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OPERATIONAL FREQUENCY STABILITY
OF RUBIDIUM AND CESIUM
FREQUENCY STANDARDS

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16. Abstract The frequency stabilities under operational conditions of several commercially available rubidium and cesium frequency standards were determined from experimental data for frequency averaging times from 10 to 10^7 s and are presented in table and graph form. For frequency averaging times between 10^5 and 10^7 s, the rubidium standards tested have a stability of between 10^{-12} and 5×10^{-12} , while the cesium standards have a stability of between 2×10^{-13} and 5×10^{-13} .			
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John E. Lavery
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INTRODUCTION

In the course of testing various rubidium and cesium frequency standards under operational conditions for use in NASA tracking stations, about 55 unit-years of relative frequency measurements for averaging times from 10 to 10^7 s have been accumulated at Goddard Space Flight Center (GSFC). Statistics on the behavior of rubidium and cesium standards under controlled laboratory conditions have been published by many institutions (see, for example, ref. 1), but it was not known to what extent the lesser controlled environments of NASA tracking stations affected the performance of the standards. The purpose of this report is to present estimates of the frequency stability of rubidium and cesium frequency standards under operational conditions based on the data accumulated at GSFC.

Table 1.—Atomic Frequency Standards Used
in Experiments

Serial no. or designation	Manufacturer
Rb 107	Varian Associates
Rb 136	Varian Associates
Rb 138	Varian Associates
Cs 110	Hewlett-Packard Co.
Cs 136	Hewlett-Packard Co.
Cs 137	Hewlett-Packard Co.
Cs 138	Hewlett-Packard Co.
Cs 139	Hewlett-Packard Co.
Cs 152	Hewlett-Packard Co.
Cs 182	Hewlett-Packard Co.
Cs 185	Hewlett-Packard Co.
Cs 186	Hewlett-Packard Co.
HM:	
H-10 no. 2	Varian Associates
NX-1	(a)

^aAn experimental hydrogen maser developed at GSFC. See ref. 2.

DATA DESCRIPTION

The three rubidium gas cells (designated Rb) and nine cesium beam frequency standards (Cs) on which the measurements were made, as well as the two hydrogen masers (HM) used as references for many of the tests, are listed in table 1 along with their serial numbers or designations and their manufacturers. During the tests the standards were kept in a laboratory at GSFC. Except for the shielding built into the standards themselves, there was no special control of the ambient magnetic, electric, vibration, and temperature conditions. The ambient magnetic and electric conditions were typically noisy. The standards were driven by ac power and were in no way isolated by transformers. Vibration from nearby air conditioning equipment and from trucks at a nearby loading platform was not shielded in any way. The ambient temperature was typically between 298 and 303 K. There

were, however, several brief excursions to temperatures as low as 291 K and as high as 313 K due to equipment problems. These conditions are less controlled than those in the NASA tracking stations. Hence the stabilities of the standards when operating in the tracking stations should be at least as good as the stabilities calculated in this paper.

The measurements made on the standards consisted of average relative frequency measurements for varying averaging times. In some of the data sets, average relative frequency measurements were missing or were bad because of ac power failure or recorder failure. All such points were a posteriori linearly interpolated from the nearest earlier (in epoch time) good average relative frequency measurement and the nearest later (in epoch time) good average relative frequency measurement.

The total number of measurements made for all types of data used in this report are given in table 2. Data sets are said to be of the same "type" when the following parameters are the same for

Table 2.—Average Relative Frequency Data Sets

Type of data				Number of data sets m	Number of measurements	
Test unit	Reference unit	Averaging time τ_0, s	Dead time d, s		Total ^a	Interpolated
Rb	Rb	3 600	0.0	2	3 090	0
Rb	Cs (10-s TC)	10	2.3	1	1 076	18
Rb	Cs (10-s TC)	100	2.2	1	538	1
Rb	Cs (10-s TC)	1 000	2.7	1	223	0
Rb	HM	10	2.3	13	8 473	67
Rb	HM	100	2.2	10	6 405	16
Rb	HM	1 000	2.7	9	5 126	15
Rb	HM	3 600	.0	7	13 320	308
Cs	Cs	3 600	.0	3	8 851	263
Cs (10-s TC)	HM	10	.2	8	4 841	0
Cs (10-s TC)	HM	100	.2	8	4 871	11
Cs (10-s TC)	HM	1 000	.2	8	4 787	25
Cs (60-s TC)	HM	10	.2	8	4 634	0
Cs (60-s TC)	HM	10	2.3	3	1 904	2
Cs (60-s TC)	HM	100	.2	8	4 706	0
Cs (60-s TC)	HM	100	2.2	3	2 496	0
Cs (60-s TC)	HM	1 000	.2	8	4 804	3
Cs (60-s TC)	HM	1 000	2.7	1	692	0
Cs	HM	3 600	.0	13	37 404	1391
Cs	HM	604 800	.0	1	88	6

TC = time constant.

