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HIGH RESOLUTION POWER SPECTRA OF DAILY ZURICH SUNSPOT NUMBERS

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16. ABSTRACT

High resolution power spectra of 77 years of Zurich daily sunspot numbers were computed using various lags and data point intervals. Major harmonic peaks of the approximately 124-month period showed up strongly as well as the 27-day solar rotational period.

17. KEY WORDS

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DEFINITION OF SYMBOLS

Symbol	Definition
L	variable lag number
M	maximum number of lags used
I	degree of term
PSD	power spectral density
N	total number of points used
n	data point number
Δt	data interval
C(L)	autocovariance
R(L)	normalized autocorrelation function
k	harmonic number
P(k)	spectral density
f	frequency

HIGH RESOLUTION POWER SPECTRA OF DAILY ZURINCH SUNSPOT NUMBERS

SUMMARY

High resolution power spectra of 77 years of daily Zurich sunspot numbers were computed using various maximum lags and data point intervals. Major harmonic peaks of the 124-month period showed up strongly. Harmonics of the 27-day solar rotational period were also present on the spectral plots.

I. INTRODUCTION

Sunspots were first observed by Galileo in 1610. Through his newly constructed telescope, Galileo watched the behavior of dark spots on the solar disk, drew pictures of them, and noted their motion across the solar disk with daily changes in number. Then, for over two hundred years there was little interest in sunspot numbers. In 1818, observers again began counting and recording sunspot numbers, but even then on an irregular basis. A continuous record was not begun until 1848 when Rudolf Wolf of the Swiss Federal Observatory in Zurich, Switzerland, began recording relative sunspot numbers on a daily basis. Wolf developed the following formula for calculating relative sunspot numbers:

$$R = k(10g + f)$$

where R is the relative daily sunspot number, k is a scale factor usually less than 1.0, g is the number of isolated cluster groups, and f is the number of individual observable spots on the visible solar disk.

From incomplete records left by previous observers, Wolf was able to estimate yearly sunspot numbers as far back as 1700 and monthly values as far back as 1749. The recording of sunspot numbers was continued after Wolf by Wolfer, Brunner, and Waldmeier. At present, we have daily Zurich sunspot numbers on a computer tape from January 1, 1890, through Dec. 31, 1971. The data used in this analysis, are

the daily, monthly, and smoothed quarterly sunspot numbers are listed in the literature by Waldmeier [1].

Provisional Zurich sunspot numbers are dependent upon observations at the Zurich Observatory and its stations at LaCarno and Arosa. These numbers are compiled by M. Waldmeier of the Swiss Federal Observatory in Zurich and are distributed monthly. At the end of the year, Waldmeier compares the results of his three observatories with reports sent to him during the year from other observatories. After a few months of the new year, Zurich Observatory personnel revise their provisional values and issue final daily Zurich sunspot numbers for the previous year.

In 1843, Schwabe noticed an average 11-year period appearing in sunspot data. Although there have been many sunspot numbers prediction studies since Schwabe's discovery, few efforts have yielded accurate long-range predictions. In this report, we will examine high resolution power spectra computed from sunspot data. These results are intended to be combined with other studies to aid solar activity predictions.

II. POWER SPECTRAL DENSITY FORMULAS

The power spectral density function $P(\omega)$ is the Fourier transform of the autocorrelation function $R(\tau)$.

$$P(\omega) = \int_{-\infty}^{\infty} R(\tau)\ e^{-i\omega\tau} d\tau. \tag{1}$$

Since the autocorrelation function $R(\tau)$ is even,

$$P(\omega) = 2\int_{0}^{\infty} R(\tau)\ e^{-i\omega\tau} d\tau. \tag{2}$$

We then use the cosine transform, where

$$P(\omega) = 2\int_{0}^{\infty} R(\tau)\ \cos\ (\omega\tau)\ d\tau. \tag{2a}$$

The Fourier transform of a function is the integral of the product of that function times an exponential. One special case of the Fourier transform is power spectra where the transformed function is the autocorrelation function. Although a statistician would recommend a standard form and symbol for the autocorrelation function, various symbols and notations are used in the literature. Here we will use a notation as used in some computer programs. An autocorrelation is a correlation of one data point of a series with another data point of the same series at an interval of time later.

The autocovariance function C(L) is computed and then divided by the variance to give the normalized autocorrelation function. The autocovariance C(L) is

$$C(L) = \frac{1}{N - L} \sum_{n=1}^{N-L} (x_n - \bar{x})(x_{n+L} - \bar{x}), \tag{3}$$

where the mean value $\bar{x}$ was subtracted from each data point before computation and L = lag number = 0, 1, 2, ..., $M \ll N$, M = maximum number of lags used beyond which the autocorrelation function is truncated in the integral summation approximation for the power spectra, n = data point number = 1, 2, 3, ..., N, N = total number of data points used.

When L = 0,

$$C(0) = \frac{1}{N - 0} \sum_{n=1}^{N-0} (x_n - \bar{x})(x_{n+0} - \bar{x}) = \frac{1}{N} \tag{4}$$

$$\sum_{n=1}^{n} (x_n - \bar{x})^2 = \text{variance} = \sigma^2, \tag{5}$$

where the variance is a measure of the variability of the data. Then the normalized autocorrelation function R(L) is the covariance divided by the variance:

$$R(L) = \frac{C(L)}{C(0)} . \tag{6}$$

For example, when the lag L = 0, the normalized autocorrelation is 1:

$$R(0) = \frac{C(0)}{C(0)} = 1. \tag{7}$$

Integrating equation (2a) by the trapezoid summation method, we obtain the raw power line spectrum P(k):

$$P(k) = 2\Delta t \left[R(0) + 2 \sum_{L=1}^{M-1} R(L) \cos (2\pi Lk/2M) + (-1)^k R(M) \right], \tag{8}$$

where k is the harmonic number.

Since the numerical procedures and data both contain errors, the line spectra estimates are averaged over a few points, using the following Hanning method to give the smooth spectra.

