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FREE VIBRATIONS OF THERMALLY STRESSED ORTHOTROPIC PLATES WITH VARIOUS BOUNDARY CONDITIONS

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16. Abstract

An analytical investigation of the vibrations of thermally stressed orthotropic plates in the prebuckled region is presented. The investigation covers the broad class of trapezoidal plates with two opposite sides parallel. Each edge of the plate may be subjected to different uniform boundary conditions. Variable thickness and arbitrary temperature distributions (analytical or experimental) for any desired combination of boundary conditions may be prescribed. Results obtained using this analysis are compared to experimental results obtained for isotropic plates with thermal stress, and to results contained in the literature for orthotropic plates without thermal stress. Good agreement exists for both sets of comparisons.

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SUMMARY

An analytical investigation of the vibrations of thermally-stressed orthotropic plates in the prebuckled region is presented. The investigation covers the broad class of trapezoidal plates with two opposite sides parallel. Each edge of the plate may be subjected to different uniform boundary conditions. Variable thickness and arbitrary temperature distributions (analytical or experimental) may be prescribed. Generality is achieved in the analysis through the treatment of boundary conditions, the choice of functions for stress distributions and deflection distributions, and the use of numerical integration for the evaluation of matrix elements. Results obtained using this analysis are compared to experimental results obtained for isotropic plates with thermal stress, and to results contained in the literature for orthotropic plates without thermal stress. Good agreement exists for both sets of comparisons. Calculations for several orthotropic plates with thermal stresses indicates that the effect of orthotropy on the frequencies may be large and should not be ignored.

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LIST OF SYMBOLS

[A]	- stress matrix from complementary energy
$A_{pq,rs}$	- elements of [A]
AR	- aspect ratio, length squared/area
a	- plate length
[a]	- matrix of material constants in equation for strains in terms of stresses
a_{11}, a_{12}, a_{22}, b_{12}	- elements of [a], Equation (3)
[B]	- generalized stiffness matrix from bending energy
$B_{ij,k\ell}$	- element of [B]
b	- plate dimension measured along left edge from x-axis to top corner
$\bar{b}_1$	- plate dimension, plate width at left edge minus b
b_1	- ratio of $\bar{b}_1/b$
b_i	- $b_i\ (x,y) = 0$, equation of i^{th} portion of plate boundary
C_{pq}	- coefficient of the pq^{th} term of the assumed stress function solution
[E]	- matrix of material constants in equation for stresses in terms of strains, $[E] = [a]^{-1}$
E_{11}, E_{12}, E_{22}, G_{12}	- elements of [E]
F	- stress function solution to the inplane equilibrium equations
f_i	- frequency of i^{th} mode, cycles per second

f	- $f(x,y)$ - function which forces the assumed stress function solution to satisfy the stress boundary conditions
g	- $g(x,y)$ - forces the assumed displacement solution to satisfy the displacement boundary conditions
h_{ij}	- coefficient of the ij^{th} term in the assumed displacement function solution
h	- $h(x,y)$ - function to represent any variation in plate thickness
h_r	- plate thickness at some reference point
$[M]$	- mid-plane energy matrix, associated with the thermal stresses moving through small out-of-plane displacements
$M_{ij,k\ell}$	- elements of $[M]$
N_1, N_2, N_{12}	- $\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_1 dz, \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_2 dz, \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{12} dz$ respectively
n	- coordinate normal to plate boundary (in-plane)
$[T]$	- generalized mass matrix from kinetic energy
$T_{ij,k\ell}$	- element of $[T]$
T	- $T(x,y)$ - difference between the temperature at a point (x,y) on the plate and the original, uniform reference temperature (non-dimensional)
T_{ref}	- difference between the temperature at some reference point on the plate and the original, uniform reference temperature
ΔT_{cr}	- The magnitude of T_{ref} at which the free vibration frequency vanishes. By definition, the thermal buckling temperature.

t	- time
u, v, w	- displacements in the x,y, and z directions respectively
x, y	- independent in-plane variables
$\bar{\alpha}$	- angle between plate leading edge and x-axis, measured positive counter-clockwise
α	- taper parameter, $\alpha = \frac{a}{b} \tan \bar{\alpha}$
α_1, α_2	- coefficient of thermal expansion in x and y directions respectively
ij	- ij^{th} term of general assumed displacement function
$\bar{\beta}$	- angle between x-axis and the line dividing the plate for thickness distribution purposes
β	- non-dimensional form of $\bar{\beta}$
$\{\Gamma\}$	- thermal loading matrix
Γ_{rs}	- element of $\{\Gamma\}$
$\bar{\gamma}$	- angle between plate trailing edge and x-axis, measured positive counter-clockwise
γ	- taper parameter, $\gamma = \frac{a}{b} \tan \bar{\gamma}$
γ_{12}	- shear strain
γ_{pq}	- pq^{th} term of general assumed stress function
ΔT	- an increment of T_{ref}, gives magnitude of the temperature distribution under consideration
$\varepsilon_1, \varepsilon_2$	- normal strains in the x and y directions respectively
η	- non-dimensional independent space variable, $\eta = y/b$

λ_i	- vibration eigenvalue, $\lambda_i = \omega_i \frac{a^2}{h_r} \sqrt{12\rho/E_{11}}$
ξ	- non-dimensional independent space variable, $\xi = x/a$
π	- energy of a system per unit time
π^*	- complimentary energy
ρ	- plate material density, mass per unit volume
σ_1, σ_2	- normal stresses in x and y directions respectively
τ_{12}	- shear stress
ω	- vibration frequency of the thermally stressed plate, radians/sec.
ω_o	- vibration frequency of the plate at T = 0, radians/sec.

NOTE ON SUBSCRIPT CONVENTION

Numeric subscripts indicate the component of a quantity in a coordinate direction (e.g., σ_1 - normal stress in the 1 or x - direction). A subscript of x,y,ξ, or η denotes differentiation with respect to that independent variable (e.g., $(\sigma_1)_x = \frac{\partial}{\partial x}(\sigma_1)$). All other alphabetic subscripts (i, j, k, , p, q, etc.) will refer to either terms in a series or elements in a matrix.

I. INTRODUCTION

Considerable work has been reported in the literature on the problem of finding the frequencies and modes of vibration of a rectangular orthotropic plate at ambient temperature. A combination of the work of Hearmon (Ref. 1, 2, 3); Hoppman, Huffington, and Magness (in various combinations, Ref. 4, 5, 6, 7, 8); Kanazawa and Kawai (Ref. 9) and Wah (Ref. 10) provide solutions for rectangular plates with any boundary condition except completely free.

In contrast, no literature was found pertaining to the free vibration frequencies of an orthotropic plate subjected to a thermal loading. For the special case of a thermally stressed isotropic plate, the torsion mode of the plate with cantilever boundary conditions has been rather thoroughly investigated (Ref. 11, 12, 13, 14, 15).

Ref. 16 presents an analysis of thermally stressed isotropic plates for various boundary conditons, ranging from plates completely clamped through several combinations of mixed boundary conditions to plates with all edges completely free. This paper extends the analysis of Ref. 16 to include orthotropic plates with a thorough discussion of the associated computer program.

In the sense that both compatability and equilibrium are satisfied as closely as one pleases at every point interior to the plate and on the boundary, this paper presents an analysis that provides, in a practical computational sense, a solution to the thermally stressed plate vibration problem for all trapezoidal plates with two opposite sides parallel, and with one of the axes of elastic symmetry parallel to these sides, restrictions that could be easily relaxed.

The analysis and associated computer program are of sufficient generality that isotropic plates are included as a special case of orthotropic plates. Various boundary conditions may be arbitrarily assigned to the different sides of the plate. Thus, the solution for the vibrations of thermally stressed plates with boundary conditions ranging from completely clamped to completely free with any combination of clamped, pinned, and/or free edges may be obtained.

A small number of quantitative strain measurements (not included herein) plus the abundance of experimental dynamic response data for various planform shapes and boundary conditions of isotropic plates indicates that the stress distributions as determined herein are correct.

No thermally stressed orthotropic plate data, either analytical or experimental, were found in the literature; however, for orthotropic plates without thermal stress, comparison is made to both analytical and experimental data from the literature. Further comparison is made with experimental data for several modes of thermally stressed isotropic plates with various planform shapes, boundary conditions, and temperature distributions.

II. APPLICATION OF THE ENERGY EQUATION

Because of a difference in notation between that used herein and that used in other sources, the derivation of the expressions for potential and complementary energy will be shown. Except for this difference, the procedures used are well known. Additional details, if desired, may be found in Ref. 17.

A. Potential Energy

The forces are taken as the independent variables and the variation of the total energy is taken with respect to the displacements. With the assumptions of plane stress and no body or surface forces,

$$\delta\pi = \iiint\{(\sigma_1\delta\varepsilon_1 + \sigma_2\delta\varepsilon_2 + \tau_{12}\delta\gamma_{12}) + \rho\,\ddot{w}\,\delta w\}\,dxdydz \quad (1)$$

The orthotropic stress-strain relations will be taken as,

$$\{\varepsilon\} = [a]\,\{\sigma\} + \{\alpha\}\,T \quad (2)$$

where

$$[a] = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & b_{12} \end{bmatrix} \quad ; \quad \{\alpha\} = \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix}$$

(3)

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \quad \text{and} \quad \{\sigma\} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

The inverse of eq. (2) is

$$\{\sigma\} = [E]\{\varepsilon\} - [E]\{\varepsilon\}T \qquad (4)$$

The Von Karman strain-displacement equations are used,

$$\varepsilon_1 = u_x + \frac{1}{2}(w_x) - z\, w_{xx}$$

$$\varepsilon_2 = v_y + \frac{1}{2}(w_y) - z\, w_{yy} \qquad (5)$$

$$\gamma_{12} = u_y + v_x + w_x w_y - 2z\, w_{xy}$$

Substitute eqs. 4 and 5 into eq. 1, carry out the indicated operations, neglect fourth order terms, and integrate through the thickness to get,

$$\delta\pi = \frac{1}{2} \delta\iint\{N_1 u_x + N_2 v_y + N_{12}(u_y + v_x)$$

$$+ \frac{h^3}{12} [E_{11}(w_{xx})^2 + E_{22}(w_{yy})^2 + 2E_{12} w_{xx} w_{yy} + 4G_{12}(w_{xy})^2]$$

$$+ N_1(w_x)^2 + N_2(w_y)^2 + 2N_{12} w_x w_y \tag{6}$$

$$+ 2\rho h \ddot{w} w\} dxdy = 0$$

where,

$$N_1 = \int_{-\frac{h}{2}}^{\frac{h}{2}} [E_{11} u_x + E_{12} v_y - T(E_{11}\alpha_1 + E_{12}\alpha_2)] dz$$

$$N_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} [E_{12} u_x + E_{22} v_y - T(E_{12}\alpha_1 + E_{22}\alpha_2)] dz \tag{7}$$

$$N_{12} = \int_{-\frac{h}{2}}^{\frac{h}{2}} G_{12}(u_y + v_x) dz ,$$

Taking the variation with respect to u gives,

$$\iint (N_1 \delta u_x + N_{12} \delta u_y) dxdy = 0.$$

Integrating this result by parts, noting that for any solution to a particular problem the boundary conditions must be satisfied, leaves the in-plane equilibrium equation in the x-direction,

$$(N_1)_x + (N_{12})_y = 0 \tag{8a}$$

Performing a similar series of operations on v gives for the y-direction,

$$(N_{12})_x + (N_2)_y = 0 \qquad (8b)$$

These eqs. (8) have the solution, F, such that,

$$N_1 = F_{yy}$$

$$N_2 = F_{xx} \qquad (9)$$

$$N_{12} = -F_{xy}$$

Thus the variational expression for potential energy of a thermally stressed, orthotropic plate becomes,

$$\begin{aligned}\delta\pi = \frac{1}{2}\ \delta\iint\{\frac{h^3}{12}\ &[E_{11}(w_{xx})^2 + E_{22}(w_{yy})^2 + 2E_{12}w_{xx}w_{yy} \\ &+ 4G_{12}(w_{xy})^2] \\ &+ [F_{yy}(w_x)^2 + F_{xx}(w_y)^2 - 2F_{xy}w_xw_y] \\ &+ 2\ \rho h\ \ddot{w}\ w\ \}\,dxdy = 0\end{aligned} \qquad (10)$$

B. Complementary Energy

In developing the expression for complementary energy, the unknown forces are varied and the displacements are held constant. Thus, with the same assumptions as used in the treatment of potential energy,

$$\delta\pi^* = \iiint\{\varepsilon_1\delta\sigma_1 + \varepsilon_2\delta\sigma_2 + \gamma_{12}\delta\tau_{12}\}dxdydz - \delta W_B = 0$$

where W_B represents the work done by the stresses on the portion of the boundary on which the displacements are specified. In this treatment, if the displacements on any part of the boundary of the plate are to be specified, they will be specified to be zero. Thus $W_B = 0$.

Substitute eq. (4) for the stresses to get,

$$\delta\pi^* = \iiint\{\varepsilon_1\delta[E_{11}\varepsilon_1 + E_{12}\varepsilon_2 - (E_{11}\alpha_1 + E_{12}\alpha_2)T]$$

$$+ \varepsilon_2\delta[E_{22}\varepsilon_2 + E_{12}\varepsilon_1 - (E_{12}\alpha_1 + E_{22}\alpha_2)T]$$

$$+ \gamma_{12}\delta G_{12}\gamma_{12}\}\ dxdydz = 0$$

Because the bending stresses have been expressed in terms of the displacement only and small deflections are assumed, the stresses that remain in the equations are not functions of the out-of-plane displacements; they are "membrane stresses" resulting only from the in-plane displacements and/or temperature. Thus, only the linear strains resulting from in-plane deformation need to be considered and eq. (5) can be simplified to,

$$\varepsilon_1 = u_x$$

$$\varepsilon_2 = v_y$$

$$\gamma_{12} = (u_y + v_x)$$

With these relations, the complementary energy can be expressed as,

$$\delta\pi^* = \iiint\{u_x\delta[E_{11}u_x + E_{12}v_y - (E_{11}\alpha_1 + E_{12}\alpha_2)T]$$

$$+ v_y\delta[E_{12}u_x + E_{22}v_y - (E_{12}\alpha_1 + E_{22}\alpha_2)T]$$

$$+ (u_y + v_x)\ \delta[G_{12}(u_y + v_x)]\ dxdydz = 0 \quad .$$

Integrate through the thickness, substitute equations (7) and (9) and define the strains to be,

$$\varepsilon_1 = u_x = \frac{1}{h}(a_{11}F_{yy} + a_{12}F_{xx}) + \alpha_1 T$$

$$\varepsilon_2 = v_y = \frac{1}{h}(a_{22}F_{xx} + a_{12}F_{yy}) + \alpha_2 T$$

$$\gamma_{12} = u_y + v_x = -\frac{1}{h} b_{12}F_{xy} \qquad ,$$

from which the complementary energy for a thermally stressed, orthotropic plate becomes,

$$\delta\pi^* = \delta\int\int\{ \frac{1}{2h}[a_{11}(F_{yy})^2 + a_{22}(F_{xx})^2 + 2a_{12}F_{xx}F_{yy} + b_{12}(F_{xy})^2] + (\alpha_1 F_{yy} + \alpha_2 F_{xx})T\} dxdy = 0 \quad . \tag{11}$$

C. The Equations in Matrix Form

Consider first the potential energy, eq. (10). Assume a displacement function of the form,

$$w(x,y,t) = \sum_{i=o}^{N} \sum_{j=o}^{M} h_{ij}(t)\, \alpha_{ij}(x,y) \tag{12}$$

where each $\alpha_{ij}(x,y)$, (1) satisfies the displacement boundary conditions, (2) is continuous, and (3) has at least continuous first derivatives.

Substitute this into eq. (10), take the variation with respect to $h_{k\ell}$ and collect coefficients of like h_{ij} to get the matrix equation,

$$[B]\{h_{ij}\} + K_1[M]\{h_{ij}\} - \lambda^2 [T]\{h_{ij}\} = 0 \qquad (13)$$

where the non-dimensionalized matrix elements and associated parameters are given in Appendix A.

Now assume a stress function of the form,

$$F(x,y) = \sum_{p=o}^{S} \sum_{q=o}^{T} C_{pq}\,\gamma_{pq}(x,y) \qquad (14)$$

where each $\gamma_{pq}(x,y)$, (1) satisfies the stress boundary conditions, (2) is continuous, and (3) has at least continuous first derivatives.

Substitute this into eq. (11), take the variation with respect to C_{rs} and collect coefficients of like C_{pq} to get the matrix equation,

$$[A]\{\hat{C}_{pq}\} + K_2\{\Gamma\} = 0 \qquad (15)$$

where the matrix elements are also given in Appendix A.

Thus, given a temperature distribution, $\{\Gamma\}$ can be calculated, eq. (15) can be solved for $\{\hat{C}_{pq}\}$, and values of the derivatives of the stress function can be found. Using this information, the elements, $M_{ij,k\ell}$, can be calculated and eq. (13) can be solved for the vibration frequencies and modes with the buckling mode and ΔT_{cr_i} obtainable as a limiting case when $\lambda_i^2 = 0$.

D. Deflection Function and Stress Function

At this point, a choice will be made concerning the form of the assumed deflection function and stress function. By observing the physical system, it can be seen that the deflected surface of the plate and the stresses within the plate are continuous and have at least continuous first derivatives. Thus, the functions to be assumed as solutions to the problem must belong to the class of functions which are continuous and have at least continuous first derivatives. The assumed solution must also satisfy the boundary conditions discussed in the next section.

A truncated power series in the independent space variables satisfies the continuity requirements. Thus, the functions assumed for the deflection, w, and for the stress function, F, will be truncated power series.

E. Boundary Conditions

The polynomial resulting from a truncated power series will not in general satisfy the boundary conditions. Therefore, the polynomial representation must be modified by an additional function which forces satisfaction of the required boundary conditions.

Let the displacement function have the form,

$$w\,(x,y) = g\,(x,y) \sum_{i=o}^{N} \sum_{j=o}^{M} h_{ij} x^i y^j$$

where g(x,y) is the boundary condition function which insures satisfaction of the displacement conditions at the boundary. The stress function will have the form,

$$F\,(x,y) = f(x,y) \sum_{p=o}^{S} \sum_{q=o}^{T} C_{pq}\, x^p y^q$$

where f(x,y) is the boundary condition function which insures satisfaction of equilibrium in the plane of the plate at the boundary, i.e., the stress boundary condition. The specific form of each will now be considered.

1. Displacement Function

Three types of displacement boundary conditions are considered herein:

(a) Both displacement and slope normal to the edge of the plate are assumed to be zero; i.e., the edge is clamped.

(b) Only the displacement is assumed to be zero and the slope is left unspecified resulting in a pinned (simply-supported) condition.

(c) Both slope and displacement are left unspecified leaving the edge completely free.

Now, given a particular plate geometry, the equation of the boundary may be expressed as a polynomial, say,

$$b\ (x,y) = 0 \quad .$$

Therefore, in order to force the displacement to be zero on the boundary, simply let,

$$g\ (x,y) = b\ (x,y) \quad ,$$

so that for any point on the boundary, (x_B, y_B), the deflection will be

$$w\ (x_B,y_B) = g\ (x_B,y_B) \sum_{i=o}^{N} \sum_{j=o}^{M} h_{ij}\ x_B^i y_B^j = 0$$

This satisfies condition (b) because the first derivative, $\frac{\partial w}{\partial n}$, will not in general be zero but will be left to take on whatever value is required for a minimum energy configuration.

Condition (a) may be satisfied by letting

$$g\ (x,y) = [b(x,y)]^2$$

The displacement will again be zero, but now the first derivative will also be zero on the boundary:

$$\frac{\partial w}{\partial n} = \sum_{i=o}^{N} \sum_{j=o}^{M} h_{ij}\ (g(x,y)\ \frac{\partial(x^i y^j)}{\partial n} + x^i y^j\ \frac{\partial g(x,y)}{\partial n})$$

$$= \sum_{i=o}^{N} \sum_{j=o}^{M} h_{ij}\ \{[b(x,y)]^2\ \frac{\partial}{\partial n}\ (x^i y^j) + 2\ x^i y^j b(x,y)\ x$$

$$\frac{\partial}{\partial n}\ b(x,y)\}$$

and at a point (x_B, y_B) on the plate boundary,

$$\frac{\partial w}{\partial n} = 0 \quad .$$

Case (c) may be satisfied simply by letting

$$g(x,y) \equiv 1 = [b(x,y)]^{0}$$

Thus, in the case of an edge free to displace out of the plane of the plate, both the displacement and slope will be left to take on whatever values are required for a minimum energy configuration.

If the plate is a polygon of N sides, write

$$g(x,y) = \prod_{i=1}^{N} [b_i(x,y)]^{k_i} \quad ,$$

where $b_i(x,y)$ is the equation of the i^{th} side of the polygon. (The sides need not be straight.) k_i will be either 0, 1, or 2 as described above.

2. Stress Function

Two types of stress boundary conditions are considered herein:

(a) The in-plane stresses normal to a boundary are specified to be zero. That is, the plate is left free to expand in the in-plane direction.

(b) The stresses are completely unspecified or, equivalently, the in-plane displacements are specified to be zero. The stresses will take on whatever values are required for satisfaction of equilibrium.

Thus, condition (a) will be termed "free" and (b) will be termed "clamped". These conditions are fulfilled in a fashion similar to that used with the deflection function.

Recall from classical elasticity theory that the stresses normal to the edge of a plate will be zero if the stress function and its first derivative (normal to the edge) vanish there. Therefore, on any portion of the boundary on which a free condition is desired, the equilibrating function is,

$$f(x,y) - [b_i(x,y)] = 0$$

where, as before, $b_i(x,y) = 0$ is the equation of that portion of the boundary.

A clamped condition on any portion of the boundary can be satisfied by setting,

$$f(x,y) = [b_i(x,y)]^0 \equiv 1 \quad .$$

Thus, as was done with the boundary condition function, the equilibrating function is

$$f(x,y) = \prod_{i=1}^{N} [b_i(x,y)]^{k_i}$$

where $k_i = 0, 2$ for clamped or free conditions respectively on the "ith" side of the polygon.

With these conditions, it is now possible to specify six different types of boundary conditions on any plate edge. The first letter of the notation used herein will denote the displacement boundary condition by using F = free, P = pinned or simply supported, and C = clamped. The second letter denotes the stress condition so that designations possible for any given edge are:

DESIGNATION	DISPLACEMENT	STRESS
(1) F - F	free	free
(2) P - F	pinned	free
(3) C - F	clamped	free
(4) F - C	free	clamped
(5) P - C	pinned	clamped
(6) C - C	clamped	clamped

III. PROGRAMMING

A. The Equations and the Plate Geometry

The equations to be programed are eqs. 13 and 15 with the matrix elements as given in Appendix A.

The planform and descriptive parameters of the plates considered herein are shown in Fig. 1. The restriction that two of the sides are parallel was made to simplify the numerical integration scheme.

The plate edges are numbered clockwise (in the top view) beginning with the edge containing the origin. Thus, the equations of the four edges as used in the boundary condition functions are:

$$
\begin{aligned}
b_1(\xi,\eta) &= \xi = 0 \\
b_2(\xi,\eta) &= (1 + \gamma\xi - \eta) = 0 \\
b_3(\xi,\eta) &= (1 - \xi) = 0 \\
b_4(\xi,\eta) &= (b_1 - \gamma\xi + \eta) = 0
\end{aligned}
\qquad (16)
$$

B. Logic Flow Diagram

The organization of the parts of the program is presented in a logic flow diagram shown in Appendix B. It should be noted first that if several sets of material properties and/or aspect ratios are to be investigated, the [B] and [A] matrices need not be integrated each time if the integrands are treated as four separate terms. Each of these terms need only be integrated once, then multiplied by the appropriate constant and added together to make up the whole integral for either isotropic or orthotropic materials. The [T] and [M] matrices are independant of both the material properties and aspect ratio. The program is structured to include this feature. A complete listing of the program is given in Appendix C.

C. Programming Boundary Conditions

The subroutine "FUNCTN" which calculates the displacement boundary condition function and the equilibrating function and their derivatives is very straightforward. Since both functions have the same form, the same subroutine can be used to simplify the user's task of calculating the exponents required. There is no need, for example, to remember that a zero exponent on the stress function means a clamped edge while the same exponent in the displacement boundary condition function means a free edge.