^aTotal number of measurements for all m data sets, including the interpolated measurements.

each set: test unit,¹ reference unit, duration or averaging time τ_0 of each average relative frequency measurement, and dead time d between successive measurements (that is, the time during which no measurement was taken). The servo time constants are indicated only for the cesium standards and only when $\tau_0 \leq 1000$ s. The difference in effect of a 10- and 60-s time constant for $\tau_0 \geq 3600$ s can be neglected because the time constants in such cases are too small with respect to τ_0 to have an appreciable effect. The rubidium standards tested all have a fixed servo time constant which is on the order of 1 ms.

Neither temperature effects nor long-term frequency drift was removed from the data before analysis because the object of the tests was to measure the stability of the frequency standards under operational conditions, where both temperature fluctuations and long-term frequency drift are present.

STATISTICAL ANALYSIS

Let there be given a set of m identical test frequency standards and a set of m identical reference frequency standards. Let $\phi_n(t)$, $1 \leq n \leq m$, denote the instantaneous fluctuations (measured in time units) of the epoch time output of the n th test standard compared to the epoch time output of the n th reference standard. Let $y_n(t)$ be the instantaneous (fractional) frequency fluctuation of the n th test standard compared with the n th reference standard; i.e.,

$$y_n(t) \equiv \frac{d\phi_n(t)}{dt} \quad (1)$$

Let $\bar{y}_n(t)$ be the average relative (fractional) frequency fluctuation of the n th test standard compared with the n th reference standard:

$$\bar{y}_n(t) = \frac{1}{\tau} \int_t^{t+\tau} y_n(t) dt = \frac{\phi_n(t+\tau) - \phi_n(t)}{\tau} \quad (2)$$

The constant τ is called the averaging time of $\bar{y}(t)$. The Allan standard deviation $\sigma(2, T, \tau)$ of the frequency fluctuations of the set of test standards compared with the set of reference standards is defined to be (ref. 3)

$$\sigma(2, T, \tau) = \sqrt{\frac{1}{m} \sum_{n=1}^m \langle \text{var} [\bar{y}_n(t+T) - \bar{y}_n(t)] \rangle} \quad (3)$$

where the symbol $\langle \rangle$ denotes infinite epoch time average. The analysis of all data listed in table 2 consisted in the calculation of an estimate, which is denoted by $s(2, T, \tau)$, of $\sigma(2, T, \tau)$ in the following manner.

Taking any type of data from table 2, let the number of average relative frequency measurements in the n th data set, $1 \leq n \leq m$, be m_n . Denote this n th set of average relative frequency measurements

¹Although there are sometimes significant differences in the frequency stabilities of various rubidium standards, the three rubidium standards listed in table 1 all had mutually close stabilities. For this reason, these rubidium standards will be considered to be identical. Because the nine cesium standards listed in table 1 all had mutually close stabilities, they too will be considered to be identical.

by $\{\bar{y}_n(i)\}_{i=1}^{m_n}$. For $i = 1, 2, \dots, m_n - 1$, denote the variance of the two average relative frequency measurements $\bar{y}_n(i)$ and $\bar{y}_n(i + 1)$ by $v_n(i)$:

$$v_n(i) = \frac{[\bar{y}_n(i + 1) - \bar{y}_n(i)]^2}{2} \quad (4)$$

The square root of the average over both i ($1 \leq i \leq m_n - 1$) and n ($1 \leq n \leq m$) of these $v_n(i)$ is the desired estimate of $\sigma(2, \tau_0 + d, \tau_0)$:

$$s(2, \tau_0 + d, \tau_0) = \sqrt{\frac{\sum_{n=1}^m \sum_{i=1}^{m_n-1} v_n(i)}{\sum_{n=1}^m (m_n - 1)}} \quad (5)$$

From the original data sets $\{\bar{y}_n\}_{i=1}^{m_n}$, $1 \leq n \leq m$, new data sets with averaging time $\tau_1 = 2\tau_0$ and dead time d (assumed small with respect to τ_0) can be approximated by defining

$$\bar{y}_n(i; 1) = \frac{\bar{y}_n(i + 1) + \bar{y}_n(i)}{2} \quad (6)$$

$i = 1, 2, \dots, m_n - 1$ and $n = 1, 2, \dots, m$. Denote the variance of $\bar{y}_n(i; 1)$ and $\bar{y}_n(i + 2; 1)$ by $v_n(i; 1)$:

$$v_n(i; 1) = \frac{[\bar{y}_n(i + 2; 1) - \bar{y}_n(i; 1)]^2}{2} \quad (7)$$

$i = 1, 2, \dots, m_n - 3$ and $n = 1, 2, \dots, m$. Estimate $\sigma(2, \tau_1 + d, \tau_1)$ by²

$$s(2, \tau_1 + d, \tau_1) = \sqrt{\frac{\sum_{n=1}^m \sum_{i=1}^{m_n-3} v_n(i; 1)}{\sum_{n=1}^m (m_n - 3)}} \quad (8)$$

Let k be the exponent of the largest power of 2 contained in any of the m_n , $1 \leq n \leq m$. For $j = 2, 3, \dots, k - 1$, the data set $\{\bar{y}_n(i; j)\}_{i=1}^{m_n-2^{j-1}+1}$ with averaging time $\tau_j = 2^j \tau_0$ and dead time d is successively calculated from the data set $\{\bar{y}_n(i; j - 1)\}_{i=1}^{m_n-2^{j-1}+1}$ by pairwise averaging:

$$\bar{y}_n(i; j) = \frac{\bar{y}_n(i + 2^{j-1}; j - 1) + \bar{y}_n(i; j - 1)}{2} \quad (9)$$

²Throughout this paper the convention is adopted that whenever a summand, e.g., $m_n - 3$ in $\sum_{n=1}^m (m_n - 3)$, is less than zero, it is treated as zero; and whenever a summation, e.g., $\sum_{i=1}^{m_n-3} v_n(i; 1)$, has an upper limit that is less than the lower limit, it also is treated as zero.