For the first point of line spectra, we use the simple average of the first two points.

$$\tilde{P}(0) = .5\ P(0) + .5\ P(1). \tag{9}$$

For the non-end points of line spectra, we use a simple three-point average with end points given half weight.

$$\tilde{P}(k) = .25\ P(k - 1) + .5\ P(k) + .25\ (k + 1), \tag{10}$$

where k = 1, 2, ..., M - 1.

For the last point of line spectra, we use the simple average of the last two points.

$$\tilde{P}(M) = .5\ P(M - 1) + .5\ P(M). \tag{11}$$

III. POWER SPECTRA PERIOD

The x-axis on the power spectra plots was originally in units of harmonic number k which was then converted to period by the following equation:

$$\text{period} = \frac{2M\Delta t}{k} , \tag{12}$$

where M is the maximum number of lags used, and Δt is the time interval between data points. Increasing M increases the length of the auto-correlation function used and also increases the resolution of the lower frequencies.

IV. POWER SPECTRA INTERPRETATION

Care must be exercised in the interpretation of the power spectra results. Present methods of numerical evaluation may produce "false" peaks that are not really present. For this reason, power spectra are usually averaged over a few points and smoothed. Even then, false peaks may still be present. Only when the amplitude exceeds the noise level of both the data and the numerical evaluation procedure will a peak be significant.

Sometimes a three-point zigzag of the power spectra may only indicate numerical errors in the program methods or random errors in the data. A significant peak, usually requires more than three or four points over which the spectra increases and decreases.

V. LAGS USED

Doubling the maximum number of lags used doubles the number of harmonics considered with a new set of harmonic numbers between the harmonic numbers of the original undoubled set. As a rule of thumb, we used a maximum lag number equal to about 10 percent of the total number of points used. The machine time increases rapidly above 10 percent. To find the low frequencies, we used many points and lags. When the machine time increased significantly, we increased our data interval, skipping over points to conserve machine time.

VI. RESULTS

1. Monthly Power Spectra Results

First, we will look at power spectra of monthly Zurich sunspot numbers over a long time interval of over 200 years, and note long-period variations. To accurately compute these long-period variations, we need a long time sample. Figure 1a shows a log-log plot of the smooth normalized power spectra of monthly Zurich sunspot numbers from January 1749 through December 1967 using 500 lags. Notice the large spectral peak near 125 months, the second harmonic peak at 67 months, and the fifth harmonic peak near 26 months. These harmonic peaks of the so-called 11-year cycle may be due to the nonsinusoidal shape of most solar cycles. The rapid rise and slow decay can be Fourier curve-fit using a fundamental plus a few odd harmonics. From other studies on Fourier curve-fits of solar cycles, there are some indications that the harmonic periods of the fundamental actually exist at least once during the time interval under consideration. Although it is difficult to detect any 26-month period in the data listings, we can pick the extreme monthly value of a solar cycle and also see a large monthly value off by itself above the smooth data about 26 months before the maximum and about 26 and 52 months after the maximum.

Figure 1b shows the raw (unsmoothed) normalized power spectra of monthly Zurich sunspot numbers from January 1749 through December 1967 using 500 lags with Δt = 1 month. Figure 1b shows a major peak around 125 months as in cycles 15, 16, and 17. Figure 1b also shows an indication of a peak near 200 months. Although none of the observed cycles were that long, cycle 4 had a fundamental period near 158 months. Figure 1b indicates a hint of a peak near 100 months. Cycles 2 and 3 had a fundamental period somewhere near 100 and 109 months, respectively.

Figure 2 shows the smooth normalized power spectra of daily Zurich sunspot numbers from 1 January 1890 through 29 December 1967 using 486 lags with Δt = 27 days. The main peak is near 3749 days, which is equal to about 122.9 months. Again, we have the first few harmonics of the main peak to show up. The second and third harmonics of 3749 days are 1874 days and 1249 days, respectively.

Figure 3 shows the smooth normalized power spectra of daily Zurich sunspot numbers from 1 January 1890 through 8 December 1967 using 162 lags with Δt = 6 days. Notice the main solar rotation period near 27.38 days with a secondary peak at 31.86 days, and also the appearance of a period near 42 and 97 days. We see that (31.86)(3) = 95.58 and that this 97-day peak could possibly be just a recurrence of an active region three solar rotations later. The existence or nonexistence of a 29-day period in solar activity data has been discussed by Shapiro and Ward [2], Bell and Defouw [3], and others.

Figure 4 shows the smooth normalized power spectra of daily Zurich sunspot numbers from 1 January 1890 through 8 December 1967 using 486 lags with the data interval $\Delta t = 6$ days. Figure 4 contains the same input data as Figure 3 except that the maximum number of lags used is increased from 162 lags to 486 lags with a data interval Δt remaining at 6 days. With this increase in resolution, we notice a period at 253.6 days. Note the band of power spectra peaks between 25 through 30 days with a major peak near 27.5 days. The 32-day peak may be due to the solar rotation of active regions at high solar latitudes near the beginning of a solar cycle with the 25-day peaks due to those regions at low latitudes near the end of a solar cycle. In figure 4, the 29-day peak does not show up clearly as a substantial peak by itself.

Figure 5a shows the smooth normalized power spectra of daily Zurich sunspot numbers from 1 March 1954 through 1 March 1966 using 500 lags with data interval $\Delta t = 1$ day, which includes cycle 19 with part of the beginning of cycle 20. Notice the twin peaks near 31.25 and 27.77 days and the broad flat peak near 90.9 days, and at 41.6 days. In figure 5a, because of the low resolution, we need to either increase the lag number or increase Δt to resolve the 90.9-day peak. Notice the 13.7-day period near the second harmonic of the fundamental.

