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for the

ADDITION OF THREE-DIMENSIONAL ISOPARAMETRIC ELEMENTS TO NASA STRUCTURAL ANALYSIS PROGRAM (NASTRAN)

Prepared by

E. I. Field
S. E. Johnson

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UNIVERSAL ANALYTICS, INC.
7151 West Manchester Avenue
Los Angeles, California 90045

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FOREWORD

This report was prepared by Universal Analytics, Inc. (UAI) under NASA Contract NAS1-11677, for the National Aeronautics and Space Administration, Langley Research Center, Hampton, Virginia. Dr. E.I. Field of UAI served as Program Manager with Mr. S.E. Johnson contributing significantly to the program's success. This work was administered under the direction of NASTRAN Systems Management Office (NSMO) with Dr. J.P. Raney and Mr. H. Adelman as the contract and technical monitors, respectively. UAI wishes to acknowledge their continued support and encouragement provided throughout the project.

This report is presented in one volume, containing unclassified material.

The objective of this contract effort was to incorporate the three-dimensional family (linear, quadratic and cubic) of isoparametric solid elements into NASTRAN, NAsa's STRuctural ANalysis computer program. The program code was designed and initially written for eventual installation into Level 16 of NASTRAN which is still in development. Actual implementation, however, was carried out in the Level 15.1 version that was made available by NSMO on the UNIVAC 1108 at the Jet Propulsion Laboratories in Pasadena, California.

The completed NASTRAN modifications and additions were successfully demonstrated by exercising three of the Rigid Formats for static, normal modes and buckling analysis. Final program documentation was prepared in the form of new sections and updates for direct insertion into the NASTRAN Theoretical, User's and Programmer's Manuals. These inserts are incorporated in the Appendices of this report. Compilation listings, program decks and the computer printout of the demonstration problems, submitted under separate cover, complete the deliverables for this contract.

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ABSTRACT

This report summarizes the work done by Universal Analytics, Inc. for NASA, Langley Research Center, to implement the three-dimensional family of linear, quadratic and cubic isoparametric solid elements into the NASA Structural Analysis program, NASTRAN. This work included program development, installation, testing and documentation. The addition of these elements to NASTRAN provides a significant increase in modeling capability particularly for structures requiring specification of temperatures, material properties, displacements and stresses which vary throughout each individual element.

Complete program documentation is presented in the form of new sections and updates for direct insertion to the three NASTRAN manuals. The results of demonstration test problems are summarized. Excellent results are obtained with the isoparametric elements for static, normal mode and buckling analyses.

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INTRODUCTION

The NAsa STRuctural ANalysis program (NASTRAN) has been developed with the intent of being responsive to a wide variety of user needs in the area of matrix structural analysis. In order to ensure the future usefulness of NASTRAN, it is essential that the best state-of-the-art finite element technology be represented in the system. New aerospace vehicle concepts, such as the Space Shuttle, have given impetus to NASA, through Langley Research Center (LRC), for undertaking the task of updating and improving NASTRAN. In response to this need, Universal Analytics, Inc. (UAI) has developed and installed the three-dimensional family of linear, quadratic and cubic isoparametric solid elements into NASTRAN. With this effort now completed, these three new elements greatly enhance NASTRAN's capability to solve any three-dimensional solid problem such as would be involved with both the structural and thermal design and analysis of the Space Shuttle.

The complete documentation of the new isoparametric solid element capability has been prepared in the form of new sections and updates to be inserted directly into each of the NASTRAN Theoretical, User's and Programmer's Manuals. These inserts are presented in this report in Appendixes A, B, and C respectively. The next section of this report discusses these documentation inserts as they were derived from the original program specification.

The remainder of this report describes the program implementation, testing and demonstration. Though the program was designed and coded for eventual implementation into Level 16 of NASTRAN when it becomes available, this report also documents the actual implementation as carried out in Level 15.1. A tabulation of typical computer run times is included for purposes of comparison. Instructions to the NASTRAN Maintenance Contractor for implementations of the new routines into Level 16 are presented in Appendix D. The excellent results obtained from the demonstration test problems are summarized from the reports presented in Appendix E. The observations made from testing the new element capability are presented in the concluding section of this report.

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DEVELOPMENT OF SPECIFICATIONS

The complete final documentation for the installation of the three isoparametric elements into NASTRAN is presented in the appendices of this report. The formal documentation of the theory, user input and program additions and modifications was prepared for direct insertion into the corresponding three NASTRAN Manuals. These inserts include new sections and appropriate up dates to the existing sections. The set of inserts for each manual is described in the paragraphs below.

The original specifications, consisting of both Theoretical and Functional Module Mathematical Specifications (FMMS), were prepared for installation of the new elements into Level 16 of NASTRAN. Level 16, when completed, will provide direct access capabilities required for efficient assembly of the isoparametric element matrices. Actual installation and testing, however, had to be carried out in Level 15.1, the latest available version of NASTRAN. Instructions to the NASTRAN maintenance contractor for installation into Level 16 when available are presented in Appendix D.

Theoretical Specifications

The documentation of the theory for isoparametric elements is presented in Appendix A along with instructions for their insertion into the NASTRAN Theoretical Manual. A complete mathematical description is provided with references. Both the structural and heat transfer theory is presented showing the distinct advantages gained by the direct integration approach to generating the element matrices. The unique advantage of the isoparametric family of elements is shown to derive from the assumption that temperatures, material properties, displacements and stresses may vary throughout the element. That is, these elements more accurately represent the structure and its response than do conventional constant strain elements.

User's Specifications

The inserts and updates for the NASTRAN User's Manual are presented in Appendix B. These include the descriptions of new Bulk Data cards (CIHEX1, CIHEX2, CIHEX3, PIHEX and PLØAD3), changes to the description of old Bulk Data cards (TEMP and TEMPD), additions to the acceptable element plot "SET" and associated symbols, and instructions to the user for modeling with this new capability. The new error messages, which may be issued during processing, are shown as additions to the User's Manual error message list.

Program Specifications

By far the bulk of the detail documentation involves updates and additions to the NASTRAN Programmer's Manual. These manual inserts are incorporated in Appendix C. They include all modifications to existing NASTRAN routines and block data descriptions as well as the description of all new subroutines. The specifications for the matrix operations and sequencing of integration steps required to implement the theoretical specifications are included as updates to the sections describing the appropriate NASTRAN modules which perform these tasks. Instructions to the NASTRAN maintenance contractor for installation into Level 16 are presented in Appendix D. The organization and development of the Level 16 routines and their adaptation for testing in Level 15.1 is summarized in the following section.

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PROGRAM IMPLEMENTATION

This section describes the development and implementation of the family of isoparametric solid elements for NASTRAN. The design of subroutines for Level 16 which was then modified to function for Level 15 is discussed. Modifications which were made to functional modules are described. The design and development of new subroutines are also described. Historical execution times are presented for an approximate comparison on computational efficiency for all three elements. The complete documentation of the program changes and additions are presented in Appendix C as inserts and updates to the NASTRAN Programmer's Manual.

Program Development

The element matrix generation subroutine XIHEX was initially developed in a stand-alone driver. It was designed to be installed Level 16. Subroutines KIHEX, MIHEX, and DIHEX were extracted from XIHEX to generate the element matrices in the Level 15 environment. These subroutines were developed as an interim measure so that the new elements could operate in Level 15. As KIHEX, MIHEX, and DIHEX were debugged, all changes and corrections made in them were also made in XIHEX. However, XIHEX could not be completely checked out because it could not operate in Level 15 and the stand-alone driver was inadequate for the task. Thus, it must be checked when it is installed in Level 16. Instructions and suggestions for this installation are contained in Appendix D. They should be helpful to the maintenance contractor in pointing out possible problem areas and incomplete interfaces with new Level 16 subroutines. Subroutine QIHEX, which computes the heat generation load vector for the isoparametric solid elements, could not be installed in Level 15. It was designed and developed for Level 16. Appendix D also describes the implementation of this subroutine. All other subroutines modified or added were developed in the environment of NASTRAN Level 15 and no significant changes are necessary to install them in Level 16.

Program Modifications

Many existing NASTRAN functional modules and subroutines were modified to implement the isoparametric solid elements. Several new subroutines were also added. These modules and subroutines are listed in Table 1 and brief descriptions of the changes to each functional module are described in Table 2. Many of the changes are those normally required when implementing new elements. However, in this case, changes were also required in the PLØT module (to plot 3-dimensional elements), the GP3 module (to process a new external load), and in various other modules to accommodate the large space requirements of the cubic element. All modifications to existing subroutines and new subroutines contain extensive in-program documentation with comment cards to assist in any future program development. Further documentation is located in the inserts and updates for the Programmer's Manual, Appendix C. These inserts and updates are based on the Programmer's Manual for Level 14 since the Level 15 Programmer's Manual was not completed during the fulfillment of this contract.

TABLE 1. SUBROUTINES MODIFIED OR ADDED TO IMPLEMENT THE FAMILY OF ISOPARAMETRIC SOLID ELEMENTS

Module	Subroutine	Module	Subroutine	Module	Subroutine	Module	Subroutine
XCSA	LD01	GP2	GP2	SSG1	EDTL	Various	GPTABD
	LD02				EXTERN		IHEXSD*
	LD03	PLTSET	CNSTRC		IHEX*		IHEXSS*
	LD04		CØMECT		PERMUT		PREMAT
	LD05	PLØT	ELELBL		PLØADS*		TKTZTK*
	LD06		SHAPE		QIHEX**		
	LD07					EMG	XIHEX**
	LD08	GP3	GP3	DSMG1	DIHEX*		
	LD09		GP3A		DS1		
	LD10		GP3BD		DS1A		
	LD11		GP3C		DS1ETD*		
	LD12		GP3D	SDR2	SDRETD		
	LD13				SDR2B		
		TA1	TA1A		SDR2BD		
IFP	IFP		TA1B		SDR2D		
	IFS5P		TA1ETD*		SDR2E		
	IFX1BD	SMA1	KIHEX*		SIHEX1*		
	IFX2BD		SMA1A		SIHEX2*		
	IFX3BD		SMA1BD				
	IFX4BD			ØFP	ØFP		
	IFX5BD	SMA2	MIHEX*		ØFP1A		
	IFX6BD		SMA2		ØFP1BD		
	IFX7BD		SMA2A		ØFP5BD		
			SMA2BD		ØF1PBD		
					ØF2PBD		
					ØF3PBD		
					ØF4PBD		

*New subroutine.

**New subroutine not implemented in Level 15.

TABLE 2. DESCRIPTIONS OF FUNCTIONAL MODULE MODIFICATIONS

XCSA	-	New entries were made in the card name table in subroutines LDi, i=1, 13 for the new Bulk Data cards
IFP	-	New Bulk Data cards were added
GP2	-	Array sizes were increased to accommodate elements with 32 grid points
PLTSET	-	Array sizes were increased to accommodate elements with 32 grid points
PLØT	-	Capability for plotting solid elements was implemented
GP3	-	PLØAD3 card processing was implemented
TA1	-	Capability to append grid point temperatures to EST/ECPT entries was implemented
SMA1	-	Stiffness and conductance matrix generation for the new elements was implemented
SMA2	-	Mass and capacitance matrix generation for the new elements was implemented
SSG1	-	Load vector generation for thermal and pressure loads on the new elements was implemented
DSMG1	-	Differential stiffness matrix generation for the new elements was implemented
SDR2	-	Stress calculations for the new elements was implemented
ØFP	-	Stress printout formats for the new elements were implemented

Program Additions

The New NASTRAN subroutines required to implement the family of isoparametric solid elements have been designed to operate in the Level 16 environment. The subroutine (XIHEX) which generates stiffness, mass, differential stiffness, conductance, and capacitance matrices was specifically designed for Level 16 Functional Module EMG (Element Matrix Generator). It was altered to create four interim subroutines (KIHEX, MIHEX, DIHEX, TKTZTK), which will operate in Level 15. When the isoparametric solids are incorporated into Level 16, these four subroutines should be discarded and subroutine XIHEX would take their place.

Subroutine XIHEX is divided into three major sections. These are initialization and geometry verification, matrix generation, and transformation and output. The first section initializes variables for the matrix generation process and checks the geometry of the element. If the geometry is acceptable, matrix generation proceeds. UAI believes that a good program design would verify the geometry of all elements, particularly isoparametric elements, before generating any element matrices. This is not possible, however in the current Level 15 design. An error in the geometry of the last element would result in the loss of all element matrices previously computed. For problems involving many isoparametric elements, this loss could be quite expensive.

In the second section of XIHEX, the stiffness (or conductance) and mass (or capacitance) and the differential stiffness matrices are generated using numerical integration. Only the lower triangular portion of these matrices is generated and the matrix is generated in the basic coordinate system. The stiffness and mass (or conductance and capacitance) matrices may be generated in the same call to subroutine XIHEX, or they may be generated in individual calls. If they are being generated in the same call, they are generated simultaneously or serially depending on the amount of open core available.

In the third section of XIHEX, the symmetric halves of the mass and stiffness matrices are first transformed to the global coordinate system. Then three complete columns, corresponding to one grid point, are extracted from the matrices and passed to subroutine EMGØUT for output. Conductance and capacitance matrices are transposed in core and the full matrix is passed to EMGØUT.

Subroutines KIHEX, MIHEX, and DIHEX are interim subroutines for use in Level 15. They perform the same operations as XIHEX. KIHEX computes stiffness and conductance matrices; MIHEX computes mass and capacitance matrices; and DIHEX computes the differential stiffness matrix. These subroutines call the interim subroutine TKTZTK, which was developed to avoid a compiler error on the UNIVAC 1108. This error was avoided in XIHEX which, therefore, does not call TKTZTK. KIHEX, MIHEX, and DIHEX were extracted from XIHEX. To improve computational efficiency, they generate only those portions of the element matrix required for each pivot point. Also, because materials with thermal capacity are not available in Level 15, MIHEX uses the value of the thermal conductivity coefficient to emulate the element "capacitance" matrix.

Two new subroutines were required to implement the thermal and pressure load for the isoparametric solid elements. Subroutine IHEX generates the thermal load vector and subroutine PLØAD3 generates the pressure load vector. Following NASTRAN programming standards, both were originally coded in single precision. However, execution of the symmetrical demonstration problems produced noticeably unsymmetric results. This problem was traced to the load vector generation, where small extraneous numbers were being generated in elements of the load vector which should have been zero. IHEX and PLØAD3 were then modified to operate in full double precision, truncating only the final element load vector on being added to the global load vector. This decreased the size of the "small extraneous numbers" by several orders of magnitude and greatly improved the symmetry of the results. The increased accuracy is attributed mainly to the more accurate computation of the isoparametric shape functions, their derivatives, and of the Jacobian matrix.

Several other new subroutines were also required. IHEXSD and IHEXSS compute the shape functions, their derivatives, and the determinant and inverse of the Jacobian matrix in double or single precision. SIHEX1 computes stress matrices and SIHEX2 computes stresses for the new elements. QIHEX calculates the heat generation load vector for a unit value of external heat per unit volume. QIHEX is not implemented in Level 15. Appendix D describes how it may be implemented in Level 16.

Program Execution

The execution times required to generate element matrices in Level 16 using subroutine XIHEX will be substantially less than the execution times of Level 15. Table 3 shows the approximate matrix generation times for subroutine XIHEX operating with the stand-alone driver used in its initial development. These execution times are considerably less than those of Level 15 which are presented in Table 4. This is so even though the Level 15 times include the overhead of executing a complete functional module and not just a single subroutine.

For complete problem execution, Table 5 presents approximate computer run times for each of three Rigid Formats (1 - Static, 3 - Normal Modes, and 5 - Buckling). These timings were for the three demonstration problems described in the following section under Testing and Demonstration. For both Rigid Formats 3 and 5, the inverse power method was used. For purposes of comparison, the characteristic dimensions of the problem are also shown in Table 5.

Of noteworthy interest is the comparison of run times for the relative accuracy obtained in the solutions. That is, the IHEX3 model in every case has the shortest run time and yielded the most accurate results when deflections were used for comparison. Comparing element stresses, however, the IHEX2 model yielded the best results for the thermal loading conditions. The next section of this report presents the details of the problems analyzed.

TABLE 3. EXECUTION TIME FOR THE LEVEL 16 ELEMENT MATRIX GENERATION SUBROUTINE XIHEX

Matrix Type	Element Type	Number of Integration Points	UNIVAC 1108 CPU Time (Minutes)
Stiffness	IHEX1	2	0.005
	IHEX2	3	0.1
	IHEX3	4	0.6
Mass	IHEX1	2	0.002
	IHEX2	3	0.05
	IHEX3	4	0.3

TABLE 4. MODULE EXECUTION TIMES FOR A ONE-ELEMENT MODEL IN NASTRAN LEVEL 15

Matrix Type	Module Name	Element Type	Number of Integration Points	UNIVAC 1108 CPU Time (Minutes)
Stiffness	SMA1	IHEX1	2	0.02
		IHEX2	3	0.3
		IHEX3	3	0.7
		IHEX3	4	2.1
Mass	SMA2	IHEX1	2	0.01
		IHEX2	3	0.1
		IHEX3	3	0.3
Differential Stiffness	DSMG1	IHEX1	2	0.05
		IHEX2	2	0.2
		IHEX3	2	0.5

TABLE 5. SOLUTION EXECUTION TIMES FOR DEMONSTRATION PROBLEMS

Model Characteristics				UNIVAC 1108 CPU Times (Minutes)		
Element Type	Number of Elements	Number of Grid Points	Half-Bandwidth (Grid Points)	RF1 - Statics 4 Load Conditions	RF3 - Modal 3 Modes#	RF5 - Buckling 3 Modes#
IHEX1	216	364	34	15	28	29
IHEX2	36	275	52	15	31	34
IHEX3	8	148	44	8	24*	18

*For the IHEX3 Modal Solution, 5 mode shapes were extracted.

#The inverse power method was used for eigenvalue extraction.

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TESTING AND DEMONSTRATION

Reports on the final demonstration testing for static normal mode and buckling analyses are incorporated in Appendix E. The demonstration problem selected and the results obtained for each model are discussed.

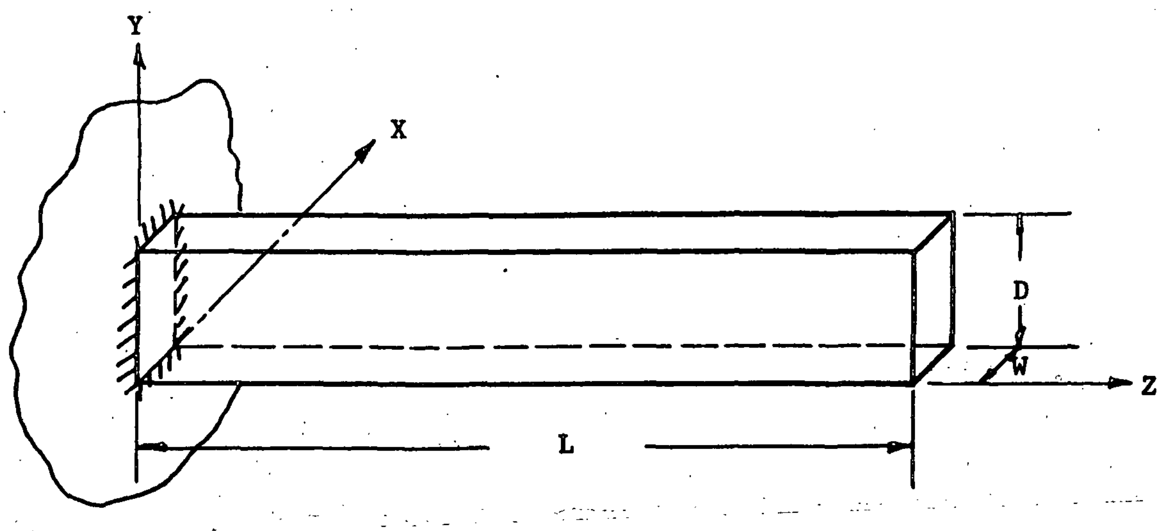
At each step in the program development, detail numerical checking was performed to qualify the logic and the computations. The routines for generating the element matrices were then integrated and exercised in a skeleton program to simulate the data blocks and control parameters to be passed from one NASTRAN module to another. At this level of development only the element matrix generation routines were tested to verify their design for implementation into Level 16 of NASTRAN.

These routines were then modified as described in the preceding section for insertion into Level 15. The necessary modifications were made to existing NASTRAN modules to prepare for their insertion and again each subroutine was tested for logic and correct computations. Whenever possible, existing programs previously developed by UAI were used to verify the final output matrices. Detail hand calculations for both structural and heat transfer computations were performed at each intermediate processing step. Checks were made to verify the shape functions, the Jacobian and its inverse, the computations of normals, the transformations, the stresses at each integration point, the strain matrix and the summation over each integration point. The final element structural matrices were then used to solve single element test problems to check the overall flow of all other module modifications, loads generation, plotting and stress computations. First, simple conditions were specified to check equilibrium. Then specific transformations were applied and the results again were checked. Combinations of a few elements were then used to verify the assembly of element structural matrices and to exercise the Rigid Formats to be used for final demonstration problem testing. The final demonstration testing was then performed, the results of which are summarized below.

Demonstration Problems

A cantilever beam was chosen for testing all three isoparametric solid elements in NASTRAN. This problem was chosen for two reasons, detail theoretical solutions are well known and solid finite elements characteristically do not perform well when used to model structures which deform primarily in a bending mode.

Three models were prepared, one with each of the three elements: CIHEX1, CIHEX2 and CIHEX3. All three beam models were of uniform crossection measuring 12 x 24 x 144 as pictured below.



Four static loading conditions were imposed:

a. Linear thermal gradient through its depth
   (T = 120° @ Y = 0, T = - 120° @ Y = 24)

b. Uniform temperature rise (ΔT = 100°)

c. Compressive axial pressure (used also for buckling)
   (P_z = 42,837 @ Z = 144)

d. Uniform transverse pressure
   (P_y = 100 @ Y = 0)

The total number of elements and grid points used, and the half-band width for each model is presented in the following table:

Model Type	Number of Elements	Number of Grid Points	Half-Band Width (Grid Points)
Model 1 - IHEX1 Elements	216	364	34
Model 2 - IHEX2 Elements	36	275	52
Model 3 - IHEX3 Elements	8	148	44

Examination of initial results which showed lack of symmetry, suggested that the load generation routines be changed to double precision, a departure from NASTRAN standard conventions. The resulting load vectors, when truncated to single precision for storage, provided excellent results. The experience gained with the thermal gradient loading condition confirmed that the selection of number of integration points (specified on the PIHEX card) is critical to the accurate computation of thermal loading vectors. The user is cautioned to request at least three integration points (NIP = 3 on the PIHEX card) for linear variations in temperature over the element. No distribution of temperature more severe than a "gentle" quadratic should be used. In this case at least four integration points are required (NIP = 4).

