



A COMPARISON OF OPEN AND CLOSED LOOP APPLICATIONS
OF THE MINIMUM DISTANCE GUIDANCE TECHNIQUE

BY
JAMES PADLEY LATTIMORE, JR.

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APPLIED MECHANICS RESEARCH LABORATORY
THE UNIVERSITY OF TEXAS AT AUSTIN AUSTIN, TEXAS

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PREFACE

An important problem associated with any mission involving a spacecraft or aircraft is that of correcting path errors in such a manner that prescribed terminal conditions which define successful completion of the mission are satisfied. Determination of these corrections is referred to as guidance. Many vehicles must be controlled by automatic or semi-automatic devices which operate without the direct intervention of a man. For non-tracked vehicles, these devices must control the path and speed of the vehicle. It is this type of vehicle (spacecraft or aircraft) which will be considered. There are several definite steps in the solution of the guidance problem. First, a nominal trajectory must be defined. Next, the errors in a current state can be evaluated by comparison to the nominal trajectory. Using information from the nominal trajectory plus the known state errors, guidance information can be generated to drive the terminal state toward the desired end conditions.

This study compares two variations of the minimum distance guidance algorithm (open and closed loop forms) with a nonlinear reoptimization scheme which is the ideal guidance procedure. This study seeks to determine the effects of loop closure on the performance of the guidance algorithm. For the numerical simulation, a minimum time low-thrust Earth-Mars transfer is used as an example problem.

The author wishes to express his deep appreciation to Dr. W. T. Fowler for serving as supervising professor and for his helpful suggestions and encouragement during the course of this investigation. He also would like to express his appreciation to Dr. D. G. Hull for serving on the supervising committee. Especially, the author wishes to thank his wife, Gaylene,

and his parents for their interest, encouragement and support during the course of his studies.

James F. Lattimore, Jr.

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ABSTRACT

A comparison is made of open and closed loop applications of a second order guidance algorithm, using the minimum distance strategy. A non-linear reoptimization procedure is used as the ideal guidance history. The system model used for the comparison is a low-thrust vehicle performing a minimum time, three-dimensional, heliocentric Earth-Mars transfer.

For the example problem considered, closed loop guidance proves to be much more accurate on satisfaction of the final state than the open loop procedure. On the other hand, closed loop guidance proves to be much more vulnerable to perturbation by highly nonlinear regions in the trajectory. Finally, the results indicate that for this problem the best loop closure interval is at each integration step, about one day, or more often, if possible.

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LIST OF SYMBOLS

The following list tabulates all significant symbols used in the main text. Each symbol is accompanied by a brief description and the equation number where the symbol is introduced.

MATRICES:

The size of a matrix is indicated in the statement immediately following the symbol. The following specific indices are used.

n - number of state variables

m - number of control variables

η - number of initially specified constraint relations

l - number of terminal constraint relations

$[A]$ $n \times n$ matrix of partial derivatives

$[X(t, t_f)]$ $2n \times (n+q)$ matrix of partial derivatives resulting from the $n+q$ backward integrations of the $2n$ vector of perturbation equations.

$[X_1(t, t_f)]$ $n \times n$ submatrix of $[X(t, t_f)]$

$[X_2(t, t_f)]$ $n \times q$ submatrix of $[X(t, t_f)]$

$[X_3(t, t_f)]$ $n \times n$ submatrix of $[x(t, t_f)]$

$[X_4(t, t_f)]$ $n \times q$ submatrix of $[X(t, t_f)]$

$[\Lambda_1]$ $m \times n$ matrix of gain functions

$[\Lambda_2]$ $m \times q$ matrix of gain functions

$[\Lambda_3]$ $m \times n$ matrix of gain functions

$[\Lambda_4]$ $m \times n$ matrix of gain functions

$[\theta(t, t_1)]$ $2n \times 2n$ matrix of partial derivatives resulting from the $2n$ forward integrations of the $2n$ vector of perturbation equations

$[\psi_i(t, t_1)]$ $n \times n$ submatrix of $[\psi(t, t_1)]$ $i = 1, 2, 3, 4$

VECTORS:

All vectors are column vectors unless otherwise noted. The vector size is indicated in the statement immediately following the symbol.

\bar{f} n -vector of state variable derivatives

\bar{F} $2n$ -vector of state variable and Euler-Lagrange variable derivatives

\bar{h} $(n+1)$ -vector of terminal constraint relations

\bar{M} q -vector of specified terminal constraint relations

\bar{u} m -vector of control variables

\bar{x} n -vector of state variables

\bar{z} $2n$ -vector of state variables and Euler-Lagrange variables

$\bar{\lambda}$ n -vector of time dependent Lagrange multipliers

\bar{v} q -vector of constant Lagrange multipliers

\bar{p} $(n-q+1)$ -vector of terminal constraints

SCALARS:

c propellant exhaust speed relative to vehicle

H variational Hamiltonian

I auxiliary functional to be extremized

J performance index

K terminal constraint obtained from variational analysis

m instantaneous mass of vehicle

m_0 initial mass of vehicle

P auxiliary functional equal to $\phi + \bar{v}^T \bar{M}$
r instantaneous radial distance of vehicle from origin of heliocentric coordinate system
t independent variable, time
T thrust of the ion engine
 β constant mass ejection rate
 μ gravitational constant for the Sun
 ϕ performance index

SUPERSCRIPTS:

($\dot{\quad}$) differentiation with respect to time
(\quad)^T matrix transpose
(\quad)⁻¹ matrix inverse
(\quad)^{*} indicates evaluation on the reference trajectory

SUBSCRIPTS:

(\quad)₀ evaluation at the initial time
(\quad)_f evaluation at the terminal time
(\quad)₁ evaluation at some time while the mission is in progress
(\quad)_M Mars
(\quad)_E Earth

ABBREVIATIONS:

AU astronomical unit
MDG minimum-distance guidance

1. INTRODUCTION

1.1. Preliminary Description

As man extends his horizons to include exploration of the solar system, large mission durations are encountered which would present some problems for a manned spacecraft. A logical step would be to shorten these mission durations (transfer times) with the use of some additional thrust during the transfer. The most likely candidate for this additional thrust is a low-thrust, continuous operation engine. This might be an ion or plasma-type engine. The force produced by this thrust requires the development of a method to control the direction of thrust so that the terminal conditions of the mission will be satisfied. This type of mission would be characterized by four phases; a) ascent and escape phase, b) midcourse phase, c) approach phase and d) terminal maneuver phase.

The midcourse phase is a heliocentric transfer with the spacecraft engine thrusting continuously at some low level. For the class of trajectories that does not contain coast periods, it is desirable to minimize the transfer time. This will allow a minimum amount of fuel to be carried through the ascent and escape phase. The constant magnitude thrust can be controlled in direction so that the transfer time is a minimum. The purpose of this study is not to look at different trajectories but to compare the performance of two guidance algorithms on the same trajectory. A nominal minimum time transfer trajectory and the associated control can be generated and stored before the comparison is started. This nominal trajectory was generated by Hart (1) and is covered in detail there.

1.2. Problem Statement

The guidance problem can be thought of as that of determining the

set of control changes necessary to get the spacecraft back to the nominal path after an off-course or off-nominal condition is recognized. Hart (1) and others indicate that rather than trying to get back on the nominal path, one should find a new optimum path to satisfy the mission and conditions. In the absence of an onboard computer capable of carrying out a total trajectory reoptimization, one might use a guidance routine. There are several algorithms under consideration for this type of problem. All of these methods are derived from the Calculus of Variations. A general comparison of several algorithms which might be used for a reoptimization approach was made by Lewallen (5). The guidance algorithm considered in this study was presented by Hart (1), along with several others for comparison. In Hart's study, the minimum distance guidance algorithm was found to be the best among those he considered. To the author's knowledge, all of the previous work done with the minimum distance guidance algorithm has been carried out employing the algorithm in an open loop form. This study applies the minimum distance guidance technique in both open and closed loop forms and compares these results. The basis for comparison is the result of the nonlinear reoptimization procedure used by Hart (1). The state history generated by the minimum distance algorithm and that generated by the nonlinear reoptimization procedure are compared to try to determine the performance of the closed loop versus the open loop application.

Several problems encountered in the application of the method are discussed. The comparison of the open and closed loop analyses consists of calculations based on perturbations in position and velocity components at several points along the trajectory. At each of these points, different loop closure periods are compared. From this information, some optimum value for the loop closure period can be determined. As a secondary goal, if the closed

loop analysis proves to be better, then some optimum value for the loop closure period will be sought.