Figure 5b has the same input data as figure 5a except the data interval has been increased from $\Delta t = 1$ to $\Delta t = 2$. By increasing this data interval, we were able to resolve the 90.9-day period shown in figure 5a. We increased the data interval Δt from 1 to 2 using every other point to conserve computer time. We now notice a 95.23-day peak, with a 125-day peak on the left of it and a 74.07 day peak on the right. Examining the computer printout, we noticed a peak at k = 231, where the period is

$$p = \frac{2(M)\Delta t}{k} = \frac{2(500)(2)}{231} = 8.65 \text{ days.}$$

When we assume that this period of 8.65 days is the third harmonic of the main rotational period during this time sample, the third harmonic shows up as a harmonic of the solar rotation frequency at a maximum of solar activity, that is, at 25.95/3 = 8.65 days.

In some of the previous figures, we saw the long period variations from spectra analysis of long-time series samples. To get the best resolution of the lower frequencies, we had to increase the number of lags. When we used many lags over a long time sample, the computer time required began to increase rapidly.

Therefore, to save computer time, we skipped over a number of points in the sample using a data interval Δt = 2, 6, and 27 days. To obtain the high frequencies or short periods shown on the right-hand side of the log-log graphs in the next figure, we need only take a short time interval sample and use a moderate number of lags of 10, 20, or 30 percent of the total number of points used.

Since, at high frequencies, we are using relatively few points, we can afford to use a greater percentage of a lag number (that is, a greater percentage of the total number of points) without increasing the machine time significantly. Figure 6 shows the smooth normalized power spectra of daily Zurich sunspot numbers from 19 January 1957 through 11 July 1962 using 200 lags. The main sunspot rotation period was about 28.57 days. On the right side of this peak, we note the second harmonic of 13.79 days. This 27/2-day period could be interpreted as (1) a harmonic of the fundamental rotational period, (2) a tendency for two sunspot groups to exist simultaneously, separated by 180 degrees of solar longitude, or (3) possible independent 13.79-day period with a phase shift from the main 28.57-day peak.

2. Interpretation of Results

High resolution power spectra of daily Zurich sunspot numbers have a major peak at 3749 days and at the second, third, fourth, fifth, and sixth harmonics. There is a possibility that these periods may not be due to independent sine wave frequencies, but rather to the non-sinusoidal wave shape of sunspot cycles. The periods centered around eleven years are non-stationary and usually change gradually from cycle to cycle. As expected, when we look at the periods centered around 27 days, we see that time series samples at the beginning of a solar cycle (when the active centers are at high latitudes) have long rotation periods of 32 days, or more than when the active centers are at low latitudes at the end of a cycle where the rotational periods are less than 27 days. Power spectra of the rotational periods also show the presence of harmonics of the solar rotational period.

CONCLUSION

Although constant periodic variations in solar activity were not found, it appears that there is some evidence of systematic behavior or modulation in the solar active regions. Of interest are the twin peaks noted near 27.38 days and 31.86 days. Also of possible interest is the peak around 95 days or 97.2 days. Of course, one possible explanation of the 97.2 day period is that it may be due to a rotation peristence where a new highly active region on the sun may continue to exist and reappear over 3 solar rotations.

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3. Bell, Barbara and Richard J. Defouw, "Discussion of Paper by M. E. Rassbach, A. J. Dessler, and A. G. W. Cameron, 'The Lunar Period, the Solar Period, and K_p'", Journal of Geophysical Research, Vol. 71, No. 23, P. 5770, December 1, 1966.

