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EFFECT OF PARALLACTIC REFRACTION CORRECTION ON STATION HEIGHT DETERMINATION

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ABSTRACT

The effect of omitting the parallactic refraction correction for satellite optical observations in the determination of station coordinates is analyzed for a large satellite data distribution. A significant error effect is seen in station heights. A geodetic satellite data distribution of 23 close earth satellites, containing 30,000 optical observations obtained by 13 principal Baker-Nunn camera sites, is employed. This distribution was used in a preliminary Goddard Earth Model (GEM 1) for the determination of the gravity field of the Earth and geocentric tracking station locations.

The parallactic refraction correction is modeled as an error on the above satellite data and a least squares adjustment for station locations is obtained for each of the 13 Baker-Nunn sites. Results show an average station height shift of +8 meters with a dispersion of ±0.7 meters for individual sites. Station latitude and longitude shifts amounted to less than a meter.

Similar results are obtained from a theoretical method employing a probability distribution for the satellite optical observations.

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TABLE OF CONTENTS

ILLUSTRATIONS

TABLES

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EFFECT OF PARALLACTIC REFRACTION CORRECTION ON STATION HEIGHT DETERMINATION

I. INTRODUCTION

Parallactic refraction correction for optical observations is modeled as an error on a large geodetic satellite data distribution. The effect of this error on the determination of station coordinates is analyzed. A description of the parallactic refraction correction is presented in the introduction including a simplified error effect on station height.

Two methods of error analysis are applied. Method A employs an actual satellite data distribution on 23 close earth satellites for 13 stations. The data for this case is considered as a discrete sample. Method B employs a satellite probability distribution for observations at an idealized station. The distribution is assumed to represent the theoretical population for the discrete sample of Method A.

Data distributions and analysis techniques are discussed in Sections II and III. Results are presented in Section IV and a special case of interest for parallactic refraction correction is discussed in the appendix.

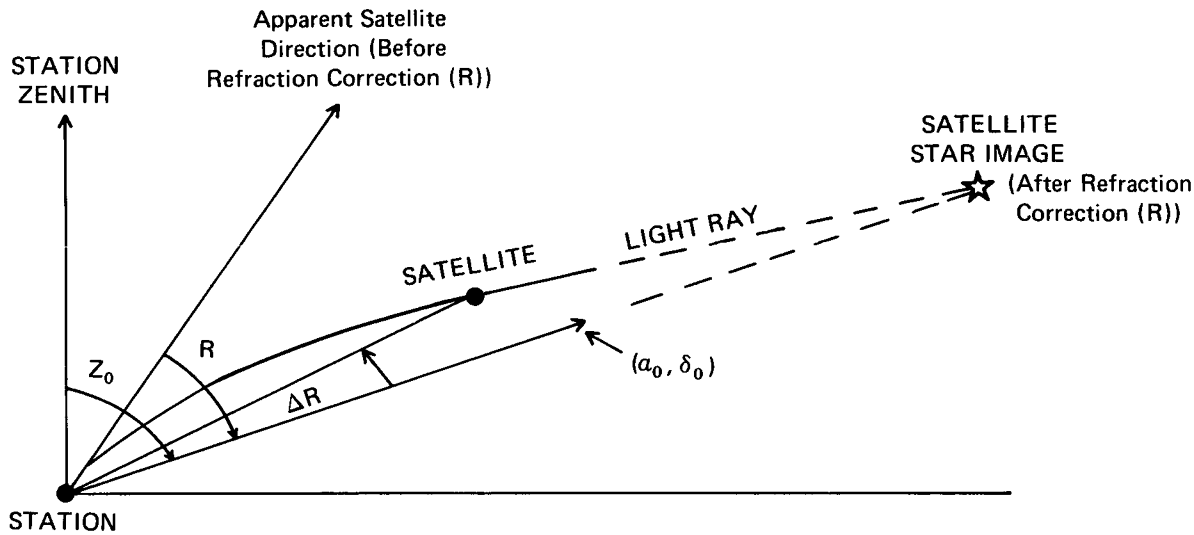
1. Parallactic Refraction

In the plate reduction of a photographic observation of an object, the observation is adjusted for normal atmospheric refraction by treating it as a star image. For a close earth object such as the satellite, see Figure 1, this amounts to an over correction for refraction. In the figure the parallactic refraction, ΔR, adjusts the normal refraction R so that the direction of the observation is pointed to the satellite.

The satellite optical observations are given in terms of right ascension (α_0) and declination (δ_0) which define the direction of the satellite star image as shown in Figure 1.

The satellite zenith, Z, is

$$Z = Z_0 - \Delta R. \tag{1}$$



(a_0, δ_0) – Right ascension (a_0) and declination (δ_0), define the direction of satellite star image corrected for normal refraction R.

Z_0 – Zenith angle corresponding to (a_0, δ_0).

