.129E-02	0.742E-03	0.553E-03	0.57
7	0.773E-03	0.305E-02	0.208E-03	0.125E-03	0.635E-04	0.95
8	0.191E-03	0.105E-02	0.235E-04	0.144E-04	0.912E-05	1.18
9	0.336E-04	0.278E-03	0.164E-05	0.101E-05	0.627E-06	1.35
10	0.387E-05	0.538E-04	0.607E-07	0.376E-07	0.232E-07	1.46
11	0.261E-06	0.695E-05	0.101E-08	0.623E-09	0.386E-09	1.55
12	0.901E-08	0.542E-06	0.597E-11	0.362E-11	0.235E-11	1.62

PERFORMANCE EVALUATION OF SYSTEMATIC RATE 4/ 5 CONVOLUTIONAL CODE

EQUIVALENT UNCODED S/N RATIO (DB.)	* * * * * BIT ERROR RATES * * * * * * * * * * UNCODED	PRE-CORRECTION	POST-CORRECTION	PROBABILITY OF UNCORRECTABLE ERROR (PUC)	PROBABILITY OF FALSE CORRECTION (PFC)	CODING GAIN (DB.)
4	0.125E-01	0.225E-01	0.440E-01	0.194E-01	0.246E-01	-2.37
5	0.595E-02	0.122E-01	0.799E-02	0.402E-02	0.396E-02	-0.37
6	0.239E-02	0.580E-02	0.132E-02	0.728E-03	0.588E-03	0.56
7	0.773E-03	0.231E-02	0.183E-03	0.106E-03	0.763E-04	1.03
8	0.191E-03	0.743E-03	0.178E-04	0.106E-04	0.720E-05	1.32
9	0.336E-04	0.182E-03	0.105E-05	0.631E-06	0.418E-06	1.51
10	0.387E-05	0.317E-04	0.316E-07	0.190E-07	0.126E-07	1.66
11	0.261E-06	0.359E-05	0.405E-09	0.243E-09	0.161E-09	1.76
12	0.901E-08	0.238E-06	0.139E-11	0.681E-12	0.708E-12	1.88

PERFORMANCE EVALUATION OF SYSTEMATIC RATE 5/ 6 CONVOLUTIONAL CODE

EQUIVALENT UNCODED S/N RATIO (DB.)	* * * * * BIT ERROR RATES * * * * * * * * * * UNCODED	PRE-CORRECTION	POST-CORRECTION	PROBABILITY OF UNCORRECTABLE ERROR (PUC)	PROBABILITY OF FALSE CORRECTION (PFC)	CODING GAIN (DB.)
4	0.129E-01	0.204E-01	0.620E-01	0.247E-01	0.373E-01	-3.27
5	0.595E-02	0.168E-01	0.111E-01	0.530E-02	0.579E-02	-0.82
6	0.239E-02	0.500E-02	0.148E-02	0.791E-03	0.688E-03	0.45
7	0.773E-03	0.193E-02	0.181E-03	0.103E-03	0.781E-04	1.04
8	0.191E-03	0.592E-03	0.159E-04	0.927E-05	0.661E-05	1.37
9	0.336E-04	0.137E-03	0.833E-06	0.491E-06	0.342E-06	1.60
10	0.387E-05	0.223E-04	0.218E-07	0.129E-07	0.895E-08	1.76
11	0.261E-06	0.232E-05	0.232E-09	0.135E-09	0.967E-10	1.88
12	0.901E-08	0.138E-06	0.735E-12	0.394E-12	0.341E-12	1.99

PERFORMANCE EVALUATION OF SYSTEMATIC RATE 6/ 7 CONVOLUTIONAL CODE

EQUIVALENT UNCODED S/N RATIO (DB.)	* * * * * BIT ERROR RATES * * * * *			PROBABILITY OF UNCORRECTABLE ERROR (PUC)	PROBABILITY OF FALSE CORRECTION (PFC)	CODING GAIN (DB.)
	UNCODED	PRE-CORRECTION	POST-CORRECTION			
4	0.125E-01	0.194E-01	0.758E-01	0.270E-01	0.468E-01	-3.88
5	0.595E-02	0.995E-02	0.162E-01	0.726E-02	0.893E-02	-1.40
6	0.239E-02	0.450E-02	0.176E-02	0.913E-03	0.645E-03	0.29
7	0.773E-03	0.169E-02	0.190E-03	0.106E-03	0.642E-04	1.00
8	0.191E-03	0.583E-03	0.154E-04	0.881E-05	0.655E-05	1.39
9	0.336E-04	0.112E-03	0.741E-06	0.429E-06	0.311E-06	1.64
10	0.387E-05	0.173E-04	0.176E-07	0.102E-07	0.738E-08	1.82
11	0.261E-06	0.170E-05	0.166E-09	0.954E-10	0.704E-10	1.96
12	0.901E-08	0.932E-07	0.390E-12	0.178E-12	0.213E-12	2.10

