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ANTENNA DIMENSIONS OF SYNTHETIC APERTURE

RADAR SYSTEMS ON SATELLITES

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ABSTRACT

Design of a synthetic aperture radar (SAR) for a satellite must take into account the limitation in weight and dimensions of the antenna. In this paper the lower limits of the antenna area are derived from the conditions of unambiguity of the SAR system. This result is applied to estimate the antenna requirements for SARs on satellites in circular orbits of various altitudes around Earth and Venus.

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INTRODUCTION

Within the past decade it has become more and more feasible to study earth resources, geological structures, atmospheric-ocean water cycle, and many other interesting features of the Earth by remote sensing with passive radiometers and active radar (including scatterometers) from either airplanes or satellites.

For mapping purposes, instruments operating in the visible and infrared region of the frequency spectrum offer high resolution at small weight and volume. Infrared measurements, however, are limited to cloudless areas, and besides this condition illumination by the sun is required when the measurements are performed in the visible region. These problems can be overcome by microwave sensors which offer all-weather capability and daylight independent measurements.

Surface mapping of the planet Venus at optical frequencies would fail completely because at these frequencies the Venusian atmosphere is opaque. However, attenuation of microwaves in the CO_2 -atmosphere of Venus is small enough to make radar mapping from a satellite feasible.

Due to the present constraints by satellites on dimension and weight microwave radiometers and conventional radar systems can not approach the

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resolution of optical systems. For higher resolution, the synthetic aperture radar concept must be taken under consideration.

Theoretically, a sidelooking synthetic aperture radar has a lower limit for the azimuthal resolution (parallel to the velocity vector of the spacecraft) which is half of the aperture dimension in the same direction. This means the smaller the azimuthal antenna dimension, the higher the resolution. It turns out that in order to avoid range- and Doppler ambiguity a decrease of the antenna in one dimension requires an increase in the orthogonal direction. For a synthetic aperture radar the product of these two dimensions of the physical antenna has to be larger than a minimum value which depends upon the diameter of the planet, the frequency, the spacecraft altitude, and the nadir angle.

RANGE- AND DOPPLER AMBIGUITY

According to the sampling theorem in synthetic aperture radar (SAR) the pulse repetition frequency, PRF, must be at least twice as high as the highest Doppler frequency within the 3-dB beamwidth of the physical antenna. For optical data processing the PRF must be at least four times the highest Doppler frequency, since positive and negative frequencies can not be differentiated.

The geometry considered is shown in Figure 1 for a spacecraft orbiting a planet with radius R_0 . The velocity is v, and the altitude above the surface is H. The radar antenna is sidelooking so that the beam axis is orthogonal to the velocity vector, and the angle toward nadir is β_0 . The beam axis intersects the surface of the planet at C, where the angle of incidence is a_0 . We define two

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orthogonal planes, one through the beam axis and the velocity vector, and the other through the beam axis and orthogonal to the velocity vector. The quantities in these two planes will be denoted by the subscripts a (azimuthal), and r (range), respectively. The 3-dB beamwidths γ_r and γ_a are small compared to $\frac{\Pi}{2}$ so that their relation to the respective antenna dimensions is given by

$$\gamma_{\rm r} = a_{\rm r} \frac{\lambda}{D_{\rm r}}$$
 , (1)

and

$$\gamma_a = a_a \frac{\lambda}{D_a}$$
 , (2)

where λ is the wavelength and a_r and a_a are constants depending upon the particular aperture illumination.

The highest Doppler frequency, Δf , originates from point E (Figure 1) and is

$$\Delta f = 2f_0 \frac{v}{c} \sin \frac{\gamma_a}{2} \approx f_0 \frac{v}{c} \gamma_a$$

and with (2)

$$\Delta f = a_a \quad \frac{v}{D_a} \qquad (3)$$

Therefore, the requirement for the PRF is

$$PRF \ge 2 pa_a \frac{v}{D_a}, p = 1, 2$$
(4)

where p=1 applies for electronic processing and p=2 for optical processing. To avoid range ambiguity, the maximum slant range difference, ΔR , for the rays through the points A and B, has to be smaller than c/(2 PRF), where c is the velocity of light. Combining this condition with that of equ. (4) one obtains

$$\Delta R \leq \frac{1}{pa_a} \frac{c}{4} \frac{D_a}{v} \qquad . \tag{5}$$

With the exception of the constant factor a_a , taking into account the aperture illumination, this equation is identical to that given by Skolnik¹.

MAXIMUM SLANT RANGE DIFFERENCE

Figure 2 shows the geometry for one ray in the range plane. The slant range for such a ray is given by

$$S_{Ri} = R_0 - \frac{\sin(a_i - \beta_i)}{\sin \beta_i}$$

with

$$\sin a_{i} = \frac{R_{0} + H}{R_{0}} \sin \beta_{i}. \qquad (6)$$

,

This becomes

$$S_{Ri} = R_0 \left(\frac{R_0 + H}{R_0} \cos \beta_i - \cos a_i \right).$$
 (7)

For the upper beam edge (i=1) the nadir angle $\beta_1 = \beta_0 + \frac{\gamma_1}{2}$, and for the lower beam edge (i=2), the angle is $\beta_2 = \beta_0 - \frac{\gamma_1}{2}$.