The first step was to write down the function and its five derivatives, leaving the exponents as variables. For example,

$$F = \xi^{I_1} (1+\alpha\xi-\eta)^{I_2} \; (1-\xi)^{I_3} \; (b_1-\gamma\xi+\eta)^{I_4} \quad ,$$

$$\frac{\partial F}{\partial \eta} = -I_2 \; [\xi^{I_1} (1+\alpha\xi-\eta)^{I_1-1} \; (1-\xi)^{I_3} \; (b_1-\gamma\xi+\eta)^{I_4}]$$

$$+ \; I_4 [\xi^{I_1} \; (1+\alpha\xi-\eta)^{I_2} \; (1-\xi)^{I_3} \; (b_1-\gamma\xi+\eta)^{I_4-1}]$$

$$\partial^2 F/\partial\eta^2 = I_2(I_2-1) \; [\xi^{I_1} (1+\alpha\xi-\eta)^{I_2-2} (1-\xi)^{I_4} (b_1-\gamma\xi+\eta)^{I_4}]$$

$$+ \; I_4(I_4-1) \; [\xi^{I_1} \; (1+\alpha\xi-\eta)^{I_2} (1-\xi)^{I_3} \; (b_1-\gamma\xi+\eta)^{I_4-2}]$$

$$- \; 2I_2I_4 \; [\xi^{I_1} (1+\alpha\xi-\eta)^{I_2-1} \; (1-\xi)^{I_3} \; (b_1-\gamma\xi+\eta)^{I_4-1}]$$

Thus, it can be seen that I_i, I_i-1, and I_i-2 are required for calculating the function and its derivatives. In the subroutine, the variable IEX(I,J) contains these quantities. The "I" refers to the four factors making up the function and "J" to I_i-0, I_i-1, or I_i-2. This is done in "DO-LOOP" number five.

Next, this information is used to calculate all the factors T (M,K) required for the function and its derivatives. For example,

$$T\;(1,1) = \xi^{I_1}$$

$$T\;(1,2) = \xi^{I_1-1}$$

$$T\;(3,1) = (1-\xi)^{I_3}$$

$$T\;(4,3) = (b\;+\gamma\xi-\eta)^{I_4-2}$$

Finally, this information is used to calculate the function and its derivatives.

D. Numerical Integration

Integration of the elements of the various matrices is performed using the Gaussian Quadrature rule. The plate is divided into two parts by the line at angle β. This provides for more accurate results when the leading or forward part of the plate has a different thickness function than does the rearward part. Since the Gaussian Quadrature is defined on the interval [-1, 1] it is necessary to transform the points and coordinates.

If $u = \phi(x)$ then $\int_a^b f(u)\,du = \int_{-1}^{1} f[\phi(x)] \frac{d\phi(x)}{dx}\,dx$

Then if

$$\int_{-1}^{1} f(x)dx \simeq \sum_{k=1}^{N} w_k f(x_k) \quad ,$$

$$\int_a^b f(u)du \simeq \sum_{k=1}^{N} W_k F(u_k)$$

where

$$W_k = \frac{d\phi(x)}{dx} w_k, \; u_k = \phi(x_k).$$

For this problem,

$$\int_0^1 \{\int_a^b f(\xi,\eta)d\eta + \int_c^a f(\xi,\eta)d\eta\}d\xi$$

$$= \int_{-1}^{1} \{\int_{-1}^{1} f[\phi(x,y) \; , \; \psi_1 (x,y)] \frac{\partial\psi_1}{\partial y} dy$$

$$+ \int_{-1}^{1} f[\phi(x,y) \; , \; \psi_2(x,y)] \frac{\partial\psi_2}{\partial y} dy \quad \frac{\partial\phi}{\partial x} dx \; ,$$

where

$$a = \beta\xi, \; b = 1+\alpha\xi, \quad c = \gamma\xi \quad -b_1$$

Thus

$$\xi = 1/2\ (x+1) \tag{17}$$

$$\eta = 1/2\ \{y[1+ (\alpha-\beta)\xi] + 1 + (\alpha+\beta)\xi\};\ (\beta\xi \le \eta \le 1+\alpha\xi)$$

$$\eta = 1/2\ \{y[b_1 + (\beta-\gamma)\xi]-b_1 + (\beta+\gamma)\}\ ;\ (\gamma\xi \le \eta \le \beta\xi)$$

Hence,

$$\frac{\partial\phi}{\partial x} = 1/2$$

$$\frac{\partial\psi_1}{\partial y} = 1/2\ [1+(\alpha-\beta)\xi]$$

$$\frac{\partial\psi_2}{\partial y} = 1/2\ [b_1 + (\beta-\gamma)\xi]$$

and the integrals may be evaluated by

$$\int_0^1 \{\int_a^b f(\xi,\eta)\ d\eta + \int_c^a f(\xi,\eta)\ d\eta\}\ d\xi$$

$$\simeq \sum_{k=1}^{N} \frac{w_k}{2} \ \{ \sum_{L=1}^{N} \frac{w_L}{2}\ [b_1 + (\beta-\gamma)\xi_k]\ f(\xi_k,\eta_L)$$

$$+ \sum_{M=N+1}^{2N} \frac{w_M}{2}\ [1 + (\alpha-\beta)\xi_k]\ f(\xi_k,\eta_M)\}$$

where w_K, w_L, and w_M are the values on [-1,1] and ξ_K, η_L, and η_M are given by equations (17).

E. Temperature Distribution

One of the assumptions made in developing the equations herein was that the material properties are not functions of temperature. This assumption was made only to conserve computer time. The assumption does, of course, restrict the maximum temperatures to around three hundred degrees Fahrenheit for aluminum. This range of temperature is, however, more than sufficient to demonstrate the validity of the theory before large deflection effects become significant.

The program can handle either an anlytical temperature "surface" or an experimentally measured temperature distribution. The analytical temperature distribution is specified in the form of a polynomial in the independent space variables as shown in Fig. 2. The only requirement for the measured temperature is that the measurements be made at a sufficient number of points to accurately define the temperature distribution.

The magnitude of the temperature distribution can be changed by inputing a series of ΔT's. In this case, since T_{ref} is used as 1.0, the ΔT's are input as the actual value of the temperature desired (in degrees Fahrenheit or Centigrade depending on the system of units used).

Any experimentally measured temperature distribution may be input. The values of the temperature at the integration points are calculated by a two-dimensional, quadratic interpolation subroutine. The temperatures are input on a rectangular grid. The points are evenly spaced in the ξ and η-directions although the respective spacings need not be equal (i.e. the elements of the grid need not be square). A sample of the grid and an explanation of the defining parameters is shown in Fig. 3.

KC (I)	=	number of the first horizontal line at the I^{th} vertical line (I = 1, 2, .., NTX)
LC(I)	=	the number of the last horizontal line at the I^{th} vertical line (I = 1, 2, .., NTX)
DTX	=	distance between vertical lines
DTY	=	distance between horizontal lines

(XT1, YT1) = coordinates of lower left hand point of the grid.

NPTS = (not input) is calculated internally. This is the total number of grid points

For the grid in Fig. 3,

NTX	= 7
KC(I)	= 1, 2, 2, 3, 3, 4, 4
LK(I)	= 14, 14, 13, 13, 12, 12, 11
DTX	= .142
DTY	= .13
(XT1, YT1)	= (0.0, - .8)

The temperatures at the grid points are input from bottom-to-top for each vertical line starting from the left side. This sequence is shown by the circled numbers in Fig. 3.

Interpolation will be attempted at any point within DTX and/or DTY of one of the grid points. This is a modification of a program contained in Ref. 18, in which a complete description is given.

The variable called TREF in the program is not actually used anywhere in the calculations. It is simply used as additional information to be output. Thus there are two ways of inputing the temperature distributions and incrementing the ΔT values.

The first method is to simply input the actual magnitudes of the temperatures on the plate. In this case the values of ΔT will be of 0 (1). At $\Delta T=1°$, then, the eigenvalues calculated will give the frequencies of the plate for the input temperature distribution.

If desired, the temperature distribution may be normalized with respect to the temperature, TREF, at some reference point on the plate. In this case as with the analytical distribution, the ΔT's are input as the actual value of the temperature desired at the reference point.

F. Thickness Distribution

The thickness distribution h/h_r, is symetric about the ξ-η plane and is described by two polynomials in ξ and η. One gives the distribution on surface 1 and the other on surface 2. These two surfaces are separated by a line from the origin of the coordinate system at the angle β. The value h_o (called TO in the program) is the thickness at the origin.

G. Miscellaneous Comments

The eigenvalue routine used here is a double precision version of the subroutine "NROOT" from the IBM Scientific Subroutine Package. (Note that this requires a double precision version of the subroutine "EIGEN" from the same source). The subroutine "DMINV" and "DGMPRD" (no listing given) are used directly from that source.

Extensive use was made of the disk storage available in writing the program. This reduced the core storage requirements to around 250,000 bytes on the IBM 370-165 computer used. Although the execution time for the program using all core storage would be about one-third of that using disk storage, the program would be limited to only thirty deflection and stress function terms and ten quadrature points. Also, core storage was a premium at the time of writing because of the large amount of business done by the Computer Center at The Ohio State University.

It should also be noted that for the coordinate system used some plates will be symmetric about the x-axis. In these cases the even and odd terms in the assumed solution uncouple. Thus, the deflection function may be separated into one function containing only even terms in η and one containing only odd terms in η. Each of these functions can then be input separately to give all even modes or all odd modes respectively. The same comments also apply to the stress function.

IV. RESULTS

To compare to results in the literature, a conversion from the notation used in most other sources to that used herein is necessary. As long as the results are presented in non-dimensional form, only ratios of the material properties are required. Thus let,

$$E_{11}/E_{22} = D_x/D_y$$

$$G_{12}/E_{22} = D_k/D_y$$

$$E_{12}/E_{22} = D_{xy}/D_y - 2\ D_k/D_y \quad .$$

All the data presented here will be converted using these relations to the notation previously described.

All of the computations presented in this section were made using a 36 mixed term deflection function,

$$w\ (\xi,\eta) = g\ (\xi,\eta) \sum_{i=o}^{5} \sum_{j=o}^{5} h_{ij}\xi^i\eta^j \quad .$$

Thus, the first thirty-six modes and frequencies were calculated. The runs took an average of three minutes (Central Processing Unit) time on the IBM 370-165.

No effort was made to optimize the program, the purpose being to obtain consistantly good results for any planform shape with any boundary conditon. e.g., acceptable results can be obtained for the torsion mode of a rectangular cantilever plate with only three terms in the deflection function. However, this number of terms is completely inadequate for any other mode of the thermally stressed cantilever plate and is inadequate for any mode of any other of the many plates investigated. Thus, the large number of terms in both the stress function and the displacement function may be much greater than required for some of the problems solved. This point is immaterial when the choice boils down to either obtaining an accurate quantitative answer in which one can have confidence or some answer that may only be in the "ball park."

A. Comparison of Orthotropic Results Without Thermal Loading

As was previously stated, the material published without thermal loading is voluminous. For the sake of brevity, only a few comparisons will be made.

Tables 1 and 2 are comparisons of calculated frequencies from the literature with those calculated by this program. It can be seen that the method under discussion here gives excellent agreement with those frequencies. The expression for λ_1^2 is given in the list of symbols and in Appendix A.

Table 3 gives a comparison with some experimental frequencies for plywood plates. Note that the experimental values are higher than the calculated frequencies. Because the method used here gives solutions which converge from above the exact solution, these errors are attributed to restraints inherent in the experimental approximation of the simply supported boundary conditions.

B. Comparison of Isotropic Results With Thermal Loading

As was stated previously, no experimental data were found in the literature on the effect of thermal stresses on orthotropic plates. Thus, a brief quantitative comparison is made with experimental data from Ref. 16 for the special case of the isotropic plate. The purpose is to show the agreement that was obtained for widely different cases. The isotropic plate elastic properties requires that,

$$E_{11} = E_{22} = E/(1-\nu^2)$$

$$E_{12} = \nu E/(1-\nu^2)$$

$$G_{12} = E/2(1+\nu)$$

As in Ref. 16, nominal values of the plate material properties used were,

$$E = 10^7 \text{ psi}$$

$$\nu = \frac{1}{3}$$

$$\alpha = 12.8\text{x}10^{-6}/^\circ\text{F}$$

Fig. 4 shows a comparison for the first five modes of a square cantilever plate.[16] It is interesting that the fourth and fifth mode frequency curves cross. A typical experimentally measured temperature distribution resulting from radiant lamp edge heating is shown in Fig. 5.

Fig. 6 presents an unsymetrical trapezoidal cantilever plate that does not appear in Ref. 16. Only the first two modes were recorded for this plate. One of the temperature distributions measured on this plate is shown in

Fig. 7.

Because of its boundary conditons and the choice of coordinate system, the plate shown in Fig. 8 is also unsymetrical. The stresses as well as the deflections are affected by the boundary conditions.[16] Here again, as in Fig. 4, two of the frequencies cross. The heating elements were centered over the diagonal from lower-left to upper-right giving a temperature distribution as shown in Fig. 9.

The frequencies of a plate with a single point clamped is shown in Fig. 10. The agreement with the four modes measured is seen to be good.

A plate with homogeneous pinned-free boundary conditions is shown in Fig. 11. Only the first two modes were recorded for this plate. The third calculated mode frequency is also shown. The temperature distribution shown in Fig. 5 is also typical of that used to calculate the frequencies for the plates in Figs. 10 and 11.

C. Comparison of Orthotropic Results With Thermal Loading

Figs. 12, 13, 14, 15, 16 and 17 constitute the results of a very brief study that indicates the large effect that orthotropy can have in the presence of thermal gradients. For the sake of brevity, only one boundary condition, the cantilever plate, and only one assumed temperature distribution, $T = \Delta T|\eta|^3$, is presented. Also, only the two lowest modes are presented although as many of the higher modes as could possibly be desired are available in the computer print-out. It should be noted that orthotropy does not change the characteristic shape of the response curves shown. However, because of the influence that the directional properties of the material can have on the stress field for a given temperature distribution, orthotropy can produce marked increases in the loss of effective stiffness for a given heating rate. In each figure the isotropic response curves are given for comparison purposes.

In Figs. 12 and 13, it can be seen that doubling the thermal expansion coefficient, α_2, in the chordwise direction has little effect on the frequency for this temperature distribution because the chordwise stress component effect is small for this plate and remains small even when α_2 is doubled. However, when the longitudinal coefficient, α_1, is doubled, the longitudinal stress component is essentially doubled and the frequency is seen to decrease at a markedly higher rate. Thermal buckling is reached at a temperature only one half as great as before. This means that, for a given heating rate, if an isotropic plate buckles within thirty seconds, an orthotropic plate with this ratio of thermal coefficients would buckle in only fifteen seconds.

In Fig. 14 it can be observed that doubling the chordwise modulus of elasticity, E_{22}, actually produces a decrease in the rate of frequency decay and an increase in ΔT_{cr} for the bending mode. That the various modes will not behave in the same way for a given material property change is shown in Fig. 15 where the doubled chordwise modulus produces the opposite effect on the torsion mode. Doubling the spanwise modulus, E_{11}, causes a higher rate of frequency decay with correspondingly lower buckling temperatures in both modes. It should be noted, however, that, although large, doubling the longitudional modulus of elasticity does not have the extreme effect that is caused by doubling the longitudinal conductivity coefficient.

Fig. 16 shows that the decrease in the rate of decay of the bending mode, achieved by doubling E_{22} in Fig. 14, is more than offset by the increase caused by doubling α_1 in Fig. 12. Both Figs. 16 and 17 show appreciable destabilizing effects when both the moduli and expansion coefficient are changed. It is recognized that the change in properties as used is large, but the effects are also large. Using the computer program presented herein, almost unlimited parametric studies may be made to determine trends or, more efficiently, calculations may be made to obtain answers for specific cases once a problem has been defined.

D. Effect of Stress Distribution

An interesting by-product of this program is an accurate calculation of the thermal stress distribution. The effect of the stress distribution function used in calculating the vibration frequency is shown in Fig. 15. The boundary conditions require a stress function containing both odd and even terms in η. Thus, the stress distribution calculated, using only even terms, does not give the correct stress distribution and hence, does not give the correct frequency. However, when the correct mixed terms were used, the correct stress distribution existing in the plate resulted and consequently, the calculated frequencies agreed more closely with the experimental values. The 36 mixed term results show that the analytically predicted stress distribution had effectively converged when 24 mixed terms were used.

The three sets of frequencies calculated were compared to the experimental response of the plate shown in Fig. 8. The results show that correct stresses are in fact necessary in order to obtain the correct frequencies.

V. CONCLUDING REMARKS

The computer program presented herein is, in effect, a general solution to the problem of the linear vibration of thermally stressed trapezoidal plates. The theory has been verified experimentally for thermally stressed isotropic plates and has been found to agree favorably with analytical data found in the literature for orthotropic plates with no thermal loading. It appears that accurate results can be obtained by the methods herein described for almost any boundary conditions of practical importance. The solutions, based on linear theory, do not hold as the buckling region is approached because of the non-linear effects of large deflections.

Experience in using the program shows that the number of terms required in the assumed solutions increases with the complexity of the geometry. However, consistantly accurate results are obtained using a 30-36 term displacement function and a 24-30 term stress function. Accurate integration can be obtained for this number of terms using ten quadrature points in the ξ-direction (twenty points in the η-direction).

A very extensive experimental program with orthotropic plates would be required to verify all the facets of the program as presented. However, the data comparisons shown combined with many cases of experimental isotropic plate data not presented herein gives the writers great confidence in the analytical results.

The generality of this program should not be overlooked. Its extension to obtain the accurate solution of the flutter of thermally stressed plates and panels can be readily made. A natural extension of this work would be to examine, without the assumption of mode identity, the large deflection effects observable as heating progresses. A recently developed method of solving large sets of non-linear equations shows great promise in the area of large deflections which has yet to be effectively investigated.

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TABLE 1

PF-CC-PF-CC

AR	E_{11}	E_{12}	E_{22}	G_{12}	λ_1 Calculated By Program	λ_1 Reference Values	Ref. No.
1.0	1.0	1.0	1.0	0.0	28.93	29.29	8
1.0	2.0	1.0	1.0	0.0	21.6	21.82	8
1.0	1.0	3.0	1.0	0.0	36.1	36.5	8
1.0	3.0	3.0	1.0	0.0	22.35	22.56	8
1.0	6.0	2.0	1.0	0.0	16.12	16.13	8
1.0	1.0	1.0	2.0	0.0	50.8	51.5	8
1.0	1.0	3.0	9.0	0.0	73.0	74.0	8
2.0	3.117	0.12	1.0	0.264	53.6	53.7	3

TABLE 2

PF-FF-PF-CF AR = 2.0

E_{11} = 3.177, E_{12}=0.12, E_{22}=1.0, G_{12}=0.264

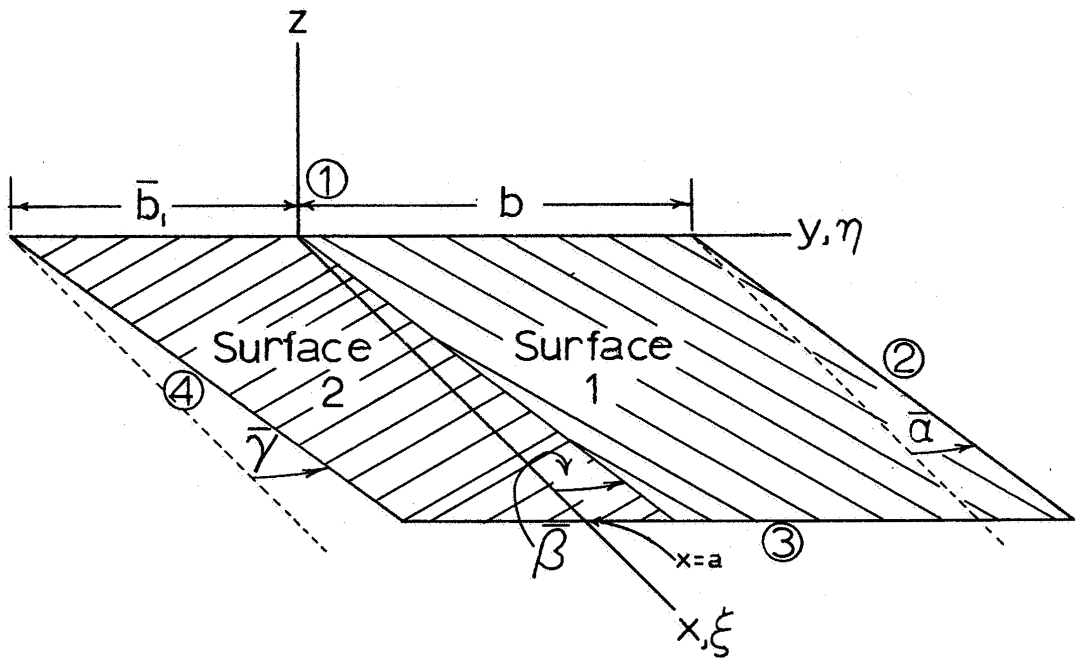
Mode No.	λ_1	
	Calculated By Program	REF 21 Values
1	14.75	14.75
3	55.35	55.4
5	93.1	91.6
6	121.4	120.0
7	144.4	144.1
12	256.1	249.0
13	295.1	278.2

TABLE 3

PF-PF-PF-PF AR = 1.0, a = 45.8 cm.

$E_{11} \times 10^{-10}$ Dynes/cm^2	$E_{12} \times 10^{-10}$ Dynes/cm^2	$E_{22} \times 10^{-10}$ Dynes/cm^2	$G_{12} \times 10^{-10}$ Dynes/cm^2	ρ gm./cc.	h cm.	f_1 (cps)	
						Calc.	REF. 2
6.9	0.17	0.17	0.30	0.33	0.291	32.06	35
7.4	0.17	0.05	0.34	0.39	0.323	33.8	39
7.9	0.19	0.19	0.45	0.399	0.310	34.0	34
13.3	0.33	0.55	0.85	0.67	0.309	34.5	41

General Planform Parameters
and
Non-Dimensionalization



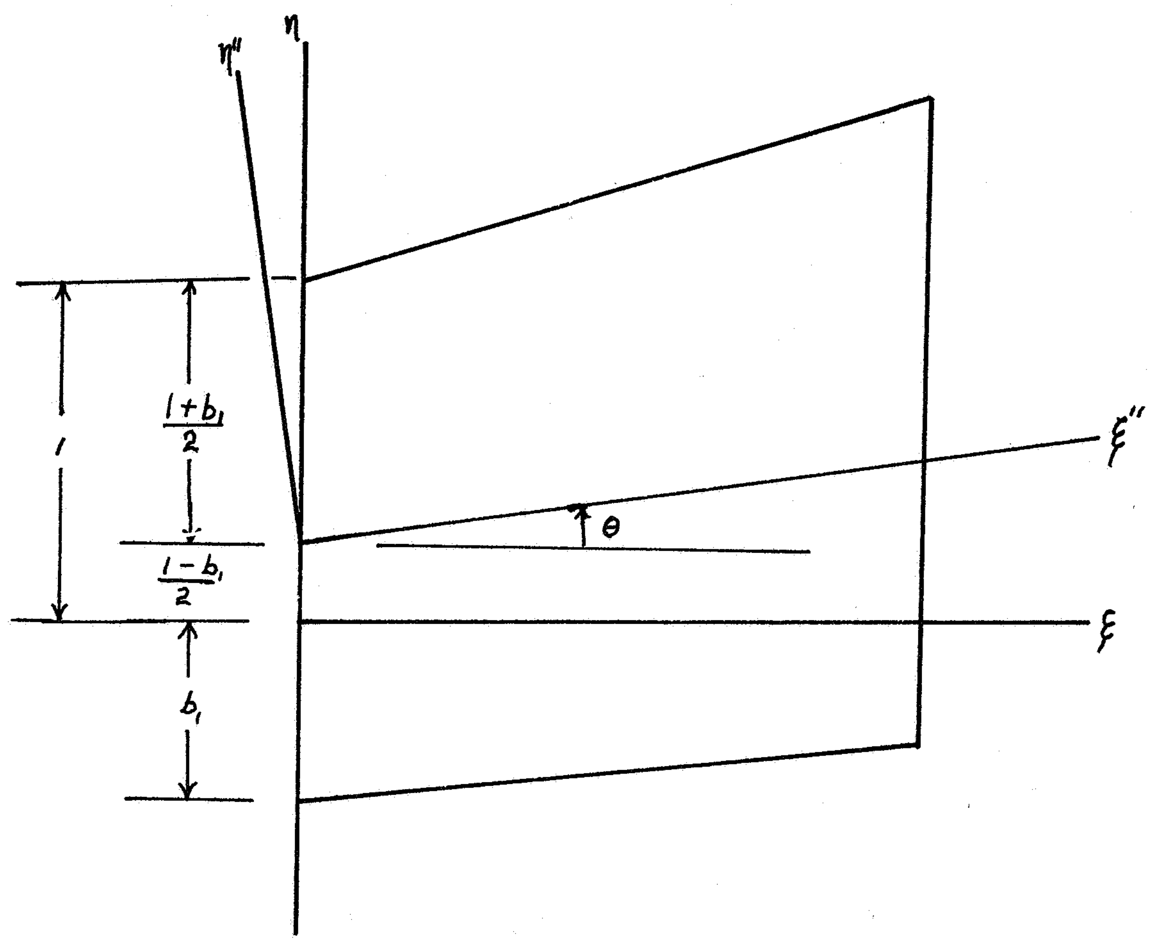
$$\xi=\frac{x}{a} \qquad \eta=\frac{y}{b}$$

$$\alpha=\frac{a}{b}\tan\bar{\alpha} \qquad \beta=\frac{a}{b}\tan\bar{\beta} \qquad \gamma=\frac{a}{b}\tan\bar{\gamma}$$

$$b_1=\frac{\bar{b}_1}{b} \qquad AR=\frac{a/b}{1+b_1-\left(\frac{\gamma-\alpha}{2}\right)}$$

