

PRELIMINARY -- NOT FOR QUOTATION

Reward and Uncertainty in Exploration Programs

by

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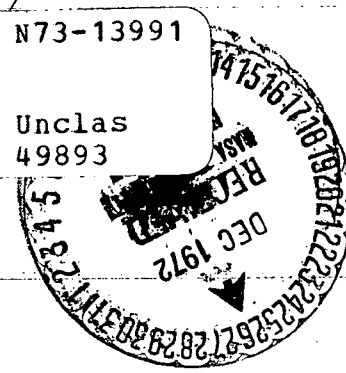
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REWARD AND UNCERTAINTY IN EXPLORATION PROGRAMS

Introduction

In our preceding paper ("Two Stochastic Models Useful in Petroleum Exploration"), we constructed a model to describe the results of wildcat drilling. Given the existence in nature of a set of targets of differing characteristics, in this case areal extent, we specified the process by which information about the targets would be accumulated. The model permits predictions about the success ratio and the size of discoveries for successive increments of drilling. Knowledge of the probability laws governing the results of petroleum exploration would make it possible to characterize the economic risks involved, but this entails a more elaborate model than the one we have proposed. It would be necessary, utilizing information obtained from wells already completed, to specify the joint probability distribution governing the entire set of variables which determine the economic return to subsequent drilling.

Previous analyses of exploratory drilling programs have emphasized particular aspects of uncertainty. The variable, size of reservoirs, has received the most attention. [Allais (1957), Arps and Roberts' (1958), Arrington (1960), Kaufman (1963)] Estimates of the expected value and standard deviation of reservoir size have been casually interpreted as measures of the economic reward and the degree of risk, respectively, of particular exploration programs. The size of reservoir found is, of course, only one aspect of the uncertainty of exploratory drilling. Among the other variables which have an important bearing on

the economics of the program are the probability of making a discovery, the depth of the producing formation, and the productivity of the wells.

In this paper we select a set of variables which are crucial to the economic outcome of petroleum exploration. These are treated as random variables; the values they assume indicate the number of successes that occur in a drilling program and determine, for a particular discovery, the unit production cost and net economic return if that reservoir is developed. In specifying the joint probability law for these variables, we are forced to make extreme and probably unrealistic assumptions. In particular, we assume the different random variables to be independently distributed, and we do not take into account changes that may occur in the probability distributions as exploration proceeds. This latter simplification, of course, ignores the thrust of our previous model, which describes a depletion process where the largest pools, having been found first, are "used up", and hence cease to be possible targets. We are conscious of, and suitably pained by, these limitations, and we stand ready to make use of any better data-generating models that come along.

The values of the independent random variables affect the economic return to exploration in relatively complicated ways. As a consequence we cannot deduce the probability functions which govern the pertinent economic measures directly from knowledge of the joint probability distribution of the physical variables. Instead we rely on a Monte Carlo type of simulation procedure. Using postulated probability functions and specified parameters, we generate values for selected random variables, such as reservoir size. From this set of values we

compute the economic magnitudes of interest, net return and unit production cost. This constitutes a single trial, and the procedure is repeated many times. The resulting histograms approximate the probability density functions of the variables which describe the economic outcomes of an exploratory drilling program.

In the next section we specify the set of physical variables whose values are critical to the economic success or failure of an exploration venture. We then present, first, a model which relates the expenditures needed to develop and produce a crude oil reservoir to this set of variables, and, second, a model which relates the output of the developed reservoir to certain of the variables. Utilizing these models we can compute for the reservoir unit production cost and total value, or net economic return, the latter conditional upon the wellhead price at which the crude can be sold.

A Model of the Returns to Exploration

In Table 1 we define the variables that will be employed in computing the returns when a reservoir is discovered. We distinguish among three classes of variables:

- (a) physical variables, which are observable upon completion of the wildcat well;
- (b) certain economic variables which we postulate to be known with certainty; and
- (c) dependent variables, whose values we will compute.

These variables take on a particular set of values for each wildcat well.

The expenditure required to produce a reservoir is resolved into four components, of which the first three comprise what is usually termed development investment:

- I_1 = drilling investment;
- I_2 = investment in surface gathering and processing facilities;
- I_3 = camp investment, required in remote locations; and
- I_4 = capitalized operating costs.