1.3. Notation Convention

All vectors are assumed to be column vectors and will be denoted by a bar; i.e., \bar{x} , \bar{z} . All matrices will be denoted by brackets; i.e., $[A]$, $[H_{xx}^{-1}]$. The superscript T indicates the transpose of a vector or matrix, and the superscript -1 indicates the inverse of a square matrix. All first partial derivatives of scalars with respect to a vector are now vectors and are denoted as:

$$\frac{\partial H}{\partial \bar{x}} = H_{\bar{x}} = \left(\frac{\partial H}{\partial x_1} \quad \frac{\partial H}{\partial x_2} \quad \dots \quad \frac{\partial H}{\partial x_n} \right)$$

All second partial derivatives of scalars with respect to vectors are matrices, i.e.,

$$\left[\frac{\partial^2 H}{\partial \bar{u} \partial \bar{x}} \right] = \left[\frac{\partial}{\partial \bar{u}} \frac{\partial H^T}{\partial \bar{x}} \right] = H_{\bar{x}\bar{u}} = \begin{bmatrix} H_{x_1 u_1} & \dots & H_{x_1 u_m} \\ \vdots & & \vdots \\ H_{x_n u_1} & \dots & H_{x_n u_m} \end{bmatrix}$$

All first partial derivatives of vectors with respect to vectors are matrices formed in the following manner:

$$\left[\frac{\partial \bar{f}}{\partial \bar{x}} \right] = \left[\bar{f}_{\bar{x}} \right] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

The variation of x is denoted by δx and the total change in x is denoted by Δx . In general, $\Delta x = \delta x + \dot{x} \delta t$ where \dot{x} is $\frac{dx}{dt}$.

2. MATHEMATICAL MODEL

The nominal trajectory employed in this study is a minimum time, low-thrust heliocentric transfer from Earth to Mars. The equations of motion describing the trajectory are expressed in terms of a heliocentric rectangular coordinate system, x_4 , x_5 , x_6 . The x_4 axis coincides with the line of the ascending node of Mars, the x_5 axis lies in the ecliptic plane and the x_6 axis coincides with the angular momentum vector of the Earth with respect to the Sun. For simplicity, the spacecraft is given the velocity of the Earth at the beginning of the mission rather than escape velocity relative to the Earth. Gravitational effects of the Earth are then neglected. Details of the arrival at Mars will be determined by specific mission requirements. For the problem under consideration, the end conditions are taken to be the position and velocity of a point on the orbit of Mars at the final time.

Gravitational effects of Mars, along with perturbing effects of the other planets, have been neglected although these forces would have to be included in an actual mission analysis. These forces are neglected so that the effects of the guidance method can be studied without complications due to the effects of small perturbing forces in the system.

The orbit of Mars is assumed to be an ellipse with an eccentricity of $e = .093393$, a semi-major axis of $a = 1.523691$ AU (Astronomical Units), and an angle of inclination of $i = .033289$ radians. The argument of perihelion is taken to be 4.932927 radians, and for the coordinate system employed, the argument of the ascending node is 0.0 . This model is the same as that used by Fowler (3), Hart (1) and others.

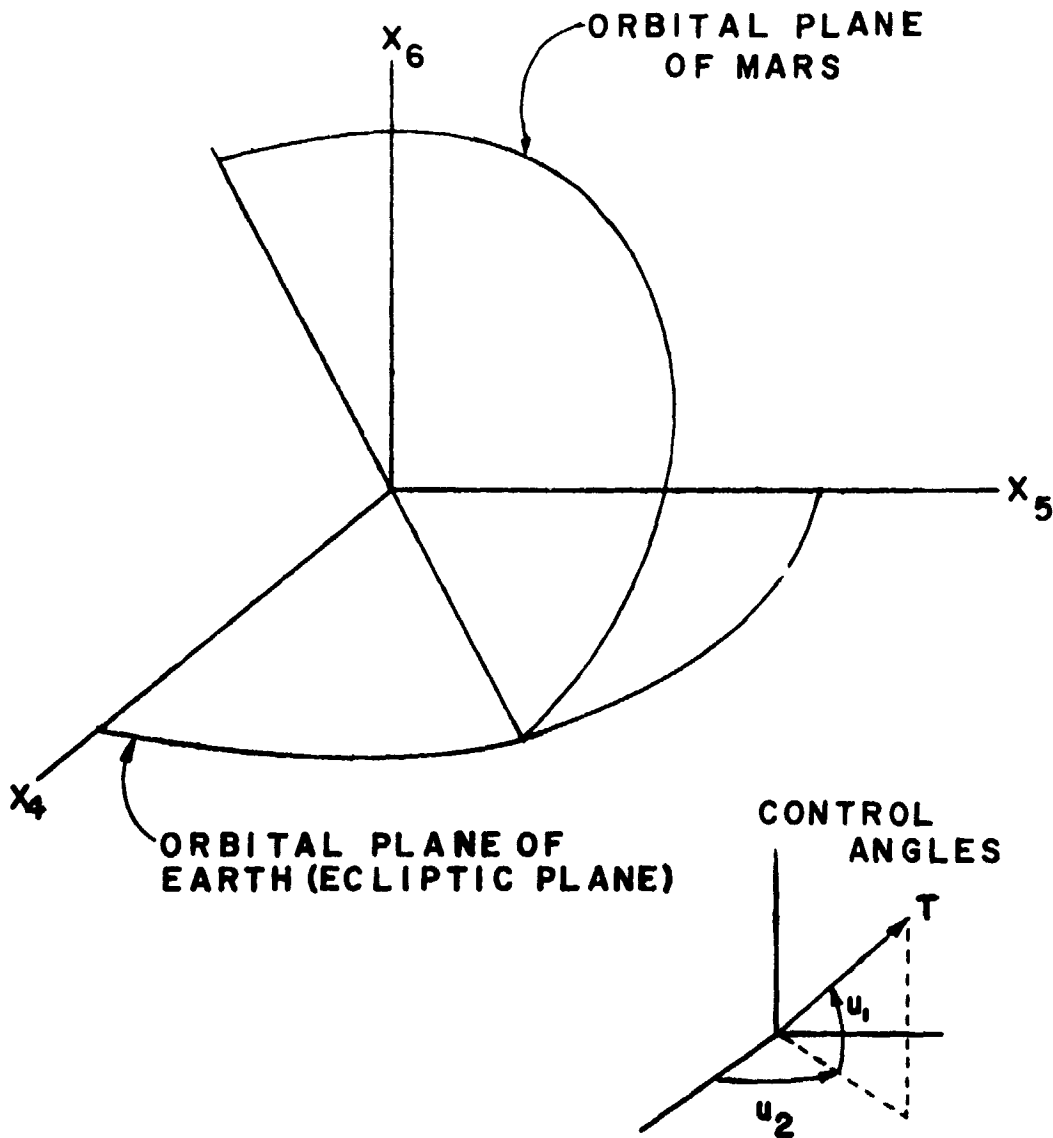


FIGURE I.
COORDINATE SYSTEM
AND
CONTROL ANGLES

3. GUIDANCE ALGORITHM

3.1. Second Order Guidance Procedures

The minimum distance guidance algorithm and the perturbation functions guidance algorithm are both based on the second variation of the Calculus of Variations. Both procedures calculate the same guidance information. The difference between the two procedures is the point at which the information is applied on the trajectory. Hart (1) gives a more detailed treatment of several second order algorithms.

The differential equations describing the system to be controlled are

$$\dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u}, t) \quad (3.1.1)$$

where

- \bar{x} is an n-vector of state variables,
- \bar{u} is an m-vector of control variables,
- t is the scalar independent variable, time, and
- \bar{f} is an n-vector of known functions.

Assume that a nominal trajectory has been obtained which minimizes a performance index

$$J = \phi(\bar{x}(t_f), t_f) \quad (3.1.2)$$

and satisfies the terminal constraints

$$\bar{M}(\bar{x}(t_f), t_f) = 0 \quad (3.1.3)$$

where ϕ is a scalar quantity evaluated at the terminal time and \bar{M} is a q-vector of terminal constraints.

This trajectory must satisfy certain differential equations; these are

$$\dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u}, t) \quad (3.1.4)$$

$$\dot{\bar{\lambda}} = -H_{\bar{x}}^T \quad (3.1.5)$$

$$H_{\bar{u}} = 0 \quad (3.1.6)$$

For $t_0 \leq t \leq t_f$ where $\bar{\lambda}$ is an n-vector of multipliers, and $H = \bar{\lambda}^T \bar{f}$. Formulation and solution of this problem is based on a Calculus of Variations approach. Since $\bar{x}(t_0)$ and t_0 are known, Equations (3.1.4) through (3.1.6) and subject to the following boundary conditions:

At t_0

$$\bar{x}(t_0) = \bar{x}_0 \quad (3.1.7)$$

At t_f

$$\bar{M}(\bar{x}(t_f), t_f) = 0 \quad (3.1.8)$$

$$\lambda^{-T}(t_f) = (\phi_{\bar{x}} + {}^{-T}\bar{M}_{\bar{x}})_{t_f} \quad (3.1.9)$$

$$(\bar{\lambda}^T \bar{f} + \phi_t + {}^{-T}\bar{M}_t)_{t_f} = 0 \quad (3.1.10)$$

After considering a perturbation of the nominal trajectory due to changes in initial and/or final conditions, Equations (3.1.4) through (3.1.6) become

$$\frac{d}{dt} (\delta \bar{x}) = [H_{\bar{x}\bar{x}}] \delta \bar{x} + [H_{\bar{x}\bar{u}}] \delta \bar{u} \quad (3.1.11)$$

$$\frac{d}{dt} (\delta \bar{\lambda}) = -[H_{\bar{x}\bar{x}}] \delta \bar{x} - [H_{\bar{x}\bar{u}}] \delta \bar{u} - [H_{\bar{x}\bar{\lambda}}] \delta \bar{\lambda} \quad (3.1.12)$$

$$[H_{\bar{u}\bar{x}}] \delta \bar{x} + [H_{\bar{u}\bar{u}}] \delta \bar{u} + [H_{\bar{u}\bar{\lambda}}] \delta \bar{\lambda} = 0 \quad (3.1.13)$$

The boundary conditions, Equations (3.1.7) through (3.1.10), become

$$\delta \bar{x}(t_0) = \delta \bar{x}_0 \quad (3.1.14)$$

$$d\bar{M}_f = \{[\bar{M}_x] \delta \bar{x} + [\bar{M}_x] \bar{f} dt + \bar{M}_t dt\}_{t_f} = 0 \quad (3.1.15)$$

A total variation of $\bar{M}(\bar{x}(t_f), t_f)$ rather than a partial variation must be considered since the final time, t_f , as well as $\bar{x}(t_f)$ is allowed to change.