Fig. 1

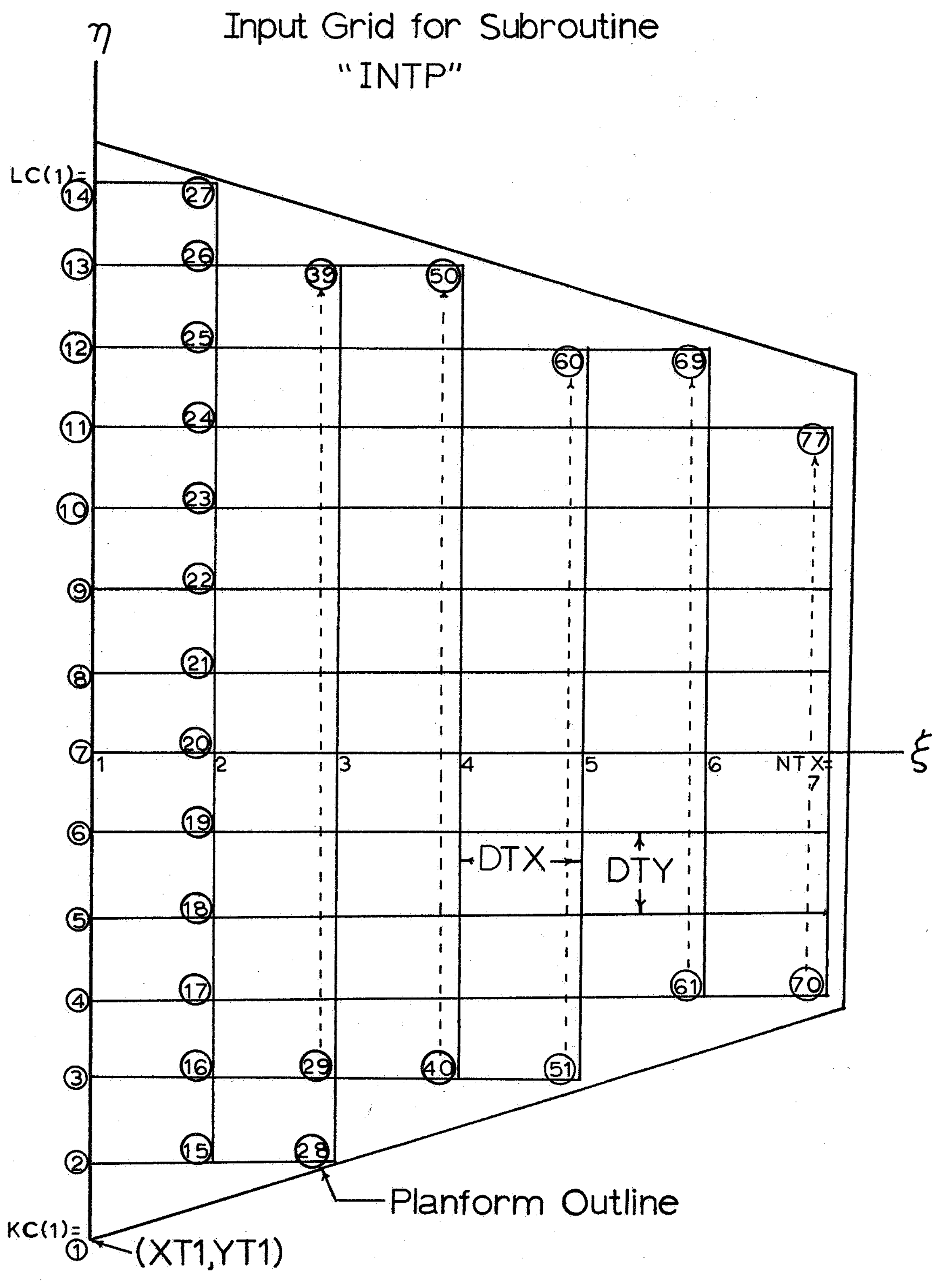
Analytical Temperature Distribution



$$T(\xi,\eta) = \sum_{I=1}^{NTEMP} TEM(I)(\bar{\eta}'')^{NTEM(I)}(\xi)^{NTEMX(I)}$$

$$\eta = \frac{|\eta''|}{\frac{1+b_1}{2}\cos\theta}$$

Fig. 2



Input Grid for Subroutine
"INTP"
η
ξ
LC(1)=
KC(1)=
1
2
3
4
5
6
NTX=
7
DTX
DTY
Planform Outline
(XT1,YT1)

Fig. 3

Comparison of Analytical and
Experimental Response of a
CC-FF-FF-FF Plate
AR=1.0 , a=18" , h=3/16"
— measured
⊙ calculated

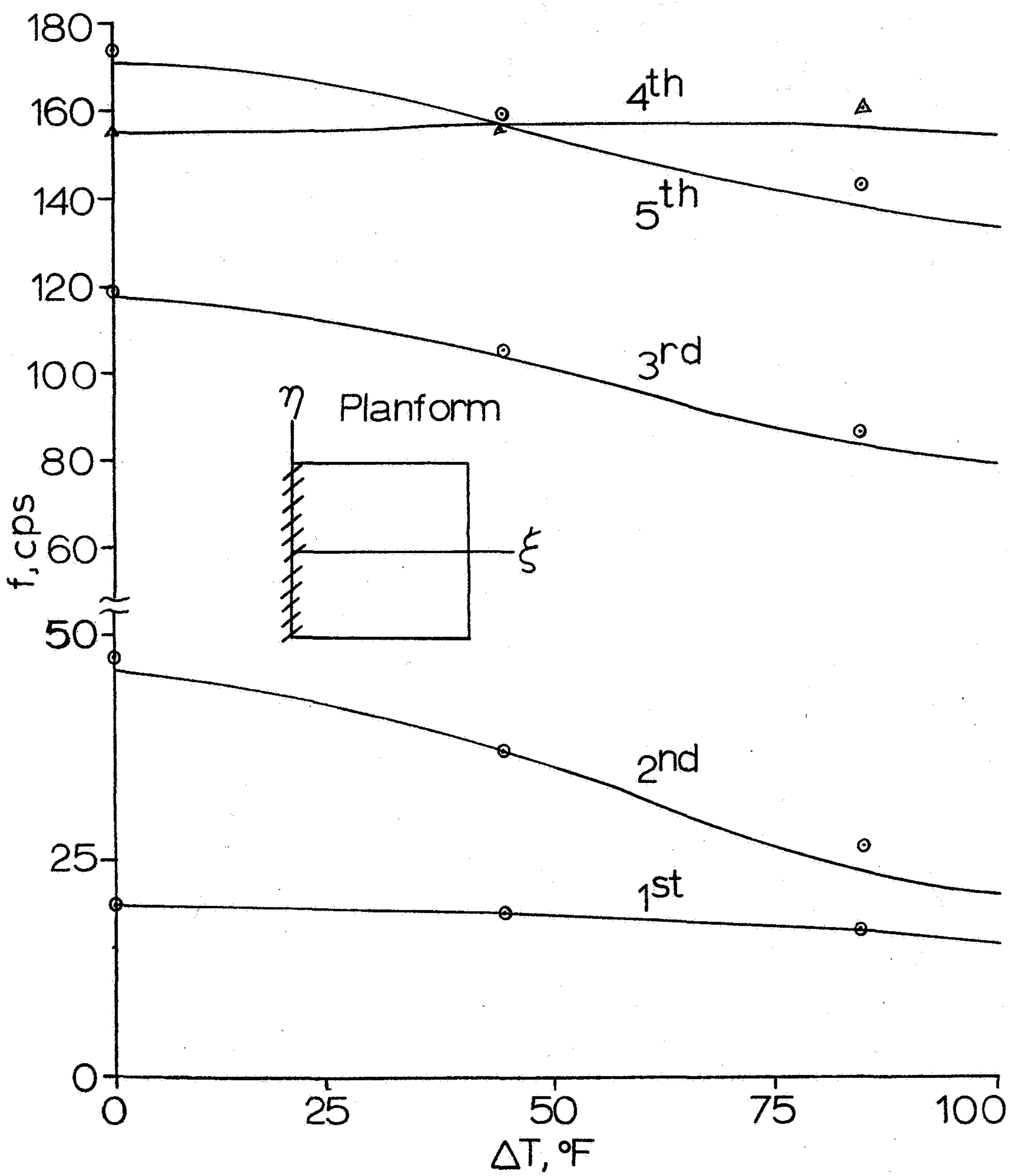


Fig. 4

Typical Temperature
Distribution used in
Fig. 4,10,11

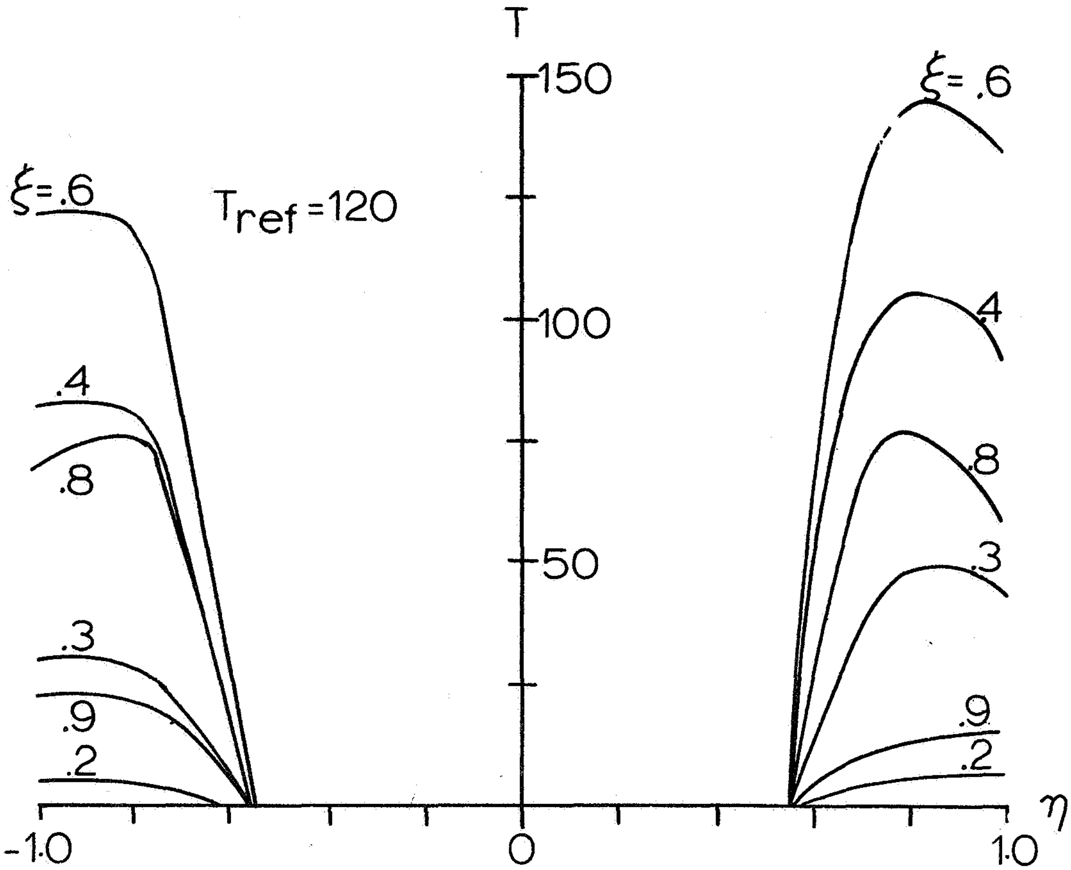


Fig. 5

Comparison of Analytical and
Experimental Response of a
CC-FF-FF-FF Plate
$\alpha=-0.6$ $\beta=0.0$ $\gamma=0.0$
AR=5/3 , a=20", h=1/4"
— measured
∘ calculated

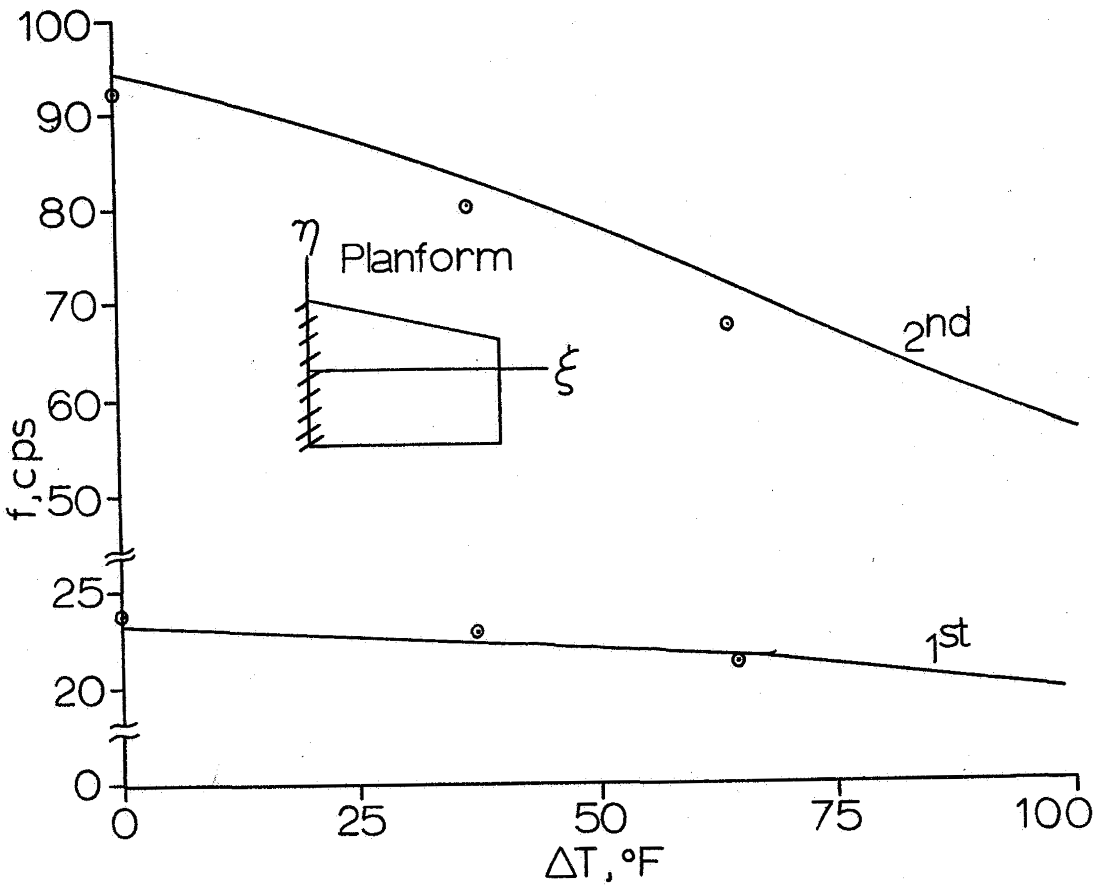


Fig. 6

Temperature Distribution for Fig. 6

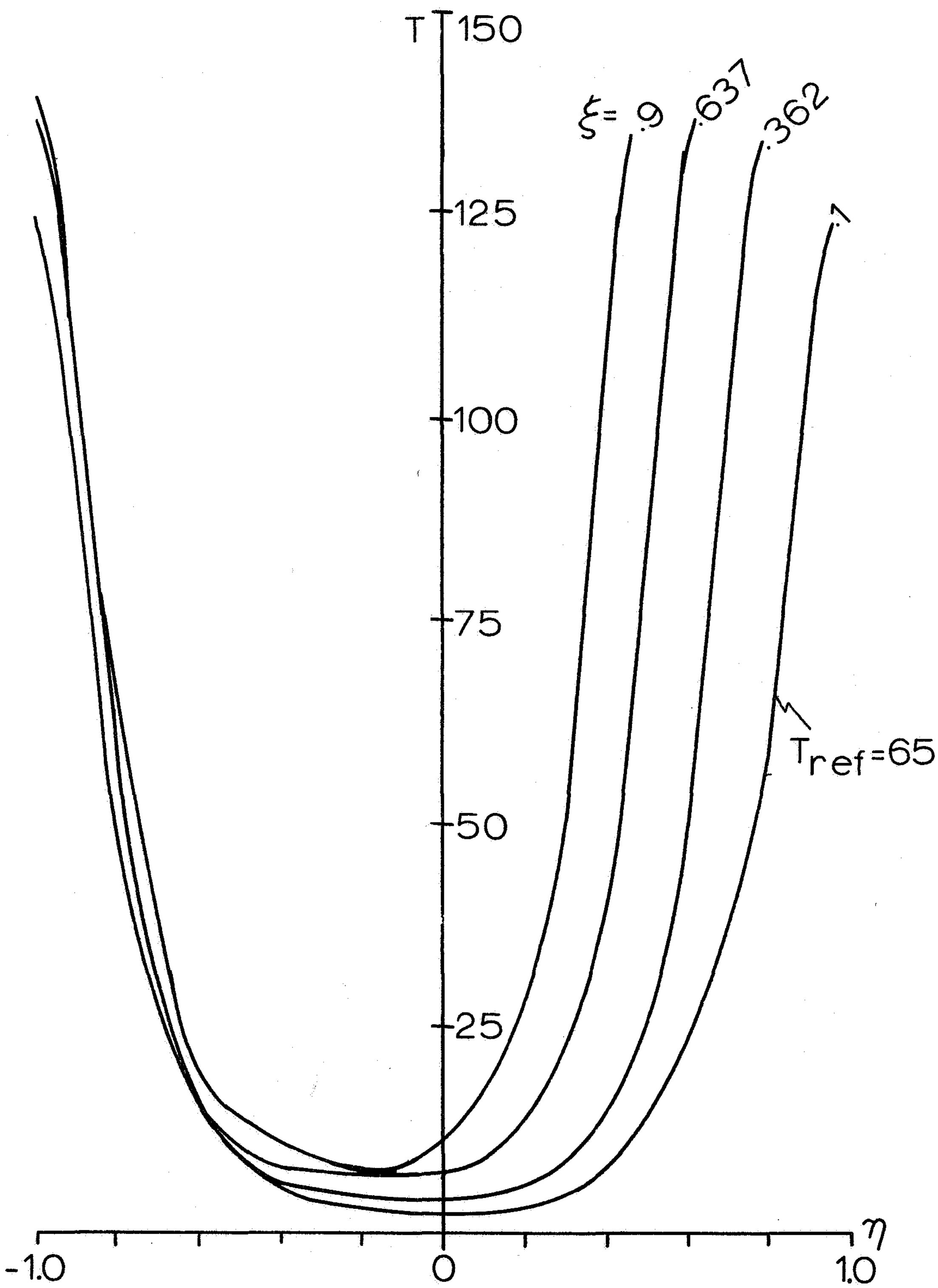


Fig. 7

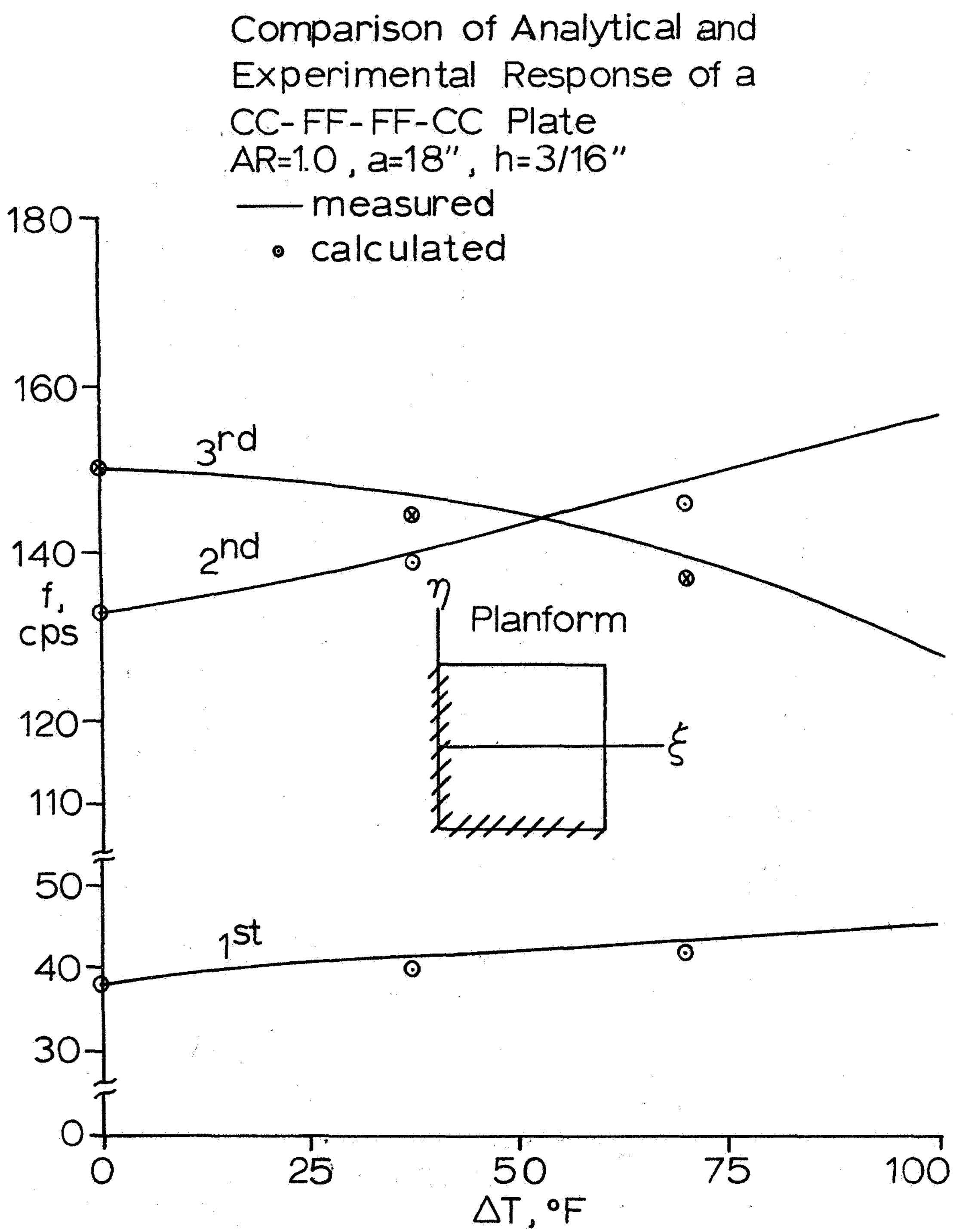

Comparison of Analytical and
Experimental Response of a
CC-FF-FF-CC Plate
AR=1.0 , a=18", h=3/16"
measured
calculated
180
160
140
120
110
50
40
30
0
f,
cps
3rd
2nd
1st
η
Planform
ξ
0
25
50
75
100
ΔT, °F

Fig. 8

Typical Temperature
Distribution used in
Fig. 8

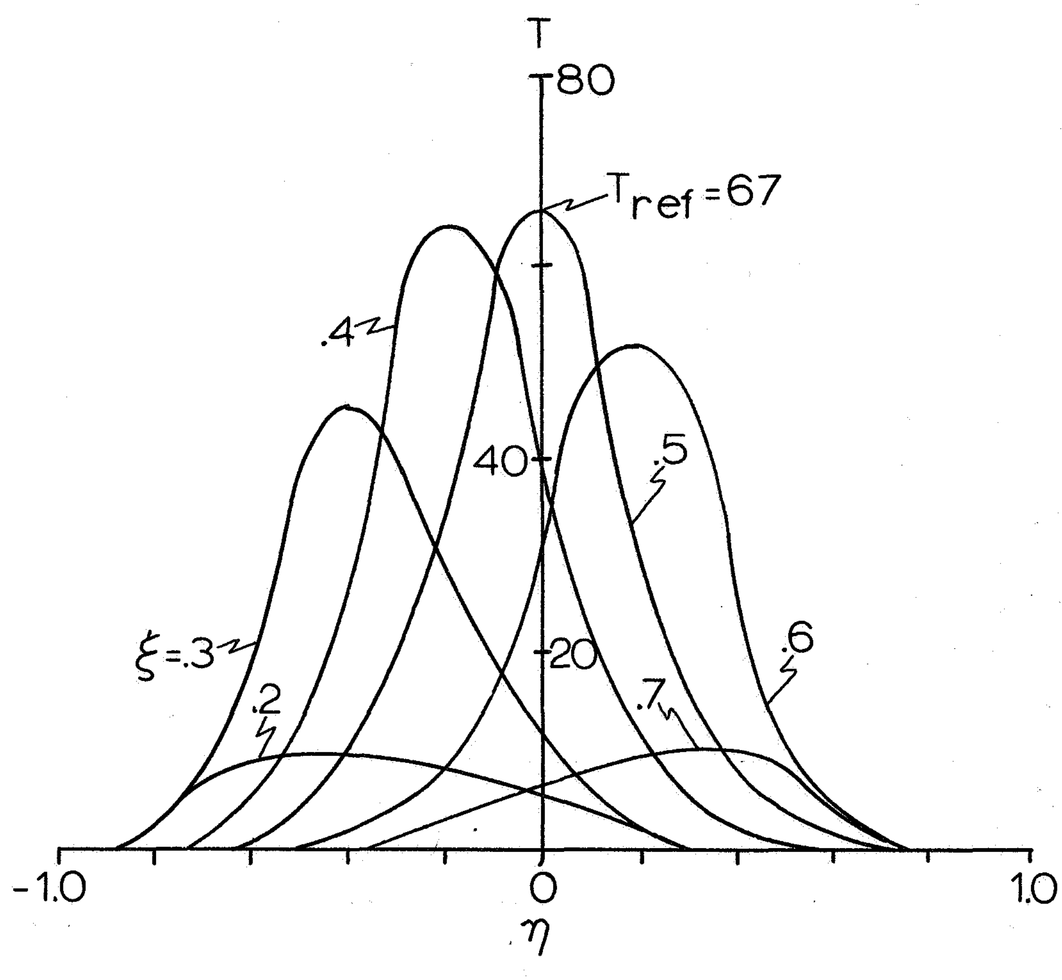


Fig. 9

Comparison of Analytical and Experimental Response of a Plate Clamped at (0,0)
AR=1.0, a=18", h=3/16"
—— measured
⊙ calculated