Typical computation times for generation and assembly of the isoparametric element matrices are presented in the preceding section. A summary of results reported in Appendix E for all three models is presented below for each of the three NASTRAN Rigid Formats used.

Static Analysis (Rigid Format - 1)

All four static loading conditions described earlier were imposed on each of the three models. The following table presents the results comparing the deflection at the tip to the theoretical solution.

Loading Case Maximum Displacement	NASTRAN			
	Model No. 1 IHEX1 Elements	Model No. 2 IHEX2 Elements	Model No. 3 IHEX3 Elements	Theoretical Solution
Y-Disp. - Load Case a	1.444	1.548	1.533	1.481
Z-Disp. - Load Case b	.2113	.2104	.2088	.2056
Z-Disp. - Load Case c	-.2039	-.2042	-.2047	-.2056
Y-Disp. - Load Case d	.1422	.1561	.1569	.1586

Note that the thermal loading results fall on either side of the theoretical and the mechanical loading results fall below theoretical, as would be expected.

Normal Modes (Rigid Format - 3)

The first three normal modes were extracted for each of the three models using the inverse power method and the coupled mass matrix generated by the isoparametric element routines. The models were too large to use the Givens method, through on smaller test cases, excellent comparisons were obtained. The natural frequencies obtained are compared with the theoretical solution in the following table.

Mode No.	Description	NASTRAN						Theoretical Solution (cps)
		Model No. 1 IHEX1 Elements		Model No. 2 IHEX2 Elements		Model No. 3 IHEX3 Elements		
		Freq.	% Error	Freq.	% Error	Freq.	% Error	
1	First Bending Mode in the X-Direction	22.0	+18	18.6	0	18.6	0	18.6
2	First Bending Mode in the Y-Direction	38.3	+4	36.5	-2	36.5	-2	37.3
3	Second Bending Mode in the X-Direction	135.3	+16	114.3	-2	113.3	-3	116.8

These results show excellent comparisons for the two higher order element models. The linear element model yields good results for the second mode only. However, because there were three elements through the thickness in the x-direction and six elements in the Y-direction, these results are as expected. They only point to the well known fact that many linear elements are required to adequately model bending behavior.

Buckling Analysis (Rigid Format - 5)

The final test for all three models was to compute the critical axial pressure buckling load (static loading c). Three buckling modes were computed using the inverse power method. These results are summarized below in comparison with the theoretical solution.

Mode No.	Description	NASTRAN						Theoretical Solution λ
		Model No. 1 IHEX1 Elements		Model No. 2 IHEX2 Elements		Model No. 3 IHEX3 Elements		
		λ	% Error	λ	% Error	λ	% Error	
1	X-Direction	1.406	40.6	1.002	.2	1.001	.1	1.0
2	Y-Direction	4.391	9.8	3.981	.5	3.979	.5	4.0
3	X-Direction	12.809	42.3	9.037	.4	8.934	.7	9.0

Again, excellent results are obtained for both the higher order element models. And again, the linear element shows much better results in bending in the Y-direction as expected. This further emphasizes the need for using many linear elements to adequately model bending behavior.

SUMMARY AND CONCLUSIONS

Three new solid elements have been implemented into NASTRAN for static, dynamic, buckling and heat transfer analysis. These are the linear, quadratic and cubic members of the isoparametric family of three-dimensional elements. The element matrices for both structural and heat transfer analyses are generated by direct integration over the element. These isoparametric elements have the unique advantage that variations in temperatures, material properties, displacements and stresses may be specified throughout the element. These elements can therefore represent a structure and it response more precisely than can the conventional constant strain elements.

The basic element matrix generation routines for these elements were designed and coded for implementation into NASTRAN Level 16 when it becomes available. For testing purposes, however, these routines were modified for and implemented into the latest available version of NASTRAN, Level 15.1 Final complete documentation, presented in Appendices A-C, was prepared for direct insertion into each of the NASTRAN Theoretical User's and Programmer's Manuals. Instructions are given in Appendix D to assist on the eventual implementation into Level 16. Reports are included in Appendix E on testing results obtained from demonstration problems exercising three Rigid Formats for static, normal modes and buckling analyses.

Complete checking at each step of the computations was performed manually and by comparison with other existing programs. Final verification of the integrated system was performed using Level 15. For each of the three NASTRAN Rigid Formats demonstrated, the results compared very favorable with the theoretical solution.

All three elements produced good demonstration test results for static, normal mode and buckling analyses. Problems for which thermal gradients were specified through the element yielded excellent results when sufficient integration points were specified for computation of the thermal load vector. As expected, the linear element results showed that it is best used when primarily a shearing type behavior is anticipated. And finally, the contention that the isoparametric three-dimensional solid elements are superior was validated by the excellent test results obtained with the quadratic and cubic elements when used to model the primarily bending type behavior of a cantilever beam. The implementation of these three isoparametric solid elements, therefore, does greatly enhance the total modeling capability of NASTRAN.

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APPENDIX A

THEORETICAL MANUAL INSERTS

The following sections contained in this appendix should be added to the NASTRAN Theoretical Manual in their entirety:

5.13 ISOPARAMETRIC HEXAHEDRON SOLID ELEMENTS

7.4 ISOPARAMETRIC HEXAHEDRON SOLID ELEMENTS

8.2.2 Three-Dimensional Isoparametric Solid Element Heat Transfer Matrices

The remaining sections are marked to indicate changes to the existing TM document.

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5.13 ISOPARAMETRIC HEXAHEDRON SOLID ELEMENTS

Hexahedron solid isoparametric elements may be used to analyze any three-dimensional continuum composed of isotropic or anisotropic materials. Examples include thick inserts in rocket engine nozzles, thermal protection system insulations, soil structure interaction problems, and geometrically complex thick-walled mechanical components such as pumps, valves, etc. The isoparametric solid elements take into account:

1. Isotropic or anisotropic temperature-dependent material properties
2. Pressure and temperature loads
3. Coupled mass matrix

Although solid elements employ only three degrees of freedom at each grid point (the three displacement components), they may be combined with all other NASTRAN elements.

The isoparametric solid elements were first presented by Irons, Ergatoudis and Zienkiewicz [Refs. 1, 2, 3 & 4]. They are also called conformal higher order elements, since the displacement of the element can be represented to any degree one desires, and still maintain interelement compatibility, by using more grid points per edge to define element geometry and deformation. In practice, however, isoparametric solid elements employing either eight, twenty or thirty-two grid points have been found to be adequate to solve most problems (Figure 1). These elements correspond to assuming a linear, parabolic and cubic variation of displacement, respectively. Clough [Ref. 5] conducted an evaluation of three-dimensional solid elements and showed that the isoparametric elements were superior to other solid elements. He further pointed out that the choice of which isoparametric element is best to use

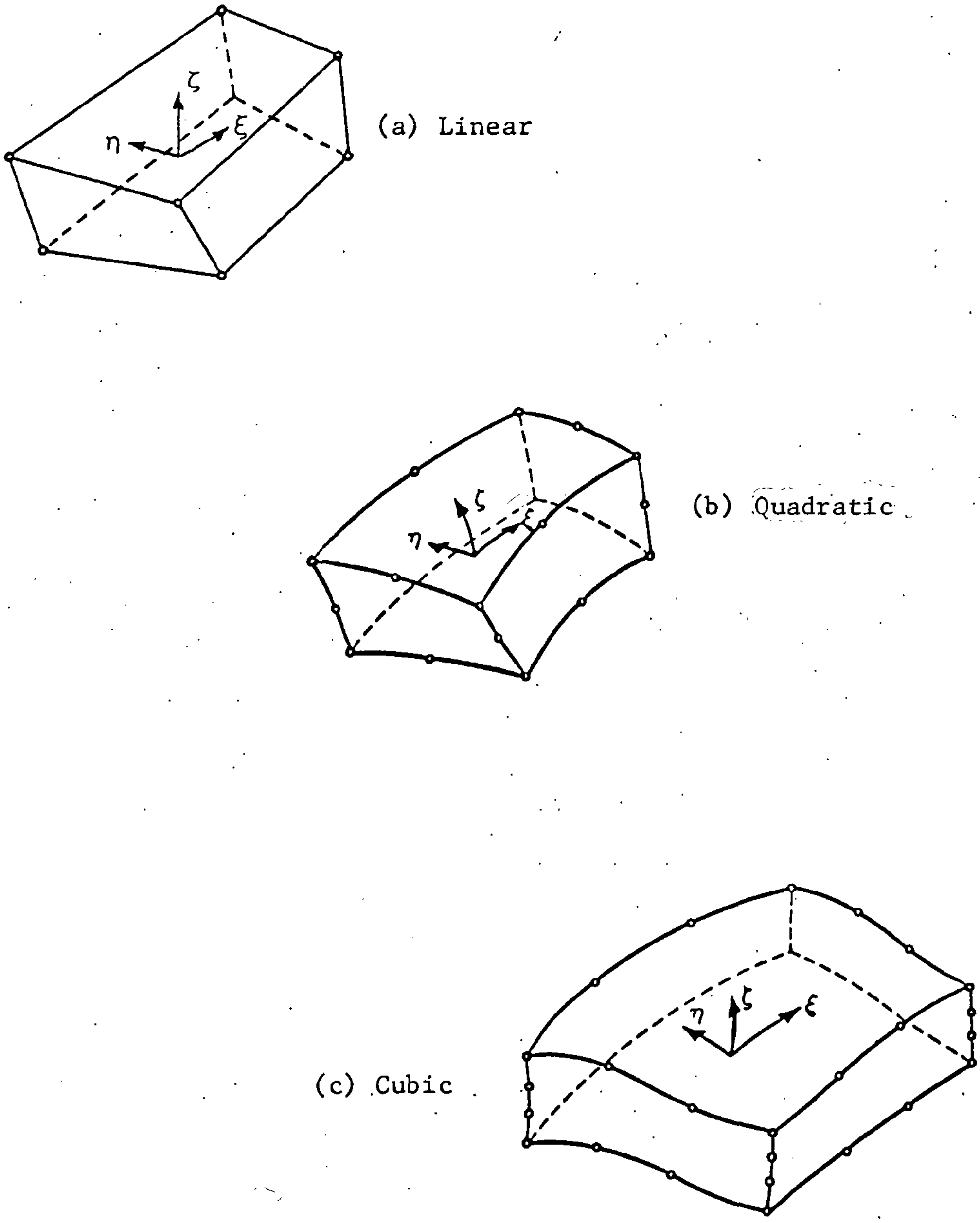

FIGURE 1. THREE ISOPARAMETRIC ELEMENTS

depends on the type of problem being solved. For problems involving plate bending type deformations, the higher order elements appear to be best, while the linear element is recommended for problems in which shear stresses are likely to be large. It is for this reason that all three isoparametric elements have been incorporated into NASTRAN.

The isoparametric elements governing equations are based on minimum energy principles. The derivation of these equations consists of assuming a displacement function for the element which depends on its grid point displacements, substitute these displacements into the Potential Energy, and minimize the energy functional to obtain the governing equations. The detailed derivation is next presented.

5.13.1 Displacement Functions for the Isoparametric Element

The name isoparametric comes from the fact that the same interpolating functions are used to represent both the geometry and the deformation of the element. This choice insures that the element displacement functions satisfy the criteria necessary for convergence of the finite element analysis [Ref. 4]. Referring to the curvilinear coordinates (ξ,η,ζ) shown in Figure 1, the (x,y,z) coordinates at any point in the element are obtained from the "basic" rectangular coordinates at each of the NGP grid points by:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \sum_{i=1}^{NGP} N_i\,(\xi,\eta,\zeta) \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_i \tag{1}$$

where $N_i(\xi,\eta,\zeta)$ are interpolating functions which depend on the number of grid points used to define the element geometry. The N_i functions are either linear, parabolic or cubic, and correspond to employing two, three or four grid points, respectively, along each edge of the element. This choice insures that there are no geometric gaps between grid points. Expressions for the interpolating functions and their derivatives are presented in Table 1.

Now, we represent the deformation of the elements with the identical interpolating functions used to define the geometry, i.e.:

$$\{\bar{u}\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{NGP} N_i(\xi, \eta, \zeta) \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_i = [N]\{u_e\} \tag{2}$$

where u, v and w are displacements along the x, y and z basic coordinate axes, and $\{u_e\}$ represents the vector of grid point displacements. The displacement functions, $N_i(\xi, \eta, \zeta)$ satisfy the required convergence criteria of adequately representing a constant strain state, and insure interelement compatibility along the complete element boundary [Ref. 4]

5.13.2 <u>Strain-Displacement Relationship</u>

The strains at any point within the element are given by the well known relations:

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{Bmatrix} \tag{3}$$

Substituting Eq. (2) into (3) yields the relationship for the strain vector in terms of the grid point displacements:

$$\{\epsilon\} = [C_1 \mid C_2 \mid \ldots\ldots \mid C_{NGP}] \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ \hline \vdots \\ \hline u_{NGP} \\ v_{NGP} \\ w_{NGP} \end{Bmatrix} = [C]\{u_e\} \tag{4}$$

TABLE 1a. ISOPARAMETRIC SHAPE FUNCTIONS AND THEIR DERIVATIVES FOR LINEAR ELEMENT - 8 GRID POINTS

CORNER GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm 1, \quad \zeta_i = \pm 1$$

$$N_i = \frac{1}{8} (1 + \xi_o) (1 + \eta_o) (1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{1}{8} \xi_i (1 + \eta_o) (1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{1}{8} \eta_i (1 + \xi_o) (1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{1}{8} \zeta_i (1 + \xi_o) (1 + \eta_o)$$

Where $\xi_o = \xi\xi_i$, $\eta_o = \eta\eta_i$, $\zeta_o = \zeta\zeta_i$

TABLE 1b. ISOPARAMETRIC SHAPE FUNCTIONS AND THEIR DERIVATIVES FOR PARABOLIC ELEMENT - 20 GRID POINTS

CORNER GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm 1, \quad \zeta_i = \pm 1$$

$$N_i = \frac{1}{8}(1+\xi_o)(1+\eta_o)(1+\zeta_o)(\xi_o+\eta_o+\zeta_o-2)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{1}{8}\xi_i(1+\eta_o)(1+\zeta_o)(2\xi_o+\eta_o+\zeta_o-1)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{1}{8}\eta_i(1+\xi_o)(1+\zeta_o)(2\eta_o+\xi_o+\zeta_o-1)$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{1}{8}\zeta_i(1+\zeta_o)(1+\xi_o)(2\zeta_o+\xi_o+\eta_o-1)$$

MID-SIDE GRID POINTS

$$\xi_i = 0, \quad \eta_i = \pm 1, \quad \zeta_i = \pm 1$$

$$N_i = \frac{1}{4}(1-\xi^2)(1+\eta_o)(1+\zeta_o)$$

$$\frac{\partial N_i}{\partial \xi} = -\frac{1}{2}\xi(1+\eta_o)(1+\zeta_o)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{1}{4}(1-\xi^2)(1+\zeta_o)\,\eta_i$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{1}{4}(1-\xi^2)(1+\eta_o)\,\zeta_i$$

MID-SIDE GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = 0, \quad \zeta_i = \pm 1$$

$$N_i = \frac{1}{4}(1-\eta^2)(1+\xi_o)(1+\zeta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{1}{4}(1-\eta^2)(1+\zeta_o)\,\xi_i$$

$$\frac{\partial N_i}{\partial \eta} = -\frac{1}{2}\eta(1+\zeta_o)(1+\xi_o)$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{1}{4}(1-\eta^2)(1+\xi_o)\,\zeta_i$$

MID-SIDE GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm 1, \quad \zeta_i = 0$$

$$N_i = \frac{1}{4}(1-\zeta^2)(1+\xi_o)(1+\eta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{1}{4}(1-\zeta^2)(1+\eta_o)\,\xi_i$$

$$\frac{\partial N_i}{\partial \eta} = \frac{1}{4}(1-\zeta^2)(1+\xi_o)\,\eta_i$$

$$\frac{\partial N_i}{\partial \zeta} = -\frac{1}{2}\zeta(1+\xi_o)(1+\eta_o)$$

Where $\xi_o = \xi\xi_i, \quad \eta_o = \eta\eta_i, \quad \zeta_o = \zeta\zeta_i$

TABLE 1c. ISOPARAMETRIC SHAPE FUNCTIONS AND THEIR DERIVATIVES FOR CUBIC ELEMENT - 32 GRID POINTS

CORNER GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm 1, \quad \zeta_i = \pm 1$$

$$N_i = \frac{9}{64}(1 + \xi_o)(1 + \eta_o)(1 + \zeta_o)(\xi^2 + \eta^2 + \zeta^2 - 19)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{9}{64}(1 + \eta_o)(1 + \zeta_o)[\xi_i(3\xi^2 + \eta^2 + \zeta^2 - 19) + 2\xi]$$

$$\frac{\partial N_i}{\partial \eta} = \frac{9}{64}(1 + \xi_o)(1 + \zeta_o)[\eta_i(3\eta^2 + \xi^2 + \zeta^2 - 19) + 2\eta]$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{9}{64}(1 + \xi_o)(1 + \eta_o)[\zeta_i(3\zeta^2 + \xi^2 + \eta^2 - 19) + 2\zeta]$$

MID-SIDE GRID POINTS

$$\xi_i = \pm\tfrac{1}{3}, \quad \eta_i = \pm 1, \quad \zeta_i = \pm 1$$

$$N_i = \frac{9}{64}(1 - \xi^2)(1 + 9\xi_o)(1 + \eta_o)(1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{9}{64}(1 + \eta_o)(1 + \zeta_o)(-2\xi + 9\xi_i - 18\xi\xi_o)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{9}{64}(1 - \xi^2)(1 + 9\xi_o)(1 + \zeta_o)\,\eta_i$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{9}{64}(1 - \xi^2)(1 + 9\xi_o)(1 + \eta_o)\,\zeta_i$$

MID-SIDE GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm\tfrac{1}{3}, \quad \zeta_i = \pm 1$$

$$N_i = \frac{9}{64}(1 - \eta^2)(1 + 9\eta_o)(1 + \xi_o)(1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{9}{64}(1 - \eta^2)(1 + 9\eta_o)\,\xi_i$$

$$\frac{\partial N_i}{\partial \eta} = \frac{9}{64}(1 + \xi_o)(1 + \zeta_o)(-2\eta + 9\eta_i - 18\eta\eta_o)$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{9}{64}(1 - \eta^2)(1 + 9\eta_o)\,\zeta_i$$

MID-SIDE GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm 1, \quad \zeta_i = \pm\tfrac{1}{3}$$

$$N_i = \frac{9}{64}(1 - \zeta^2)(1 + 9\zeta_o)(1 + \xi_o)(1 + \eta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{9}{64}(1 - \zeta^2)(1 + 9\zeta_o)\,\xi_i$$

$$\frac{\partial N_i}{\partial \eta} = \frac{9}{64}(1 - \zeta^2)(1 + 9\zeta_o)\,\eta_i$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{9}{64}(1 + \xi_o)(1 + \eta_o)(-2\zeta + 9\zeta_i - 18\zeta\zeta_i)$$

Where $\xi_o = \xi\xi_i$, $\eta_o = \eta\eta_i$, $\zeta_o = \zeta\zeta_i$

and

$$[C_i] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{bmatrix} \tag{5}$$

In order to evaluate the strain matrix [C], we must calculate the derivatives of the shape functions, N_i, with respect to x, y and z. Since N_i is defined in terms of ξ, η and ζ, it is necessary to use the relation that:

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \\ \frac{\partial N_i}{\partial \zeta} \end{Bmatrix} \tag{6}$$

where [J] is the Jacobian matrix and is easily evaluated by noting that:

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix} = \begin{bmatrix} \frac{\partial N_1}{\partial \xi}, & \frac{\partial N_2}{\partial \xi}, & \cdots & \frac{\partial N_{NGP}}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta}, & \frac{\partial N_2}{\partial \eta}, & \cdots & \frac{\partial N_{NGP}}{\partial \eta} \\ \frac{\partial N_1}{\partial \zeta}, & \frac{\partial N_2}{\partial \zeta}, & \cdots & \frac{\partial N_{NGP}}{\partial \zeta} \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots \\ x_{NGP} & y_{NGP} & z_{NGP} \end{bmatrix} \tag{7}$$

The derivatives of the shape functions with respect to ξ, η and ζ are given in Table 1, and x_i, y_i and z_i are the coordinates of the element grid points, and NGP is the number of element grid points.

5.13.3 Stress-Strain Relations

The stress-strain relations for a general elastic anisotropic material are:

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix} = [G_e]\{\epsilon - \epsilon_t\} \tag{8}$$

where for an isotropic material

$$[G_e] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

and $\{\sigma\}$ is the stress vector in the basic rectangular Cartesian coordinate system, $[G_e]$ is, in general, a full symmetric material elastic modulus matrix, ε is the total strain vector given by Eq. (3), and ε_t is the thermal strain:

$$\{\epsilon_t\} = \{\alpha_e\}\bar{T} = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ \alpha_{xy} \\ \alpha_{yz} \\ \alpha_{zx} \end{Bmatrix} \bar{T} \tag{9}$$

where $\{\alpha_e\}$ is a vector of thermal expansion coefficients, and $\bar{T}$ is the temperature distribution within the element, and is determined from user-specified grid point temperatures, $\{T_e\}$ by applying the interpolating relation:

$$\bar{T} = \sum_{i=1}^{n} N_i(\xi, \eta, \zeta) T_i = [N]\{T_e\} \tag{10}$$

In NASTRAN, the user may specify either isotropic or anisotropic material properties. Furthermore, for anisotropic materials, the user may specify the properties with respect to a particular orientation that does not necessarily coincide with the basic rectangular coordinate system. NASTRAN will then transform the material properties to the basic coordinate system by constructing a transformation matrix, T^M, such that the material properties in the basic coordinate system are given by:

$$\left.\begin{aligned} [G_e] &= [T^M]^T [G_m] [T^M] \\ \{\alpha_e\} &= [T^M] \{\alpha_m\} \end{aligned}\right\} \tag{11}$$

where $[G_m]$ and $\{\alpha_m\}$ are the material modulus matrix and thermal expansion coefficient vector specified by the user.