Thus,

$$\delta \bar{x}_f = \{[P_{xx}] \delta \bar{x} + ([P_{xx}] \bar{f} + (P_{tx})^T + H_x^T) dt + [\bar{M}_x]^T d\bar{v}\}_{t_f} \quad (3.1.16)$$

$$\{\delta \bar{\lambda}^T \bar{f} + (H_x + P_{tx}) \delta \bar{x} + (H_t + P_{tx} \bar{f} + P_{tt}) dt + d\bar{v}^T \bar{M}_f\}_{t_f} = 0 \quad (3.1.17)$$

where $P = \phi + \bar{v}^T \bar{M}$.

This set of Equations, (3.1.11) through (3.1.17), must be satisfied over the entire trajectory, for $t_0 \leq t_1 \leq t_f$.

Now, assuming that $[H_{uu}]$ is nonsingular, Equation (3.1.13) can be solved for $\delta \bar{u}$ to give

$$\delta \bar{u} = -[H_{uu}]^{-1} \{ [H_{ux}] \delta \bar{x} + [H_{u\lambda}] \delta \bar{\lambda} \} \quad (3.1.18)$$

Using this result, $\delta \bar{u}$ can be eliminated from Equations (3.1.11) and (3.1.12).

This will yield a set of $2n$ first order linear differential equations, with time dependent coefficients in $\delta \bar{x}$ and $\delta \bar{\lambda}$. For simplicity, let this system of $2n$ equations be written as

$$\frac{d}{dt} \begin{pmatrix} \delta \bar{x} \\ \delta \bar{\lambda} \end{pmatrix} = \begin{bmatrix} A_1(t) & A_2(t) \\ A_3(t) & A_4(t) \end{bmatrix} \begin{pmatrix} \delta \bar{x} \\ \delta \bar{\lambda} \end{pmatrix} \quad (3.1.19)$$

where

$$[A_1(t)] = [H_{\lambda x}] - [H_{\lambda u}] [H_{uu}]^{-1} [H_{ux}] ,$$

$$[A_2(t)] = -[H_{\lambda u}] [H_{uu}]^{-1} [H_{u\lambda}] ,$$

$$[A_3(t)] = -[H_{xx}^-] + [H_{xu}^-][H_{uu}^-]^{-1}[H_{ux}^-] \quad , \text{ and}$$

$$[A_4(t)] = -[H_{x\lambda}^-] + [H_{xu}^-][H_{uu}^-]^{-1}[H_{u\lambda}^-] \quad .$$

Numerical integration of Equation (3.1.19) with proper boundary conditions will now determine $\delta\bar{x}$ and $\delta\bar{\lambda}$. These results can be used in Equation (3.1.18) to evaluate the control variation history, $\delta\bar{u}(t)$. The boundary conditions can be obtained from Equations (3.1.15) through (3.1.17). The $n+q+1$ equations contain the $2n+q+1$ unknowns $\delta\bar{x}_f$, $\delta\bar{\lambda}_f$, $\Delta\bar{v}$ and Δt_f . If the $n+q+1$ equations are linearly independent, then there are n independent quantities among the unknowns. Therefore, if n of the unknowns are chosen arbitrarily and independent of one another, the $2n+q+1$ unknowns are uniquely determined. Choose the q components of $\Delta\bar{v}$ and $n-q$ of the components of $\delta\bar{x}_f$ as the arbitrarily chosen unknowns. By specifying each of these values, all $2n+1$ components of $\delta\bar{x}_f$, $\delta\bar{\lambda}_f$ and Δt_f can be determined. This now allows backward integration of Equation (3.1.19) from t_f to t_1 . The value of t_1 may be any arbitrary value in the interval $t_0 \leq t_1 \leq t_f$.

The n arbitrarily chosen unknowns and the q components of $\Delta\bar{M}_f$ are all specified quantities at t_f . Combining these vectors will give an $n+q$ vector of specified quantities. Equation (3.1.19) is now integrated $n+q$ times with $n+q$ linearly independent starting conditions. The $n+q$ vectors of solutions for $\delta\bar{x}(t)$ and $\delta\bar{\lambda}(t)$ form a $(2n \times n+q)$ transition matrix. After some manipulation the desired solution for $\delta\bar{x}(t)$ and $\delta\bar{\lambda}(t)$ is obtained.

$$\begin{pmatrix} \delta\bar{x}(t) \\ \delta\bar{\lambda}(t) \end{pmatrix} = \begin{bmatrix} x_1(t, t_f) : x_2(t, t_f) \\ \vdots : \vdots \\ x_q(t, t_f) : x_{n+q}(t, t_f) \end{bmatrix} \quad (3.1.20)$$

$$\begin{bmatrix} [x_1(t_1, t_f)]^{-1} : -[x_1(t_1, t_f)]^{-1}[x_2(t_1, t_f)] \\ \vdots : \vdots \\ \underset{q \times n}{0} : \underset{q \times q}{1} \end{bmatrix} \begin{pmatrix} \delta\bar{x}(t_1) \\ \vdots \\ \Delta\bar{M}_f \end{pmatrix}$$

where

- $[x_1(t, t_f)]$ is an $n \times n$ matrix,
- $[x_2(t, t_f)]$ is an $n \times q$ matrix,
- $[x_3(t, t_f)]$ is an $n \times n$ matrix,
- $[x_4(t, t_f)]$ is an $n \times q$ matrix,
- $0_{q \times n}$ is a $q \times n$ null matrix, and
- $I_{q \times q}$ is a $q \times q$ identity matrix.

When Equation (3.1.20) is substituted into Equation (3.1.18), the control variation, $\delta \bar{u}(t)$, in terms of the state error, $\delta \bar{x}(t_1)$, and terminal constraint error, $\Delta \bar{M}_f$ is obtained. Making this substitution and rewriting gives the following guidance equation

$$\delta \bar{u}(t) = -[\Lambda_1(t), \Lambda_2(t)] \quad (3.1.21)$$

$$\begin{bmatrix} [x_1(t_1, t_f)]^{-1} & : & -[x_1(t_1, t_f)]^{-1}[x_2(t_1, t_f)] \\ \dots & \dots & \dots \\ 0_{q \times n} & : & I_{q \times q} \end{bmatrix} \begin{pmatrix} \delta \bar{x}(t_1) \\ \dots \\ \Delta \bar{M}_f \end{pmatrix}$$

where

$$[\Lambda_1(t)] = [H_{uu}^-]^{-1} \{ [H_{ux}^-][x_1(t, t_f)] + [H_{u\lambda}^-][x_3(t, t_f)] \} ,$$

and

$$[\Lambda_2(t)] = [H_{uu}^-]^{-1} \{ [H_{ux}^-][x_2(t, t_f)] + [H_{u\lambda}^-][x_4(t, t_f)] \} .$$

After the nominal path has been determined which minimizes $J = \phi(\bar{x}(t_f), t_f)$ and satisfies the necessary boundary conditions, the computational procedure for this algorithm can be summarized as follows:

1. Solve the $n+q+1$ Equations (3.1.15) through (3.1.17) $n+q$ times. From the solutions of these equations, $n+q$ initial conditions are obtained for the integration of Equation (3.1.19).
2. Now integrate Equation (3.1.19) $n+q$ times from t_f to t_1 . Store the integrated values at each step.
3. The gain matrices, $[\Lambda_1(t)]$ and $[\Lambda_2(t)]$ can now be evaluated. Substitute these results into Equation (3.1.21) to give the control variation history, $\delta u(t)$.

This presents a general algorithm for generating second order guidance information. The perturbation functions approach and the minimum distance procedure differ only in the way this guidance information is applied.

3.2. Variations in the Procedures

The state variations for the perturbation functions guidance algorithm and the minimum distance algorithm are determined in the same manner. For the control variation, however, the calculation is the same, but the guidance information is applied in a slightly different way.

In the minimum distance guidance algorithm, an index time is used for calculating the gain matrices which minimizes a metric function of the difference between the current perturbed states and the states on the nominal trajectory. In other words, the index time is taken to be the time of the closest point on the nominal trajectory. The perturbation functions guidance algorithm uses the time based on the time of the perturbation on the nominal path for calculation of the gain matrices. This procedure was originally proposed by Powers (2) and is covered in detail in Hart (1). The correction to the time for calculation of the gain matrices is given as

$$\Delta t = \sum_{i=4}^6 \left\{ \frac{-\ddot{x}_{i_c}^* (x_i - x_{i_c}^*)}{\ddot{x}_{i_c}^* (x_i - x_{i_c}^*) - \ddot{x}_{i_c}^*} \right\} \quad (3.2.22)$$

where

- ()_c indicates evaluation at the current time and
- ()* indicates evaluation on the nominal path.