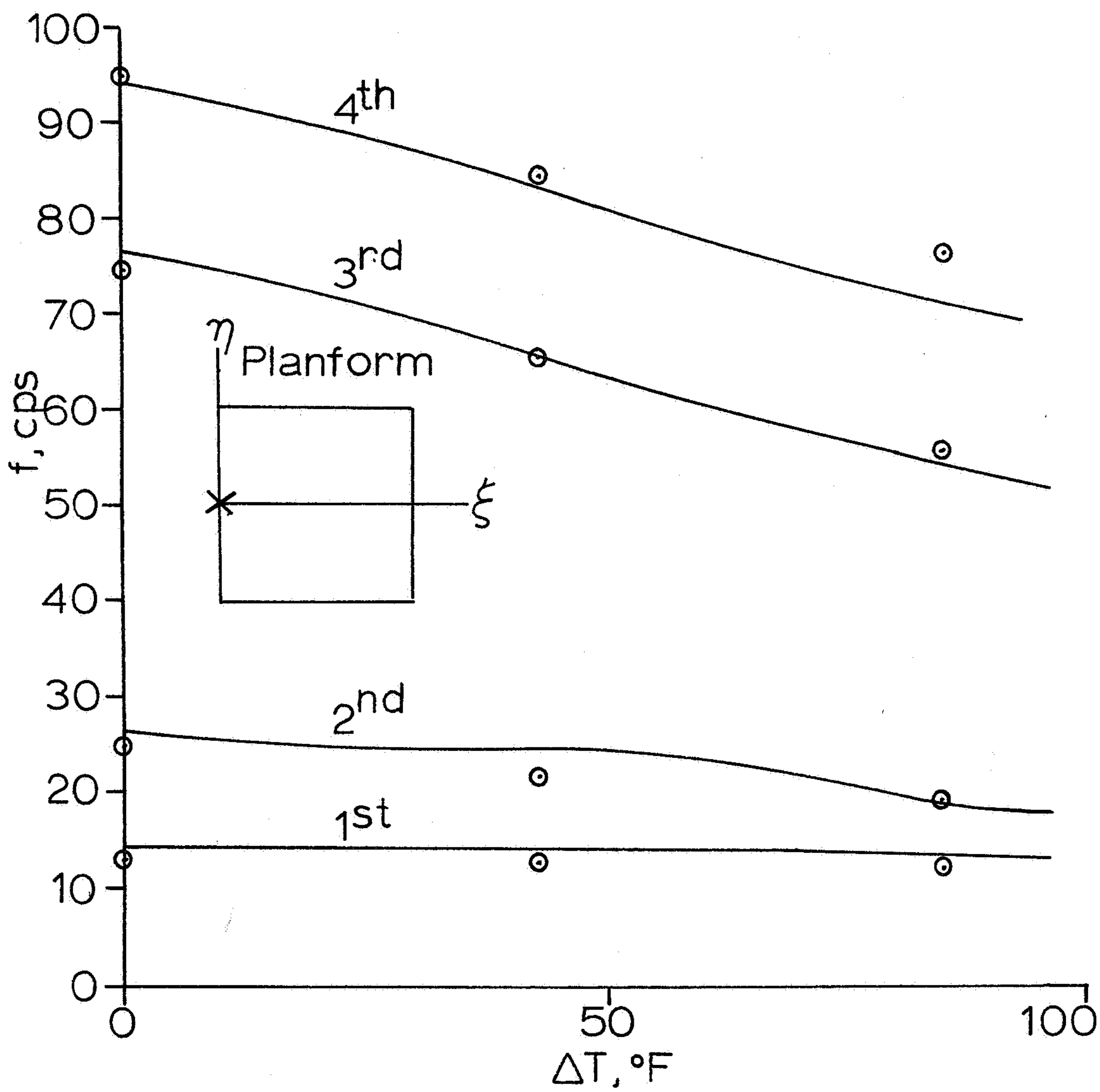


Fig. 10

Comparison of Analytical and Experimental Response of a PF-PF-PF-PF Plate

AR=1.0 , a=18", h=3/16"

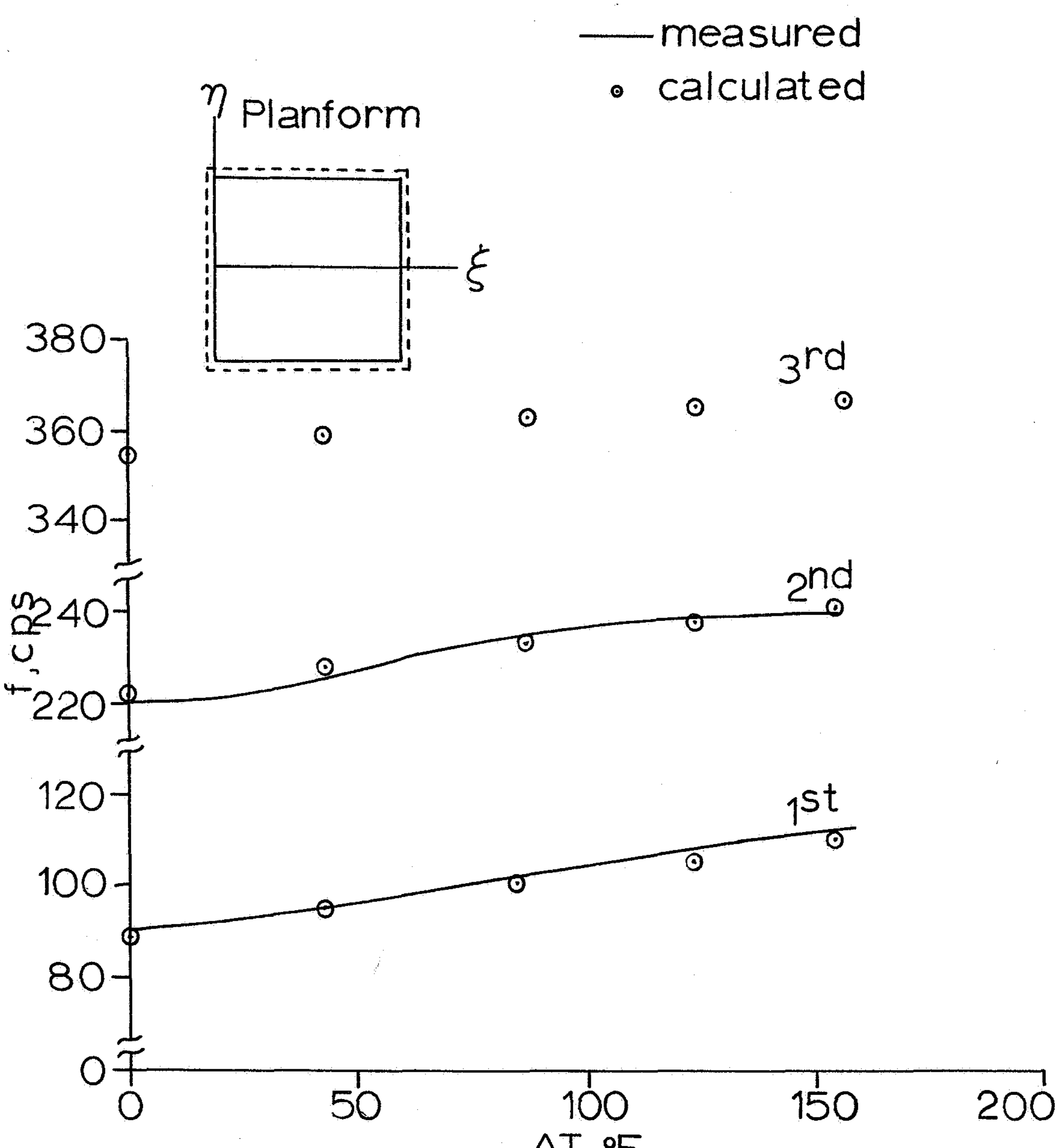


Fig. 11

Effect of α on Plate Vibration

$T = \Delta T |\eta|^3$

Constant Thickness

AR=5/3

1^{st} mode

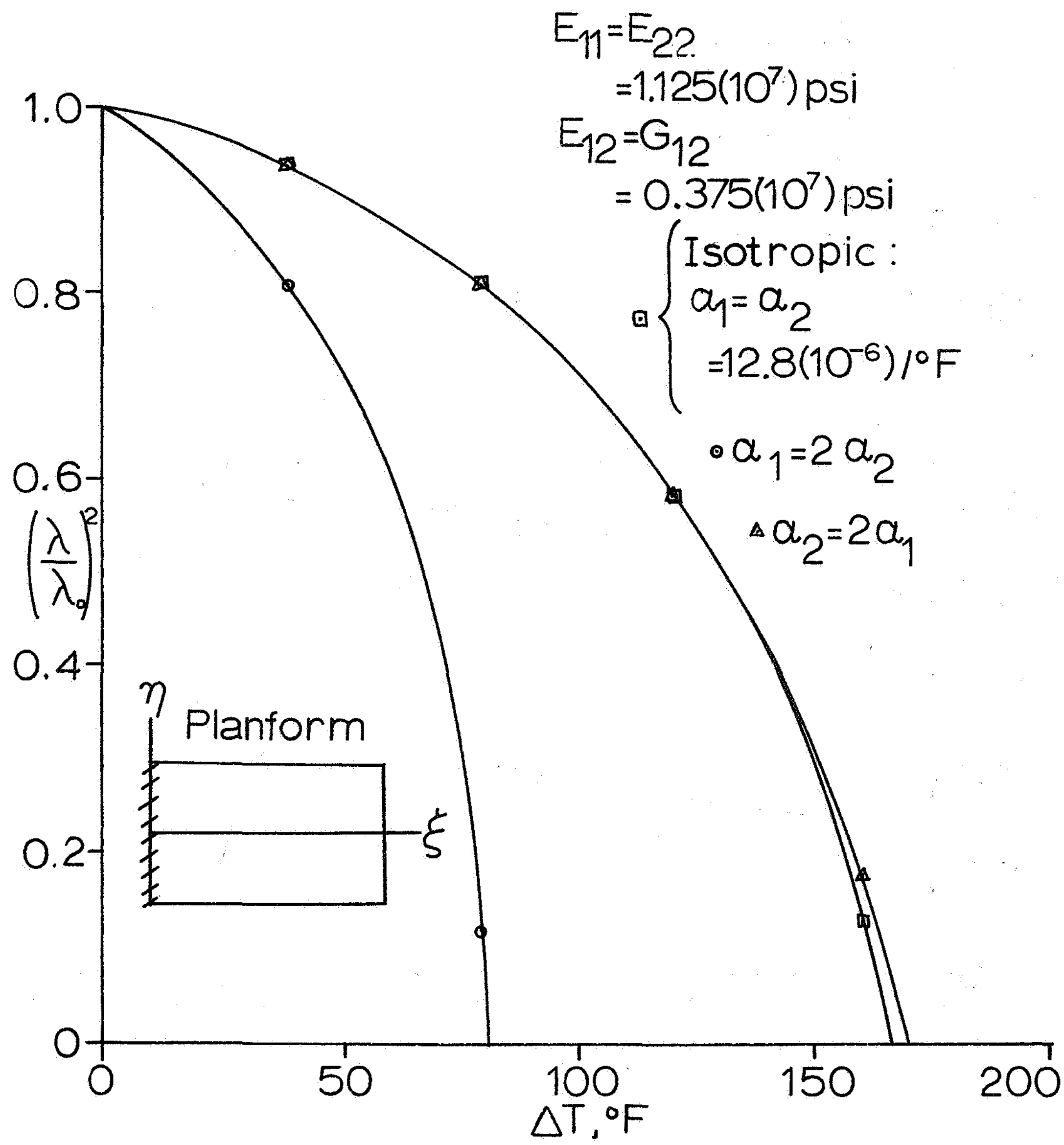


Fig. 12

Effect of α on Plate Vibration

2^{nd} mode

(See Fig.12 for notation)

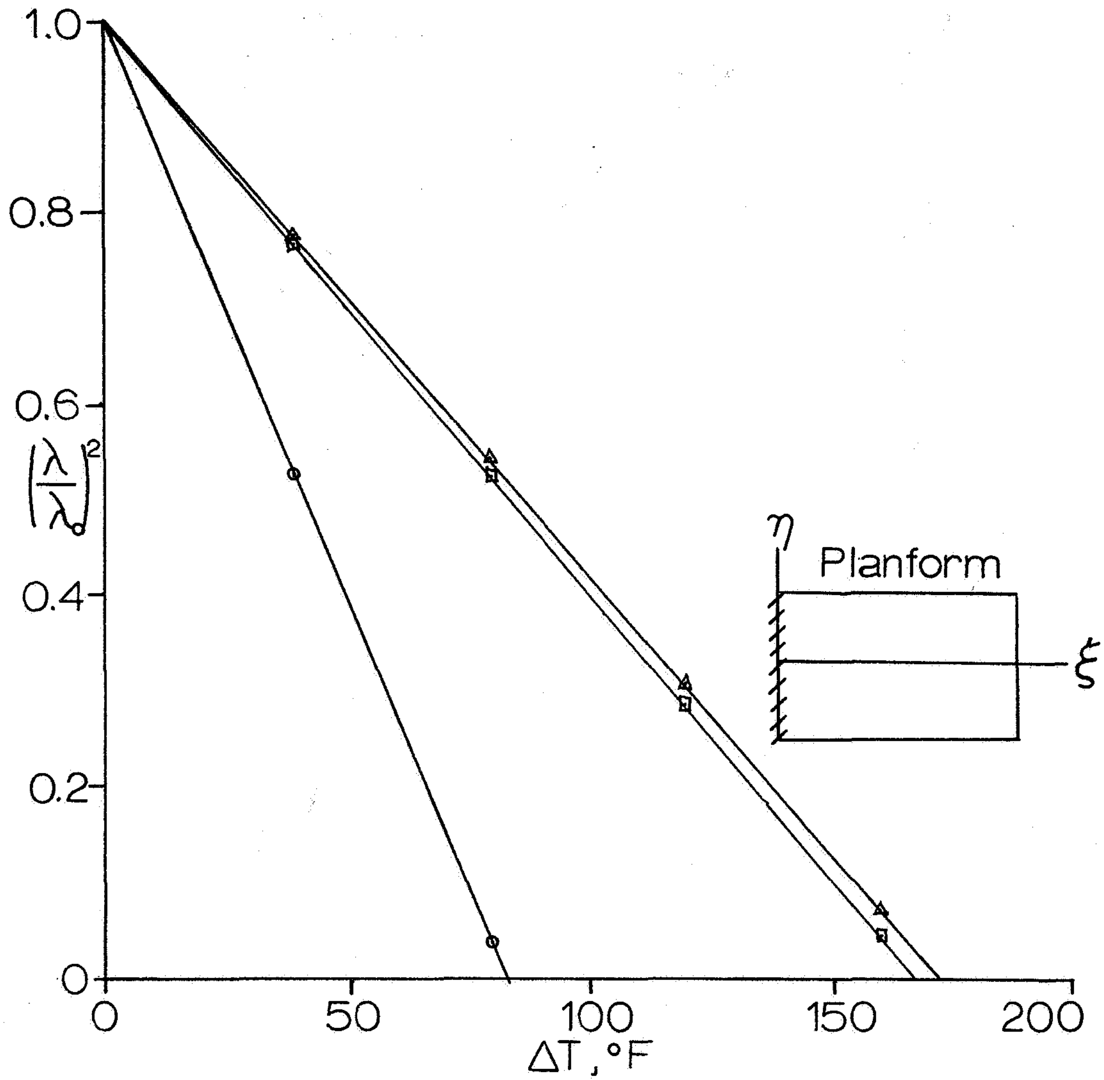
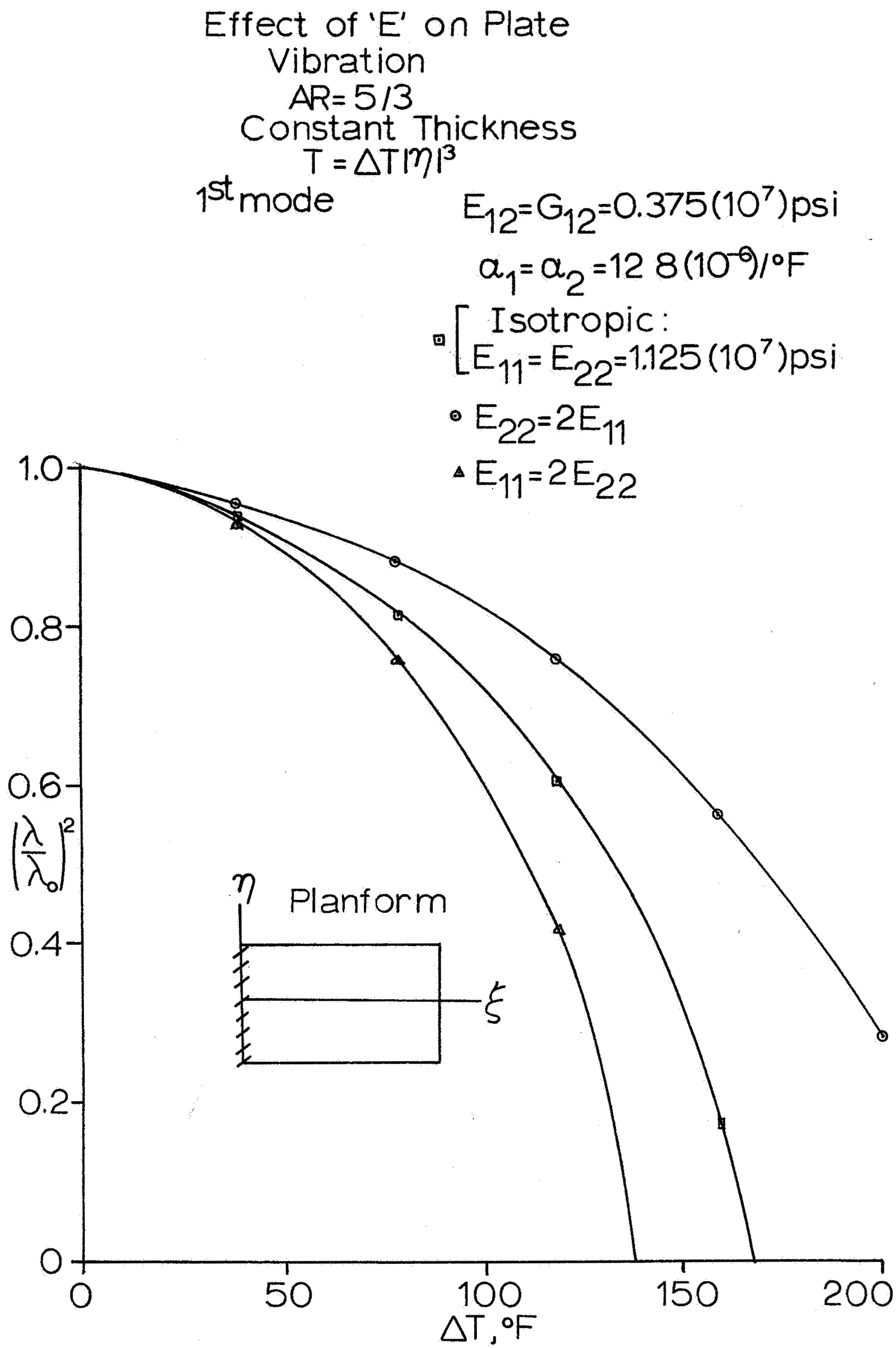


Fig.13



Effect of 'E' on Plate Vibration
AR= 5/3
Constant Thickness
T = ΔT|η|³
1st mode
E12=G12=0.375(10⁷)psi
α1=α2=12 8(10⁻⁶)/°F
Isotropic:
E11= E22=1.125(10⁷)psi
E22=2E11
E11=2E22
1.0
0.8
0.6
0.4
0.2
0
(λ/λo)²
η
Planform
ξ
0
50
100
150
200
ΔT, °F

Fig. 14

Effect of 'E' on Plate Vibration
2nd mode
(See Fig 14 for notation)