The maximum range difference within the 3-dB beamwidth is then

$$\Delta R = R_0 \left[\frac{R_0 + H}{R_0} (\cos \beta_1 - \cos \beta_2) - (\cos a_1 - \cos a_2) \right] .$$
 (8)

Expressing β_1 , β_2 , a_1 , and a_2 by the nadir angle β_0 , the incidence angle a_0 , and the half power beamwidth γ_r , one obtains

$$\Delta R = R_0 \left[-\gamma_r \sin a_0 - \sqrt{1 - \left(\frac{R_0 + H}{R_0}\right)^2 \left(\sin \beta_0 + \frac{\gamma_r}{2} \cos \beta_0\right)^2} + \sqrt{1 - \left(\frac{R_0 + H}{R_0}\right)^2 \left(\sin \beta_0 - \frac{\gamma_r}{2} \cos \beta_0\right)^2} \right].$$
(9)

If the following conditions

$$\frac{\gamma_{\rm r}}{2} \ll \tan \beta_0 \quad , \tag{10}$$

and

$$\gamma_{\rm r} \ll \frac{\tan \beta_0}{\tan^2 a_0} \tag{11}$$

hold, (9) can be simplified to

$$\Delta R = \gamma_r R_0 \sin a_0 \left(\frac{\tan a_0}{\tan \beta_0} - 1 \right) .$$
 (12)

MINIMUM ANTENNA AREA

Using (5), (12), and (1), we obtain

$$D_{r} D_{a} \ge 4 p a_{r} a_{a} \frac{v}{f_{0}} R_{0} \sin a_{0} \left(\frac{\tan a_{0}}{\tan \beta_{0}} - 1 \right).$$
(13)

For a satellite in a circular orbit the velocity is

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$$v = \sqrt{\frac{\mu}{R_0 + H}}$$

Substituting in (13), we finally obtain

$$D_r D_a \ge 4 p a_r a_a \frac{\sqrt{\mu}}{f_0} \sqrt{R_0 + H} \sin \beta_0 \left(\frac{\tan a_0}{\tan \beta_0} - 1\right) .$$
 (14)

The product $D_r D_a$ is a characteristic area proportional to the physical antenna area and some measure for the minimum requirements can be obtained from this equation.

Figure 3 shows the results for Earth ($R_0 = 6370 \text{ km}, \mu = 3.99 \ 10^5 \text{ km}^3 \text{ s}^{-2}$) for an SAR in a 500 km and 1000 km orbit for different nadir angles. Figure 4 is for an SAR in a 500 km orbit around Venus ($R_0 = 6050 \text{ km}, \mu = 3.21 \ 10^5 \text{ km}^3 \text{ s}^{-2}$). For these values it must be kept in mind that the results are from (14) which holds only as long as the conditions in (10) and (11) are maintained. These conditions can be expressed by the requirement of a minimum ratio D_r / λ when γ_r is substituted by (1) and assuming that 0.1 is small enough compared to 1. Then the conditions write as

$$\frac{D_{r}}{\lambda} \geq \frac{5a_{r}}{\tan \beta_{0}} , \qquad (15)$$

and/or

$$\frac{D_r}{\lambda} \ge 10 \frac{a_r \tan^2 a_0}{\tan \beta_0}$$
(16)

Figure 5 shows the minimum ratio $D_r/(a_r \lambda)$ as function of the angle of incidence on the surface for 500 km and 1000 km altitude. The curves can be used for both Earth and Venus, since the difference of the radii of the planets is only 5% resulting in a negligible deviation.

CONCLUSION

Equation (14) was derived neglecting the refraction of the radar beam in the planetary atmosphere. However, if the incidence angle is not too large (14) is still a good approximation for the required minimum antenna area. If there is any particular reason that D_r must be chosen so small that one of the conditions in (15) and (16) is violated D_a must be calculated from (5) using the exact formula for the slant range difference in (8). For most applications, however, the required antenna area can be determined from Figures 3 and 4.

These figures also show reasonable antenna dimensions for about 30° incidence angle, so that synthetic aperture radars are feasible even on smaller satellites. This is of importance for the purpose of mapping Venus to provide information on the appearance of the surface of Venus.

REFERENCE

 Skolnik, M. J., (1970), Radar Handbook, McGraw-Hill Book Co., Chapter 23, p. 23-25.



Figure 1. Footprint of a Satellite Antenna



Figure 2. Geometry of a Ray in The Range Plane

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Figure 3. Minimum Antenna Area Versus Frequency for an SAR at 500 km (broken lines) and 1000 km (solid lines) Above Earth Surface



Figure 4. Minimum Antenna Area Versus Frequency for an SAR at 500 km Above Venus Surface



Figure 5. Minimum Antenna Dimension D_r Required Versus Angle of Incidence For 500 km and 1000 km Orbits