R – Normal optical atmospheric refraction.

ΔR – Parallactic refraction.

Figure 1. Parallactic Refraction

The parallactic refraction correction used in the analysis as a modeling error is

$$\Delta R = 2.1 \tan Z/\rho \cos Z \quad \text{(radians)} \tag{2}$$

where ρ is the distance (meters) from the station to the satellite. A more complete representation accounting for variations in atmospheric conditions and short range distances is given in references 1 and 2.

2. Station Height Adjustment

If the station is constrainted from any horizontal adjustment, a simple relationship exists between parallactic refraction error and station height adjustment as illustrated in Figure 2. The relationship is given in equation (3) and results in a formula, equation (4), of height adjustment (Δh) as a function of zenith angle only. The formula is shown to be applicable in Section II, 2.1 for the case of the theoretical satellite data distribution of Method B, because of symmetry in satellite geometry about the local vertical of the station. In the case of the discrete sample of Method A for an actual satellite data distribution, no assumptions are made about the distribution and all three components of the station coordinate are allowed to adjust.

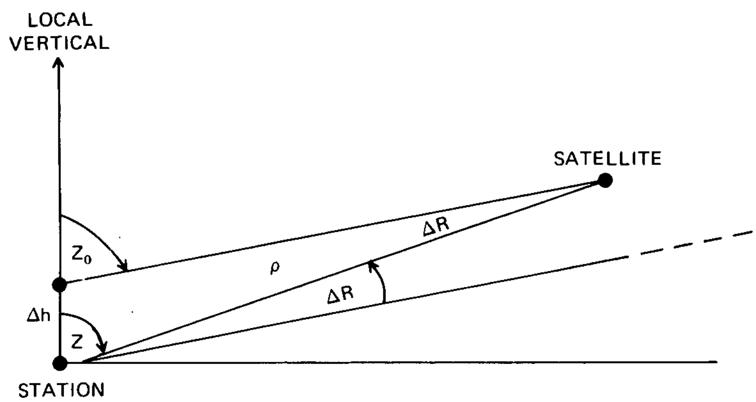


Figure 2. Station Height Adjustment

Using the law of sines for the triangle in Figure 2 and the fact that ΔR is small,

$$\Delta h = \rho\, \Delta R / \sin Z \quad \text{(meters)} \tag{3}$$

where ρ is in meters and ΔR in radians. From ΔR in equation (2) above, equation (3) becomes

$$\Delta h = 2.1/\cos^2 Z \quad \text{(meters)} \tag{4}$$

Table 1 is presented to show values of Δh and ΔR as a function of zenith angle. For ΔR, ρ = 1000 kilometers is used for simplicity. It is seen that Δh increases with zenith angle to a value as large as 18 meters for zenith Z = 70°.

The height relationship of equation (4) is used in Method B to estimate an expected height adjustment $\Delta \overline{h}$ over the theoretical satellite frequency distribution.

Table 1. Values of Δh and ΔR As a Function of Zenith

Z°	Δh (meters)	ΔR (seconds of arc, ρ = 1000 km.)
30	2.8	0.3
40	3.6	0.5
50	5.1	0.8
60	8.4	1.5
70	18.0	3.5

Values of ΔR as seen in Table 1 are of the order of size of the accuracy (σ) of optical observations, namely 2" of arc. In view of this a separate random error from a normal population (σ = 2") is applied to each of the observation components of right ascension and declination for the case of the discrete sample of Method A.

II. SATELLITE DATA DISTRIBUTIONS AND ANALYSIS – METHODS A AND B

1. Method A – Actual Satellite Data Distribution (Discrete Sample)

The parallactic refraction ΔR in equation (2) is modeled as an error on satellite optical observations of right ascension and declination (α, δ). The effect of this error is analyzed on the determination of station coordinates for a large geodetic satellite data distribution. The satellite data distribution consists of 30,000 optical observations (α, δ) taken by 13 Baker-Nunn camera sites on 23 close earth satellites, ranging in altitude from about 400 to 5000 kilometers. A least squares adjustment in station latitude, longitude, and height are determined for each of the 13 stations. The mathematical equations for the model are developed in Section III. The satellite data distribution is presented in Table 2 and its distribution as a function of zenith angle is presented in Figure 3. Figure 3 is discussed in the next section where a comparison is made between the discrete and theoretical frequency distributions for zenith angle.