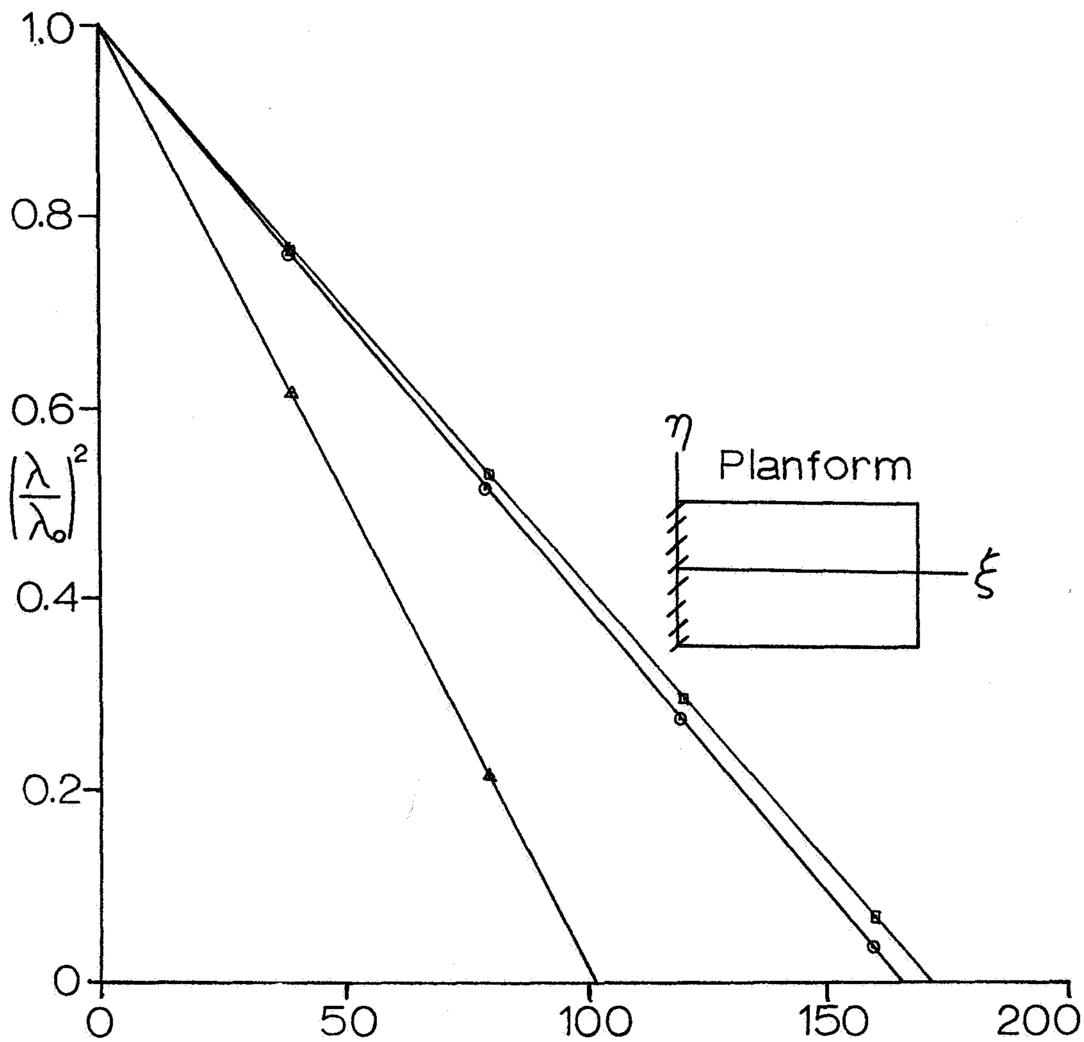
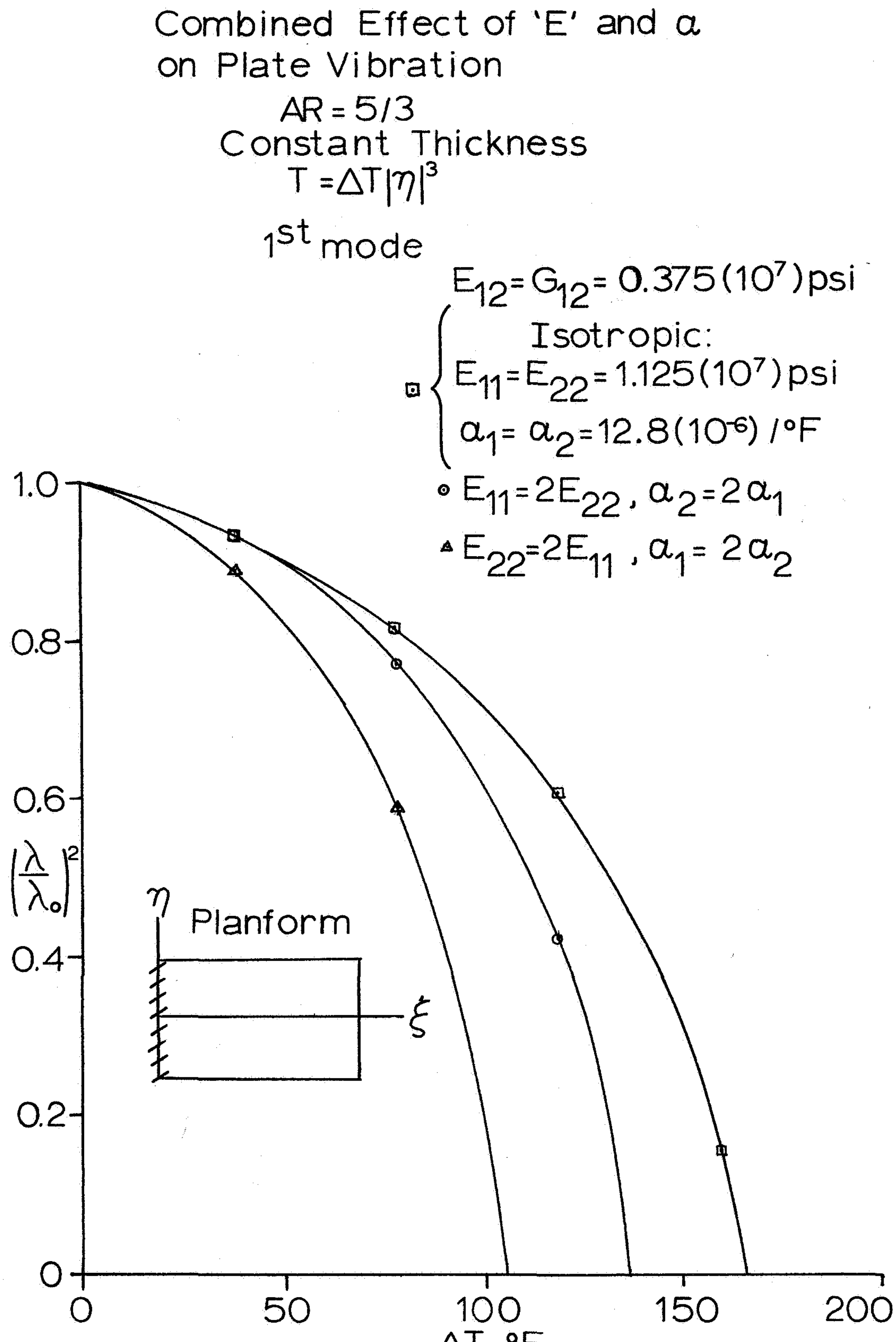


Fig. 15



Combined Effect of 'E' and α on Plate Vibration
AR = 5/3
Constant Thickness
T = ΔT|η|³
1st mode
E12 = G12 = 0.375(10⁷) psi
Isotropic:
E11 = E22 = 1.125(10⁷) psi
α1 = α2 = 12.8(10⁻⁶) /°F
E11 = 2E22 , α2 = 2α1
E22 = 2E11 , α1 = 2α2
1.0
0.8
0.6
(λ/λ₀)²
0.4
0.2
0
η
Planform
ξ
0
50
100
150
200
ΔT, °F

Fig. 16

Combined Effect of 'E' and α on Plate Vibration

2^{nd} mode

(See Fig. 16 for notation)

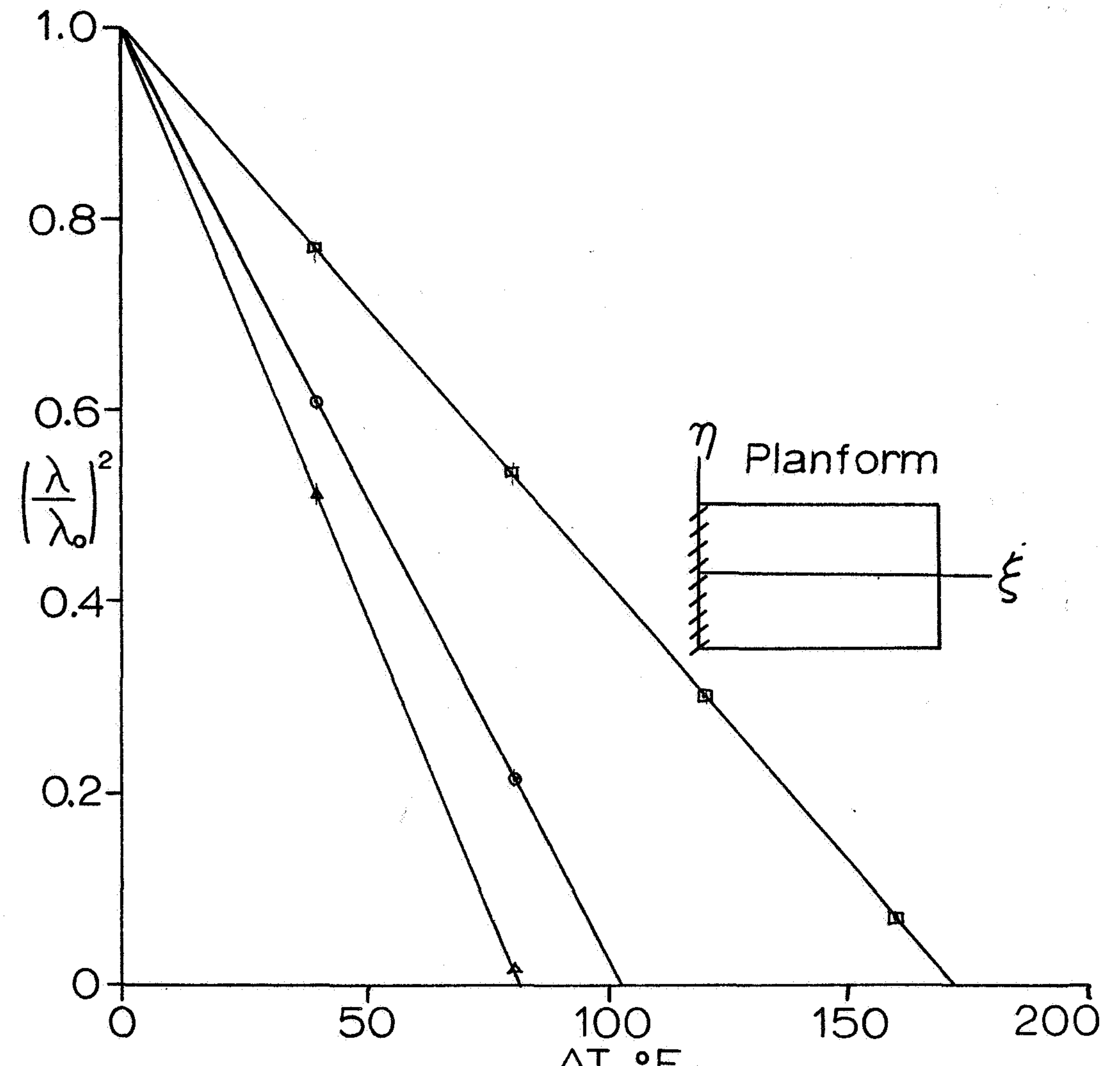
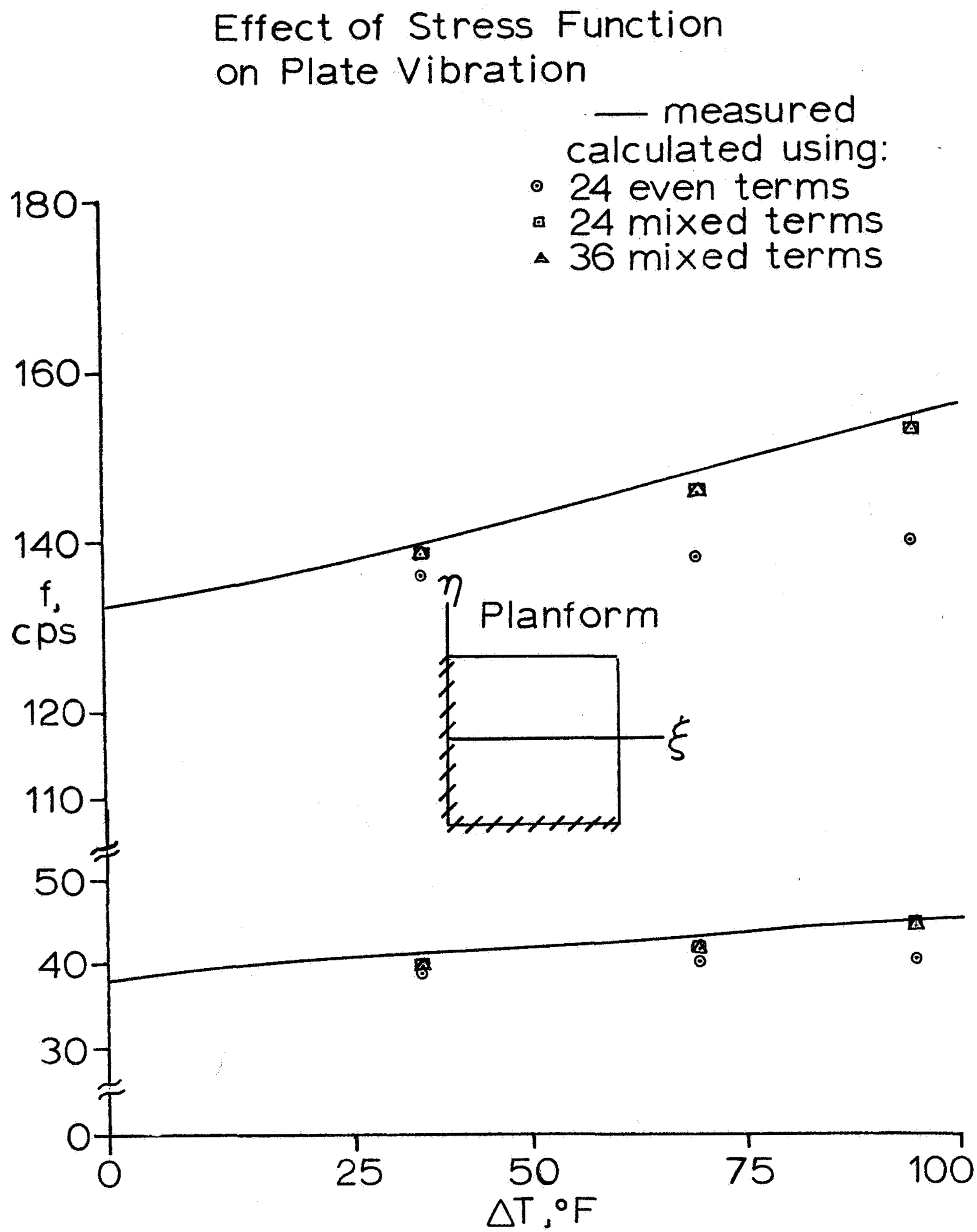


Fig. 17


Effect of Stress Function
on Plate Vibration
— measured
calculated using:
24 even terms
24 mixed terms
36 mixed terms
180
160
140
120
110
50
40
30
0
f,
cps
η
Planform
ξ
0
25
50
75
100
ΔT, °F
Fig. 18

Appendix A

Matrix Elements and Parameters

$$B_{ij,k\ell} = \iint (\frac{h}{h_r})^3 \{(\alpha_{ij})_{\xi\xi} (\alpha_{k\ell})_{\xi\xi} + \frac{E_{22}}{E_{11}} (\frac{a}{b})^4 (\alpha_{ij})_{\eta\eta} (\alpha_{k\ell})_{\eta\eta}$$

$$+ (\frac{a}{b})^2 \frac{E_{12}}{E_{11}} [(\alpha_{ij})_{\xi\xi} (\alpha_{k\ell})_{\eta\eta} + (\alpha_{ij})_{\eta\eta} (\alpha_{k\ell})_{\xi\xi}]$$

$$+ 4(\frac{a}{b})^2 \frac{G_{12}}{E_{11}} (\alpha_{ij})_{\xi\eta} (\alpha_{k\ell})_{\xi\eta}\} \, d\xi d\eta$$

$$M_{ij,k\ell} = \iint \{F_{\eta\eta}(\alpha_{ij})_{\xi} (\alpha_{k\ell})_{\xi} + F_{\xi\xi} (\alpha_{ij})_{\eta} (\alpha_{k\ell})_{\eta}$$

$$- F_{\xi\eta}[(\alpha_{ij})_{\xi} (\alpha_{k\ell})_{\eta} + (\alpha_{ij})_{\eta} (\alpha_{k\ell})_{\xi}]\} \, d\xi d\eta$$

$$T_{ij,k\ell} = \iint \frac{h}{h_r} (\alpha_{ij}) (\alpha_{k\ell}) \, d\xi d\eta$$

$$A_{pq,rs} = \iint \frac{h_r}{h} \{(\frac{a}{b})^4 (\gamma_{pq})_{\eta\eta}(\gamma_{rs})_{\eta\eta} + \frac{a_{22}}{a_{11}} (\gamma_{pq})_{\xi\xi}(\gamma_{rs})_{\xi\xi}$$

$$+ \frac{a_{12}}{a_{11}} (\frac{a}{b})^2 [(\gamma_{pq})_{\xi\xi}(\gamma_{rs})_{\eta\eta} + (\gamma_{pq})_{\eta\eta} (\gamma_{rs})_{\xi\xi}]$$

$$+ \frac{b_{12}}{a_{11}} (\frac{a}{b})^2 (\gamma_{pq})_{\xi\eta}(\gamma_{rs})_{\xi\eta}\} \, d\xi d\eta$$

$$\Gamma_{rs} = \iint [(\frac{a}{b})^2 (\gamma_{rs})_{\eta\eta} + \frac{\alpha_2}{\alpha_1} (\gamma_{rs})_{\xi\xi}] \, T(\xi,\eta) \, d\xi d\eta$$

$$\lambda^2 = \omega^2 \; 12\rho a^4 / E_{11} h_r^2$$

$$k_1 = \frac{12}{E_{11}} \left(\frac{a}{b}\right)^2 \left(\frac{a}{h_r}\right)^2$$

$$k_2 = ({}^{\alpha}1 \quad \Delta T/a_{11})$$

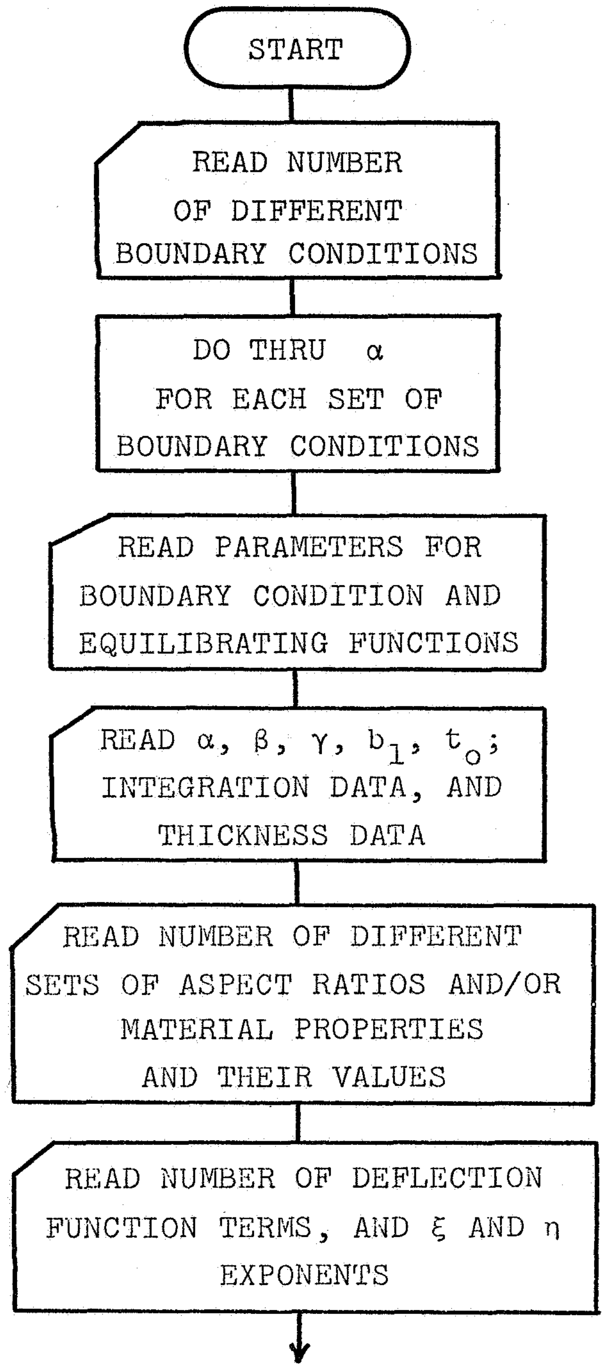
$$\{\hat{C}\} = \frac{1}{a^2 h_r} \{C\}$$

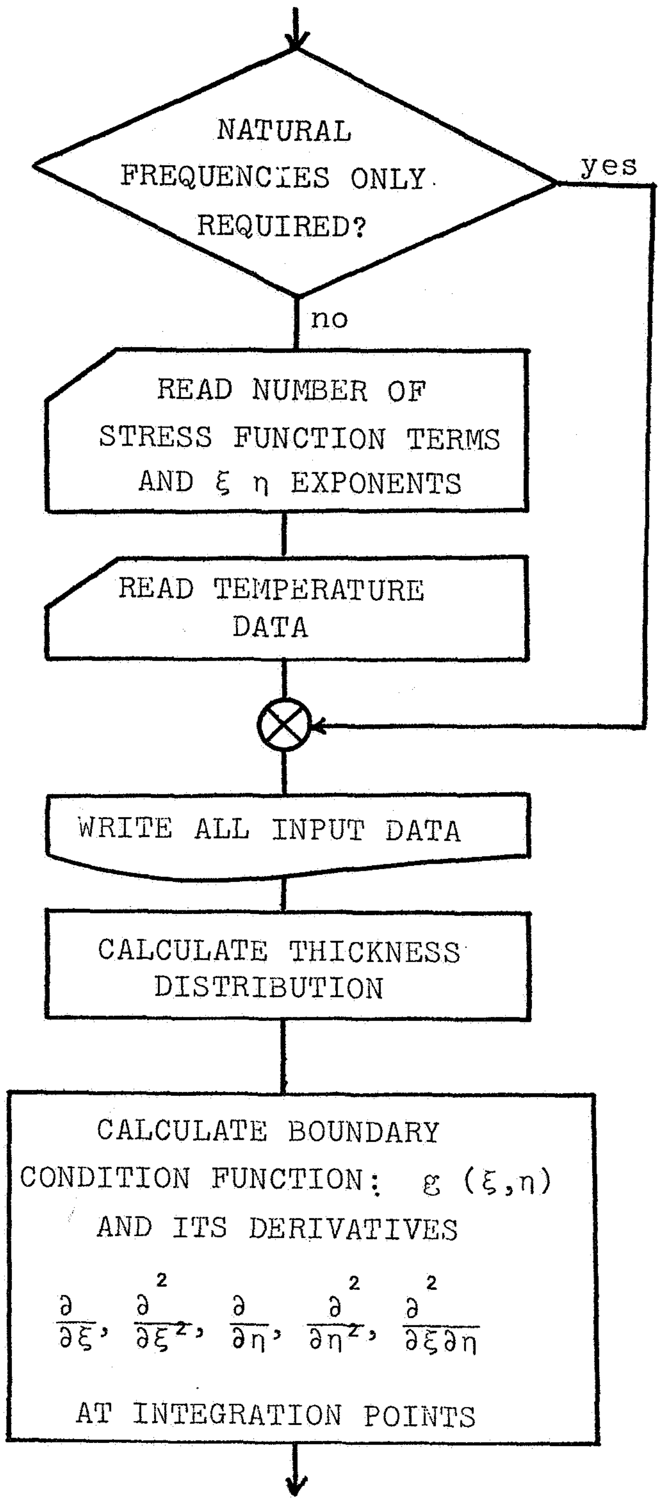
$$\alpha_{ij} = g(\xi,\eta) \; \xi^i \eta^j$$

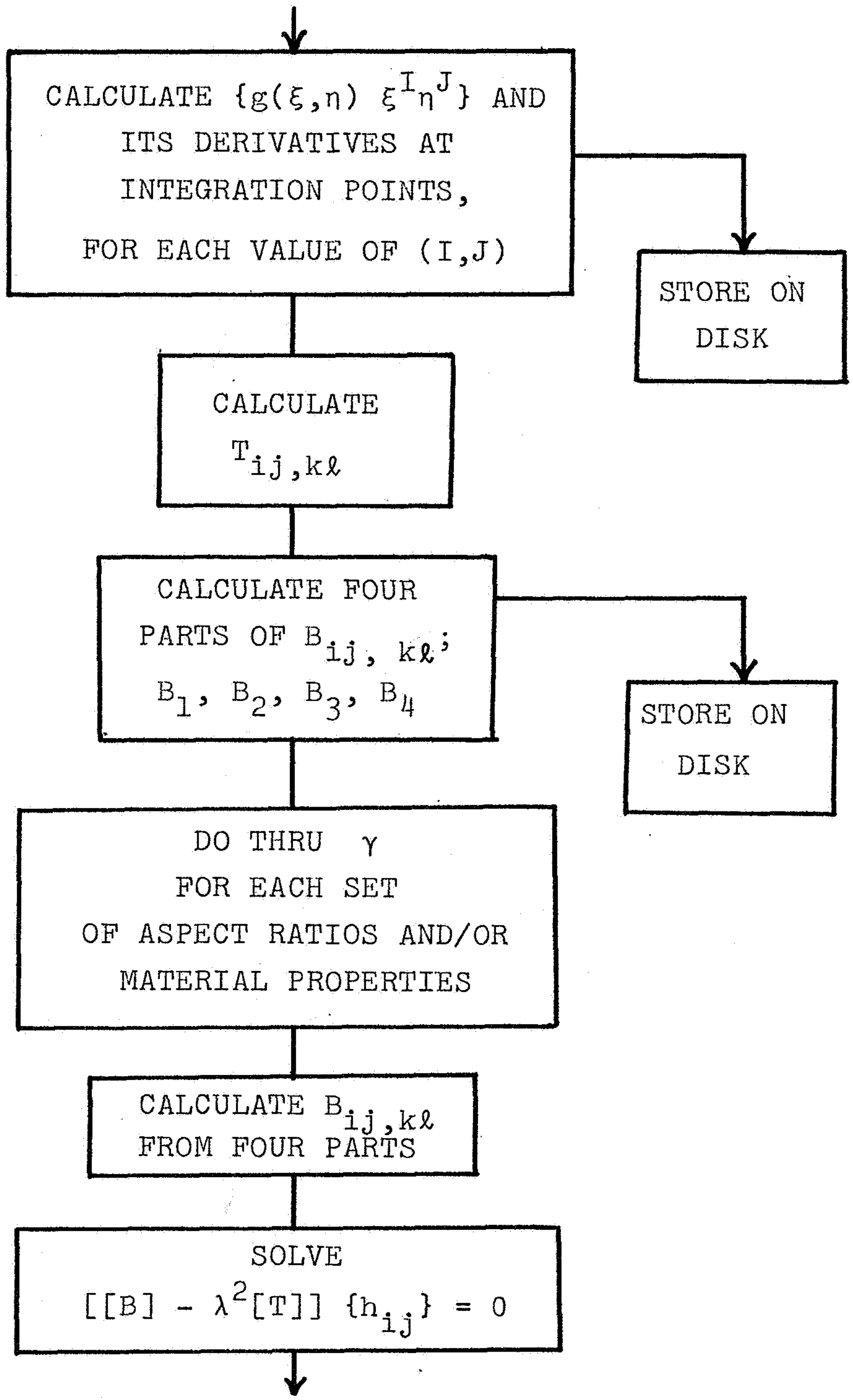
$$\gamma_{pq} = f(\xi,\eta)\xi^p \eta^q$$