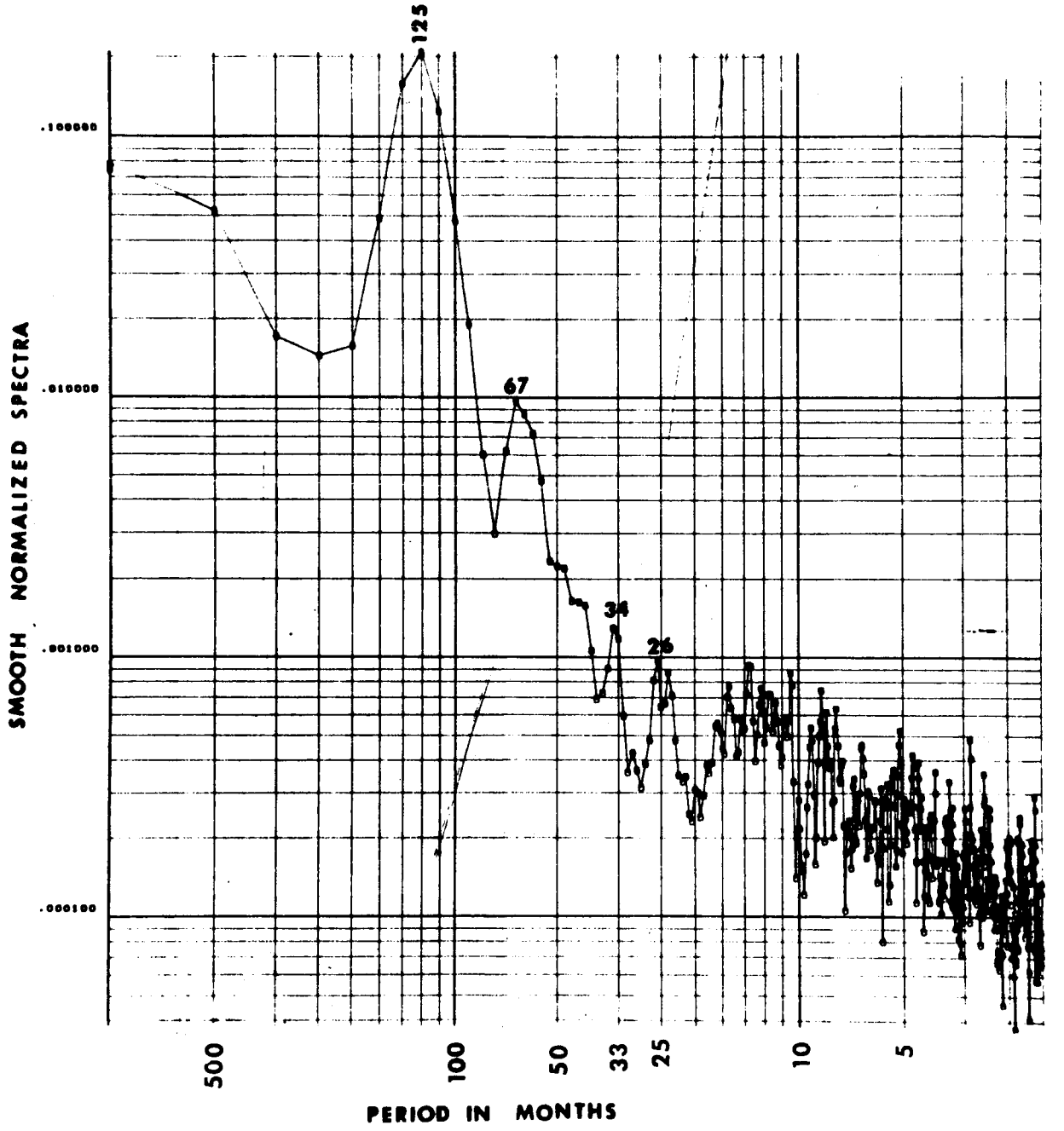

Figure 1a. Smooth Normalized Power Spectra of Monthly Zurich Sunspot Numbers from Jan. 1749 through Dec. 1967 Using 500 Lags

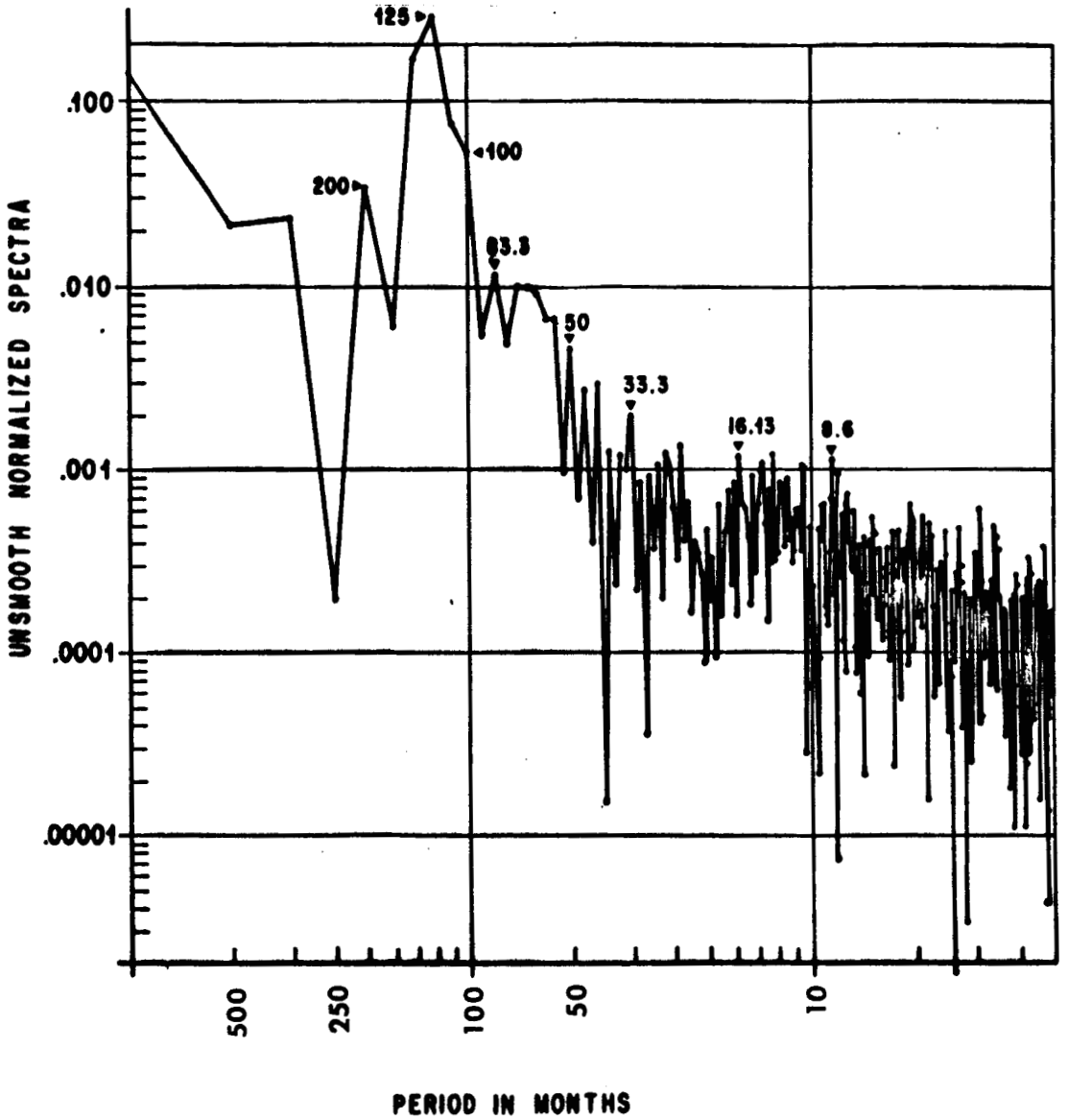

Figure 1b. Unsmooth Normalized Power Spectra of Monthly Zurich Sunspot Numbers from Jan 1749 Through Dec 1967 Using 500 Lags