This calculation is based on the position only, as indicated by the summation from four to six. A value of Δt is calculated at each point along the nominal path beyond t_1 . The index time, which is used for calculating the gain matrices, is now determined by $t_c + \Delta t$. The nominal state variables at this index time are used to calculate the gain matrices. In order to make an evaluation of the performance of the algorithm, the guidance information obtained from the technique is added to the nominal control program. The nonlinear equations of motion are then integrated, using this augmented control history as the control program. This integration gives the state history.

3.3. Closed Loop Application of the Algorithm

The previous section covers only open loop control, using the minimum distance guidance procedure. A closed loop modification of this algorithm is now considered. The procedure remains basically the same as far as solution of the $n+q+1$ Equations (3.1.15) through (3.1.17) and the backward integration of Equation (3.1.19) from t_f to t_1 . The gain matrices, $[\Lambda_1(t)]$ and $[\Lambda_2(t)]$, are now evaluated. This allows calculation of the control variation history and the state variation history. The guidance deviations predicted at t_1 are now added to the nominal guidance history. The nonlinear equations of motion for the vehicle are integrated using this

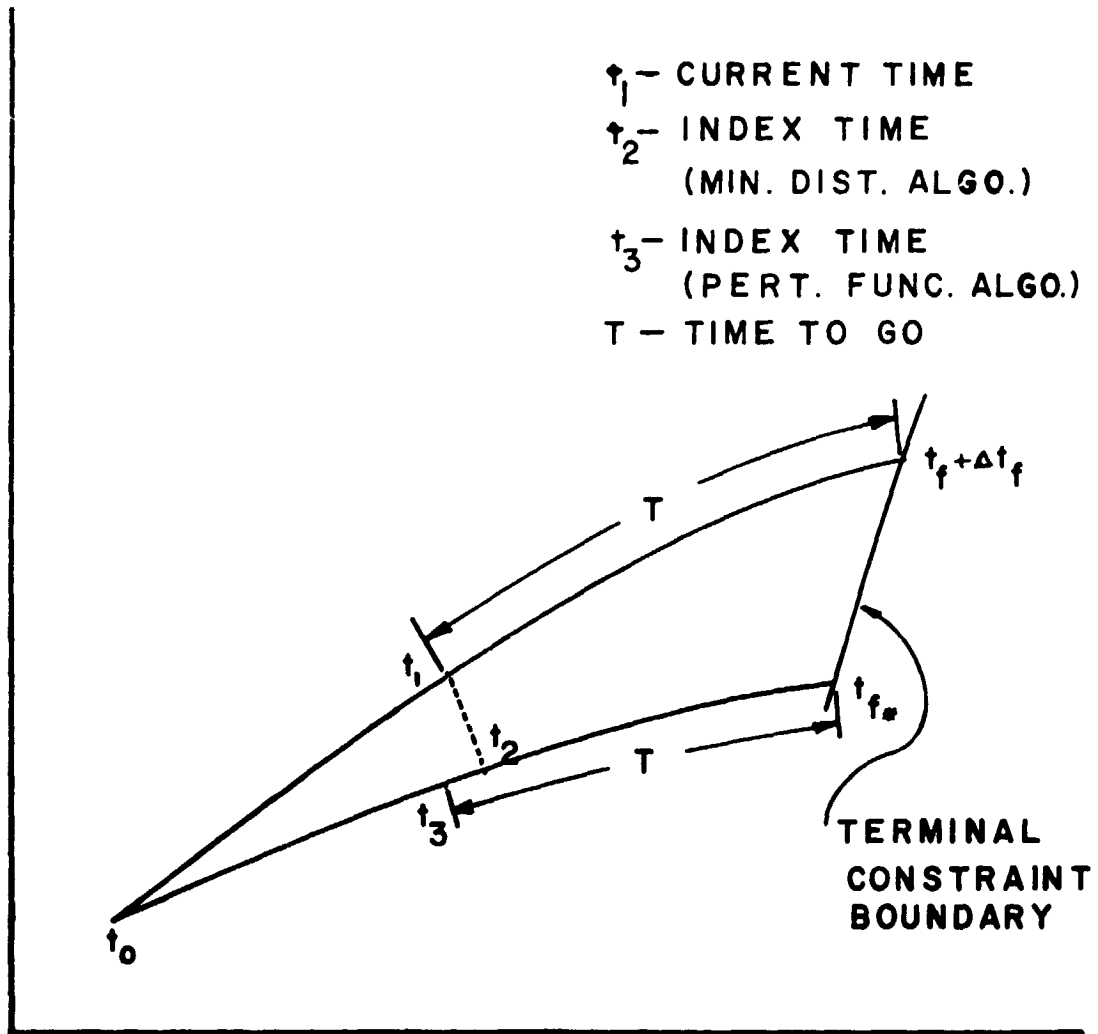


FIGURE 2.
TIME PARAMETERS ASSOCIATED WITH
MINIMUM DISTANCE GUIDANCE,
OPEN LOOP.

augmented guidance information. At this point, rather than allowing the guidance information to be used throughout the rest of the mission, one can create a closed loop system to update the guidance. The guidance loop is closed at periodic intervals by evaluating the state errors at this time. The gain matrices are then re-evaluated. A new control variation history is calculated, along with a new state deviation history. The new augmented control history is formed, and the nonlinear equations of motion are once again integrated. The advantage of this closed loop method is that it continually updates the control variation history as the mission proceeds. The computational procedure for the minimum distance algorithm, open and closed loop can be summarized as follows:

1. Solve the $n+q+1$ Equations (3.1.15) through (3.1.17), $n+q$ times. From the solutions of these equations, $n+q$ initial conditions are obtained for the integration of Equations (3.1.19).
2. Now integrate Equation (3.1.19) $n+q$ times from t_f to t_1 . Store the integrated values at each step.
3. Evaluate the gain matrices, $[\Lambda_1(t)]$ and $[\Lambda_2(t)]$, using the states for the closest point on the nominal trajectory.
4. Substitute $[\Lambda_1(t)]$ and $[\Lambda_2(t)]$ into Equation (3.1.21) to give the control variation history, $\delta\bar{u}(t)$.
5. Evaluate the state variation history, $\delta\bar{x}(t)$ from Equation (3.1.20).
6. Form the augmented control history and integrate the nonlinear equations of motion using the new control.
7.
 - a. If the control is to be open loop, the algorithm is finished.
 - b. If the control is to be closed loop, integrate to the time of the next loop closure, evaluate the state deviations and go back to Step 3.

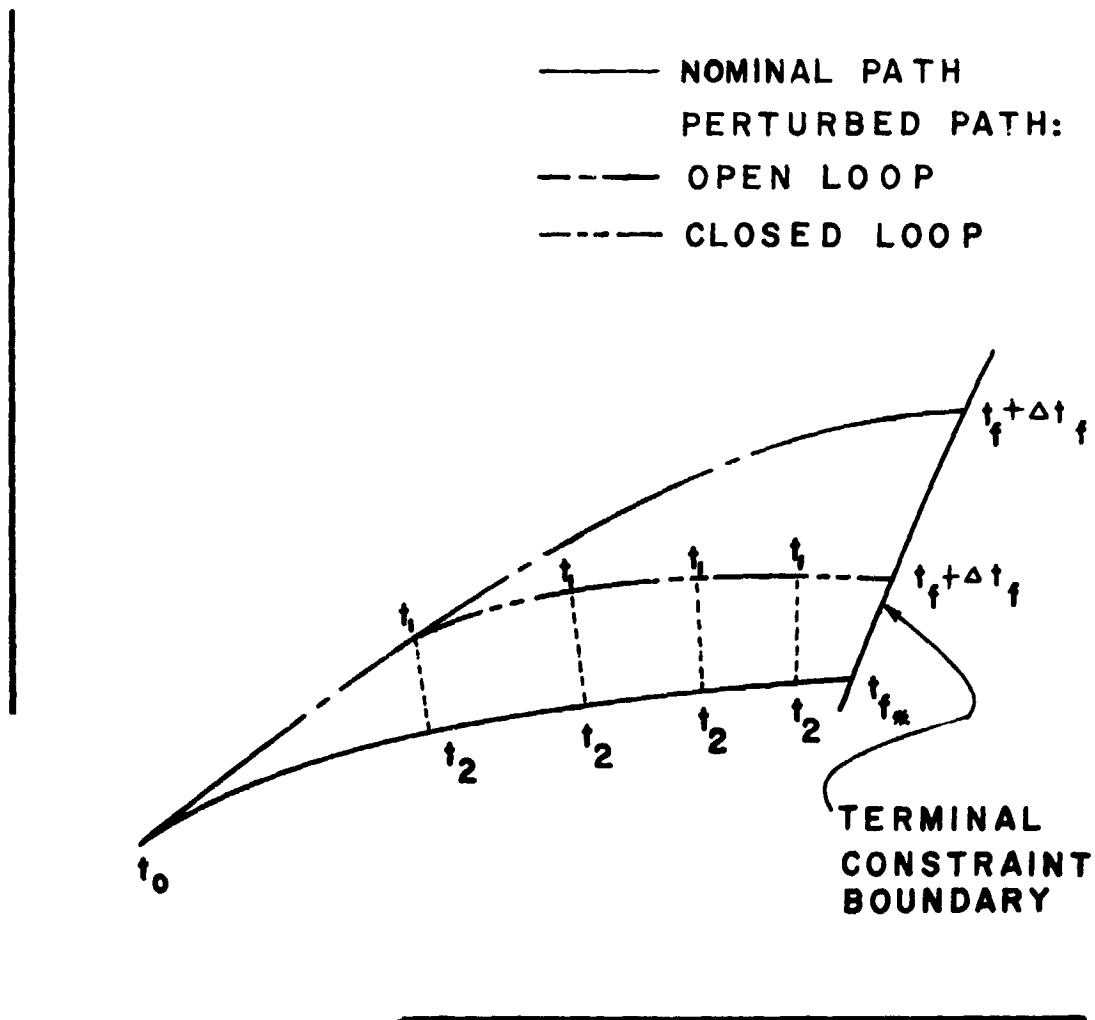


FIGURE 3.
TIME PARAMETERS ASSOCIATED WITH
MINIMUM DISTANCE GUIDANCE,
COMPARING OPEN AND CLOSED LOOP.
(SEE FIGURE 2 FOR DEFINITION OF t_2)