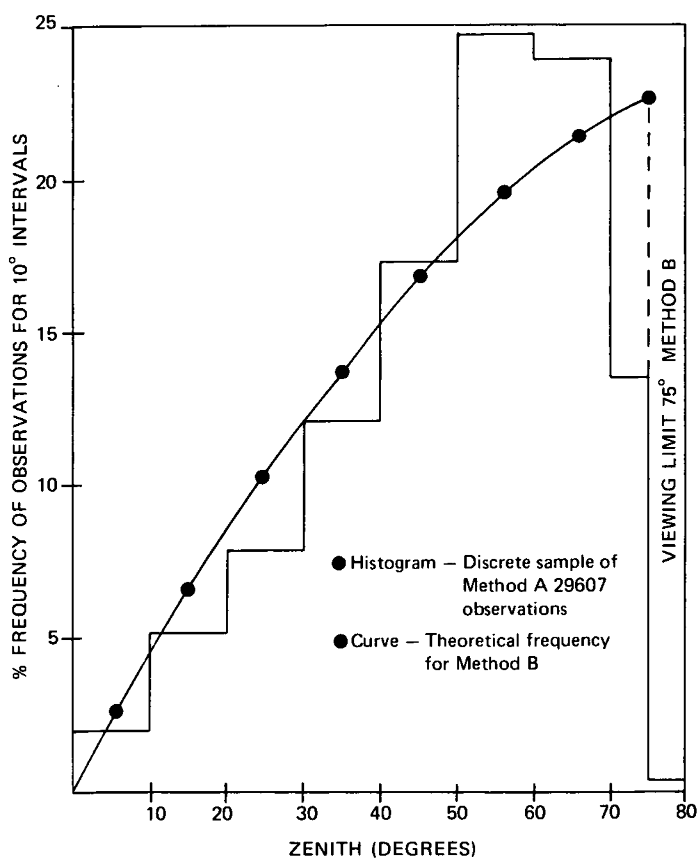


Figure 3. Satellite Frequency Distribution in Zenith Angle

Table 2. Satellite Data Distribution for Method A
(Baker-Nunn Data)

Satellite		No. of OBS	Mid Altitude	Perigee Height	Apogee Height
			(Kilometers*)		
AGENA-R	640011	477	900	900	950
ANNA-1E	620601	1630	1150	1000	1200
BE-B	640841	126	1000	900	1100
BE-C	650321	2286	1100	950	1300
COURIER	600131	1447	1050	950	1200
DI-C	670111	425	950	600	1350
DI-D	670141	2999	1250	600	1900
ECHO	600092	1014	1600	1500	1700
GEOS-1	650891	5812	1700	1100	2300
GEOS-2	680021	3660	1350	1100	1600
GRS	630261	174	850	400	1300
INJUN-1	610162	321	950	900	1000
MIDAS-4	610281	4451	3600	3500	3750
OGO-2	650811	363	1000	450	1500
OSCAR-7	660051	622	1000	850	1200
OVI-2	650781	315	1950	400	3500
SECOR-5	650631	124	1800	1150	2450
TELSTAR	620291	1620	3300	950	5600
TRANSIT	610151	728	950	900	1000
VAN-2R	590012	34	1950	550	3300
VAN-2S	590011	472	1950	550	3300
VAN-3S	590071	329	2100	500	3700
5BN-2	630492	178	1100	1050	1150

* Heights rounded to nearest 50 kilometers

The satellite data distribution in Table 2 was used in a preliminary Goddard Earth Model GEM 1, reference 3. A special case of interest in connection with parallactic refraction correction for the GEM 1 data distribution is presented in the appendix.

2. Method B – Satellite Probability Distribution

The satellite data distribution of Method A is regarded as a discrete sample from a theoretical population.

The probability distribution assumes the satellite population has equal area probability on any given sphere centered at the station, with a limiting condition on scheduling the satellite to within 75° in zenith angle. The probability distribution has complete symmetry about the local vertical of an idealized station.

Consider the line of sight to the satellite at zenith Z and distance ρ as in Figure 2. Form an element of area dA at zenith Z in a ring of width dz on the sphere of radius ρ, then

$$dA = 2\pi\rho^2 \sin Z \, dZ \tag{5}$$

Thus the probability (P) for the satellite to occur at zenith Z is

$$dP = \frac{dA}{A} = \frac{\sin Z \, dZ}{\int_0^{75^\circ} \sin Z \, dZ} \tag{6}$$

The satellite frequency distribution is then

$$f(Z) = \frac{\sin Z}{1 - \cos 75^\circ} \tag{7}$$

The distribution f(Z) is plotted in Figure 3 for comparison with the histogram of zenith frequency for the discrete sample of Method A. The histogram is given in terms of percent frequency at 10° zenith intervals. In order to compare with the histogram, the theoretical frequency f(Z), which has units of percent per radian, is plotted with a scale factor of 10° in radian measure. The behavior of the sample case as given in the histogram is irregular beyond 70° and is plotted at 5° intervals for 70° to 80° to show a negligible frequency beyond 75°. The probability that the satellite will have a zenith angle as large as Z is equal to the area under the frequency plot from zero out to Z (after the above factor of 10° in radian measure is inversely applied). Thus the integrated frequency distribution over the domain of Z for each plot will equal to unity.