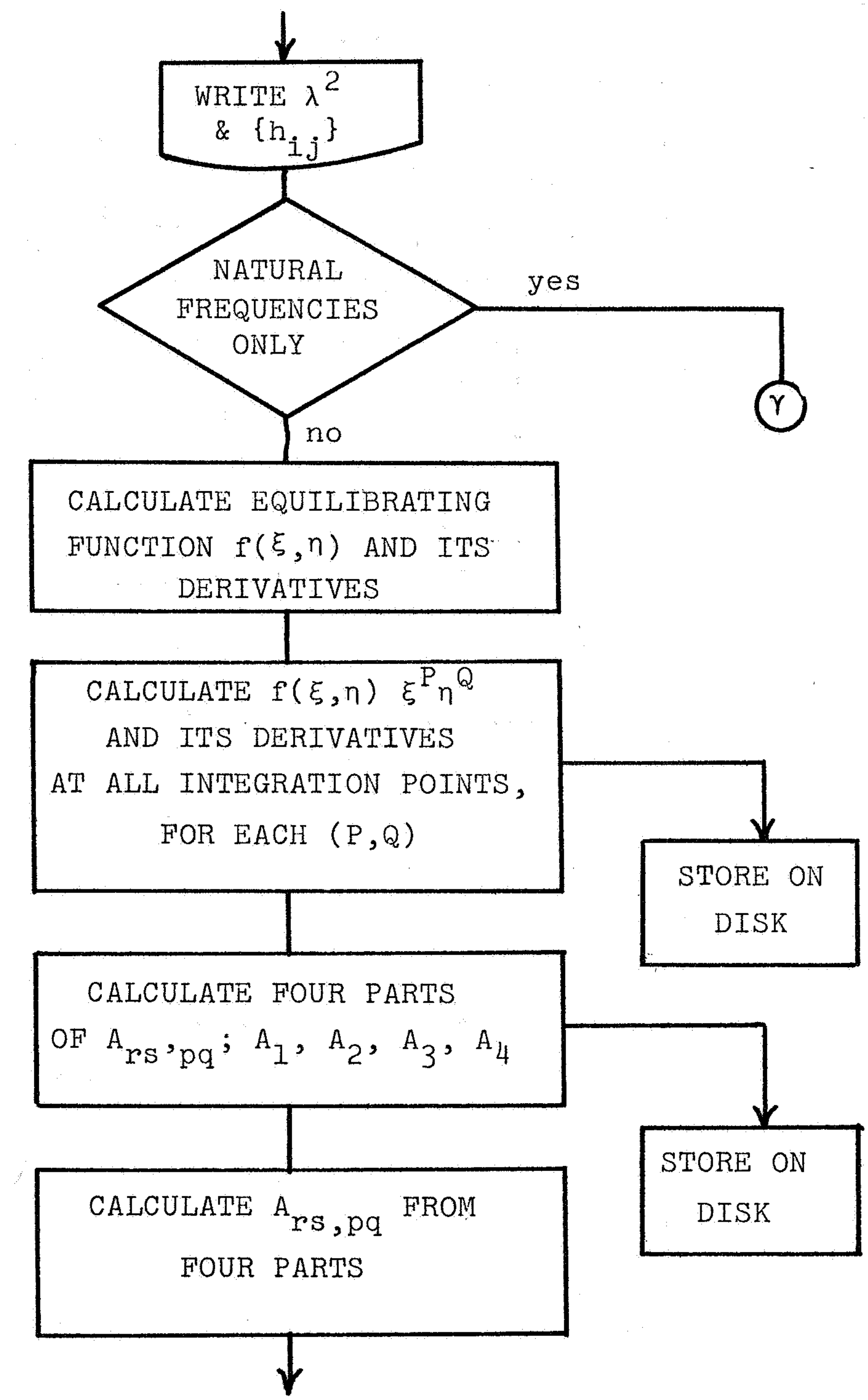
Appendix B

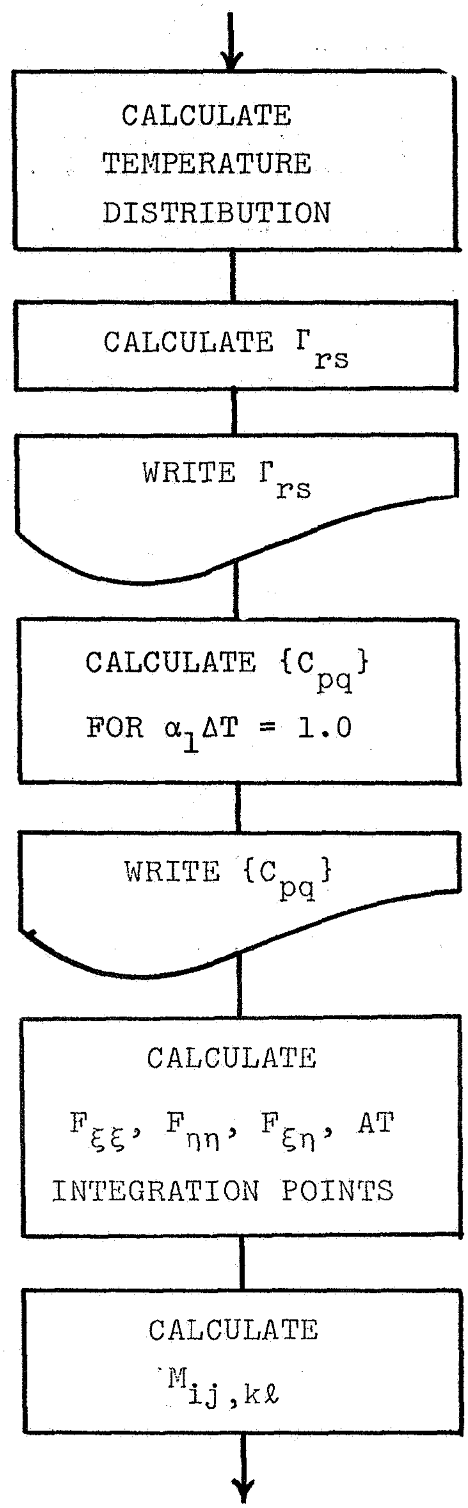
Logic Flow Diagram

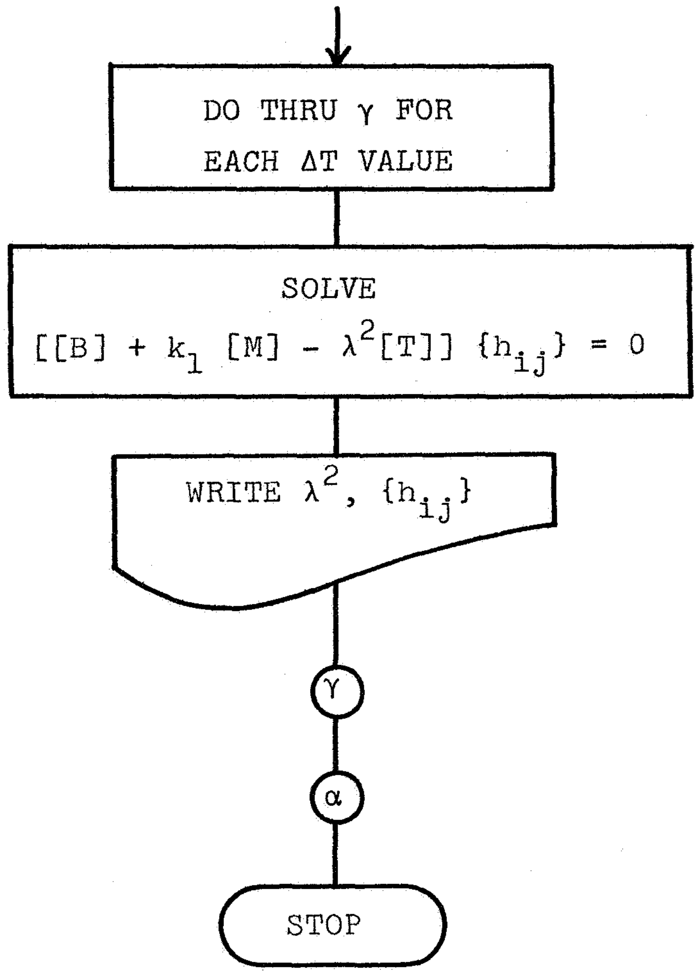



NATURAL FREQUENCIES ONLY REQUIRED?
yes
no
READ NUMBER OF STRESS FUNCTION TERMS AND ξ η EXPONENTS
READ TEMPERATURE DATA
WRITE ALL INPUT DATA
CALCULATE THICKNESS DISTRIBUTION
CALCULATE BOUNDARY CONDITION FUNCTION: g (ξ,η) AND ITS DERIVATIVES
∂/∂ξ, ∂²/∂ξ², ∂/∂η, ∂²/∂η², ∂²/∂ξ∂η
AT INTEGRATION POINTS


CALCULATE {g(ξ,η) $\xi^I \eta^J$} AND
ITS DERIVATIVES AT
INTEGRATION POINTS,
FOR EACH VALUE OF (I,J)
STORE ON
DISK
CALCULATE
$T_{ij,k\ell}$
CALCULATE FOUR
PARTS OF $B_{ij,k\ell}$;
B_1, B_2, B_3, B_4
STORE ON
DISK
DO THRU γ
FOR EACH SET
OF ASPECT RATIOS AND/OR
MATERIAL PROPERTIES
CALCULATE $B_{ij,k\ell}$
FROM FOUR PARTS
SOLVE
$[[B] - \lambda^2[T]] \{h_{ij}\} = 0$


WRITE λ^2 & $\{h_{ij}\}$
NATURAL FREQUENCIES ONLY
yes
Y
no
CALCULATE EQUILIBRATING FUNCTION $f(\xi,\eta)$ AND ITS DERIVATIVES
CALCULATE $f(\xi,\eta)\ \xi^P \eta^Q$ AND ITS DERIVATIVES AT ALL INTEGRATION POINTS, FOR EACH (P,Q)
STORE ON DISK
CALCULATE FOUR PARTS OF $A_{rs,pq}$; A_1, A_2, A_3, A_4
STORE ON DISK
CALCULATE $A_{rs,pq}$ FROM FOUR PARTS


CALCULATE
TEMPERATURE
DISTRIBUTION
CALCULATE Γ_{rs}
WRITE Γ_{rs}
CALCULATE $\{C_{pq}\}$
FOR $\alpha_1 \Delta T = 1.0$
WRITE $\{C_{pq}\}$
CALCULATE
$F_{\xi\xi}$, $F_{\eta\eta}$, $F_{\xi\eta}$, AT
INTEGRATION POINTS
CALCULATE
$M_{ij,k\ell}$


DO THRU γ FOR EACH ΔT VALUE
SOLVE
$[[B] + k_1 [M] - \lambda^2[T]] \{h_{ij}\} = 0$
WRITE λ^2, $\{h_{ij}\}$
γ
α
STOP

