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antenna dimensions
OF SYNTHETIC APERATURE RADAR SYSTEMS ON SATELLITES


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# ANTENNA DIMENSIONS OF SYNTHETIC APERTURE RADAR SYSTEMS ON SATELLITES 

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# ANTENNA DIMENSIONS OF SYNTHETIC APERTURE RADAR SYSTEMS ON SATELLITES 

Kurt R. Richter*


#### Abstract

Design of a synthetic aperture radar (SAR) for a satellite must take into account the limitation in weight and dimensions of the antenna. In this paper the lower limits of the antenna area are derived from the conditions of unambiguity of the SAR system. This result is applied to estimate the antenna requirements for SARs on satellites in circular orbits of various altitudes around Earth and Venus.

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# ANTENNA DIMENSIONS OF SYNTHETIC APERTURE <br> RADAR SYSTEMS ON SATELLITES 

## INTRODUCTION

Within the past decade it has become more and more feasible to study earth resources, geological structures, atmospheric-ocean water cycle, and many other interesting features of the Earth by remote sensing with passive radiometers and active radar (including scatterometers) from either airplanes or satellites.

For mapping purposes, instruments operating in the visible and infrared region of the frequency spectrum offer high resolution at small weight and volume. Infrared measurements, however, are limited to cloudless areas, and besides this condition illumination by the sun is required when the measurements are performed in the visible region. These problems can be overcome by microwave sensors which offer all-weather capability and daylight independent measurements.

Surface mapping of the planet Venus at optical frequencies would fail completely because at these frequencies the Venusian atmosphere is opaque. However, attenuation of microwaves in the $\mathrm{CO}_{2}$-atmosphere of Venus is small enough to make radar mapping from a satellite feasible.

Due to the present constraints by satellites on dimension and weight microwave radiometers and conventional radar systems can not approach the
resolution of optical systems. For higher resolution, the synthetic aperture radar concept must be taken under consideration.

Theoretically, a sidelooking synthetic aperture radar has a lower limit for the azimuthal resolution (parallel to the velocity vector of the spacecraft) which is half of the aperture dimension in the same direction. This means the smaller the azimuthal antenna dimension, the higher the resolution. It turns out that in order to avoid range- and Doppler ambiguity a decrease of the antenna in one dimension requires an increase in the orthogonal direction. For a synthetic aperture radar the product of these two dimensions of the physical antenna has to be larger than a minimum value which depends upon the diameter of the planet, the frequency, the spacecraft altitude, and the nadir angle. RANGE- AND DOPPLER AMBIGUITY

According to the sampling theorem in synthetic aperture radar (SAR) the pulse repetition frequency, PRF, must be at least twice as high as the highest Doppler frequency within the $3-\mathrm{dB}$ beamwidth of the physical antenna. For optical data processing the PRF must be at least four times the highest Doppler frequency, since positive and negative frequencies can not be differentiated.

The geometry considered is shown in Figure 1 for a spacecraft orbiting a planet with radius $R_{0}$. The velocity is $v$, and the altitude above the surface is H. The radar antenna is sidelooking so that the beam axis is orthogonal to the velocity vector, and the angle toward nadir is $\beta_{0}$. The beam axis intersects the surface of the planet at $C$, where the angle of incidence is $a_{0}$. We define two
orthogonal planes, one through the beam axis and the velocity vector, and the other through the beam axis and orthogonal to the velocity vector. The quantities in these two planes will be denoted by the subscripts a (azimuthal), and $\mathbf{r}$ (range), respectively. The $3-\mathrm{dB}$ beamwidths $\gamma_{\mathrm{r}}$ and $\gamma_{\mathrm{a}}$ are small compared to $\frac{\Pi}{2}$ so that their relation to the respective antenna dimensions is given by

$$
\begin{equation*}
\gamma_{r}=a_{r} \frac{\lambda}{D_{r}} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
\gamma_{a}=a_{a} \frac{\lambda}{D_{a}} \tag{2}
\end{equation*}
$$

where $\lambda$ is the wavelength and $a_{r}$ and $a_{a}$ are constants depending upon the particular aperture illumination.