It is shown analytically in section 2.1 that use of equation (4), where $\Delta h = 2.1/\cos^2 Z$, is applicable for Method B. This is so because of complete symmetry in the satellite probability distribution about the local vertical at the station. The expected value of Δh, over the frequency distribution in equation (7) is

$$\Delta \bar{h} = \int_0^{75^\circ} \Delta h \, f(Z) \, dZ \tag{8}$$

$$= \frac{2.1}{\cos 75^\circ} = 8.1 \text{ meters.}$$

The following table is presented to show variation in $\Delta \bar{h}$ for a 2° offset on each side of the limiting zenith angle of $Z_c = 75°$.

Table 3. Variation of $\Delta\bar{h}$ for Selected Zenith Limits

Zc	$\Delta\bar{h}$ meters
73°	7.2
75°	8.1
77°	9.3

The table shows sensitivity to the zenith limit, but a value of 75° appears to serve the best viewing limit based upon the histogram in Figure 3.

2.1 Simplified Height Relationship for Method B

The simplified formula used in Method B for $\Delta h = 2.1/\cos^2 Z$ resulted from the relationship shown in Figure 2, Section I, between the parallactic refraction ΔR and Δh. In this relationship it was assumed, as mentioned above, that no adjustment was necessary in the lateral direction of the station for the theoretical distribution of Method B. This condition is shown to be valid, although it may be intuitively clear. With the aid of Figure 2 consider a second point of satellite position placed symmetrically to the left of the station's local vertical. These two points of satellite position have equal probability of occurring in Method B. Through use of Figure 2, denote the vertical and lateral coordinates of the two satellite positions by $(y, \pm x)$ and denote the station coordinates by (v, ℓ). From the relationship,

$$\tan Z = (x - \ell)/(y - v) \tag{9}$$

the variation in Z gives respectively for the two points (y, ±x) of satellite position the two equations

$$\Delta Z = \Delta R = \frac{\sin Z}{\rho} \Delta v \mp \frac{\cos Z}{\rho} \Delta \ell \qquad (10)$$

where $\Delta R = 2.1 \tan Z/\rho \cos Z$ from equation (2).

Solving these two equations give $\Delta \ell = 0$ and the desired result for Δh, where $\Delta h = \Delta v$. Since the vertical plane in Figure 2 is arbitrary along with an arbitrary ρ and Z, the above result is valid for the entire distribution of Method B.

III. MATHEMATICAL MODEL FOR METHOD A

1. Method A - GEM 1 Satellite Data Distribution

The GEM 1 satellite data distribution, was described above in Section II. Table 2 above presented the distribution of data by satellites and Figure 3 gave the observation frequency distribution in zenith angle.

For the error model, observation residuals for right ascension (α) and declination (δ) are formed to contain the error of parallactic refraction ΔR. The residuals include a random error from a normal distribution with standard deviation of 2 seconds of arc. These residuals are expressed in terms of a linear adjustment in station components. A least squares reduction is then performed on the satellite data for each of 13 stations. The satellite position relative to the stations was obtained from the satellite orbits reduced in the GEM 1 solution, reference 3.

The observation residual equations in the form of orthogonal components are formally

$$\epsilon_1 + \cos\delta\,\Delta\alpha \simeq \cos\delta\left(\frac{\partial\alpha}{\partial h}\Delta h + \frac{\partial\alpha}{\partial N}\Delta N + \frac{\partial\alpha}{\partial E}\Delta E\right) \qquad (11)$$

$$\epsilon_2 + \Delta\delta \simeq \frac{\partial\delta}{\partial h}\Delta h + \frac{\partial\delta}{\partial N}\Delta N + \frac{\partial\delta}{\partial E}\Delta E \qquad (12)$$

where

$\Delta\alpha$ $= \alpha_0 - \alpha$, transformed effect of ΔR

$\Delta\delta$ $= \delta_0 - \delta$, transformed effect of ΔR

ϵ_1, ϵ_2 - the random errors described above

$\Delta h, \Delta N, \Delta E$ - orthogonal station component adjustments in metric length, respectively in the directions of the local vertical, North and East.

The equations for transforming ΔR to the observation residuals for α, δ and the equations for the partial derivatives may be developed with the aid of the geometry presented in Figure 4. The satellite/station orientation in the figure is presented on a unit sphere where arcs of great circles are employed. The angles defined in Figure 4 denote the following quantities:

B

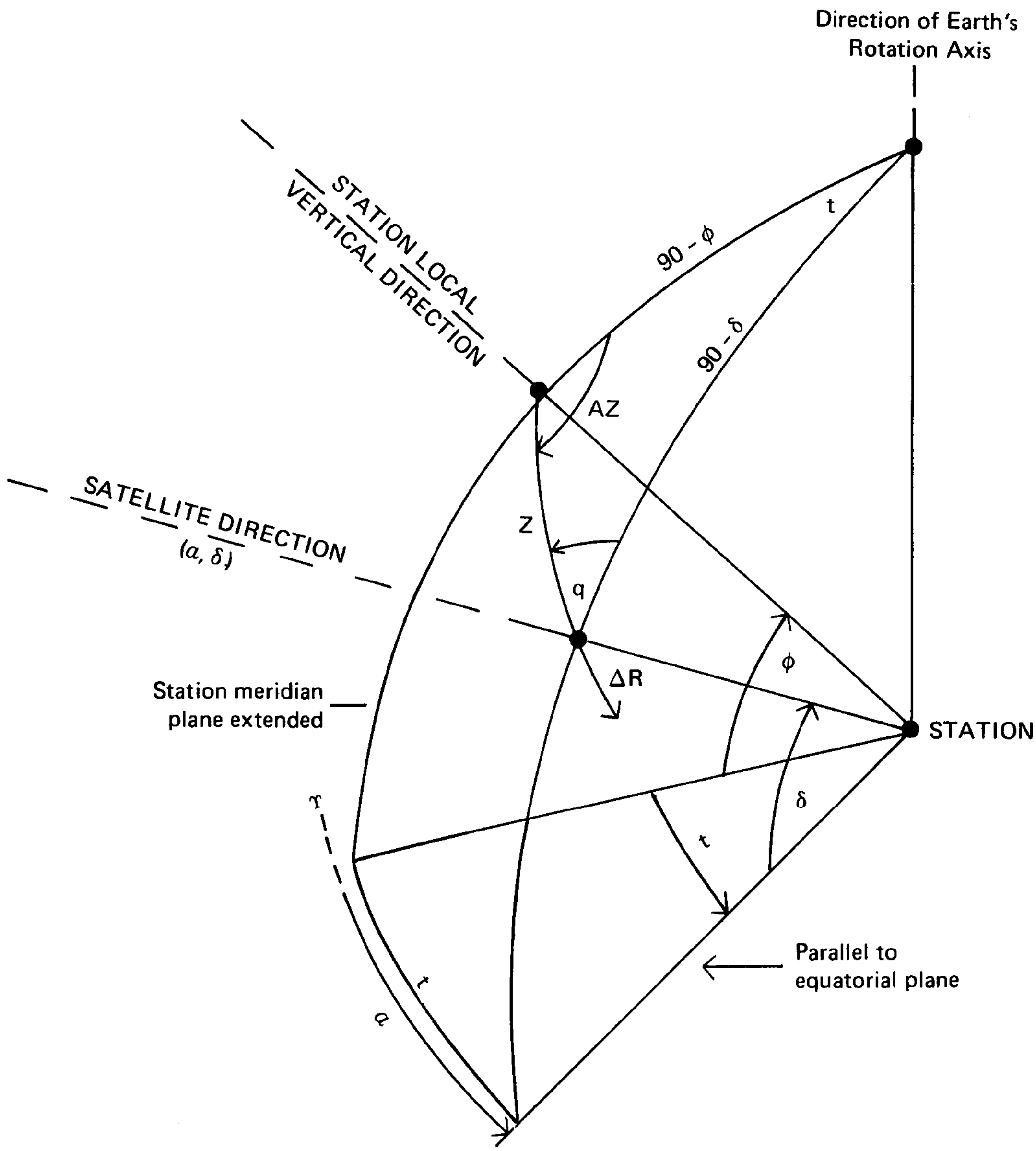


Figure 4. Satellite/Station Orientation Geometry on Unit Sphere

ϕ - station latitude

A_z - satellite azimuth as given in the local horizon plane at the station and measured eastward from true north

Z - satellite zenith

Z_0 - $Z + \Delta R$ modeled observation of satellite with parallactic refraction error ΔR

α - right ascension of satellite measured eastward from Aries (γ)

δ - declination of satellite measured northward from equatorial plane.

t - angle between the vertical planes of the satellite and station and measured in the direction of the right ascension α of the satellite

q - parallactic angle used to provide the orientation of ΔR and its projections in the direction of increasing δ and α.

Thus from the definition of q as given above and as shown in Figure 4, the orthogonal observation error components due to ΔR are

$$\cos \delta \, (\alpha_0 - \alpha) = \Delta R \sin q \tag{13}$$

$$\Delta \delta = - \Delta R \cos q. \tag{14}$$

From the spherical triangle in the figure, the laws of sines and cosines give

$$\sin q = \sin t \cos \phi / \sin Z \tag{15}$$

$$\cos q = (\sin \phi - \cos Z \sin \delta)/\sin Z \sin \delta. \tag{16}$$

Denoting the station and satellite geocentric vectors by $\vec{r}$ and $\vec{s}$ respectively, then the satellite observing direction relative to the station is along the vector

$$\vec{\rho} = \vec{r} - \vec{s}. \tag{17}$$

The variation of (17) due to the effect of parallactic refraction error on station adjustment is