Appendix C

Program Listing

```
      IMPLICIT REAL*8 (A,B,C,D,E,F,G,H,M,O,P,Q,R,S,T,U,V,W,X,Y,Z)
      DIMENSION ARRAY1(1300),ARRAY2(1300),ARRAY3(1300),ARRAY4(1300),
     *          ARRAY5(1300),ARRAY6(1300),BMAT(1300),TMAT(1300),
     *          RMMAT(1300),
     *          IC(36),JC(36),
     *          AMAT(1300),CPQ(36),GAMRS(36),IP(36),IQ(36),
     *          MAT1(200),MAT2(200),MAT3(200),MAT4(200),MAT5(200),
     *          MAT6(200),M1(200),M2(200),M3(200),M4(200),M5(200),
     *          M6(200),T(200),ETA(200),TEMPT(200),        HKER(10),
     *          ZKER(10),
     *          BC(4),RES(4),TCC1(5),NTX1(5),NTY1(5),TCO2(5),NTX2(5),
     *          NTY2(5),AR(10),TEM(5),NTEMX(5),
     *          NTEMY(5),TEMP(200,5),KC(20),LC(20),TREF(5),DT(20,5),
     *          NDT(5),TITLE(10  ),
     *          E11(10),E12(10),E22(10),G12(10),AL1(10),AL2(10)
C
C        ARRAYS ON LINES 1-4 OF DIMENSION STATEMENT PERTAIN TC DEFLECTION
C     FUNCTION, LINE 5 PERTAINS TO STRESS FUNCTION AND LINES 6-9 TO QUADRATURE
C     POINTS. DIMENSION OF QUADRATURE ARRAYS MUST BE AT LEAST THE NUMBER OF
C     POINTS SQUARED TIMES 2. STRESS FUNCTION AND DEFLECTION FUNCTION ARRAYS
C     EITHER THE NUMBER OF TERMS SQUARED OR THE NUMBER OF TERMS ITSELF.
C
      LOGICAL LAMDAO,EXPT,SIN
      DATA ONE,TWO,ZERO/1.0D0,2.0D0,0.0D0/
C
C                                  FORMAT LISTING
C
   10 FORMAT(6E12.6)
   20 FORMAT(36I2)
   30 FORMAT(4A1)
   40 FORMAT(4E18.16)
   50 FORMAT(L12)
   60 FORMAT(I12,5E12.6)
   80 FORMAT(//44X
     *,'ALPHA =',E16.8/45X,'BETA =',E16.8/44X,'GAMMA =',E16.8/46X,
     *'A/H =',E16.8/47X,'B1 =',E16.8/36X,'MAX. THICKNESS =',E16.8/40X,
```

```
   *I3,' QUADRATURE POINTS')
100 FORMAT(1H /30X,'ASPECT RATIO =',E17.8/(45X,E16.8))
130 FORMAT(1H /16X,'I= ',16I4/(19X,16I4))
140 FORMAT(1H ,15X,'J= ',16I4/(19X,16I4))
150 FORMAT(1H /16X,'P= ',16I4/(19X,16I4))
160 FORMAT(1H ,15X,'Q= ',16I4/(19X,16I4))
170 FORMAT(1H /37X,'THICKNESS DISTRIBUTION',/,23X,'COEFFICIENTS',5X,
   *'X EXPONENTS',5X,'Y EXPONENTS',/,47X,'SURFACE 1',/,(19X,E16.8,7X,
   *I3,11X,I3))
180 FORMAT(1H ,46X,'SURFACE 2',/,(19X,E16.8,7X,I3,11X,I3))
190 FORMAT(10A8)
200 FORMAT(1H /37X,'TEMPERATURE DISTRIBUTION',/,20X,'COEFFICIENTS',5X,
   *'X EXPONENTS',5X,'Y EXPONENTS',/,(16X,E16.8,7X,I3,11X,I3))
210 FORMAT(1H /20X,10A8)
220 FORMAT(1H /30X,'DELTA-T =',E17.8/(40X,E16.8))
230 FORMAT(1H ,25X,'INITIAL DEFLECTION COEFFICIENTS'/(  20X,5E16.8))
240 FORMAT(1H ,25X,'TRANSVERSE LOADING COEFFICIENTS'/(  20X,5E16.8))
250 FORMAT(1H1,42X,'ARRAY T')
260 FORMAT(1H ,5X,'ROW',I3/(10X,5E16.8))
270 FORMAT(1H1,35X,'ASPECT RATIO =',E16.8,/,42X,'ARRAY B')
280 FORMAT(1H1,10X,'FUNDAMENTAL VIBRATION EIGENVALUES SQUARED - ASPECT
   * RATIO=',E16.8                       /(10X,5E18.8))
290 FORMAT(1H ,10X,'VIBRATION EIGENVECTORS')
300 FORMAT(1H1,35X,'ASPECT RATIO =',E16.8/42X,'ARRAY A')
310 FORMAT(1H1,37X,'THERMAL LOADING'/(10X,5E16.8))
320 FORMAT(///30X,'STRESS FUNCTION COEFFICIENTS'/(10X,5E16.8))
330 FORMAT(1H1,42X,'ARRAY M')
340 FORMAT(1H1,30X,'LINEAR VIBRATION EIGENVALUES SQUARED @ DELTA-T= ',
   *E16.8                        /(10X,5E18.8))
350 FORMAT(1H ,25X,'VIBRATION EIGENVECTORS')
360 FORMAT(1H1,56X,A1,',',A1/55X,'-------'/55X,'|     |'/30X,'BOUNDARY
   * CONDITIONS:  ',A1,',',A1,'|     |',A1,',',A1/55X,'|     |'/55X,
   *'-------'/57X,A1,',',A1)
400 FORMAT(1H1//12X,'STRESS VARIATION AT X=',E13.5//12X,'Y',15X,'  NX
   * ',13X,'  NY  ',13X,' NXY ',13X,'THICK',13X,'TEMP')
410 FORMAT(1X,6E19.8)
```

```
  480 FORMAT(1H1,'X')
  490 FORMAT(1H1,20X,'TEMPERATURE DISTRIBUTION NO.',I3,' TREF= ',
     *E13.5///)
  500 FORMAT(10X,5E15.5)
  510 FORMAT(1H1,////////////////////,40X,'TEMPERATURE DISTRIBUTION NO.'
     *,I3,' OF',I3)
  560 FORMAT(1H ,11X,'E11',14X,'E12',14X,'E22',14X,'G12',12X,'ALPHA1',
     *11X,'ALPHA2'/(4X,6E17.7))
      READ(5,20)NBC
      DO 1000 NCOND=1,NBC
      REWIND 2
      REWIND 3
      REWIND 4
      READ(5,30)(BC(I),I=1,4)
C        BC(I) GIVES THE DISPLACEMENT BOUNDARY CONDITIONS
C                  BC(I)= P - SIMPLY SUPPORTED
C                         C - CLAMPED
C                         F - FREE
      READ(5,30) (RES(I),I=1,4)
C        RES(I) GIVES THE STRESS BOUNDARY CONDITIONS
C                  RES(I)= C - CLAMPED
C                          F - FREE
C        EDGES ARE NUMBERED CLOCKWISE STARTING WITH THE EDGE CONTAINING
C       THE ORIGIN
      READ(5,10) ALPHA,BETA,GAMMA,ACH,B1,TO
C        PLATE GEOMETRIC PARAMETERS
      READ(5,20)NX2
C        NX2= 1/2 THE NUMBER OF QUADRATURE POINTS
      READ(5,40)(HKER(I),I=1,NX2)
C          HKER(I)= QUADRATURE COEFFICIENTS IN ASCENDING ORDER
      READ(5,40)(ZKER(I),I=1,NX2)
C        ZKER(I)= QUADRATURE POINTS IN DESCENDING ORDER
      READ(5,20)NTHIC1
C        NTHIC1= NUMBER OF THICKNESS TERMS ON SURFACE 1
      READ(5,10)(TCO1(I),I=1,NTHIC1)
C        TCO1= THICKNESS FUNCTION COEFFICIENTS ON SURFACE 1
```

```
      READ(5,20)(NTX1(I),I=1,NTHIC1)
C        NTX1= X-EXPONENTS OF THICKNESS FUNCTION ON SURFACE 1
      READ(5,20)(NTY1(I),I=1,NTHIC1)
C        NTY1= Y-EXPONENTS OF THICKNESS FUNCTION ON SURFACE 1
      READ(5,20)NTHIC2
      READ(5,10)(TCO2(I),I=1,NTHIC2)
      READ(5,20)(NTX2(I),I=1,NTHIC2)
      READ(5,20)(NTY2(I),I=1,NTHIC2)
C        NTHIC2, TCO2, NTX2, NTY2 ARE SAME AS ABOVE BUT ON SURFACE 2
      READ(5,20)NAR
C        NAR = NUMBER OF DIFFERENT SETS OF ASPECT RATIO AND MATERIAL
C      PROPERTIES.
      DO 2 J=1,NAR
      READ(5,10) E11(J),E12(J),E22(J),G12(J),AL1(J),AL2(J)
C        MATERIAL PROPERTIES
C
C          FOR AN ISOTROPIC MATERIAL ;
C               E11=E22=E/(1-NU**2)
C               E12=NU*E/(1-NU**2)
C               G12=E/2*(1+NU)
C               AL1=AL2=AL    -    (THERMAL EXPANSION COEFF.)
C
    2 READ(5,10) AR(J)
C        ASPECT RATIO
      READ(5,20)NDEFL
C        NUMBER OF DEFLECTION FUNCTION TERMS
      READ(5,20)(IC(I),I=1,NDEFL)
C        X-EXPONENTS OF DEFLECTION FUNCTION
      READ(5,20)(JC(I),I=1,NDEFL)
C        Y-EXPONENTS OF DEFLECTION FUNCTION
      READ(5,50)LAMDAO
C        LAMDAO IS A LOGICAL VARIABLE: = T - END OF INPUT AND ONLY
C                                            FUNDAMENTAL FREQUENCIES ARE
C                                            CALCULATED.
C                                        = F - INPUT CONTINUES.
      IF(LAMDAO) GO TO 3
```

```
C          NUMBER OF STRESS FUNCTION TERMS
       READ(5,20)NSTRES
       READ(5,20)(IP(I),I=1,NSTRES)
C          X-EXPCNENTS OF STRESS FUNCTION
       READ(5,20)(IQ(I),I=1,NSTRES)
C          Y-EXPONENTS OF STRESS FUNCTION
       READ(5,50)EXPT
C          EXPT IS A LCGICAL VARIABLE: = T - INPUT UP TC 5 EXPERIMENTAL
C                                            TEMPERATURE DISTRIBUTIONS.
C                                        F - INPUT 1 ANALYTICAL DISTRIBUTION.
       IF(EXPT) GO TO 4
       NTEMP=1
       READ(5,60)NTEM,TREF(1)
C          NTEM= NUMBER OF TERMS IN TEMPERATURE POLYNOMIAL,
C          TREF= REFERENCE TEMPERATURE
       READ(5,10)(TEM(I),I=1,NTEM)
C          COEFFICIENTS OF TEMPERATURE POLYNOMIAL
       READ(5,20)(NTEMX(I), I=1,NTEM)
C          X-EXPCNENTS OF TEMPERATURE PCLYNOMIAL
       READ(5, 20)(NTEMY(I), I=1,NTEM)
C          Y-EXPCNENTS OF TEMPERATURE PCLYNOMIAL
       READ(5,20)NDT(1)
C          NUMBER OF DELTA-T'S
       NT=NCT(1)
       DO 41 J=1,NT
    41 READ(5,10) DT(J,1)
C          DT= VALUES OF DELTA-T
       GO TO 3
     4 READ(5,60) NTX,DTX,DTY,XT1,YT1
C          INPUTS FOR SUBROUTINE 'INTP'
       READ(5,20)(KC(I), I=1,NTX)
C          INPUTS FOR SUBRCUTINE 'INTP'
       READ(5,20)(LC(I),I=1,NTX)
C          INPUTS FOR SUBROUTINE 'INTP'
       NPTS=0
       DO 6 I=1,NTX
```

```
      6 NPTS=NPTS+LC(I)-KC(I)+1
        READ(5,19C)(TITLE(I  ),I=1,10)
C          TITLE= SOME DESCRIPTIVE INFORMATION ABOUT THE TEMPERATURES INPUT.
        READ(5,20)NTEMP
        DO 7 I=1,NTEMP
        READ(5,10)TREF(I)
C          REFERENCE TEMP. FCR THE I'TH CISTRIBUTION
        READ(5,10)(TEMP(J,I),J=1,NPTS)
C          VALUE CF THE I'TH TEMP. CIST. AT EACH OF THE GRID POINTS.
        READ(5,20)NDT(I)
C          NUMBER OF DELTA-T'S TO BE CONSIDERED FOR I'TH TEMP. DIST.
        NT=NCT(I)
        DO 7 J=1,NT
      7 READ(5,10) DT(J,I)
C          VALUES OF DELTA-T FOR I'TH DISTRIBUTION
      3 CONTINUE
        NX=2*NX2
        NX4=2*NX*NX
        WRITE(6,360) BC(2),RES(2),BC(1),RES(1),BC(3),RES(3),BC(4),RES(4)
        WRITE(6,80)                 ALPHA,BETA,GAMMA,AOH,B1,TO,NX
        WRITE(6,1CC) (AR(I),I=1,NAR)
        WRITE(6,560) (E11(I),E12(I),E22(I),G12(I),AL1(I),AL2(I),I=1,NAR)
        WRITE(6,130)(IC(I),I=1,NDEFL)
        WRITE(6,140)(JC(I),I=1,NDEFL)
        WRITE(6,170)(TCO1(I),NTX1(I),NTY1(I),I=1,NTHIC1)
        WRITE(6,180)(TCC2(I),NTX2(I),NTY2(I),I=1,NTHIC2)
        IF(LAMDAO) GO TO 8
        WRITE(6,150)(IP(I),I=1,NSTRES)
        WRITE(6,160)(IQ(I),I=1,NSTRES)
        IF(EXPT) GO TO 15
        WRITE(6,200)(TEM(I),NTEMX(I),NTEMY(I),I=1,NTEM)
        GO TO 16
     15 WRITE(6,210)(TITLE(I  ),I=1,10)
     16 CCNTINUE
        IF(EXPT) GO TO 12
        DC 32 J=1,NTEMP
```

```
      NT=NDT(J)
   32 WRITE(6,220) (DT(I,J),I=1,NT)
   12 CCNTINUE
   14 CONTINUE
      WRITE(6,480)
      IF (.NOT.EXPT) GC TC 8
      DO 11 J=1,NTEMP
      WRITE(6,490) J,TREF(J)
      NT=NDT(J)
      WRITE(6,220) (DT(I,J),I=1,NT)
   11 WRITE(6,500) (TEMP(I,J),I=1,NPTS)
    8 CONTINUE
C
C       TRANSFORM QUADRATURE COEFFICIENTS AND POINTS TO OUR COORDINATE SYSTEM
C
      CO 17 I=1,NX2
      HKER(I)=HKER(I)/TWC
      J=NX-I+1
      HKER(J)=HKER(I)
      ZKER(I)=CNE-((ZKER(I)+CNE)/TWC)
   17 ZKER(J)=ONE-ZKER(I)
C
C       CALCULATE INTEGRATICN PCINTS, AND THICKNESS @ POINTS
C
      I1=0
      DO 18 I=1,NX
      XB=BETA*ZKER(I)
      AA=ONE-ZKER(I)
      QP=ONE+ALPHA*ZKER(I)
      CD=B1-GAMMA*ZKER(I)
      DC 18 NSEC=1,2
      DO 18 J=1,NX
      J1=NX+1-J
      I1=I1+1
      ETA(I1)=(QP-XB)*ZKER(J1)+XB
      IF(NSEC.EQ.2) ETA(I1)=-(XB+CD)*ZKER(J)+XB
```

```
      X=ZKER(I)
      Y=ETA(I1)
      T(I1)=CTHIC(NTHIC1,TCO1,NTX1.NTY1,NTHIC2,TCO2,NTX2,NTY2,X,Y,BETA)
     * *TO
   18 CCNTINUE
C
C        CALCULATE BOUNDARY CONDITION FUNCTION @ QUADRATURE  POINTS
C
      CALL FUNCTN(ARRAY1,ARRAY2,ARRAY3,ARRAY4,ARRAY5,ARRAY6,ZKER,ETA,
     *            BC,.FALSE.,NX,ALPHA,GAMMA,B1)
C
C        ARRAY1=F, ARRAY2=FX, ARRAY3=FXX, ARRAY4=FY, ARRAY5=FYY, ARRAY6= FXY
C
C       CALCULATE DERIVATIVES OF DISPLACEMENT FUNCTION
C
      DO 29 IJ=1,NDEFL
      I1=0
      DO 19 I=1,NX
      X=ZKER(I)
      X2=X**(IC(IJ)-2)
      X1=X2*X
      X0=X1*X
      DC 19 NSEC=1,2
      DO 19 J=1,NX
      I1=I1+1
      Y=ETA(I1)
      Y2=Y**(JC(IJ)-2)
      Y1=Y2*Y
      Y0=Y1*Y
      F=ARRAY1(I1)
      FX=ARRAY2(I1)
      FXX=ARRAY3(I1)
      FY=ARRAY4(I1)
      FYY=ARRAY5(I1)
      FXY=ARRAY6(I1)
      MAT1(I1)  =F*X0*Y0
```

```
      MAT2(I1)   =FX*XO*YO+F*IC(IJ)*X1*YO
      MAT3(I1)   =(FXX*XO+TWO*IC(IJ)*X1*FX+F*IC(IJ)*(IC(IJ)-1)*X2)*YO
      MAT4(I1)   =FY*XO*YO+F*JC(IJ)*XO*Y1
      MAT5(I1)   =(FYY*YC+TWC*JC(IJ)*Y1*FY+F*JC(IJ)*(JC(IJ)-1)*Y2)*XO
      MAT6(I1)   =FXY*XC*YO+FX*JC(IJ)*XO*Y1+FY*IC(IJ)*X1*YO+F*JC(IJ)*
     *                                                IC(IJ)*X1*Y1
   19 CONTINUE
   29 WRITE(2) (MAT1(I),MAT2(I),MAT3(I),MAT4(I),MAT5(I),MAT6(I),I=1,NX4)
C                                                                  FT02F001
      END FILE 2                                                    =WIJ
      REWINC 2
C       READ FRCM FT02F001
C       CALCULATE FOUR PARTS OF B MATRIX, AND T MATRIX
C
      DC 21 IJ=1,NDEFL
      READ(2)(M1(I),M2(I),M3(I),M4(I),M5(I),M6(I),I=1,NX4)
      REWIND 2
      DO 21 KL=1,IJ
      READ(2)(MAT1(I),MAT2(I),MAT3(I),MAT4(I),MAT5(I),MAT6(I),I=1,NX4)
      I3=(KL-1)*NDEFL+IJ
      I4=(IJ-1)*NDEFL+KL
      I1=0
      TAAD=ZERC
      BAAD=ZERO
      BAAC2=ZERO
      BAAC3=ZERO
      BAAD4=ZERO
      CO 22 I=1,NX
      TAD1=ZERC
      BAD1=ZERO
      BAC2=ZERC
      BAD3=ZERC
      BAD4=ZERO
      X=ZKER(I)
      DO 23 NSEC=1,2
      TAD=ZERO
```

```
      BADC1=ZERC
      BADD2=ZERO
      BADC3=ZERO
      BADC4=ZERC
      DY=ONE+(ALPHA-BETA)*X
      IF(NSEC EQ 2) DY=B1+(BETA-GAMMA)*X
      DC 24 J=1,NX
      I1=I1+1
      Y=ETA(I1)
      TGRAND=T(I1)*MAT1(I1)*M1(I1)
      BGRAN1 =(M3(I1)*MAT3(I1))*T(I1)**3
      BGRAN2 =(M5(I1)*MAT5(I1))*T(I1)**3
      BGRAN3 =(M3(I1)*MAT5(I1)+M5(I1)*MAT3(I1))*T(I1)**3
      BGRAN4 =M6(I1)*MAT6(I1)*T(I1)**3
      TAC=TAD+TGRAND*HKER(J)*DY
      BADD1=BADD1+BGRAN1*HKER(J)*DY
      BADC2=BADC2+BGRAN2*HKER(J)*DY
      BADD3=BADD3+BGRAN3*HKER(J)*DY
   24 BADD4=BADD4+BGRAN4*HKER(J)*DY
      TAD1=TAC+TAC1
      BAD1=BAD1+BADD1
      BAC2=BAC2+BADD2
      BAD3=BAC3+BADC3
   23 BAD4=BAD4+BADD4
      TAAC=TAAC+TAC1*HKER(I)
      BAAD=BAAC+BAC1*HKER(I)
      BAAD2=BAAD2+BAD2*HKER(I)
      BAAC3=BAAD3+BAD3*HKER(I)
   22 BAAC4=BAAC4+BAD4*HKER(I)
      TMAT(I3)=TAAD
      TMAT(I4)=TAAC
      ARRAY1(I3)=BAAD
      ARRAY1(I4)=BAAD
      ARRAY2(I4)=BAAC2
      ARRAY2(I3)=BAAD2
      ARRAY3(I3)=BAAD3
```

```
      ARRAY3(I4)=BAAD3
      ARRAY4(I3)=BAAD4
   21 ARRAY4(I4)=BAAD4
      NDF2=NDEFL**2
      IF(NAR.EQ.1) GO TO 34
      WRITE(4)(ARRAY1(I),ARRAY2(I),ARRAY3(I),ARRAY4(I),I=1,NDF2)           FT04F001
   34 CONTINUE
      CC 999 NARS=1,NAR
      ACB=AR(NARS)*(ONE+B1-(GAMMA-ALPHA)/TWO)
      AOB2=AOB**2
      AOB4=AOB**4
C
C     CALCULATE B MATRIX FROM THE FOUR PARTS PREVIOUSLY CALCULATED
C
      IF(NARS.EC.1) GC TO 33
      REWIND 4
      READ(4)(ARRAY1(I),ARRAY2(I),ARRAY3(I),ARRAY4(I),I=1,NDF2)
   33 CONTINUE
      Z2=AOB4*E22(NARS)/E11(NARS)
      Z3=ACB2*E12(NARS)/E11(NARS)
      Z4=ACB2*4.0D0*G12(NARS)/E11(NARS)
      DO 26 I=1,NDF2
   26 BMAT(I)=ARRAY1(I)+Z2*ARRAY2(I )+Z3*ARRAY3(I)+Z4*ARRAY4(I)
C
C        CALCULATE THE FUNDAMENTAL FREQUENCIES AND MODE SHAPES
C
      MODE=C
      DO 35I=1,NDF2
      ARRAY5(I)=BMAT(I)
   35 ARRAY2(I)=TMAT(I)
      CALL SING(ARRAY5,ARRAY2,NDEFL,ZERO,M1,ARRAY6,SIN)
      IF(SIN) GC TC 145
      CALL DNROOT(NDEFL,ARRAY2,ARRAY5,M1,ARRAY6,MODE)
      DC 148 I=1,NDEFL
  148 M1(I)=ONE/M1(I)
  145 WRITE(6,280) AR(NARS),(M1(I),I=1,NDEFL)
```

```
      WRITE(6,290)
      I2=0
      DO 31 I=1,NDEFL
      I1=I2+1
      I2=I2+NDEFL
   31 WRITE(6,260) I,(ARRAY6(J),J=I1,I2)
      IF(LAMDA0) GC TC 999
C
C        SCLVE LINEAR PROBLEM WITH TEMPERATURE
C
      IF(NARS NE 1) GO TO 51
C
C        CALCULATE EQUILIBRATING FUNCTICN AT QUADRATURE POINTS
C
      REWINC 3
      CALL FUNCTN(MAT1,MAT2,MAT3,MAT4,MAT5,MAT6,ZKER,ETA,RES, TRUE ,NX,
     *ALPHA,GAMMA,B1)
C
C        MAT1=F, MAT2=FX, MAT3=FXX, MAT4=FY, MAT5=FYY, MAT6=FXY
C
C
C        READ EQUILIBRATING FUNCTION AND CALCULATE THE THREE SECOND
C     DERIVATIVES OF THE STRESS FUNCTICN
      REWINC 3
      DC 43 IPQ=1,NSTRES
      I1=0
      DC 53 I=1,NX
      X=ZKER(I)
      DO 53 NSEC=1,2
      DC 53 J=1,NX
      I1=I1+1
      Y=ETA(I1)
      CALL PC(IP,IQ,MAT1,MAT2,MAT3,MAT4,MAT5,MAT6,ZKER,ETA,IPQ,I1,I,
     *FPQXX,FPQYY,FPQXY)
      M1(I1)=FPQXX
      M2(I1)=FPQYY
```

```
      M3(I1)=FPQXY
   53 CCNTINUE
   43 WRITE(3) (M1(I),M2(I),M3(I),I=1,NX4)                          FT03F001
      ENC FILE 3
C
C        NCW CALCULATE THE FCUR PARTS CF THE A MATRIX
C
      REWIND 3
      DO 54 NM=1,NSTRES
      READ(3) (MAT1(I),MAT2(I),MAT3(I),I=1,NX4)
      REWIND 3
      DO 54 N=1,NM
      READ(3) (M1(I),M2(I),M3(I),I=1,NX4)
      I3=(N-1)*NSTRES+NM
      I4=(NM-1)*NSTRES+N
      AAD1=ZERO
      AAD2=ZERO
      AAD3=ZERO
      AAD4=ZERO
      I1=0
      DO 55 I=1,NX
      EAD1=ZERO
      EAD2=ZERO
      EAD3=ZERO
      EAD4=ZERO
      DO 56 NSEC=1,2
      FAD1=ZERO
      FAD2=ZERO
      FAD3=ZERO
      FAD4=ZERO
      DY=CNE+(ALPHA-BETA)*ZKER(I)
      IF(NSEC.EQ.2)DY=B1+(BETA-GAMMA)*ZKER(I)
      DO 57 J=1,NX
      I1=I1+1
      EGRAN1=MAT1(I1)*M1(I1)/T(I1)
      EGRAN2=MAT2(I1)*M2(I1)/T(I1)
```

```
      EGRAN3=(MAT1(I1)*M2(I1)+M1(I1)*MAT2(I1))/T(I1)
      EGRAN4=(MAT3(I1)*M3(I1))/T(I1)
      FAD1=FAD1+EGRAN1*HKER(J)*DY
      FAD2=FAD2+EGRAN2*HKER(J)*DY
      FAD3=FAD3+EGRAN3*HKER(J)*DY
   57 FAD4=FAD4+EGRAN4*HKER(J)*DY
      EAD1=EAD1+FAD1
      EAD2=EAD2+FAD2
      EAD3=EAD3+FAD3
   56 EAD4=EAD4+FAD4
      AAD1=AAD1+EAD1*HKER(I)
      AAD2=AAD2+EAD2*HKER(I)
      AAD3=AAD3+EAD3*HKER(I)
   55 AAD4=AAD4+EAD4*HKER(I)
      ARRAY1(I3)=AAD1
      ARRAY1(I4)=AAD1
      ARRAY2(I4)=AAD2
      ARRAY2(I3)=AAD2
      ARRAY3(I4)=-AAD3
      ARRAY3(I3)=-AAD3
      ARRAY4(I3)=AAD4
      ARRAY4(I4)=AAD4
   54 CONTINUE
C
C        STORE THESE MATRICES IN DSRN=4                              FT04F001
      NSTRS2=NSTRES**2
      IF(NAR.EQ.1) GO TO 51
      WRITE(4) (ARRAY1(I),ARRAY2(I),ARRAY3(I),ARRAY4(I),I=1,NSTRS2)
      END FILE 4
   51 CONTINUE
      IF(NARS.NE.1) READ(4)(ARRAY1(I),ARRAY2(I),ARRAY3(I),ARRAY4(I),
     *                      I=1,NSTRS2)
C
C        CALCULATE A MATRIX FROM ITS FOUR PARTS
      Y1=E11(NARS)/E22(NARS)
      Y2=AOB4
```

```
      Y3=AOB2*E12(NARS)/E22(NARS)
      Y4=AOB2*(E11(NARS)*E22(NARS)-E12(NARS)**2)/(G12(NARS)*E22(NARS))
      DC 58NM=1,NSTRES
      DO 58 N=1,NM
      I3=(N-1)*NSTRES+NM
      I4=(NM-1)*NSTRES+N
      AMAT(I3)=Y1*ARRAY1(I3)+Y2*ARRAY2(I3)+Y3*ARRAY3(I3)+Y4*ARRAY4(I3)
   58 AMAT(I4)=AMAT(I3)
C
C        CALCULATE TEMPERATURE AT QUADRATURE POINTS
C
      DO 998 NTS=1,NTEMP
      WRITE(6,510) NTS,NTEMP
      N2X=NX*2
      IF(EXPT) GO TO 62
      I1=0
      DO 63 I=1,NX
      X=ZKER(I)
      DC 63 J=1,N2X
      I1=I1+1
      Y=ETA(I1)
   63 TEMPT(I1)=CTEM(NTEM ,TEM,NTEMX,NTEMY,X,Y,ALPHA,B1,GAMMA)
      GO TC 64
   62 CONTINUE
      DC 65 I=1,NPTS
   65 MAT1(I)=TEMP(I,NTS)
      I1=0
      DC 66 I=1,NX
      X=ZKER(I)
      DC 66 J=1,N2X
      I1=I1+1
      Y=ETA(I1)
      CALL INTP(MAT1,KC,LC,X,Y,DTY,DTX,NTX,YT1,XT1,WS,WXS)
   66 TEMPT(I1)=WS
   64 CONTINUE
C
```

```
C        CALCULATE GAMRS - THERMAL LOACING
      REWIND 3
      DC 67NM=1,NSTRES
      READ(3)(M1(I),M2(I),M3(I),I=1,NX4)
      I1=0
      GAD1=ZERC
      DO 68 I=1,NX
      GAC2=ZERO
      X=ZKER(I)
      DO 69 NSEC=1,2
      GAC3=ZERO
      DY=ONE+(ALPHA-BETA)*X
      IF (NSEC.EQ.2) DY=B1+(BETA-GAMMA)*X
      DC 71  J=1,NX
      I1=I1+1
      Y=ETA(I1)
   71 GAD3=GAD3+(AL2(NARS)/AL1(NARS)*M1(I1)+ADB2*M2(I1))*TEMPT(I1)*
     *HKER(J)*DY
   69 GAC2=GAC2+GAD3
   68 GAD1=GAD1+GAD2*HKER(I)
   67 GAMRS(NM)=(E11(NARS)*E22(NARS)-E12(NARS)**2)/E22(NARS)*GAD1
      WRITE(6,310) (GAMRS(I),I=1,NSTRES)
C
C        CALCULATE CPQ DUE TO TEMPERATURE WITH ALPHA1*DT = 1 0
C
      IF(NTS.EQ.1) CALL CMINV(AMAT,NSTRES,D,M1,M2)
C        'AMAT' NOW CONTAINS THE INVERSE OF THE A-MATRIX
      CO 36 I=1,NSTRES
   36 GAMRS(I)=-GAMRS(I)
      CALL DGMPRD(AMAT,GAMRS,CPQ,NSTRES,NSTRES,1)
      WRITE(6,320) (CPQ(I),I=1,NSTRES)
C
C        CALCULATE THE THERMAL STRESSES
C
      REWIND 3
      READ(3) (MAT1(I),MAT2(I),MAT3(I),I=1,NX4)
```

```
      DC201 I=1,NX4
      M2(I)=MAT1(I)*CPQ(1)
      M1(I)=MAT2(I)*CPQ(1)
  201 M3(I)=MAT3(I)*CPC(1)
      DO202 IPQ=2,NSTRES
      READ(3) (MAT1(I),MAT2(I),MAT3(I), I=1,NX4)
      DO202 I=1,NX4
      M2(I)=M2(I)+MAT1(I)*CPQ(IPQ)
      M1(I)=M1(I)+MAT2(I)*CPC(IPQ)
  202 M3(I)=M3(I)+MAT3(I)*CPQ(IPQ)
      A=E22(NARS)/(E11(NARS)*E22(NARS)-E12(NARS)**2)
      I1=C
      DO 42 I=1,NX
      WRITE(6,400) ZKER(I)
      DO 42 NSEC=1,2
      DO 42 J=1,NX
      I1=I1+1
      SXX=M1(I1)*TO*ACB2*A
      SYY=M2(I1)*TO*A
      SXY=-M3(I1)*TO*ACB*A
   42 WRITE(6,410) ETA(I1),SXX,SYY,SXY,T(I1),TEMPT(I1)
C
C        CALCLLATE THE 2-D M-MATRIX
C
      REWINC 2
      DC 44 IJK=1,NCEFL
      READ(2) (MAT2(I),MAT1(I),MAT4(I),MAT3(I),MAT5(I),MAT6(I),I=1,NX4)
      REWINC 2
      DC 44 IKL=1,IJK
      READ(2) (ARRAY1(I),ARRAY2(I),ARRAY3(I),ARRAY4(I),ARRAY5(I),
     *ARRAY6(I),I=1,NX4)
      DO 45 I=1,NX4
      MAT2(I)=ARRAY2(I)
   45 MAT4(I)=ARRAY4(I)
      I3=(IJK-1)*NDEFL+IKL
      I4=(IKL-1)*NDEFL+IJK
```

```
      EAD1=ZERC
      I1=0
      DC 47  I=1,NX
      EAD2=ZERC
      X=ZKER(I)
      DC 48  NSEC=1,2
      DY=ONE+(ALPHA-BETA)*X
      IF(NSEC EQ 2) DY=B1+(BETA-GAMMA)*X
      EAD3=ZERC
      DO 49 J=1,NX
      I1=I1+1
      EGRAND=M1(I1)*MAT2(I1)*MAT1(I1)+M2(I1)*MAT3(I1)*MAT4(I1)-M3(I1)*
     **(MAT1(I1)*MAT4(I1)+MAT2(I1)*MAT3(I1))
   49 EAD3=EAD3+EGRANC*HKER(J)*DY
   48 EAD2=EAD2+EAD3
   47 EAC1=EAC1+EAD2*HKER(I)
      RMMAT(I3)=EAD1
   44 RMMAT(I4)=EAD1
C
      NT=NDT(NTS)
      DO 997 NDTS=1,NT
C
C         SCLVE LINEAR RESPONSE PRCBLEM
C
      Q=12 0D0*AOB2*AOH**2*AL1(NARS)*DT(NDTS,NTS)/E11(NARS)
      DO 76 I=1,NDF2
      ARRAY1(I)=TMAT(I)
   76 ARRAY2(I)=BMAT(I)+RMMAT(I)*Q
      CALL SING(ARRAY2,ARRAY1,NDEFL,ZERO,M1,ARRAY3,SIN)
      IF(SIN) GO TO 146
      CALL DNROOT(NDEFL,ARRAY1,ARRAY2,M1,ARRAY3,MODE)
      DO 149 I=1,NDEFL
  149 M1(I)=ONE/M1(I)
  146 FDT=DT(NDTS,NTS)
      WRITE(6,340) FDT,(M1(I),I=1,NDEFL)
      WRITE(6,350)
```

```
      I 2=0
      DC 78 I=1,NDEFL
      I1=I2+1
      I 2=I 2+NDEFL
   78 WRITE(6,260) I,(ARRAY3(J),J=I1,I2)
  996 CCNTINUE
  997 CONTINUE
  998 CCNT INUE
  999 CONTINUE
 1000 CONTINUE
      STCP
      END
```

```
      SUBROUTINE FUNCTN(F,FX,FXX,FY,FYY,FXY,X,Y,S,   STRESS,NX,A,G,B1)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION F(1),FX(1),FXX(1),FY(1),FYY(1),FXY(1),X(1),Y(1),S(1),Q(1
     *),B(4),IEX(4,3),IR(4),IE(4),T(4,3)
      LOGICAL STRESS
C
C                                ( TRUE.-STRESS FUNCTION
C    STRESS IS A LOGICAL VARIABLE=(
C                                ( FALSE.-DISPLACEMENT FUNCTION
C
C    S(I) IS AN ALPHAMERIC VARIABLE:
C         ('C' IF EDGE(I) IS CLAMPED
C    S(I)=('P' IF EDGE(I) IS PINNED
C         ('F' IF EDGE(I) IS FREE
C
      DATA P,FF,CC/'P','F','C'/
C
C        CALCULATE EXPONENTS REQUIRED
C
      DO 1 I=1,4
    1 IE(I)=0
      IF (STRESS) GO TO 2
      DO 21 I=1,4
      IF(S(I).EQ.CC) IE(I)=2
      IF(S(I).EQ.P) IE(I)=1
   21 CONTINUE
      GO TO 3
    2 CONTINUE
      DO 4 I=1,4
      IF(S(I).EQ.FF.OR.S(I).EQ.P) IE(I)=2
    4 CONTINUE
    3 CONTINUE
      DO 5 I=1,4
      MP=IE(I)
      DO 5 J=1,3
      IEX(I,J)=MP-(J-1)
```

```
      IF(IEX(I,J) LT 0) IEX(I,J)=0
    5 CONTINUE
      I1=0
      DO 7 I=1,NX
      DO 7 NSEC=1,2
      DC 7 J=1,NX
      I1=I1+1
C
C        CALCULATE THE FACTORS REQUIRED FOR THE FUNCTION AND ITS
C        DERIVATIVES
C
      DC 8 K=1,3
      T(1,K)=X(I)**IEX(1,K)
      T(2,K)=(1 ODO+A*X(I)-Y(I1))**IEX(2,K)
      T(3,K)=(1 ODO-X(I))**IEX(3,K)
      T(4,K)=(B1-G*X(I)+Y(I1))**IEX(4,K)
   13 CCNTINUE
    8 CCNTINUE
      DO 9 K=1,3
      DC 9 L=1,4
      IF(IEX(L,K) EQ 0) T(L,K)=1 ODO
    9 CCNTINUE
C
C        CALCULATE THE FUNCTICN AND ITS DERIVATIVES
C
      F(I1)=T(1,1)*T(2,1)*T(3,1)*T(4,1)
C
      FX(I1)=(IEX(1,1)*T(1,2)*T(2,1)*T(3,1)*T(4,1))+(A*IEX(2,1)*T(1,1)*T
     *(2,2)*T(3,1)*T(4,1))-(IEX(3,1)*T(3,2)*T(4,1)*T(1,1)*T(2,1))-(G*IEX
     *(4,1)*T(4,2)*T(1,1)*T(2,1)*T(3,1))
C
      FY(I1)=(IEX(4,1)*T(4,2)*T(1,1)*T(2,1)*T(3,1))-(IEX(2,1)*T(2,2)*T(1
     *,1)*T(3,1)*T(4,1))
C
      FYY(I1)=IEX(2,1)*IEX(2,2)*(T(1,1)*T(2,3)*T(3,1)*T(4,1))
     *        +IEX(4,1)*IEX(4,2)*(T(1,1)*T(2,1)*T(3,1)*T(4,3))
```

```
*        -2.0D0*IEX(2,1)*IEX(4,1)*T(1,1)*T(2,2)*T(3,1)*T(4,2)        FUN
C
   FXY(I1)=(IEX(4,1)*((IEX(1,1)*T(1,2)*T(2,1)*T(3,1)*T(4,2))+(A*IEX(2FUN
  *,1)*T(1,1)*T(2,2)*T(3,1)*T(4,2))-(IEX(3,1)*T(1,1)*T(2,1)*T(3,2)*T(FUN
  *4,2))-(G*IEX(4,2)*T(1,1)*T(2,1)*T(3,1)*T(4,3))))-(IEX(2,1)*((IEX(1FUN
  *,1)*T(1,2)*T(2,2)*T(3,1)*T(4,1))+(A*IEX(2,2)*T(1,1)*T(2,3)*T(3,1)*FUN
  *T(4,1))-(IEX(3,1)*T(1,1)*T(2,2)*T(3,2)*T(4,1))-(G*IEX(4,1)*T(1,1)*FUN
  *T(2,2)*T(3,1)*T(4,2))))                                           FUN
C
   FXX(I1)=IEX(1,1)*IEX(1,2)*T(1,3)*T(2,1)*T(3,1)*T(4,1)             FUN
  *        +A**2*IEX(2,1)*IEX(2,2)*T(1,1)*T(2,3)*T(3,1)*T(4,1)       FUN
  *        +IEX(3,1)*IEX(3,2)*T(1,1)*T(2,1)*T(3,3)*T(4,1)            FUN
  *        +G**2*IEX(4,1)*IEX(4,2)*T(1,1)*T(2,1)*T(3,1)*T(4,3)       FUN
  *        +2.0D0*A*IEX(1,1)*IEX(2,1)*T(1,2)*T(2,2)*T(3,1)*T(4,1)    FUN
  *        -2.0D0*IEX(1,1)*IEX(3,1)*T(1,2)*T(2,1)*T(3,2)*T(4,1)      FUN
  *        -2.0D0*IEX(1,1)*IEX(4,1)*T(1,2)*T(2,1)*T(3,1)*T(4,2)*G
  *        -2.0D0*A*IEX(2,1)*IEX(3,1)*T(1,1)*T(2,2)*T(3,2)*T(4,1)    FUN
  *        -2.0D0*A*G*IEX(2,1)*IEX(4,1)*T(1,1)*T(2,2)*T(3,1)*T(4,2)  FUN
  *        +2.0D0*G*IEX(3,1)*IEX(4,1)*T(1,1)*T(2,1)*T(3,2)*T(4,2)    FUN
 7 CONTINUE                                                          FUN
   RETURN                                                            FUN
   END                                                               FUN
```

```
      SUBROUTINE PQ(IP,IQ,FUN,FUNX,FUNXX,FUNY,FUNYY,FUNXY,XI,ETA,M,I1,I,
     *SXPQ,SYPQ,SXYPQ)
      IMPLICIT REAL*8 (F,X,E,S,Y)
      DIMENSION FUN(1),FUNX(1),FUNXX(1),FUNY(1),FUNYY(1),FUNXY(1),XI(1),
     *ETA(1),IP(1),IQ(1)
      X2=XI(I)**(IP(M)-2)
      X1=X2*XI(I)
      X0=X1*XI(I)
      Y2=ETA(I1)**(IQ(M)-2)