Based on F. M. Fisher's investigation of drilling costs [Fisher (1964)], we assume that well cost increases exponentially with depth and that:

$$\tilde{I}_d = N \beta_{11} (e^{\beta_{12}d} - 1) + \tilde{\epsilon}_1 \quad (1)$$

where $\tilde{\epsilon}_1$ is an error term. In a similar vein, it has been shown that the relation between investment and capacity in a chemical process plant

Table 1

DEFINITIONS OF VARIABLES

(a) physical variables

| | |
|-------|--|
| v | oil in place in reservoir, barrels |
| d | mean well depth, feet |
| q_0 | mean initial well productivity, barrels per day per well |

(b) economic variables

| | |
|---|---|
| D | ratio of initial developed production capacity of reservoir to total proved reserves (or production decline rate) |
| p | expected price, assumed constant, dollars per barrel |
| r | discount (interest) rate |
| T | economic time horizon, years |

(c) dependent variables

| | |
|---|---|
| W | cost of an exploratory well, dollars |
| N | number of development wells drilled |
| G | gross value of reservoir, present value dollars |
| I | total expenditure required to establish and maintain production, development investment plus operating costs, present value dollars |
| Y | net value of reservoir, present value dollars |
| X | unit production cost, dollars per barrel |

may often be well approximated by the so-called "six-tenths factor" [Chilton (1950), Williams (1947)]. Employing this we specify the investment for surface equipment to be:

$$\tilde{I}_2 = \beta_{21} (Nq_0)^{\beta_{22}} + \beta_{23} N + \tilde{\epsilon}_2 \quad (2)$$

The first term on the lefthand side, with $\beta_{22}=0.6$, represents investment which is dependent on the scale of operations, or throughput. The second component relates to expenditures which are dependent on the number of development wells, such as roads, gathering lines, and drilling pads, all of which are costly under Arctic permafrost conditions. Assuming that investment in a field base camp is allocated to individual pools in proportion to their developed capacity we have:

$$\tilde{I}_3 = \beta_{31} (Nq_0) + \tilde{\epsilon}_3 \quad (3)$$

Finally, capitalized operating costs are assumed to be represented by:

$$\tilde{I}_4 = \beta_{41} + \beta_{42} N + \tilde{\epsilon}_4 \quad (4)$$

Eq. (4) resolves total operating costs into a component which depends on the number of development wells and a component which is constant for the reservoir.

Summing the various investment components we have:

$$\begin{aligned} \tilde{I} = N [& \beta_{11} (e^{\beta_{12}d} - 1) + \beta_{23} + \beta_{42}] \\ & + \beta_{21} (Nq_0)^{\beta_{22}} + \beta_{31} (Nq_0) + \beta_{41} \\ & + \tilde{\epsilon}_1 + \tilde{\epsilon}_2 + \tilde{\epsilon}_3 + \tilde{\epsilon}_4 \end{aligned} \quad (5)$$

We assume the individual error terms comprise over a number of reservoirs a sequence of mutually independent random variables, identically distributed according to a known probability law. Writing $\tilde{u} = \tilde{\epsilon}_1 + \tilde{\epsilon}_2 + \tilde{\epsilon}_3 + \tilde{\epsilon}_4$ in Eq. (5) yields:

$$\begin{aligned} \tilde{I} = N [& \beta_{11} (e^{\beta_{12}d} - 1) + \beta_{23} + \beta_{42}] \\ & + \beta_{21} (Nq_0)^{\beta_{22}} + \beta_{31} (Nq_0) + \beta_{41} + \tilde{u} \end{aligned} \quad (6)$$

The output which can be obtained from a given reservoir is directly related to the volume of reserves that can be proved, V . Postulating that the mean recovery factor, F , is known with certainty:

$$V = F v \quad (7)$$

A number of models describing the production decline behavior of a reservoir have been studied, the most notable of which are exponential decline and hyperbolic decline. We use the former, anticipating that it describes output over time sufficiently well for our purposes and

gratefully accepting its mathematical convenience. Consequently, cumulative production, Q , at any given time is:

$$Q_t = \int_0^t N_t q_t dt = Nq_0 \int_0^t e^{-Dt} dt \quad (8)$$

where D is the production declines rate. The term on the extreme left incorporates the assumption that installed capacity is not increased at later stages in the productive life of the pool.