$i = 1, 2, \dots, m_n - 2^j + 1; n = 1, 2, \dots, m; j$ fixed. Denote the variance of $\bar{y}_n(i; j)$ and $\bar{y}_n(i + 2^j; j)$ by $v_n(i; j)$:

$$v_n(i; j) = \frac{[\bar{y}_n(i + 2^j; j) - \bar{y}_n(i; j)]^2}{2} \quad (10)$$

$i = 1, 2, \dots, m_n - 2^{j+1} + 1$ and $n = 1, 2, \dots, m$. Estimate $\sigma(2, \tau_j + d, \tau_j)$ by

$$s(2, \tau_j + d, \tau_j) = \sqrt{\frac{\sum_{n=1}^m \sum_{i=1}^{m_n - 2^{j+1} + 1} v_n(i; j)}{\sum_{n=1}^m (m_n - 2^{j+1} + 1)}} \quad (11)$$

An example of this procedure for zero dead time is presented in figure 1. The quantity v represents the variance between the ordinates of the two lines to which the dotted line near v points.

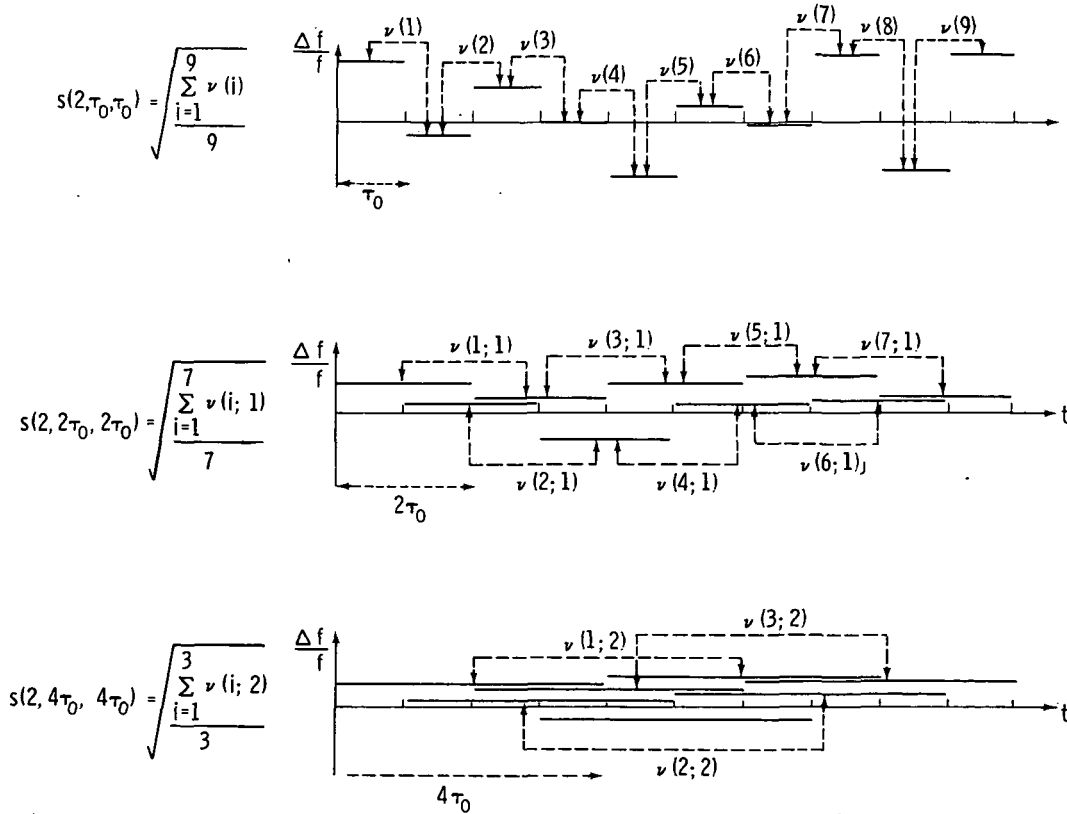


Figure 1.—Calculation of $s(2, \tau, \tau)$.

RESULTS

For each type of data listed in table 2 and for each averaging time $\tau_j = 2^j \tau_0$, $0 \leq j \leq k - 1$ (τ_0 and k change with the type of data), the estimate $s(2, \tau_j + d, \tau_j)$ of $\sigma(2, \tau_j + d, \tau_j)$ was calculated.³ The results are presented in table 3 and figure 2 for all data involving a rubidium standard as either the test or the reference unit and in table 4 and figure 3 for the cesium versus cesium and cesium versus hydrogen maser data.