5.13.4 Stiffness, Mass and Load Matrices

The stiffness, mass and load matrices for the isoparametric element may now be derived by application of the Virtual Work Principle:

$$\delta U - \delta W = 0 \tag{12}$$

where δU is the internal strain energy in the element due to a virtual displacement, $\delta\bar{u}$, and δW is the work performed by the external loads during the virtual displacement, i.e.:

$$\left.\begin{aligned} \delta U &= \int_V \{\sigma\}^T \{\delta\epsilon\}\, dV \\ \delta W &= \int_S \{\delta\bar{u}\}^T \{p\}\, dS + \int_V \{\delta\bar{u}\}^T \{F\}\, dV \end{aligned}\right\} \tag{13}$$

where $\{p\}$ and $\{F\}$ are 3 x 1 vector representing surface pressure and body forces, respectively, in the x, y and z directions, and the integrations are performed over the element volume, V, and surface area, S, on which the pressure load p acts.

Substituting Eqs. (2), (4) and (8) into (13) and applying the Virtual Work Principle, Eq. (12), yields:

$$\{\delta u_e\}^T \left([K_{gg}]\{u_e\} - \{F_e\}\right) = 0 \tag{14}$$

or for any virtual displacement, we have:

$$[K_{gg}]\{u_e\} = \{F_e\} \tag{15}$$

where $[K_{gg}]$ is the element stiffness matrix and $\{F_e\}$ is the element load vector to surface pressures and temperature.

Stiffness Matrix

$$[K_{gg}] = \int [C]^T [G_e] [C] \, dV \tag{16}$$

where the infinitesimal volume is in terms of the curvilinear coordinates, ξ, η and ζ,

$$dV = dx \, dy \, dz = \det. [J] \, d\xi \, d\eta \, d\zeta \tag{17}$$

Surface Pressure Load

$$\{F_p\} = \int_S [N]^T \{p\} \, dS \tag{18}$$

Thermal Load

$$\{F_T\} = \int_V [C]^T [G_e] \{\alpha_e\} [N] \{T_e\} \, dV \tag{19}$$

For dynamic problems, the mass matrix is also required and is easily derived by adding the kinetic energy to Eq. (12). The result is:

Mass Matrix

$$[M] = \int_V [N]^T [N] \rho \, dV \tag{20}$$

where ρ is the mass density.

5.13.5 Numerical Integration

The integrals in the isoparametric element stiffness, mass and load matrices are evaluated by the use of numerical integration using Gaussian Quadrature [Ref. 6]. Thus, for example, the stiffness matrix is calculated by the triple summation:

$$[K] = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} [C]^T [G_e] [C] \, |J| \, d\xi \, d\eta \, d\zeta = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} H_i H_j H_k \left([C]^T [G_e] [C]\right) |J| \qquad (21)$$

where the weight coefficients H_ℓ and abscissa S_ℓ are given in Table 2. Note that the triple product matrix operation in Eq. (16) as well as the determinent of the Jacobian, $|J|$, must be evaluated at each integration point. This process could be very time consuming, and requires that efficient programming practices and mathematical techniques be used to minimize this time. In NASTRAN, for isotropic materials, the triple product in Eq. (21) is explicitly evaluated to avoid calculating zeros and thereby minimize the number of mathematical operations performed.

The number of integration points needed to evaluate the stiffness, mass and load matrices depends on the element geometry, displacement function, and material property variations. Elements which are extremely distorted from a rectangular shape require more integration points. Best results, however, are obtained using rectangular elements as far as possible, and therefore extremely distorted elements should be avoided [Refs. 8 & 9]. It has been found that for most problems, satisfactory results may be obtained using a $2 \times 2 \times 2$ integration mesh for the linear element and a $3 \times 3 \times 3$ integration mesh for the quadratic and cubic elements. These meshes are used as default values in NASTRAN. However, since good results have also been reported using smaller

TABLE 2. GAUSSIAN QUADRATURE FORMULA

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f(x, y, z)dx\,dy\,dz = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n} H_j\, f(s_j, s_k, s_i)$$

n	Abscissa (s)	Weight Coefficient (H)
2	±0.57735026919	1.0
3	±0.77459666924	0.55555555555
	0.0	0.88888888888
4	±0.86113631159	0.34785484514
	±0.33998104358	0.65214515486

meshes than suggested above [Refs. 7 & 8], the user has the option of specifying the integration mesh size.

5.13.6 Transformation from Basic to Global Coordinates

As previously stated, all computations for the isoparametric elements are carried out in the basic coordinate system. If the global coordinate system at any grid point is different from the basic system, NASTRAN transforms the final matrices and vectors into the global coordinate system using the appropriate transformation matrix $[T_i]$, corresponding to grid point i. This calculation is identical to that performed for the other elements in NASTRAN.

5.13.7 Stress Recovery

Element stresses may be obtained by combining Eqs. (4), (8) and (9) to yield:

$$\{\sigma\} = [G_e] \left([C] \{u_e\} - \{\alpha_e\} [N] \{T_e\} \right) \tag{22}$$

where the matrices [C] and [N] are functions of the element coordinates ξ, η, and ζ. in NASTRAN these stresses are calculated in the basic coordinate system and they are printed at the following locations, depending on element type:

a. Linear Element - Eight corner points and at center of element.

b. Quadratic and Cubic Element - Eight corner points, center of each edge, and at center of element.

The principal stresses, principal angles, mean stress, and octohedral shear stress are also computed and output at every point at which the basic stresses are computed. The mean stress, or hydrostatic pressure, is given by:

$$\sigma_n = -\frac{1}{3} \left(\sigma_x + \sigma_y + \sigma_z \right) . \tag{23}$$

The octahedral shear stress is given by:

$$\sigma_0 = \left\{ \frac{1}{3} \left[(S_x + \sigma_n)^2 + (S_y + \sigma_n)^2 + (S_z + \sigma_n)^2 \right] \right\}^{1/2} \tag{24}$$

where S_x, S_y, and S_z are the three principal stresses.

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7.4 ISOPARAMETRIC HEXAHEDRON THREE-DIMENSIONAL SOLID ELEMENTS

The differential stiffness matrix for the isoparametric solid elements are obtained by adding the energy due to initial stresses to the potential energy function. This additional energy is derived in Section 7.1, and is given by:

$$W_p = \frac{1}{2}\int_V \Big[\omega_x^2(\sigma_y + \sigma_z) + \omega_y^2(\sigma_x + \sigma_z) + \omega_z^2(\sigma_x + \sigma_y) - 2\omega_x\omega_y\tau_{xy} - 2\omega_y\omega_z\tau_{yz} - 2\omega_z\omega_x\tau_{zx}\Big]\, dV \tag{1}$$

where the rotations are given by the relations:

$$\left.\begin{aligned} \omega_x &= \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \\ \omega_y &= \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\ \omega_z &= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \end{aligned}\right\} \tag{2}$$

and may be expressed in terms of the grid point displacements by using Eq. (1) of Section 5.13.1, i.e.,

$$\begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} = \left[\bar{C}_1 \,\vdots\, \bar{C}_2 \,\vdots\, \bar{C}_3 \,\vdots\, \cdots\cdots\, \bar{C}_{NGP}\right] \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ \hline u_2 \\ v_2 \\ w_2 \\ \hline \vdots \\ u_{NGP} \\ v_{NGP} \\ w_{NGP} \end{Bmatrix} = [\bar{C}]\{u_e\} \tag{3}$$

and

$$\bar{C}_i = \begin{bmatrix} 0 & -\dfrac{\partial N_i}{\partial z} & \dfrac{\partial N_i}{\partial y} \\ \dfrac{\partial N_i}{\partial z} & 0 & -\dfrac{\partial N_i}{\partial x} \\ -\dfrac{\partial N_i}{\partial y} & \dfrac{\partial N_i}{\partial x} & 0 \end{bmatrix}$$

Substituting Eq. (3) into (1) and adding this function to the Potential Energy expression yields the differential stiffness matrix:

$$\left[K^d_{ee}\right] = \int_V \left[\bar{C}\right]^T \left[K^d_{\omega\omega}\right] \left[\bar{C}\right] dV \tag{4}$$

where

$$\left[K^d_{\omega\omega}\right] = \begin{bmatrix} \sigma_y + \sigma_z & -\tau_{xy} & -\tau_{zx} \\ -\tau_{xy} & \sigma_x + \sigma_z & -\tau_{zy} \\ -\tau_{zx} & -\tau_{zy} & \sigma_x + \sigma_y \end{bmatrix} \tag{5}$$

As in the structural stiffness matrix, the evaluation of the integral in Eq. (4) is obtained by application of the Gaussian Quadrature Formula (Table 2, Section 5.13.5).

8.2 VOLUME HEAT CONDUCTION ELEMENTS

The volume heat conduction elements are the same as NASTRAN structural elements. The elements for which heat conduction is available are listed in the following table:

Heat Conduction Elements	
Type	Elements
Linear	BAR, RØD, CØNRØD, TUBE
Planar	TRMEM, TRIA1, TRIA2, QDMEM, QUAD1, QUAD2
Solid of Revolution	TRIARG, TRAPRG
Solid	TETRA, WEDGE, HEXA1, HEXA2 IHEX1, IHEX2, IHEX3

Scalar elements, single point constraints, and multipoint constraints are also available for heat transfer analysis. The same connection and property cards are used for heat transfer and structural analysis. Linear elements have a constant cross-sectional area. For the planar elements, the heat conduction thickness is the membrane thickness. Elements with bending properties, such as BAR and TRIA1, have been included so that the user may use the same elements for the thermal and structural analyses of a given structure. The bending characteristics of the elements do not enter into heat conduction problems. The trapezoidal solid of revolution element, TRPRG, has been generalized to accept general quadrilateral rings (i.e., the top and bottom need not be perpendicular to the z-axis) for heat conduction only.

The heat conduction elements are composed of constant gradient lines, triangles and tetrahedra. The quadrilaterals are composed of overlapping triangles, and the wedges and hexahedra are formed from sub-tetrahedra in exactly the same way as for the structural case. The IHEXi elements are isoparametric hexahedron elements and are similar to the isoparametric solid elements described for structural analyses.

Thermal conductivity and capacity are specified on MAT4 (isotropic) and MAT5 (anisotropic) bulk data cards.

The heat conduction matrix for a volume heat conduction element may be derived from a thermal potential function in the same way that the stiffness matrix of a structural element is derived from the strain energy function. The thermal potential function is

8.2.2 Three-Dimensional Isoparametric Solid Element Heat Transfer Matrices

(add this new section)

The heat transfer conduction matrix for the three-dimensional isoparametric elements [Refs. 1, 2, 3] are derived by using Equation (8). For these three elements, the temperature u at an exterior point is given by:

$$u = [L_e]\{u_e\} \tag{23}$$

where L_e is a function of the curvilinear coordinates ξ, η and ζ shown in Figure 3, and are identical to the N_i functions given in the structural analysis section. The derivatives of L_e with respect to the basic Cartesian coordinates X, Y, Z are calculated in exactly the same manner as presented in the structural analysis section for these elements, i.e.,

$$\begin{Bmatrix} \dfrac{\partial L_i}{\partial X} \\ \dfrac{\partial L_i}{\partial Y} \\ \dfrac{\partial L_i}{\partial Z} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \dfrac{\partial L_i}{\partial \xi} \\ \dfrac{\partial L_i}{\partial \eta} \\ \dfrac{\partial L_i}{\partial \zeta} \end{Bmatrix} \tag{24}$$

where [J] is the Jacobian Matrix, and the derivatives of L_i with respect to ξ, η and ζ are listed in Tables 1 and 2 of the structural analysis section. Since, in general, the matrix $[L_{e,i}]$ is a function of ξ, η and ζ, the integration of Equation (8) is carried out by Gaussian Quadrature numerical integration, and the heat conduction matrix is calculated from the equation:

$$[K^e] = \sum_{i=1} \sum_{j=1} \sum_{k=1} H_i H_j H_k \left([L_{e,i}] [k_{ij}] [L_{e,j}]^T \right) |J| \tag{25}$$

where

$$\left[L_{e,j}\right] = \begin{bmatrix} \frac{\partial L_1}{\partial X} & \frac{\partial L_1}{\partial Y} & \frac{\partial L_1}{\partial Z} \\ \frac{\partial L_2}{\partial X} & \frac{\partial L_2}{\partial Y} & \frac{\partial L_2}{\partial Z} \\ \vdots & \vdots & \vdots \\ \frac{\partial L_{NGP}}{\partial X} & \frac{\partial L_{NGP}}{\partial Y} & \frac{\partial L_{NGP}}{\partial Z} \end{bmatrix} \qquad (26)$$

and NGP is the number of grid points in the element. NGP equals eight, twenty and thirty-two for the linear, quadratic and cubic isoparametric elements, respectively. The weight coefficients H_ℓ and abscissa S_ℓ are given in Table 1 in the structural analysis Section 5.13.

Elements of the heat capacity matrix [B] are calculated by the coupled mass method,(see Section 5.13.4). The equation for this matrix is:

$$[B] = \int_V [L_e]^T [L_e] c_p \, dV \qquad (27)$$

where c_p is the heat capacity per unit volume. Examination of this equation shows that [B] is identical to the structural mass matrix with the single exception that the heat capacity c_p is used instead of the material density. The heat capacity matrix is also evaluated using the Gaussian Quadrature formula:

$$[B] = \sum_{i=1} \sum_{j=1} \sum_{k=1} H_i H_j H_k ([L_e]^T [L_e]) c_p |J| \qquad (28)$$

REFERENCES

1. O. C. Zienkiewicz and Y. K. Cheung, The Finite Element Method in Structural and Continuum Mechanics, McGraw Hill Publishing Company, New York, 1967.

2. O. C. Zienkiewicz and C. J. Parekh, "Transient Field Problems: Two-Dimensional and Three-Dimensional Analysis by Isoparametric Finite Elements," International Journal for Numerical Methods in Engineering, January-March 1970.

3. O. C. Zienkiewicz and Y. K. Cheung, "Finite Elements in the Solution of Field Problems," The Engineer, pp. 507-10, September 24, 1965.

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APPENDIX B

USER'S MANUAL UPDATES

The following sections contained in this appendix should be added to the NASTRAN User's Manual in their entirety:

1.3.10 Isoparametric Solid Hexahedron Elements (including Figure 13)

2.4.2 Bulk Data Card Descriptions: Add CIHEX1, CIHEX2, CIHEX3, PIHEX, and PLØAD3

The remaining sections are marked to indicate changes to the existing UM document.

blank

The following new section and figure should be added to the User's Manual:

1.3.10 Isoparametric Solid Hexahedron Elements

Three types of isoparametric solid hexahedron elements are provided for general solid structures. These elements (see Figure 13) are a linear, a quadratic, and a cubic isoparametric hexahedron. The theory is given in section 5.13 of the Theoretical Manual. These elements can be used with all other NASTRAN elements, except the axisymetric elements. Connections are made only to the translational degrees of freedom at the grid points. The elements are defined by CIHEXI, CIHEX2, and CIHEX3 connection cards. All three of these cards reference the PIHEX property card.

The isoparametric solid hexahedron elements are sophisticated elements. They allow the user to accurately define a structure with fewer elements and grid points than might otherwise be necessary with simple constant strain solid elements. The linear element generally gives best results for problems involving mostly shear deformations, and the higher order elements give good results for problems involving both shearing and bending deformations. Only a coupled mass matrix is generated to retain the inherent accuracy of the elements. Temperature, temperature dependent material properties, displacements, and stresses may vary through the volume of the elements. The values at interior points of the element are interpolated using the isoparametric shape function. For best results, the applied grid point temperatures should not have more than a "gentle" quadratic variation in each of the three dimensions of the element. If the element has non-uniform applied temperatures, or if it is not a rectangular parallelopiped, 3 or more integration points should be specified on the PIHEX card. Severely distorted element shapes should be avoided.

Stiffness, mass, differential stiffness, structural damping, conductance, and capacitance matrices may be generated with these elements. Piecewise linear analysis has not been implemented.

The output stresses are given in the basic coordinate system. The stresses are assumed to vary through the element. Therefore, stresses are computed at the center and at each corner grid point of these elements. For the quadratic and cubic elements, they are also computed at the midpoint of each edge of the element. In addition to the six normal and shear stresses, output also includes the principal stresses, the direction cosines of the principal planes, the mean stress

$$\sigma_n = -\frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)^*$$

and the octahedral shear stress

$$\sigma_o = \left\{\frac{1}{3}[(S_x + \sigma_n)^2 + (S_y + \sigma_n)^2 + (S_z + \sigma_n)^2]\right\}^{1/2}$$

where S_x, S_y, and S_z are the three principal stresses.

1.5.1 Static Loads

The following pages should be modified as indicated.

*The minus sign is used to be consistent with the hydrostatic pressure computations for elements TETRA, WEDGE, HEXA1 and NEXA2.

Pressure loads on triangular and quadrilateral elements are defined with a PLØAD2 card. The positive direction of the loading is determined by the order of the grid points on the element connection card, using the right hand rule. The magnitude and direction of the load is automatically computed from the value of the pressure and the coordinates of the connected grid points. The load is applied to the connected grid points. The PLØAD card is used in a similar fashion to define the loading of any three or four grid points regardless of whether they are connected with two-dimensional elements. The PRESAX card is used to define a pressure loading on a conical shell element.

Pressure loads on the isoparametric solid elements are defined with the PLØAD3 card. The pressure is defined positive outward from the element. The magnitude and direction of the equivalent grid point forces is automatically computed using the isoparametric shape functions of the element to which the load has been applied.

The GRAV card is used to specify a gravity load by providing the components of the gravity vector in any defined coordinate system. The gravity load is obtained from the gravity vector and the mass matrix assembled by the Structural Matrix Assembler (see Section 4.28 of the Programmer's Manual). The gravitational acceleration is not calculated at scalar points. The user is required to introduce gravity loads at scalar points directly.

The RFØRCE card is used to define a static loading condition due to a centrifugal force field. A centrifugal force load is specified by the designation of a grid point that lies on the axis of rotation and by the components of rotational velocity in any defined coordinate system. In the calculation of the centrifugal force, the mass matrix is regarded as pertaining to a set of distinct rigid bodies connected to grid points. Deviations from this viewpoint, such as the use of scalar points or the use of mass coupling between grid points, can result in errors.

Temperatures may be specified for selected elements. The temperatures for a RØD, BAR, CØNRØD or TUBE element are specified on the TEMPRB data card. This card specifies the average temperature on both ends and, in the case of the BAR element, is used to define temperature gradients over the cross section. Temperatures for two dimensional plate and membrane elements are specified on a TEMPP1, TEMPP2, or TEMPP3 data card. The user defined average temperature over the volume is used to produce in-plane loads and stresses. Thermal gradients over the depth of the bending elements, or the resulting moments, may be used to produce bending loads and stresses.

element are temperature-dependent by use of the MATTi card, they are always calculated from the "average" temperature of the element, except for the isoparametric solid elements. The temperature is allowed to vary throughout the isoparametric solid elements. Hence, the material coefficients and the integration of the thermal loads require that the temperature at any point in the element be interpolated from the grid point temperatures specified. The mere presence of a thermal field does not imply the application of a thermal load. A thermal load will not be applied unless the user makes a specific request in the Case Control Deck.

Enforced axial deformations can be applied to rod and bar elements. They are useful in the simulation of misfit and misalignment in engineering structures. As in the case of thermal expansion, the equivalent loads are calculated by separate subroutines for each type of structural

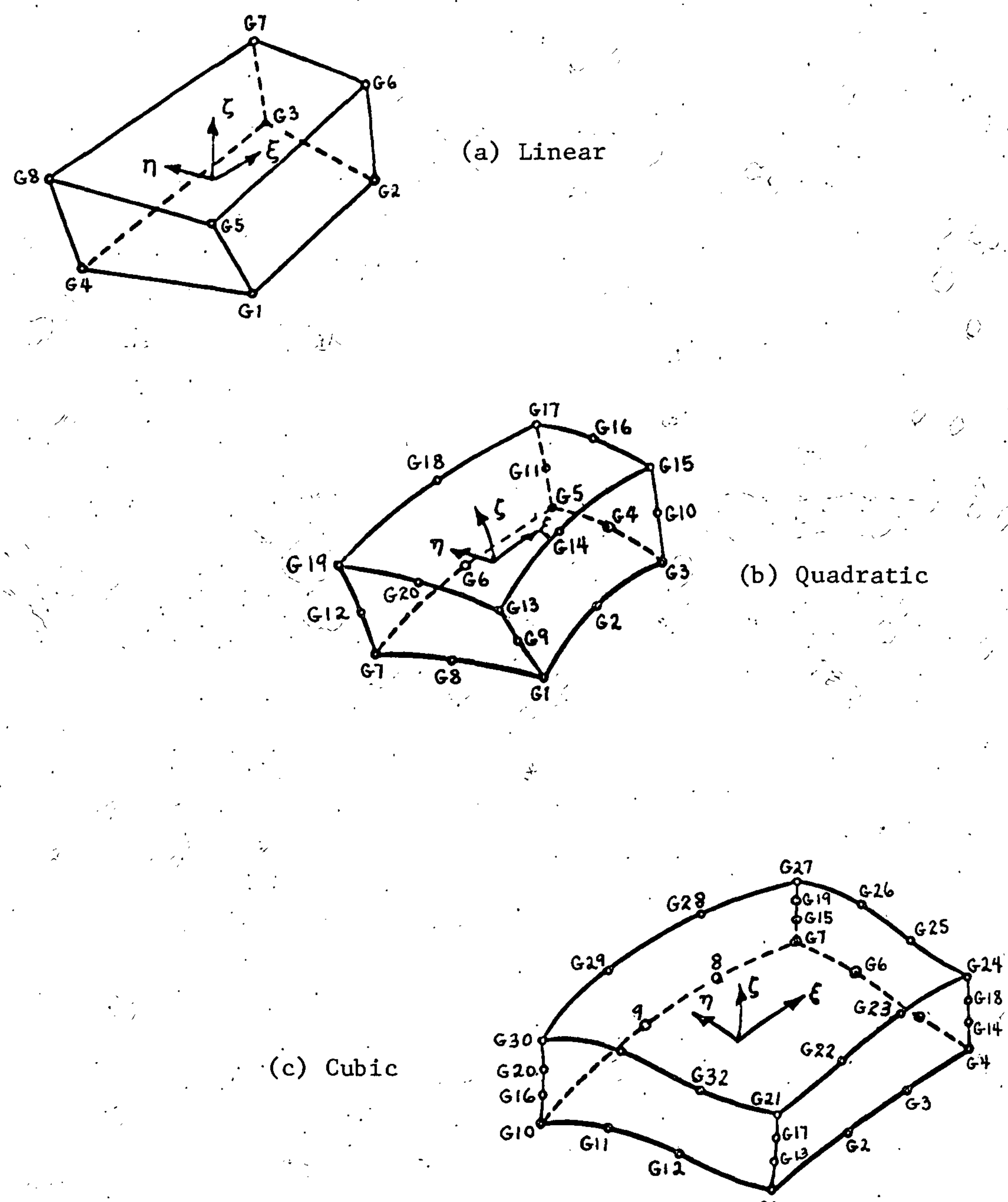


FIGURE 13. ISOPARAMETRIC SOLID HEXAHEDRON ELEMENTS

2.4 Bulk Data Deck

Section 2.4.2, Bulk Data Card Descriptions. Five new descriptions for the new bulk data cards should be inserted in this section. The new descriptions are contained in Figures 1 to 5. In addition, remark 4 for the TEMP, and TEMPD cards should be changed to read as follows:

4. If the element material is temperature dependent, its properties are evaluated at the average temperature. The isoparametric solid elements are exceptions to this. Their properties are evaluated at the temperature computed by interpolating the grid point temperatures.