$$\Delta \vec{\rho} = - \Delta \vec{s}. \tag{18}$$

The orthogonal metric components of $\Delta \vec{\rho}$ in the observation coordinate system of (ρ, α, δ) and $\Delta \vec{s}$ in the local coordinate system (V, N, E) defined above are

$$\Delta \vec{\rho} = (\Delta \rho,\ \rho \Delta \delta,\ \rho \cos \delta \Delta \alpha) \tag{19}$$

$$\Delta \vec{s} = (\Delta h,\ \Delta N,\ \Delta E) \tag{20}$$

Using Figure 4 the components of $-\Delta \vec{s}$ may be transformed to the components of $\Delta \vec{\rho}$ through the following rotation matrices,

$$\begin{bmatrix} \Delta \rho \\ \rho \Delta \delta \\ \rho \cos \delta \Delta \alpha \end{bmatrix} = - [\delta] \cdot [t] \cdot [-\phi] \begin{bmatrix} \Delta h \\ \Delta N \\ \Delta E \end{bmatrix} \tag{21}$$

where the angles φ, t, and δ are defined above. The matrix relation of interest in (21) for the observation equations (11) and (12) is given as follows:

$$\begin{bmatrix} \cos \delta \Delta \alpha \\ \\ \Delta \delta \end{bmatrix} = -\frac{1}{\rho} \begin{bmatrix} -s\,t\,c\,\phi & c\,t & s\,t\,s\,\phi \\ \\ c\,\delta\,s\,\phi - s\,\delta\,c\,t\,c\,\phi & -s\,t\,s\,\delta & s\,\delta\,c\,t\,s\,\phi + c\,\delta\,c\,\phi \end{bmatrix} \begin{bmatrix} \Delta h \\ \Delta E \\ \Delta N \end{bmatrix} \tag{22}$$

where s ≡ sin and c ≡ cos for the trigonometric functions.

Equation (22) provides the observation residual equations for the least squares adjustment of the station coordinates, for height Δh, for ΔN in the direction of latitude, and for ΔE in the direction of longitude. As indicated in equations (11) and (12) random errors $\epsilon 1$ and $\epsilon 2$ are added to the above residuals in the reduction (since the parallactic refraction error is the order of accuracy of the observations).

Results are obtained for each of the 13 Baker-Nunn stations and presented in Section IV.

IV. RESULTS

Error analysis results are presented for the effect of parallactic refraction error on station coordinates. Results for 13 Baker-Nunn stations are given in Table 4 using the technique of Method A, where the GEM 1 satellite data distribution was employed. Station height adjustments show an average or mean shift of 8.1 meters with an rms of 0.7 meters about the mean value. Horizontal components show very little adjustment. In Table 4 the mean value for the station height adjustment is the same as that for the idealized result of Method B, where a limiting value of zenith angle of 75° was employed.

The small horizontal station adjustments of Method A reflect a good geometrical distribution of data for each of the 13 Baker-Nunn sites in the GEM 1 data sample. In the idealized distribution of Method B it was shown in Section II, 2.1 that no adjustment results in the horizontal station components from the error effect of parallactic refraction correction.

Table 4. Station Adjustments for Method A and Method B

Method A

Station Name	ϕ° (Latitude)	Δ h(m) (Height)	Δ E(m) (Longitude)	Δ N(m) (Latitude)
1. ORGAN	32	8.0	-0.1	0.8
2. OLFAN	-26	8.2	-0.6	0.0
3. WOOMR	-31	10.2	0.0	-0.5
4. SPAIN	36	8.5	0.6	0.0
5. TOKYO	35	7.0	-0.5	0.4
6. NATAL	29	7.4	0.7	0.3
7. QUIPA	-16	9.5	0.0	-0.9
8. SHRAZ	29	8.1	-0.4	0.7
9. CURAC	12	8.1	0.2	-0.3
10. JUPTR	27	7.4	-0.2	0.6
11. VILDO	-32	7.6	0.4	0.5
12. MAUIO	21	8.5	-0.4	-0.2
13. AUSBK	-31	7.5	0.0	0.1
Mean shift		8.1	0.0	0.1
rms (about mean)		0.7	–	–

Method B (Idealized Station)

Zenith Limit	Δ h(m)	Δ E(m)	Δ N(m)
Z_c = 73°	7.2	0	0
Z_c = 75°	8.1	0	0
Z_c = 77°	9.3	0	0

V. SUMMARY

The effect of omission of parallactic refraction correction for optical observations in the determination of station coordinates was shown to have a significant effect on station height. Error analysis was made on a large geodetic satellite data distribution on 23 satellites for each of 13 Baker-Nunn camera sites. An average height adjustment for the 13 stations amounted to +8.1 meters with an rms dispersion of ±0.7 meters for individual sites. The effect on horizontal station components for each site was less than a meter. The latter effect results from a good geometrical distribution of data at a station. Similar results for station adjustments were obtained from a satellite probability distribution which was assumed to represent the theoretical population for the above discrete satellite sample.