      Y1=Y2*ETA(I1)
      Y0=Y1*ETA(I1)
      SXPQ=(FUNXX(I1)*X0+2.0*IP(M)*X1*FUNX(I1)+FUN(I1)*IP(M)*(IP(M)-1)*X
     *2)*Y0
      SYPQ=(FUNYY(I1)*Y0+2.0*FUNY(I1)*IQ(M)*Y1+FUN(I1)*IQ(M)*(IQ(M)-1)*Y
     *2)*X0
      SXYPQ=FUNXY(I1)*X0*Y0+FUNX(I1)*IQ(M)*X0*Y1+FUNY(I1)*IP(M)*X1*Y0+FU
     *N(I1)*IP(M)*IQ(M)*X1*Y1
      RETURN
      END
```

```
 DOUBLE PRECISION FUNCTION CTHIC(NTHIC1,TCO1,NTX1,NTY1,NTHIC2,TCO2,
*NTX2,NTY2,X,Y,BETA)
 IMPLICIT REAL*8 (X,B,Y,T,C)
 DIMENSION  TCO1(1),NTX1(1),NTY1(1),TCO2(1),NTX2(1),NTY2(1)
 X1=BETA*X
 Y1=Y-X1
 IF(Y LT.X1) GO TO 1
 T=0 0
 DO2 I=1,NTHIC1
2 T=T+TCO1(I)*(X**NTX1(I))*(Y1**NTY1(I))
 GC TC 4
1 T=0 C
 DO3 I=1,NTHIC2
3 T=T+TCO2(I)*(X**NTX2(I))*(Y1**NTY2(I))
4 CTHIC=T
 RETURN
 END
```

```
      DOUBLE PRECISION FUNCTION CTEM(NTEMP,TEM,NTEMX,NTEMY,X,Y,ALPHA,B1,
     *GAMMA)
      IMPLICIT REAL*8(A-H,O-Z)
      CIMENSION TEM(1),NTEMX(1),NTEMY(1)
      A1=B1+1 ODO
      YP=(-  X*((ALPHA+GAMMA)           )+2. ODO*Y-1 ODO+B1)/A1
      YP=DABS(YP)
      TADD=C CDC
      DC1 K=1,NTEMP
    1 TADD=TACD+TEM(K)*(YP**NTEMY(K))*(X**NTEMX(K))
      CTEM=TADD
      RETURN
      END
```

```
      SUBROUTINE SING(A1,A2,N,ZERO,EVAL,EVECT,SIN)
      IMPLICIT REAL*8 (A-H,C-Z)
      DIMENSION A1(1),A2(1),EVAL(1),EVECT(1)
      LCGICAL SIN
      SIN= FALSE
      I2=0
      DC 1 I=1,N
      I1=I2+1
      I2=I2+N
      DC 2 J=I1,I2
      IF(A1(J).NE.ZERO) GO TO 1
    2 CCNTINUE
      GC TC 3
    1 CONTINUE
      GO TO 4
    3 CALL DNRCCT(N,A1,A2,EVAL,EVECT,0)
      SIN= TRUE
    4 CCNTINUE
      RETURN
      END
```

```
      SUBROUTINE DNROOT (M,A,B,XL,X,MODE)
      DIMENSION A(1),B(1),XL(1),X(1)
      DOUBLE PRECISION A,B,XL,X,SUMV
      IF (MODE .EQ. 1) GC TC 101
      K=1
      DC 100 J=2,M
      L=M*(J-1)
      DO 100 I=1,J
      L=L+1
      K=K+1
  100 B(K)=B(L)
C
C        THE MATRIX B IS A REAL SYMMETRIC MATRIX
C
      MV=0
      CALL DEIGEN (B,X,M,MV)
C
C     FCRM RECIPROCALS OF SQUARE ROOT OF EIGENVALUES   THE RESULTS
C     ARE PREMULTIPLIED BY THE ASSOCIATED EIGENVECTORS
C
      L=0
      DO 110 J=1,M
      L=L+J
  110 XL(J)=1 C/DSQRT(DABS(B(L)))
      K=0
      DC 115 J=1,M
      DO 115 I=1,M
      K=K+1
  115 B(K)=X(K)*XL(J)
  101 CONTINUE
C
C     FCRM (B**(-1/2))PRIME * A * (B**(-1/2))
C
      DO 120 I=1,M
      N2=0
      DO 120 J=1,M
```

```
      N1=M*(I-1)
      L=M*(J-1)+I
      X(L)=0.0
      DC 120 K=1,M
      N1=N1+1
      N2=N2+1
  120 X(L)=X(L)+B(N1)*A(N2)
      L=C
      DC 130 J=1,M
      DO 130 I=1,J
      N1=I-M
      N2=M*(J-1)
      L=L+1
      A(L)=0.0
      DC 130 K=1,M
      N1=N1+M
      N2=N2+1
  130 A(L)=A(L)+X(N1)*B(N2)
C
C     CCMPUTE EIGENVALUES AND EIGENVECTORS OF A
C
      CALL DEIGEN (A,X,M,MV)
      L=0
      DO 140 I=1,M
      L=L+I
  140 XL(I)=A(L)
C
C     CCMPUTE THE NORMALIZED EIGENVECTORS
C
      DO 150 I=1,M
      N2=0
      DO 150 J=1,M
      N1=I-M
      L=M*(J-1)+I
      A(L)=0.0
      DO 150 K=1,M
```

```
      N1=N1+M
      N2=N2+1
  150 A(L)=A(L)+B(N1)*X(N2)
      L=C
      K=0
      DC 180 J=1,M
      SUMV=C C
      DC 170 I=1,M
      L=L+1
  170 SUMV=SUMV+A(L)*A(L)
  175 SUMV=DSQRT(SUMV)
      DO 180 I=1,M
      K=K+1
  180 X(K)=A(K)/SUMV
      RETURN
      END
```

```
      SUBROUTINE INTP(W,K,L,YO,XO,DX,DY,NY,X1,Y1,WANS,WXANS)            INTP0015
C                                                                       INTP0020
      IMPLICIT REAL*8 (A,B,C,D,E,F,G,H,O,P,Q,R,S,T,U,V,W,X,Y,Z)
C                                                                       INTP0030
      DIMENSION II(4),JJ(4),K(1),L(1),WW(4),WWX(4),W(1)                 INTP0025
C     W  FUNCTIONAL VALUES ON A CLOSED RECTANGULAR NODAL POINT SET      INTP0035
C     NY  NUMBER OF STRIPS IN RECTANGULAR NODAL POINT SET               INTP0040
C     K,L  LIMITS DEFINING NODAL POINT BOUNDARIES OF THE NY STRIPS      INTP0045
C     DX,DY  SPACING OF THE NODAL POINTS                                INTP0050
C     X1,Y1  COORDINATES OF REFERENCE NODAL POINT IN XO,YO SYSTEM       INTP0055
C     WANS  FUNCTIONAL VALUE AT (XO,YO)                                 INTP0060
C     WXANS  FUNCTIONAL DERIVATIVE OF WANS IN THE XO DIRECTION          INTP0065
C                                                                       INTP0070
C                                                                       INTP0075
C     SHIFT COORDINATES TO NODAL POINT COORDINATE SYSTEM                INTP0080
C                                                                       INTP0085
C                                                                       INTP0090
      YY=YO-Y1                                                          INTP0095
      XX=XO-X1                                                          INTP0100
C                                                                       INTP0105
C     DETERMINE IDENTIFICATION NUMBERS OF SURROUNDING NODAL POINTS      INTP0110
C                                                                       INTP0115
   12 KMIS=0                                                            INTP0120
      XI=XX/DX                                                          INTP0125
      YI=YY/DY                                                          INTP0130
      II(1)=XI                                                          INTP0135
      II(1)=II(1)+1                                                     INTP0140
      II(3)=II(1)                                                       INTP0145
      II(2)=II(1)+1                                                     INTP0150
      II(4)=II(2)                                                       INTP0155
      JJ(1)=YI                                                          INTP0160
      JJ(1)=JJ(1)+1                                                     INTP0165
      JJ(2)=JJ(1)                                                       INTP0170
      JJ(3)=JJ(2)+1                                                     INTP0175
      JJ(4)=JJ(3)                                                       INTP0180
C                                                                       INTP0185
```

```
      DO 15 M=1,4                                                       INTP0190
      JC=JJ(M)                                                          INTP0195
      IF(II(M).LT K(JO))GO TO 100                                       INTP0200
      IF(II(M).GT L(JO))GO TO 100                                       INTP0205
      GO TO 16                                                          INTP0210
C                                                                       INTP0215
C                                                                       INTP0220
C     POINT II(M),JJ(M) IS NCT AN INPUT POINT                           INTP0225
C                                                                       INTP0230
  100 IF(M EQ 1) KMIS=KMIS+1                                            INTP0235
      IF(M EQ 2) KMIS=KMIS+2                                            INTP0240
      IF(M EQ 3) KMIS=KMIS+4                                            INTP0245
      IF(M EQ 4) KMIS=KMIS+8                                            INTP0250
C                                                                       INTP0255
C     KMIS DETERMINES WHICH SURROUNDING NODAL POINTS ARE PRESENT        INTP0260
C                                                                       INTP0265
      GO TO 15                                                          INTP0270
C                                                                       INTP0275
C     POINT II(M),JJ(M) IS AN INPUT POINT                               INTP0280
C                                                                       INTP0285
C                                                                       INTP0290
C                                                                       INTP0295
C     PROCEED NOW TO FIT TWO DIMENSIONAL PARABOLA                       INTP0300
C     ABOUT THE POINTII(M),JJ(M)                                        INTP0305
C                                                                       INTP0310
   16 JCJ=JJ(M)                                                         INTP0315
      IOI=II(M)                                                         INTP0320
      JM1=JOJ-1                                                         INTP0325
      JP1=JCJ+1                                                         INTP0330
      IM1=IOI-1                                                         INTP0335
      IP1=IOI+1                                                         INTP0340
C                                                                       INTP0345
C                                                                       INTP0350
C     L1  LOCATION OF W(IOI,JM1)                                        INTP0355
C     L2  LOCATION OF W(IM1,JOJ)                                        INTP0360
C     L3  LOCATION OF W(ICI,JOJ)                                        INTP0365
```

```
C      L4  LOCATICN OF W(IP1,JOJ)                                    INTP0370
C      L5  LOCATION OF W(IOI,JP1)                                    INTP0375
C                                                                    INTP0380
       L3=0                                                          INTP0385
       IF(JM1.LT.1)GO TO 505                                         INTP0390
       DO 19 KK=1,JM1                                                INTP0395
    19 L3=L3+L(KK)-K(KK)                                             INTP0400
   505 L3=L3+IOI+JOJ-K(JOJ)                                          INTP0405
       L1=L3+K(JCJ)-L(JM1)-1                                         INTP0410
       L5=L3+L(JOJ)-K(JP1)+1                                         INTP0415
       L2=L3-1                                                       INTP0420
       L4=L3+1                                                       INTP0425
C                                                                    INTP0430
C                                                                    INTP0435
C      IF KB1=0  W(IOI,JM1) NCT AN INPUT POINT                       INTP0440
C      IF KB2=0  W(IP1,JOJ) NCT AN INPUT POINT                       INTP0445
C      IF KB3=0  W(IM1,JOJ) NCT AN INPUT POINT                       INTP0450
C      IF KB4=0  W(IOI,JP1) NCT AN INPUT POINT                       INTP0455
C                                                                    INTP0460
       IF(JM1.LT.1) GO TO 501                                        INTP0465
       KB1=1                                                         INTP0470
       IF(IOI.LT.K(JM1))KB1=0                                        INTP0475
       IF(ICI.GT.L(JM1))KB1=0                                        INTP0480
       GO TO 502                                                     INTP0485
   501 KB1=0                                                         INTP0490
   502 CCNTINUE                                                      INTP0495
C                                                                    INTP0500
       KB2=1                                                         INTP0505
       IF(IP1.LT.K(JCJ))KB2=0                                        INTP0510
       IF(IP1.GT.L(JOJ))KB2=0                                        INTP0515
C                                                                    INTP0520
       KB3=1                                                         INTP0525
       IF(IM1.LT.K(JOJ))KB3=0                                        INTP0530
       IF(IM1.GT.L(JCJ))KB3=0                                        INTP0535
C                                                                    INTP0540
       IF(JP1.GT.NY ) GO TO 503                                      INTP0545
```

```
      KB4=1                                                             INTP0550
      IF(IOI LT K(JP1))KB4=0                                            INTP0555
      IF(IOI GT L(JP1))KB4=0                                            INTP0560
      GC TO 5C4                                                         INTP0565
  503 KB4=0                                                             INTP0570
C                                                                       INTP0575
C                                                                       INTP0580
C     DETERMINE COEFFICIENTS OF LOCAL PARABOLIC FIT                     INTP0585
C                                                                       INTP0590
  504 IF(KB2 EC 1) GC TC 24                                             INTP0595
      C=0 0                                                             INTP0600
      IF(KP3 EC 1)GC TC 27                                              INTP0605
      B=C C                                                             INTP0610
      GC TO 26                                                          INTP0615
   24 IF(KB3 EQ 1)GC TC 25                                              INTP0620
      D=C C                                                             INTP0625
      P=W(L4)-W(L3)                                                     INTP0630
      GO TC 26                                                          INTP0635
   25 B=0 5*(W(L4)-W(L2))                                               INTP0640
      D=(W(L4)+W(L2)-2 0*W(L3))/2 0                                     INTP0645
      GC TC 26                                                          INTP0650
   27 B=W(L3)-W(L2)                                                     INTP0655
C                                                                       INTP0660
   26 IF(KB1 FQ 1)GO TO 28                                              INTP0665
      E=0 0                                                             INTP0670
      IF(KR4 EC 1)GC TC 31                                              INTP0675
      C=C C                                                             INTP0680
      GO TO 30                                                          INTP0685
   28 IF(KB4 EQ 1)GC TC 29                                              INTP0690
      E=C C                                                             INTP0695
      C=W(L3)-W(L1)                                                     INTP0700
      GC TC 30                                                          INTP0705
   29 C=0 5*(W(L5)-W(L1))                                               INTP0710
      E=(W(L1)+W(L5)-2 0*W(L3))/2 0                                     INTP0715
      GC TO 3C                                                          INTP0720
   31 C=W(L5)-W(L3)                                                     INTP0725
```

```
C                                                                       INTP0730
C                                                                       INTP0735
C     LOCAL PARABOLA GIVEN BY                                           INTP0740
C       WW(M)=WW(ICI,JCJ)=A+B*X+C*Y+0 5*D*X*X+0 5*E*Y*Y                 INTP0745
C                                                                       INTP0750
   30 A=W(L3)                                                           INTP0755
      XIX=IOI                                                           INTP0760
      YJY=JOJ                                                           INTP0765
      XCX=(XX/DX)-XIX+1 0                                               INTP0770
      YOY=(YY/DY)-YJY+1 0                                               INTP0775
      SW=((XOX*D)+B)*XOX                                                INTP0780
      WW(M)=((YCY*E)+C)*YOY+SW+A                                        INTP0785
      WWX(M)=((2 0*D)*XOX+B)/DX                                         INTP0790
   15 CONTINUE                                                          INTP0795
C                                                                       INTP0800
      KMIS=KMIS+1                                                       INTP0805
      XII=II(1)                                                         INTP0810
      YII=JJ(1)                                                         INTP0815
      YOY=YI-YII+1 0                                                    INTP0820
      XCX=XI-XII+1 0                                                    INTP0825
C                                                                       INTP0830
C                                                                       INTP0835
C     KMIS DETERMINES MANNER IN WHICH LOCAL PARABOLIC FITS ARE WEIGHTED INTP0840
C                                                                       INTP0845
      GO TO (17,180,181,182,183,184,185,186,187,188,189,190,191,192,193,INTP0850
     1101),KMIS                                                         INTP0855
C                                                                       INTP0860
  180 WW(1)=0 5*(WW(2)+WW(3))                                           INTP0865
      WWX(1)=0.5*(WWX(2)+WWX(3))                                        INTP0870
      GO TO 17                                                          INTP0875
  181 WW(2)=0 5*(WW(1)+WW(4))                                           INTP0880
      WWX(2)=0.5*(WWX(1)+WWX(4))                                        INTP0885
      GO TO 17                                                          INTP0890
  182 WW(1)=WW(3)                                                       INTP0895
      WWX(1)=WWX(3)                                                     INTP0900
      WW(2)=WW(4)                                                       INTP0905
```

```
      WWX(2)=WWX(4)                                                     INTP0910
      GO TO 17                                                          INTP0915
  183 WW(3)=0.5*(WW(1)+WW(4))                                           INTP0920
      WWX(3)=0.5*(WWX(1)+WWX(4))                                        INTP0925
      GO TO 17                                                          INTP0930
  184 WW(1)=WW(2)                                                       INTP0935
      WWX(1)=WWX(2)                                                     INTP0940
      WW(3)=WW(4)                                                       INTP0945
      WWX(3)=WWX(4)                                                     INTP0950
      GO TO 17                                                          INTP0955
  185 WW(3)=0.5*(WW(1)+WW(4))                                           INTP0960
      WWX(3)=0.5*(WWX(1)+WWX(4))                                        INTP0965
      WW(2)=WW(3)                                                       INTP0970
      WWX(2)=WWX(3)                                                     INTP0975
      GO TO 17                                                          INTP0980
  186 WW(1)=WW(4)                                                       INTP0985
      WW(2)=WW(4)                                                       INTP0990
      WW(3)=WW(4)                                                       INTP0995
      WWX(1)=WWX(4)                                                     INTP1000
      WWX(2)=WWX(4)                                                     INTP1005
      WWX(3)=WWX(4)                                                     INTP1010
      GO TO 17                                                          INTP1015
  187 WW(4)=0.5*(WW(2)+WW(3))                                           INTP1020
      WWX(4)=0.5*(WWX(2)+WWX(3))                                        INTP1025
      GO TO 17                                                          INTP1030
  188 WW(1)=0.5*(WW(2)+WW(3))                                           INTP1035
      WWX(1)=0.5*(WWX(2)+WWX(3))                                        INTP1040
      WW(4)=WW(1)                                                       INTP1045
      WWX(4)=WWX(1)                                                     INTP1050
      GO TO 17                                                          INTP1055
  189 WW(2)=WW(1)                                                       INTP1060
      WWX(2)=WWX(1)                                                     INTP1065
      WW(4)=WW(3)                                                       INTP1070
      WWX(4)=WWX(3)                                                     INTP1075
      GO TO 17                                                          INTP1080
  190 WW(1)=WW(3)                                                       INTP1085
```

```
      WW(2)=WW(3)                                                       INTP1090
      WW(4)=WW(3)                                                       INTP1095
      WWX(1)=WWX(3)                                                     INTP1100
      WWX(2)=WWX(3)                                                     INTP1105
      WWX(4)=WWX(3)                                                     INTP1110
      GC TC 17                                                          INTP1115
  191 WW(3)=WW(1)                                                       INTP1120
      WWX(3)=WWX(1)                                                     INTP1125
      WW(4)=WW(2)                                                       INTP1130
      WWX(4)=WWX(2)                                                     INTP1135
      GO TC 17                                                          INTP1140
  192 WW(1)=WW(2)                                                       INTP1145
      WW(3)=WW(2)                                                       INTP1150
      WW(4)=WW(2)                                                       INTP1155
      WWX(1)=WWX(2)                                                     INTP1160
      WWX(3)=WWX(2)                                                     INTP1165
      WWX(4)=WWX(2)                                                     INTP1170
      GC TO 17                                                          INTP1175
  193 WW(2)=WW(1)                                                       INTP1180
      WW(3)=WW(1)                                                       INTP1185
      WW(4)=WW(1)                                                       INTP1190
      WWX(2)=WWX(1)                                                     INTP1195
      WWX(3)=WWX(1)                                                     INTP1200
      WWX(4)=WWX(1)                                                     INTP1205
      GO TO 17                                                          INTP1210
C                                                                       INTP1215
  101 WRITE(6,111)                                                      INTP1220
      WRITE(6,112) XO,YO                                                INTP1225
  111 FORMAT(49HONO NEIGHBORING POINTS-NO INTERPOLATION ATTEMPTED)      INTP1230
  112 FORMAT(1HO,32HCOORDINATES OF POINT IN QUESTION//25X,3HXO ,E17 8,3HINTP1235
     1YO=,E27.8)                                                        INTP1240
      GC TC 13                                                          INTP1245
C                                                                       INTP1250
C                                                                       INTP1255
C     COEFFICIENTS OF WEIGHTING FUNCTION WANS                           INTP1260
C                                                                       INTP1265
```

```
   17 AA=WW(1)                                                    INTP1270
      AAX=WWX(1)                                                  INTP1275
      BB=WW(2)-WW(1)                                              INTP1280
      BBX=WWX(2)-WWX(1)                                           INTP1285
      CC=WW(3)-WW(1)                                              INTP1290
      CCX=WWX(3)-WWX(1)                                           INTP1295
      DD=(WW(4)-WW(3))-(WW(2)-WW(1))                              INTP1300
      DDX=(WWX(4)-WWX(3))-(WWX(2)-WWX(1))                         INTP1305
C                                                                 INTP1310
   18 WANS=AA+BB *XOX+CC*YOY+DD*XOX*YOY                           INTP1315
      WXANS=AAX+BBX*XOX+CCX*YOY+DDX*XOX*YOY+((BB+DD*YOY)/DX)      INTP1320
   13 CONTINUE                                                    INTP1325
      RETURN                                                      INTP1330
      END                                                         INTP1335
```

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