Integration of Eq. (8) yields:

$$Q_t = \frac{Nq_0}{D} (1 - e^{-Dt}) \quad (9)$$

If we consider a long period of time, such that proved reserves are essentially recovered ($Q_t \rightarrow V$, $t \rightarrow \infty$), Eq. (9) becomes:

$$V = \frac{Nq_0}{D} \quad (10)$$

Hence, for a specified decline rate, D , initial producing capacity can be related to reservoir size by applying Eqs. (10) and (7):

$$Nq_0 = D F v \quad (11)$$

In addition, it can readily be seen that the number of development wells drilled depends on reservoir size and the mean initial capacity (or productivity) of a well.

$$N = DF \frac{V}{q_0} \quad (12)$$

Eq. (6) in conjunction with Eqs. (11) and (12) relates the investment required to produce the crude in a reservoir to the set of physical parameters which describe that reservoir, as listed in Table 1.

We can now turn to computation of the economic return that will be gained by developing the newly discovered reservoir. With output as described by Eq. (8) and assuming continuous discounting, gross revenue must be:

$$G = pNq_0 \int_0^T e^{-(D+r)t} dt \quad (13)$$

Integration yields:

$$G = pNq_0 \left[\frac{1 - e^{-(D+r)T}}{D + r} \right] \quad (14)$$

Representing the factor in brackets by A, the expression for gross revenue becomes:

$$G = pNq_0 A \quad (15)$$

Net revenue, or the economic return resulting from the discovery of the reservoir, is the difference between gross revenue and total investment:

$$\tilde{Y} = G - \tilde{I} \quad (16)$$

Our basic model for the returns to exploration thus consists of Eqs. (16), (15), (6), (11), and (12). This model relates the economic payoff of a discovery to the set of observable physical parameters in Table 1. When a wildcat is dry, the direct payoff is zero. The required economic parameters, also shown in Table 1, are assumed to be determined exogenously, to be known with certainty, and to be fixed for all reservoirs discovered by a series of wildcat wells. At the outset of exploration in a region (and therefore before the development of any producing capacity) the vector $\underline{\beta}$ of parameters as well as the probability law governing \tilde{u} would not be known. These would have to be estimated from sample data.

Before turning to the applications of this model, it will be useful to describe the final dependent variable listed in Table 1, unit production cost, denoted by X . This will be particularly useful in our analysis because it is not dependent upon economic expectations; specifically, it is calculated without reference to expected wellhead price. By focusing on cost we properly restrict ourselves to geological and technical forms of risk.¹ We define unit cost, X , as the amount that must be realized on each barrel of crude produced in order to recover the investment in the reservoir, including capitalized operating costs.²

1. M. A. Adelman (1966) has distinguished among commercial, geological, engineering, and political risks. Changes in selling price would represent commercial risk, narrowly defined. Our estimates of expected return and variability of returns neglect the risk of possible changes in selling price and hence may be misleading. We have not treated political risk here, but would do so through the revenue side, for example, by specifying the probability of getting any returns after a given year.

2. This measure of cost has been used elsewhere in analyzing crude oil production. See Adelman (1966), Bradley (1967).

In the notation we have used:

$$\tilde{X} = \frac{\tilde{I}}{Nq_o A} \quad (17)$$

Since the denominator of Eq. (17) represents discounted, or present value, output (measured in barrels), unit cost depends on the investment needed to obtain production and the resulting pattern of output over time.

Applying the Model to the Arctic

In using this model to gain insights into the economics of Arctic exploration we encounter formidable problems. At the outset we noted the need to specify the probability laws governing petroleum exploration in order to properly use the data which are collected to make inferences about the underlying parameters. As a makeshift substitute for a more comprehensive model we have postulated probability functions for the independent random variables. In the case of reservoir size it is possible to employ a hypothesis which has received considerable testing in the literature;³ in other cases we were compelled to perform our own rough tests on the selected probability functions. Once the required probability functions are specified, we need estimates of parameters which characterize the Arctic area under study. Since we have not had access to information on which such estimates could be based, we conjecture possible values. It is also necessary to know the vector of cost parameters which determine investment, denoted \underline{g} in the previous sections. To meet this need we have made some rough calculations which employ the estimates of another panelist, C. A. Norman.