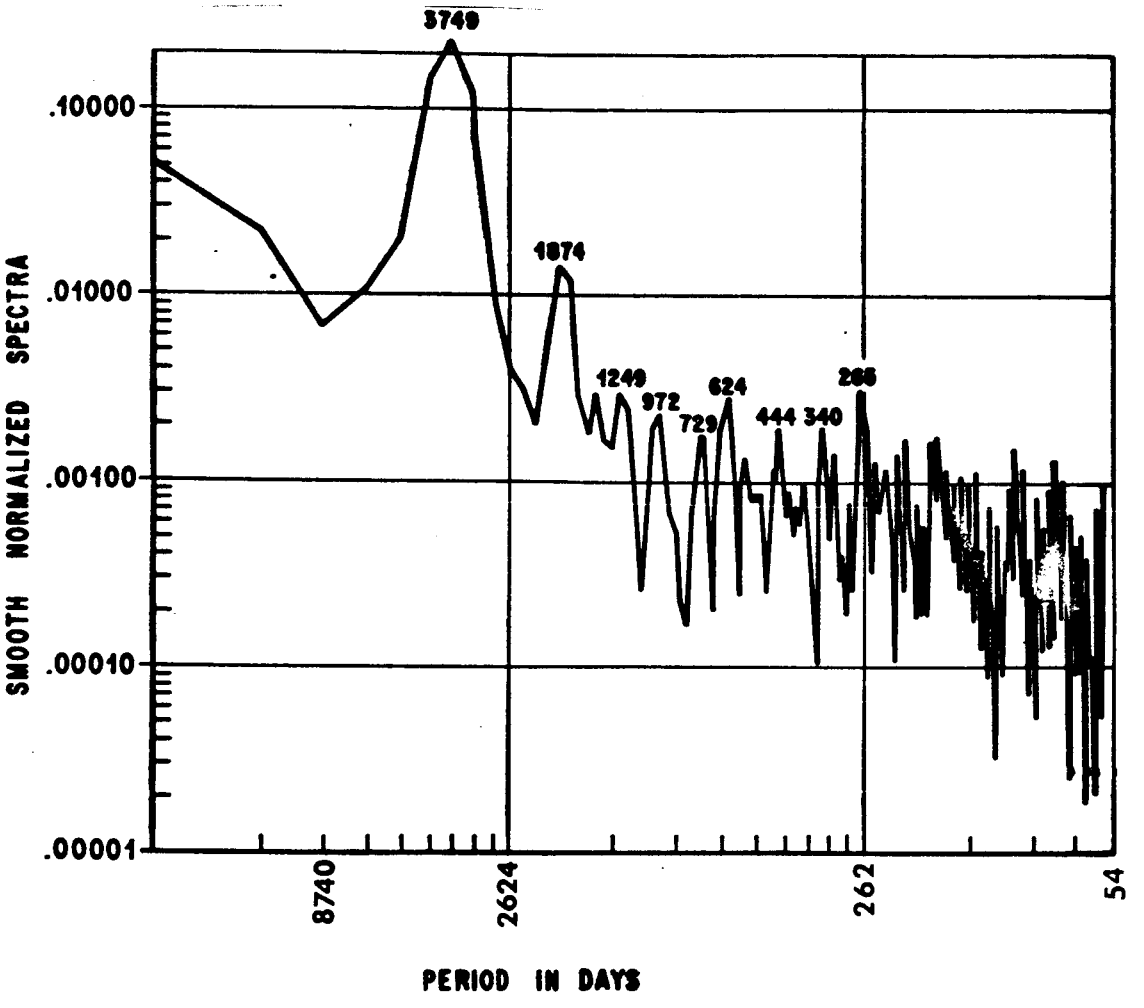

Figure 2 Smooth Normalized Power Spectra of Daily Zurich Sunspot Number from 1 January 1890 through 29 December 1967 Using 486 Lags with Δt = 27 Days