Input Data Card CIHEX1 - Linear Isoparametric Hexahedron Element Connection

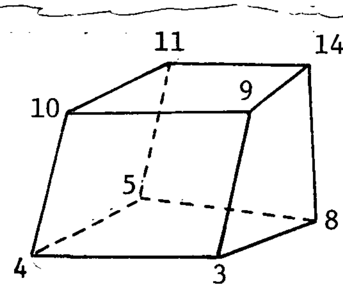
Description: Defines a linear isoparametric hexahedron element of the structural model.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CIHEX1	EID	PID	G1	G2	G3	G4	G5	G6	abc
CIHEX1	137	5	3	8	5	4	9	14	ABC

+bc	G7	G8							
+BC	11	10							

Field	Contents
EID	Element identification number (Integer > 0)
PID	Identification number of a PIHEX property card (Integer > 0)
G1,...,G8	Grid point identification numbers of connection points (Integer > 0, G1 ≠ G2 ≠ ... ≠ G8)



Remarks:

1. Element identification numbers must be unique with respect to all other element identification numbers.
2. Grid points G1, G2, G3, G4 must be given in counter-clockwise order about one quadrilateral face when viewed from inside the element. G5, G6, G7, G8 are in order in the same direction around the opposite quadrilateral, with G1 and G5 along the same edge.
3. There is no non-structural mass.
4. The quadrilateral faces need not be planar.
5. Stresses are given in the basic coordinate system.
6. For structural problems, material must be defined by a MAT1 card.
7. For heat transfer problems, material may be defined with either a MAT4 or MAT5 card.

FIGURE 1

Input Data Card CIHEX2 - Quadratic Isoparametric Hexahedron Element Connection

Description: Defines a quadratic isoparametric hexahedron element of the structural model.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CIHEX2	EID	PID	G1	G2	G3	G4	G5	G6	abc
CIHEX2	110	7	3	8	12	13	14	9	ABC

+bc	G7	G8	G9	G10	G11	G12	G13	G14	def
+BC	5	4	16	19	20	17	23	27	DEF

+ef	G15	G16	G17	G18	G19	G20			
+EF	31	32	33	28	25	24			

Field	Contents
EID	Element identification number (Integer > 0)
PID	Identification number of a PIHEX property card (Integer > 0)
G1,...,G20	Grid point identification numbers of connection points (Integer > 0, G1 $\neq$ G2 $\neq$ ... $\neq$ G20)

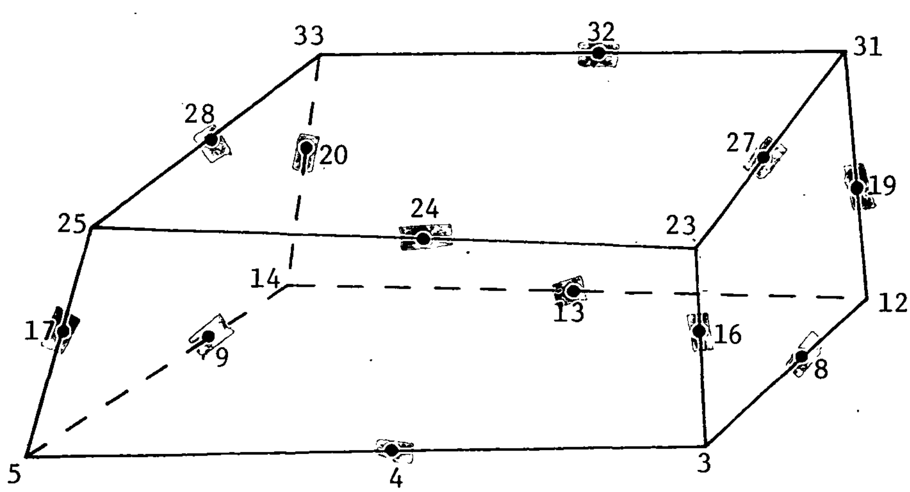


FIGURE 2

Remarks:
1. Element identification numbers must be unique with respect to all other element identification numbers.
2. Grid points G1,...,G8 must be given in counter-clockwise order about one quadrilateral face when viewed from inside the element. G9,...,G12 and G13,...,G20 are in the same direction with G1, G9 and G13 along the same edge.
3. There is no nonstructural mass.
4. The quadrilateral faces need not be planar.
5. Stresses are given in the basic coordinate system.
6. For structural problems, material must be defined by a MAT1 card.
7. For heat transfer problems, material may be defined with either a MAT4 or MAT5 card.

FIGURE 2 (cont'd)

Input Data Card CIHEX3 - Cubic Isoparametric Hexahedron Element Connection

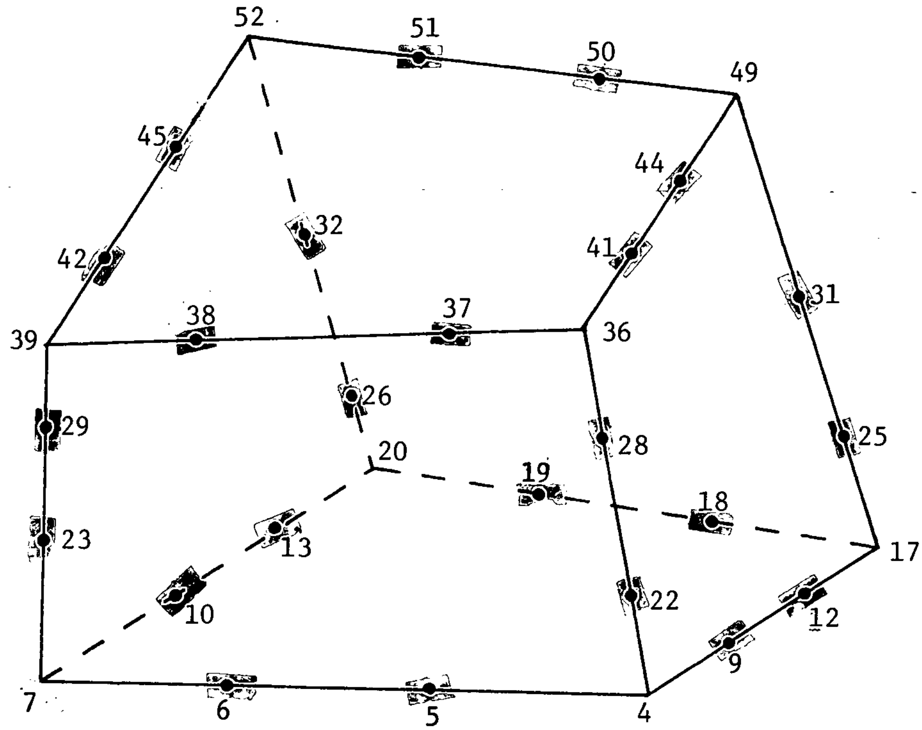
Description: Defines a cubic isoparametric hexahedron element of the structural model.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CIHEX3	EID	PID	G1	G2	G3	G4	G5	G6	abc
CIHEX3	15	3	4	9	12	17	18	19	ABC
+bc	G7	G8	G9	G10	G11	G12	G13	G14	def
+BC	20	13	10	7	6	5	22	25	DEF
+ef	G15	G16	G17	G18	G19	G20	G21	G22	ghi
+EF	26	23	28	31	32	29	36	41	GHI
+hi	G23	G24	G25	G26	G27	G28	G29	G30	jkl
+HI	44	49	50	51	52	45	42	39	JKL
+kl	G31	G32							
+KL	38	37							

Field	Contents
EID	Element identification number (Integer > 0)
PID	Identification number of a PIHEX property card (Integer > 0)
G1,...,G32	Grid point identification number of connection points (Integer > 0, G1 $\neq$ G2 $\neq$ ... $\neq$ G32)

FIGURE 3



Remarks:

1. Element identification numbers must be unique with respect to all other element identification numbers.
2. Grid points G1,...,G12 must be given in counter-clockwise order about one quadrilateral face when viewed from inside the element. G13,...,G16; G17,...,G20; and G21,...,G32 are in the same direction with G1, G13, G17, G21 along the same edge.
3. There is no nonstructural mass.
4. The quadrilateral face need not be planar.
5. Stresses are given in the basic coordinate system.
6. For structural problems, material must be defined by a MAT1 card.
7. For heat transfer problems, material may be defined with either a MAT4 or MAT5 card.

FIGURE 3 (cont'd)

Input Data Card PIHEX - Isoparametric Hexahedron Properties

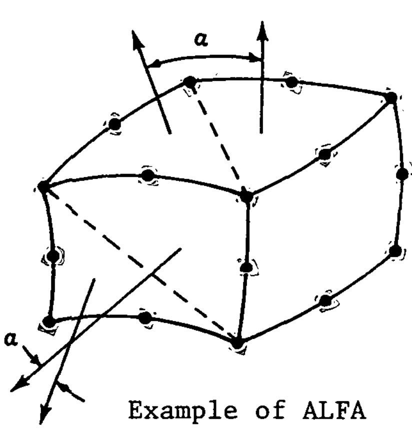
Description: Defines the properties of an isoparametric solid element of the structural mode, including a material reference and the number of integration points. Referenced by the CIHEX1, CIHEX2, and CIHEX3 cards.

Format and Example:

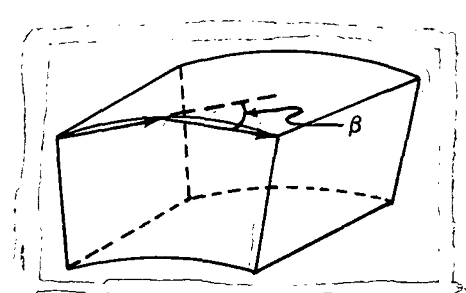
1	2	3	4	5	6	7	8	9	10
PIHEX	PID	MID	CID	NIP	AR	ALFA	BETA	X	
PIHEX	15	3		3			5.0		

Field	Contents
PID	Property identification number (Integer > 0)
MID	Material identification number (Integer > 0)
CID	Identification number of the coordinate system in which the material referenced by MID is defined (Integer $\geq$ 0 or blank)
NIP	Number of integration points along each edge of the element (Integer = 2, 3, 4 or blank)
AR	Maximum aspect ratio (ratio of longest to shortest edge) of the element (Real > 1.0 or blank)
ALFA	Maximum angle between the normals of two sub-triangles comprising a quadrilateral face (Real, 0.0 $\leq$ ALFA $\leq$ 180.0, or blank)
BETA	Maximum angle between the vector connecting a corner point to an adjacent midside point and the vector connecting that midside point and the other midside or corner point (Real, 0.0 $\leq$ BETA $\leq$ 180.0, or blank)

Examples of Field Definitions:



Example of ALFA



Example of BETA

FIGURE 4

Remarks:

1. All PIHEX cards must have unique identification numbers.
2. CID is not used for isotropic materials.
3. The default for CID is the basic coordinate system.
4. The default for NIP is 2 for IHEXI and 3 for IHEX2 and IHEX3.
5. AR, ALFA, and BETA are used for checking the geometry of the element. The defaults are:

	AR	ALFA (degrees)	BETA (degrees)
CIHEX1	5.0	45.0	-
CIHEX2	10.0	45.0	45.0
CIHEX3	15.0	45.0	45.0

FIGURE 4 (cont'd)

Input Data Card PLOAD3 - Pressure Load on a Face of an Isoparametric Element

Description: Defines a uniform static pressure load applied to a surface of an isoparametric element only.

Format and Example:

1	2	3	4	5	6	7	8	9	10
PLOAD3	SID	P	EID1	G11	G12	EID2	G21	G22	
PLOAD3	3	-15.1	15	7	25	16	117	135	

Field	Contents
SID	Load set identification number (Integer > 0)
P	Pressure value (Real, force per unit area)
EID1, EID2	Element identification number (Integer > 0)
G11, G12, G21, G22	Grid point identification number of two grid points at diagonally opposite corners of the face on which the pressure acts (Integers > 0)

Remarks:

1. Load sets must be selected in the Case Control Deck (LOAD = SID) to be used by NASTRAN.
2. At least one EID must be present on each PLOAD3 card.
3. All elements referenced must exist.
4. Computations consider the pressure to act positive outward on specified face of element.

FIGURE 5

4.2 Structure Plotting

Section 4.2.2.1 SET Definition Cards, the following element types are also now permissible:

IHEX1, IHEX2, IHEX3, TETRA, HEXA1, HEXA2, WEDGE

Section 4.2.2.3 PLØT Execution Card, the plot labels for the solid elements are:

Element Type	Label
IHEX1	XL
IHEX2	XQ
IHEX3	XC
TETRA	TT
HEXA1	H1
HEXA2	H2
WEDGE	WG

6.2 NASTRAN System and User Messages

Section 6.2.3, Functional Module Messages, the following new messages should be added to this section:

2141 *** USER FATAL MESSAGE 2141, IHEX* ELEMENT NUMBER *** INSUFFICIENT CØRE TØ CØMPUTE ELEMENT MATRIX

2142 *** USER FATAL MESSAGE 2142, IHEX* ELEMENT NUMBER *** ILLEGAL GEØMETRY, text

The type of geometry error is identified in "text". The possibilities are:

AR EXCEEDED ALFA EXCEEDED BETA EXCEEDED	Either correct the element or increase the allowable value on the PIHEX card for this element.
REVERSED NUMBERING	The element was numbered in a clockwise fashion rather than counter-clockwise as required. This would result in a left-handed element coordinate system. Correct the numbering sequence on the CIHEXi card for this element.
CØØRDINATES ØF TWØ PØINTS ARE THE SAME	The coordinates of all connections of the element must be different.

2143 *** USER FATAL MESSAGE 2143, SINGULAR JACØBIAN MATRIX FØR ISØPARAMETRIC ELEMENT NUMBER ***

The element is severely warped or the outer surface of the element is folded through itself. Check the connection card for this element and the coordinates of the points it connects.

2144 *** USER FATAL MESSAGE 2144, PLØAD3 CARD FRØM LØAD SET *** REFERENCES MISSING ØR NØN-ISØPARAMETRIC ELEMENT ***

2145 *** USER FATAL MESSAGE 2145, INVALID GRID PØINTS ØN PLØAD3 CARD FØR ELEMENT ***

Either the element does not connect the specified grid points, or the grid points do not identify the diagonal of a face of the element.

4024 *** USER FATAL MESSAGE 4024, STRESSES REQUESTED FØR SET *** WHICH CØNTAINS NØ VALID ELEMENT ID-S

The set of elements for which stresses were requested in this subcase contains only ID's for nonexistent elements.

blank

2.3.2 Data Blocks Output from Module IFP

Section 2.3.2.2 GEØM2 (TABLE), the following data should be added to this section:

Card Types and Header Information:

Card Type	Header Word 1 Card Type	Header Word 2 Trailer Bit Position	Header Word 3 Internal Card Number
CIHEX1	7108	71	251
CIHEX2	7208	72	252
CIHEX3	7308	73	253

Card Type Formats:

CIHEX1 (10 words)	EID	PID	G1
	G2	...	G8
CIHEX2 (22 words)	EID	PID	G1
	G2	...	G20
CIHEX3 (34 words)	EID	PID	G1
	G2	...	G32

Section 2.3.2.3 GEØM3 (TABLE), the following data should be added to this section:

Card Types and Header Information:

Card Type	Header Word 1 Card Type	Header Word 2 Trailer Bit Position	Header Word 3 Internal Card Number
PLØAD3	7109	71	255

Card Type Formats:

PLØAD3 (5 words)	SID	P	EID
	G1	G2	

Section 2.3.2.5 EPT (TABLE), the following data should be added to this section:

Card Types and Header Information:

Card Type	Header Word 1 Card Type	Header Word 2 Trailer Bit Position	Header Word 3 Internal Card Number
PIHEX	7002	70	254

Card Type Formats:

PIHEX (7 words)	PID	MID	CID
	NIP	AR	ALFA
	BETA		

2.3.7 Data Blocks Output from Module GP3

Section 2.3.7.1 SLT (TABLE), the following data should be added to this section under "Notes", item 2:

13 = PLØAD3

Word	Type	Item
1-6	R	Pressures
7-38	I	Internal grid numbers of which the last 12 or the last 24 may be zero

Under "Notes", item 3 should begin:

3. With the exception of GRAV, PLØAD, and PLØAD3 card types, ...

2.3.8 Data Blocks Output from Module TA1

The EST table should be amended as follows:

2.3.8.1 EST (TABLE)

Description

Element Summary Table.

The EST is a collection of data for all elements of the structural model. It contains one logical record for each element type. For each element: connection data, properties data, basic grid point data and ~~the~~ element temperature data are grouped. General elements and elements that belong to super elements are not included in the EST.

Table Format

Record	Word	Type	Item		
0			Header record		
1	1	I	Element type		repeated for each element type
	2-i+1		ECT section	repeated for each element	
	i+2-i+j+1		EPT section		
	i+j+2-i+j+k+1		BGPDT section		
	i+j+k+2-i+j+k+m+1		~~Element temperature~~ ETT section		
n+1			End-of-file		

Notes

1. i = number of words in ECT section.
   j = number of words in EPT section.
   k = number of words in BGPDT section.
   m = number of words in ETT section.
2. The number of records in the EST corresponds to the number of separate element types in the model.
3. The EST is generated in subroutine TA1A.

Summary of EST Formats

Element Type	Mnemonic	ECT Section number of Words	EPT Section Number of Words	BGPDT Section Number of Words	ETT Section number of Words ~~Element Temperature~~	Total Words Per Element
1	RØD	3	5	8	Yes	17
2	BEAM	19	19	8	Yes	47
3	TUBE	3	4	8	Yes	16
4	SHEAR	5	3	16	Yes	25
5	TWIST	5	3	16	Yes	25
6	TRIA1	5	9	12	Yes	27
7	TRBSC	5	7	12	Yes	25
8	TRPLT	5	7	12	Yes	25
9	TRMEM	5	3	12	Yes	21
10	CØNRØD	8	0	8	Yes	17
11	ELAS1	5	3	0	No	8
12	ELAS2	8	0	0	No	8
13	ELAS3	3	3	0	No	6
14	ELAS4	4	0	0	No	4
15	QDPLT	6	7	16	Yes	30
16	QDMEM	6	3	16	Yes	26
17	TRIA2	5	3	12	Yes	21

Change Yes to 1
Change No to 0

Element Type	Mnemonic	ECT Section Number of Words	EPT Section (cont'd) Number of Words	BGPDT Section Number of Words	ETT Section number of words ~~Element Temperature~~	Total Words Per Element
18	QUAD2	6	3	16	~~Yes~~	26
19	QUAD1	6	9	16	~~Yes~~	32
20	DAMP1	5	1	0	~~No~~	6
21	DAMP2	6	0	0	~~No~~	6
22	DAMP3	3	1	0	~~No~~	4
23	DAMP4	4	0	0	~~No~~	4
24	VISC	3	2	8	~~Yes~~	14
25	MASS1	5	1	0	~~No~~	6
26	MASS2	6	0	0	~~No~~	6
27	MASS3	3	1	0	~~No~~	4
28	MASS4	4	0	0	~~No~~	4
29	CØNM1	24	0	4	~~Yes~~	29
30	CØNM2	13	0	4	~~Yes~~	18
31	PLØTEL	3	0	8	~~Yes~~	12
32	REACT	19	0	4	~~Yes~~	24
33	QUAD3	7	1	16	~~Yes~~	24
34	BAR	15	18	8	~~Yes~~	42
35	CONEAX	3	23	8	~~Yes~~	35
36	TRIARG	6	0	12	~~Yes~~	19
37	TRAPRG	7	0	16	~~Yes~~	24
38	TØRDRG	6	3	2	~~Yes~~	18
43	FLUID2	6	0	8	~~No~~	14
44	FLUID3	7	0	12	~~No~~	19
45	FLUID4	8	0	16	~~No~~	24
46	MFREE	5	0	8	~~No~~	13
62	IHEX1	9	6	32	8	57
63	IHEX2	21	6	80	20	127
64	IHEX3	33	6	128	32	199

Change Yes to 1
Change No to 0

The following data should be added for "Detailed EST Formats":

ECT section for element type = 62:

Word	Type	Item
1	I	Element ID
2-9	I	SIL numbers for grid points 1-8

ECT section for element type = 63:

Word	Type	Item
1	I	Element ID
2-21	I	SIL numbers for grid points 1-20

ECT section for element type = 64:

Word	Type	Item
1	I	Element ID
2-33	I	SIL numbers for grid points 1-32

2.3.8 Data Blocks Output From Module TA1 (cont'd)

EPT section for element type = 62, 63, 64:

Word	Type	Item
1	I	Material ID
2	I	CID
3	I	NIP
4	R	AR
5	R	ALFA
6	R	BETA

ETT section for element type = 1-61:

Word	Type	Item
1	R	Average element temperature

ETT section for element type = 62:

Word	Type	Item
1-8	R	Temperature at grid points 1-8

ETT section for element type = 63:

Word	Type	Item
1-20	R	Temperature at grid points 1-20

ETT section for element type = 64:

Word	Type	Item
1-32	R	Temperature at grid points 1-32

2.3.51 Element Stress Output Data Description

The following data should be added to this section:

Element Type	Element Name	Real: Word or Component	Real: Item	Complex: Word or Component	Complex: Item	Real Imag.
61	CIHEX1†	2	External grid point ID	2	External grid point ID	
		3	Normal - x	3	Normal - x	R
		4	Shear - xy	4	Normal - y	R
		5	First principal	5	Normal - z	R
		6	First principal x cosine	6	Shear - xy	R
		7	Second principal x cosine	7	Shear - yz	R
		8	Third principal x cosine	8	Shear - zx	R
		9	Mean stress	9	Normal - x	I
		10	Octahedral shear stress	10	Normal - y	I
		11	Normal - y	11	Normal - z	I
		12	Shear - yz	12	Shear - xy	I
		13	Second principal	13	Shear - yz	I
		14	First principal y cosine	14	Shear - zx	I
		15	Second principal y cosine			
		16	Third principal y cosine			
		17	Normal - z			
		18	Shear - zx			
		19	Third principal			
		20	First principal z cosine			
		21	Second principal z cosine			
		22	Third principal z cosine			
62	CIHEX2†		Note CIHEX1		Note CIHEX1	
63	CIHEX3†	2	First external grid point ID	2	First external grid point ID	
		3	Normal - x	3	Normal - x	R
		4	Shear - xy	4	Normal - y	R
		5	First principal	5	Normal - z	R
		6	First principal x cosine	6	Shear - xy	R
		7	Second principal x cosine	7	Shear - yz	R
		8	Third principal x cosine	8	Shear - zx	R
		9	Mean stress	9	Second external grid point ID	
		10	Octahedral shear stress	10	Normal - x	I
		11	Second external grid point ID	11	Normal - y	I
		12	Normal - y	12	Normal - z	I
		13	Shear - yz	13	Shear - xy	I
		14	Second principal	14	Shear - yz	I
		15	First principal y cosine	15	Shear - zx	I
		16	Second principal y cosine			
		17	Third principal y cosine			

2.3.51 Element Stress Output Data Description (cont'd)

Element		Real Element Stresses		Complex Element Stresses		
Type	Name	Word or Component	Item	Word or Component	Item	Real Imag.
		18	Normal - z			
		19	Shear - zx			
		20	Third principal			
		21	First principal z cosine			
		22	Second principal z cosine			
		23	Third principal z cosine			

†The stresses are repeated for each stress point within each element.