In order to use the data in tables 3 and 4 to estimate the frequency stability of the rubidium and cesium standards tested, rather than the relative frequency stability of a comparison of two of these standards or of a comparison of one of these standards to a hydrogen maser, the following procedure is used. Denote the Allan standard deviations of the test standard versus a hypothetical perfect standard, the reference standard versus a hypothetical perfect standard, and the test standard versus the reference standard by $\sigma_T(2, \tau + d, \tau)$, $\sigma_R(2, \tau + d, \tau)$, and $\sigma_{T-R}(2, \tau + d, \tau)$, respectively. Because the variances $\sigma_T^2(2, \tau + d, \tau)$ and $\sigma_R^2(2, \tau + d, \tau)$ are linear functions (in fact, weighted integrals) of the respective power spectral densities of the test and reference standards (ref. 3), and because the power spectral density of the comparison of two frequency standards is the sum of the power spectral densities of each of the standards, the following relation occurs:

$$\sigma_{T-R}^2(2, \tau + d, \tau) = \sigma_T^2(2, \tau + d, \tau) + \sigma_R^2(2, \tau + d, \tau) \quad (12)$$

For comparisons of two identical standards (rubidium standard versus rubidium standard and cesium standard versus cesium standard), $\sigma_R(2, \tau + d, \tau) = \sigma_T(2, \tau + d, \tau)$. Hence, from relation (12),

$$\sigma_T(2, \tau + d, \tau) = \frac{\sigma_{T-R}(2, \tau + d, \tau)}{\sqrt{2}} \quad (13)$$

For all data for which a hydrogen maser was used as a reference, it is assumed that the instabilities of the maser were sufficiently small so as to have

$$\sigma_T(2, \tau + d, \tau) \approx \sigma_{T-R}(2, \tau + d, \tau) \quad (14)$$

The normalized standard deviation $\sigma_T(2, \tau, \tau)$ can be calculated from $\sigma_T(2, \tau + d, \tau)$ by the relation

$$\sigma_T(2, \tau, \tau) = \frac{\sigma_T(2, \tau + d, \tau)}{\sqrt{B_2(r, \mu)}} \quad (15)$$

where $B_2(r, \mu)$ is a bias function (defined in ref. 4); $r = (\tau + d)/\tau$; and μ , representing the type of noise of the standard for the fixed averaging time τ and fixed dead time d , is determined from

$$\sigma_T(2, \tau + d, \tau) \propto \tau^{\mu/2} \quad (16)$$

³The analysis was carried out by programs E00016 and E00036 of the GSFC Computer Program Library. Program E00016 is for input relative phase data; program E00036 is for input relative frequency data. Although program E00016 reads relative phase data as input, its output is the Allan standard deviation of relative frequency $s(2, \tau_j + d, \tau_j)$ defined in eqs. (5), (8), and (11). These two programs are based on a program written by David W. Allan of the National Bureau of Standards, Boulder, Colo.