3.4. Numerical Implementation

The open and closed loop forms of the minimum distance guidance algorithm were compared. The performance of each method was referenced to the ideal guidance history produced by the nonlinear guidance scheme. The results of this procedure are considered to be the new nominal or ideal guidance history considering the known state errors. By integration of the nonlinear equations of motion, position and velocity histories are obtained. The Euclidean norms of the differences in the position and velocity components were computed at each point along the three trajectories generated by these methods. The deviations in velocity and position between the minimum distance techniques and the ideal guidance were then plotted as a function of time. The open loop form of the minimum distance technique was found to perform as indicated by Hart (1). However, it is of interest to discuss some of the problems encountered with the implementation of the closed loop control algorithm. One problem area was the period at about 100 days into the mission when it is necessary to turn the spacecraft around for the deceleration for Mars rendezvous. The perturbed control causes the time of the turn-around to shift slightly. This time shift, due to the rapid turn-around causes predictions of large control changes, as can be seen in Figure 4. These large control changes exceed linearity and cause the algorithm to break down. To avoid this problem, nominal control is used during the turn-around period.

The second problem area occurs when the spacecraft is nearing the final time. At this point in the trajectory, the effect of a control change is small. The algorithm then predicts a large control change to cause the necessary results. This region is sensed by the increasing magnitude of the control changes. When these deviations reach a magnitude of .15 radians,

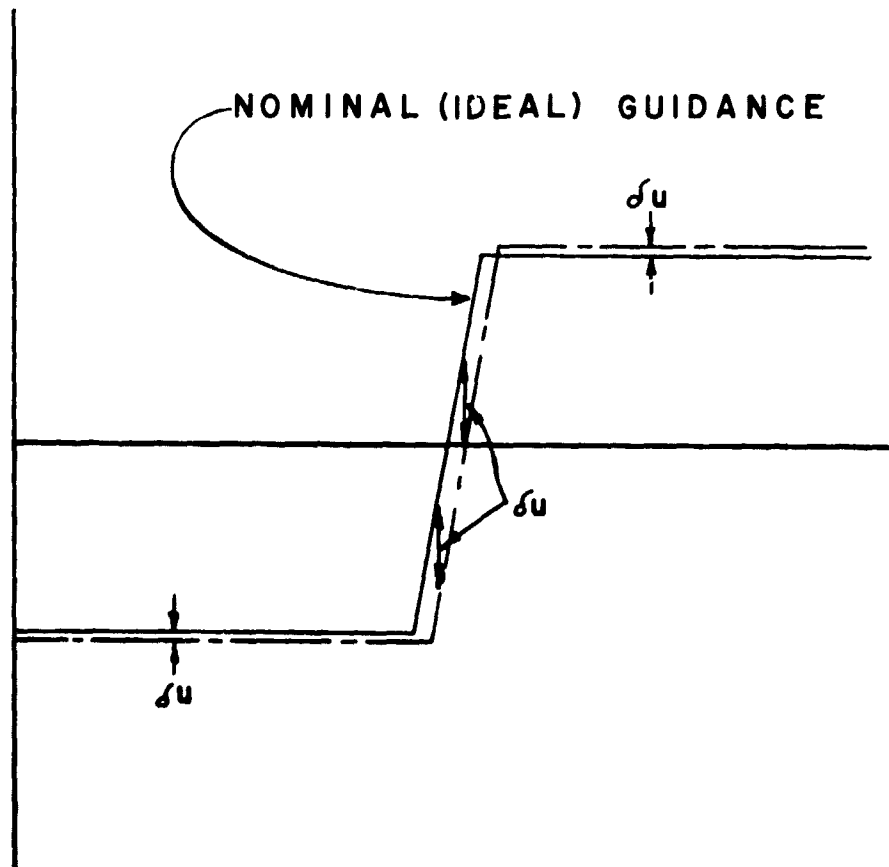


FIGURE 4.
CONTROL DURING TURNAROUND

nominal control was used since calculations of control could not be made within the linear range. This limit of .15 radians was found by trying several values. Beyond the time at which a .15 radian control change was sensed, predicted control changes quickly approached a value of about .3 radians. This limit is the same as that used by Hart (1).

3.5. Numerical Methods

Accuracy, time and storage requirements for these methods are all highly dependent on the numerical processes involved. Matrix inversion and numerical integration account for the majority of round-off and truncation errors. The numerical integration routine used in this study is a fourth-order Adams-Bahsforth predictor and a fifth-order Adams-Moulton corrector. A fourth-order Runge-Kutta routine is used as a starter to obtain the initial and three succeeding derivatives. The dependent variables are carried in double precision during integration in an attempt to control round-off errors. Matrix inversion is performed by Gaussian elimination. All numerical calculations were carried out on the CDC 6600/6400 digital computer in the FORTRAN IV language. Storage requirements for the minimum distance algorithm is 167,000₈ words while the reoptimization (ideal guidance) requires 60,000₈ words. Computation time for the minimum distance guidance was around 12 seconds. The reoptimization (ideal guidance) requires about 15 seconds of computation time.

4. RESULTS OF THE NUMERICAL SIMULATION

The first case considered involved a state error of 5×10^{-6} AU/Day in the x_1 velocity component. Position deviation as a function of time is shown in Figure 5, for open loop and closed loop control. The loop closure periods are 30, 10, 5 and 1 day. A loop closure period of 30 days results in a larger error at the final time than does the open loop guidance. Steady improvement is shown, until at a loop closure period of 1 day, the minimum error in the final position for the closed loop method is generated. For simplicity, none of the plots in this study show complete results below a time of 80 days. The reason for doing this is that the loop closure period has very little effect on velocity or position deviations from initiation until this time is reached. A representative curve is shown in each case for the general closed loop results, since all of these results look basically the same.

Figure 6 gives a summary of velocity deviations between the reoptimized trajectory and the closed and open loop techniques. The initial state error is again in the x_1 (velocity) component and loop closure periods of 30, 10, 5 and 1 day are considered. Once again, the 30-day loop closure period is actually worse than the open loop control with gradual improvement as the loop closure interval decreases. A loop closure period of 1 day gives the least error in the final velocity deviation.

Position deviations between open and closed loop control and the reoptimized for a position error of 5×10^{-6} AU are shown in Figure 7. This error occurs in the x_4 component. The loop closure periods considered are 10, 5 and 1 days. Results in this case are the same as for previous cases in that the trend is toward increased accuracy with the smaller loop closure

period.

For all test cases investigated, the results are comparable. The closed loop results show continued improvement over the open loop down to the minimum loop closure period of 1 day. This limit of 1 day corresponds to the length of one integration step.

Figures 9 and 10 show results for an error introduced at 65 days into the mission. The closed loop results appear somewhat better than open loop, with the errors in position corrected more accurately than those in velocity.

Figure 11 gives a listing of the actual terminal errors for the analyses shown in Figures 5 through 8. There is actually only a small difference between open and closed loop results at this point. Figure 12 compares the computation time for open and closed loop analyses. There is very little difference between the two procedures. This, in effect, means that the closed loop procedure is comparable to the open loop procedure as far as computer costs are concerned.