2.3.52 Element Force Output Data Description

The following data should be added to this section:

Element		Real Element Stresses		Complex Element Stresses		
Type	Name	Word or Component	Item	Word or Component	Item	Real Imag.
62	CIHEX1	-	Undefined	-	Undefined	-
63	CIHEX2	-	Undefined	-	Undefined	-
64	CIHEX3	-	Undefined	-	Undefined	-

2.4.2 Executive Tables Not Permanently Core Resident

Section 2.4.2.8 IFPX1 (Master Card Name Table), the following data should be added to this section:

Word No. In IFPX1	Bit No. In IFPX0	Contents	Output Only (PMR)	Supported
502	251	CIHEX1	-	-
504	252	CIHEX2	-	-
506	253	CIHEX3	-	-
508	254	PIHEX	-	-
510	255	PLØAD3	-	-

3. SUBROUTINE DESCRIPTIONS

3.4.36 PREMAT (Material Property Utility)

Section 3.4.36.3 Calling Sequence, common block /MATOUT/ has been lengthened to 21 words. Its description should be altered as follows:

CØMMØN/MATØUT/ - (Output Common Block). Length 21 words. Depending upon the values of INFLAG, the output common block is defined variously as follows:

1. MAT1 Format (INFLAG = 1)

Word	Symbol	Definition
1	E	Young's modulus (modulus of elasticity)
2	G	Shear Modulus
3	ν	Poisson's ratio
4	ρ	Density
5	α	Thermal expansion coefficient
6	T_o	Thermal expansion reference temperature
7	g_e	Structural element damping coefficient
8	σ_t	Stress limit for tension
9	σ_c	Stress limit for compression
10	σ_s	Stress limit for shear
11-20	-	Undefined
21	TDEP	Temperature dependence flag (logical) T - material referenced was temperature dependent F - material referenced was not temperature dependent

For the tables in items 2 through 8, corresponding to values of INFLAG from 2 to 8, the entries like

16-20	-	Undefined

should be changed to

16-21	-	Undefined

4. MODULE FUNCTIONAL DESCRIPTIONS

4.5 Executive Preface Module IFP (Input File Processor)

The following data should be added to Table 1 and Table 2:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	Ø IJHK
251	CIHEX1	8	GEØM2	-2	12	16	951	1	7108	71	S5	5000	-1	33B5
252	CIHEX2	8	GEØM2	-2	24	28	951	1	7208	72	S5	5100	-1	33B6
253	CIHEX3	8	GEØM2	-2	36	4	951	1	7308	73	S5	5200	-1	33C1
254	PIHEX	2	EPT	-2	4	12	981	1	7002	70	S5	5300	-1	33C2
255	PLØAD3	9	GEØM3	-2	8	12	949	0	7109	71	S5	5400	-1	33C3

4.24 Functional Module PLØT (Structural Plotter)

Section 4.24.8.8 Subroutine name: SHAPE, the last sentence of item 4 should read:

> As each element is read, it is drawn, taking into account whether the element is one-, two-, or three-dimensional.

4.25 <u>Functional Module GP3 (Geometry Processor-Phase 3)</u>

Section 4.25.8.2 Subroutine: GP3C, this entire section should be replaced with the following:

1. Entry Point: GP3C

2. Purpose:

   To convert PLOAD2 data (if present) to PLOAD format, merge PLOAD2 data with PLOAD data (if present) and write the resulting data on SCR2, a scratch file. Then, to convert PLOAD3 data (if present) to an expanded format and write the resulting data on SCR2.

3. Calling Sequence: CALL GP3C

4. Method:

   A. The trailer bits for PLOAD2 and PLOAD3 cards are checked. If no PLOAD2 cards exist, Step B is executed. Otherwise, PLOAD2 cards are read into core from GEOM3. Six words are used for each entry. The first word (set identification) is set negative and the sixth word of each entry is set to zero. Step C is executed.

   B. If no PLOAD3 cards exist, a return to GP3 is issued. Otherwise, PLOAD3 cards are read into core from GEOM3. 39 words are used for each entry, which contains all pressures on one element. As each PLOAD3 card is read, the element ID is checked against those of PLOAD3 cards previously read. If a match is found, the data is added to that entry (pressures on the same face are summed). If no match is found, the data is stored in the next unused entry. The first word (set identification) is set negative and unused words are set to zero.

   C. GEOM2 is opened and the header record is skipped. The following steps occur for each record on GEOM2.

      C1. The three-word header is read. /GPTA1/ (see Section 2.5, P.M.) is searched for a match. If no match is found, the record is skipped and the process is repeated. If an end-of-file is encountered, Step 4 is executed. If a match is found, at test on element type is made. If not a two-dimensional element and PLOAD2 data is being processed or if not an isoparametric solid element and PLOAD3 data is being processed, the record is skipped and the process is repeated. Otherwise, step C2 is executed.

C2. An entry of the current element type is read. A linear search through the PLOAD2 or PLOAD3 data is made to find a match on element identification. If no match is found, the next entry is read. For each match which is found for PLOAD2 data, the grid identification numbers which connect the element are stored in the corresponding PLOAD2 entry and the first word of that entry is set positive. For each match which is found for PLOAD3 data, the pressure factors are rearranged according to element face, the grid identification numbers which connect the element are stored in the corresponding PLOAD3 entry, and the first word of that entry is set positive.

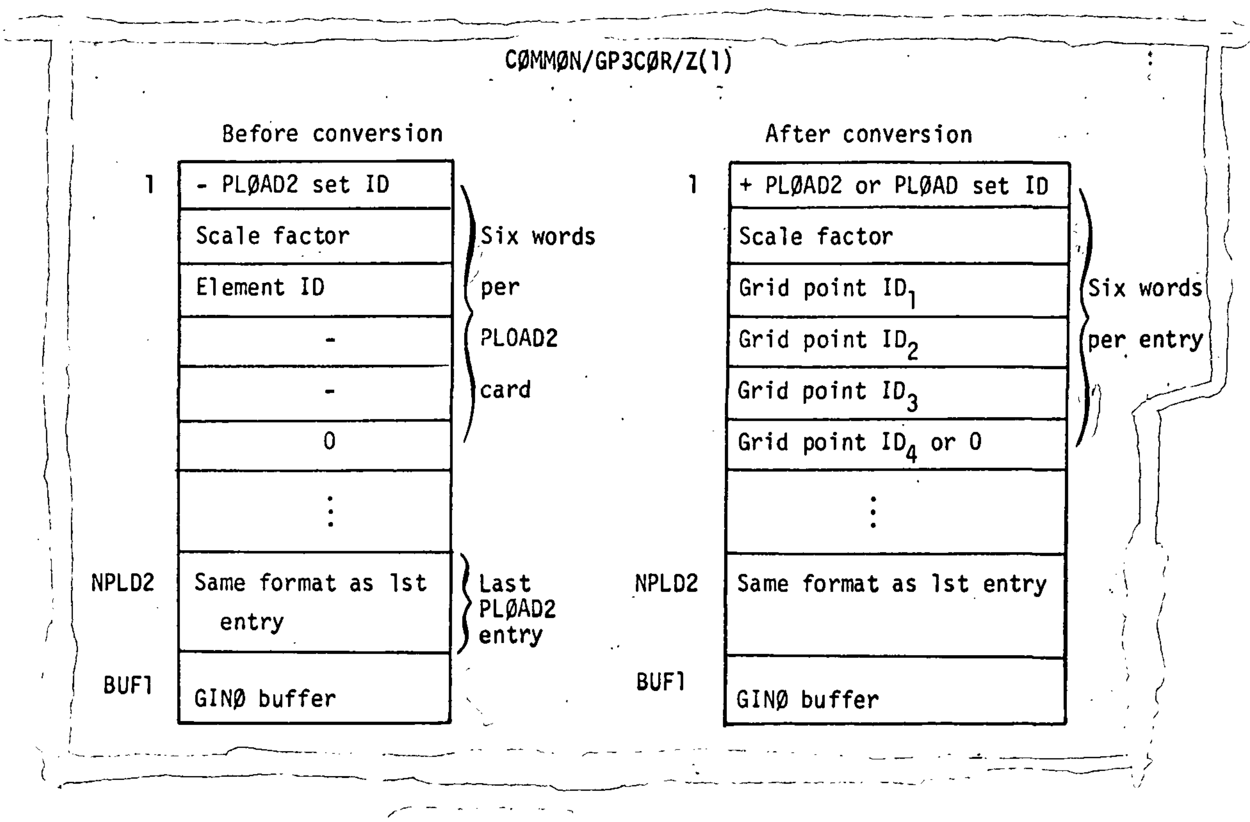
D. A pass through each entry in the PLOAD2 or PLOAD3 data is made. For each entry for which the first word is negative, an error message is queued or written and the NOGO flag is turned on. If upon completion of the pass, the NOGO flag is on and all PLOAD2 and PLOAD3 data has been processed, PEXIT is called. If the NOGO flag is on and all data has not been processed, control is passed to Step B. Otherwise Step E is executed.

E. If PLOAD3 data is being processed, Step F is executed. Otherwise LOCATE is called to position GEOM2 to PLOAD data. If none exists, Step F is executed. Otherwise, the PLOAD data is read into core following the PLOAD2 data. The combined list is sorted by SORT on set identification number.

F. The data in core is written on SCR2 (All PLOAD2/PLOAD data is written as one logical record and all PLOAD3 is written as the next logical record). If all PLOAD3 data has been processed, a return to GP3 is given.

Allocation of open core storage in GP3C is as follows:

1. For processing PLOAD2 data:



2. For processing PLOAD3 data:

COMMON/GP3COR/Z(1)

Before Conversion

1	- PLOAD3 set ID	39 words per entry
	Element ID	
	Pressure 1	
	⋮	
	Pressure 12	
	Grid point ID_1 for pressure 1	
	Grid point ID_2 for pressure 1	
	⋮	
	Grid point ID_1 for pressure 12	
	Grid point ID_2 for pressure 12	
	0	
	⋮	
NPLD2	Same format as first entry	
BUF1	GINO buffer	

After Conversion

1	+ PLOAD3 set ID	39 words per entry
	Pressure on face 1	
	⋮	
	Pressure on face 6	
	Grid point ID_1	
	⋮	
	Grid point ID_8	
	Grid point ID_9 or 0	
	⋮	
	Grid point ID_{32} or 0	
	⋮	
NPLD2	Same format as first entry	
BUF1	GINO buffer	

Section 4.25.8.3 Subroutine: GP3A, the parenthetical expression in item 4 in the second sentence should read:

(or SCR2 for PLØAD and PLØAD3 data)

Also for item 4, the parenthetical expression in the next-to-last paragraph should read:

(except GRAV, PLØAD, and PLØAD3 data)

Section 4.25.9.1 Allocation of Core Storage, the entry for GP3C should be changed to:

GP3C: Maximum requirement = MAX [6*(number of PLØAD2 + number of PLØAD cards), 39*(number of elements with PLØAD3 pressures)] + one GINØ buffer.

Section 4.25.10 Diagnostic Messages, the following messages may additionally be issued by GP3:

2144, 2145.

Figure 1 - Flowchart for module GP3, this figure should be modified as shown:

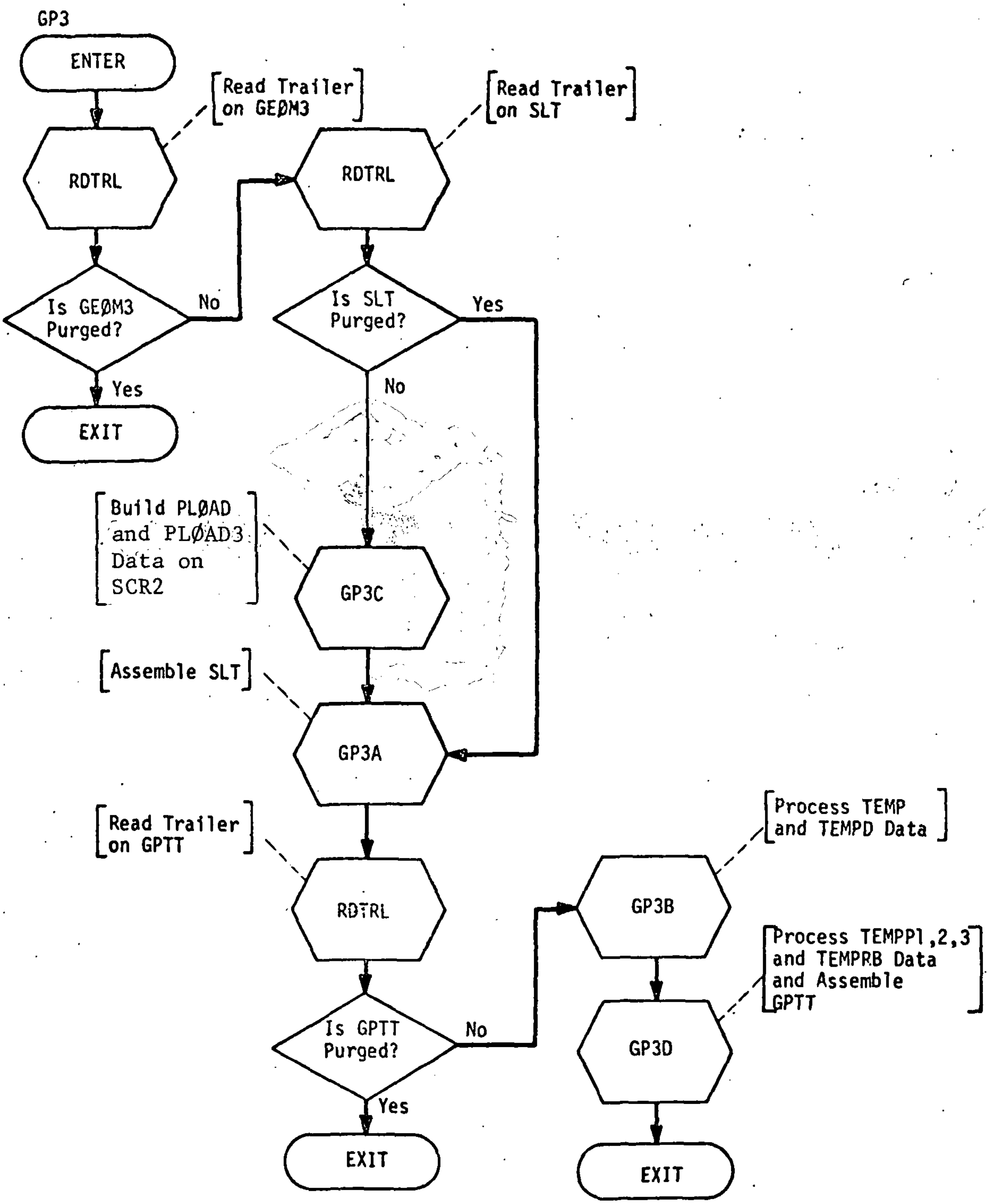


Figure 1. Flowchart for module GP3

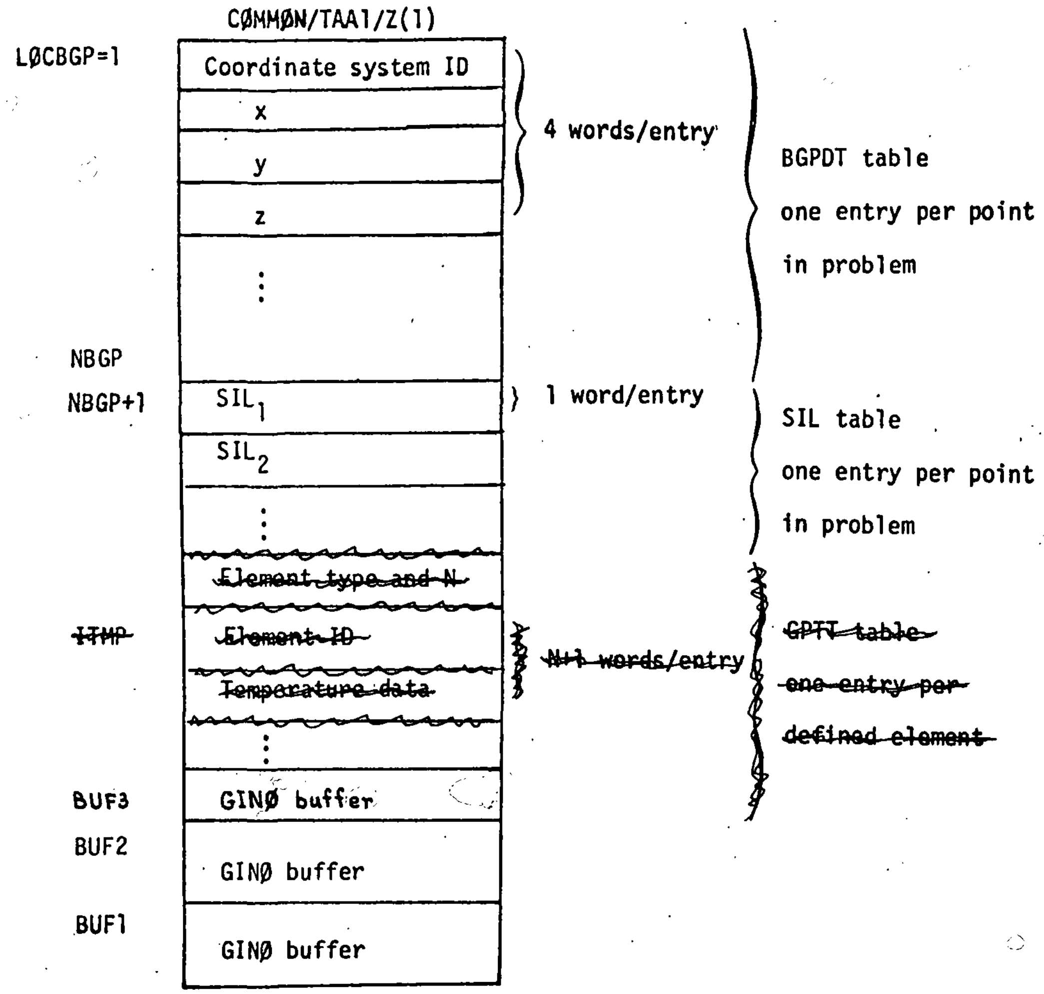
4.26 Functional Module TA1 (Table Assembler)

Section 4.26.7.2 TA1A, this section should be replaced by:

Assembly of the Element Summary Table is performed in two steps. For the first step, the EPT is read into core one property at a time. The ECT is read one element at a time. For each element the referenced property data are found by performing a binary search in the EPT in core. The ECT and EPT data are written on SCR1, a scratch file, one element at a time, one logical record per element type.

To initiate the second step, the BGPDT and SIL data blocks are read into core. Data from SCR1 are read one element at a time. Internal indices for the grid points are used as pointers into the BGPDT and SIL tables. The temperature data for the element is extracted from the GPTT data block using subroutine TA1ETD. The internal indices are now replaced with corresponding scalar index values. A line comprising ECT, EPT, BGPDT and GPTT data for the element is written on the EST. Each logical record of the EST comprises all data of one element type.

Allocation of core storage during the second step is as follows:



Section 4.26.7.3 TA1B, the fifth sentence in the next to last paragraph should read:

If the BGPDT and SIL can be held in core ....

The following section should be added:

4.26.8.8 Subroutine Name: TA1ETD

1. Entry Point: TA1ETD

2. Purpose: To extract the temperature data for an element from the GPTT data block.

3. Calling Sequence: CALL TA1ETD (ELID, TI, GRIDS)

   ELID - ID of the element for which temperature data is desired - integer-input

   TI - Array of temperature data - real-output

   GRIDS - Number of grid points of the element. If GRIDS=0, average element temperature is returned in TI(1), integer-input

Section 4.26.9.1 Allocation of Core Storage, step (2) for TA1A should read:

Step (2): Maximum core storage equals 5* (number of grid and scalor points in model) plus three GINØ buffers.

