X-621-72-496
PREPRINT

NASA TM X 66158
66157

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ABSTRACT

The electron temperatures which would be determined (using the conventional single-temperature analysis) by the electrostatic probe, the diffuse resonance, and the radar backscatter techniques in an isotropic two temperature plasma are presented. Plasma models corresponding to the addition of a minor component of energetic electrons, and models corresponding to a process that cools a fraction of the ionospheric electrons are considered. The diffuse resonance temperature is found to lie between the probe and radar backscatter temperatures. The isotropic models corresponding to the addition of energetic electrons cannot support the reported discrepancies between radio wave and probe electron temperature measurements. Temperature differences similar to the observed differences can be produced by models with a fraction of the electrons at a temperature cooler than that of the main component of electrons. These models, however, are difficult to explain in terms of our present understanding of the ionospheric plasma.

INTRODUCTION

A discrepancy has been observed between the electron temperature deduced from in situ electrostatic probe measurements and from ground-based radar backscatter measurements; the probe temperature T_p being greater than the backscatter temperature T_b (Hanson et al., 1969; Carlson and Sayers, 1970). A similar discrepancy has been observed in a comparison of simultaneous in situ Alouette 2 electron temperature measurements based on the rf sounder stimulated plasma resonances and the cylindrical probe experiment (Oya and Benson, 1972). The resonance measurements are based on the diffuse resonance which occurs at the frequency f_{D1}, between f_H and $2f_H$ (Oya, 1970), where f_H is the electron cyclotron frequency; the probe measurements are based on the experiment of Findlay and Brace (1969). Again, the radio wave temperature, i.e., the diffuse resonance electron temperature $T_{f_{D1}}$, is less than the probe temperature T_p.

An assumption inherent in the determination of T_p, T_b, and $T_{f_{D1}}$ is that the electron velocity distribution function $f(v)$ is Maxwellian. The purpose of the present paper is to determine the effect of an isotropic non-Maxwellian velocity distribution on the three temperature measuring techniques. For illustrative purposes we represent the distorted velocity distribution by a simple, isotropic two temperature distribution – a superposition of two Maxwellian distributions. Hoegy (1971) applied this distribution to a study of the probe-radar temperature difference for distortions in $f(v)$ at high energies

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(photoelectron distortion) and at low energies (thermal distortion). It was concluded that significant temperature differences could be produced by a thermal distortion but not by a distortion due to realistic photoelectron populations. This paper represents an extension of the work of Hoegy (1971) to include the diffuse resonance technique.

Basically, we pose the following question: if an isotropic two temperature plasma is present, but the data are interpreted in terms of a one temperature Maxwellian plasma, what temperature will be computed? A description of the method for obtaining T_p and T_b for such a plasma has been given by Hoegy (1971); the procedure for obtaining $T_{f_{D}}$ is given in the Appendix.

ILLUSTRATIVE EXAMPLES

The effect of an isotropic distorted $f(v)$ on the temperatures deduced by the probe, radar, and diffuse resonance techniques is illustrated in Figure 1 for a wide range of conditions. In all cases the bulk of the electrons are considered to be at $3,000°K$ and the perturbing electron component extends over a range of temperatures above (top row) and below (bottom row) this value. By convention, T_1 denotes the temperature of the cooler electron component regardless of its role as the major or minor constituent. The three columns of figures from left to right in Figure 1 represent perturbing electron populations of 10%, 20%, and 30%, respectively.

The following trends are evident from Figure 1:

(1) $T_{f_{D}}$ is between T_p and T_b in all cases.

(2) A larger perturbation in the probe temperature can be obtained when a fraction of the electrons are warmer than the main component rather than cooler than the main component; the converse is true in the case of the diffuse resonance and radar backscatter temperatures (compare Figure 1a with 1d, 1b with 1e, or 1c with 1f).

(3) All three techniques give very similar results when warmer electrons (at T_2) are present up to $T_2 \approx 10,000°K$; when electrons with $T_2 \gtrsim 10,000°K$ are present the radio wave temperatures ($T_{f_{D}}$ and T_b) are greater than the probe temperature (see Figures 1a, 1b, and 2c).

(4) The radio wave temperatures are always less than the probe temperatures ($T_b < T_{f_{D}} < T_p$) when cooler electrons are present (see Figures 1d, 1e, and 1f).