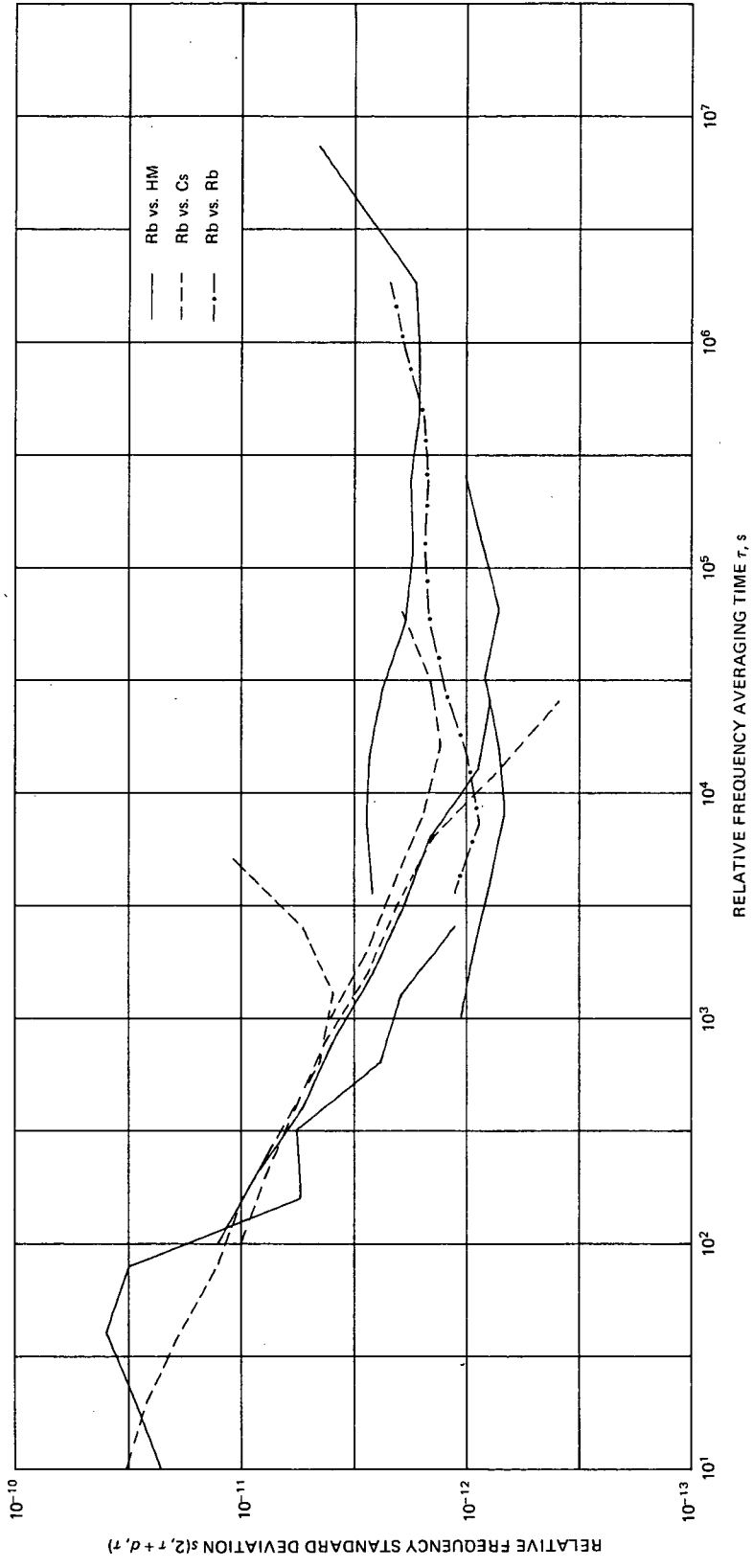


Figure 2.—Rubidium standard relative frequency stability.