Section 4.26.10 Diagnostic Messages, the following message may additionally be issued by TA1:

4016

4.27 Functional Module SMA1 (Structural Matrix Assembly - Phase 1)

The following two sections should be added:

4.27.8.42 Subroutine Name: KIHEX

1. Entry Point: KIHEX

2. Purpose: To generate the element stiffness matrix for an IHEX1, IHEX2, or IHEX3 element.

3. Calling Sequence: CALL KIHEX (ITYPE)

$$\text{ITYPE} = \left\{ \begin{array}{l} 1 - \text{IHEX1} \\ 2 - \text{IHEX2} \\ 3 - \text{IHEX3} \end{array} \right\} - \text{integer-input}$$

4.27.8.43 Subroutine Name: IHEXSD

1. Entry Point: IHEXSD

2. Purpose: To generate isoparametric shape function data.

3. Calling Sequence: CALL IHEXSD (ITYPE, SHP, DSHP, JINV, DETJ, EID, XI, ETA, ZETA, CØRD)

ITYPE - same as in KIHEX calling sequence - integer-input

SHP - array of values of the shape functions - double precision-output

DSHP - array of values of the derivatives with respect to element coordinates of the shape functions - double precision-output

JINV - array of the inverse of the Jacobian matrix - double precision-output

DETJ - determinant of the Jacobian matrix - double precision-output

EID - Element ID - integer-input

XI, ETA, ZETA - Element coordinates at which shape functions are evaluated - double precision-input

CØRD - array of basic coordinates of the grid points of the element - real-input

Section 4.27.10 Diagnostic Messages, the following additional user fatal error messages may be issued by the isoparametric solid element routines:

2141, 2142, 2143.

4.28 Functional Module SMA2 (Structural Matrix Assembler - Phase 2)

The following two sections should be added:

4.28.8.27 Subroutine Name: MIHEX

1. Entry Point: MIHEX
2. Purpose: To generate the coupled mass matrix for an IHEX1, IHEX2, or IHEX3 element.
3. Calling Sequence: CALL MIHEX (ITYPE)

$$\text{ITYPE} = \left\{ \begin{array}{l} 1 - \text{IHEX1} \\ 2 - \text{IHEX2} \\ 3 - \text{IHEX3} \end{array} \right\} - \text{integer-input}$$

4.28.28 Subroutine Name: IHEXSD

See Section 4.27.8.43

4.41 Functional Module SSG1 (Static Solution Generator - Phase 1)

Section 4.41.7 Overview of the Method Used in SSG1, the second sentence in item 1 should read:

Pressure loads may be applied to an area defined by three or four grid points or to a face of an isoparametric solid element.

Section 4.41.8 Direct Applied Loads, the first sentence of this section should read:

Direct loads are applied to the structural model by means of FØRCE, FØRCE1, FØRCE2, GRAV, MØMENT, MØMENT1, PLØAD, PLØAD2, PLØAD3, RFØRCE, and SLØAD Bulk Data Cards and the PRESAX card which is used for the axisymmetric conical shell problem only.

The following four sections should be added:

4.41.8.9 PLØAD3 Card Processing

The data contents from a PLØAD3 card, as converted in GP3, are:

$P_1, P_2, \ldots, P_6$ = Normal pressure value on faces 1 to 6

$N_1^g, N_2^g, \ldots, N_{32}^g$ = Internal grid point numbers defining element on which pressure acts (N_9^g to N_{32}^g may be zero depending on element type).

The pressure load vector on the $i^{\underline{th}}$ element grid point is:

$$\{F_i\} = \sum_{k=1}^{6} P_k \sum_{m=1}^{2} \sum_{n=1}^{2} N_i^s \{B_k\}$$

Where P_k is the normal pressure on the k^{th} face, N_i^s is the isoparametric shape function, and $\{B_k\}$ is an area factor times a vector of direction cosines defining the outward normal to the k^{th} face at each integration point. The summation on m and n are for integration over the area of the k^{th} face using the method of Gaussian Quadrature. The load vector transformed to global coordinates for the i^{th} point is:

$$\{P_{g_i}\} = [T_i]^T \{F_i\}$$

where $[T_i]$ is the basic to global transformation matrix.

The computational method used in computing the load vector is as follows: Each element of a vector {F} of length three times the number of grid points is set to zero. The following steps are repeated at each integration point for each face which has a non-zero applied pressure:

1. Isoparametric utility routines are called to compute N_i^s's and $[J]^{-1}$ (See Section 4.87.16.2).

2. For each grid point i, and element face k, $\{B_k\}$ is multiplied by $N_i^s P_k$ and added to $\{F_i\}$. $\{B_k\}$ is simply ± 1 times det [J] times a column of $[J]^{-1}$. The column and sign are dependent on the face numbers as follows:

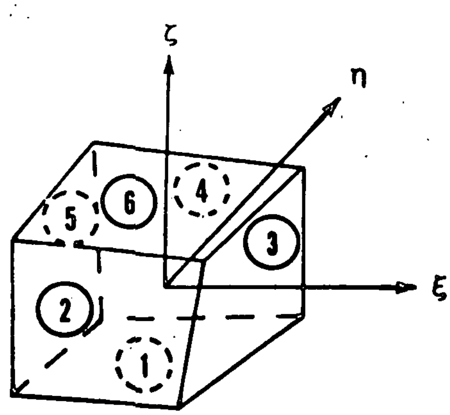


TABLE 5

Face	Sign	Column Number of $[J]^{-1}$
1	-	3
2	-	2
3	+	1
4	+	2
5	-	1
6	+	3

The {F} vector is then transformed to global coordinates and added to the global static load vector $\{P_g\}$.

4.41.11.38 Subroutine Name: PLØAD3

1. Entry Point: PLØAD3

2. Purpose: To apply loads due to PLØAD3 cards

3. Calling Sequence: CALL PLØAD3

   CØMMØN/LØADX/

   LØADX - See description of /LØADX/ above (section 4.41.11.8)

4.41.11.39 Subroutine Name: IHEX

1. Entry Point: IHEX

2. Purpose: To calculate an element thermal load vector for the IHEX1, IHEX2, and IHEX3 elements in the SSG1 module.

3. Calling Sequence: CALL IHEX (TEMPS, PG, TYPE)

   CØMMØN/TRIMEX/

   TEMPS - Array of grid point temperatures - real-input

   PG - Load vector array - real-input

   $$\text{TYPE} = \begin{Bmatrix} 1 - \text{IHEX1} \\ 2 - \text{IHEX2} \\ 3 - \text{IHEX3} \end{Bmatrix} - \text{integer-input}$$

   TRIMEX - EST entry for the element - mixed-input

4.41.11.40 Subroutine Name: QIHEX

1. Entry Point: QIHEX

2. Purpose: To calculate an element heat generation vector for the isoparametric solid elements for a unit external heat per unit volume.

3. Calling Sequence: CALL QIHEX (TYPE,P)

   $$\text{TYPE} = \begin{Bmatrix} 1 - \text{IHEX1} \\ 2 - \text{IHEX2} \\ 3 - \text{IHEX3} \end{Bmatrix} - \text{integer-input}$$

   P - Array containing force vector - real-output

Section 4.41.13 Diagnostic Messages, the following message may additionally be issued by SSG1:

2143

4.46 Functional Module SDR2 (Stress Data Recovery - Phase 2)

Section 4.46.8.21 Subroutine Name: SDR2D, paragraph 3 of item 4 should read:

> Core and GINØ buffers are then allocated as required for a) the Case Control data block, b) the Element Deformation Table, c) the Grid Point Temperature Table, d) the element stress matrices, and e) the EQEXIN Data Block. If there is insufficient space in core for the element stress matrices, they are maintained on the scratch data block generated in stage III.

The following three sections should be added.

4.46.8.37 Subroutine Name: SIHEX1

1. Entry Point: SIHEX1

2. Purpose: To generate element stress matrices for the IHEX1, IHEX2, and IHEX3 elements, SIHEX1 must be called once for each stress point in an element.

3. Calling Sequence: CALL SIHEX1 (TYPE, STRSPT)

$$\text{TYPE} = \left\{ \begin{matrix} 1 - \text{IHEX1} \\ 2 - \text{IHEX2} \\ 3 - \text{IHEX3} \end{matrix} \right\} - \text{integer-input}$$

STRSPT - stress point number - integer-input

4.46.8.38 Subroutine Name: IHEXSS

1. Entry Point: IHEXSS

2. Purpose: To generate isoparametric shape function data. (Single precision version of subroutine IHEXSD, section 4.27.8.43)

3. Calling Sequence: CALL IHEXSS (ITYPE, SHP, DSHP, JINV, DETJ, EID, XI, ETA, ZETA, CØRD)

See section 4.27.8.43 for description of arguments

4.46.8.39 Subroutine Name: SIHEX2

1. Entry Point: SIHEX2

2. Purpose: To perform final stress computations for the IHEX1, IHEX2, and IHEX elements. SIHEX2 must be called once for each stress point in an element.

3. Calling Sequence: CALL SIHEX2 (TYPE, GPT)

$$\text{TYPE} = \left\{ \begin{array}{l} 1 - \text{IHEX1} \\ 2 - \text{IHEX2} \\ 3 - \text{IHEX3} \end{array} \right\} - \text{integer-input}$$

GPT - Array of grid point temperatures - real-input

Section 4.46.10 Diagnostic Messages, the following messages may additionally be issued by SDR2:

2143, 4024

4.49 Functional Module DSMG1 (Differential Stiffness Generator - Phase 1)

Section 4.49.7 Method, the first paragraph of this section should be replaced by the following:

The module driver, DSMG1, is a very short routine whose only function is to call the two principal subroutines of the module, DS1 and DS1A, which accomplish the two phases of the module. The first phase of the module is incorporated in subroutine DS1. This routine creates the scratch file ECPTDS (GINØ file number 301) by appending to each element in the ECPT data block for which differential stiffness is defined an element deformation, an average element loading temperature for all elements except the conical shell element and the isoparametric solid elements (a loading temperature at each grid point is appended for these elements) and the proper components of the displacement vector, UGV. It should be noted that although element deformations are defined only for rods, tubes, beams, and bars, an element deformation is attached to each element written on the ECPTDS scratch data block. The elements admissible to the ECPTDS scratch data block are: rods, beams, tubes, shear (but not twist) panels, triangular and quadrilateral elements (TRMEM's and QDMEM's), the combination membrane and plate triangular and quadrilateral elements (TRIA1, TRIA2, QUAD1, QUAD2), the conical shell element, and the isoparametric solid elements (IHEX1, IHEX2, and IHEX3).

Item 3 should read:

3. If there is no temperature load, go to step 4. If there is a temperature load, the Grid Point Temperature Table data block, GPTT, is positioned to the proper thermal record.

Sentence 10 of the first paragraph of item 6 should read:

Then an element deformation number, an average element loading temperature (grid point temperatures for the conical shell and isoparametric solid elements), and the displacement vector components are appended in core to the ECPT entry for the element.

The following two sections should be added:

4.49.8.11 Subroutine Name: DIHEX

1. Entry Point: DIHEX

2. Purpose: To generate the element differential stiffness matrix for the isoparametric solid elements.

3. Calling Sequence: CALL DIHEX (TYPE)

$$\text{TYPE} = \begin{Bmatrix} 1 - \text{IHEX1} \\ 2 - \text{IHEX2} \\ 3 - \text{IHEX3} \end{Bmatrix} - \text{integer-input}$$

4.49.8.12 Subroutine Name: IHEXSD

See section 4.27.8.43

4.87 Structural Element Descriptions

The following sections should be added:

4.87.16 The IHEX1, IHEX2, and IHEX3 Elements

4.87.16.1 Input Data for the IHEXi Elements

1. The EST entries for IHEXi are:

Symbol	Description
SIL_i, i=1,2,...,NGP	Scalar indices of the connected grid points. NGP = number of grid points per element.
N_i	
X_i i=1,2,...,NGP	Referenced local coordinate system and location in basic coordinates of
Y_i	connected grid points.
Z_i	
MID	Material identification number
CID	Coordinate system identification number in which anisotropic material is defined.
NIP	Number of integration points in each coordinate direction.
AR	Maximum aspect ratio of element
ALFA	Maximum angle (in degrees) for face normals
BETA	Maximum angle (in degrees) for mid-edge points
GPT_i, i=1,2,...,NGP	Grid point temperatures

2. Coordinate System Data

The numbers N_i, X_i, Y_i, and Z_i are used to calculate the three by three global-to-basic coordinate transformation matrix $[T_i]$ for the $i^{\underline{th}}$ grid point via calls to either TRANSS or TRANSD.

The CID is used to calculate the six by six global-to-basic coordinate transformation matri $[T^M]$ for anisotropic material. If the CID references a cylindrical or spherical coordinate system, $[T^M]$ must be computed at each integration point. Otherwise, it need be computed once only for the element.

3. Material Data

Symbol	Description
$[G_m]$	Six by six stress-strain matrix defined in coordinate system CID
ρ	Mass density
$\{\alpha_m\}$	Vector of six thermal expansion coefficients defined in coordinate system CID
T_o	Reference temperature
g_e	Structural damping coefficient

4.87.16.2 Basic Equations for IHEXi Elements

1. Numerical Integration

   The method of Gaussian Quadrature is used to numerically integrate the element matrices. The weighting coefficients H_i and abscissas s_i are listed in Table 1 at the conclusion of this Section 4.87.16.

2. Element Coordinates

   The coordinates of the grid points in element coordinates are:

   For the IHEX1 element:

i	ξ_i	η_i	ζ_i
1	-1	-1	-1
2	1	-1	-1
3	1	1	-1
4	-1	1	-1
5	-1	-1	1
6	1	-1	1
7	1	1	1
8	-1	1	1

For the IHEX2 element:

i	ξ_i	η_i	ζ_i
1	-1	-1	-1
2	0	-1	-1
3	1	-1	-1
4	1	0	-1
5	1	1	-1
6	0	1	-1
7	-1	1	-1
8	-1	0	-1
9	-1	-1	0
10	1	-1	0
11	1	1	0
12	-1	1	0
13	-1	-1	1
14	0	-1	1
15	1	-1	1
16	1	0	1
17	1	1	1
18	0	1	1
19	-1	1	1
20	-1	0	1

For the IHEX3 element:

i	ξ_i	η_i	ζ_i
1	-1	-1	-1
2	-1/3	-1	-1
3	1/3	-1	-1
4	1	-1	-1
5	1	-1/3	-1
6	1	1/3	-1
7	1	1	-1
8	1/3	1	-1
9	-1/3	1	-1
10	-1	1	-1
11	-1	1/3	-1
12	-1	-1/3	-1
13	-1	-1	-1/3
14	1	-1	-1/3
15	1	1	-1/3
16	-1	1	-1/3
17	-1	-1	1/3
18	1	-1	1/3
19	1	1	1/3
20	-1	1	1/3
21	-1	-1	1
22	-1/3	-1	1
23	1/3	-1	1
24	1	-1	1
25	1	-1/3	1
26	1	1/3	1
27	1	1	1
28	1/3	1	1
29	-1/3	1	1
30	-1	1	1
31	-1	1/3	1
32	-1	-1/3	1

3. Interpolating Functions and Their Derivatives

The interpolating function, or isoparametric shape function N_i and the derivative of this function with respect to the element's coordinates is given for the $i^{\underline{th}}$ grid point by the following tables: 2, 3, and 4.

4. Jacobian Matrix

The Jacobian matrix at any point (x,y,z) within the element is:

$$[J] = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \cdots & \frac{\partial N_{NGP}}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \cdots & \frac{\partial N_{NGP}}{\partial \eta} \\ \frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} & \cdots & \frac{\partial N_{NGP}}{\partial \zeta} \end{bmatrix} \begin{bmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ \vdots & \vdots & \vdots \\ X_{NGP} & Y_{NGP} & Z_{NGP} \end{bmatrix} \qquad (1)$$

5. Strain Displacement Relations

The transformations from displacements to strain for the $i^{\underline{th}}$ grid point are:

$$[C_i] = \begin{bmatrix} C_{1i} & 0 & 0 \\ 0 & C_{2i} & 0 \\ 0 & 0 & C_{3i} \\ C_{2i} & C_{1i} & 0 \\ 0 & C_{3i} & C_{2i} \\ C_{3i} & 0 & C_{1i} \end{bmatrix} \qquad (2)$$

where

$$\begin{Bmatrix} C_{1i} \\ \\ C_{2i} \\ \\ C_{3i} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \dfrac{\partial N_i}{\partial x} \\ \\ \dfrac{\partial N_i}{\partial y} \\ \\ \dfrac{\partial N_i}{\partial z} \end{Bmatrix} \qquad (3)$$

4.87.16.3 Stiffness and Mass Matrix Calculation for IHEXi Elements

The equation used in the stiffness matrix generation in global coordinates is:

$$[K_{ij}] = [T_i]^T \left\{ \sum_{\ell=1}^{NIP} \sum_{m=1}^{NIP} \sum_{n=1}^{NIP} H_\ell\, H_m\, H_n\, |J|\, [C_i]^T\, [T^M]^T\, [G_m]\, [T^M]\, [C_j] \right\} [T_j] \qquad (4)$$

The equation used in the coupled mass matrix generation in global coordinates is:

$$[M_{ij}] = [T_i]^T \left\{ \sum_{\ell=1}^{NIP} \sum_{m=1}^{NIP} \sum_{n=1}^{NIP} \rho H_\ell\, H_m\, H_n\, |J| \begin{bmatrix} N_iN_j & 0 & 0 \\ 0 & N_iN_j & 0 \\ 0 & 0 & N_iN_j \end{bmatrix} \right\} [T_j] \qquad (5)$$

$[K_{ij}]$ and $[M_{ij}]$ are three by three partitions of the element matrices coupling the i^{th} and j^{th} element grid points.*

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* No provision is made for the computation of a lumped mass matrix for the isoparametric hexahedron elements. Such a computation would necessitate an arbitrary distribution of mass reducing the accuracy advantages of these type elements.

4.87.16.4 Element Load Calculations for IHEXi Elements

1. Thermal Force Vector (Subroutine IHEX of Module SSG1)

   The thermal force vector is generated with the following equation:

$$\{P_i\} = [T_i]^T \left\{ \sum_{\ell=1}^{NIP} \sum_{m=1}^{NIP} \sum_{n=1}^{NIP} H_\ell \, H_m \, H_n \, |J| \, [C_i]^T \, [T^M]^T \, [G_e] \, \{\alpha_e\} * \left(\sum_{j=1}^{NGP} N_j \, (t_j - T_o) \right) \right\} \tag{6}$$

   where $\{P_i\}$ is the load vector of length three for the $i^{\underline{th}}$ element grid point and t_j is the temperature at the $j^{\underline{th}}$ element grid point.

2. Pressure Force Vector (Subroutine PLØAD3 of Module SSGI)

   The generation of the pressure force vector is described in Section 4.41.8.9.

3. Heat Generation Vector for Heat Transfer Problems (Subroutine QIHEX of Module SSG1)

   The terms of the heat generation vector are given by:

$$P_i = -Q \sum_{\ell=1}^{2} \sum_{m=1}^{2} \sum_{n=1}^{2} |J| \, N_i \tag{7}$$

   where Q is the external heat per unit volume.

4.87.16.5 Element Stress Calculations for IHEXi Elements

1. Calculations performed in SIHEX1 (Phase 1 calculations)

   The Phase 1 calculations include the computation of stress matrices at each point within the element at which stresses are to be evaluated.

   The transformation matrix from displacements at the $i^{\underline{th}}$ grid point to stresses at the $j^{\underline{th}}$ stress point is:

$$[S_{ji}] = [T^M]^T [G_m] [T^M] [C_i] [T_i] \tag{8}$$

   where $[T_i]$ is the global to basic coordinate transformation matrix. All matrices on the right-hand side of this equation are computed at stress point j.

   The temperature to stress relation at the $j^{\underline{th}}$ stress point is:

$$\{S_{tj}\} = -[T^M]^T [G_m] \{\alpha_m\} \tag{9}$$

2. Calculations performed in SIHEX2 (Phase 2 calculations)

   The Phase 2 calculations include the computation of stresses at each stress point within the element.

The equation for stress at the $j^{\underline{th}}$ stress point is:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix}_j = \sum_{i=1}^{NGP} [S_{ji}] \{U_{gi}\} + \{S_{tj}\} \left[\sum_{k=1}^{NGP} N_k \, (t_k - T_o) \right] \qquad (10)$$

Where $\{U_{gi}\}$ is the three by one global displacement vector at the $i^{\underline{th}}$ grid point and t_k is the loading temperature at the $k^{\underline{th}}$ grid point.

The three principal stresses are the roots of the following cubic equation in S:

$$S^3 - (\sigma_x + \sigma_y + \sigma_z)\, S^2 + (\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \sigma_{xy}^{\ 2} - \sigma_{yz}^{\ 2} - \sigma_{zx}^{\ 2})\, S -$$

$$(\sigma_x\sigma_y\sigma_z + 2\sigma_{xy}\sigma_{yz}\sigma_{zx} - \sigma_x\sigma_{yz}^{\ 2} - \sigma_y\sigma_{zx}^{\ 2} - \sigma_z\sigma_{xy}^{\ 2}) = 0 \qquad (11)$$

The direction cosines of the normals to the principal plane on which each of the principal stresses is acting are found by solving:

$$\begin{bmatrix} (S - \sigma_x) & -\sigma_{xy} & -\sigma_{zx} \\ -\sigma_{xy} & (S - \sigma_y) & -\sigma_{yz} \\ -\sigma_{zx} & -\sigma_{yz} & (S - \sigma_z) \end{bmatrix} \begin{Bmatrix} \ell \\ m \\ n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (12)$$

using the constraint equation:

$$\ell^2 + m^2 + n^2 = 1 \qquad (13)$$

The mean stress or hydrostatic pressure is:

$$\sigma_n = -\frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)^* \qquad (14)$$

The octahedral shear stress is:

$$\sigma_o = \left\{\frac{1}{3}[(S_x + \sigma_n)^2 + (S_y + \sigma_n)^2 + (S_z + \sigma_n)^2]\right\}^{1/2} \qquad (15)$$

4.87.16.6 Differential Stiffness Calculations for IHEXi Elements

The differential stiffness matrix in generalized coordinates is (the stresses are in the basic system):

$$[K_w^d] = \begin{bmatrix} \sigma_y + \sigma_z & -\sigma_{xy} & -\sigma_{xz} \\ -\sigma_{xy} & \sigma_x + \sigma_z & -\sigma_{yz} \\ -\sigma_{xz} & -\sigma_{yz} & \sigma_x + \sigma_y \end{bmatrix} \qquad (16)$$

The transformation from displacements at the nodal points to rigid body rotation are:

$$[C_i^d] = \frac{1}{2}\begin{bmatrix} 0 & -C_{3i} & C_{2i} \\ C_{3i} & 0 & -C_{1i} \\ -C_{2i} & C_{1i} & 0 \end{bmatrix} \qquad (17)$$

where C_{1i}, C_{2i}, and C_{3i} are given in Section 4.87.16.2.