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APPENDIX

A SPECIAL CASE OF PARALLACTIC REFRACTION ERROR

1. Modified Method A – Combined Parallactic Refraction Error

A special case of interest is analyzed because of a program modeling error of parallactic refraction correction that was made in processing the Baker-Nunn observations for the preliminary GEM 1 solution. This case is analyzed with the error analysis model for Method A given in the report. The error is associated with the projections for the parallactic angle q, given in equation (13) through (16) and shown in Figure 4 of the report. This material of Method A should be referred to for the following discussion.

A modified parallactic angle q', defined below, was used instead of q in equations (15) and (16) and was applied in the opposite sense of a correction. This resulted in a combined error on the observation components, consisting of the normal refraction error as given by equations (13) and (14) plus the effect of the modified correction, namely

$$\cos \delta \, (\alpha_0' - \alpha) = \Delta R \sin q + \Delta R \sin q' \tag{A-1}$$

$$\delta_0' - \delta = - \Delta R \cos q - \Delta R \cos q' \tag{A-2}$$

The modified parallactic angle q' is similar to q which is shown in Figure 4 of the report. The modified angle resulted by using the geocentric direction of the satellite in place of its topocentric direction (α, δ). The geometry for this case may be represented in Figure 4 by making the following replacements:

replace δ by ϕs	geocentric latitude of satellite
replace α by λs	longitude of satellite
replace t by t'	$t' = \lambda s - \lambda$, λ station longitude
replace z by θ	angle between the local vertical direction of the station and the geocentric direction of the satellite.

From similiarity to equations (15) and (16) in the report,

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$$\sin q' = \sin t' \cos \phi / \sin \theta \qquad \text{(A-3)}$$

$$\cos q' = (\sin \phi - \cos \theta \sin \phi_s)/\sin \theta \sin \phi_s. \qquad \text{(A-4)}$$

The modified observation error components, given by equations (A-1) and (A-2), are substituted in the matrix relation (22) of the report. A least squares adjustment is made for station coordinates as in the case of the normal parallactic refraction error of Method A. Results for the modified case are presented in the next section.

2. Results of Modified Method A

Table A1 presents the station adjustments due to the combined parallactic refraction error. The average station height adjustment is shown to be +13.5 meters with little adjustment for the horizontal station components. A systematic effect of the height adjustment Δh for individual stations may be seen in Figure A-1 where Δh is plotted as a function of station latitude. This effect on station height corresponds to (a) nearly twice the error of that obtained for the normal parallactic refraction error (Table 4 of the report) for stations near the equator and (b) diminishes to approximately 1-1/2 times the normal case for stations remote from the equator. The maximum deviation in station latitude as shown in Table A1 occurs for SPAIN at 36° North Latitude.

The reason for the latitude variation in Δh may be seen from the difference between q and q' as stations deviate from the equator. The effect of the observation errors in equations (A-3) and (A-4) on the zenith component is

$$\Delta Z = \Delta R \left[1 + \cos (q - q')\right] \qquad \text{(A-5)}$$

For the normal case of parallactic refraction error, $\Delta Z = \Delta R$. In the present case for the combined error of equation (A-5), if q is near q' then $\Delta Z \simeq 2 \Delta R$ and the height adjustment is nearly twice the effect for the normal case. As q separates from q' then ΔZ diminishes and in turn Δh reduces. A qualitative discussion for the behavior between q and q' is given.

For a station on the equator it may be shown, with the aid of Figure 4 and the associated equations for q and q', that q = q' for satellite azimuths in the North/South and East/West directions and that $\cos (q - q') \simeq 1$ for values of azimuth in between. However, for a station at +30° latitude and for large satellite

Table A-1. Station Adjustments for the Modified Case of Method A

Station Name	ϕ° (Latitude)	Δ h(m) (Height)	Δ E(m) (Longitude)	Δ N(m) (Latitude)	Δ h*(m) (Height)
1. ORGAN	32	12.3	0.4	-1.4	14
2. OLFAN	-26	14.6	1.0	0.9	7
3. WOOMR	- 31	16.9	0.6	1.6	12
4. SPAIN	36	11.5	0.9	-1.2	15
5. TOKYO	35	10.1	0.0	-0.8	11
6. NATAL	29	13.0	-0.4	-0.6	15
7. QUIPA	-16	16.3	-0.4	1.3	7
8. SHRAZ	29	12.1	0.7	-1.2	15
9. CURAC	12	15.3	-1.3	-0.4	11
10. JUPTR	27	12.5	-1.3	-0.3	14
11. VILDO	-32	12.7	-0.1	1.0	7
12. MAUIO	21	15.5	-0.3	-0.8	14
13. AUSBK	-31	13.9	0.2	0.9	7
Mean shift		13.5	0.4	-0.8	11.5