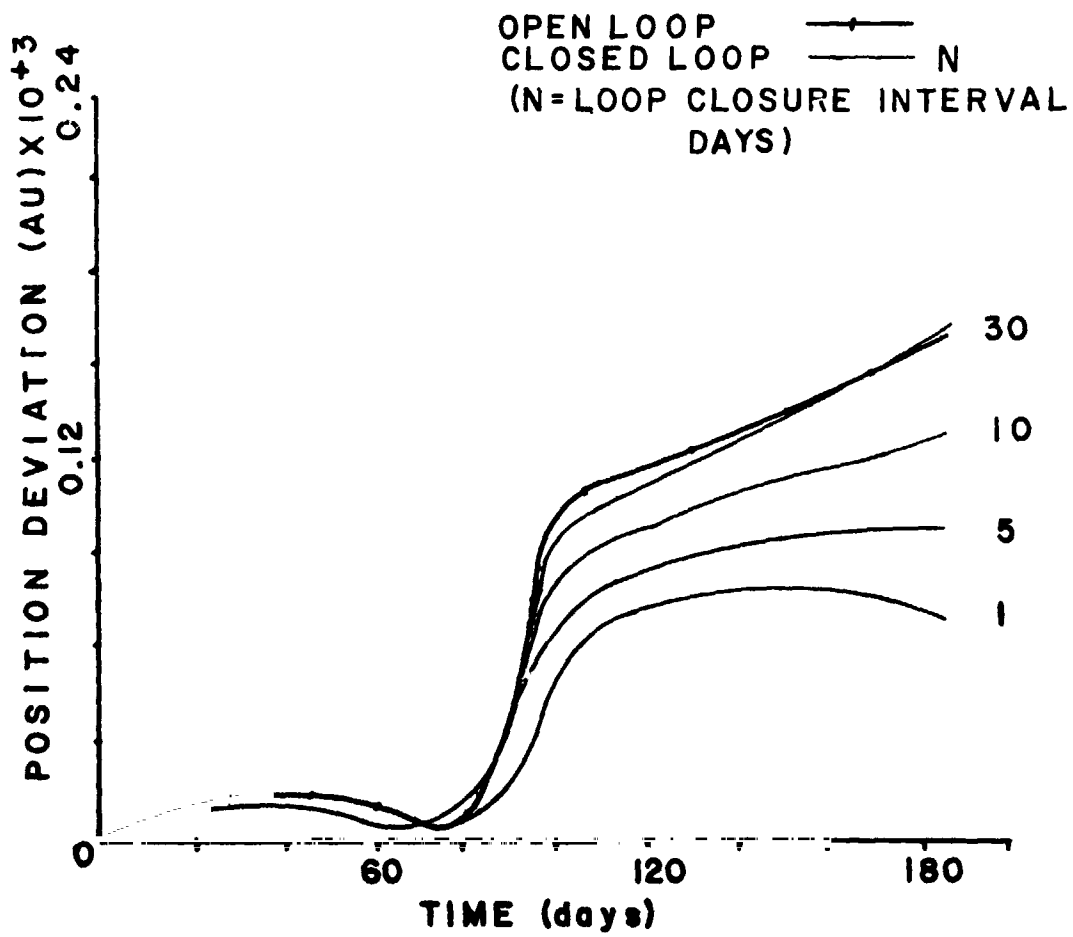


FIGURE 5.
POSITION DEVIATION FOR ERROR
OF 5×10^{-6} AU/DAY IN X_1 VELOCITY
INTRODUCED AT 0 DAYS

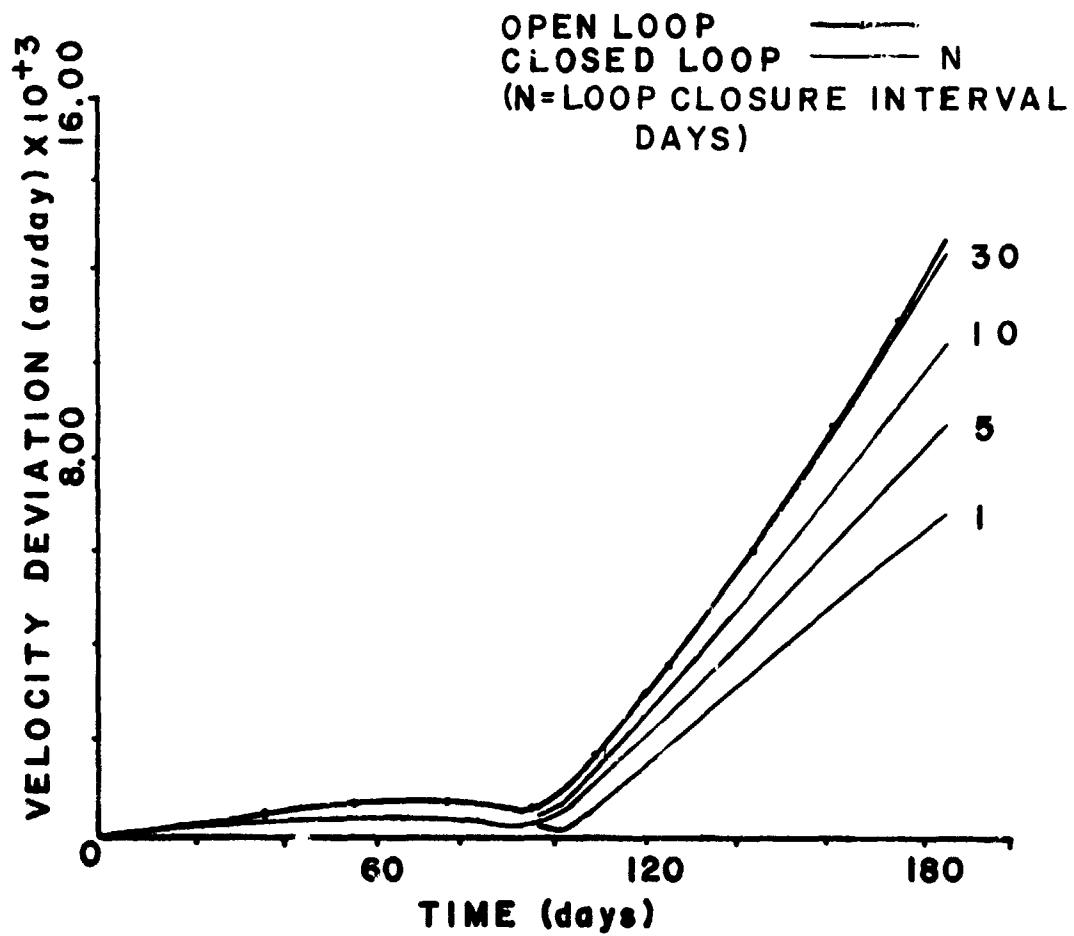


FIGURE 6.
VELOCITY DEVIATION FOR ERROR
OF 5×10^{-6} AU/DAY IN X_1 VELOCITY
INTRODUCED AT 0 DAYS

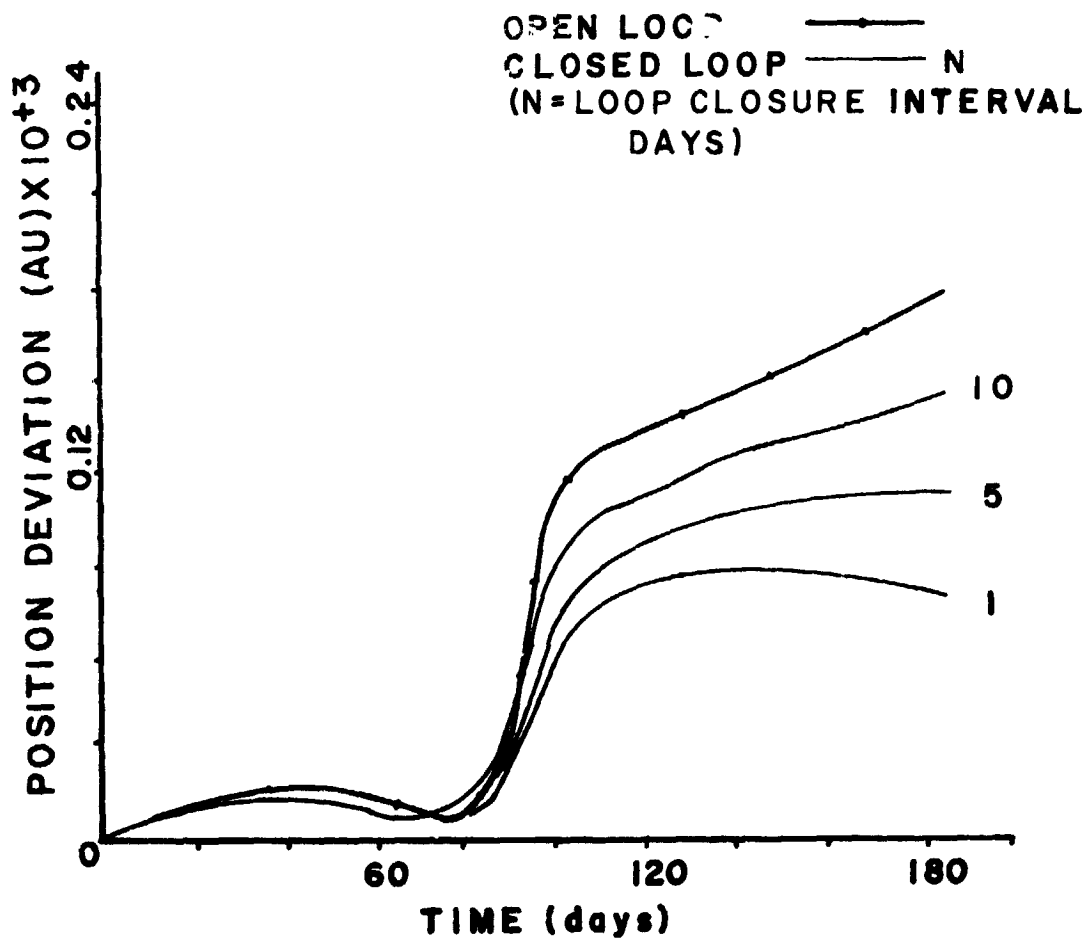


FIGURE 7.