The three by three partitions of the global element differential stiffness matrix couping the $i^{\underline{th}}$ and $j^{\underline{th}}$ element grid points are given by:

$$[K_{ij}^d] = [T_i]^T \left\{ \sum_{\ell=1}^{2} \sum_{m=1}^{2} \sum_{n=1}^{2} |J| \; [C_i^d]^T [K_w^d] [C_j^d] \right\} [T_j] \qquad (18)$$

---

* The minus sign is used to be consistent with the hydrostatic pressure computations for elements CTETRA, CWEDGE, CHEXA1, and CHEXA2.

4.87.16.7 Heat Transfer Calculations for IHEXi Elements

The terms of the heat conductance matrix are given by:

$$K_{ij} = \sum_{\ell=1}^{NIP} \sum_{m=1}^{NIP} \sum_{n=1}^{NIP} H \, H_m H_n \, |J| \, \{C_i\}^T \, [T^M]^T \, [k] \, [T^M] \, \{C_j\} \tag{19}$$

where

$$\{C_i\} = \begin{Bmatrix} \dfrac{\partial N_i}{\partial x} \\ \dfrac{\partial N_i}{\partial y} \\ \dfrac{\partial N_i}{\partial z} \end{Bmatrix} \tag{20}$$

and [k] is the three by three matrix of thermal conductivity coefficients in global coordinates.

The terms of the heat capacitance matrix are given by:

$$B_{ij} = \sum_{\ell=1}^{NIP} \sum_{m=1}^{NIP} \sum_{n=1}^{NIP} H_\ell H_m H_n \, |J| \, N_i c_p N_j \tag{21}$$

where c_p is the thermal capacity per unit volume.

TABLE 1. GAUSSIAN QUADRATURE FORMULA

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f(x, y, z)dx\, dy\, dz = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n} H_j\, f(s_j, s_k, s_i)$$

n	Abscissa (s)	Weight Coefficient (H)
2	±0.57735026919	1.0
3	±0.77459666924	0.55555555555
	0.0	0.88888888888
4	±0.86113631159	0.34785484514
	±0.33998104358	0.65214515486

TABLE 2. ISOPARAMETRIC SHAPE FUNCTIONS AND THEIR DERIVATIVES FOR THE IHEX1 ELEMENT - 8 GRID POINTS

CORNER GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm 1, \quad \zeta_i = \pm 1$$

$$N_i = \frac{1}{8} (1 + \xi_o) (1 + \eta_o) (1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{1}{8} \xi_i (1 + \eta_o) (1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{1}{8} \eta_i (1 + \xi_o) (1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{1}{8} \zeta_i (1 + \xi_o) (1 + \eta_o)$$

Where $\xi_o = \xi\xi_i, \quad \eta_o = \eta\eta_i, \quad \zeta_o = \zeta\zeta_i$

TABLE 3. ISOPARAMETRIC SHAPE FUNCTIONS AND THEIR DERIVATIVES FOR THE IHEX2 ELEMENT - 20 GRID POINTS

CORNER GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm 1, \quad \zeta_i = \pm 1$$

$$N_i = \frac{1}{8}(1 + \xi_o)(1 + \eta_o)(1 + \zeta_o)(\xi_o + \eta_o + \zeta_o - 2)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{1}{8}\xi_i (1 + \eta_o)(1 + \zeta_o)(2\xi_o + \eta_o + \zeta_o - 1)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{1}{8}\eta_i (1 + \xi_o)(1 + \zeta_o)(2\eta_o + \xi_o + \zeta_o - 1)$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{1}{8}\zeta_i (1 + \xi_o)(1 + \eta_o)(2\zeta_o + \xi_o + \eta_o - 1)$$

MID-SIDE GRID POINTS

$$\xi_i = 0, \quad \eta_i = \pm 1, \quad \zeta_i = \pm 1$$

$$N_i = \frac{1}{4}(1 - \xi^2)(1 + \eta_o)(1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \xi} = -\frac{1}{2}\xi(1 + \eta_o)(1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{1}{4}(1 - \xi^2)(1 + \zeta_o)\eta_i$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{1}{4}(1 - \xi^2)(1 + \eta_o)\zeta_i$$

MID-SIDE GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = 0, \quad \zeta_i = \pm 1$$

$$N_i = \frac{1}{4}(1 - \eta^2)(1 + \xi_o)(1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{1}{4}(1 - \eta^2)(1 + \zeta_o)\xi_i$$

$$\frac{\partial N_i}{\partial \eta} = -\frac{1}{2}\eta(1 + \zeta_o)(1 + \xi_o)$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{1}{4}(1 - \eta^2)(1 + \xi_o)\zeta_i$$

MID-SIDE GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm 1, \quad \zeta_i = 0$$

$$N_i = \frac{1}{4}(1 - \zeta^2)(1 + \xi_o)(1 + \eta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{1}{4}(1 - \zeta^2)(1 + \eta_o)\xi_i$$

$$\frac{\partial N_i}{\partial \eta} = \frac{1}{4}(1 - \zeta^2)(1 + \xi_o)\eta_i$$

$$\frac{\partial N_i}{\partial \zeta} = -\frac{1}{2}\zeta(1 + \xi_o)(1 + \eta_o)$$

Where $\xi_o = \xi\xi_i, \quad \eta_o = \eta\eta_i, \quad \zeta_o = \zeta\zeta_i$

TABLE 4. ISOPARAMETRIC SHAPE FUNCTIONS AND THEIR DERIVATIVES FOR THE IHEX3 ELEMENT - 32 GRID POINTS

CORNER GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm 1, \quad \zeta_i = \pm 1$$

$$N_i = \frac{9}{64}(1 + \xi_o)(1 + \eta_o)(1 + \zeta_o)(\xi^2 + \eta^2 + \zeta^2 - \frac{19}{9})$$

$$\frac{\partial N_i}{\partial \xi} = \frac{9}{64}(1 + \eta_o)(1 + \zeta_o)[\xi_i(3\xi^2 + \eta^2 + \zeta^2 - \frac{19}{9}) + 2\xi]$$

$$\frac{\partial N_i}{\partial \eta} = \frac{9}{64}(1 + \xi_o)(1 + \zeta_o)[\eta_i(3\eta^2 + \xi^2 + \zeta^2 - \frac{19}{9}) + 2\eta]$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{9}{64}(1 + \xi_o)(1 + \eta_o)[\zeta_i(3\zeta^2 + \xi^2 + \eta^2 - \frac{19}{9}) + 2\zeta]$$

MID-SIDE GRID POINTS

$$\xi_i = \pm\frac{1}{3}, \quad \eta_i = \pm 1, \quad \zeta_i = \pm 1$$

$$N_i = \frac{9}{64}(1 - \xi^2)(1 + 9\xi_o)(1 + \eta_o)(1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{9}{64}(1 + \eta_o)(1 + \zeta_o)(-2\xi + 9\xi_i - 27\xi\xi_o)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{9}{64}(1 - \xi^2)(1 + 9\xi_o)(1 + \zeta_o)\ \eta_i$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{9}{64}(1 - \xi^2)(1 + 9\xi_o)(1 + \eta_o)\ \zeta_i$$

MID-SIDE GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm\frac{1}{3}, \quad \zeta_i = \pm 1$$

$$N_i = \frac{9}{64}(1 - \eta^2)(1 + 9\eta_o)(1 + \xi_o)(1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{9}{64}(1 - \eta^2)(1 + 9\eta_o)\ \xi_i\ (1 + \zeta_o)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{9}{64}(1 + \xi_o)(1 + \zeta_o)(-2\eta + 9\eta_i - 27\eta\eta_o)$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{9}{64}(1 - \eta^2)(1 + 9\eta_o)\ \zeta_i\ (1 + \xi_o)$$

MID-SIDE GRID POINTS

$$\xi_i = \pm 1, \quad \eta_i = \pm 1, \quad \zeta_i = \pm\frac{1}{3}$$

$$N_i = \frac{9}{64}(1 - \zeta^2)(1 + 9\zeta_o)(1 + \xi_o)(1 + \eta_o)$$

$$\frac{\partial N_i}{\partial \xi} = \frac{9}{64}(1 - \zeta^2)(1 + 9\zeta_o)\ \xi_i\ (1 + \eta_o)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{9}{64}(1 - \zeta^2)(1 + 9\zeta_o)\ \eta_i\ (1 + \xi_o)$$

$$\frac{\partial N_i}{\partial \zeta} = \frac{9}{64}(1 + \xi_o)(1 + \eta_o)(-2\zeta + 9\zeta_i - 27\zeta\zeta_o)$$

Where $\xi_o = \xi\xi_i$, $\eta_o = \eta\eta_i$, $\zeta_o = \zeta\zeta_i$

APPENDIX D

INSTRUCTIONS FOR IMPLEMENTATION

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INSTRUCTIONS FOR IMPLEMENTATION INTO NASTRAN LEVEL 16

There are three major tasks involved in the implementation of the family of isoparametric solid elements into NASTRAN Level 16. These tasks are in addition to the normal implementation of subroutines and updates as delivered. Each of these tasks will involve changes in the source code. The first task is the installation of subroutine XIHEX for element matrix generation. The second is the installation of subroutine QIHEX for heat generation load computation. The third is the addition of the capability to generate element matrices and loads using general, 3-dimensional anisotropic materials, when such materials become available in NASTRAN.

Subroutine XIHEX

All program interfaces for this subroutine are based on MacNeal-Schwendler Corporation memo number DNH-4, dated June 21, 1972. The variables which control program branching to generate the various types of matrices are presented in Table D.1. The GINØ files which are passed to subroutine EMGØUT for writing the various matrices are shown in Table D.2. The work items which must be performed to implement XIHEX are listed below:

1. Discard subroutines KIHEX, MIHEX, DIHEX, and TKTZTK.
2. The local array GPTLD should contain the grid point temperatures for computing stresses for the differential stiffness matrix. GPTLD must be initialized either by placing it in the argument list or in a common block. If GPTLD (1) = -1 (integer), the program assumes there is no temperature load.
3. An integer pointer, UGV, must be initialized. UGV indicates the location of the first element of the single precision global displacement vector in open core. It is also used as the flag for computing the differential stiffness matrix (see Table D.1). UGV>0 to compute differential stiffness; UGV≤0 otherwise.
4. The calling sequences for the four calls to EMGØUT should be checked.
5. The matrices generated by XIHEX should be thoroughly checked against those generated by the Level 15 subroutines.

Subroutine QIHEX

No documentation on how this subroutine was to interface with the calling routine was available. Thus, the interfaces below were assumed. These must be modified as required to implement QIHEX.

1. The element's EST entry is in common block ESTQ.
2. The element load vector is generated and stored in array P. P is passed back to the calling routine in the argument list. The load is not inserted into a global load vector.

3. The load vector generated is for a unit value of Q, the external heat per unit volume. Therefore, the load vector passed to the calling routine must be multiplied by the actual value of Q.

Anisotropic Materials

When a 3-dimensional anisotropic material becomes available in NASTRAN, changes may be effected in the isoparametric solid element routines to make use of it. The following subroutines use the element material properties and thus would have to be changed: XIHEX, IHEX, and SIHEX1. In each of these subroutines, the logic for branching to computations for isotropic materials or anisotropic materials has already been implemented. However, certain changes and additions must still be made. First the interfaces with the material property utility PREMAT/MAT will have to be modified as required. Second, new sections of code involving the actual computation with an anisotropic material must be added where indicated. The locations where new code must be added are indicated by comment cards with equal signs in columns 2-72. The computations which must be performed there are also indicated on comment cards. These computations are documented in the Theoretical Manual inserts in Appendix A and in the Programmer's Manual inserts in Appendix C. The actual amount of code to be added is small and involves only the computation of small intermediate matrices using the anisotropic material properties.

TABLE D.1 RULES FOR MATRIX GENERATION

Rule	Matrix Generated
KGG>0, UGV≤0, HEAT=0	Stiffness
KGG>0, UGV>0, HEAT=0	Differential Stiffness
KGG>0, UGV≤0, HEAT=1	Conductance
MGG>0, UGV≤0, HEAT=0	Mass
MGG>0, UGV≤0, HEAT=1	Capacitance
KGG>0, MGG>0, UGV≤0, HEAT=0	Stiffness and Mass
KGG>0, MGG>0, UGV≤0, HEAT=1	Conductance and Capacitance

TABLE D.2 MATRIX OUTPUT FILES

Matrix	Output File
Stiffness	KGG
Mass	MGG
Differential Stiffness	KGG
Conductance	KGG
Capacitance	MGG

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APPENDIX E

DEMONSTRATION PROBLEM REPORTS

Rigid Format - 1	Static Analysis
Rigid Format - 3	Normal Modes
Rigid Format - 5	Buckling Analysis

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RIGID FØRMAT No. 1, Static Analysis

Static Analysis of a Long Beam Using Hexahedron Isoparametric Elements

A. Description

This problem illustrates the use of NASTRAN's three types of hexahedron isoparametric solid elements (IHEX1, IHEX2, and IHEX3) to solve a long beam problem. The beam, shown in Figure 1, is fully constrained at the left end and free at the right end. Four separate loading conditions are considered in the analysis. In addition, solutions are calculated for each loading condition using three distinct finite element models of the beam. Each model is comprised of a different type of hexahedron isoparametric solid element (IHEX1, IHEX2 or IHEX3). Results are presented for each of the three models, and compared with closed form solutions.

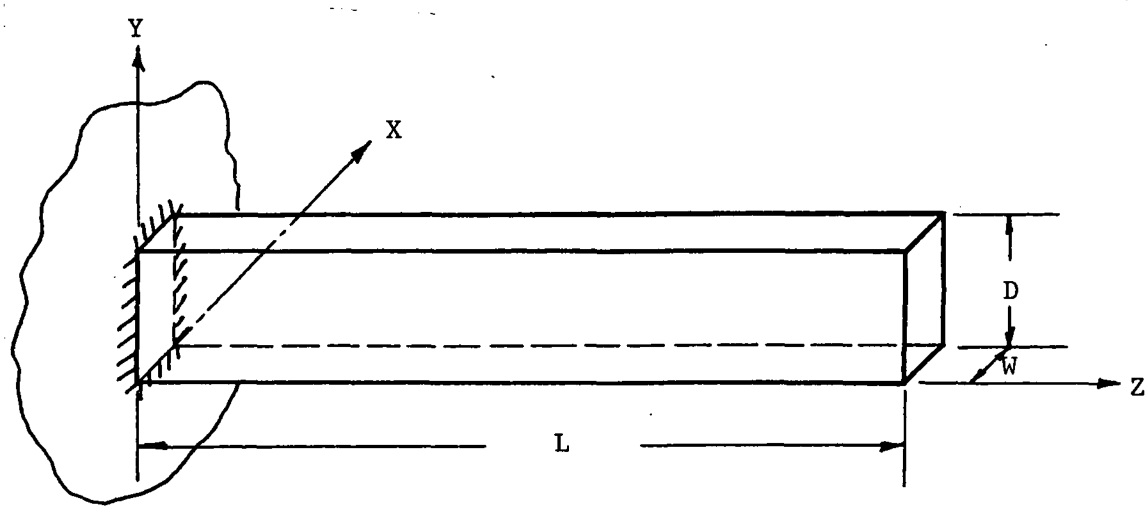

Figure 1. Cantilever Beam

B. Input

1. Parameters:

   $L = 144$ - Length

   $D = 24$ - Depth

   $W = 12$ - Width

   $E = 30 \times 10^6$ - Modulus of Elasticity

   $\mu = .3$ - Poisson's Ratio

   $\alpha = 1.428 \times 10^{-5}$ - Coefficient of Thermal Expansion

   $\rho = 7.535 \times 10^{-4}$ - Mass Density

2. Loads:

   a) Linear Thermal Gradient (Y-Direction)

      $T = 120°$ @ $Y = 0$, $T = -120°$ @ $Y = 24$

   b) Uniform Temperature Rise

      $\Delta T = 100°$

   c) Compressive Axial Pressure Load (Z-Direction) @ $Z = 144$

      $P_Z = -42,837$

   d) Transverse Pressure Load (Y-Direction) @ $Y = 0$

      $P_Y = 100$

3. Constraints:

   All grid points in the plane $Z = 0$ are constrained in the X, Y and Z directions.

C. Modeling Techniques

The finite element model for each type of isoparametric hexahedron element is shown in Figures 2, 3 and 4. A summary of the number of elements and grid points used in each model is given in Table 1 below.

Table 1. Total Number of Elements and Grid Points for Each Type of Isoparametric Hexahedron Model

Model Type	Number of Elements	Number of Grid Points	Half-Band Width (Grid Points)
Model 1 - IHEX1 Elements	216	364	34
Model 2 - IHEX2 Elements	36	275	52
Model 3 - IHEX3 Elements	8	148	44

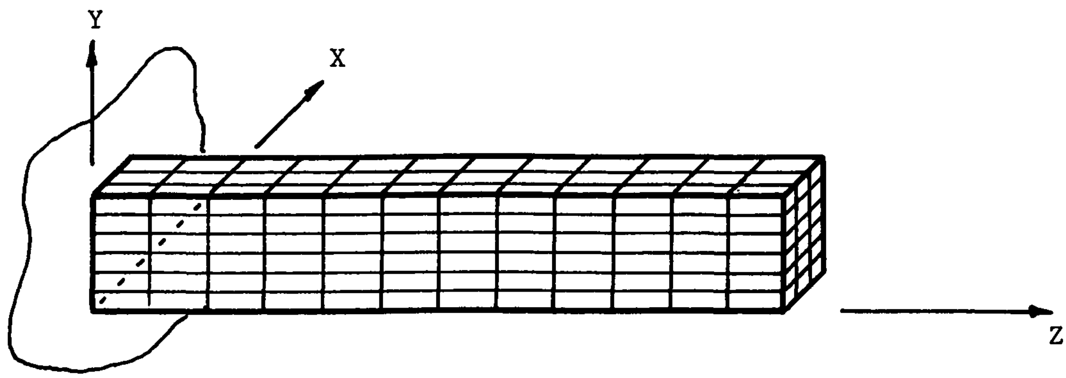

FIGURE 2. IHEX1 MODEL (216 ELEMENTS & 364 GRID POINTS)

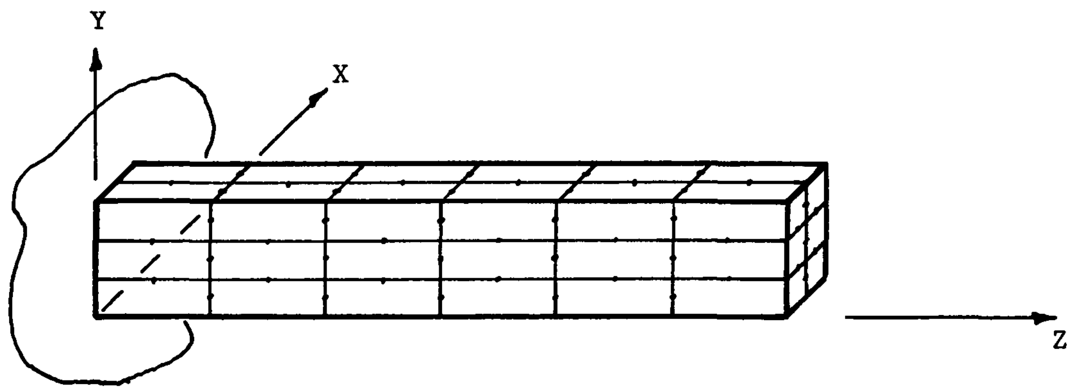

FIGURE 3. IHEX2 MODEL (36 ELEMENTS & 275 GRID POINTS)

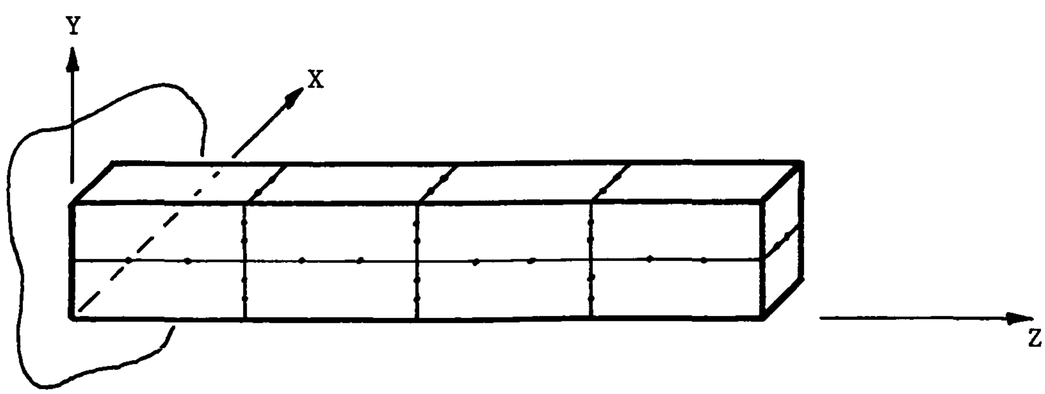

FIGURE 4. IHEX3 MODEL (8 ELEMENTS & 148 GRID POINTS)

D. Results

Table 2 summarizes the maximum displacement obtained by NASTRAN for each of the three finite element models, and the approximate theoretical solution obtained by presented formulas. The results for the IHEX2 and IHEX3 models show a maximum error of 4 per cent of the theoretical solutions, and the IHEX1 model is within 8 per cent of the theoretical solutions.

Table 2. Comparison of NASTRAN and Theoretical Solutions

Loading Case Maximum Displacement	NASTRAN			Theoretical* Solution
	Model No. 1 IHEX1 Elements	Model No. 2 IHEX2 Elements	Model No. 3 IHEX3 Elements	
Y-Disp. - Load Case a	1.444	1.548	1.533	1.481
Z-Disp. - Load Case b	.2113	.2104	.2088	.2056
Z-Disp. - Load Case c	-.2039	-.2042	-.2047	-.2056
Y-Disp. - Load Case d	.1422	.1561	.1569	.1586

*Theoretical Solutions

Load Case a

$$\delta_Y = \frac{\alpha T L^2}{2D}$$

$$\delta_Y = 1.481$$

Load Case b

$$\delta_Z = \alpha \Delta T L$$

$$\delta_Z = .2056$$

Load Case c

$$\delta_Z = \frac{P_Z L}{E}$$

$$\delta_Z = -.2056$$

Load Case d

$$\delta_Y = \frac{3P_Y L^4}{2ED^3}\left[1 + \frac{4D^2}{5L^2}\right]$$

$$\delta_Y = .1586$$

RIGID FORMAT No. 3, Real Eigenvalue Analysis

Vibration Analysis of a Long Beam Using Hexahedron Isoparametric Elements

A. Description

This problem illustrates the use of NASTRAN's three types of hexahedron isoparametric solid elements (IHEX1, IHEX2 and IHEX3) to obtain the vibration modes and frequencies of a cantilevered beam. The beam geometry and the three finite element models employed in the analysis are identical to the models used in the previous static analysis problem.