The probability functions and parameters for variables used in the simulations are summarized in Table 2. Parameter estimates are based on data describing petroleum occurrence in the Province of Alberta, except for initial well productivity where the Alberta data were not appropriate to our needs. After inspecting data for Algerian, Iran, and

3. Two examples are G. M. Kaufman (1963) and R. G. McCrossan (1969).

Table 2
PROBABILITY FUNCTIONS AND PARAMETERS FOR INDEPENDENT RANDOM VARIABLES

| Variable | Probability Density Function | Parameter Values | | Source of Parameter Values |
|---|------------------------------|------------------|----------|--|
| | | μ | σ | |
| Depth of well, \hat{y} | Normal | 5610 | 1615 | Estimated by authors from data on 106 light and medium crude reservoirs in Alberta. [Oil and Gas Conservation Board, Reserves of Crude Oil, Gas, ..., Province of Alberta, 1968.] |
| Volume of crude discovered, \hat{v} | Log-normal | (1) 16.15 | 2.14 | Uhler and Bradley (unpublished) estimated $\mu=16.15$, $\sigma=2.14$ for limestone reef pools in Alberta (143 pools, 1968 data). Item (2) is parameter set where pools are postulated to be twice as big, i.e., $(v_1)_2 = 2(v_1)_1$. Items (3) and (4) represent successive doublings of pool size. |
| | | (2) 16.84 | 2.14 | |
| | | (3) 17.54 | 2.14 | |
| | | (4) 18.23 | 2.14 | |
| Mean initial well productivity, \hat{q}_0 | Log-normal | 7.51 | 1.18 | Estimated by authors from data on 25 reservoirs in Libya. [Oil and Gas Journal, "Worldwide Oil Report", Dec. 29, 1969.]** |
| Error of estimate in investment function (development), \hat{u} | Normal | 0 | 44000 | Estimated by authors. Standard error of estimate in regression relating expenditures to footage of development drilling; Alberta, E.C., Saskatchewan, Manitoba. |

*For \hat{y} normally distributed, density function is:

$$f = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[-\frac{1}{2\sigma^2} (y - \mu)^2 \right].$$

**For \hat{x} lognormally distributed:

$$f = \frac{1}{x\sigma\sqrt{2\pi}} \exp \left[-\frac{1}{2\sigma^2} (\log x - \mu)^2 \right].$$

**Data could not be obtained for Alberta. Because of prorating, initial reported production would understate the M.E.R. of production for Alberta pools.

Libya we based our rough estimates of productivity parameters on the Libyan information. It scarcely need be said that our hypotheses about the distributions of the depth and productivity variables require further testing. It would be of great interest to test for possible correlation between these variables. The important figures used in computing the cost parameters, β , are listed in Table 3.

With this information in hand, we return to the model of the previous section. The outcome of drilling a wildcat well is determined: (a) by whether the well is a success, that is, whether it finds crude oil, and (b) if it does, by the observed values of the variables v , d , and q_0 . Considering for the present only successes, we simulate wildcat drilling by treating v , d , and q_0 as random variables, recognizing this distinction by the notation \tilde{v} , \tilde{d} , and \tilde{q}_0 . A single outcome is evaluated by generating values for \tilde{v} , \tilde{d} , and \tilde{q}_0 according to the probability density functions and parameters shown in Table 2, and then by computing -- using Eqs. (6), (11), (12), (15), and (16) -- the corresponding values for the dependent variables listed in Table 1. For each set of conditions examined, 2000 outcomes were evaluated. The results were displayed in the form of histograms, two of which are illustrated in the Appendix. In the next section we describe the results of this procedure where the underlying physical and economic parameters were chosen to represent Arctic conditions.

Table 3

COMPONENTS OF COST PARAMETERS
(Derived from Estimates for the Prudhoe Bay Field)*

| | | |
|-------|--|--|
| I_1 | Investment in producing wells: | |
| | (a) Drilling costs, per well | $C_1 = \beta_{11} (e^{\beta_{12}d} - 1)$ where $\beta_{11} = 119,000$, $\beta_{12} = .0002$ (cost of drilling only, 9000 ft. development well, estimated to be \$600,000) |
| | (b) Cost increment for slant drilling of development wells, per well | \$75,000 |
| | (c) Cost of drilling pad, per well | \$50,000 |
| | (d) Cost of connecting roads, per mile | \$200,000 |
| I_2 | Investment in surface facilities: | |
| | (a) Processing plant [inc. oil and gas separation, gas compression for reinjection] | $C_2 = \beta_{21} (Nq_0)^{\beta_{22}}$ where $\beta_{21} = 291$, $\beta_{22} = 0.6$ (cost of plant with 100,000 bpd capacity estimated to be \$8.62 million). |
| | (b) Cost of injection wells, per well | I_1 , (a,c,d) above. |
| | (c) Cost of gathering lines, per mile | \$120,000 |
| I_3 | Operating costs, per well per year | $E_1 = \gamma_1 + \gamma_2/N$ where $\gamma_1 = 6000$, $\gamma_2 = 554,000$, and $N =$ number of development wells. |
| I_4 | Camp investment [assumed to serve several reservoirs in field and allocated to a given reservoir according to share of field output over production period.] | \$24,300,000 |