The following comments should be kept in mind when interpreting Figure 1. (1) The actual temperature values for T_p and $T_{f_{D}}$ depend on the simulated experimental conditions - the voltage interval for the probe and the ambient plasma conditions for the diffuse resonance. (2) There is a double-valued behavior of T_p as a function of the perturbing temperature because the perturbing current can be recognized and rejected in the temperature reduction process when the perturbing temperature differs by a large amount from the temperature of the

major component of electrons. (3) The peak observed in $T_{(D)}$ in the top row of Figure 1 corresponds to the conditions giving the maximum distortion in the two temperature dispersion curves (see Figure A1 of the Appendix).

DISCUSSION

Hanson et al. (1969) observed T_p to be as much as 70% greater than T_b, and Oya and Benson (1971) found T_p to be typically 20% to 30% greater than $T_{(D)}$. It is clear that these discrepancies cannot be explained in terms of an isotropic plasma with a component of hot electrons present (see comment 3 of the previous section).

If an isotropic plasma exists with a fraction of the electrons at a temperature cooler than the main component, then it is possible to produce a temperature difference resembling the observed discrepancies (see comment 4 of the previous section). For example, consider the case of a 10% perturbation (Figure 1d) with $T_1 = 1,200°K$; the two temperature model then gives the following values: $T_p = 2930°K$, $T_{(D)} = 2740°K$, and $T_b = 2610°K$. The calculated ratios for $T_p/T_{(D)}$ and T_p/T_b (1.07 and 1.12, respectively) are smaller than the observed ratios, but larger values can be obtained by using a lower T_1 and/or a higher minor constituent population (see Figures 1d, 1e, and 1f).

In order to maintain such large distortions of the low energy electron population, there must exist heating and cooling processes with rates comparable in magnitude to the electron-electron energy exchange rate. Since this time is three orders of magnitude smaller than the electron-ion or electron-neutral

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energy exchange time, the main effect of Coulomb collisions with cooler particles is to modify the average energy of the electrons rather than to modify their Maxwellian velocity distribution. Other processes, however, may be important enough to modify this picture-such as non-local processes or wave-particle interactions which could enhance the selective heating and cooling mechanisms at the expense of the electron-electron equilibration. This possibility merits future investigation.

The present work has treated the isotropic case. Recent observations by Clark et al. (1973) indicate that significant anisotropies, with the electron temperature parallel to the earth's magnetic field vector $\vec{B}$ greater than the temperature perpendicular to $\vec{B}$, are common in the upper ionosphere. Such anisotropies could contribute to the observed discrepancy between radio wave and probe measurements (for example, $T_{f_{D_1}}$ corresponds to electron motion perpendicular to $\vec{B}$, whereas T_p is not very sensitive to direction). Thus, in addition to considering mechanisms which could produce a distortion in the isotropic distribution of electron velocities, future work should consider the anisotropic case.

APPENDIX

$T_{f_{D_1}}$ IN A TWO TEMPERATURE PLASMA

In order to determine $T_{f_{D_1}}$ in a two temperature plasma it is necessary to derive the two temperature dispersion relation for plasma waves propagating perpendicular to the earth's magnetic field vector $\vec{B}$. (The diffuse resonance

experiment is interpreted in terms of perpendicular propagation (Oya and

Benson, 1972).)

The dispersion relation for longitudinal plasma waves with the wave vector

$\vec{k}$ perpendicular to $\vec{B}$ is obtained by setting the expression for the dielectric

constant $(\epsilon_{long})_\perp$ equal to zero, ie.,

$$0 = \left(\epsilon_{long}\right)_\perp = 1 + \frac{4\pi e^2}{mk^2} \int_0^\infty 2\pi v_\perp \; dv_\perp \sum_{n=-\infty}^{+\infty} \frac{\frac{n\Omega}{v_\perp} \frac{\partial F(v_\perp)}{\partial v_\perp} J_n^2\left(\frac{kv_\perp}{\Omega}\right)}{\omega - n\Omega}$$ (A1)

where

$$F(v_\perp) = \int_{-\infty}^{+\infty} dv_\parallel \quad f(v)$$

is the distribution function for electron motion perpendicular to $\vec{B}$, e and m are

the electron charge and mass, $v_\parallel$ and $v_\perp$ are the components of electron velocity