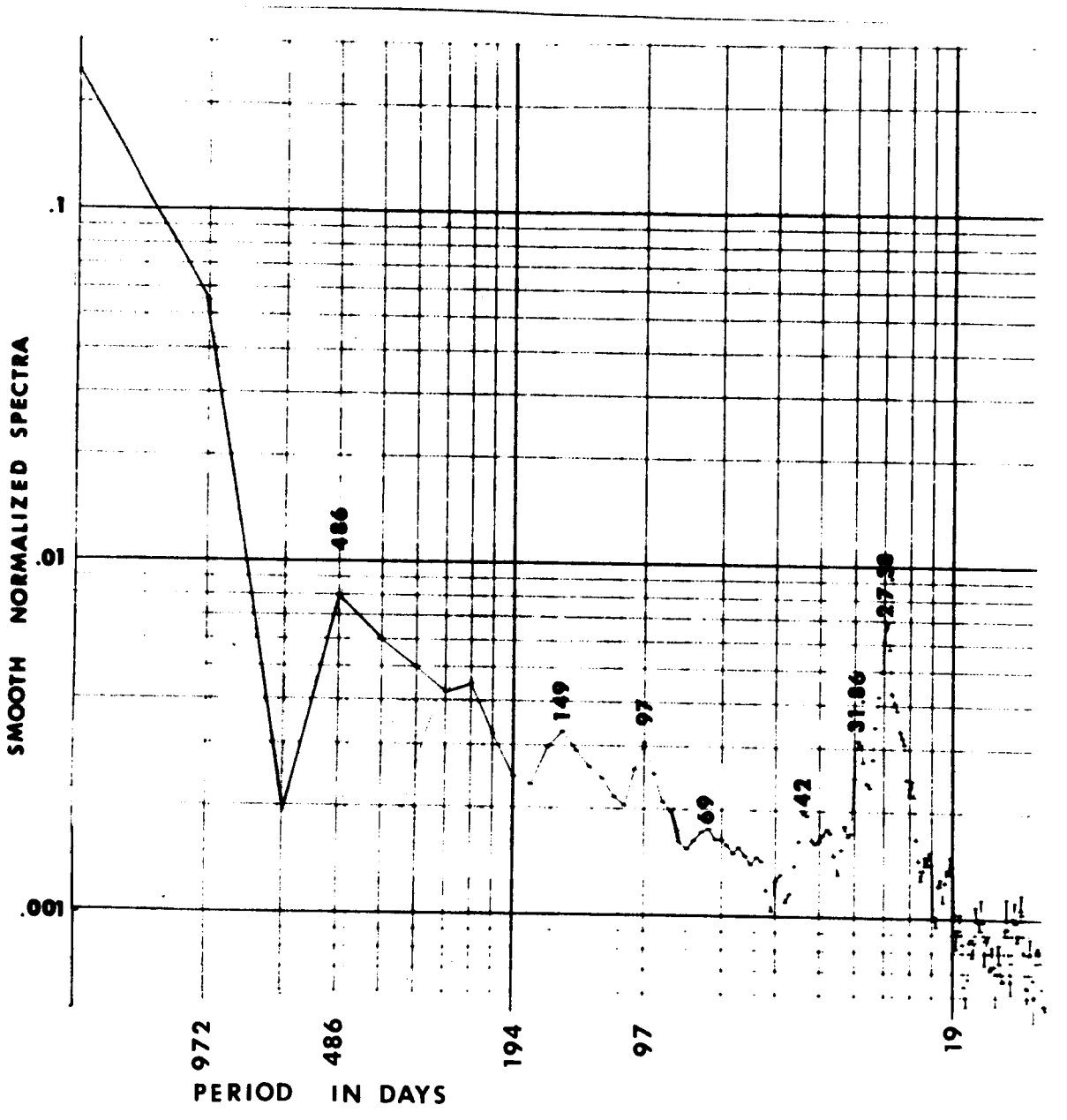

Figure 3 Smooth Normalized Power Spectra of Daily Zurich Sunspot Numbers from 1 Jan 1890 thru 8 Dec 1967 using 162 Lags with $\Delta t = 6$

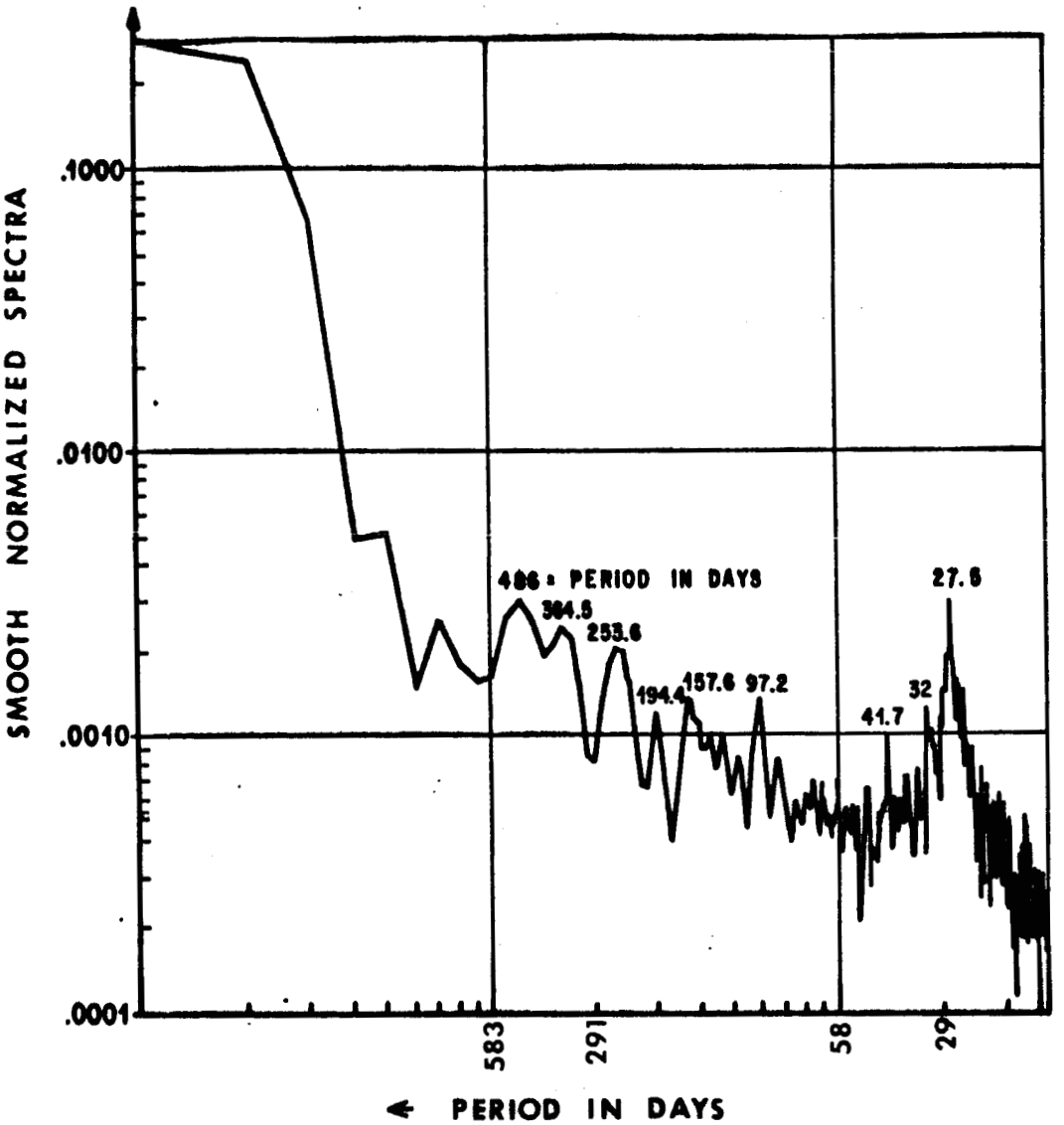

Figure 4 Smooth Normalized Power Spectra of Daily Zurich Sunspot Numbers from 1 January 1890 through 8 December 1967 Using 486 Lags with $\Delta t = 6$.