The highest Doppler frequency, $\Delta \mathrm{f}$, originates from point E (Figure 1) and is

$$
\Delta \mathrm{f}=2 \mathrm{f}_{0} \frac{\mathrm{v}}{\mathrm{c}} \sin \frac{\gamma_{\mathrm{a}}}{2} \approx \mathrm{f}_{0} \frac{\mathrm{v}}{\mathrm{c}} \gamma_{\mathrm{a}}
$$

and with (2)

$$
\begin{equation*}
\Delta \mathrm{f}=\mathrm{a}_{\mathrm{a}} \frac{\mathrm{v}}{\mathrm{D}_{\mathrm{a}}} \tag{3}
\end{equation*}
$$

Therefore, the requirement for the PRF is

$$
\begin{equation*}
\operatorname{PRF} \geqslant 2 \mathrm{pa}_{\mathrm{a}} \frac{\mathrm{v}}{\mathrm{D}_{\mathrm{a}}}, \mathrm{p}=1,2 \tag{4}
\end{equation*}
$$

where $p=1$ applies for electronic processing and $p=2$ for optical processing. To avoid range ambiguity, the maximum slant range difference, $\Delta \mathrm{R}$, for the rays through the points $A$ and $B$, has to be smaller than $c /(2 P R F)$, where $c$ is the
velocity of light. Combining this condition with that of equ. (4) one obtains

$$
\begin{equation*}
\Delta R \leqslant \frac{1}{p a_{a}} \frac{c}{4} \frac{D_{a}}{v} \tag{5}
\end{equation*}
$$

With the exception of the constant factor $a_{a}$, taking into account the aperture illumination, this equation is identical to that given by Skolnik ${ }^{1}$.

MAXIMUM SLANT RANGE DIFFERENCE
Figure 2 shows the geometry for one ray in the range plane. The slant range for such a ray is given by

$$
S_{R i}=R_{0} \frac{\sin \left(a_{i}-\beta_{i}\right)}{\sin \beta_{i}}
$$

with

$$
\begin{equation*}
\sin a_{i}=\frac{R_{0}+H}{R_{0}} \sin \beta_{i} \tag{6}
\end{equation*}
$$

This becomes

$$
\begin{equation*}
\mathrm{S}_{\mathrm{Ri}}=\mathrm{R}_{0}\left(\frac{\mathrm{R}_{0}+\mathrm{H}}{\mathrm{R}_{0}} \cos \beta_{\mathrm{i}}-\cos a_{\mathrm{i}}\right) \tag{7}
\end{equation*}
$$

For the upper beam edge ( $\mathrm{i}=1$ ) the nadir angle $\beta_{1}=\beta_{0}+\frac{\gamma_{r}}{2}$, and for the lower beam edge ( $\mathrm{i}=2$ ), the angle is $\beta_{2}=\beta_{0}-\frac{\gamma_{\mathrm{r}}}{2}$.

The maximum range difference within the $3-\mathrm{dB}$ beamwidth is then

$$
\begin{equation*}
\Delta R=R_{0}\left[\frac{\mathrm{R}_{0}+\mathrm{H}}{\mathrm{R}_{0}}\left(\cos \beta_{1}-\cos \beta_{2}\right)-\left(\cos a_{1}-\cos a_{2}\right)\right] \tag{8}
\end{equation*}
$$

Expressing $\beta_{1}, \beta_{2}, a_{1}$, and $a_{2}$ by the nadir angle $\beta_{0}$, the incidence angle $a_{0}$, and the half power beamwidth $\gamma_{r}$, one obtains

$$
\begin{align*}
\Delta \mathrm{R}=\mathrm{R}_{0}\left[-\gamma_{\mathrm{r}} \sin a_{0}\right. & -\sqrt{1-\left(\frac{\mathrm{R}_{0}+\mathrm{H}}{\mathrm{R}_{0}}\right)^{2}\left(\sin \beta_{0}+\frac{\gamma_{\mathrm{r}}}{2} \cos \beta_{0}\right)^{2}} \\
& \left.+\sqrt{1-\left(\frac{\mathrm{R}_{0}+\mathrm{H}}{\mathrm{R}_{0}}\right)^{2}\left(\sin \beta_{0}-\frac{\gamma_{\mathrm{r}}}{2} \cos \beta_{0}\right)^{2}}\right] \tag{9}
\end{align*}
$$

If the following conditions

$$
\begin{equation*}
\frac{\gamma_{I}}{2} \ll \tan \beta_{0} \tag{10}
\end{equation*}
$$

and

$$
\begin{equation*}
\gamma_{\mathrm{I}} \ll \frac{\tan \beta_{0}}{\tan ^{2} a_{0}} \tag{11}
\end{equation*}
$$

hold, (9) can be simplified to

$$
\begin{equation*}
\Delta \mathrm{R}=\gamma_{\mathrm{r}} \mathrm{R}_{0} \sin a_{0}\left(\frac{\tan a_{0}}{\tan \beta_{0}}-1\right) \tag{12}
\end{equation*}
$$