Table 4.—Cesium Standard Frequency Stability

Type of data				Type of data				Type of data				Type of data			
Test unit	Reference unit	τ, s	d, s	Test unit	Reference unit	τ, s	d, s	Test unit	Reference unit	τ, s	d, s	Test unit	Reference unit	τ, s	d, s
Cs	Cs	3 600	0.0	Cs (60-s TC)	HM	256 000	0.2	Cs (60-s TC)	HM	1 000	0.2	Cs (60-s TC)	HM	1 000	0.2
		7 200	.0			10	.2			2 000	.2			2 000	.2
		14 400	.0			20	.2			4 000	.2			4 000	.2
		28 800	.0			40	.2			8 000	.2			8 000	.2
		57 600	.0			80	.2			16 000	.2			16 000	.2
		115 200	.0			160	.2			32 000	.2			32 000	.2
		230 400	.0			320	.2			64 000	.2			64 000	.2
		460 800	.0			640	.2			128 000	.2			128 000	.2
		921 600	.0			1 280	.2			256 000	.2			256 000	.2
		1 843 200	.0			2 560	.2			512 000	.2			512 000	.2
		3 686 400	.0			5 120	.2			1 024 000	.2			1 024 000	.2
Cs (10-s TC)	HM	10	.2	Cs (60-s TC)	HM	10	2.3	Cs (60-s TC)	HM	2 000	2.7	Cs (60-s TC)	HM	2 000	2.7
		20	.2			20	2.3			4 000	2.7			4 000	2.7
		40	.2			40	2.3			8 000	2.7			8 000	2.7
		80	.2			80	2.3			16 000	2.7			16 000	2.7
		160	.2			160	2.3			32 000	2.7			32 000	2.7
		320	.2			320	2.3			64 000	2.7			64 000	2.7
		640	.2			640	2.3			128 000	2.7			128 000	2.7
		1 280	.2			1 280	2.3			256 000	2.7			256 000	2.7
		2 560	.2			2 560	2.3			512 000	2.7			512 000	2.7
		5 120	.2			5 120	2.3			1 024 000	2.7			1 024 000	2.7
		10 240	.2			10 240	2.3			2 048 000	2.7			2 048 000	2.7
		20 480	.2			20 480	2.3			4 096 000	2.7			4 096 000	2.7
		40 960	.2			40 960	2.3			8 192 000	2.7			8 192 000	2.7
		81 920	.2			81 920	2.3			16 384 000	2.7			16 384 000	2.7
		163 840	.2			163 840	2.3			32 768 000	2.7			32 768 000	2.7
		327 680	.2			327 680	2.3			65 536 000	2.7			65 536 000	2.7
		655 360	.2			655 360	2.3			1 310 720	2.7			1 310 720	2.7
		1 310 720	.2			1 310 720	2.3			2 621 440	2.7			2 621 440	2.7
		2 621 440	.2			2 621 440	2.3			5 242 880	2.7			5 242 880	2.7
		5 242 880	.2			5 242 880	2.3			10 485 760	2.7			10 485 760	2.7
		10 485 760	.2			10 485 760	2.3			20 971 520	2.7			20 971 520	2.7
		20 971 520	.2			20 971 520	2.3			41 943 040	2.7			41 943 040	2.7
		41 943 040	.2			41 943 040	2.3			83 886 080	2.7			83 886 080	2.7
		83 886 080	.2			83 886 080	2.3			167 772 160	2.7			167 772 160	2.7
		167 772 160	.2			167 772 160	2.3			335 544 320	2.7			335 544 320	2.7
		335 544 320	.2			335 544 320	2.3			671 088 640	2.7			671 088 640	2.7
		671 088 640	.2			671 088 640	2.3			1 342 177 280	2.7			1 342 177 280	2.7
		1 342 177 280	.2			1 342 177 280	2.3			2 684 354 560	2.7			2 684 354 560	2.7
		2 684 354 560	.2			2 684 354 560	2.3			5 368 709 120	2.7			5 368 709 120	2.7
		5 368 709 120	.2			5 368 709 120	2.3			10 737 418 240	2.7			10 737 418 240	2.7
		10 737 418 240	.2			10 737 418 240	2.3			21 474 836 480	2.7			21 474 836 480	2.7
		21 474 836 480	.2			21 474 836 480	2.3			42 949 672 960	2.7			42 949 672 960	2.7
		42 949 672 960	.2			42 949 672 960	2.3			85 899 345 920	2.7			85 899 345 920	2.7
		85 899 345 920	.2			85 899 345 920	2.3			171 798 691 840	2.7			171 798 691 840	2.7
		171 798 691 840	.2			171 798 691 840	2.3			343 597 383 680	2.7			343 597 383 680	2.7
		343 597 383 680	.2			343 597 383 680	2.3			687 194 767 360	2.7			687 194 767 360	2.7
		687 194 767 360	.2			687 194 767 360	2.3			1 374 389 534 720	2.7			1 374 389 534 720	2.7
		1 374 389 534 720	.2			1 374 389 534 720	2.3			2 748 779 069 440	2.7			2 748 779 069 440	2.7
		2 748 779 069 440	.2			2 748 779 069 440	2.3			5 497 558 138 880	2.7			5 497 558 138 880	2.7
		5 497 558 138 880	.2			5 497 558 138 880	2.3			10 995 116 277 760	2.7			10 995 116 277 760	2.7
		10 995 116 277 760	.2			10 995 116 277 760	2.3			21 990 232 555 520	2.7			21 990 232 555 520	2.7
		21 990 232 555 520	.2			21 990 232 555 520	2.3			43 980 465 111 040	2.7			43 980 465 111 040	2.7
		43 980 465 111 040	.2			43 980 465 111 040	2.3			87 960 930 222 080	2.7			87 960 930 222 080	2.7
		87 960 930 222 080	.2			87 960 930 222 080	2.3			175 921 860 444 160	2.7			175 921 860 444 160	2.7
		175 921 860 444 160	.2			175 921 860 444 160	2.3			351 843 720 888 320	2.7			351 843 720 888 320	2.7
		351 843 720 888 320	.2			351 843 720 888 320	2.3			703 687 441 776 640	2.7			703 687 441 776 640	2.7
		703 687 441 776 640	.2			703 687 441 776 640	2.3			1 407 374 883 552 280	2.7			1 407 374 883 552 280	2.7
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		5 629 499 534 209 120	.2			5 629 499 534 209 120	2.3			11 258 999 068 418 240	2.7			11 258 999 068 418 240	2.7
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		22 517 998 136 836 480	.2			22 517 998 136 836 480	2.3			45 035 996 273 672 960	2.7			45 035 996 273 672 960	2.7
		45 035 996 273 672 960	.2			45 035 996 273 672 960	2.3			90 071 992 547 345 920	2.7			90 071 992 547 345 920	2.7
		90 071 992 547 345 920	.2			90 071 992 547 345 920	2.3			180 143 985 094 691 840	2.7			180 143 985 094 691 840	2.7
		180 143 985 094 691 840	.2			180 143 985 094 691 840	2.3			360 287 970 189 383 680	2.7			360 287 970 189 383 680	2.7
		360 287 970 189 383 680	.2			360 287 970 189 383 680	2.3			720 575 940 378 767 360	2.7			720 575 940 378 767 360	2.7
		720 575 940 378 767 360	.2			720 575 940 378 767 360	2.3			1 441 151 880 757 534 720	2.7			1 441 151 880 757 534 720	2.7
		1 441 151 880 757 534 720	.2			1 441 151 880 757 534 720	2.3			2 882 303 761 515 069 440	2.7			2 882 303 761 515 069 440	2.7
		2 882 303 761 515 069 440	.2			2 882 303 761 515 069 440	2.3			5 764 607 523 030 138 880	2.7			5 764 607 523 030 138 880	2.7
		5 764 607 523 030 138 880	.2			5 764 607 523 030 138 880	2.3			11 529 215 046 060 277 760	2.7			11 529 215 046 060 277 760	2.7
		11 529 215 046 060 277 760	.2			11 529 215 046 060 277 760	2.3			23 058 430 092 120 555 520	2.7			23 058 430 092 120 555 520	2.7
		23 058 430 092 120 555 520	.2			23 058 430 092 120 555 520	2.3			46 116 860 184 241 111 040	2.7			46 116 860 184 241 111 040	2.7
		46 116 860 184 241 111 040	.2			46 116 860 184 241 111 040	2.3			92 233 720 368 482 222 080	2.7			92 233 720 368 482 222 080	2.7
		92 233 720 368 482 222 080	.2			92 233 720 368 482 222 080	2.3			184 467 440 736 964 444 160	2.7			184 467 440 736 964 444 160	2.7
		184 467 440 736 964 444 160	.2			184 467 440 736 964 444 160	2.3			368 934 881 473 928 888 320	2.7			368 934 881 473 928 888 320	2.7