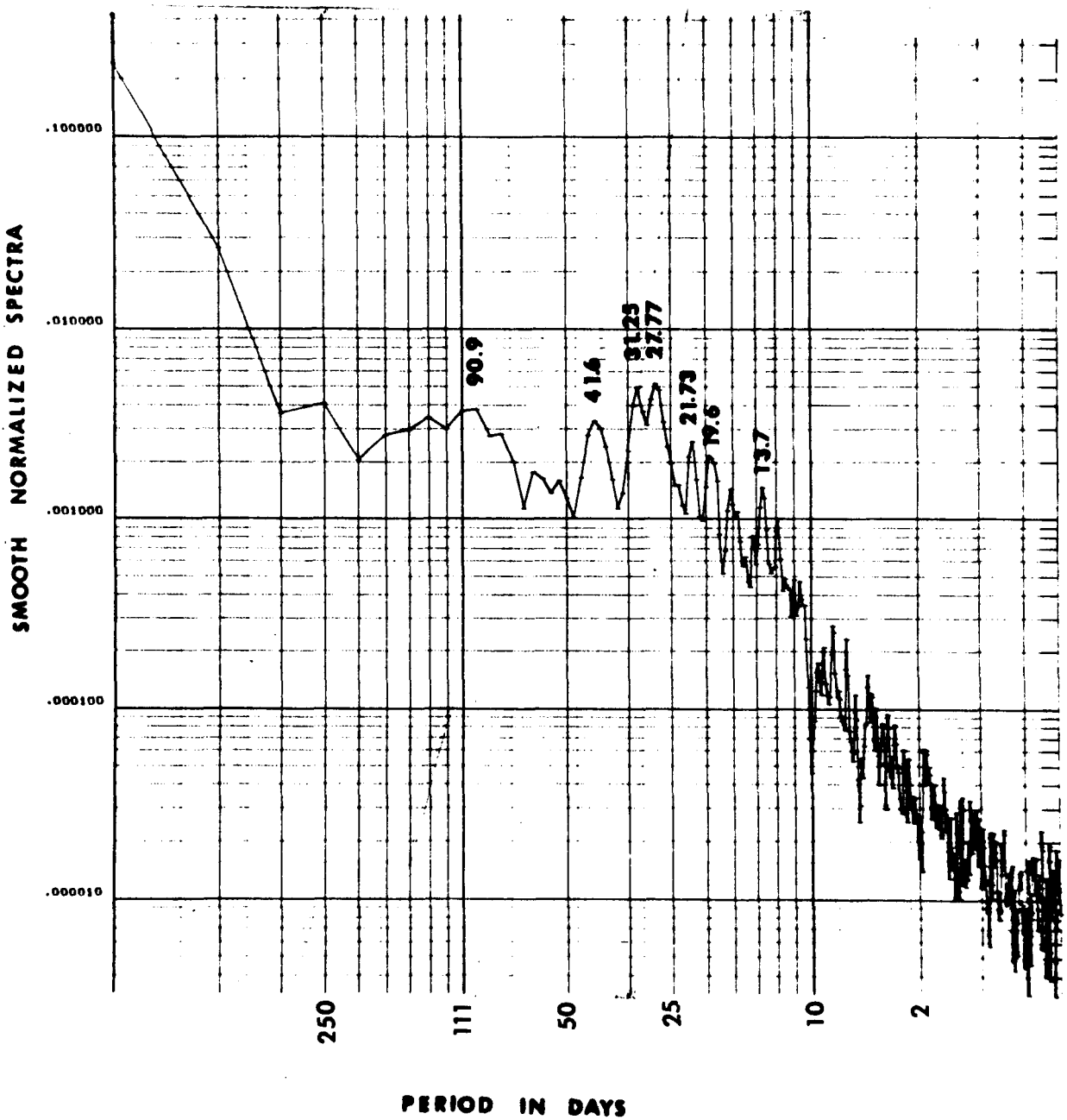

Figure 5a Smooth Normalized Power Spectra of Daily Zurich Sunspot Numbers From 1 March 1954 through 1 March 1966 Using 500 Lags with $\Delta t = 1$.