* Difference between the GEM1 solution and a later solution which employed the proper parallatic refraction correction.

zenith and declination angles, q and q' separate significantly. For instance if azimuth is true North, then q' = 0 and q = 180° for zenith angles greater than 60° (60° $\leq \delta \leq$ 90). A similar case exists for a station at -30° latitude. The reason for the separation of q and q' is due to the convergent meridians toward the poles at $\delta = \pm 90°$, which affects the orientation on q as seen by Figure 4. In a limited region about the poles, corresponding to satellite declination near ±90°, values of q and q' separate widely and hence ΔZ reduces by equation A-5. Since the scheduling opportunity is greater in this region for the stations that are closer to the poles, the reduction in Δ Z and thus Δ h is expected to be greater for stations that depart more from the equator as shown in Figure A-1.

A final result is given in the last column of Table A-1. It represents the height differences between the preliminary GEM 1 solution and a new solution in which the proper parallactic refraction correction was applied. The average height difference for the 13 Baker-Nunn stations is 11.5 meters. This result may be compared with the error analysis result of 13.5 meters for combined parallactic refraction correction. The two mean height errors are not completely comparable since much additional data was used in the new solution.

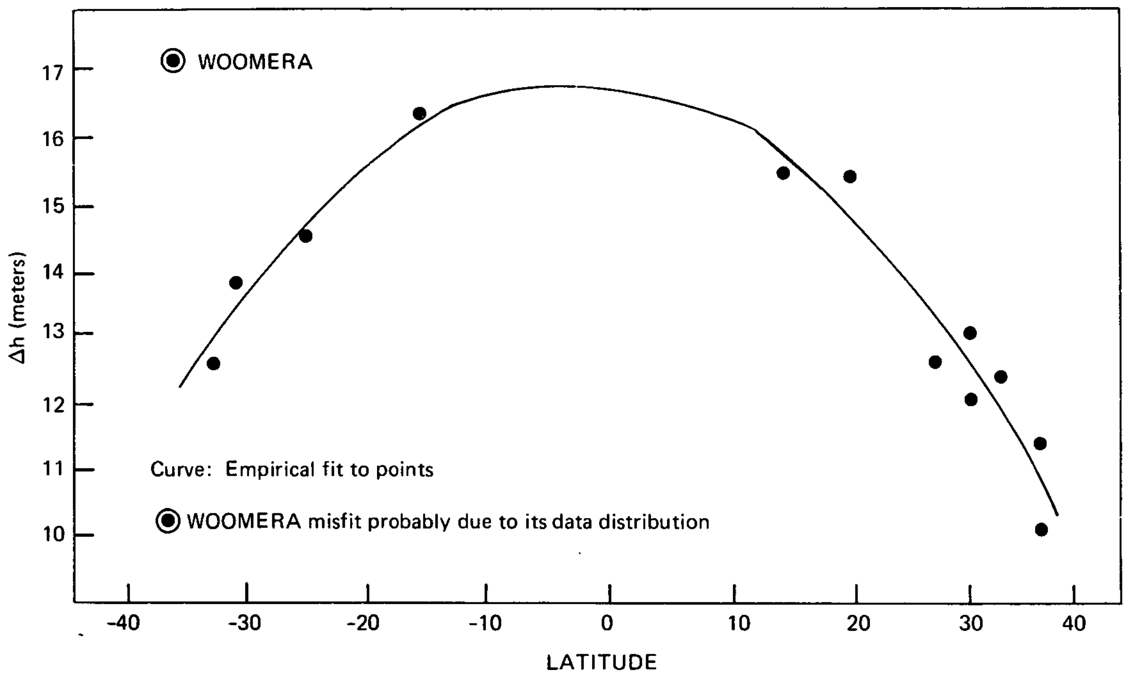

Figure A-1. Variation of Δ h with Station Latitude for Modified Method A

In the case of the preliminary GEM 1 solution a nominal correction of -15 meters in height was applied to each of the Baker-Nunn stations at the time of the solution. The nominal value at that time was based upon the technique of Method B for normal parallactic refraction error. A factor of slightly less than two was applied to the derived mean height adjustment from Method B in order to account for the effect of the combined parallactic refraction error given by equation A-5.

REFERENCES

1. Gaposchkin, E. M., "Empirical Data and the Variance - Covariance Motion for the 1969 Smithsonian Standard Earth (II)," SAO Special Report 342, pp. 5-7, April 1972.

2. Schmid, Hellmut, "The Influence of Atmospheric Refraction and Directions Measured to and from a Satellite," GIMRADA Research Note 10, 1963.

3. Lerch, F. J., Wagner, C. A., Smith, D. E., Sandson, M. L., et al., "Gravitational Field Models for the Earth (GEM 1 and 2)," GSFC X-553-72-146, May, 1972.

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