$\vec{v}$ parallel and perpendicular to $\vec{B}$, $\omega = 2\pi f$, $\Omega = 2\pi f_H$, $k = |\vec{k}|$, f (v) is the electron

velocity distribution function, and J_n is the Bessel function of order n (e.g.,

see the review by Dougherty and Watson (1971)). Inserting the isotropic two-

temperature velocity distribution function

$$f(v) = (1 - p) \; N \; \left(\frac{m}{2\pi\kappa T_1}\right)^{3/2} \exp \left(-\frac{mv^2}{2\kappa T_1}\right)$$

$$+ \; pN \; \left(\frac{m}{2\pi\kappa T_2}\right)^{3/2} \exp \left(-\frac{mv^2}{2\kappa T_2}\right)$$ (A2)

into A1 (where p represents the population of electrons at T_2) gives the two

temperature dispersion relation

$$\frac{1}{2\left(\frac{\omega_N}{\Omega}\right)^2} = \frac{(1-p)}{\lambda_1} \sum_{n=1}^{\infty} \frac{e^{-\lambda_1} I_n(\lambda_1)}{\left(\frac{\omega}{n\Omega}\right)^2 - 1} + \frac{p}{\lambda_2} \sum_{n=1}^{\infty} \frac{e^{-\lambda_2} I_n(\lambda_2)}{\left(\frac{\omega}{n\Omega}\right)^2 - 1}$$

$$\omega_N^2 = 4\pi N e^2/m \qquad\qquad\qquad \text{(A3)}$$

$$\lambda_1 = \frac{\kappa T_1 k^2}{m\Omega^2}$$

$$\lambda_2 = \frac{\kappa T_2 k^2}{m\Omega^2}$$

The standard single-temperature dispersion relation is obtained by setting

p=0 or 1 or by setting $T_1 = T_2$ in (A3). Note: When very energetic electrons

are considered, large λ_2 values are encountered and the second summation

term in (A3) converges very slowly. A much faster convergence is obtained by

making use of the identity

$$\sum_{n=-\infty}^{+\infty} e^{-\lambda} I_n(\lambda) = 1$$

in order to make the following replacement

$$\sum_{n=1}^{\infty} \frac{e^{-\lambda} I_n(\lambda)}{\left(\frac{\omega}{n\Omega}\right)^2 - 1} = \sum_{n=1}^{\infty} \frac{e^{-\lambda} I_n(\lambda)}{1 - \left(\frac{n\Omega}{\omega}\right)^2} + \frac{e^{-\lambda} I_0(\lambda) - 1}{2} \quad .$$

The dispersion curves based on (A3) are presented in Figure A1 in the

form of the frequency f, normalized by the electron cyclotron frequency f_H,

versus the wave number k normalized by 1/R, where R represents the electron

cyclotron radius. In Figures 1a-1c, R is expressed as $R = R(T_1) = \left(\kappa T_1/m(2\pi f_H)^2\right)^{1/2}$

where T_1 is the temperature of the major component; in Figures 1d-1f it is expressed as $R = R(T_2) = \left(\kappa T_2 / m(2\pi f_H)^2\right)^{1/2}$ where T_2 is the temperature of the major component. The first case (Figures 1(a)-1(c)) corresponds to the addition of energetic photoelectrons, while the second case (Figures 1(d)-1(f)) corresponds to a process that cools the ionospheric electrons. The solid line in each figure corresponds to the single temperature dispersion relation that was used by Oya and Benson (1972) in the determination of $T_{f_{D1}}$.

The procedure for determining $T_{f_{D1}}$ from a single-temperature dispersion relation in a two temperature plasma is as follows: Select the two temperature parameters p, T_1, and T_2, and the ionospheric parameters ω_N, Ω, k, that correspond to the conditions of the observations. Then determine the normalized diffuse resonance frequency from the two temperature dispersion relation (A3). Finally, use the standard single temperature dispersion relation to obtain $\lambda = (kR)^2$ and hence $T_{f_{D1}} = \lambda m \Omega^2 / \kappa k^2$.

ACKNOWLEDGEMENTS

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Figure 1. The electron temperature T_p, T_b, $T_{(D)}$ deduced from a single temperature analysis using a two temperature plasma for the electrostatic probe, the radar backscatter, and the diffuse resonance techniques, respectively. T versus T_2 with $T_2 > T_1 = 3,000\,°K$ in the top row, and T versus T_1 with $T_1 < T_2 = 3,000°K$ in the bottom row. The left column corresponds to the condition when 10% of the electrons deviate from the model temperature of 3,000°K, the center column corresponds to a 20% perturbation, and the right column to a 30% perturbation. Note the abscissa scale change between the top and bottom rows. In each column, the bottom figure represents an enlarged extension of the top figure to the temperature range below 3,000°K. The T_p values correspond to the 0.3 to 1.3v interval of the probe current-voltage curve; the $T_{(D)}$ values correspond to the plasma conditions given in the caption of Figure A1.