POSITION DEVIATION FOR ERROR
OF 5×10^{-6} AU IN X_4 POSITION
INTRODUCED AT 0 DAYS

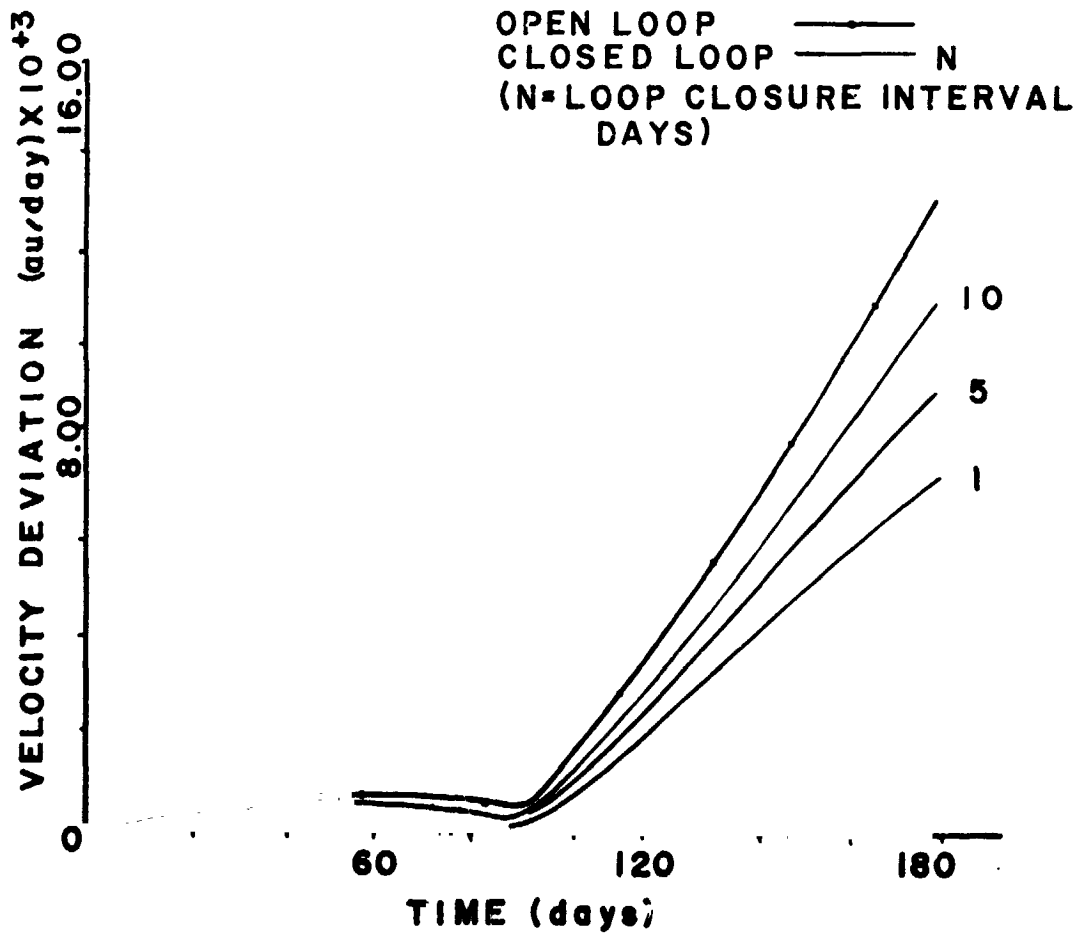
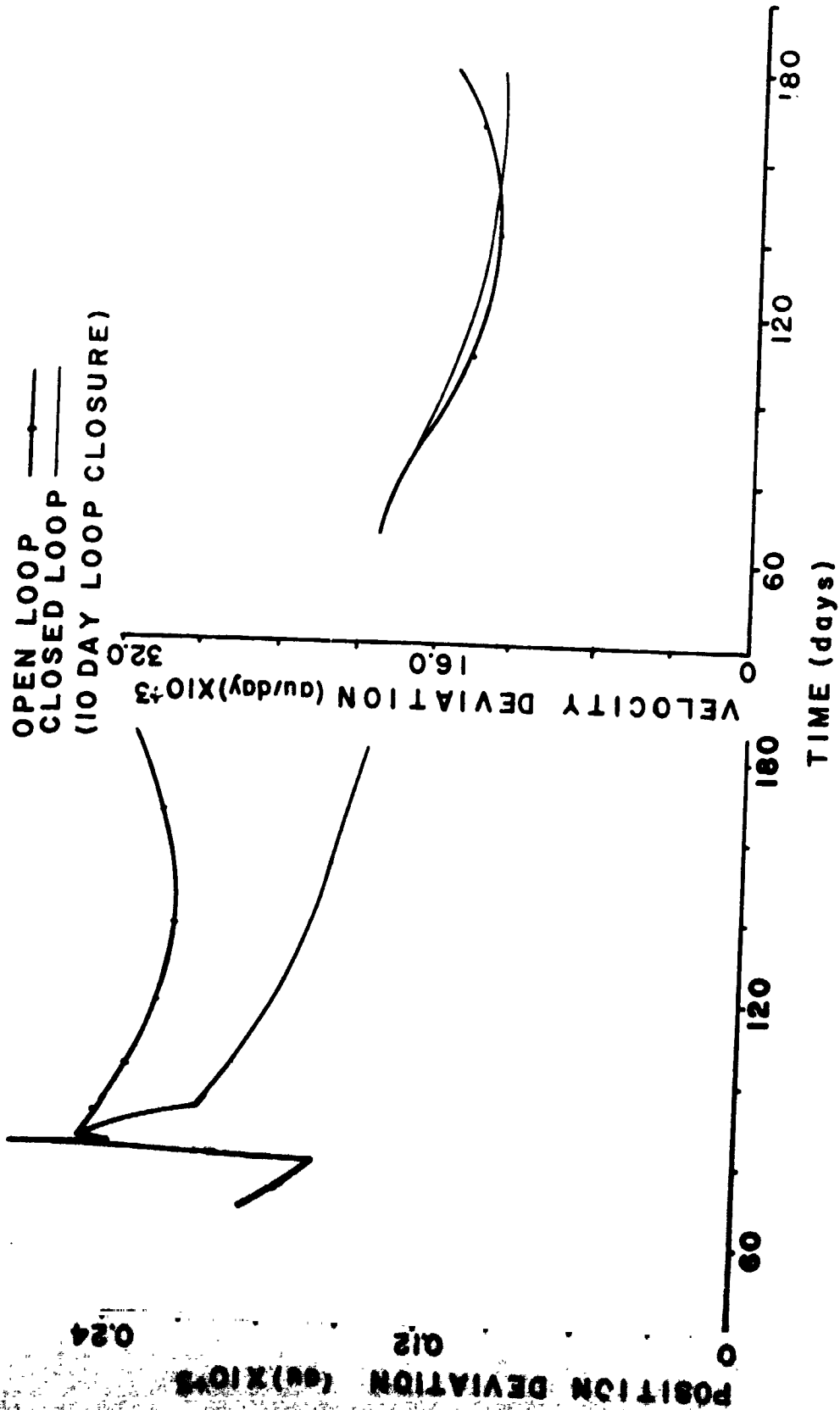


FIGURE 8.

**VELOCITY DEVIATION FOR ERROR
 OF 5×10^{-6} AU IN X_4 POSITION
 INTRODUCED AT 0 DAYS**



STATE DEVIATION FOR ERROR OF 5×10^{-6} AU/DAY IN X_1 VELOCITY INTRODUCED AT 65 DAYS.