* Estimates of various expenditures required to produce crude at Prudhoe Bay were made by C. A. Norman. We are not aware of any other estimates available to the public which are as carefully detailed. In adapting the original figures to obtain the ones shown here and ~~and~~, we have combined categories and made simplifying assumptions for which the author of the original estimates should not be held responsible.

Simulation Results

For the probability functions specified in Table 2, the expected values for depth and well productivity are, respectively, 5610 feet and 3640 barrels per well per day. The latter figure is close to the 4000 barrels per well per day postulated in the Norman paper; apparently the discoveries at Prudhoe are deeper, however. Suppose it were known with certainty that all Arctic discoveries would be at about this depth, with wells flowing at this output rate. The returns in such a situation were simulated in the method just described, with results that are reported in Table 4. Discoveries under these conditions were uniformly profitable at a wellhead price of one dollar per barrel.

Looking more closely at Table 4, the rows correspond to progressively more optimistic assumptions about the size of reservoirs. At the low end the postulated lognormal distribution of reservoir sizes yields pools whose median size is around 10 million barrels (oil in place), a figure corresponding to experience with the attractive Devonian reefs in Alberta. At the high end the postulated distribution yields pools whose median size is about 8 times as big, a very generous assumption indeed. Production costs vary between 60 and 80 cents a barrel. The principal cause of cost variation lies with diseconomies of scale in surface facilities. This effect would be stronger, were it not that the calculations permitted a sharing of the costly items included under the category of base camp expenditures (camp, airstip, vehicles, power plant, rig mobilization) with other pools assumed to exist in the field. Net returns increased as would be expected, roughly in proportion to the volume of reserves discovered.

We now consider the situation where it is not certain that the development wells in the pools discovered will produce initially at

Table 4

EXPECTED RETURNS TO DEVELOPMENT

Case I: reservoir size variable; depth and mean initial well productivity fixed.

| Reservoir Size, \bar{v} (millions of barrels of oil in place) Median | Production Cost, \bar{X} (dollars per barrel) | | Net Returns, \bar{Y} (millions of present value dollars) | | | |
|--|---|--------------------|--|--------------------|-------------------------------|------|
| | Mean | Standard Deviation | Mean | Standard Deviation | Fraction with Positive Return | |
| (1) 10.3 | .791 | .817 | 8.23 | 39.9 | 485 | 1.00 |
| (2) 20.6 | .724 | .765 | 19.4 | 118. | 608 | 1.00 |
| (3) 41.4 | .669 | .721 | 42.7 | 405. | 948 | 1.00 |
| (4) 82.6 | .585 | .650 | 77.4 | 455. | 588 | 1.00 |

*Reservoir recovery factor (F) = 0.35.

**The following economic variables are treated as known with certainty:

- (a) declines rate (D) = 8 percent per year,
- (b) discount rate (r) = 15 percent per year,
- (c) economic time horizon (T) = 25 years,
- (d) expected wellhead price (p) = 1.00 dollars.

3600 barrels per day. We postulate this to be the expected rate, but permit the degree of dispersion in rates which we observed in the Libyan data. The simulated outcomes of exploratory drilling are displayed in Table 5. A striking feature of this new situation is that now not all the discoveries are commercial -- defined under our assumptions as being capable of producing profitably at a wellhead price of one dollar per barrel. The fraction of fields with positive returns is under 0.5 for the most conservative assumption about reservoir sizes. This means that production cost was observed to be under one dollar per barrel less than half the time; the corresponding mean production cost is about \$1.80. As would be expected, the dispersion of outcomes is much greater than previously; this is seen by comparing the coefficients of variation (defined as the standard deviation measured as a percentage of the mean).

In Table 6 we postulate reservoir size, depth, and well productivity to all be variable, defined by the corresponding probability functions shown in Table 2. The differences between Tables 5 and 6 are not very pronounced. The coefficients of variation in Table 6 are generally higher; the fact that they are not universally higher suggests that we should increase the number of trials evaluated under each set of conditions beyond 2000; the observed moments of the outcome distributions are not yet quite stable, a consequence of the extreme skewness of the distributions. The means and standard deviations in the initial row of Table 6 are computed from the histograms shown in the Appendix.