MINIMUM ANTENNA AREA
Using (5), (12), and (1), we obtain

$$
\begin{equation*}
\mathrm{D}_{\mathrm{r}} \mathrm{D}_{\mathrm{a}} \geqslant 4 \mathrm{p} \mathrm{a}_{\mathrm{r}} \mathrm{a}_{\mathrm{a}} \frac{\mathrm{v}}{\mathrm{f}_{0}} \quad \mathrm{R}_{0} \sin a_{0}\left(\frac{\tan a_{0}}{\tan \beta_{0}}-1\right) . \tag{13}
\end{equation*}
$$

For a satellite in a circular orbit the velocity is

$$
v=\sqrt{\frac{\mu}{R_{0}+H}} .
$$

Substituting in (13), we finally obtain

$$
\begin{equation*}
\dot{\mathrm{D}}_{\mathrm{r}} \mathrm{D}_{\mathrm{a}} \geqslant 4 \mathrm{p} \mathrm{a}_{\mathrm{r}} \mathrm{a}_{\mathrm{a}} \frac{\sqrt{\mu}}{\mathrm{f}_{0}} \sqrt{\mathrm{R}_{0}+\mathrm{H}} \sin \beta_{0}\left(\frac{\tan a_{0}}{\tan \beta_{0}}-1\right) \tag{14}
\end{equation*}
$$

The product $D_{r} D_{a}$ is a characteristic area proportional to the physical antenna area and some measure for the minimum requirements can be obtained from this equation.

Figure 3 shows the results for Earth ( $\mathrm{R}_{0}=6370 \mathrm{~km}, \mu=3.9910^{5} \mathrm{~km}^{3} \mathrm{~s}^{-2}$ ) for an SAR in a 500 km and 1000 km orbit for different nadir angles. Figure 4 is for an SAR in a 500 km orbit around Venus ( $\left.\mathrm{R}_{0}=6050 \mathrm{~km}, \mu=3.2110^{5} \mathrm{~km}^{3} \mathrm{~s}^{-2}\right)$. For these values it must be kept in mind that the results are from (14) which holds only as long as the conditions in (10) and (11) are maintained. These conditions can be expressed by the requirement of a minimum ratio $D_{r} / \lambda$ when $\gamma_{r}$ is substituted by (1) and assuming that 0.1 is small enough compared to 1 . Then the conditions write as

$$
\begin{equation*}
\frac{\mathrm{D}_{\mathrm{r}}}{\lambda} \geqslant \frac{5 \mathrm{a}_{\mathrm{r}}}{\tan \beta_{0}} \tag{15}
\end{equation*}
$$

and/or

$$
\begin{equation*}
\frac{D_{r}}{\lambda} \geqslant 10 \frac{a_{r} \tan ^{2} a_{0}}{\tan \beta_{0}} \tag{16}
\end{equation*}
$$

Figure 5 shows the minimum ratio $D_{r} /\left(a_{r} \lambda\right)$ as function of the angle of incidence on the surface for 500 km and 1000 km altitude. The curves can be used for both Earth and Venus, since the difference of the radii of the planets is only $5 \%$ resulting in a negligible deviation.

## CONCLUSION

Equation (14) was derived neglecting the refraction of the radar beam in the planetary atmosphere. However, if the incidence angle is not too large (14) is still a good approximation for the required minimum antenna area.

If there is any particular reason that $D_{r}$ must be chosen so small that one of the conditions in (15) and (16) is violated $D_{a}$ must be calculated from (5) using the exact formula for the slant range difference in (8). For most applications, however, the required antenna area can be determined from Figures 3 and 4.

These figures also show reasonable antenna dimensions for about $30^{\circ}$ incidence angle, so that synthetic aperture radars are feasible even on smaller satellites. This is of importance for the purpose of mapping Venus to provide information on the appearance of the surface of Venus.

## REFERENCE

1. Skolnik, M. J., (1970), Radar Handbook, McGraw-Hill Book Co., Chapter 23, p. 23-25.


Figure 1. Footprint of a Satellite Antenna


Figure 2. Geometry of a Ray in The Range Plane


Figure 3. Minimum Antenna Area Versus Frequency for an SAR at 500 km (broken lines) and 1000 km (solid lines) Above Earth Surface


Figure 4. Minimum Antenna Area Versus Frequency for an SAR at 500 km Above Venus Surface


Figure 5. Minimum Antenna Dimension $D_{r}$ Required Versus Angle of Incidence For 500 km and 1000 km Orbits