PERFORMANCE EVALUATION OF SYSTEMATIC RATE 7/ 8 CONVOLUTIONAL CODE

EQUIVALENT UNCODED S/N RATIO (DB.)	* * * * * BIT ERROR RATES * * * * * * * * * UNCODED	PRE-CORRECTION	POST-CORRECTION	PROBABILITY OF UNCORRECTABLE ERROR (PUC)	PROBABILITY OF FALSE CORRECTION (PFC)	CODING GAIN (DB.)
4	0.125E-01	0.18[illegible]E-01	0.859E-01	0.276E-01	0.584E-01	-4.30
5	0.595E-02	0.933E-02	0.229E-01	0.940E-02	0.135E-01	-2.00
6	0.239E-02	0.415E-02	0.217E-02	0.109E-02	0.107E-02	0.10
7	0.773E-03	0.153E-02	0.206E-03	0.113E-03	0.936E-04	0.95
8	0.191E-03	0.445E-03	0.155E-04	0.879E-05	0.676E-05	1.38
9	0.336E-04	0.964E-04	0.703E-06	0.401E-06	0.301E-06	1.66
10	0.387E-05	0.144E-04	0.155E-07	0.886E-08	0.662E-08	1.86
11	0.2[illegible]1E-06	0.134E-05	0.133E-09	0.755E-10	0.576E-10	2.00
12	0.9[illegible]1E-08	0.695E-07	0.287E-12	0.133E-12	0.155E-12	2.15

PERFORMANCE EVALUATION OF SYSTEMATIC RATE 8/ 9 CONVOLUTIONAL CODE

EQUIVALENT UNCODED S/N RATIO (DB.)	* * * * * BIT ERROR RATES * * * * * * * * * * UNCODED	PRE-CORRECTION	POST-CORRECTION	PROBABILITY OF UNCORRECTABLE ERROR (PUC)	PROBABILITY OF FALSE CORRECTION (PFC)	CODING GAIN (DB.)
4	0.125E-01	0.173E-01	0.937E-01	0.277E-01	0.660E-01	-4.61
5	0.599E-02	0.987E-02	0.295E-01	0.109E-01	0.186E-01	-2.49
6	0.239E-02	0.390E-02	0.277E-02	0.136E-02	0.141E-02	-0.15
7	0.773E-03	0.142E-02	0.229E-03	0.123E-03	0.106E-03	0.88
8	0.191E-03	0.405E-03	0.162E-04	0.903E-05	0.715E-05	1.37
9	0.336E-04	0.657E-04	0.594E-06	0.392E-06	0.302E-06	1.67
10	0.387E-05	0.124E-04	0.144E-07	0.814E-08	0.625E-08	1.88
11	0.261E-06	0.112E-05	0.114E-09	0.629E-10	0.506E-10	2.04
12	0.591E-08	0.554E-07	0.124E-12	0.0	0.124E-12	2.28

PERFORMANCE EVALUATION OF SYSTEMATIC RATE 9/10 CONVOLUTIONAL CODE

EQUIVALENT UNCODED S/N RATIO (DB.)	* * * * * BIT ERROR RATES * * * * * * * * * UNCODED	PRE-CORRECTION	POST-CORRECTION	PROBABILITY OF UNCORRECTABLE ERROR (PUC)	PROBABILITY OF FALSE CORRECTION (PFC)	CODING GAIN (DB.)
4	0.125E-01	0.167E-01	0.996E-01	0.278E-01	0.718E-01	-4.84
5	0.595E-02	0.852E-02	0.357E-01	0.117E-01	0.240E-01	-2.89
6	0.239E-02	0.372E-02	0.364E-02	0.173E-02	0.191E-02	-0.43
7	0.773E-03	0.133E-02	0.258E-03	0.137E-03	0.121E-03	0.80
8	0.191E-03	0.376E-03	0.171E-04	0.945E-05	0.766E-05	1.34
9	0.336E-04	0.783E-04	0.702E-06	0.393E-06	0.309E-06	1.66
10	0.387E-05	0.110E-04	0.139E-07	0.778E-08	0.611E-08	1.89
11	0.261E-06	0.966E-06	0.106E-09	0.590E-10	0.467E-10	2.05
12	0.901E-08	0.462E-07	0.107E-12	0.0	0.107E-12	2.31

PERFORMANCE EVALUATION OF SYSTEMATIC RATE 10/11 CONVOLUTIONAL CODE

EQUIVALENT UNCODED S/N RATIO (DB.)	* * * * * BIT ERROR RATES: UNCODED	PRE-CORRECTION	POST-CORRECTION * * * * * * * * *	PROBABILITY OF UNCORRECTABLE ERROR (PUC)	PROBABILITY OF FALSE CORRECTION (PEC)	CODING GAIN (DB.)
4	0.125E-01	0.163E-01	0.104E 00	0.279E-01	0.762E-01	-5.01
5	0.596E-02	0.825E-02	0.412E-01	0.120E-01	0.292E-01	-3.21
6	0.239E-02	0.357E-02	0.485E-02	0.221E-02	0.264E-02	-0.76
7	0.773E-03	0.127E-02	0.293E-03	0.153E-03	0.140E-03	0.72
8	0.191E-03	0.353E-03	0.183E-04	0.100E-04	0.828E-05	1.31
9	0.336E-04	0.723E-04	0.723E-06	0.401E-06	0.322E-06	1.65
10	0.387E-05	0.104E-04	0.137E-07	0.763E-08	0.611E-08	1.89
11	0.261E-06	0.858E-06	0.985E-10	0.540E-10	0.445E-10	2.07
12	0.901E-08	0.398E-07	0.958E-13	0.0	0.958E-13	2.33