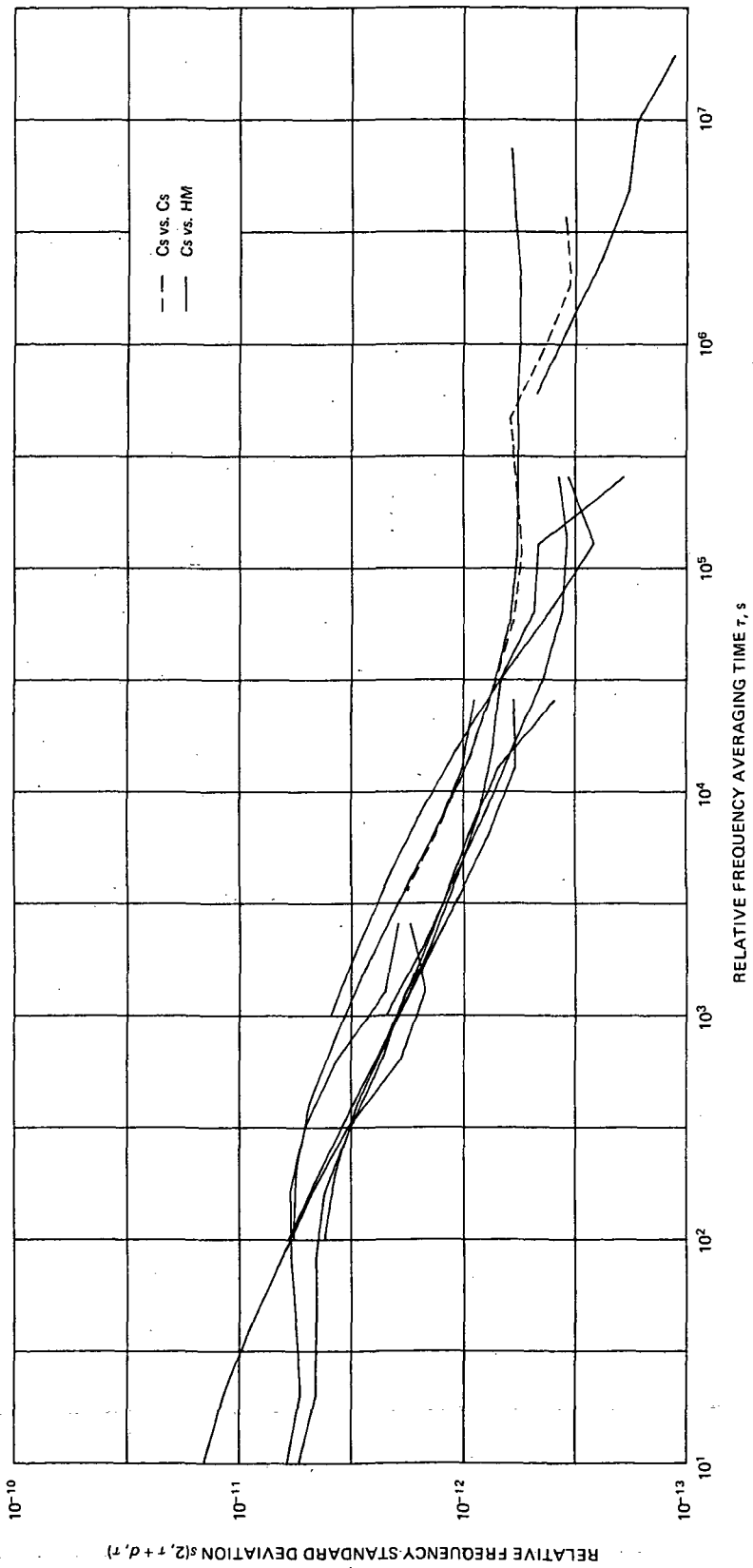


Figure 3.—Cesium standard relative frequency stability.