B. Input

1. Parameters:

   L = 144 - Length

   D = 24 - Depth

   W = 12 - Width

   E = 30 x 10^6 - Modulus of Elasticity

   μ = .3 - Poisson's Ratio

   α = 1.428 x 10^{-5} - Coefficient of Thermal Expansion

   ρ = 7.535 x 10^{-4} - Mass Density

2. Constraints:

   All grid points in the plane Z = 0 are constrained in the Z direction. In addition, the points on the line Z = 0, Y = 12 are constrained in the Y direction, the points on the line Z = 0 X = 0 are constrained in the X direction for the IHEX1 and IHEX3 models, and the points on the line Z = 0, X = 6 are constrained in that direction for the IHEX2 model.

3. Eigenvalue extraction data:

   Method: Inverse power

   Region of interest: 0 < f < 120

   Number of desired roots: 3

C. Results

Table 1 and Figures 1 and 2 summarize the NASTRAN results for the three lowest natural frequencies and associated mode shapes. Table 1 lists the theoretical frequencies and the NASTRAN frequencies for each type of hexahedron isoparametric element model. The results for IHEX2 and IHEX3 elements are within 3 per cent of the theoretical solution. The results for the IHEX1 elements are within 4 per cent of the theoretical solution for bending in the Y direction, but overestimate the first and second mode frequencies for bending in the X direction by 14 and 17 per cent, respectively. The latter is probably due to not having sufficient grid points through the beam in the X direction. This problem demonstrates that the IHEX2 and IHEX3 elements are better choices for beam bending problems than the IHEX1 element.

Plots of the first three beam bending modes are shown in Figures 1 and 2 for each type of finite element model, and are compared with the theoretical solution. All NASTRAN results show excellent correlation.

Table 1. Comparison of Natural Frequencies from NASTRAN with Theoretical

Mode No.	Description	NASTRAN						Theoretical Solution* (cps)
		Model No. 1 IHEX1 Elements		Model No. 2 IHEX2 Elements		Model No. 3 IHEX3 Elements		
		Freq.	% Error	Freq.	% Error	Freq.	% Error	
1	First Bending Mode in the X-Direction	22.0	+18	18.6	0	18.6	0	18.6
2	First Bending Mode in the Y-Direction	38.3	+4	36.5	-2	36.5	-2	37.3
3	Second Bending Mode in the X-Direction	135.3	+16	114.3	-2	113.3	-3	116.8

*W. C. Hurty and M. F. Rubenstein, Dynamics of Structures, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1964, (Chapter 5).

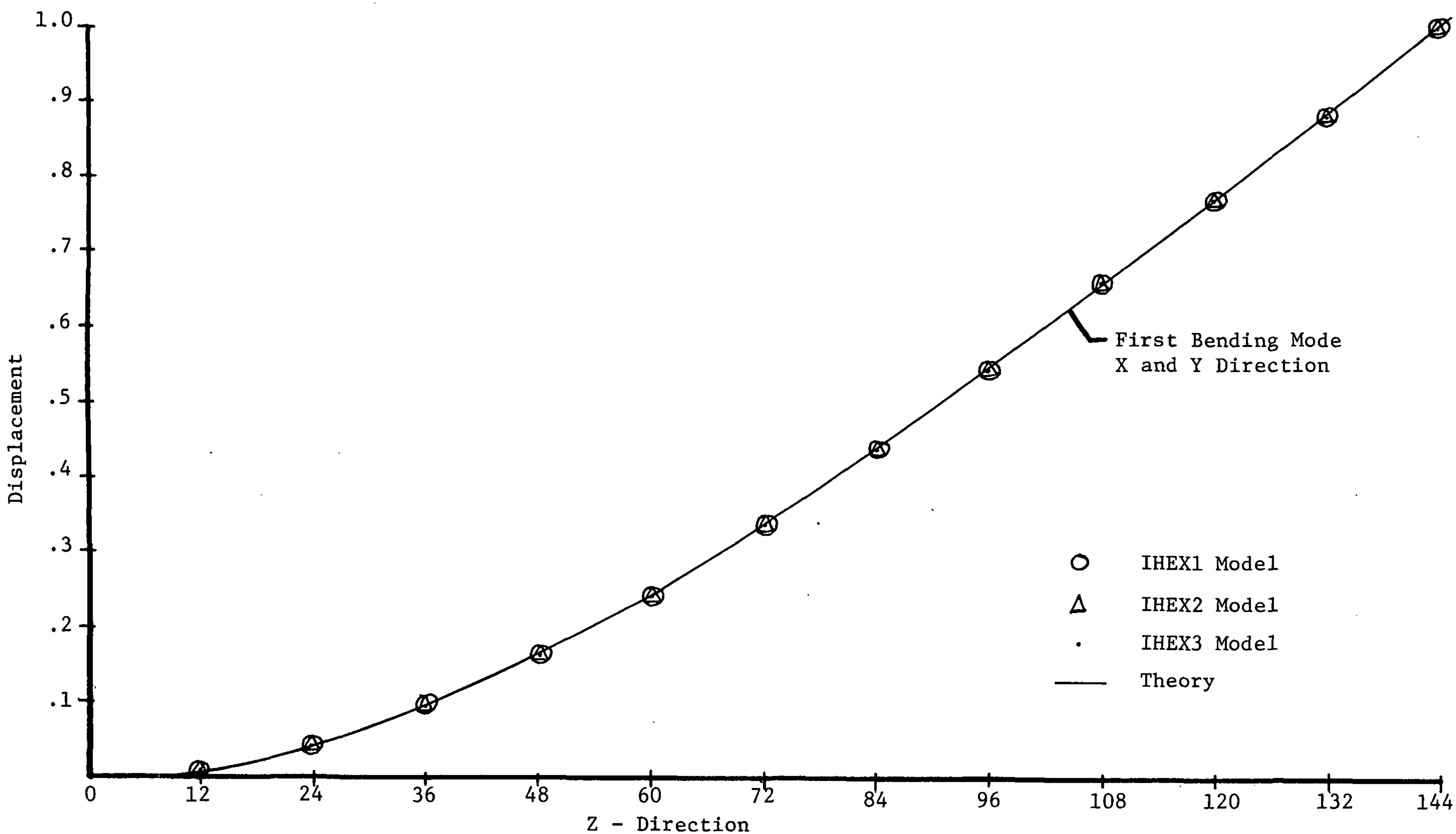

FIGURE 1. FIRST BEAM VIBRATION BENDING MODES, X AND Y DIRECTIONS

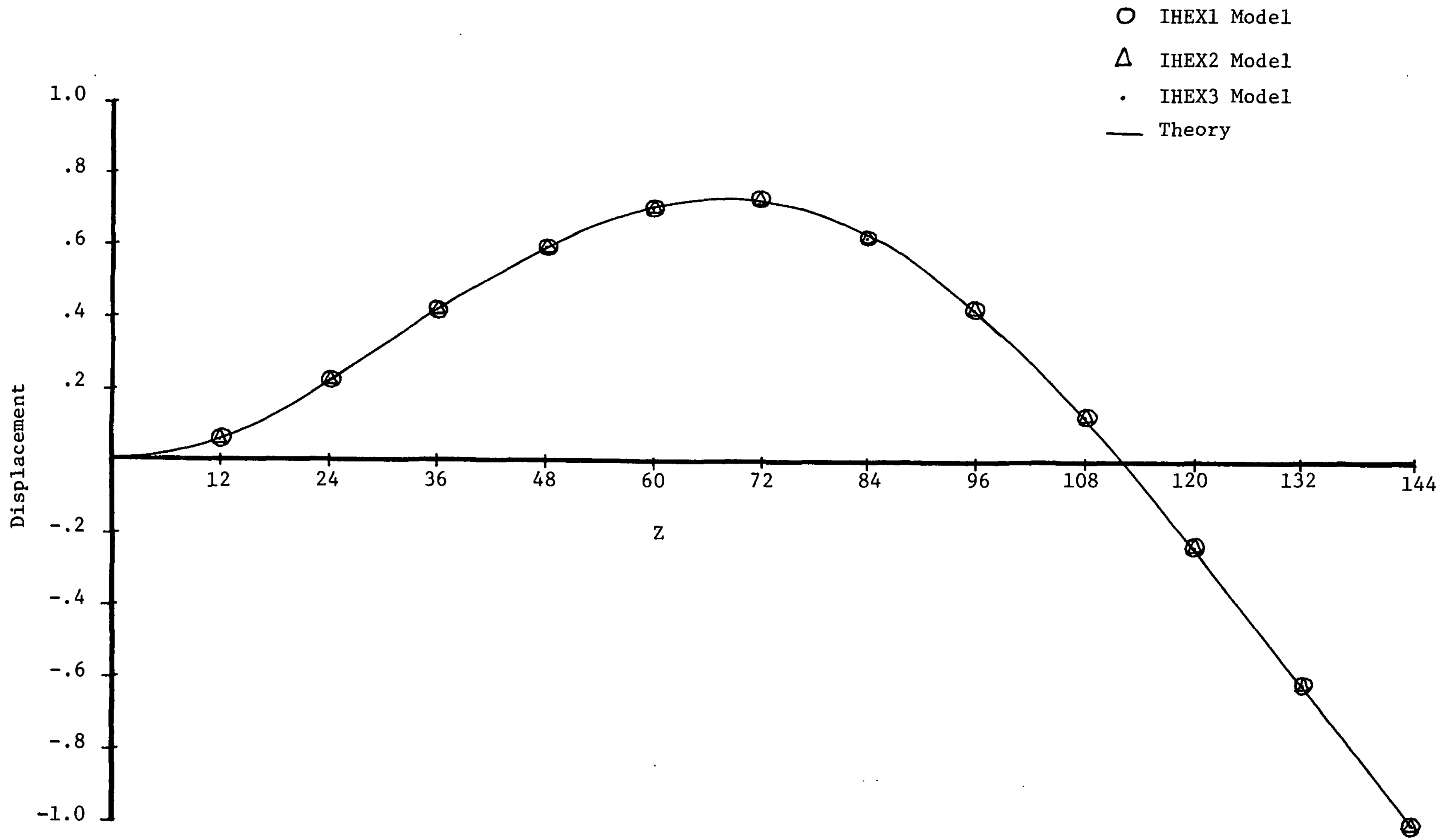

FIGURE 2. SECOND BEAM VIBRATION BENDING MODE, X DIRECTION

RIGID FORMAT No. 5, Buckling Analysis

Buckling Analysis of a Long Beam Using Hexahedron Isoparametric Elements

A. Description

This problem illustrates the use of NASTRAN's three types of hexahedron isoparametric solid elements (IHEX1, IHEX2 and IHEX3) to obtain the axial buckling load of a cantilevered beam. The beam geometry and the three finite element models employed in the analysis are identical to the models used in the previous static analysis problem.

B. Input

1. Parameters:

   L = 144 - Length

   D = 24 - Depth

   W = 12 - Width

   E - 30 x 10^6 - Modulus of Elasticity

   μ = .3 - Poisson's Ratio

   α = 1.428 x 10^{-5} - Coefficient of Thermal Expansion

   ρ = 7.535 x 10^{-4} - Mass Density

2. Loads:

   Compressive Axial Pressure Load (Z-Direction) @ Z = 144

   P_z = 42,837 psi

3. Constraints:

   All grid points in the plane Z = 0 are constrained in the Z direction. In addition the points on the line Z = 0, Y = 12 are constrained in the Y - direction, the points on the line Z = 0, X = 0 are constrained in the X - direction for the IHEX1 and IHEX3 models and the points on the line Z = 0, X = 6 are constrained in that direction for the IHEX2 model.

4. Eigenvalue extraction data:

   Method: Inverse Power

Region of Interest: $0 < \lambda < 10$

Number of desired roots: 3

C. Results

The critical buckling load* for a long cantilevered beam is given by

$$\text{Buckling Load} = \frac{m^2 \pi^2 EI}{4L^2} \tag{1}$$

Substituting the appropriate parameters into (1) yield:

For m = 1, Buckling Load (X - Dir.) = 1.234×10^7 lbs.

For m = 1, Buckling Load (Y - Dir.) = 4.955×10^7 lbs.

For m = 2, Buckling Load (X - Dir.) = 1.110×10^8 lbs.

or First Mode $P_{z\ critical}$ (X - Dir.) = 42837 psi

First Mode $P_{z\ critical}$ (Y - Dir.) = 4(42837) = 170,948 psi

Second Mode $P_{z\ critical}$ (X - Dir.) = 9(42837) = 385,533 psi

Since the compressive pressure load applied to the beam is equal to the minimum buckling load, the eigenvalue which NASTRAN should obtain is $\lambda = 1.0$ for buckling in the X-direction, $\lambda = 4.0$ for buckling in the Y-direction, and $\lambda = 9.0$ for buckling in the second mode, X-direction.

Table 1 and Figure 1 and 2 summarize the NASTRAN results for buckling in the X and Y directions. Table 1 lists the NASTRAN calculated buckling parameter, λ for each type of isoparametric element model. The IHEX2 and IHEX3 element results are excellent and are within .7 percent of the theoretical solution. However, as in the vibration problem, the IHEX 1 element model overestimates the buckling load in the Y direction by 10 percent, and the buckling load in the X-direction by 41 and 42 percent respectively. Again, the latter is most likely due to the use of insufficient grid points through the X-direction of the beam.

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* S.P. Timoshenko and J. M. Gere, Theory of Elastic Stability, McGraw-Hill Book Company, Inc., New York, 1961 (Chapter 2).

The fundamental buckling modes in the X and Y directions are compared with the theoretical in Figure 1. The second buckling mode in the X direction is shown in Figure 2. The correlation with theory is excellent.

Table 1. Comparison of Buckling Loads from NASTRAN with Theoretical

Mode No.	Description	NASTRAN						Theoretical Solution λ
		Model No. 1 IHEX1 Elements		Model No. 2 IHEX2 Elements		Model No. 3 IHEX3 Elements		
		λ	% Error	λ	% Error	λ	% Error	
1	X-Direction	1.406	40.6	1.002	.2	1.001	.1	1.0
2	Y-Direction	4.391	9.8	3.981	.5	3.979	.5	4.0
3	X-Direction	12.809	42.3	9.037	.4	8.934	.7	9.0

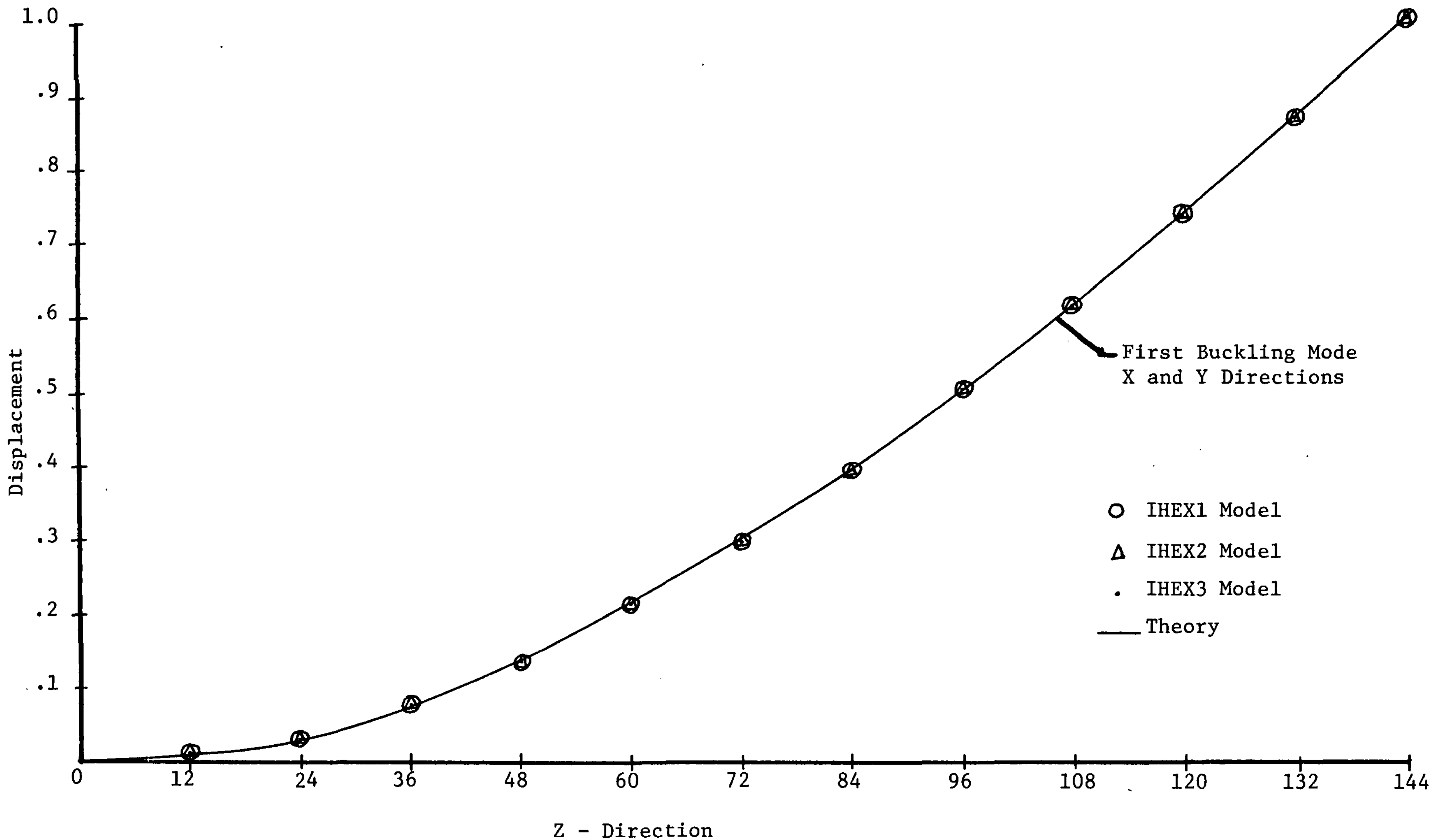

FIGURE 1. FIRST BEAM BUCKLING BENDING MODE, X AND Y DIRECTIONS

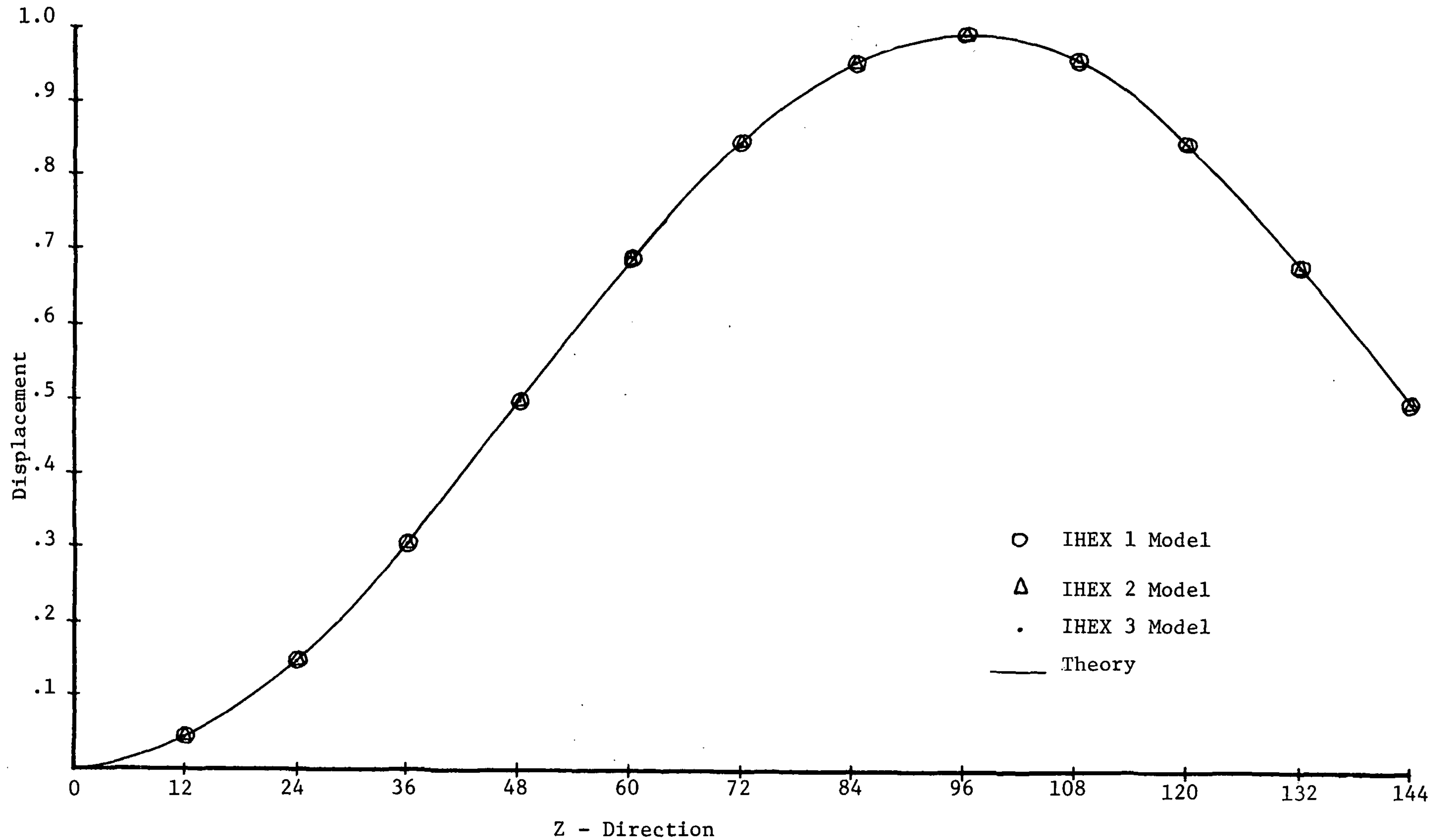

FIGURE 2. SECOND BEAM BUCKLING BENDING MODE, X DIRECTION