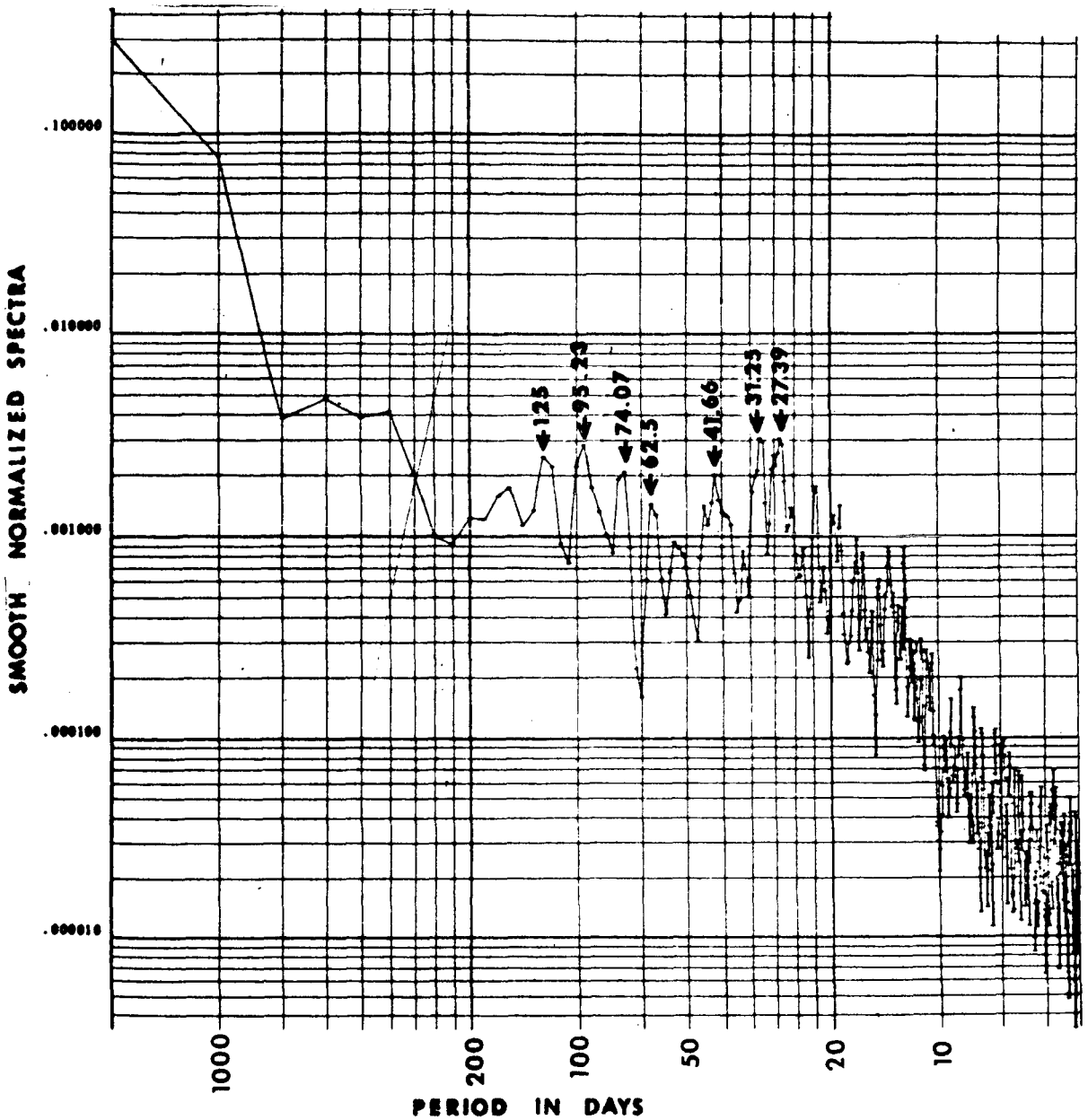

Figure 5b Smooth Normalized Power Spectra of Daily Zurich Sunspot Numbers From 1 March 1954 through 1 March 1966 using 500 Lags with $\Delta t = 2$

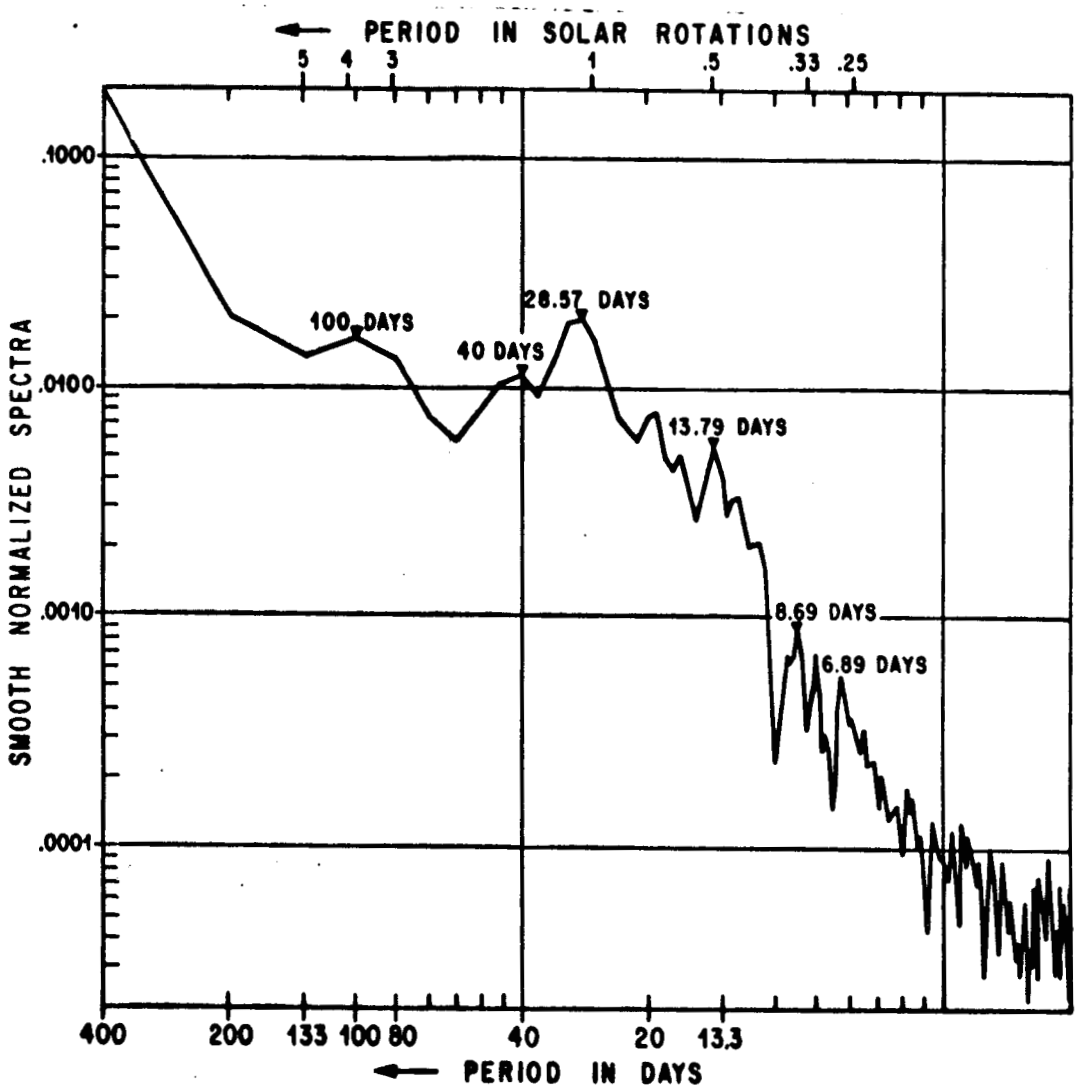

Figure 6 Smooth Normalized Power Spectra of Daily Zurich Sunspot Numbers from 19 Jan 1957 through 11 July 1962 Using 200 Lags

APPROVAL

HIGH RESOLUTION POWER SPECTRA OF
DAILY ZURICH SUNSPOT NUMBERS

By Harold C. Euler

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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