The results just considered were conditional upon the exploratory well striking oil. To compute expected returns before drilling begins, these figures must be modified to take account of the probability that

Table 5

EXPECTED RETURNS TO DEVELOPMENT

Case II: reservoir size, mean initial well productivity variable; depth fixed.

| Reservoir Size, \tilde{v} (millions of barrels of oil in place) Median | Production Cost, \tilde{X} (dollars per barrel) | | Net Returns, \tilde{Y} (millions of present value dollars) | | Fraction with Positive Return |
|---|---|--------------------|--|--------------------|-------------------------------|
| | Mean | Standard Deviation | Mean | Standard Deviation | |
| (1) 10.3 | 1.79 | 2.89 | 6.92 | 25.7 | .449 |
| (2) 20.6 | 1.56 | 2.42 | 13.8 | 57.2 | .523 |
| (3) 41.4 | 1.33 | 1.95 | 44.1 | 597. | .552 |
| (4) 82.6 | 1.15 | 1.75 | 69.7 | 471. | .637 |

* Reservoir recovery factor (F) = 0.35.

** The following economic variables are treated as known with certainty:

- (a) decline rate (D) = 8 percent per year,
- (b) discount rate (r) = 15 percent per year,
- (c) economic time horizon (T) = 25 years,
- (d) expected wellhead price (p) = 1.00 dollars.

Table 6

EXPECTED RETURNS TO DEVELOPMENT

Case III: reservoir size, depth, and mean initial well productivity variable.

| Reservoir Size, \bar{V} (millions of barrels of oil in place) Median | Production Cost, \bar{X} (dollars per barrel) | | Net Returns, \bar{Y} (millions of present value dollars) | |
|---|---|--------------------|--|--------------------|
| | Mean | Standard Deviation | Mean | Standard Deviation |
| (1) 10.3 | 1.83 | 2.78 | 6.83 | 28.7 |
| (2) 20.6 | 1.67 | 2.78 | 16.8 | 150. |
| (3) 41.4 | 1.37 | 2.35 | 30.3 | 247. |
| (4) 82.6 | 1.19 | 1.99 | 87.2 | 1168. |

Coef. of Vari- ation

Coef. of Vari- ation

Fraction with Positive Return

420

893

815

1339

.435

.481

.568

.643

* Reservoir recovery factor (F) = 0.35.

** The following economic variables are treated as known with certainty:

- (a) decline rate (D) = 8 percent per year,
- (b) discount rate (r) = 15 percent per year,
- (c) economic time horizon (T) = 25 years,
- (d) expected wellhead price (p) = 1.00 dollars.

the well will in fact turn out to be a success. This might be done by treating successes and failures as Bernouli trials, although this clearly is an oversimplification. It should also be noted that the net return figures we have computed relate to development investment and operating costs, and do not include the cost of exploratory wells, which have been reported to run from two to three million dollars and higher on the North Slope. It therefore appears from Table 6 that with the figures we have used some combination of very favorable success ratio and large median reservoir size (high mean of the distribution of \tilde{V}) is needed to make expected returns to a sequence of wildcats positive. Given the skewness of the distributions, there will of course be some very profitable finds even where the expected return is low.

With regard to the need to find large reservoirs, for any existing distribution of pool sizes in nature we might expect the initial finds to be relatively large. This is the line of reasoning formalized by the model in our first paper -- that the probability of finding a big pool is higher than the probability of finding a small one. Against this, though, is the benefit of better knowledge about the geology of the region, acquired as data accumulate from exploration. This might permit better selection among available prospects in later periods.

The results presented in this paper are intended to be suggestive. They cannot be treated as more because we have not had Arctic data from which to derive our parameter estimates, and therefore have relied on possible similarities with already developed basins. We also feel that it will be necessary to make progress along the lines suggested in the previous paper before we can confidently characterize the uncertainties of the exploration process.