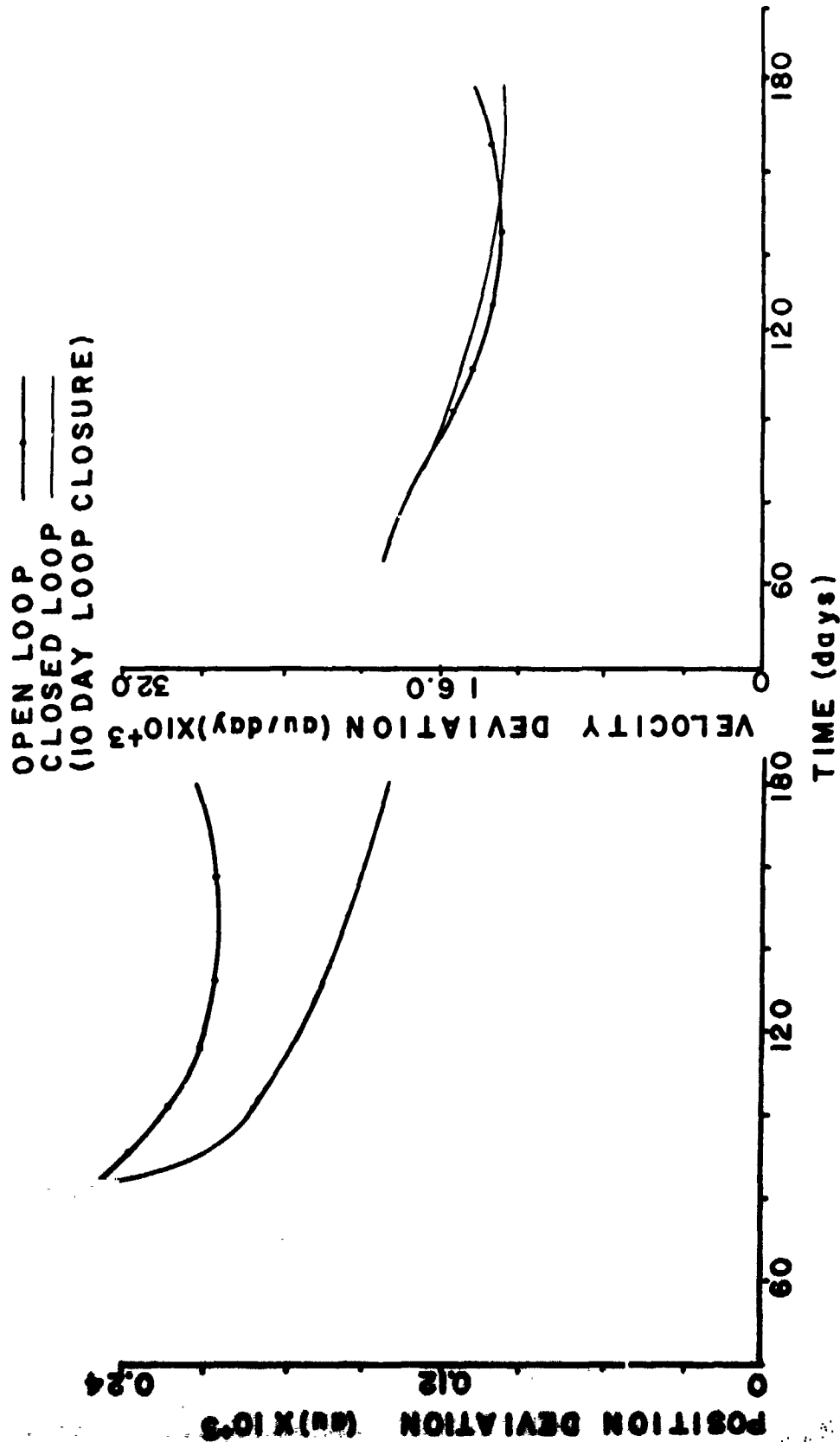


FIGURE 10.
 STATE DEVIATION FOR ERROR OF 5×10^{-6} AU IN X_4 POSITION
 INTRODUCED AT 65 DAYS.

Differences Between Final States of
Reoptimized Trajectory and Minimum Distance Technique

Perturbation	Error		Loop Closure Period (Days)
	Position (AU)	Velocity (AU/Day)	
$\delta x_1(0) = 5.0 \times 10^{-6}$ AU	$1.6854386101 \times 10^{-4}$	$1.3356785238 \times 10^{-3}$	Open Loop
"	$7.5959967022 \times 10^{-5}$	$7.5581577845 \times 10^{-3}$	1
"	$7.5959967022 \times 10^{-5}$	$7.5581577845 \times 10^{-3}$	5
"	$1.3712271024 \times 10^{-4}$	$1.1165335824 \times 10^{-4}$	10
$\delta x_4(0) = 5.0 \times 10^{-6}$ AU/Day	$1.6388290871 \times 10^{-4}$	$1.2626059427 \times 10^{-2}$	Open Loop
	$7.1368155158 \times 10^{-5}$	$6.8088261401 \times 10^{-3}$	1
	$1.0093910324 \times 10^{-4}$	$8.7410210064 \times 10^{-3}$	5
	$1.3037531342 \times 10^{-4}$	$1.0469032389 \times 10^{-2}$	10
	$1.6468665832 \times 10^{-4}$	$1.2406315644 \times 10^{-2}$	30

Figure 11. ERRORS AT FINAL TIME FOR
ERROR INTRODUCED AT INITIATION OF THE MISSION.

Velocity Perturbation in x_1 Component at

Initial Point of Trajectory:

Loop Closure Period (Days)	Time (Sec.)
Open loop	11.763
1	12.652
5	11.980
10	11.308
30	11.490

$$\delta x_1(0) = 5.0 \times 10^{-6} \text{ AU/Day.}$$

Position Perturbation in x_4 Component at

Initial Point of Trajectory:

Loop Closure Period (Days)	Time (Sec.)
Open loop	11.545
1	12.171
5	12.143
10	12.041

$$\delta x_4(0) = 5.0 \times 10^{-6} \text{ AU.}$$

Figure 12. COMPUTATION TIME REQUIRED

5. CONCLUSIONS AND RECOMMENDATIONS

In this study, a comparison of open and closed loop applications of the minimum distance guidance algorithm is presented. The comparison is based on the differences between the results of these algorithms and the results generated by a nonlinear reoptimization algorithm. The trajectory used is for a low-thrust spacecraft on a three-dimensional, heliocentric Earth-Mars transfer.

5.1. Summary of Conclusions

The major conclusions resulting from this study are:

- 1) The errors in the final state are somewhat smaller for the closed loop procedure than for the open loop. However, the difference is not large enough to indicate a choice of the closed loop procedure over the open loop procedure.
- 2) The position errors are somewhat smaller for both procedures than are the velocity errors at the final time.
- 3) The errors in the final state seem to become smaller as the loop closure interval becomes smaller. The results indicate that the choice of a continuous loop closure would decrease the errors in the final state but would increase computation time.
- 4) Reoptimization appears to be the best method of guidance. Considering the fact that the terminal errors are still relatively large, the closed loop procedure has only small merit above the open loop analysis. Since the computation times are on the same order for the minimum distance guidance and the reoptimization (ideal guidance), little is gained by using the second order guidance method. This result is the same as that obtained

by Hart (1) in his study.

5.2. Recommendations for Further Study

It is suggested that the study of low-thrust guidance techniques be extended as follows:

- 1) The results of this study indicate that the nonlinear reoptimization technique is much more accurate with only a small difference in computation costs. Therefore, it is recommended that further effort be directed at perfecting the reoptimization techniques for use in low-thrust guidance.

- 2) An extension of this study should be made to trajectories which contain coast arcs.

In addition, trajectories for which reoptimization is not feasible are of interest. An example of this would be the reentry problem.

APPENDIX A

Earth Orbital Data

Semi-major axis, a_E	1.0 AU
Eccentricity, e_E	.016726
Argument of perihelion, ω_E	0.0°
Angle of inclination, i_E	0.0°
Argument of ascending node, Ω_E	0.0°
Time of perihelion	Jan. 3.022307069, 1950
Period	365.198084 days

Mars Orbital Data

Semi-major axis, a_m	1.523691 AU
Eccentricity, e_m	.093393
Argument of perihelion, ω_m	286.07366°
Angle of inclination, i_m	1.84991°
Argument of ascending node, Ω_m	0.0°
Time of perihelion	March 17.490627, 1959
Period	686.968886 days

Vehicle Thrust Constants

Mass flow rate, β	.00108 initial vehicle mass/day
Exhaust speed, c	.045365 AU/day
Solar gravitational constant, μ	$2.96007536 \times 10^{-6}$ AU ³ /day ²
Initial spacecraft mass	1.0

Time of mission initiation	12:00 noon, Jan. 9, 1982
Mission duration	196.76594763 days

The spacecraft leaves the vicinity of the Earth with position and velocity equal to that of Earth. The mission is carried out under the influence of the inverse gravitational force of the Sun. After a 196-day heliocentric transfer, the spacecraft matches position and velocity with Mars.

APPENDIX B

THE DIFFERENTIAL EQUATIONS OF MOTION AND
EULER-LAGRANGE DIFFERENTIAL EQUATIONS

$$\dot{x}_1 = -\frac{\mu x_4}{r^3} + \frac{T}{M} \cos u_1 \cos u_2$$

$$\dot{x}_2 = -\frac{\mu x_5}{r^3} + \frac{T}{M} \cos u_1 \sin u_2$$

$$\dot{x}_3 = -\frac{\mu x_6}{r^3} + \frac{T}{M} \sin u_1$$

$$\dot{x}_4 = x_1$$

$$\dot{x}_5 = x_2$$

$$\dot{x}_6 = x_3$$

$$\dot{\lambda}_1 = -\lambda_4$$

$$\dot{\lambda}_2 = -\lambda_5$$

$$\dot{\lambda}_3 = -\lambda_6$$

$$\dot{\lambda}_4 = \frac{\mu \lambda_1}{r^3} - \frac{3\mu x_4}{r^5} (\lambda_1 x_4 + \lambda_2 x_5 + \lambda_3 x_6)$$

$$\dot{\lambda}_5 = \frac{\mu \lambda_2}{r^3} - \frac{3\mu x_5}{r^5} (\lambda_1 x_4 + \lambda_2 x_5 + \lambda_3 x_6)$$

$$\dot{\lambda}_6 = \frac{\mu \lambda_3}{r^3} - \frac{3\mu x_6}{r^5} (\lambda_1 x_4 + \lambda_2 x_5 + \lambda_3 x_6)$$

The augmented vector, $\bar{z} = \begin{pmatrix} \bar{x} \\ \bar{\lambda} \end{pmatrix}$, is initially

$$\begin{aligned}z_1 &= -1.4835073 \times 10^{-2} \\z_2 &= 9.2714508 \times 10^{-3} \\z_3 &= 0.0 \\z_4 &= 5.199345 \times 10^{-1} \\z_5 &= 8.3463802 \times 10^{-1} \\z_6 &= 0.0 \\z_7 &= 1.0058717029 \times 10^1 \\z_8 &= -2.1350450338 \times 10^1 \\z_9 &= -6.7014133502 \times 10^{-1} \\z_{10} &= -5.1681265185 \times 10^{-2} \\z_{11} &= -4.3276807729 \times 10^{-1} \\z_{12} &= -1.3232925942 \times 10^{-3}\end{aligned}$$

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