Figure A1. Dispersion curves based on the two-temperature dispersion equation (A3) for plasma models with $T_2 > T_1 = 3,000\,°K$ (a), (b), and (c); and for models with $T_1 < T_2 = 3,000°K$ (d), (e), and (f). The parameter p represents the fraction of electrons at T_2. The solid curves (p=0 and 1) correspond to the single temperature dispersion curve. The plasma conditions used in these models correspond to a typical observation from the Ottawa data sample of Oya and Benson (1972) ($f_N = \omega_N/2\pi = 3.475$ MHz and $f_H = 1.178$ MHz so that $f_N/f_H = 2.95$; the corresponding value for kR when the temperature is taken as 3,000°K is 0.97).
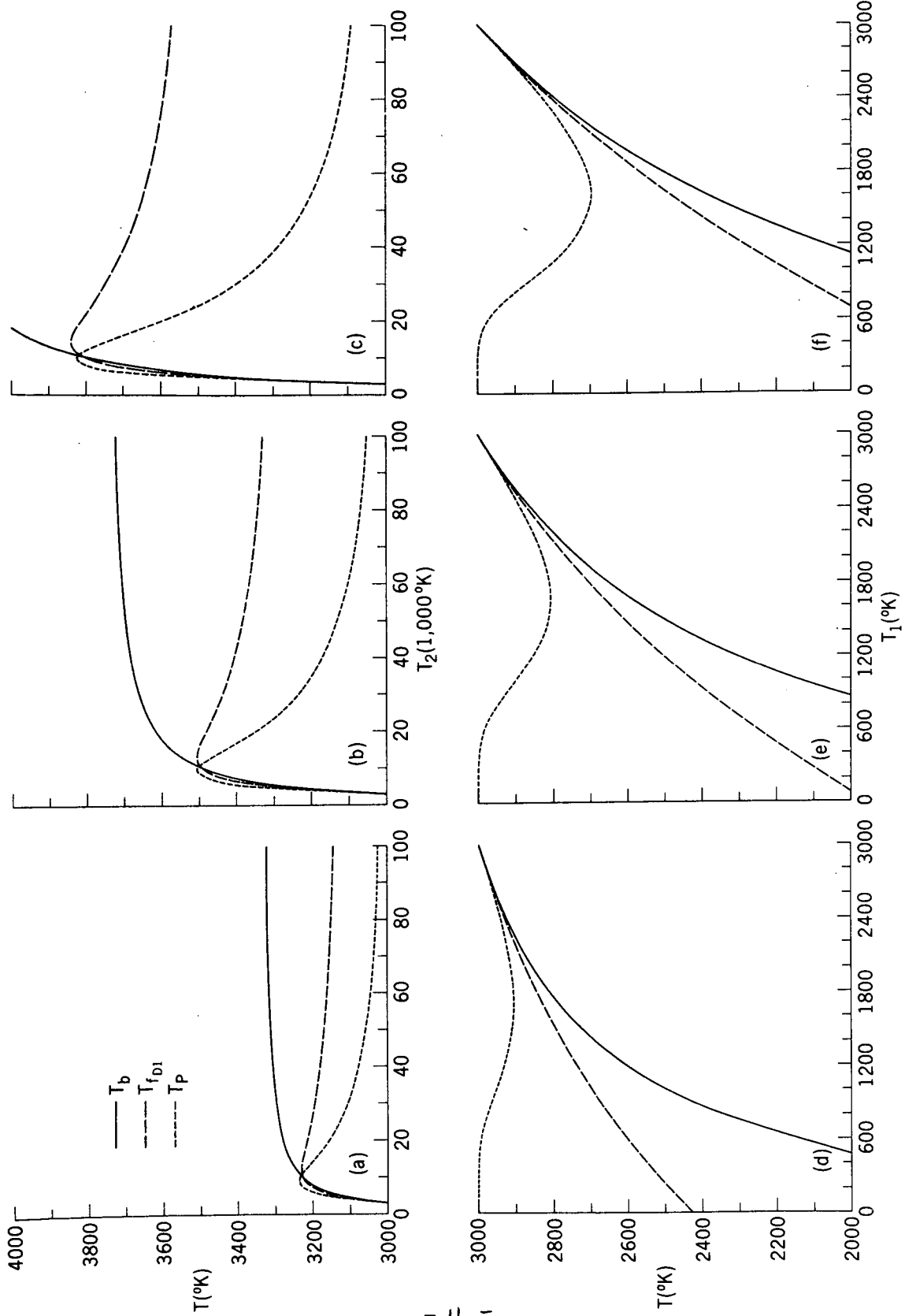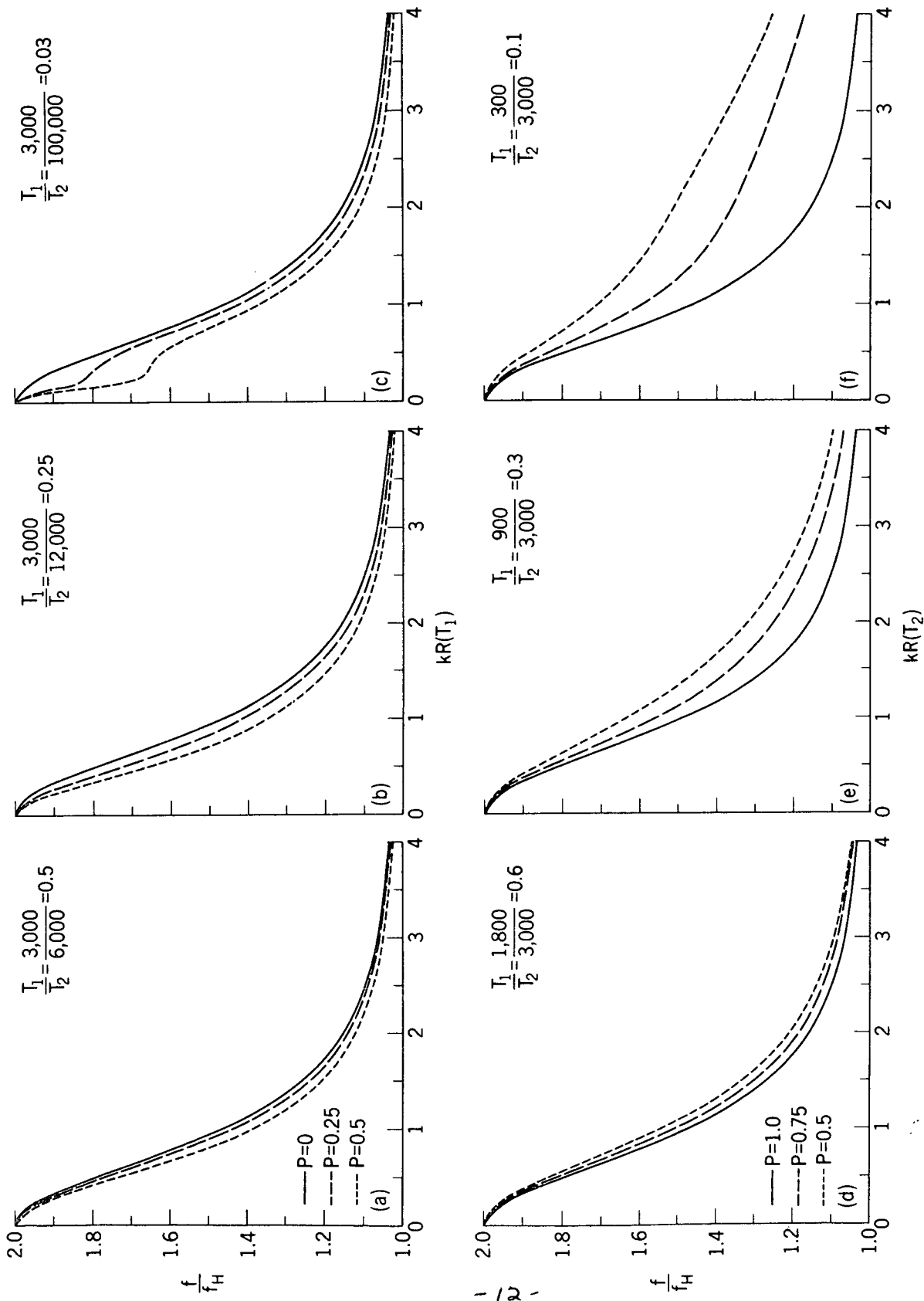
Figure 1

Figure A1

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