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Appendix

Simulation of Unit Production Costs
(2000 discoveries)

| Frequency | 0. | 1. | 2. | 4. | 5. | 6. |
|-----------|-----|----|----|----|----|----|
| 250 | *** | | | | | |
| 225 | ** | | | | | |
| 200 | * | | | | | |
| 175 | * | | | | | |
| 150 | * | | | | | |
| 125 | * | | | | | |
| 100 | * | | | | | |
| 75 | * | | | | | |
| 50 | * | | | | | |
| 25 | * | | | | | |
| 0 | * | * | * | * | * | * |

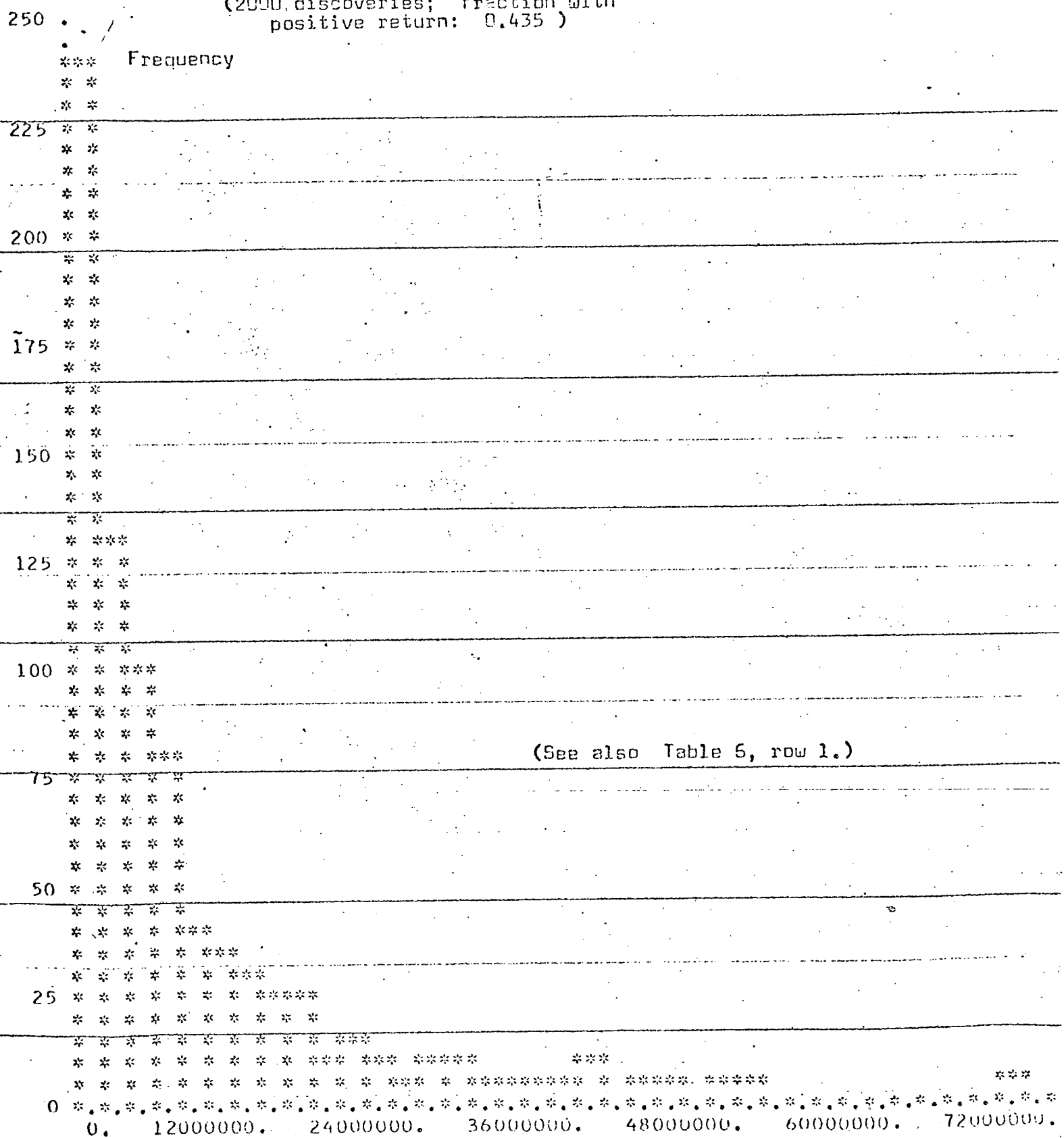
(See also Table 6, row 1.)

Unit production cost, dollars per barrel

Appendix

Simulation of Net Returns

(2000 discoveries; fraction with positive return: 0.435)



Net return, present value dollars