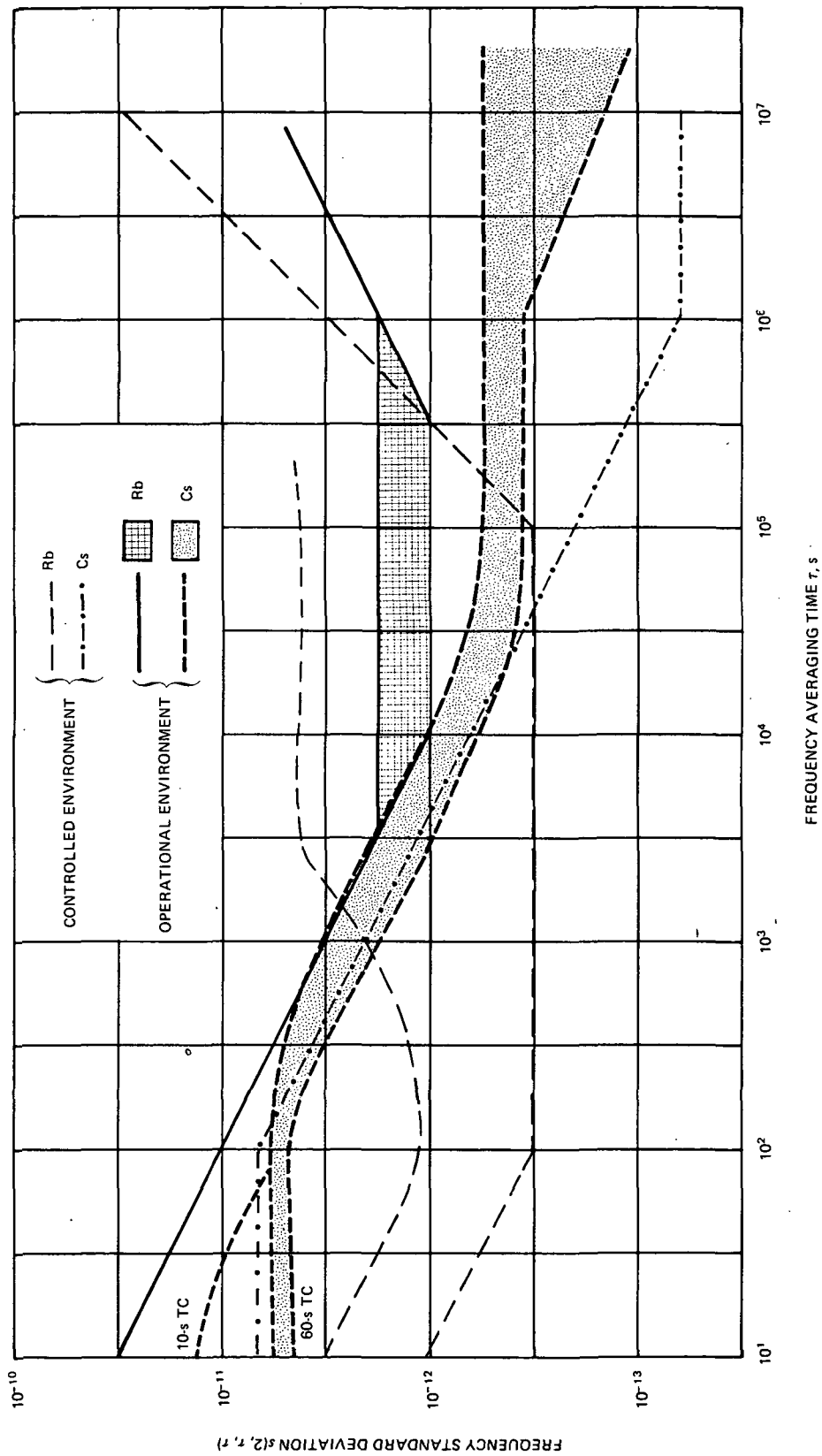


Figure 4.—Rubidium and cesium standard frequency stabilities.

It is of interest to note that for the r and μ of the data analyzed in this report, $B_2(r, \mu)$ differs from unity by less than 0.1 percent and can be ignored. Hence, for the data in this report,

$$\sigma_T(2, \tau, \tau) \approx \sigma_T(2, \tau + d, \tau) \quad (17)$$

Of course, relation (17) is an exact equality whenever $d = 0$.

Using the estimates $s(2, \tau + d, \tau)$ of $\sigma_{T-R}(2, \tau + d, \tau)$ from figures 2 and 3 in relations (13) and (14) and using relation (17), the standard deviations $\sigma_T(2, \tau, \tau)$ of the rubidium and cesium standards tested can be estimated. These estimates of $\sigma_T(2, \tau, \tau)$ are presented in figure 4 as the "operational environment" curves. Also shown in figure 4 are curves taken from references 1 and 5 representing the performance of rubidium and cesium standards in a "controlled environment." By "controlled environment" is meant an experimental environment shielded from magnetic, electric, vibration, and temperature effects much more than the "operational" environment in which the data presented in figures 2 and 3 were taken.⁴ The upper curve for rubidium standards under a controlled environment in figure 4 is taken from reference 5 and represents the measured performance of Varian rubidium standards under controlled conditions. The lower curve for rubidium standards under a controlled environment and the curve for cesium standards under a controlled environment in figure 4 are taken from reference 1 and represent the measured performance of Hewlett-Packard rubidium and cesium standards under controlled conditions.

CONCLUSIONS

From figure 4 it is apparent that an operational environment degrades the performance of the rubidium standards (by up to one order of magnitude) for frequency averaging times between 10 and 10^3 s and that it degrades the performance of the cesium standards (by up to one order of magnitude) for frequency averaging times between 3×10^4 and 2×10^7 s. For all other averaging times in the range covered by the data in figure 4, the stabilities of the standards are not degraded by the operational conditions.

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Goddard Space Flight Center
National Aeronautics and Space Administration
Greenbelt, Maryland, December 22, 1971
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⁴That this was the case for the data presented in reference 1 was verified by private communication with G. M. R. Winkler. That the environment in which the data presented in reference 5 was taken was "controlled" is assumed.

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